Exact Three-Point Functions in $\mathcal{N} = 2$ Superconformal Field Theories

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adapted from arXiv:2503.07925 with S. Komatsu, G. Lefundes, and D. Serban

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Motivation

A non-perturbative solution of planar $\mathcal{N} = 4$ SYM should be reachable thanks to integrability.

We already have access to the spectrum of conformal dimensions (QSC).

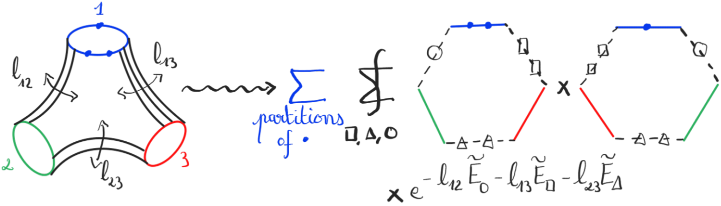
For three- and higher-point correlation functions, there is still work to be done. Various approaches: hexagons, T-functions, quantum spectral curve (QSC)/separation of variables (SoV).

In this talk, we will apply the hexagons to the $\mathbb{Z}_K$ orbifolds of $\mathcal{N} = 4$ SYM and make contact with localisation.

Outline

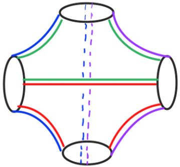
- Hexagonalisation of correlation functions
- $\mathbb{Z}_K$ orbifolds of $\mathcal{N} = 4$ SYM and localisation results
- Twisted Hexagons
- Regularisation using a genus-two surface
- Outlook

Hexagonalisation



Hexagon form factors = building blocks for n-point correlators.
Gluing along a seam = sum over a complete basis of mirror magnons.
[Basso, Komatsu, and Vieira (2015)] [Fleury and Komatsu (2016,2017)]
[Eden and Sfondrini (2016)]

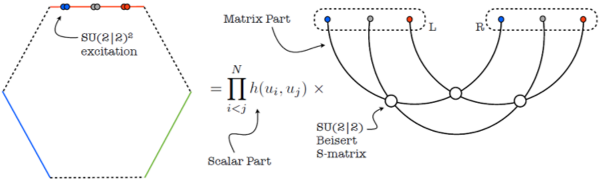
The hexagon expansion is the analogue of the Lüscher expansion for the spectrum.



$\longrightarrow$ 4 hexagons, 6 cuts

Excitations over the vacuua $\mathrm{Tr}(Z^k)$ transform under the bi-fundamental representation of $\mathrm{PSU}(2|2)_L \times \mathrm{PSU}(2|2)_R$; the hexagon is built using the SU(2|2) R-matrix. [Beisert (2005,2006)] [Arutyunov, de Leeuw, and Torrielli (2009)]

Start with all excitations on the same edge:

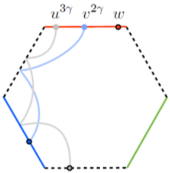


[Basso, Komatsu, and Vieira (2015)]

$$\mathcal{H}(u_1, u_2, u_3, \dots) = \left(\prod_{i<j} h(u_i, u_j) \right) (\cdots \mathcal{S}_{23} \mathcal{S}_{13} \mathcal{S}_{12}) ,$$

Scalar factor solves some crossing relation.

Move excitations from one edge to another using mirror transformation



[Basso, Komatsu, and Vieira (2015)]

Energy of mirror magnons with bound-state index a: $\widetilde{E}_a = \ln\left(x^{[+a]}x^{[-a]}\right)$ in terms of the Zhukovsky variables

$$x(u) + \frac{1}{x(u)} = \frac{u}{g}, \qquad x^{[a]}(u) = x\left(u + \mathrm{i}\,\frac{a}{2}\right) .$$

In the mirror:

$$x^{[+a]}(u^\gamma) = 1/x^{[+a]}(u), \quad x^{[-a]}(u^\gamma) = x^{[-a]}(u).$$

Physical momentum becomes mirror energy $p_a(u^\gamma) = \mathrm{i}\,\widetilde{E}_a$.

The Octagon

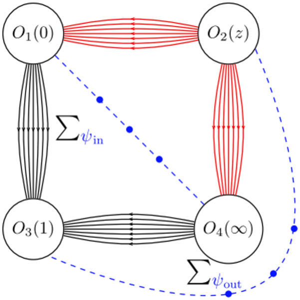
Take $\mathcal{O}_1 = \mathrm{Tr}\big(Z^K (X^\dagger)^K\big) + \ldots$, $\mathcal{O}_2 = \mathrm{Tr}\big((Z^\dagger)^{2K}\big)$, $\mathcal{O}_3 = \mathrm{Tr}\big(X^{2K}\big)$.
Then, when $K \to +\infty$, one has

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_1(x_4)\rangle \sim \frac{\mathbb{O}_0^2(z,\bar{z})}{(x_{12}^2 x_{24}^2 x_{13}^2 x_{34}^2)^K} \,.$$

Conformal cross-ratios:

$$z\bar{z} = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} \,, \qquad \frac{z}{\bar{z}} = e^{2\,\mathrm{i}\,\phi} \,,$$

$$(1-z)(1-\bar{z}) = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2} \,.$$



[Coronado (2018)]

Generalisation with arbitrary bridge length ℓ and R-symmetry polarisation vectors: [Coronado (2018)]

$$\mathbb{O}_\ell(z,\bar{z},\kappa,\bar{\kappa}) = 1 + \sum_{n=1}^{+\infty} \frac{\lambda_+^n + \lambda_-^n}{2\; n!} \sum_{a_1,\dots,a_n=1}^{+\infty} \prod_{k=1}^{n} 4\frac{\sin(a_k\phi)}{\sin(\phi)}$$

$$\times \int \prod_{i<j} H_{a_i,a_j}(u_i,u_j) \prod_{k=1}^{n} (z\bar{z})^{-\mathrm{i}\, p_{a_k}(u_k)}\, \mathrm{e}^{-\ell \widetilde{E}_{a_k}(u_k)}\, \mu_{a_k}(u_k) \frac{\mathrm{d}u_k}{2\pi}\,,$$

where

$$\tilde{p}_a = g\left(x^{[+a]} + x^{[-a]} - \frac{1}{x^{[+a]}} - \frac{1}{x^{[-a]}}\right)\,, \quad H_{a,b}(u,v) = \prod_{\delta,\epsilon=\pm} \frac{x^{[\delta a]}(u) - x^{[\epsilon b]}(v)}{1 - x^{[\delta a]}(u) x^{[\epsilon b]}(v)}\,,$$

$$\mu_a = \frac{\mathrm{i}\left(x^{[+a]} - x^{[-a]}\right) x^{[+a]} x^{[-a]}}{g\left((x^{[+a]})^2 - 1\right)\left((x^{[-a]})^2 - 1\right)\left(1 - x^{[+a]} x^{[-a]}\right)}\,.$$

$$\mathbb{O}_\ell(z,\bar{z},\kappa,\bar{\kappa}) = \frac{\det(1-\lambda_+ K_\ell) + \det(1-\lambda_- K_\ell)}{2}\,,$$

where K_ℓ is a semi-infinite matrix: for $m, n \geqslant 0$,

$$(K_{\ell-1}(g))_{mn} = 4\sqrt{(\ell+2m)(\ell+2n)}\int_0^{+\infty} \frac{J_{\ell+2m}(2gt)J_{\ell+2n}(2gt)}{\cos\phi - \operatorname{ch}\sqrt{t^2+z\bar{z}}}\frac{\mathrm{d}t}{t}\,.$$

[Kostov, Petkova, and Serban (2019)] [Belitsky and Kortchemsky (2019)]

This object occurs in several other situations.

[Beisert, Eden, and Staudacher (2006)] [Basso, Dixon, and Papathanasiou (2020)]
[Sever, Tumanov, and Wilhelm (2020,2021)] [Basso and Tumanov (2024)]

$\mathbb{Z}_K$ orbifolds of $\mathcal{N} = 4$ SYM

- $\mathcal{N} = 2$ SCFT with gauge group $SU(N)_0 \times \cdots \times SU(N)_{K-1}$, K vector multiplets and K bifundamental hypermultiplets.
- In $\mathcal{N} = 4$ language, fields are $KN \times KN$ matrices that satisfy

$$[A_\mu, \gamma] = [Z, \gamma] = \{X, \gamma\} = \{Y, \gamma\} = 0\,,$$

where the $\mathbb{Z}_K$ twist is $\gamma = \text{Diag}(I_N, \rho I_N, \ldots, \rho^{K-1} I_N)$ for $\rho = e^{2\,i\,\pi/K}$. Thus, $Z = \text{Diag}(Z_0, Z_1, \ldots, Z_{K-1})$ for instance.

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- At the orbifold point $g_0 = \cdots = g_{K-1}$, theory expected to be integrable in the planar limit.
- The twist breaks $PSU(2,2|4)$ down to $PSU(2,2|2) \times SU(2)$ (for $K = 2$) or $PSU(2,2|2) \times U(1)$ (for $K > 2$).

Twisted BPS Operators

We focus on correlation functions of the following BPS operators:

$$\mathcal{O}_\ell^{(\alpha)} = \frac{1}{\sqrt{K}} \operatorname{Tr}\left(\gamma^\alpha Z^\ell\right) ,$$

where $\alpha \in \{0, \ldots, K-1\}$. The operator $\mathcal{O}_\ell^{(0)}$ is called untwisted, the others are twisted operators.

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$$\langle \mathcal{O}_\ell^{(\alpha),\dagger}(x) \mathcal{O}_\ell^{(\alpha)}(0) \rangle = \frac{\ell \, G_\ell^{(\alpha)}(g)}{x^{2\ell}} \, ,$$

where the normalisation is expressed in terms of the octagon for $z = \bar{z} = 1$ and $\kappa = \rho^\alpha, \bar{\kappa} = \rho^{-\alpha}$:

$$G_\ell^{(\alpha)}(g) = \frac{\det(1 - s_\alpha K_{\ell+1})}{\det(1 - s_\alpha K_{\ell-1})} \, ,$$

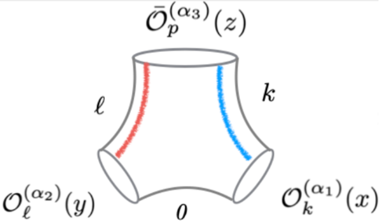
where $s_\alpha = \sin^2(\pi\alpha/K)$.

[Galvagno and Preti (2020)]

[Billò, Frau, Galvagno, Lerda, and Pini (2021)]

Three-Point Functions

$$\frac{\langle \mathcal{O}_k^{(\alpha_1)}(x)\mathcal{O}_\ell^{(\alpha_2)}(y)\mathcal{O}_{k+\ell}^{(\alpha_3),\dagger}(z)\rangle}{\sqrt{G_k^{(\alpha_1)}(g)G_\ell^{(\alpha_2)}(g)G_{k+\ell}^{(\alpha_3)}(g)}} = \frac{k\ell(k+\ell)}{\sqrt{K}N}\,\frac{C_k^{(\alpha_1)}C_\ell^{(\alpha_2)}C_{k+\ell}^{(\alpha_3)}}{|x-z|^{2k}|y-z|^{2\ell}}$$



$$C_\ell^{(\alpha)}(g) = \sqrt{1+\frac{g}{2\ell}\partial_g \ln\frac{\det(1-s_\alpha K_{\ell+1})}{\det(1-s_\alpha K_{\ell-1})}}$$

[Billò, Frau, Lerda, Pini, and Vallarino (2022)]

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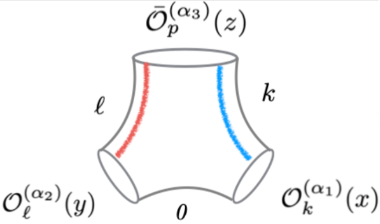

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[Billò, Frau, Lerda, Pini, and Vallarino (2022)]

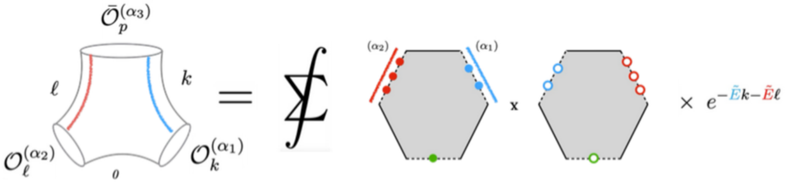
$$C_\ell^{(\alpha)}(g) = \frac{\det(1-s_\alpha K_\ell)}{\sqrt{\det(1-s_\alpha K_{\ell-1})\det(1-s_\alpha K_{\ell+1})}}$$

[Ferrando, Komatsu, Lefundes, and Serban (2025)]

[Korchemsky and Testa (2025)] [Korchemsky (2025)]

How can we recover this result in the hexagon framework?

Twisted Hexagons



Twisting is implemented by inserting powers of $1_L \times \mathrm{Diag}(\rho, \rho^{-1}, 1, 1)_R$.

This easily explains part of the result:

$$\frac{\det(1 - s_{\alpha_1} K_k)\,\det(1 - s_{\alpha_2} K_\ell)\,\det(1 - s_{\alpha_3} K_{k+\ell})}{\prod_{i=1}^{3} \sqrt{\det(1 - s_{\alpha_i} K_{\ell_i - 1})\,\det(1 - s_{\alpha_i} K_{\ell_i + 1})}}$$

Indeed, one has $\det(1 - s_\alpha K_\ell) = \mathbb{O}_\ell(z = \bar{z} = 1, \kappa = \rho^\alpha, \bar{\kappa} = \rho^{-\alpha})$.

Regularisation

The naïve hexagon formula contains some divergences when the quantum numbers of magnons living on different bridges coincide. These divergences stem from the infinite volume of the mirror theory.

Difficult to regularise in general. [Basso, Gonçalves, and Komatsu (2017)]
[Basso, Georgoudis, and Klemenchuk Sueiro (2022)]

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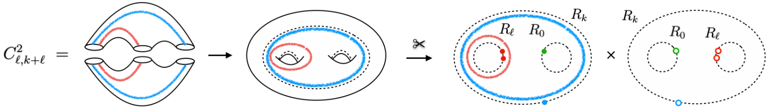
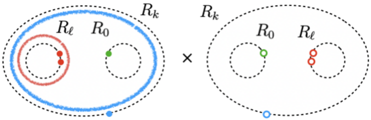
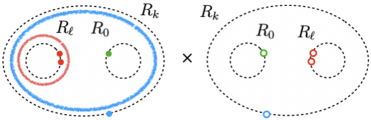
In our case, the operators are protected and we can do something similar to the usual torus regularisation for the ground state energy.



$$\left(C_k^{(\alpha_1)} C_\ell^{(\alpha_2)} C_{k+\ell}^{(\alpha_3)}\right)^2 = \sum_{n_u, n_v, n_w \geqslant 0} \mathcal{C}_{(n_u, n_v, n_w)}$$



The two new objects are themselves 3-pt functions in the mirror theory. And we only need them in the infinite volume limit = asymptotic 3-pt functions.



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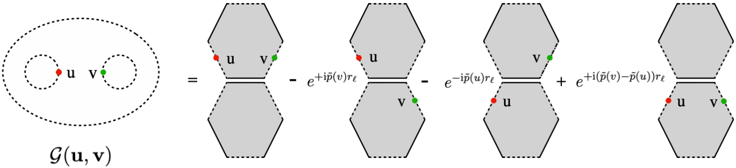
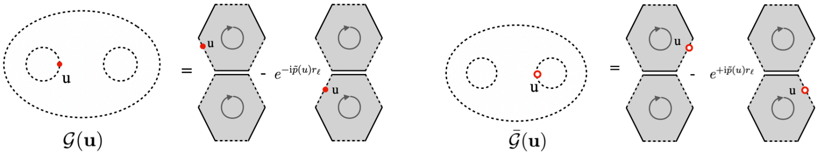
We can simply compute them using the asymptotic hexagon formula: sum over partitions of the (mirror) excitations between 2 hexagons.



Bridge Contributions

$$\mathcal{C}_{(1,0,0)} = \lim_{r_\ell \to \infty} \sum_{a=1}^{\infty} \int \frac{du}{2\pi} \mu_a(u) e^{-\ell \widetilde{E}_a(u)} \, \mathrm{STr}_a \left[\tau_a^{\alpha_2} \, \mathcal{G}(u) \, \overline{\mathcal{G}}(u) \right]$$



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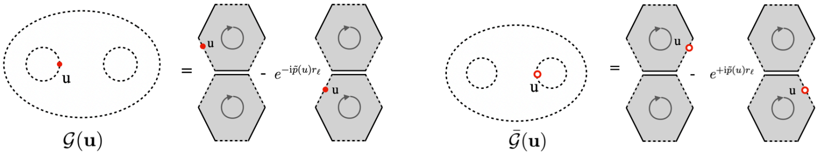


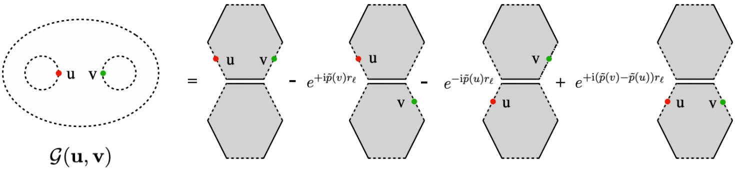
$$\mathcal{C}_{(1,0,0)} = \lim_{r_\ell \to \infty} \sum_{a=1}^{\infty} \int \frac{\mathrm{d}u}{2\pi} \mu_a(u)\, e^{-\ell \widetilde{E}_a(u)}\, T_a^{(\alpha_2)} \left(1 - e^{-\,i\,\tilde{p}_a(u) r_\ell} - e^{i\,\tilde{p}_a(u) r_\ell} + 1 \right)$$

$$= 2 \sum_{a=1}^{\infty} \int \frac{\mathrm{d}u}{2\pi}\, e^{-\ell \widetilde{E}_a(u)}\, T_a^{(\alpha_2)}\,, \qquad T_a^{(\alpha)} = \mathrm{STr}\; \tau_a^{\alpha} = 4 a s_\alpha\,.$$

First contribution to $(\det(1 - s_{\alpha_2} K_\ell))^2$. Easily generalised to

$$\sum_{n=0}^{\infty} \mathcal{C}_{(n,0,0)} = (\det(1 - s_{\alpha_2} K_\ell))^2\,.$$

Wrapping Contributions



$$\mathcal{C}_{(1,1,0)} = \lim_{r_\ell \to \infty} \sum_{a,b=1}^{\infty} \int \frac{\mathrm{d}u\,\mathrm{d}v}{(2\pi)^2} \mu_a(u)\,\mu_b(v)\, e^{-\ell \widetilde{E}_a(u)}\, \mathrm{STr}_{ab}[\tau_a^{\alpha_2}\, \mathcal{G}(u,v)\, \bar{\mathcal{G}}(u,v)]$$

Compute (one of) the integrals by closing the contour in the lower/upper half-plane. We choose the order $\mathrm{Im}(u) > \mathrm{Im}(v) > \mathrm{Im}(w)$.
Only decoupling poles $(a,u) = (b,v)$ survive:

$$\mathcal{C}_{(1,1,0)} = \sum_{a=1}^{\infty} \int \frac{\mathrm{d}u}{2\pi} e^{-\ell \widetilde{E}_a(u)} \left(-\mathrm{i}\,\partial_v\, \mathrm{STr}_{ab}\, \tau_a^{\alpha_2}\, (\mathcal{S}_{ab}(u,v))^2 \right) \Big|_{v \to u}$$

We conjecture that

$$\sum_{n=0}^{\infty} \mathcal{C}_{(n,n,0)} = \frac{1}{\det(1 - s_{\alpha_2} K_{\ell-1}) \det(1 - s_{\alpha_2} K_{\ell+1})} ,$$

$$\mathcal{C}_{(n,n,0)} = \frac{\mathrm{i}^{-n}}{n!} \prod_{k=1}^{n} \left(\sum_{a_k=1}^{\infty} \int \frac{\mathrm{d}u_k}{2\pi} e^{-\ell \widetilde{E}_{a_k}(u_k)} \right) \partial_{\vec{v}} \, \mathrm{STr}\big[\tau_{\vec{a}}^{\alpha_2} \, \mathcal{S}_{\vec{a}\vec{b}}(\vec{u}, \vec{v}) \, \mathcal{S}_{\vec{a}\vec{b}}(\vec{u}, \vec{v}) \big] \Big|_{\vec{v} \to \vec{u}} ,$$

where

$$\tau_{\vec{a}}^{\alpha} = \prod_{k=1}^{n} \tau_{a_k}^{\alpha} , \quad \mathcal{S}_{\vec{a}\vec{b}}(\vec{u}, \vec{v}) = \prod_{i=1}^{n} \prod_{j=1}^{n} \mathcal{S}_{a_i, b_j}(u_i, v_j) .$$

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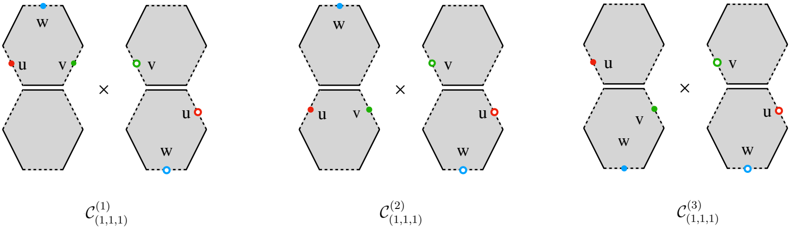
For comparison, recall that $\sum_{n=0}^{\infty} \mathcal{C}_{(n,0,0)} = (\det(1 - s_{\alpha_2} K_\ell))^2$,

$$\mathcal{C}_{(n,0,0)} = \frac{\mathrm{i}^{-n}}{n!} \prod_{k=1}^{n} \left(\sum_{a_k=1}^{\infty} \int \frac{\mathrm{d}u_k}{2\pi} e^{-\ell \widetilde{E}_{a_k}(u_k)} \mu_{a_k}(u_k) 4 a_k s_{\alpha_2} \right) \prod_{i<j} H_{a_i, a_j}(u_i, u_j) ,$$

Bridge-Like Contribution

There are 3-magnon contact terms, simplest example $= \mathcal{C}_{(1,1,1)}$.



$$\mathcal{C}^{(2)}_{(1,1,1)} = \lim_{r_\ell, r_p \to \infty} \sum_{a,b,c=1}^{\infty} \int \frac{\mathrm{d}u\,\mathrm{d}v\,\mathrm{d}w}{(2\pi)^3}\, \mu_a(u)\, \mu_b(v)\, \mu_c(w)\; \mathrm{e}^{-\ell \widetilde{E}_a(u) - k\widetilde{E}_c(w)}$$

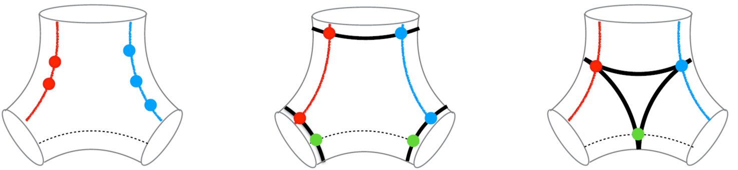
$$\times\, \mathrm{e}^{\mathrm{i}(\tilde{p}_c(w)-\tilde{p}_a(u))r_p + \mathrm{i}(\tilde{p}_b(v)-\tilde{p}_a(u))r_\ell}\, \frac{\mathrm{STr}_{abc}\ \tau_a^{\alpha_2}\, \mathcal{S}_{ab}(u,v)\, \tau_c^{\alpha_1}\, \mathcal{S}_{ac}(u,w)}{h_{ab}(v,u)\, h_{ca}(w,u)}\,,$$

Decoupling poles at $(c,w) = (a,u)$ and $(b,v) = (a,u)$. Eventually,

$$\mathcal{C}_{(1,1,1)} = 2\left(\mathcal{C}^{(1)}_{(1,1,1)} + \mathcal{C}^{(2)}_{(1,1,1)} + \mathcal{C}^{(3)}_{(1,1,1)}\right) = 2\sum_{a=1}^{\infty} \int \frac{\mathrm{d}u}{2\pi}\, e^{-(k+\ell)\widetilde{E}_a(u)} \mu_a(u) 4 a s_{\alpha_3}\,.$$

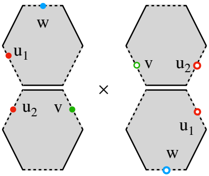
Complete Result

$$\frac{\det(1 - s_{\alpha_1} K_k)\ \det(1 - s_{\alpha_2} K_\ell)\ \det(1 - s_{\alpha_3} K_{k+\ell})}{\prod_{i=1}^{3} \sqrt{\det(1 - s_{\alpha_i} K_{\ell_i - 1})\,\det(1 - s_{\alpha_i} K_{\ell_i + 1})}}$$

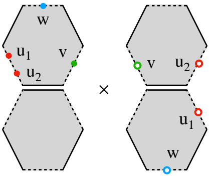


We explained the appearance of each building block separately, but we still need to make sure that the full result is indeed factorised.

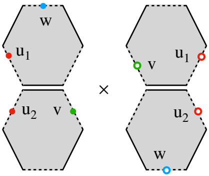
Factorisation: an Example



$\frac{1}{2}\,\mathcal{C}_{(1,1,0)}\,\mathcal{C}_{(1,0,1)}$



$\frac{1}{4}\,\mathcal{C}_{(1,0,0)}\,\mathcal{C}^{(1)}_{(1,1,1)}$



$\frac{1}{4}\,\mathcal{C}_{(1,0,0)}\,\mathcal{C}^{(2)}_{(1,1,1)}$

$$\mathcal{C}_{(2,1,1)} = \mathcal{C}_{(1,1,0)}\,\mathcal{C}_{(1,0,1)} + \mathcal{C}_{(1,0,0)}\,\mathcal{C}_{(1,1,1)}$$

Conclusion and Outlook

We proposed an extension of the hexagon formalism to the $\mathbb{Z}_K$ orbifold $\mathcal{N} = 2$ SCFT, and a regularisation applicable to 3-pt functions of operators corresponding to (twisted) vacua.

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Further possible directions:

- Some of our results are still conjectural, how can we prove them?
- 3-pt functions of non-BPS operators or higher-point correlation functions [le Plat and Skrzypek (2025)]
- Can we interpret the result in the language of T-functions? of Q-functions? [Basso, Georgoudis, and Klemenchuk Sueiro (2022)] [Bercini, Homrich, and Vieira (2022)][Bargheer, Bercini, Cavaglià, Lai, and Ryan (2025)]
- Extension to other twisted theories?

Thank you!