Black holes, naturalness, and the renormalization group

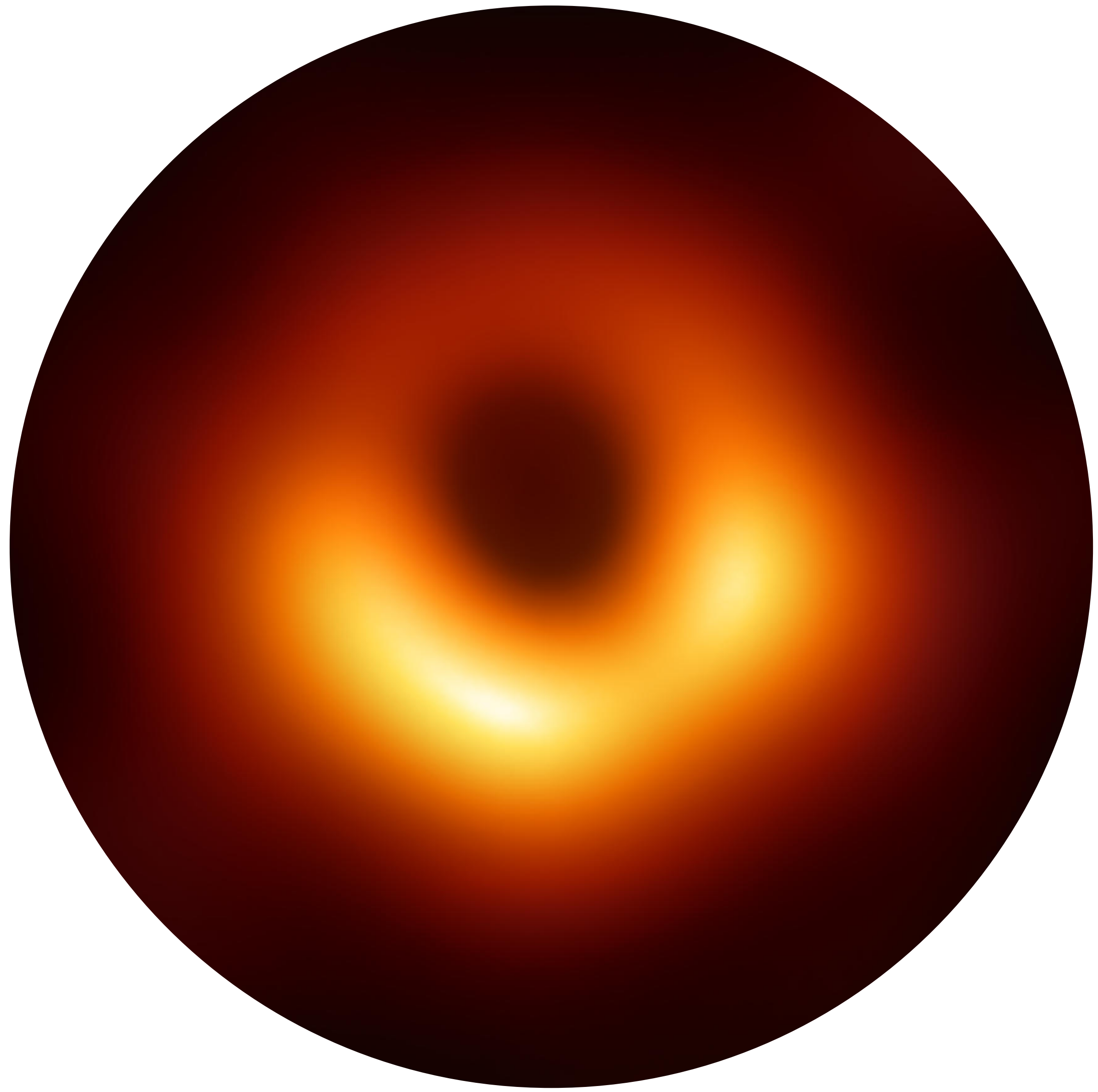
Julio Parra-Martinez (IHES)

Institut des Hautes Études Scientifiques

Observables in Gravity and Gauge Theory workshop

IPhT/CEA Saclay, December 2025

Black holes are some of the most mysterious objects we have observed in the universe



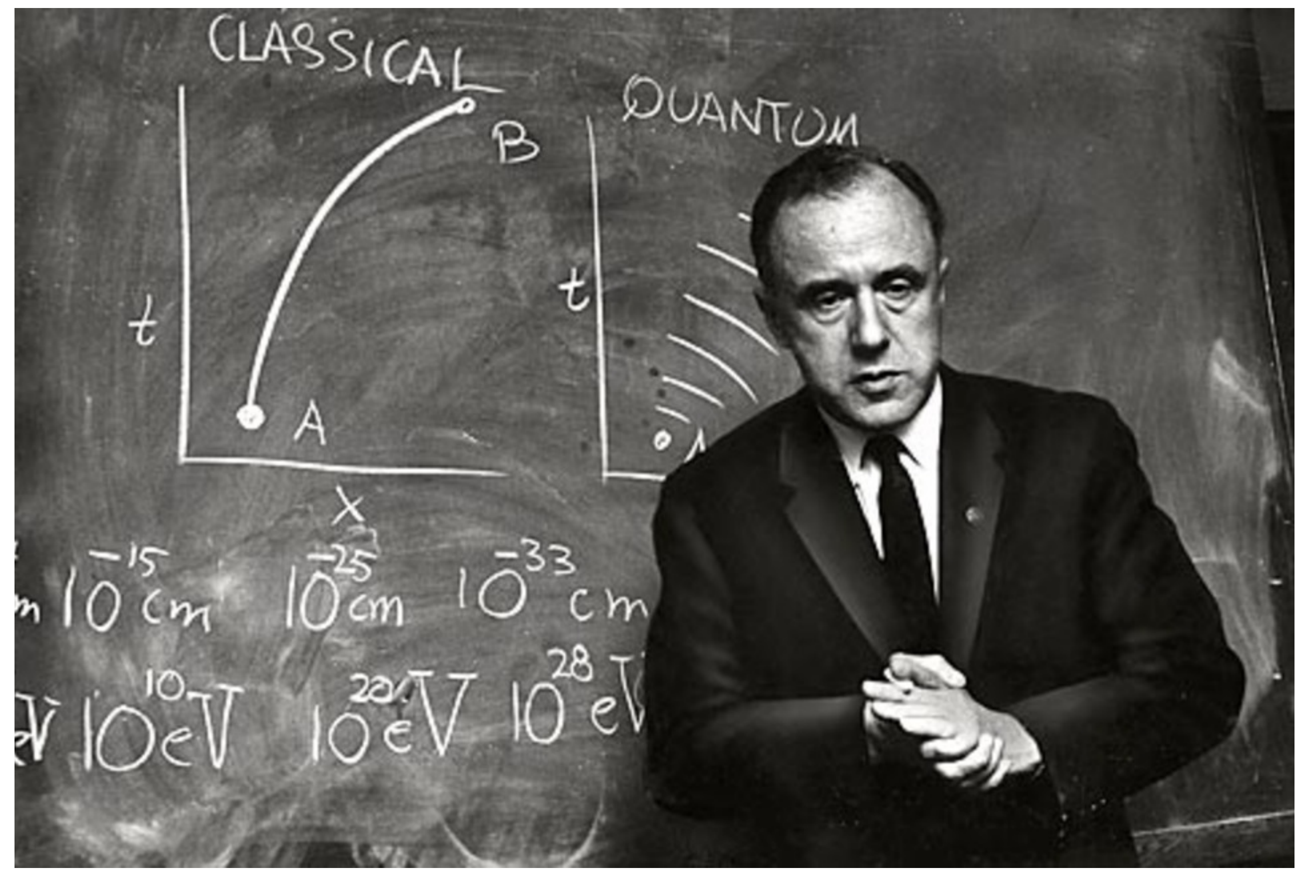
Classically they have many surprising properties

“Black holes have no hair”

Black holes behave very much as fundamental particles

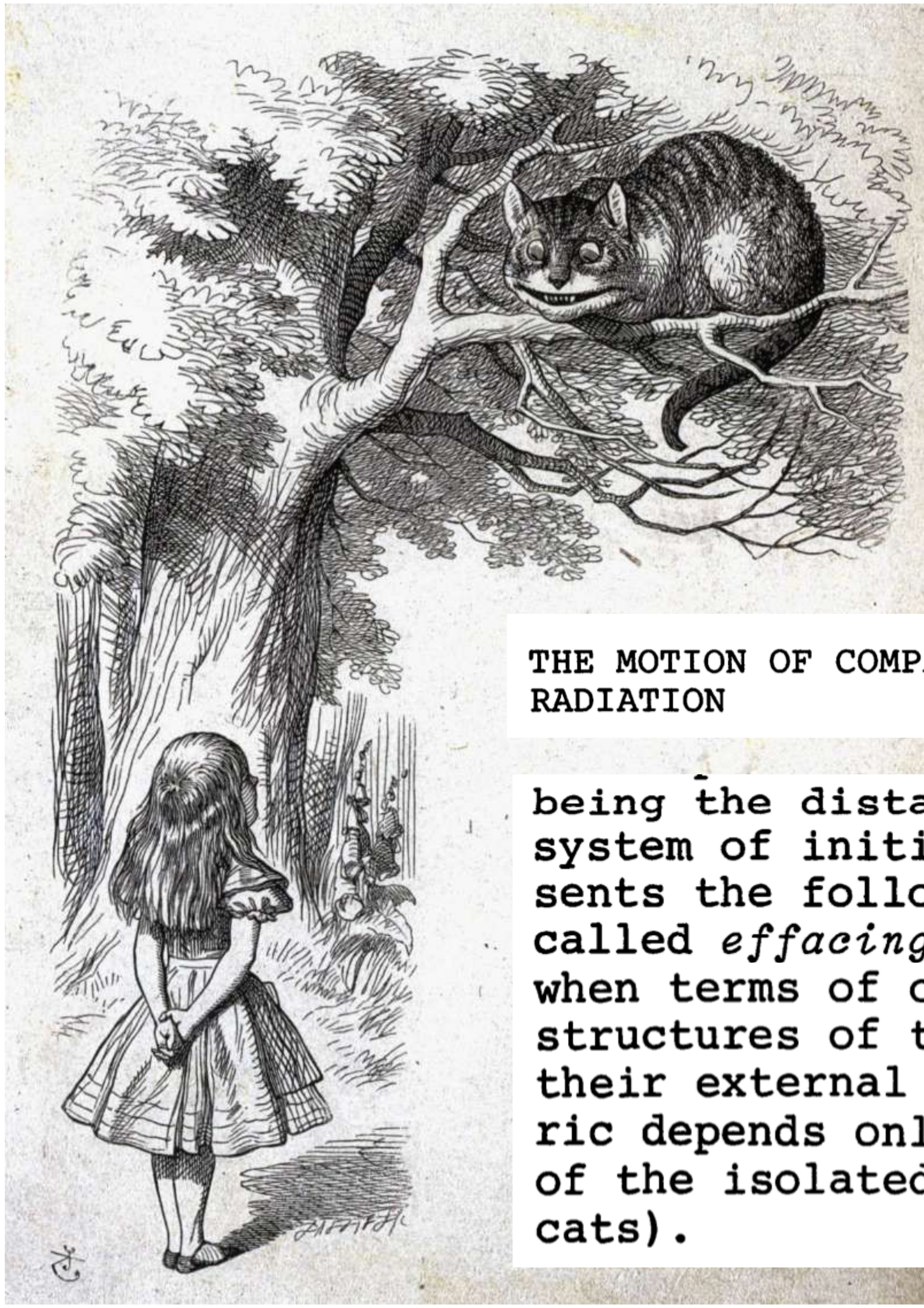
Parameterized by
just three quantities:

M	S	Q
mass	spin	charge



Only a theorem in GR with further assumptions (e.g., matter),
but tantalizing and has inspired many developments

"Black holes do not adiabatically deform"



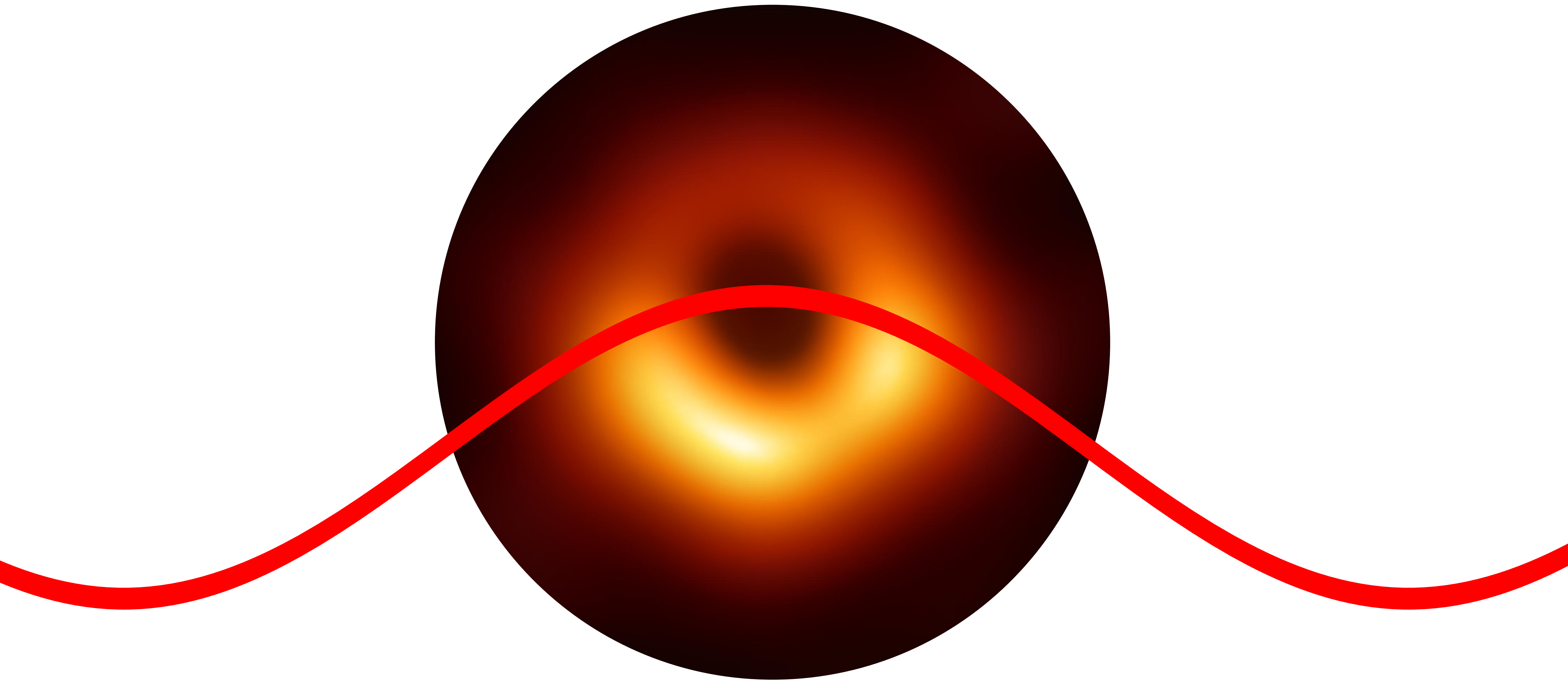
When adiabatically perturbed, still parameterized just by M, S, Q!

THE MOTION OF COMPACT BODIES AND GRAVITATIONAL RADIATION

being the distance to the companion. Therefore a system of initially non-rotating compact bodies presents the following remarkable property (which can be called *effacing principle* or *Cheshire cat principle*): when terms of order G^6/c^{12} are neglected the internal structures of the compact bodies do not show up in their external metric and therefore the external metric depends only on the constant Schwarzschild masses of the isolated compact bodies (the "grins" of the cats).

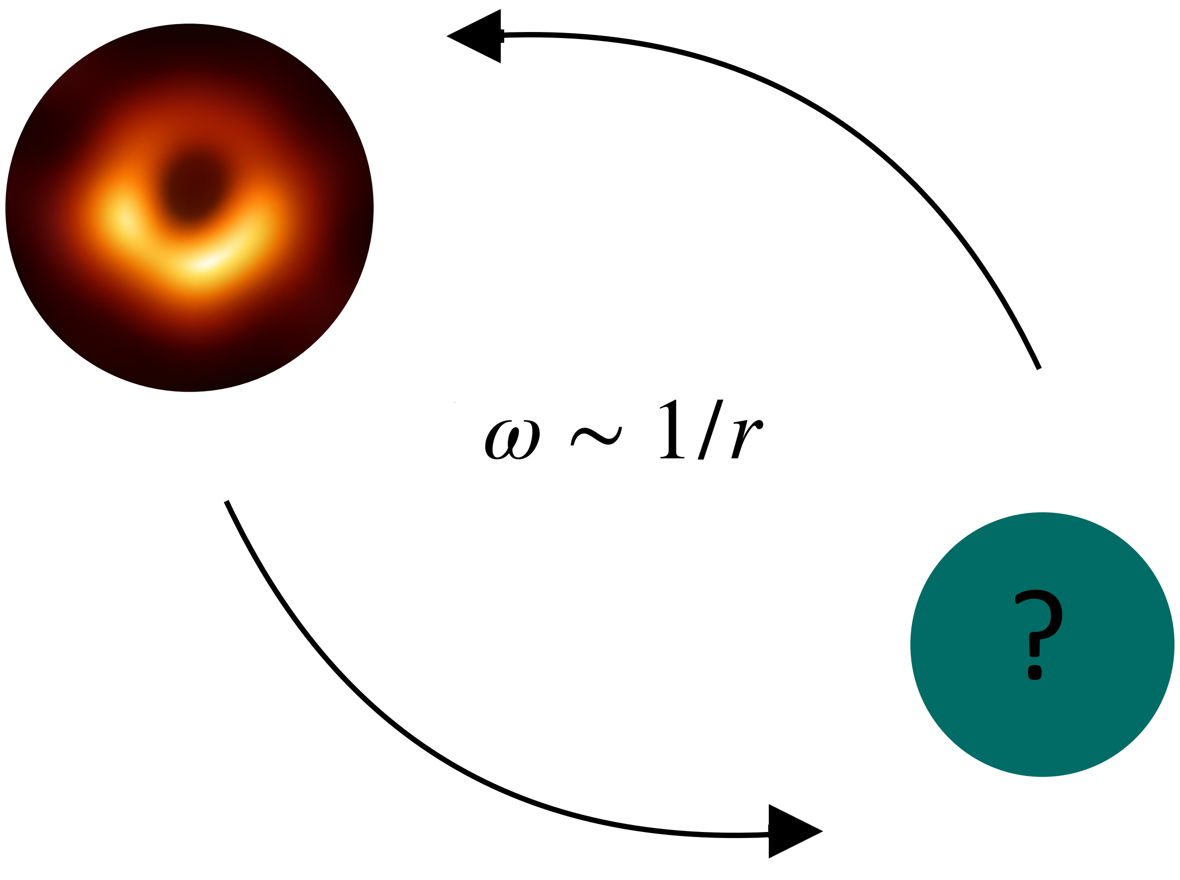
[Thibault Damour, Les Houches Lectures 1983]

We will make this precise using the language of effective field theory for $\lambda \gg R_s$ (or $\omega \ll 1/R$)

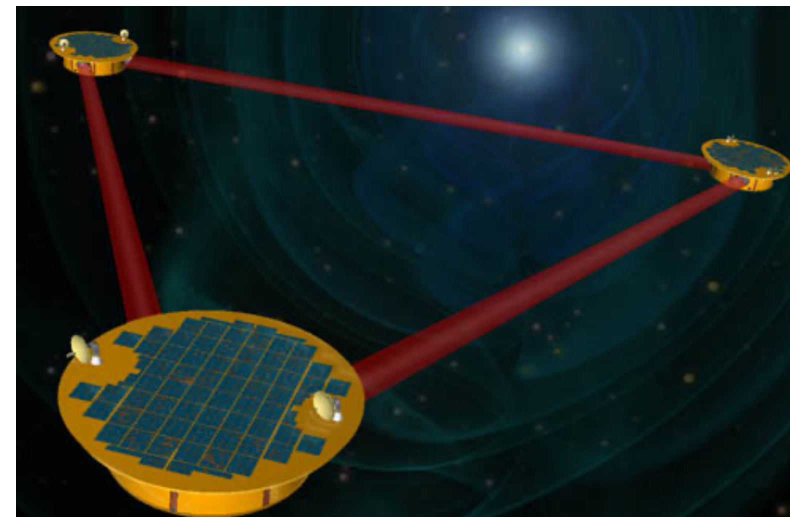


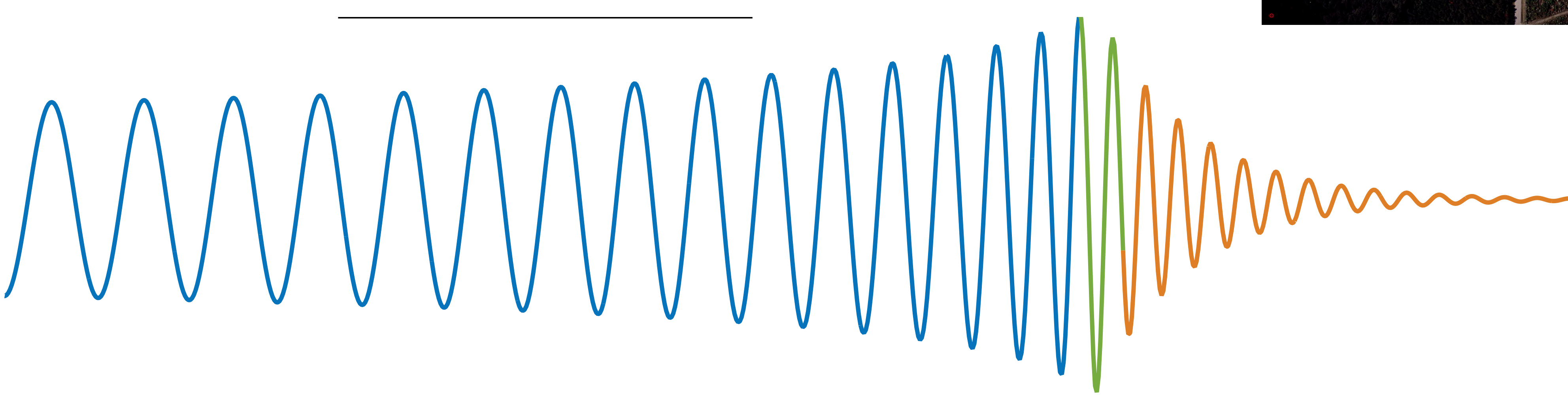
Key modern ideas: Naturalness, RG, Universality

This is the regime probed by GW experiments!



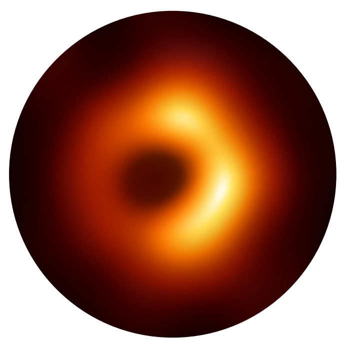
$$\omega \sim 1/r$$

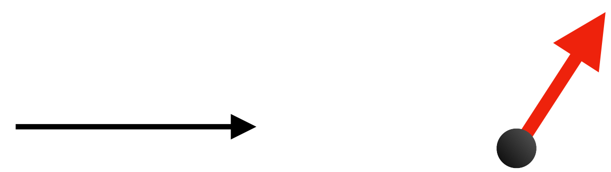




The simplest EFT

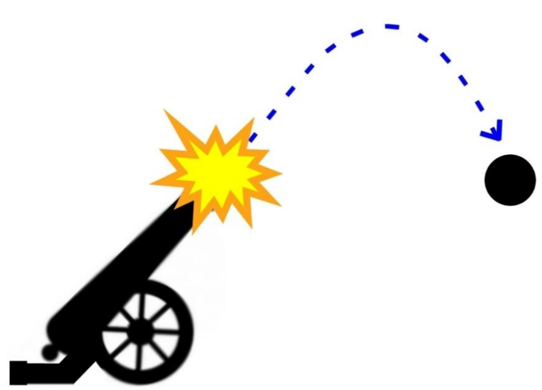
In the low-frequency regime we can replace a black hole, or any object, by a spinning point particle

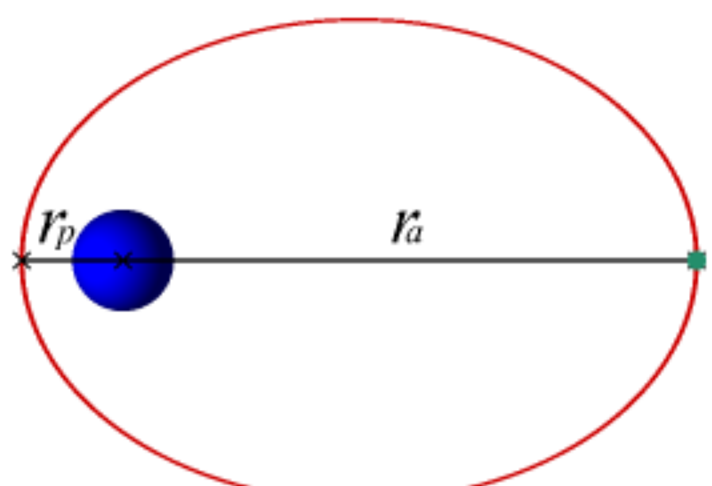




$$(M, S, Q)$$

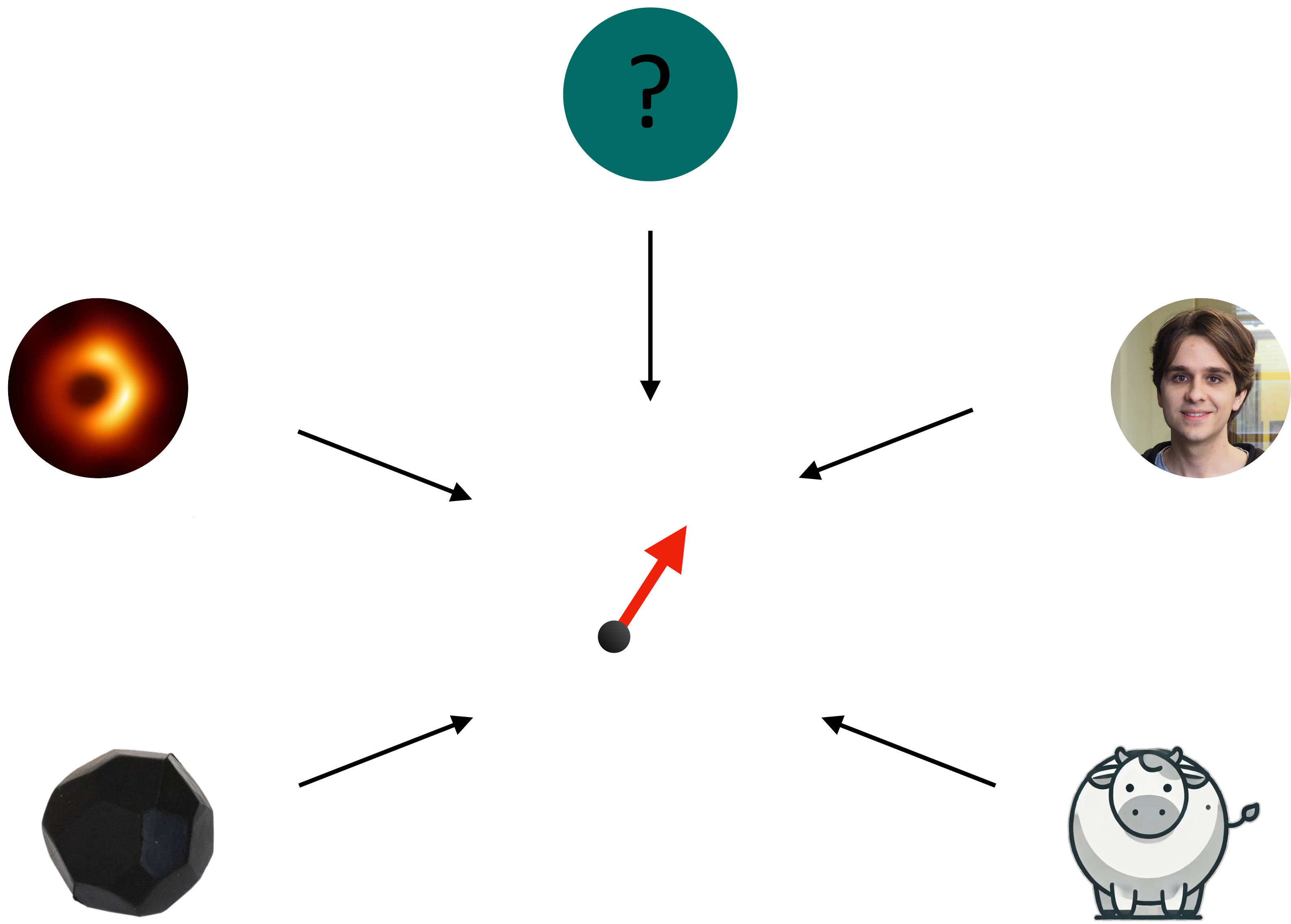
We use this EFT in undergraduate, but it's not complete!





The simplest EFT

$$S = M \int d\tau\, g_{\mu\nu}(x)\dot{x}^{\mu}\dot{x}^{\nu} + Q \int d\tau A_{\mu}(x)\dot{x}^{\mu} + \cdots$$



The simplest EFT

Point particle must be augmented by multipoles encoding microscopics [Goldberger, Rothstein; Porto]

$$E_{ij} = C_{i0j0}$$

$$S = \int d\tau \left[M + \omega_i S^i + Q_{ij}(\tau) E^{ij} + Q_{ijk}(\tau) \nabla^{(i} E^{jk)} + \cdots \right]$$

spin

octupole

mass/energy

quadrupole

So the field far from the objects takes the familiar form of a multipole expansion

$$g_{\mu\nu} \sim 1 + \frac{M}{r} + \frac{S}{r^2} + \frac{M^2}{r^2} + \frac{Q_2}{r^3} + \cdots$$

Nonlinearities

“No-hair” revisited

In the EFT “no-hair” is simply the statement that black holes do not have intrinsic multipole moments

$$Q_\ell = 0 \qquad \longrightarrow \qquad g_{\mu\nu} \sim 1 + \frac{M}{r} + \frac{M^2}{r^2} + \frac{M^3}{r^3} + \cdots$$

For spinning black holes the statement is that all multipoles are spin-induced

$$Q_\ell = M\, a^\ell = M \frac{S^\ell}{M^\ell}$$

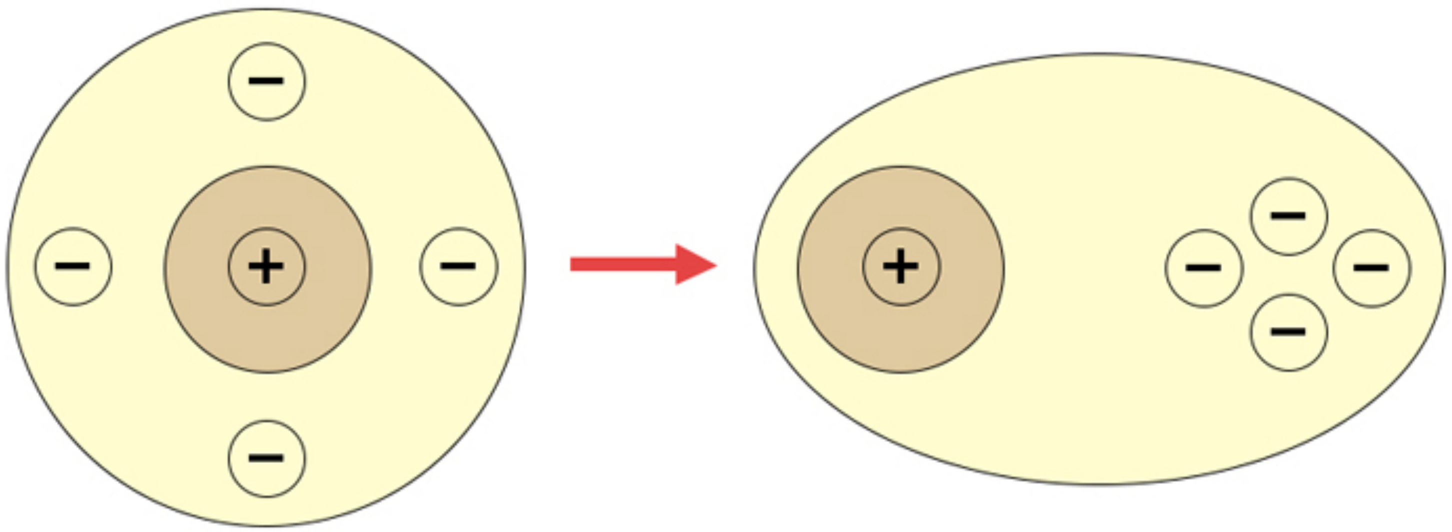
We will see later that these constitute puzzles

What about no deformation?

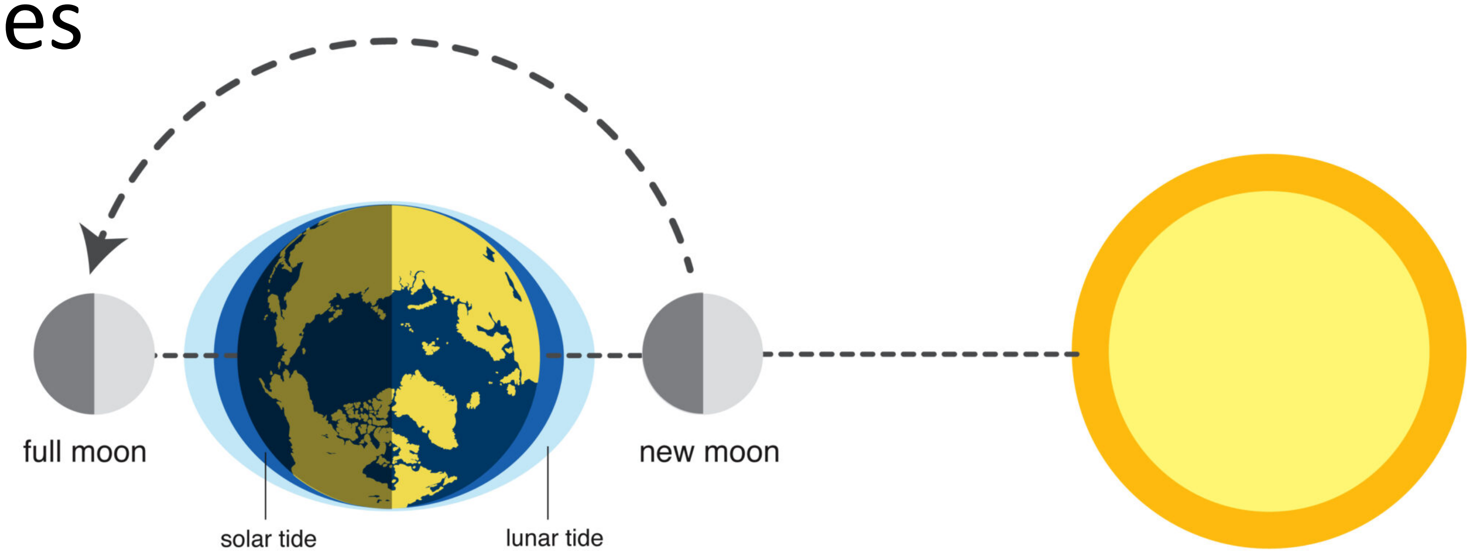
Tides

We need to parameterize response to external fields, such as polarizability

$$D_i^{\text{ind}} = \alpha_1 E_i^{\text{ext}} \qquad \alpha_1 \sim QR$$

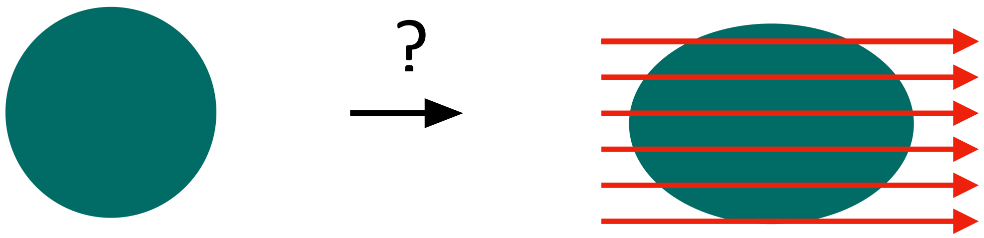


Or more generally, tides



Tidal Love numbers

Love numbers = "gravitational polarizabilities"



$$Q_{ij}^{\mathrm{ind}} = \lambda_2 E_{ij}^{\mathrm{ext.}}$$

Linear response to applied external gravitational field.

$$\delta g \sim \mathcal{G} r^{\ell} \left(1 + \cdots + \frac{\lambda_\ell}{r^{2\ell+1}} + \cdots \right)$$

Non-minimal couplings in EFT

$$\Delta S^{\mathrm{con.}} = \alpha_1 \int d\tau E_i^2 + \lambda_2 \int d\tau E_{ij}^2 + \lambda_{2\omega^2} \int d\tau (\dot{E}_{\mu\nu})^2 + \cdots$$

Experimental probes

The modify the post-Newtonian potential & fluxes

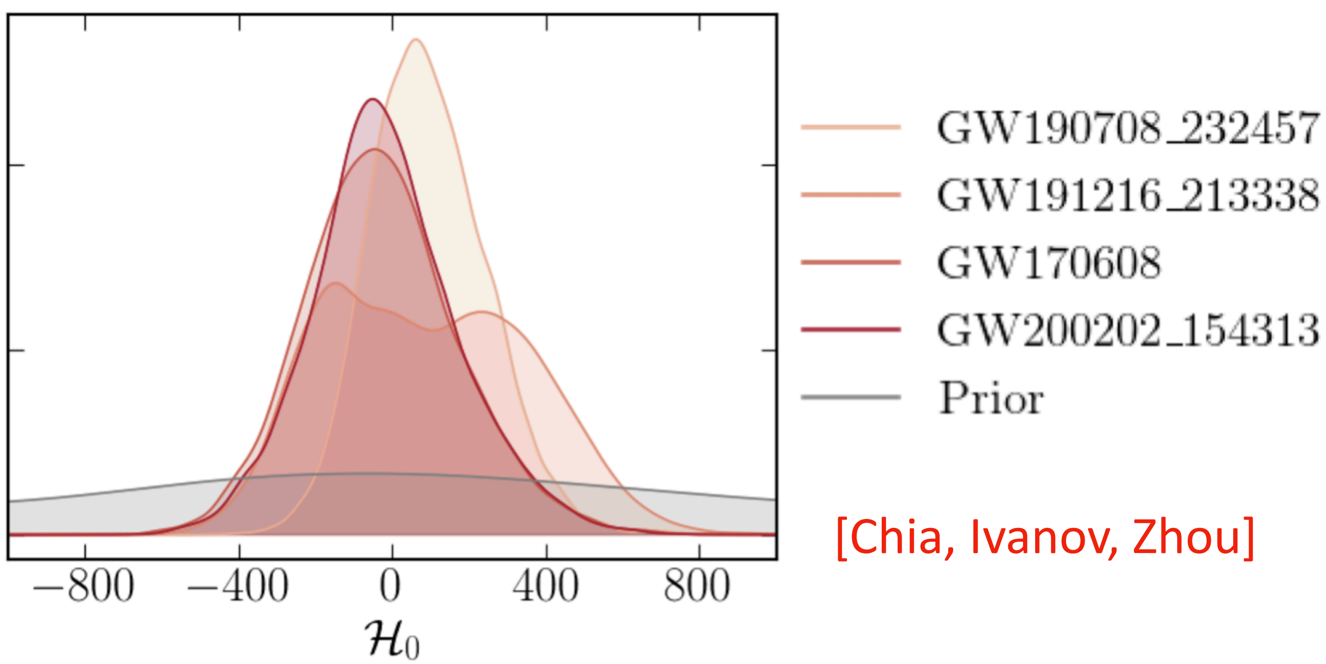
e.g.,

$$V(r) \sim \frac{\lambda_2}{r^4}\frac{R_s^2}{r^2}$$

"5PN"

[Damour; Flanagan, Hinderer; Cheung, Solon; Bern, **JPM**, Roiban, Sawyer, Shen; Porto, many others]

We should be able to test with GW detectors.

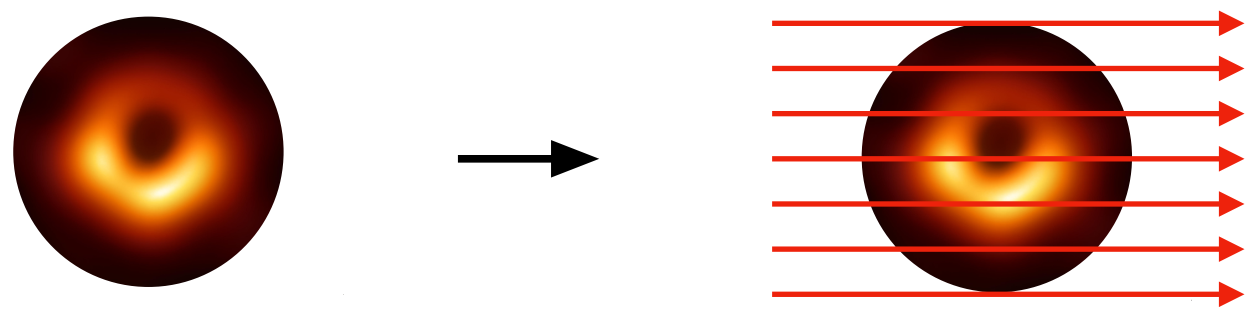


[Chia, Ivanov, Zhou]

Excellent possible window into new physics! [Cardoso, Franzin, …]

“No-deformation” revisited

Polarizabilities are zero for BH in D=4 GR! [Damour ’83]



Induced multipole moments also vanish at linear order!

$$Q_\ell^{\mathrm{ind}} = 0 \quad \longrightarrow \quad \delta g \sim \mathscr{G} r^\ell \left(1 + \cdots + \frac{0}{r^{2\ell+1}} + \cdots\right)$$

People say “it must have to do with no-hair” but unclear

Note for later: dynamical tides do not! [Ivanov, Li, **JPM**, Zhou]

Static tides from linear response

Regge-Wheeler/Teukolsky equation at zero freq $\nabla^2_{BH}\delta g_{\mu\nu} = 0$

$$\delta g \sim \sum_{\ell m} {}_2Y_{\ell m}(\Omega)\, h_{\ell m}(r,\omega)$$

$$\Delta^{-2}\frac{d}{dr}\left(\Delta^3 \frac{d}{dr} h_{\ell m}(r)\right) + V_{eff}(r)\, h_{\ell m}(r) = 0$$

regularity

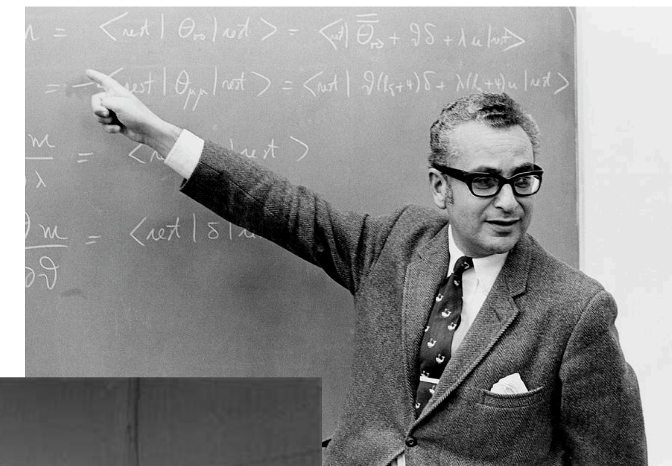
$$h_{\ell m} \to \mathcal{G} r^{\ell} + \cdots + \mathcal{G}\frac{0}{r^{\ell+1}}$$

Naturalness problem

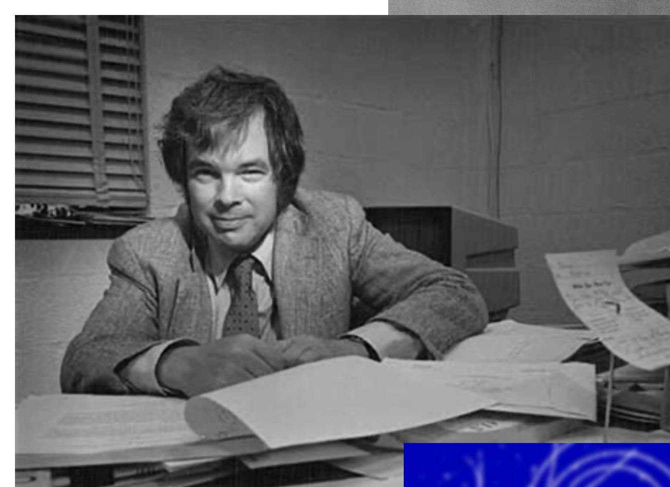
Gell-Mann, Wilson, and ’t Hooft

Gell-Mann’s totalitarian principle:

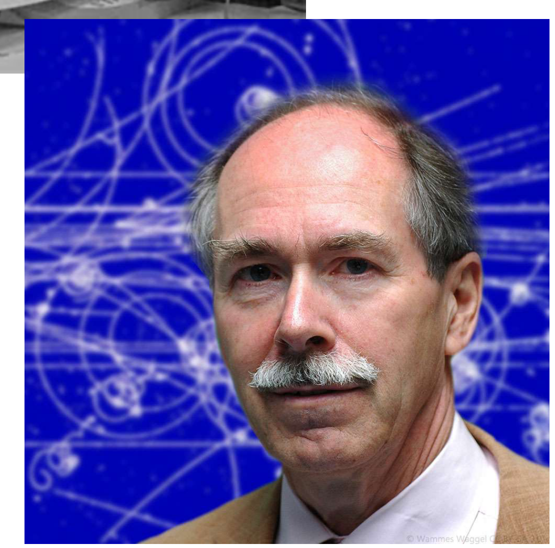
“In physics, everything not forbidden is compulsory”



Made precise by Wilsonian paradigm relating physics at different scales



’t Hooft’s naturalness:



“A parameter is only allowed to be small or zero if in the limit there is an enhanced symmetry”

A classical naturalness problem

Dimensional analysis dictates the size of corrections to point particle picture, which justify EFT

$$Q_\ell \sim MR^\ell \qquad \text{c.f.,} \int d^3x\, \rho(x) x^\ell \quad > 0\,?$$

$$\frac{Q_\ell^{\text{ind.}}}{E_\ell} = \lambda_\ell \sim MR^{2\ell} \qquad \text{c.f.,} \sum_n \frac{|\langle n | Q_\ell | 0 \rangle|^2}{E_n - E_0} \quad > 0\,?$$

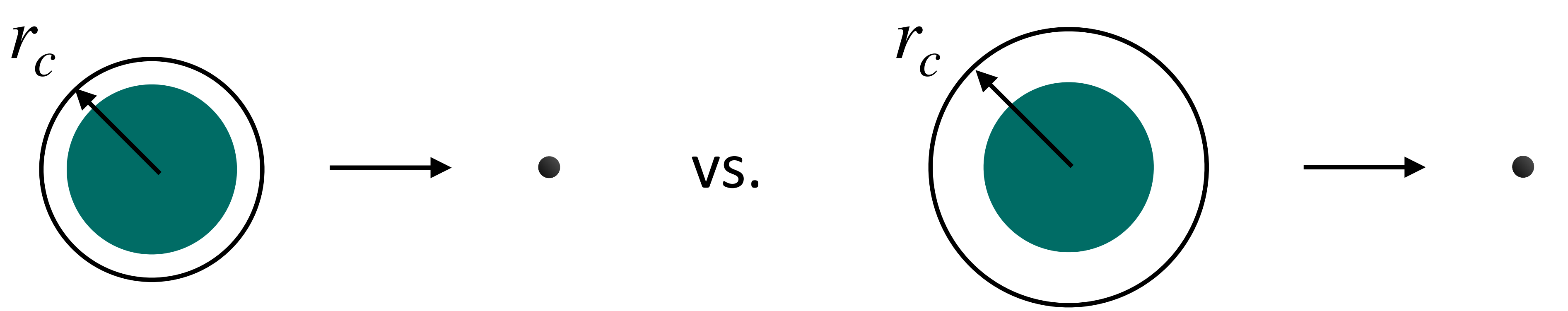
Vanishing for black holes seems to violate naturalness.

[Porto '16]

$$Q_\ell^{BH} = M\, a^\ell \qquad \lambda_\ell^{BH} = 0$$

Tides and multipoles can run!

A compact object cannot be cleanly separated from its gravitational field



Tides depend on "how much of the spacetime is integrated out together with the microscopic d.o.f of the object"

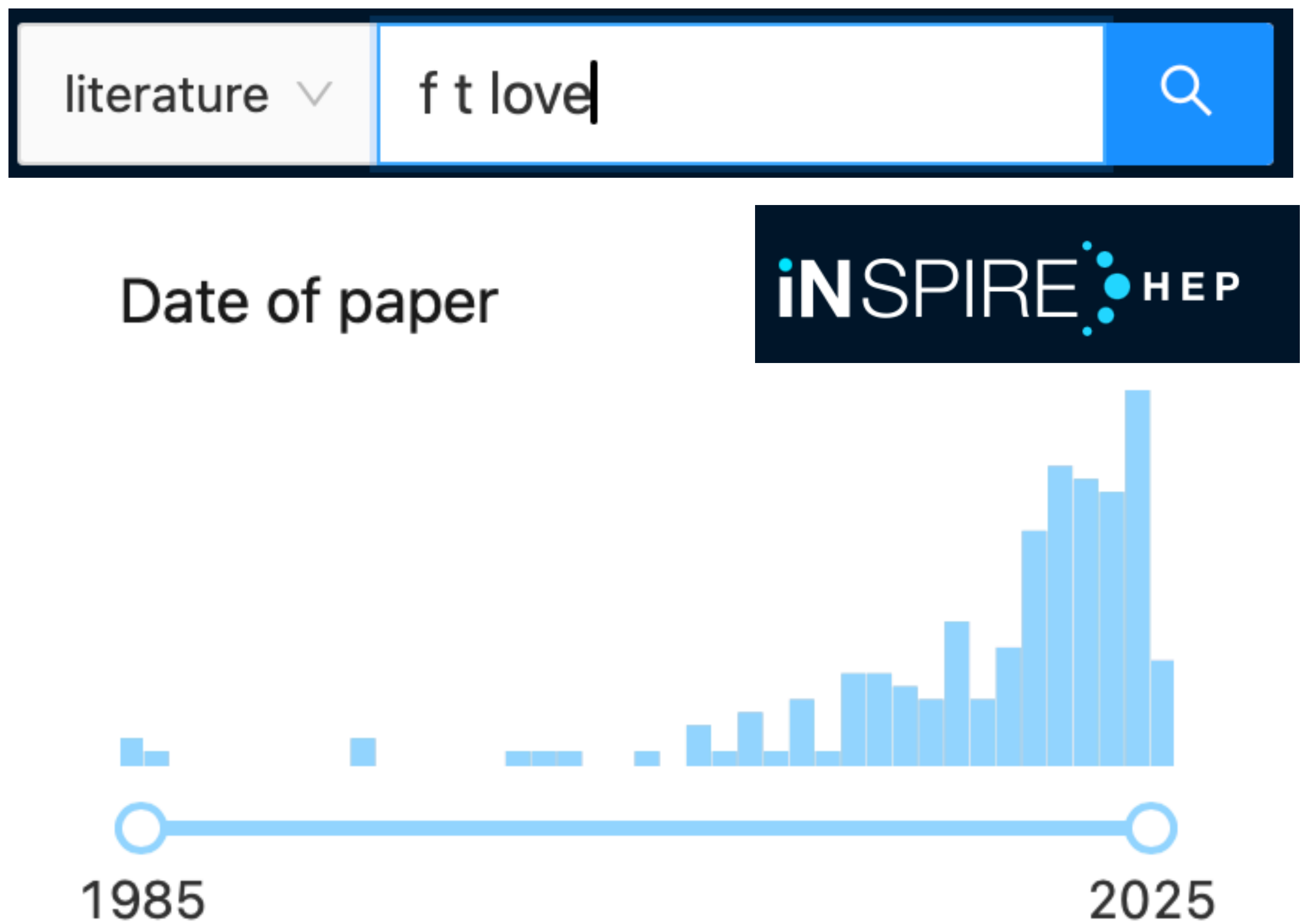
Generically we should expect: $Q_\ell(r_c)$ $\lambda_\ell(r_c)$

$$\mu \frac{dQ_\ell(\omega)}{d\mu} = \gamma_\ell(\omega) Q_\ell(\omega) \qquad \mu \sim 1/r_c$$

This makes the naturalness problem even worse!

Love explosion

Lots of activity but no consensus…



Most proposed explanations only valid at linear order and not in the EFT.
Loopholes abound.

Our proposed solution

[**JPM**, Podo]

Accidental symmetries of static GR

Let's choose a parameterization of the metric adapted to $\omega \ll 1/R$

$$ds^2 = -\, e^{2\phi}\left(dt - A_i dx^i\right)^2 + e^{-2\phi}\gamma_{ij}dx^i dx^j \,.$$

Focusing on static sector, ignoring ∂_t, we can write a magnetic potential

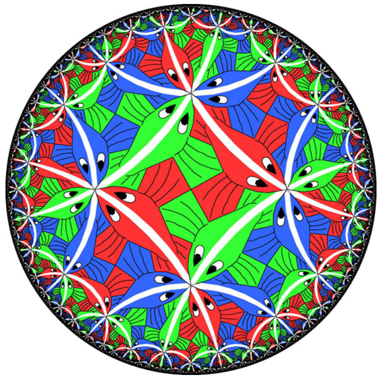
$$B_i = \frac{1}{2}\epsilon^{ijk}\partial_j A_k \sim \partial^i a$$

And do a change of variables/field redef.

$$z = a + ie^{2\phi} \qquad\qquad w = i\frac{z-i}{z+i}$$

Accidental symmetries of static GR

$$S_{\mathrm{EH}} = -\int d^4x\sqrt{\gamma}\ \left(-R^{(3)}[\gamma] + 2\frac{\partial_i w\, \partial^i \overline{w}}{(1-\overline{w}w)^2} + \mathcal{O}(\partial_t^2) \right)$$



Has full $PSL(2,\mathbb{R})$ isometries of the hyperbolic disk!

$$w \to \frac{Aw+B}{\bar{B}w+\bar{A}} \qquad \text{with } |A|^2 - |B^2| = 1$$

This was known in the work of Ehlers, Geroch and others going back more than 50 years as a solution-generating symmetry.

It is actually an off-shell accidental symmetry of static GR!

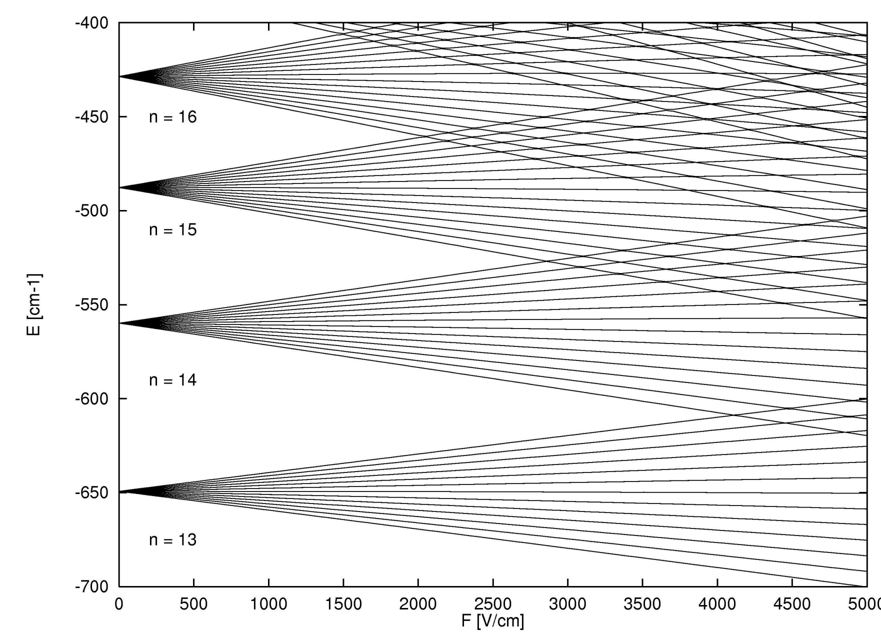
Spurions

Can upgrade couplings to "spurion" fields to restore symmetry

c.f. Stark effect $\Delta H = -QzE = -Q\vec{r}\cdot\overrightarrow{E}$

Use symmetry to derive selection rules

$$\langle n|\vec{r}|m\rangle \propto \overrightarrow{E}$$



Other famous examples:

- Quark masses and chiral symmetry in pion EFT
- Yukawa couplings and flavor symmetry in Standard Model

Spurionic mass

We focus on a simple subgroup of $PSL(2,\mathbb{R})$: $\quad w \to -w$

In GR ("the UV") this is broken spontaneously by the mass

$$w = \frac{-M}{r\left(1+\frac{M^2}{4r^2}\right)} \xrightarrow{w\to -w} \frac{M}{r\left(1+\frac{M^2}{4r^2}\right)} \xrightarrow{M\to -M} \frac{-M}{r\left(1+\frac{M^2}{4r^2}\right)}$$

In the EFT ("the UV") this is broken explicitly by the mass, and hence multipoles and and tides $\quad Q_\ell \sim MR^\ell \qquad \lambda_\ell \sim MR^{2\ell}$

$$S = \int d\tau \left(M\, w(x) + Q_{ij}\partial_i\partial_j w + \lambda_2(\partial_i\partial_j w)^2 - (c\,.\,c) + \cdots \right)$$

Can restore symmetry be treating mass as spurion! $\quad M \to -M$

Vanishing non-linear tides

[**JPM**, Podo]

Spurionic symmetry enforces vanishing of multipoles and tides, even non-linearly!

$$\delta w \sim w^{\rm src}\left(1 + \cdots + \frac{M^{2\ell+1} \text{ or } \lambda_\ell}{r^{2\ell+1}} + \cdots\right) + \cdots$$

$$+ \left(w^{\rm src}\right)^n \left(1 + \cdots + \frac{M^{(n+1)\ell+1} \text{ or } \lambda_{n\ell}}{r^{(n+1)\ell+1}} + \cdots\right) + \cdots$$

Full argument requires also γ_{ij} response.

Conclusion:

"Black holes do not polarize, no matter how strong a external static field is applied"

*static = time-indep

Extensions

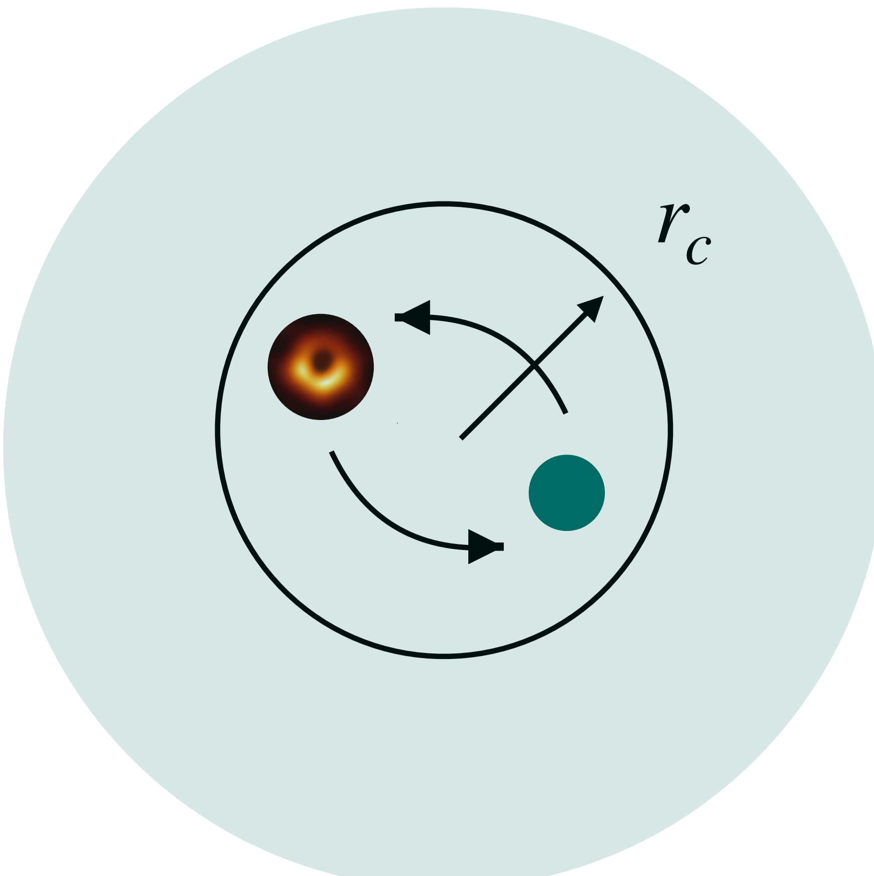
- Vanishing tides (only a subset) in $D > 4$
- Vanishing tides for theories with matter — in 1-to-1 correspondence with no-hair theorem
- Vanishing tides for Taub-NUT solutions
- Vanishing tides for Reissner-Nordstrom with em charges
- Non-vanishing in EFT extensions, (A)dS, fermionic and charged perturbations (in agreement with known results)
- Spinning black holes WIP

Tides and the Renormalization Group

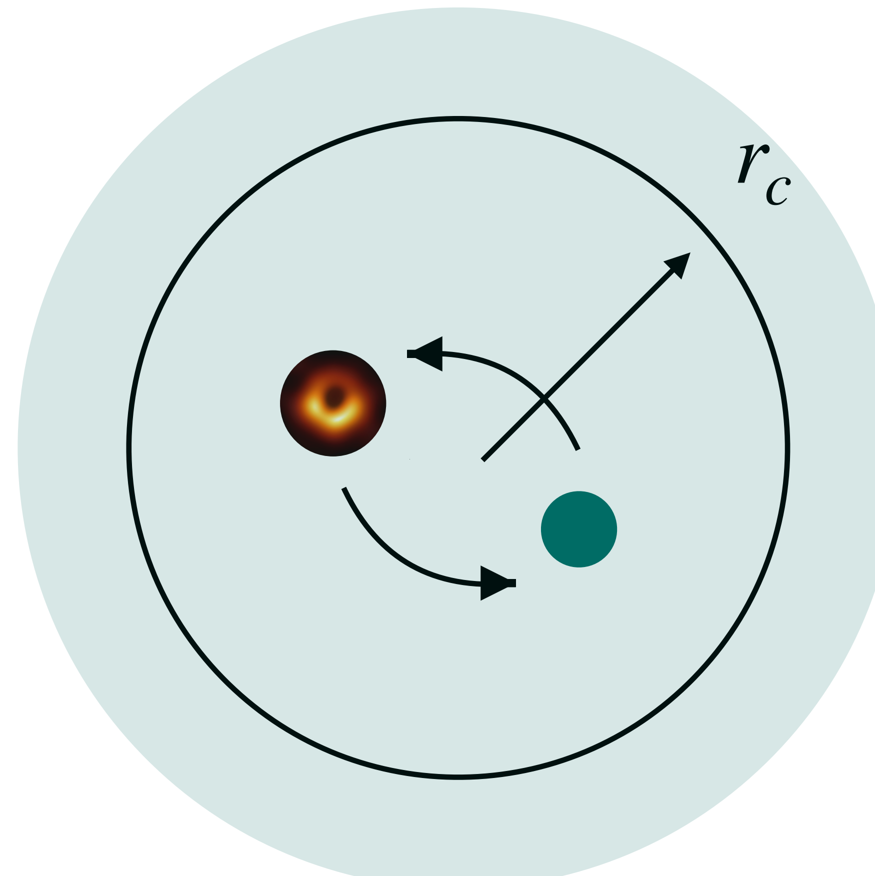
  

[Ivanov, Li, **JPM**, Zhou]

Renormalization of multipoles



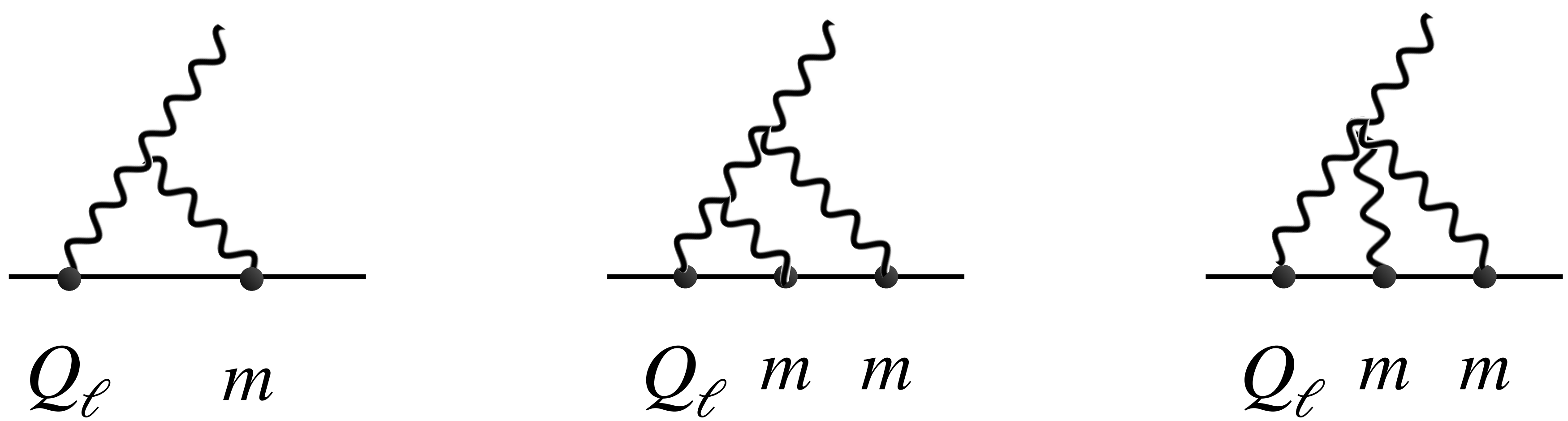
vs.



$$Q_\ell^{\text{ren.}}(\omega) = \left(\omega r_c\right)^{\gamma_\ell(\omega)} Q_\ell(\omega, \mu)$$

Binary tails: dissipative

Gravitational waves interact with potential on their way to the observer



These give rise to logarithms in the spectral waveform

$$h_{\ell m}(\omega) \supset (GE\omega)^{n+k} \log^k \omega$$

$n = 0$ IR tails

$n > 0$ UV tails

This is a classical RG, how much of it can we understand?

$$\mu \frac{dQ^{\text{ren.}}_{\ell m}(\omega)}{d\mu} = \gamma_{\ell m}(\omega) Q^{\text{ren.}}_{\ell m}(\omega)$$

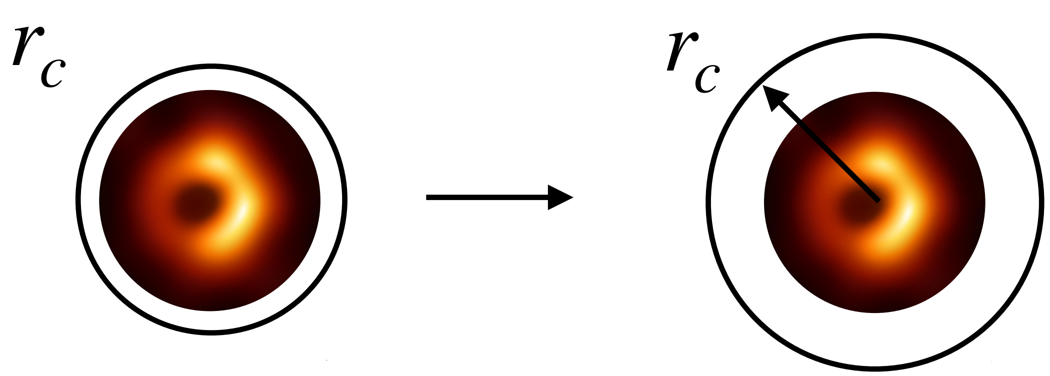
$(\mu \sim 1/r_c)$

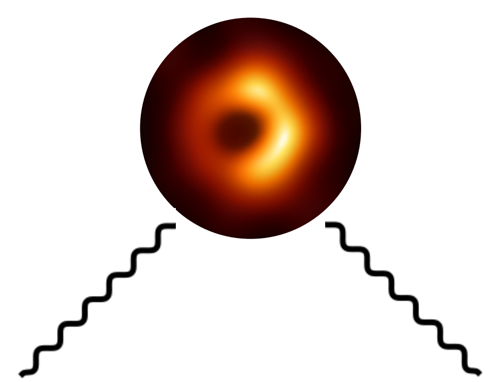
New non-perturbative relation

Anomalous dimension from phase shift: [Ivanov, Li, **JPM**, Zhou]

$$\boxed{\gamma_\ell(\omega) = -\frac{1}{\pi}\Big(\delta_\ell(\omega) + \delta_\ell(-\omega)\Big)}$$

"B2B for tails"





Thanks to this we will compute the all-orders running of BH multipoles.

γ from δ

[Ivanov, Li, **JPM**, Zhou]

Instead consider symmetric Green's function $\quad O(\tau) = Q_\ell E^\ell$

$$G_S(\omega) = \tfrac{1}{2}\langle \mathrm{BH} \,|\, \{O(\omega), O(-\omega)\} \,|\, \mathrm{BH}\rangle \sim \mathrm{Im} G_R$$

Analyticity + dim. Analysis:

$$e^{i\pi D} G_S(\omega) = G_S(e^{i\pi}\omega) = G_S(\omega)^* = \tfrac{1}{2}\langle \mathrm{BH} \,|\, \{O^\dagger(\omega), O^\dagger(-\omega)\} \,|\, \mathrm{BH}\rangle$$

Unitarity: $\quad O^\dagger = S^\dagger O S^\dagger \qquad G_S(\omega)^* = e^{-2i(\delta(\omega)+\delta(-\omega))} G_S(\omega)$

$$\boxed{\gamma_\ell(\omega) = -\frac{1}{\pi}(\delta_\ell(\omega) + \delta_\ell(-\omega))}$$

Inspired by: [Caron-Huot, Wilhelm]

Scattering phase-shift in BHPT

Scattering amplitude:

$$f_s(\theta) = \frac{2\pi}{i\omega} \sum_{\ell=s}^{\infty} {}_{-s}S_\ell^s(1, a\omega)\, {}_{-s}S_\ell^s(\cos\theta, a\omega)(\eta_{\ell s} e^{2i\delta_{\ell s}} - 1)$$

Phase shifts receive contributions from "near zone" ($r \sim R_s$) and "far zone" $r \gg R_s$

$$\delta_{\ell s} = \delta_{\ell s}|^{NZ} + \delta_{\ell s}|^{FZ}$$

Contains information about tides starting at $\mathcal{O}(G^{2\ell+1})$

Computable in EFT, modulo counterterms

Wave scattering off BH

Regge-Wheeler/Teukolsky equation $\quad \Box_{BH}\, h_{\mu\nu} = 0$

$$\Delta^{-s}\frac{d}{dr}\left(\Delta^{s+1}\frac{d}{dr}\,{}_sU_{\ell m}(r)\right) + V_{eff}(r)\,{}_sU_{\ell m}(r) = 0$$

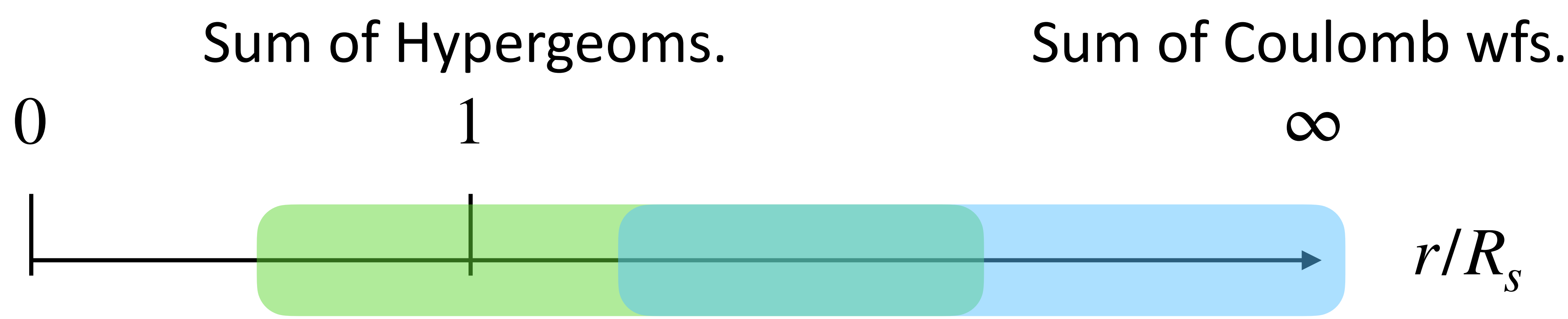
incoming b.c. $\quad {}_{-s}U_{\ell m}(r) \to B^{(\mathrm{inc})}_{-s\ell m} r^{-1} e^{-i\omega r} + B^{(\mathrm{refl})}_{-s\ell m} r^{-1+2s} e^{i\omega r}$

"Connection coefficients for
the confluent Heun equation"

$$\eta_{\ell s} e^{2i\delta_{\ell s}} \sim \frac{B^{(\mathrm{refl})}_{-s\ell m}}{B^{(\mathrm{inc})}_{-s\ell m}}$$

Variou methods

1. Matched asymptotic expansions (improved) [Mano, Suzuki, Takasugi]



Match both asymptotic series to determine coefficients.

2. Relation to Seiberg-Witten theory: [Aminov, Grassi, Hatsuda; ...]

Radial Teukolsky equation $\Delta^{-s}\frac{d}{dr}\left(\Delta^{s+1}\frac{d}{dr}{}_sU_{\ell m}(r)\right) + V_{eff}(r)\,{}_sU_{\ell m}(r) = 0$

=

Quantum Curve for $SU(2)$ $\mathcal{N} = 2$ SYM with $N_f = 3$ hypers

Scattering Solution

[Bautista, Bonelli, Iossa, Tanzini, Zhou]

Coulomb phase

TS constants

Renormalized ℓ

$$\delta_\ell|^{FZ} = \epsilon \log(2|\epsilon|) - \frac{1-\kappa}{2}\epsilon + \frac{1}{2}\mathrm{Im}\partial_{m_3}F + \frac{1}{2}\mathrm{Arg}[A_s^P] + \frac{1}{2}\mathrm{Arg}\frac{\Gamma(1+\nu-m_3)}{\Gamma(1+\nu+m_3)} + \boxed{\frac{\pi}{2}(\ell-\nu)}$$

NS function

NS function, F, can be computed combinatorially, and also A-period

$$a^2 - \frac{1}{4} = \nu(\nu+1) = L\partial_L F(\nu) - u$$

"Matone relation"

$f(m, \chi, s)$

Near zone later.

A ν perspective

$$\gamma_\ell^{\mathrm{BH}}(\omega,\chi) = -\frac{1}{\pi}(\delta_\ell^{\mathrm{BH}}(\omega,\chi) + \delta_\ell^{\mathrm{BH}}(-\omega,\chi))$$

Plugging in we find that this is precisely the renormalized angular momentum!

$$\boxed{\gamma_\ell^{\mathrm{BH}}(\omega,\chi) = \nu(\omega,\chi) - \ell}$$

[Ivanov, Li, **JPM**, Zhou]

MST called $\nu(\omega)$ the "renormalized angular momentum" but nobody knew what it actually renormalized.

Now we do! The BH multipoles!

Perturbative expansion

Can be computed recursively to any order

$$\nu_2 = -\frac{2\left(15\lambda^2 + 13\lambda + 24\right)}{(2\ell+1)\ell(\ell+1)(4\ell(\ell+1)-3)}\,, \tag{S7}$$

$$\nu_3 = \frac{8m\chi\left(5\lambda^3 - \lambda^2 + 18\lambda + 108\right)}{(\ell-1)\ell^2(\ell+1)^2(\ell+2)(2\ell-1)(2\ell+1)(2\ell+3)}\,, \tag{S8}$$

$$\nu_4 = \frac{2(-18480\lambda^8 + 61320\lambda^7 - 2415\lambda^6 + 85775\lambda^5 + 123233\lambda^4 + 51522\lambda^3 - 953424\lambda^2 + 102816\lambda + 51840)}{(\ell-1)\ell^3(\ell+1)^3(\ell+2)(2\ell-3)(2\ell-1)^3(2\ell+1)^3(2\ell+3)^3(2\ell+5)}\,, \tag{S9}$$

$$\nu_5 = \frac{48m\chi(3696\lambda^9 - 13944\lambda^8 + 18347\lambda^7 - 22136\lambda^6 - 42625\lambda^5 - 145050\lambda^4 - 650274\lambda^3 + 1450620\lambda^2 - 125064\lambda - 77760)}{(\ell-1)^2\ell^4(\ell+1)^4(\ell+2)^2(2\ell-3)(2\ell-1)^3(2\ell+1)^3(2\ell+3)^3(2\ell+5)} \tag{S10}$$

$$\begin{aligned}\nu_6 = -&\frac{4}{(\ell-1)^2\ell^5(\ell+1)^5(\ell+2)^2(2\ell-5)(2\ell-3)^2(2\ell-1)^5(2\ell+1)^5(2\ell+3)^5(2\ell+5)^2(2\ell+7)}\Big[104552448\lambda^{15} \\ &- 1671301632\lambda^{14} + 8204035840\lambda^{13} - 15243669056\lambda^{12} + 13732238520\lambda^{11} - 12944646946\lambda^{10} - 13002690896\lambda^9 \\ &- 24635974293\lambda^8 + 887441317\lambda^7 + 30247168320\lambda^6 + 680072616180\lambda^5 - 1013061463920\lambda^4 + 111802065696\lambda^3 \\ &+ 82127701440\lambda^2 - 12975033600\lambda - 3919104000\Big]\,,\end{aligned} \tag{S11}$$

$$\begin{aligned}\nu_7 = &\frac{48m\chi}{(\ell-2)(\ell-1)^3\ell^6(\ell+1)^6(\ell+2)^3(\ell+3)(2\ell-5)(2\ell-3)^2(2\ell-1)^5(2\ell+1)^5(2\ell+3)^5(2\ell+5)^2(2\ell+7)} \\ \times &\Big[74680320\lambda^{17} - 1644797440\lambda^{16} + 13439345920\lambda^{15} - 53051339968\lambda^{14} + 115693152168\lambda^{13} - 153954147622\lambda^{12} \\ &+ 104796913232\lambda^{11} - 46104555329\lambda^{10} + 51933011989\lambda^9 + 352999107060\lambda^8 - 571463718576\lambda^7 \\ &+ 11287693868616\lambda^6 - 33483100996872\lambda^5 + 28193417777664\lambda^4 - 1752702484032\lambda^3 - 2381917337280\lambda^2 \\ &+ 293009011200\lambda + 105815808000\Big]\,,\end{aligned} \tag{S12}$$

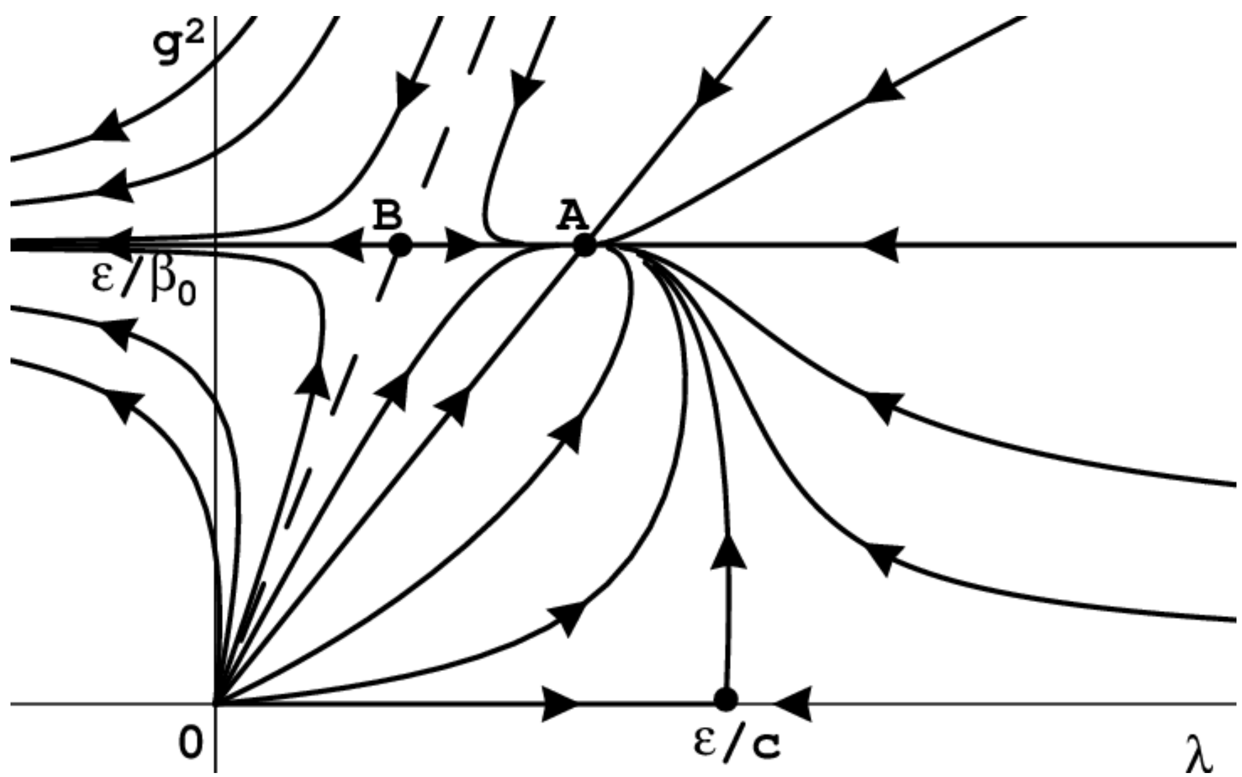
$$\gamma_{\ell m} = \sum_{\ell} \nu_\ell \epsilon^n$$

$$\lambda = \ell(\ell+1)$$

Universality

RG & Universality

Key insight from RG: Universality = Only a few "relevant" couplings



Same in the gravitational case: only mass, m, and spin, S are relevant in the technical sense (c.f., "no hair")

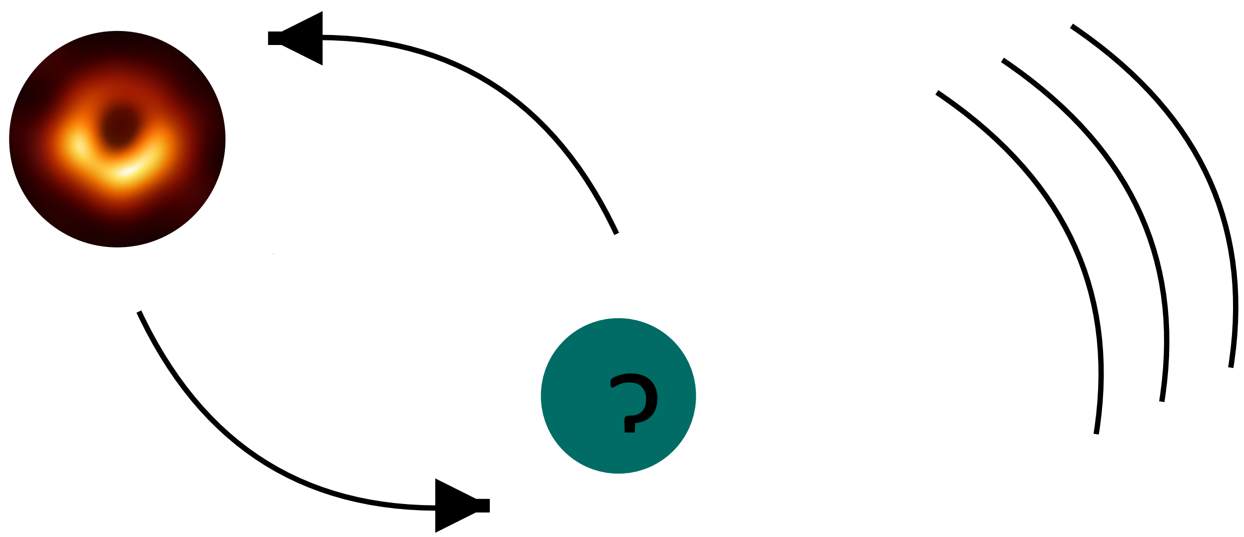
Universality

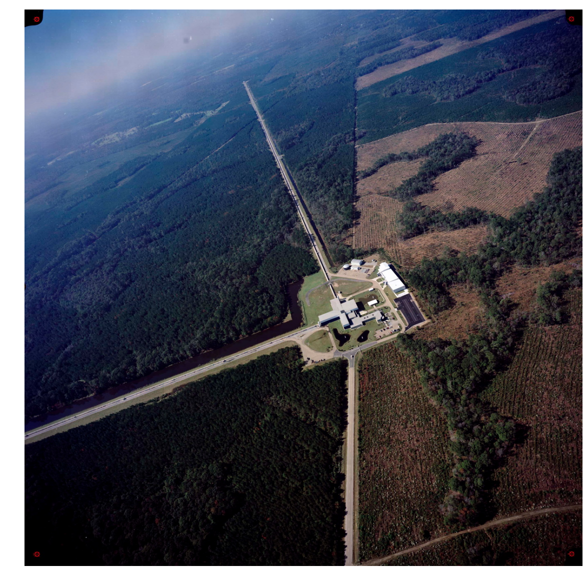
$$S = \int d\tau \, \Big[\underbrace{E(\tau) + \omega_i L^i}_{\text{Universal!}} + \underbrace{\mathcal{O}(L^2) + \mathcal{O}(R^5)}_{\text{Non-universal}} \Big]$$

Leading tide is $\lambda_2 \sim R^5$, so EFT predicts that we cannot tell apart point particle from BH until at least $O(R^5, RL^2)$

Radiation from a binary

Binary itself is point-like for $\omega \ll 1/r$





(E, L, Q_ℓ)

c.f. quadrupole formula

$$\frac{dE}{d\omega} = \frac{G}{5}\dddot{Q}_2^2 + \cdots$$

Universality

Universality of leading couplings in EFT means anomalous dimensions also contain universal pieces

$$S = \int d\tau \left[E(\tau) + \omega_{ij} L^{ij} + \mathcal{O}(L^2) + \mathcal{O}(R^{2\ell+1}) \right]$$

$$\gamma_\ell^{\mathrm{BH}}(\omega,\chi) = \gamma_\ell^{\mathrm{BH}}(\omega,0) + \chi\,\partial_\chi \gamma_\ell^{\mathrm{BH}}(\omega,0) + \mathcal{O}(\chi^2)$$

Universal up to $\mathcal{O}(G^{2\ell+1})$!

Not universal

With the simple replacements $m \to E$ and $\chi \to L/(GE^2)$!

Running of the quadrupole

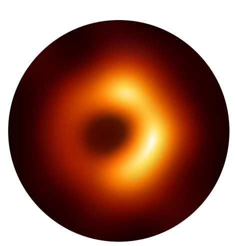
[Ivanov, Li, **JPM**, Zhou]

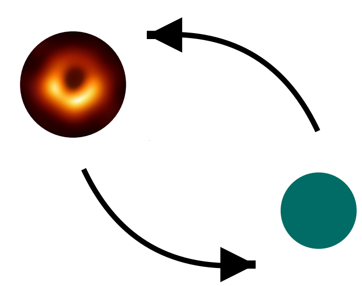
E.g., at low frequency

$$\gamma_{2m}(\omega) = -\frac{214}{115}\epsilon^2 + \frac{2m\chi}{3}\epsilon^3 - \frac{3390466}{1157625}\epsilon^4 + \frac{381863m\chi}{99225}\epsilon^5 + \mathcal{O}(\epsilon^7, \chi^2)$$

With $\epsilon = GE\omega$ and $\chi = L/GE^2$, and omitting non-universal terms

Universal terms of BH anomalous dimension also tells us about the running of generic objects. Including NS and binaries!





Waveform resumation

We can use it to resum logarithms in the multipolar binary waveform itself! This is the observable measured in LVK.

$$h_{\ell m} \sim (r\omega)^{\hat{\nu}(\omega)}\, h^{\text{finite}}_{\ell m} \qquad \hat{\nu}(\omega) = \ell + \gamma^{\text{univ.}}(\omega)$$

For quasi-circular binaries this allows us to propose a formula for "tail-resummed" multipolar waveform (improving upon [Damour, Nagar '09])

$$h_{\ell m} = (-ir\omega)^{\hat{\nu}(\omega)} e^{i2iGE\omega \log(2r\omega) + GE\omega\pi} \frac{\Gamma(\hat{\nu} + 1 - 2iGE\omega)}{\Gamma(\hat{\nu} + 1)} h^{\text{finite}}_{\ell m}$$

[Ivanov, Li, **JPM**, Zhou]

In the probe limit $m_1 \ll m_2$, this agrees to all orders with a recent calculation, which also uses relation to SW theory [Fucito, Morales, Russo]

Comparison with state-of-the-art

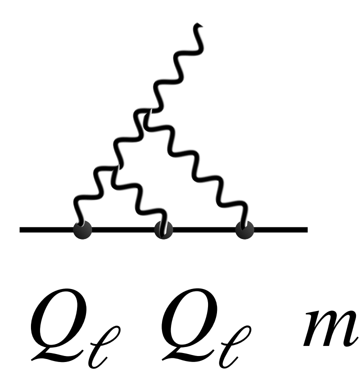
We can compare with state-of-the-art post-Newtonian waveforms used for LIGO/Virgo/Kagra (4PN) [Blanchet et al. 2023]

$$
\begin{aligned}
H_{22} = 1 &+ \left[-\frac{107}{42}+\frac{55}{42}\eta\right]x + 2\pi x^{\frac{3}{2}} + \left[-\frac{2173}{1512}-\frac{1069}{216}\eta+\frac{2047}{1512}\eta^2\right]x^2 + \left[-\frac{107\pi}{21}+\left(\frac{34\pi}{21}-24i\right)\eta\right]x^{\frac{5}{2}} \\
&+ \left[\left(-\frac{428}{105}\log(16x)+\frac{2\pi^2}{3}-\frac{856\gamma_E}{105}+\frac{428i\pi}{105}+\frac{27027409}{646800}\right)+\left(\frac{41\pi^2}{96}-\frac{278185}{33264}\right)\eta-\frac{20261}{2772}\eta^2+\frac{114635}{99792}\eta^3\right]x^3 \\
&+ \left[-\frac{2173\pi}{756}+\left(-\frac{2495\pi}{378}+\frac{14333i}{162}\right)\eta+\left(\frac{40\pi}{27}-\frac{4066i}{945}\right)\eta^2\right]x^{\frac{7}{2}} + \left[\left(\frac{22898\log(16x)}{2205}+\frac{45796\gamma_E}{2205}-\frac{22898i\pi}{2205}-\frac{107\pi^2}{63}\right.\right. \\
&\left.-\frac{846557506853}{12713500800}\right)+\left(\frac{7642}{441}\log(16x)-\frac{336005827477}{4237833600}+\frac{15284\gamma_E}{441}-\frac{219314i\pi}{2205}-\frac{9755\pi^2}{32256}\right)\eta \\
&\left.+\left(\frac{256450291}{7413120}-\frac{1025\pi^2}{1008}\right)\eta^2-\frac{81579187}{15567552}\eta^3+\frac{26251249\eta^4}{31135104}\right]x^4+\mathcal{O}(x^{\frac{9}{2}}),
\end{aligned}
$$

$$x \sim v^2 \sim (GM\omega)^{2/3}$$

$$\eta = \frac{m_1 m_2}{(m_1+m_2)^2}$$

“tails-of-memory”



Comparison with state-of-the-art

Same for other values of (ℓ, m), e.g.,

$$H_{33} = -\frac{3}{4} i \sqrt{\frac{15}{14}} \sqrt{1-4\eta} \left[\sqrt{x} + [-4+2\eta]\, x^{\frac{3}{2}} + \left[3\pi + 6i \log\left(\frac{3}{2}\right) - \frac{21i}{5} \right] x^2 + \left[\frac{123}{110} - \frac{1838}{165}\eta + \frac{887}{330}\eta^2 \right] x \right.$$
$$+ \left[-12\pi - 24i \log\left(\frac{3}{2}\right) + \frac{84i}{5} + \left(\frac{9\pi}{2} + 9i\log\left(\frac{3}{2}\right) - \frac{48103i}{1215} \right)\eta \right] x^3 + \left[\left(-\frac{39}{7}\log(16x) + \frac{3\pi^2}{2} - \frac{78\gamma_E}{7} \right.\right.$$
$$\left.\left.\left. + 6i\pi \left(3\log\left(\frac{3}{2}\right) - \frac{41}{35} \right) - 18\log^2\left(\frac{3}{2}\right) + \frac{19388147}{280280} \right) + \left(\frac{41\pi^2}{64} - \frac{7055}{3432} \right)\eta - \frac{318841}{17160}\eta^2 + \frac{8237}{2860}\eta^3 \right] x^{\frac{7}{2}} \right],$$

$$H_{31} = i \frac{\sqrt{1-4\eta}}{12\sqrt{14}} \left[\sqrt{x} + \left[-\frac{8}{3} - \frac{2}{3}\eta \right] x^{\frac{3}{2}} + \left[-\frac{7i}{5} + \pi - 2i\log(2) \right] x^2 + \left[\frac{607}{198} - \frac{136}{99}\eta - \frac{247}{198}\eta^2 \right] x^{\frac{5}{2}} \right.$$
$$+ \left[\left(\frac{56i}{15} - \frac{8\pi}{3} + \frac{16}{3} i \log(2) \right) + \left(-\frac{i}{15} - \frac{7\pi}{6} + \frac{7}{3} i \log(2) \right)\eta \right] x^3 + \left[-\frac{13\log(16x)}{21} + \frac{\pi^2}{6} - \frac{26\gamma_E}{21} - \frac{82i\pi}{105} \right.$$
$$\left.\left. -2\log^2(2) - 2i\pi\log(2) - \frac{164\log(2)}{105} + \frac{10753397}{1513512} + \left(\frac{41\pi^2}{64} - \frac{1738843}{154440} \right)\eta + \frac{327059}{30888}\eta^2 - \frac{17525}{15444}\eta^3 \right] x^{\frac{7}{2}} \right].$$

Resums most transcendental parts! Predictions to all orders in G!

Conclusion

- Black holes have remarkable properties, which hopefully will be experimentally tested in near future

- Apparent puzzles about their static tides can be resolved using EFT toolset

- Dynamical tides run and their running is related to simpler scattering process. Exact running for BH!

- Leveraging universality, one can translate BH results to understand signals from binary mergers!

Thank you!