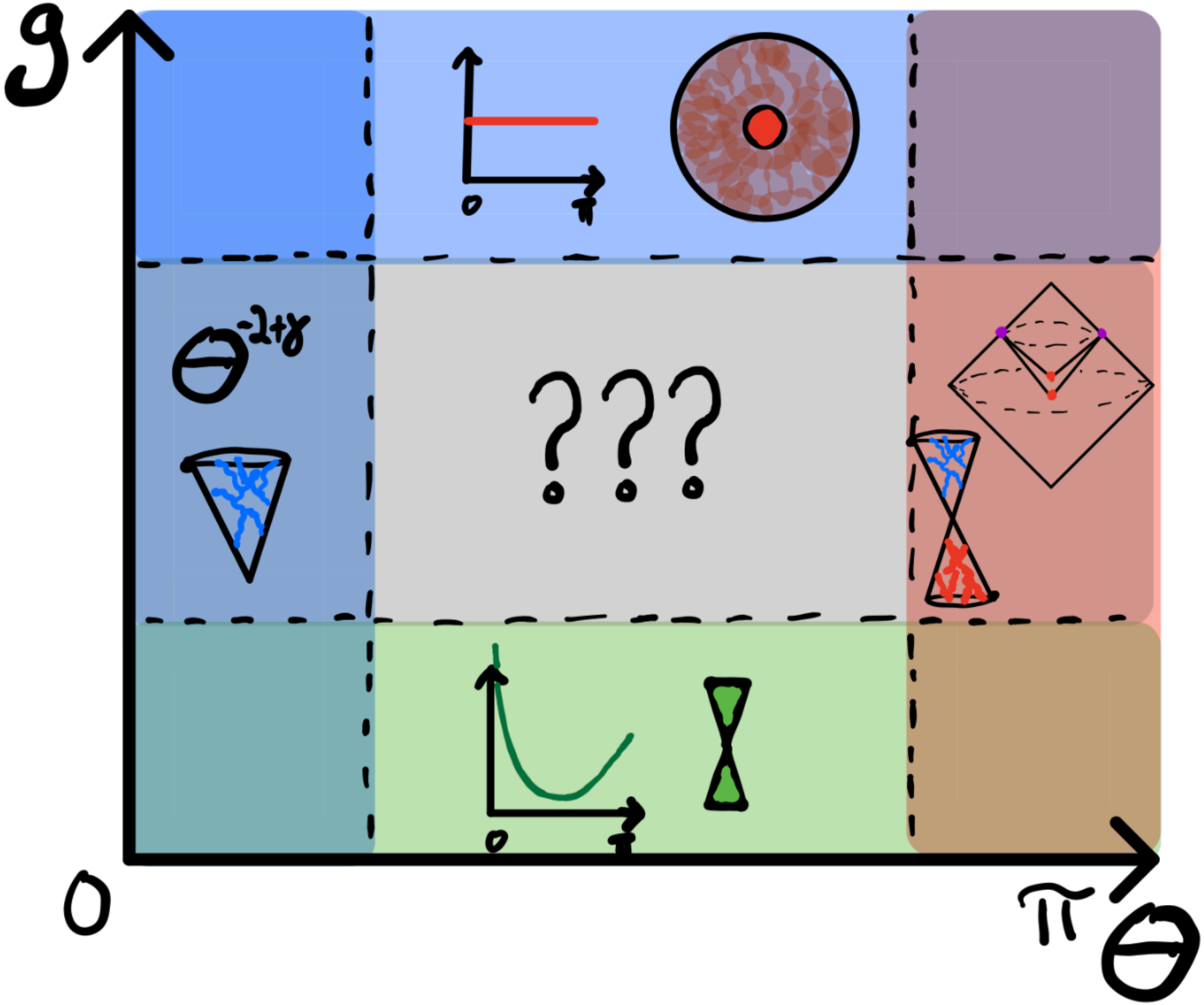
Conformal collider bootstrap in $\mathcal{N} = 4$ SYM

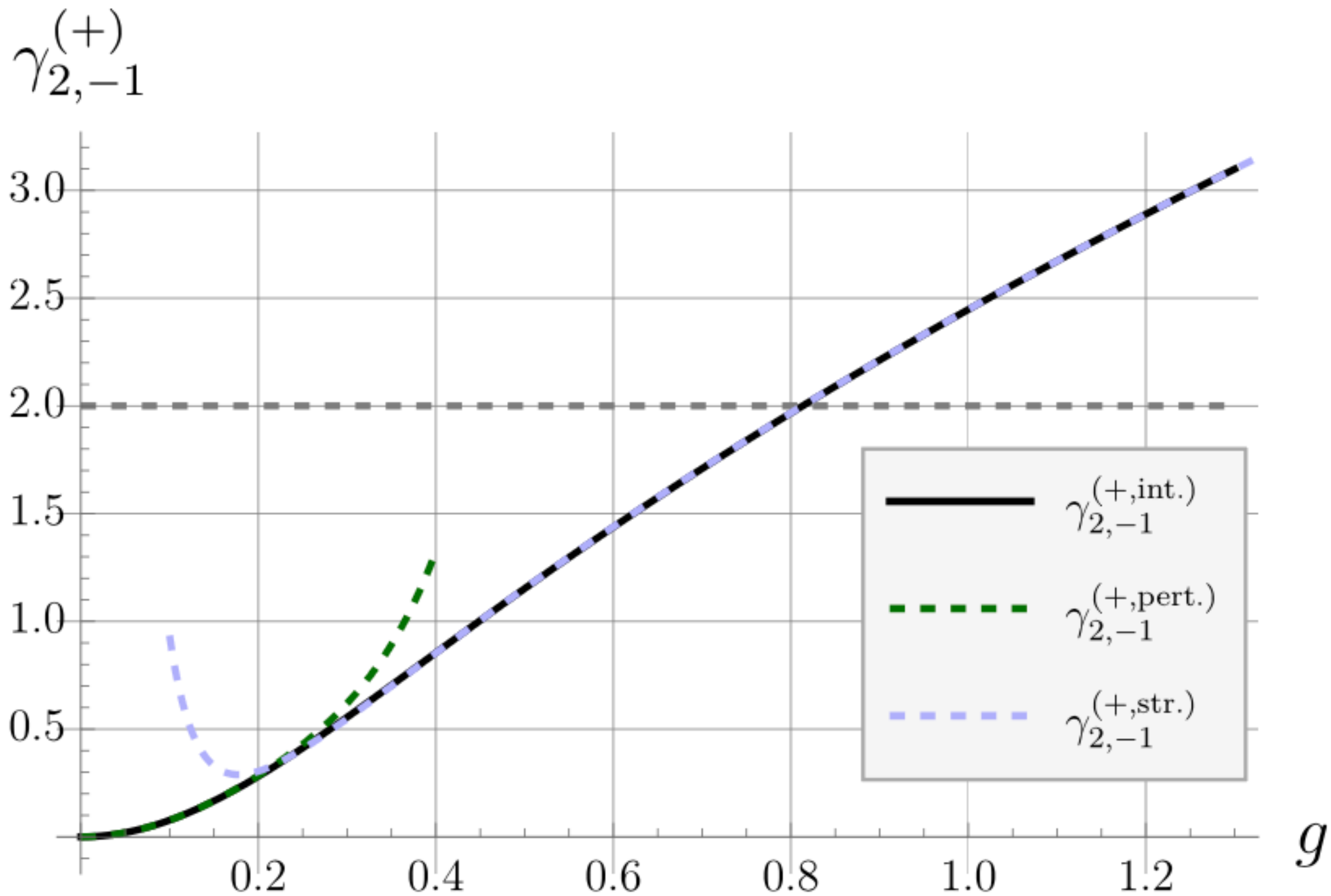
Ross Dempsey,[a] **Robin Karlsson,**[b] **Silviu S. Pufu,**[c,d] **Zahra Zahraee,**[e] **Alexander Zhiboedov**[f]

Saclay, 2025



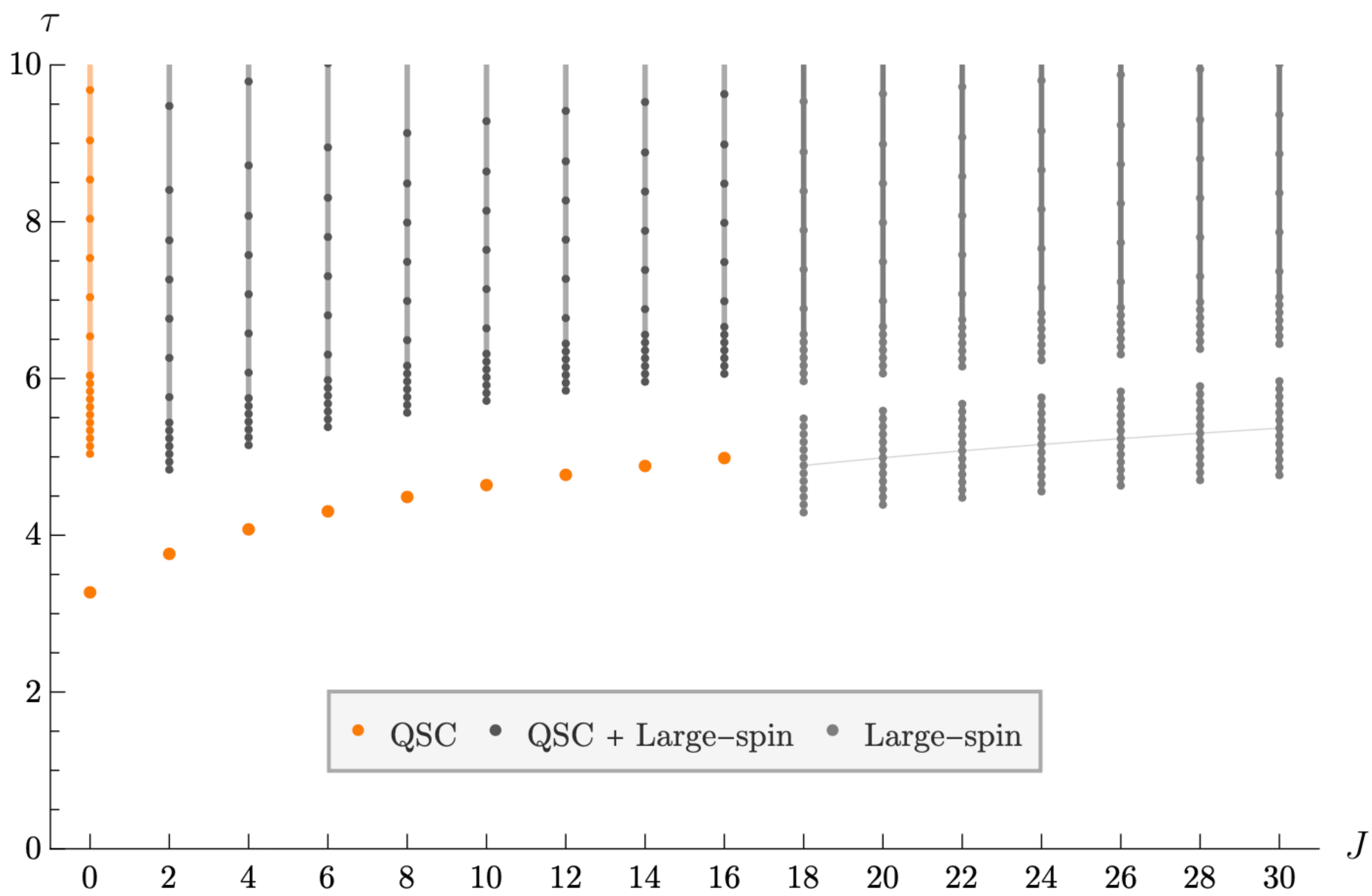
$$\lambda \equiv g_{\mathrm{YM}}^2 N_c = (4\pi g)^2$$

Light-ray OPE (integrability)



$$\text{EEC}(z) \underset{z\to 0}{\sim} \frac{1}{z^{1-\gamma^{(+)}_{2,-1}/2}}$$

Source-detector (integrability)



Bootstrap problems

finite Nc

$$\begin{array}{lll} \text{min/max} & \sum \beta_{\tau,J}\lambda^2_{\tau,J} & \\ \text{subject to} & \sum \lambda^2_{\tau,J} F_{\tau,J;m,n} + F^{\text{protected}}_{m,n}(N_c) = 0, & m+n \text{ odd}, m<n \\ & \sum \lambda^2_{\tau,J} I_{k,\tau,J} + (I^{\text{protected}}_{k,N_c} - \mathcal{F}_k(N_c, g_{\text{YM}})) = 0, & k = 2,4 \\ & \sum \lambda^2_{\tau,J} \sin^2 \frac{\pi\tau}{2} \alpha(\tau,J) f_{J+2,s}(\tau) - \max c_s \le 0, & s = 2,3,\ldots \\ & \lambda^2_{\tau,J} \ge 0 & \forall \tau, J \end{array}$$

+ANEC

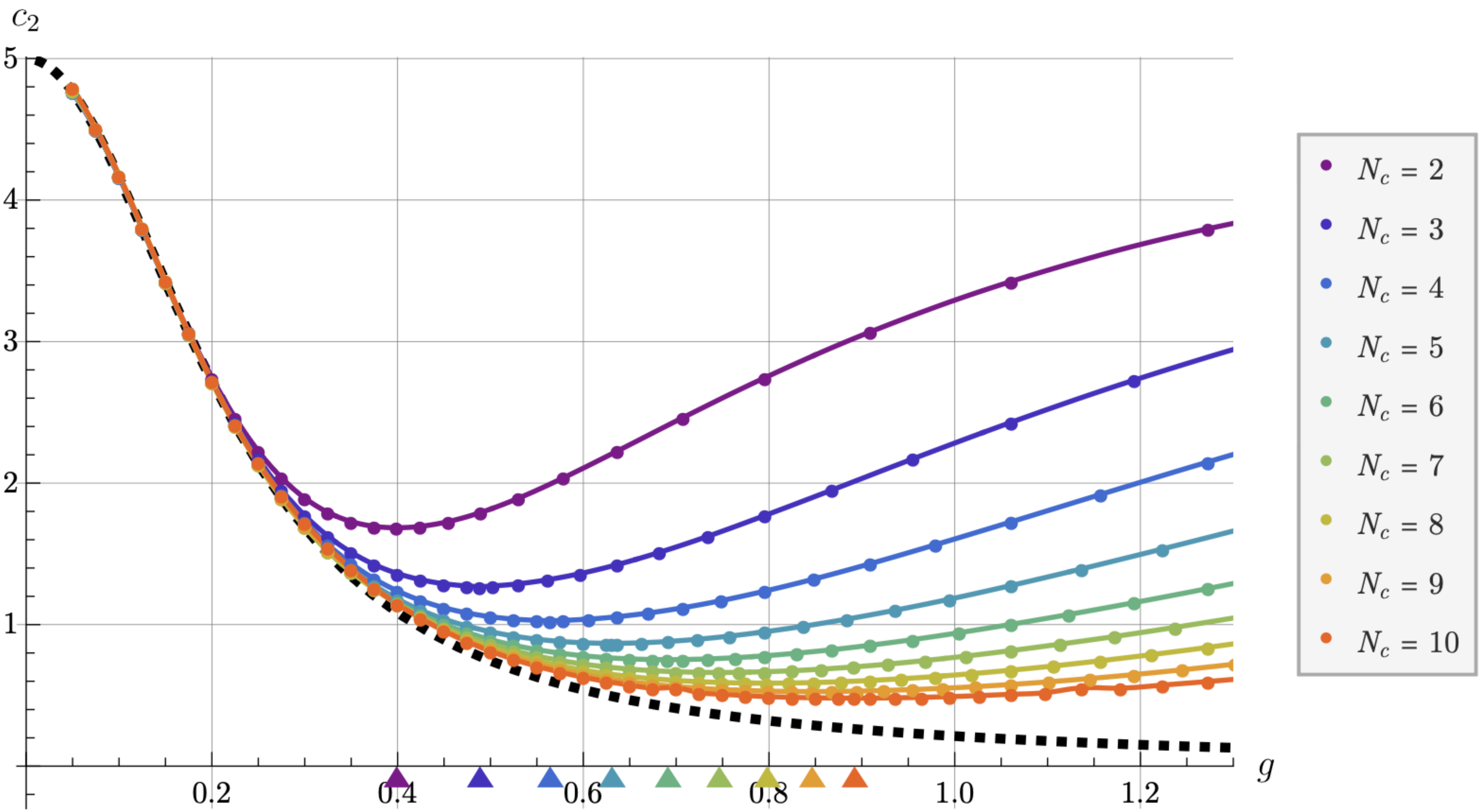
[Bender,Chester,Dempsey,Pufu,Wang]

planar

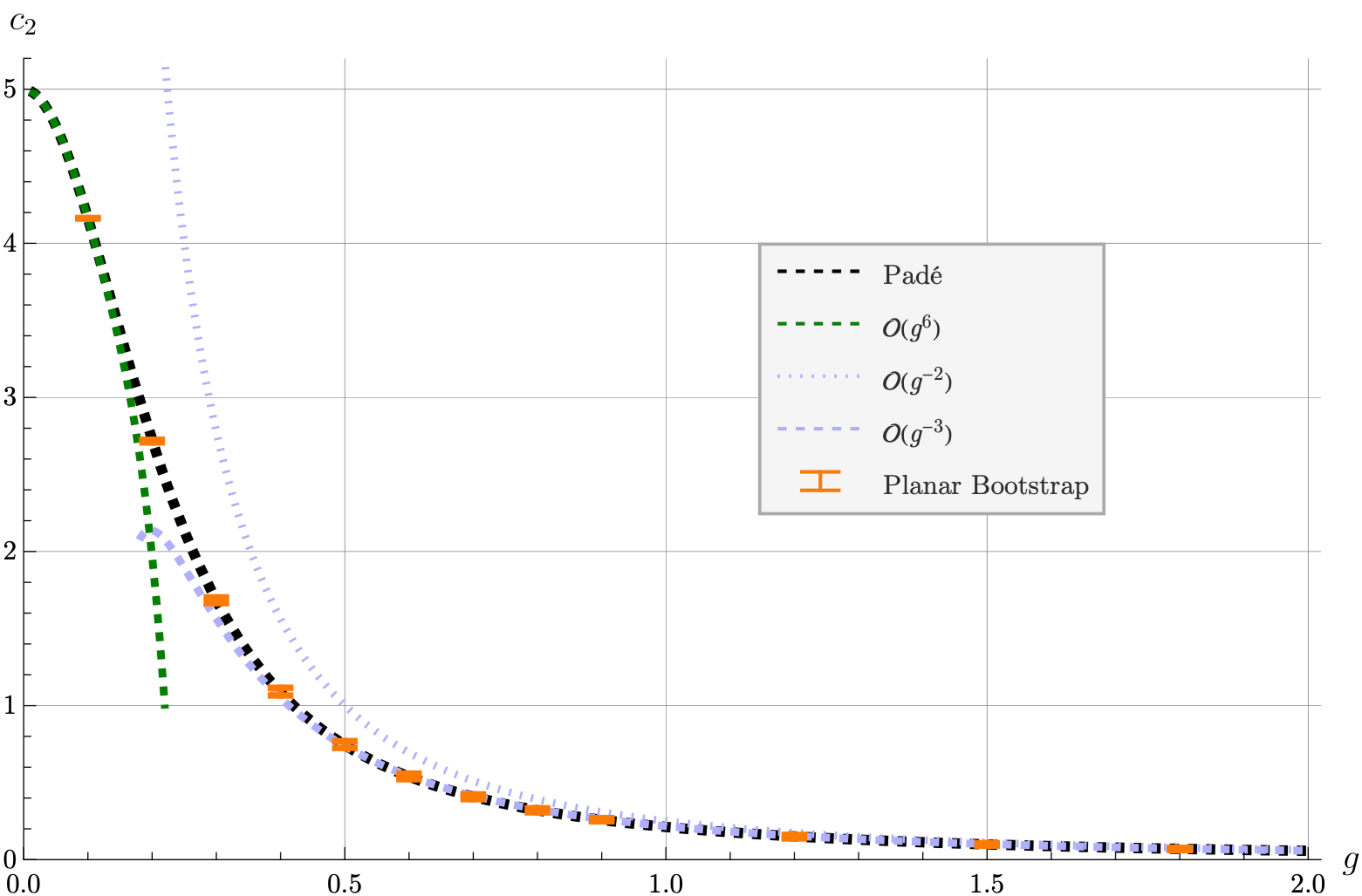
$$\begin{array}{lll} \text{min/max} & \sum \beta_{\tau,J}\lambda^2_{\tau,J} & \\ \text{subject to} & \sum \tilde\lambda^2_{\tau,J} X_{\tau,J}(u,v) = 0, & (u,v) \text{ Euclidean } (6.14)\,, \\ & \sum \tilde\lambda^2_{\tau,J} B_{\tau,J}(v) + B^{\text{protected}}_v = 0, & v > 0 \text{ real}\,, \\ & \sum \tilde\lambda^2_{\tau,J} \hat B_{\tau,J}(t) + \hat B^{\text{protected}}_t = 0, & 4 < \text{Re}\, t < 6\,, \\ & \sum \tilde\lambda^2_{\tau,J} \Psi_{\ell;\tau,J} + \Psi^{\text{protected}}_\ell = 0, & \ell = 0,2,\ldots\,, \\ & \sum \tilde\lambda^2_{\tau,J} \Phi_{\ell,\ell+2;\tau,J} = 0, & \ell = 0,2,\ldots\,, \\ & \sum \tilde\lambda^2_{\tau,J} I_{k,\tau,J} + (I^{\text{strong}}_k - I_k(g)) = 0, & k = 2,4\,, \\ & \tilde\lambda^2_{\tau,J} \ge 0 & \forall J = 0,2,\ldots,\ \tau \in \tau(g,J)\,. \end{array}$$

[Caron-Huot,Coronado,Trinh,Zahraee]

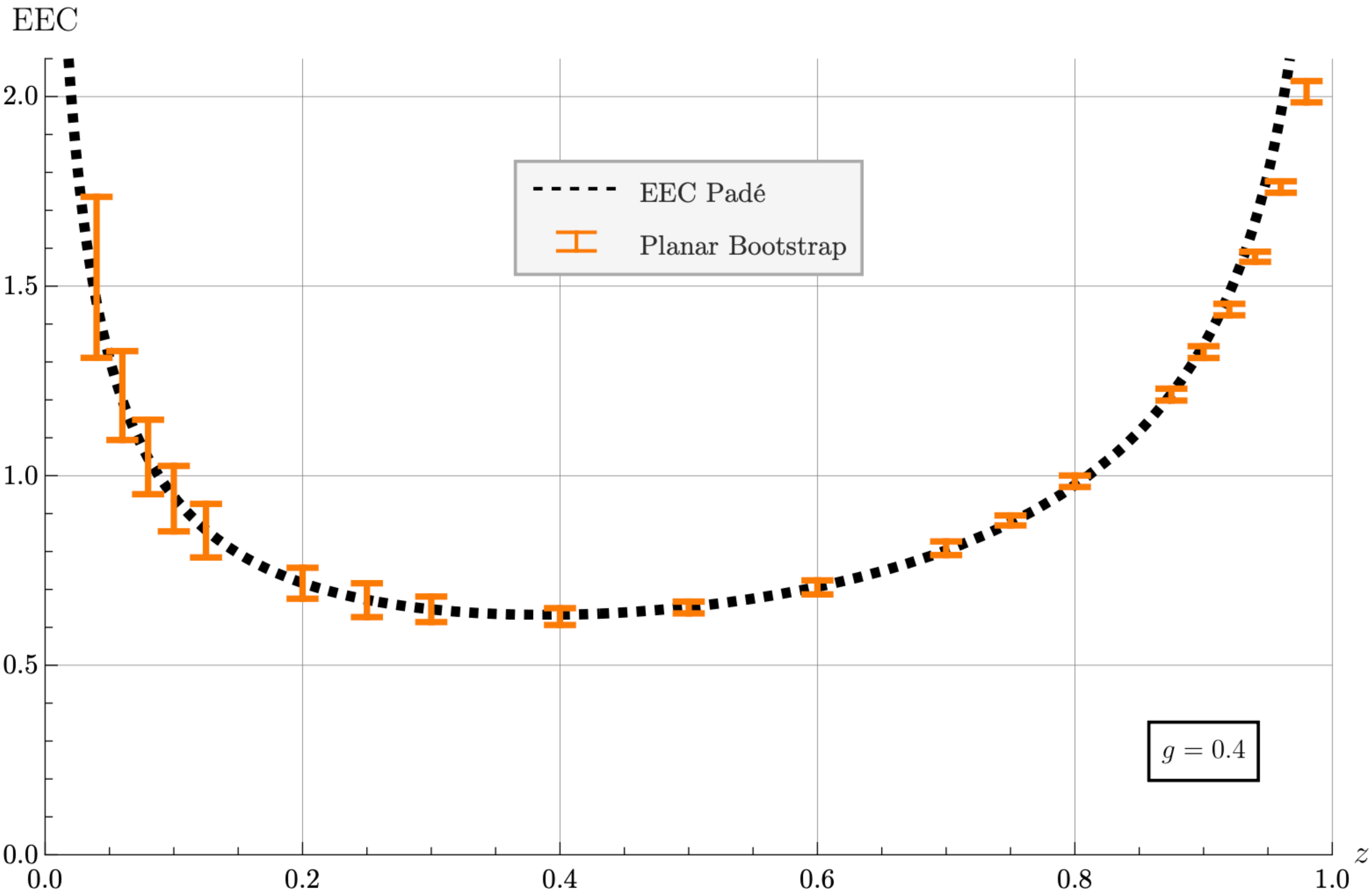
Lower bound (finite Nc)



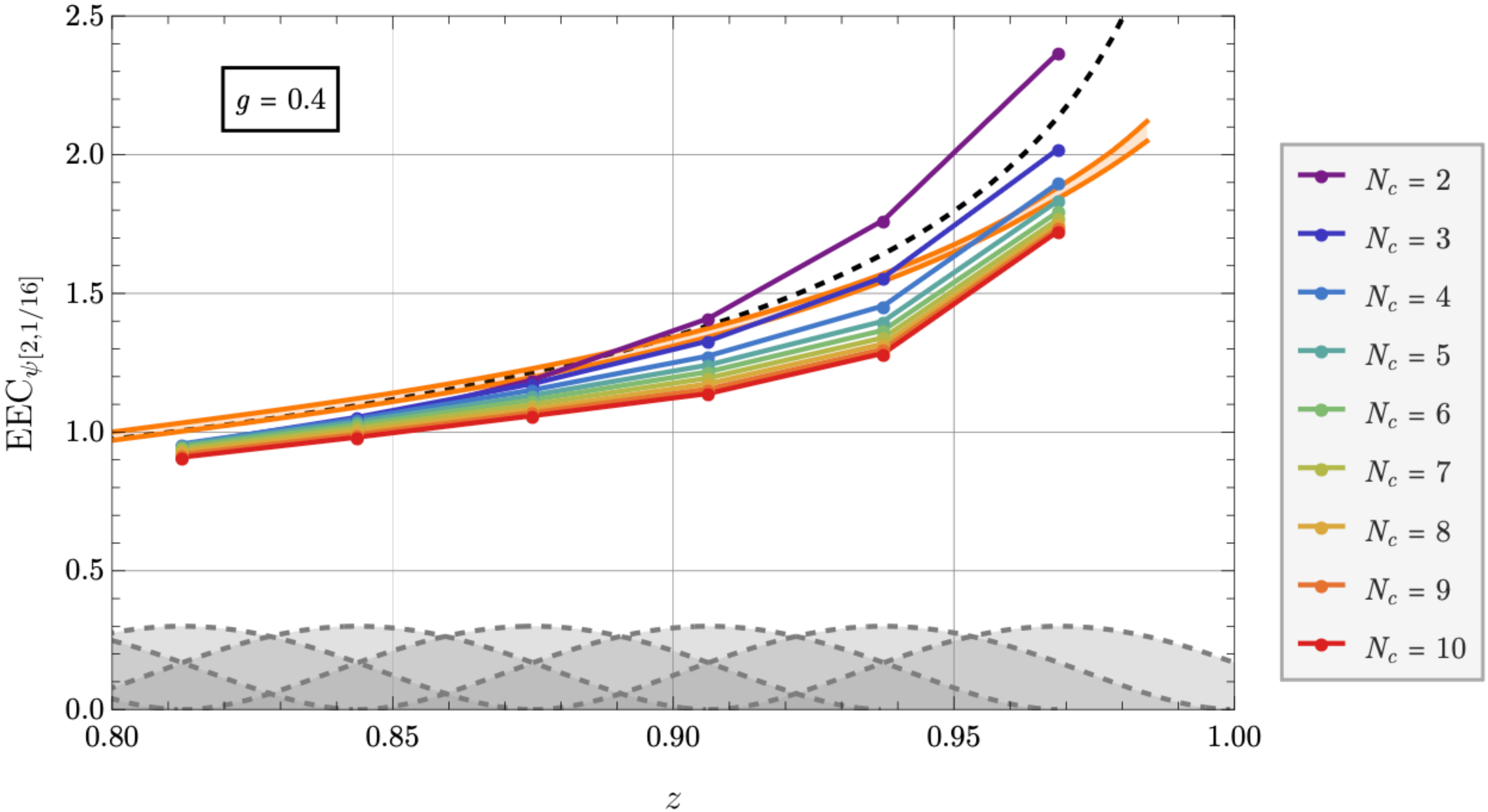
Two-sided bound (planar)



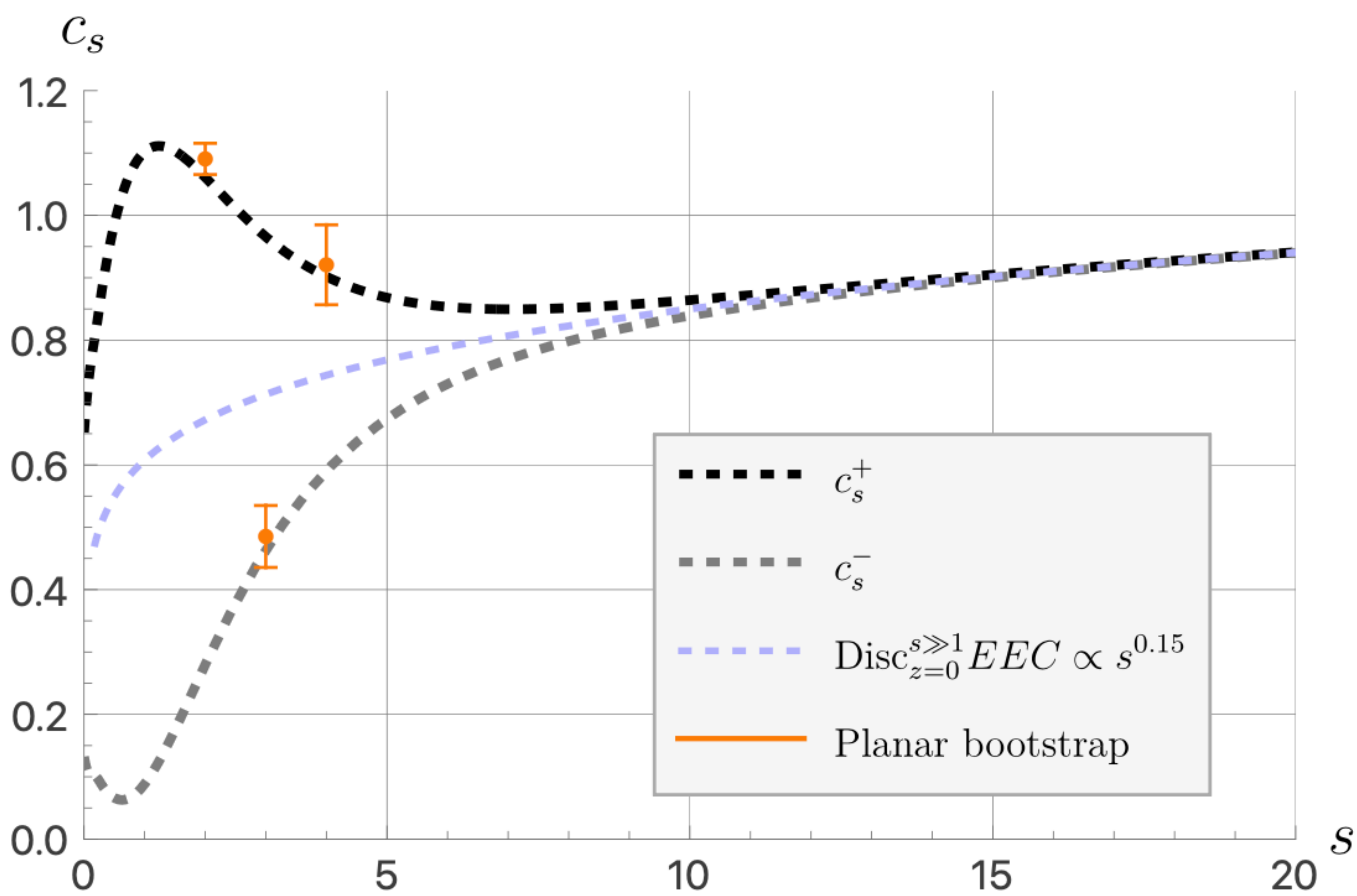
EEC two-sided bound (planar)



Smeared EEC lower bound (finite Nc)

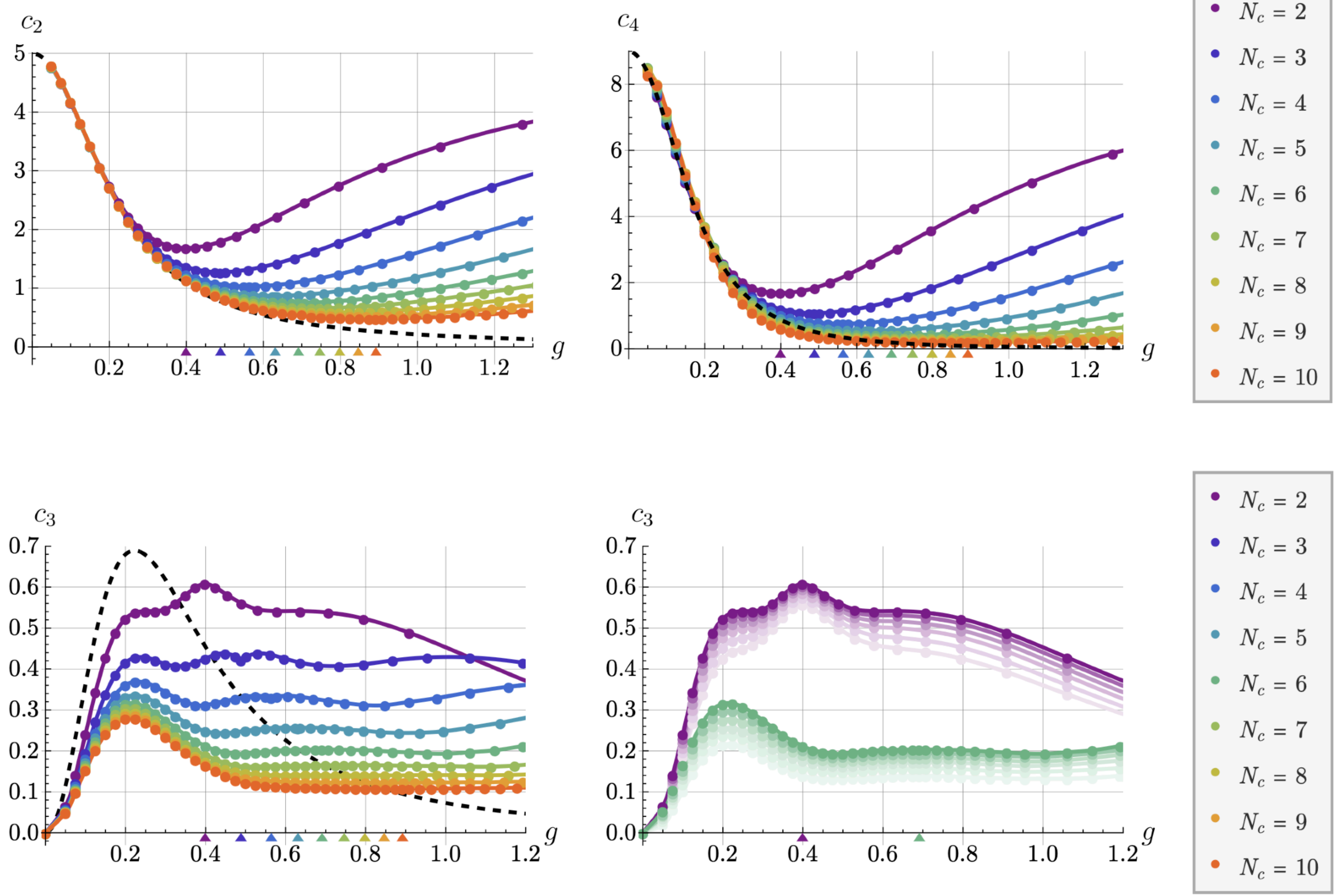


Inversion formula ($g = 0.4$)

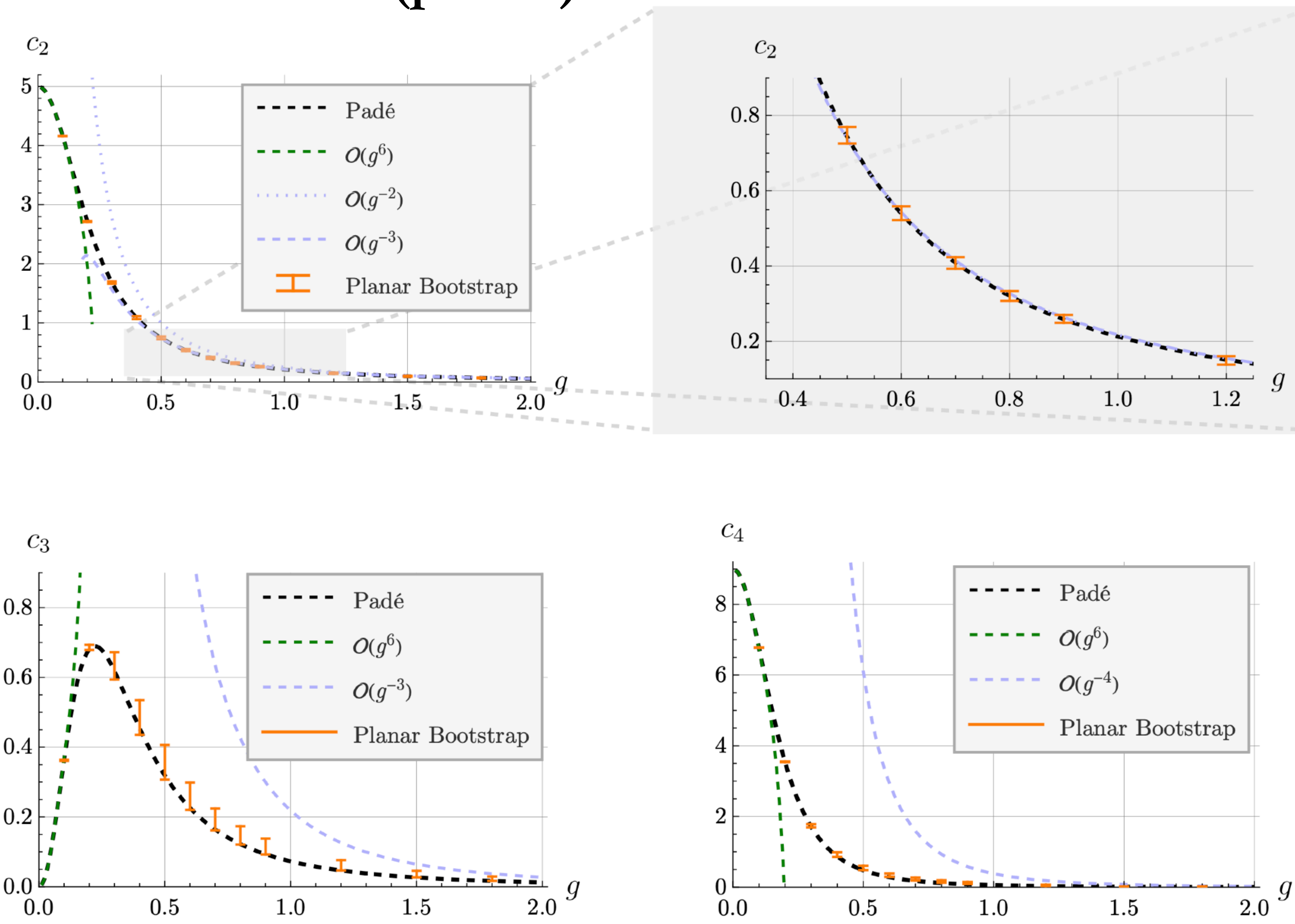


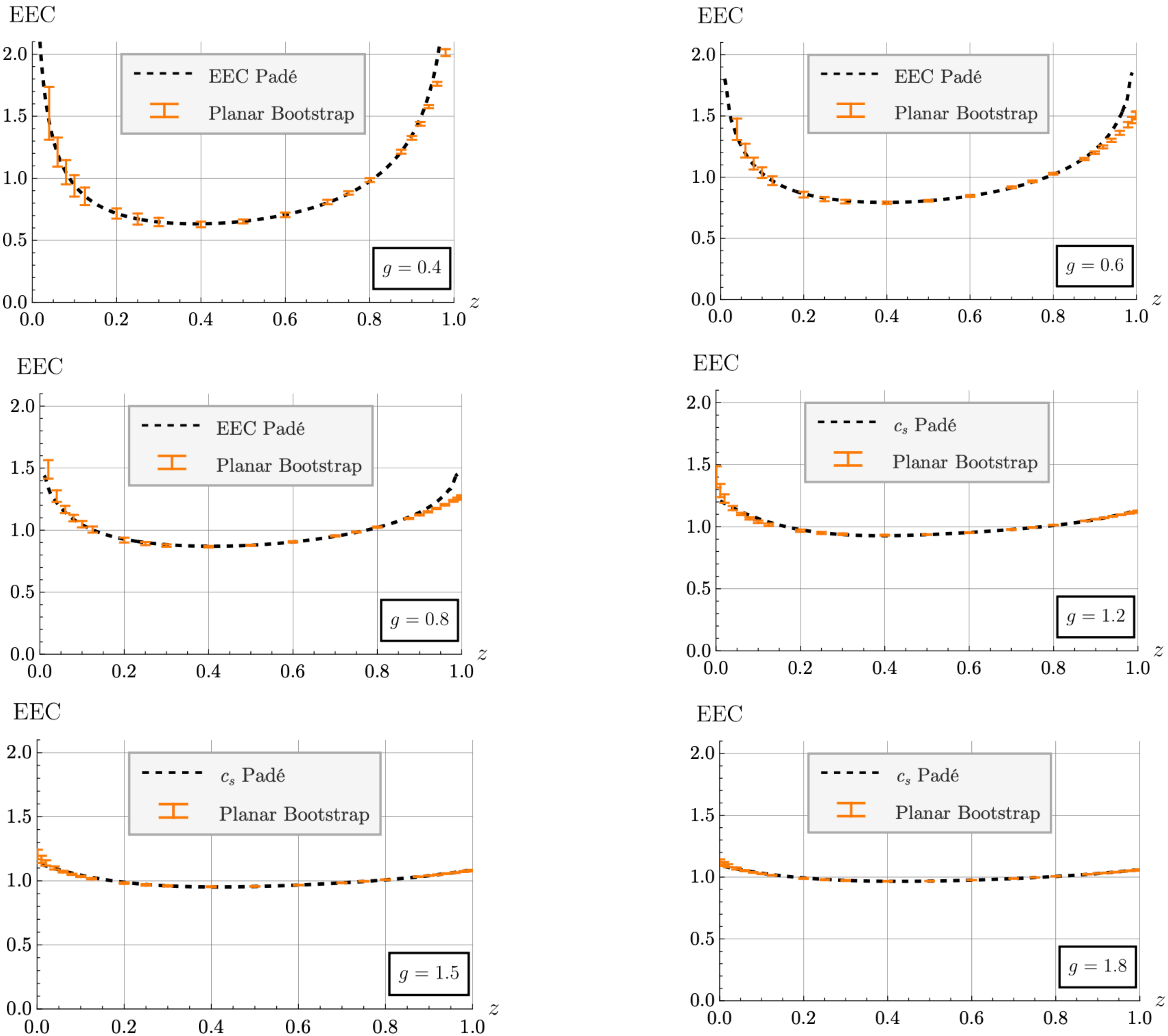
$$c_s^{\pm} = 2\frac{2s+1}{\pi}\int_{-\infty}^{0} d\tilde{z}\, Q_s(1-2\tilde{z})\Big(\mathrm{Disc}_{z=0}\mathrm{EEC}(\tilde{z}) \pm \mathrm{Disc}_{z=1}\mathrm{EEC}(1-\tilde{z})\Big)$$

Lower bound (finite Nc)



Two-sided bound (planar)



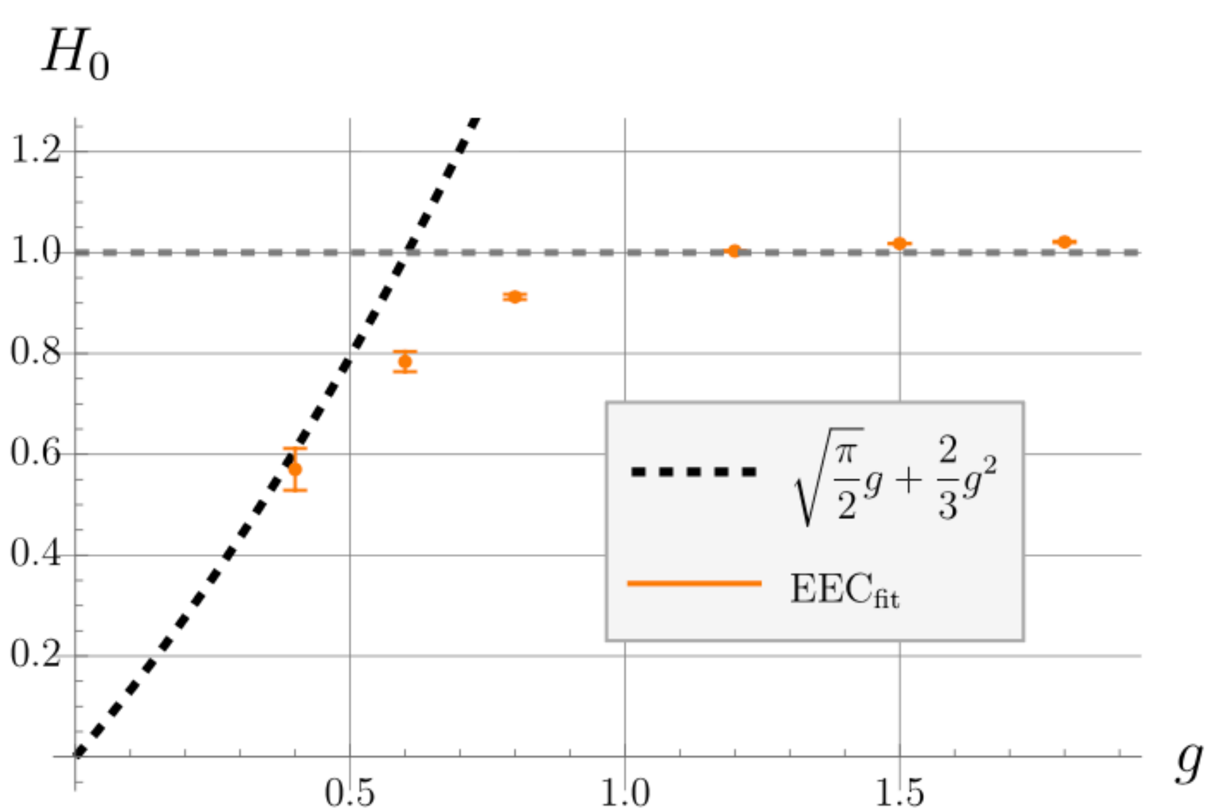

EEC
EEC Padé
Planar Bootstrap
$g = 0.4$
z
EEC
EEC Padé
Planar Bootstrap
$g = 0.6$
z
EEC
EEC Padé
Planar Bootstrap
$g = 0.8$
z
EEC
c_s Padé
Planar Bootstrap
$g = 1.2$
z
EEC
c_s Padé
Planar Bootstrap
$g = 1.5$
z
EEC
c_s Padé
Planar Bootstrap
$g = 1.8$
z

Back-to-back limit (finite coupling)

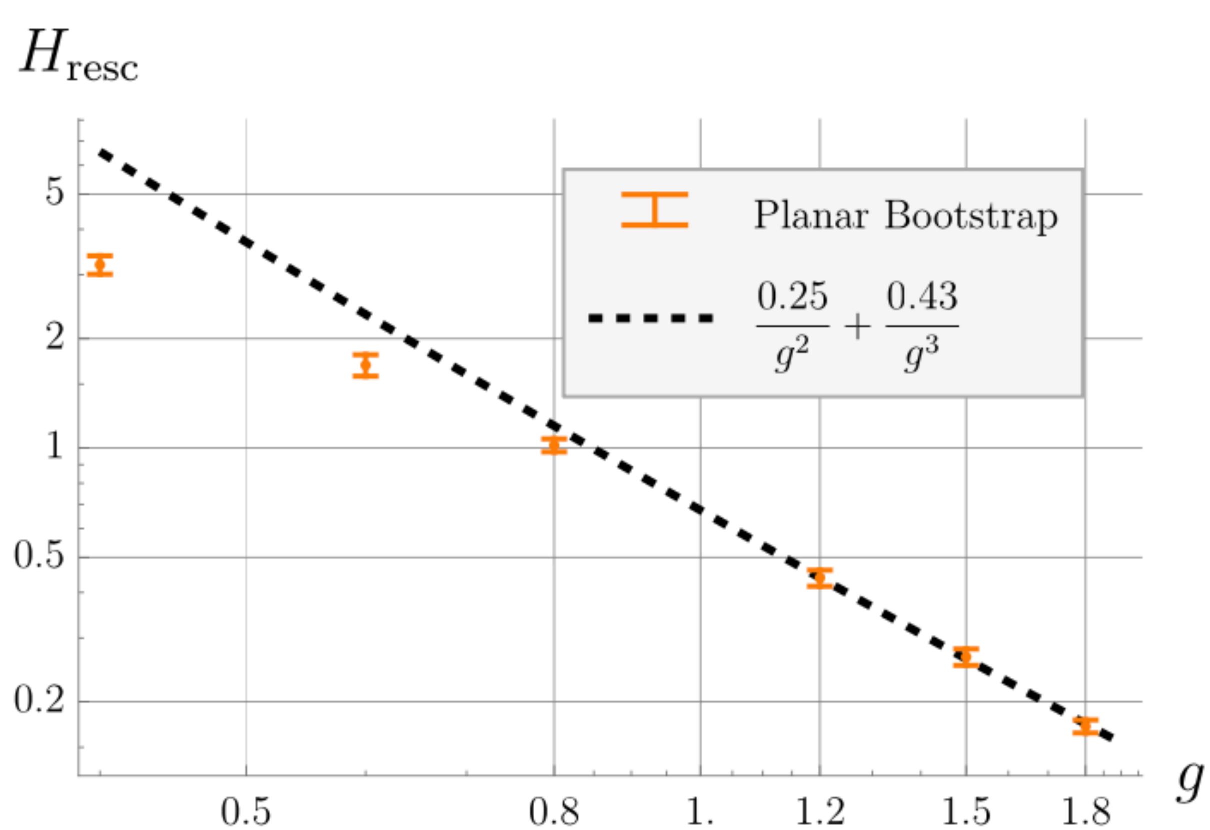
$$\mathrm{EEC}_{\mathrm{fit}}(y \equiv 1-z) = \frac{H(g)}{4y}\int_0^\infty db\, b J_0(b)$$
$$\times \exp\Big[-\frac{1}{2}\Gamma_{\mathrm{cusp}}(g)\log^2(b^2/(yb_0^2)) - \Gamma_{\mathrm{coll}}(g)\log(b^2/(yb_0^2))\Big]$$
$$+ H_0(g)\,.$$

[Korchemsky]

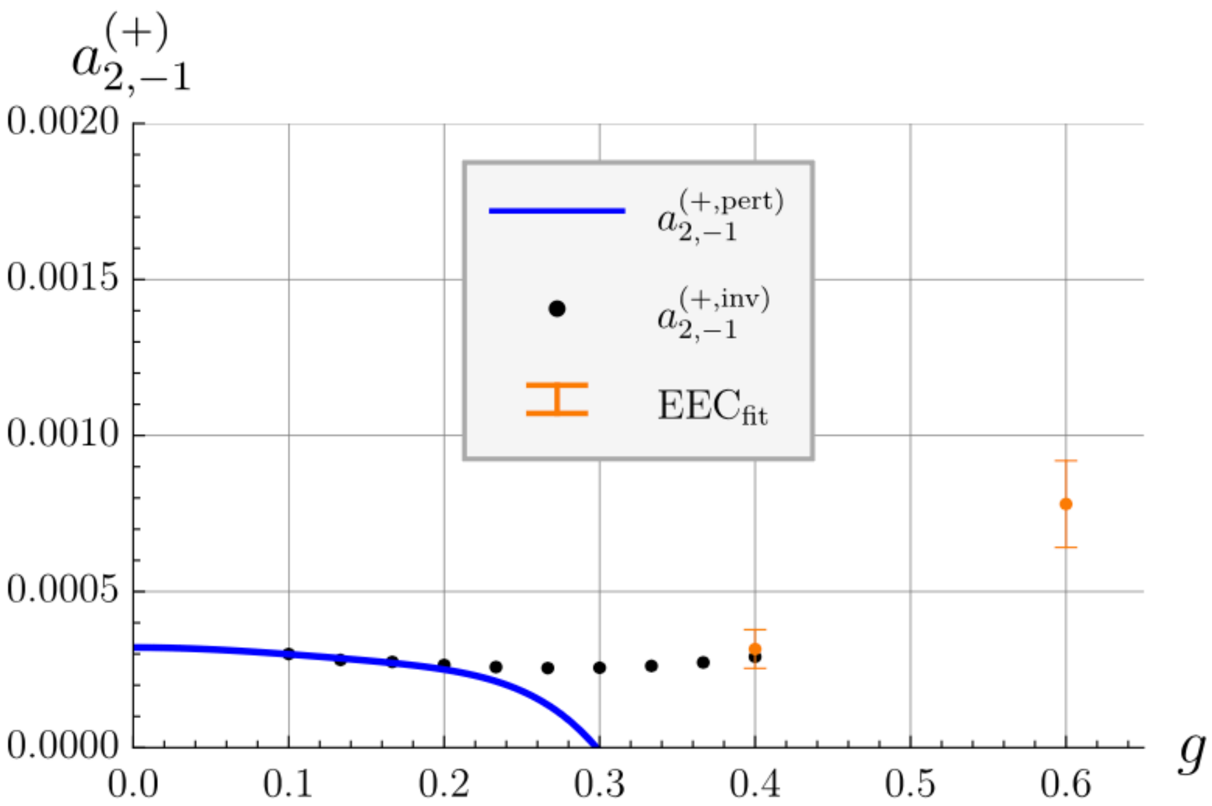
$$H_{\mathrm{resc}}(g) \equiv H(g)e^{\frac{(-1+\Gamma_{\mathrm{coll}}(g))^2}{2\Gamma_{\mathrm{cusp}}(g)}}$$
$$\approx H(g)e^{4g(\log(g/2)+\gamma_E+1)+\cdots} \qquad \text{for } g \gg 1.$$



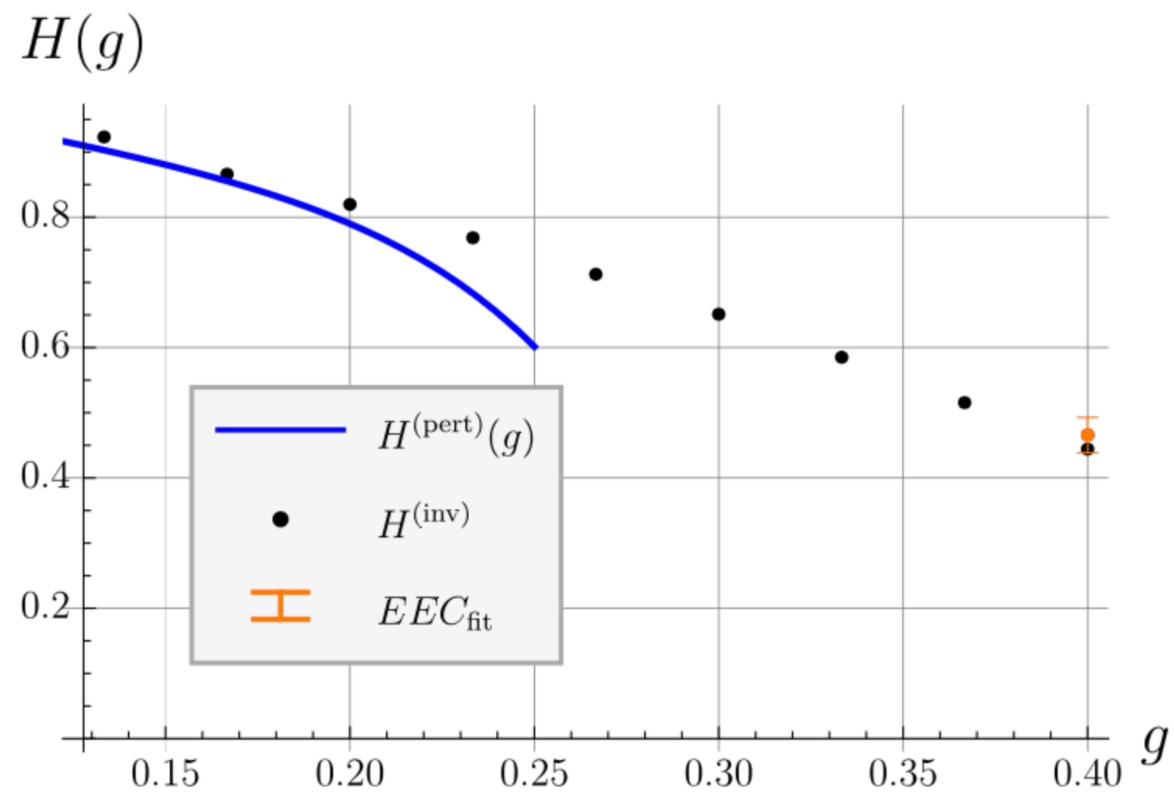
[Chen,Zhou,Zhu]



Intermediate couplings $(0.2 \lesssim g \lesssim 0.4)$



small angle

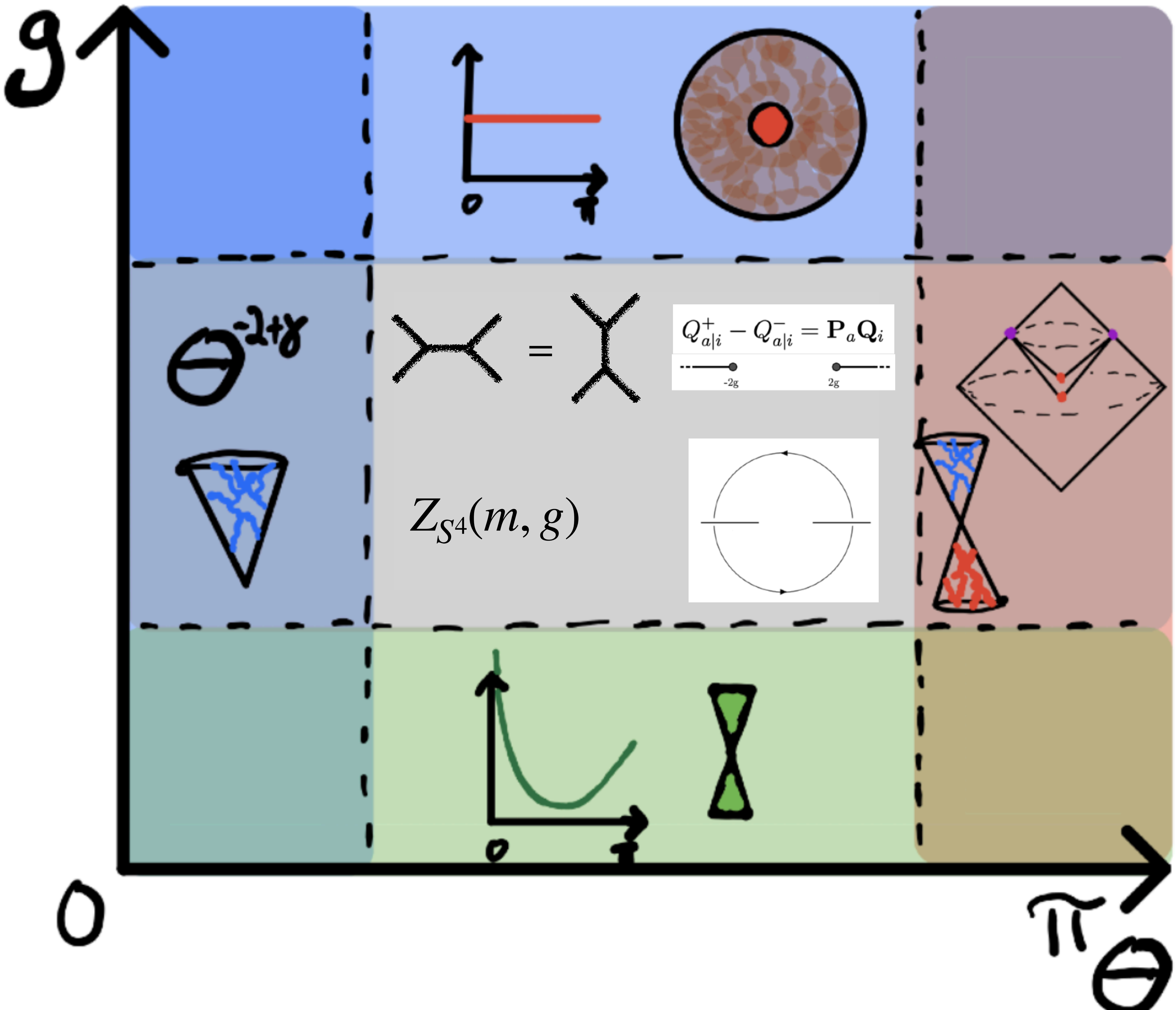


back-to-back

$$\mathrm{EEC}(z) \propto \frac{a^{+}_{2,-1}}{z^{1-\gamma^{+}_{2,-1}/2}}$$

Discussion

- CFT bootstrap for Lorentzian observables
- Other theories
- Dispersive functionals at finite Nc
- Higher-point functions
- Finite temperature, density, etc



Thank you for your attention!