Towards delensing of BICEP with SPT-3G

J. Carron, with SPT-3G lensers (K. Wu, Y. Omori, …), SPT-3G and SPO

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1. Broader context — BICEP + SPT-3G

   and then a couple of new things from early work on these maps:

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   Reinecke & JC in prep.

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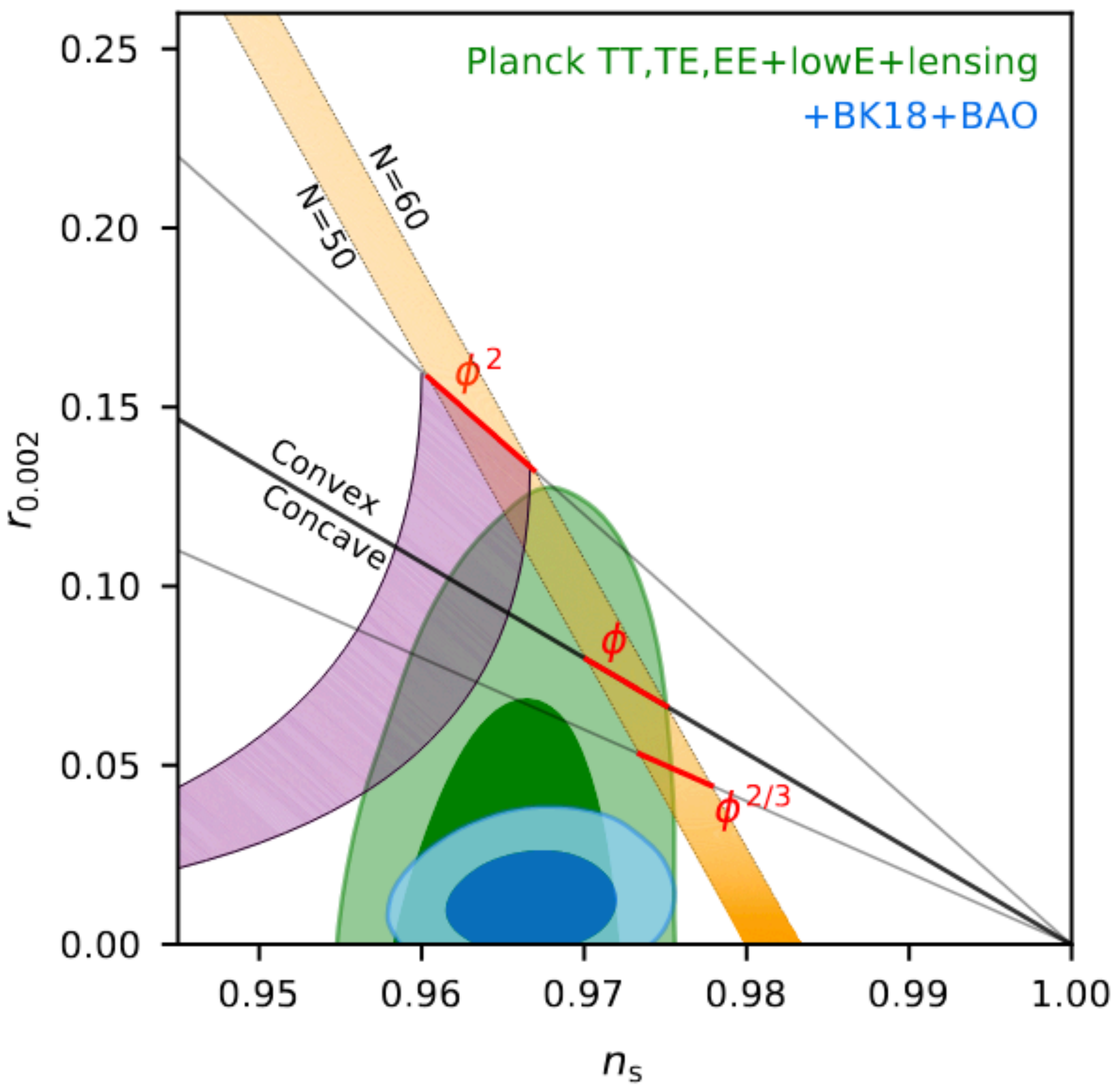
   JC 2025

Characterising inflation: BICEP/Keck r-n_s plot

BK18 2110.00483

+ BK18

+BAO



$$\text{BK18}: \sigma(r_{0.05}) = 0.009, \quad r_{0.05} < 0.036\ (95\ \%\ \text{c.l.}),$$

Characterising inflation: BICEP/Keck r-n_s plot

BK18 2110.00483

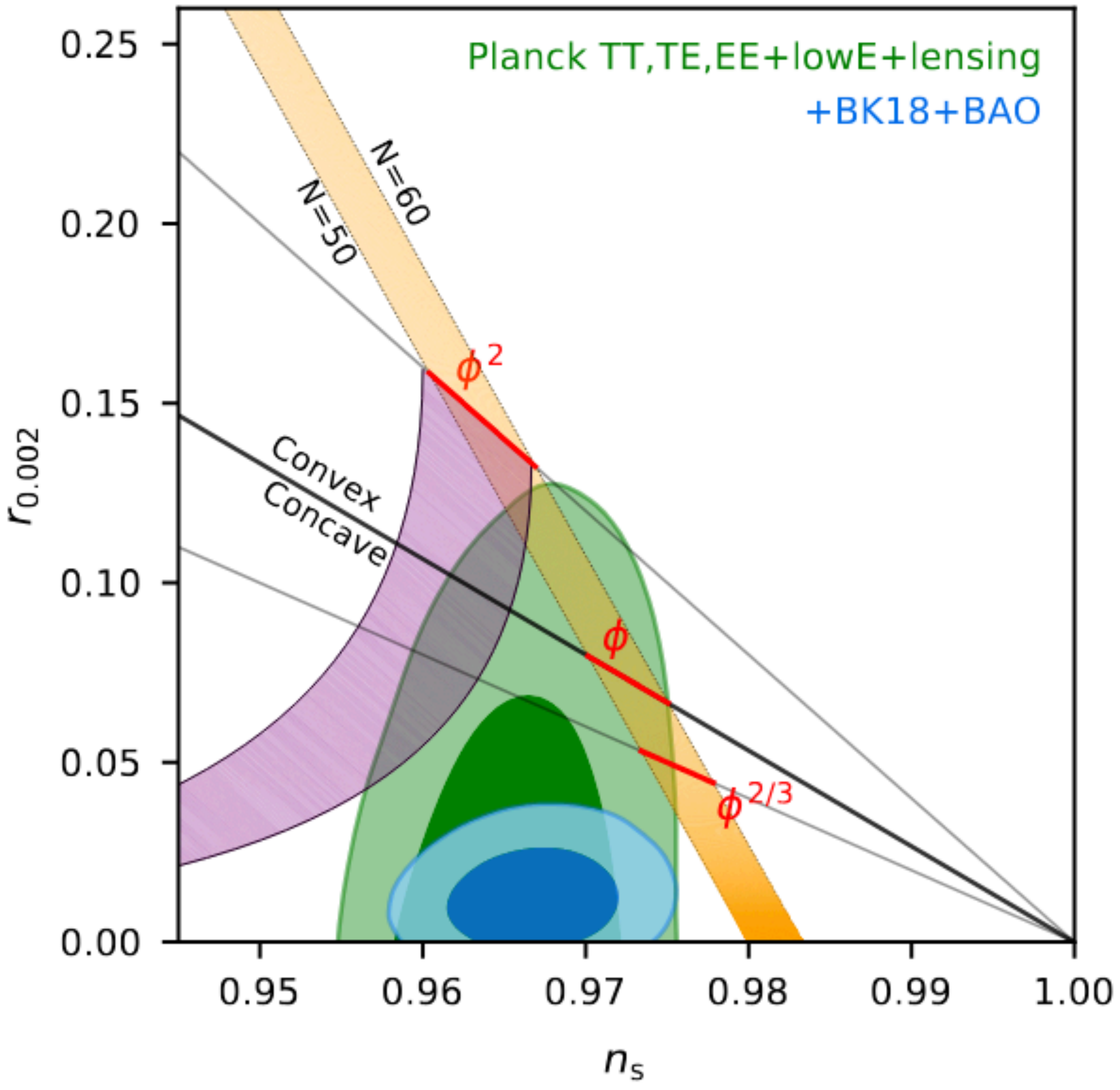
- CMB observations will tighten n_s significantly from wide-area CMB spectra (SPT-3G Ext-10k, and then Simons Observatory)

+ BK18

- But now, $\sigma(r)$ is dominated by lensing B-mode variance. (no-lensing $\sigma(r)$ from BK18 would be a factor of 2 tighter)

+BAO

- BK can only progress by removing the lensing signal in some way

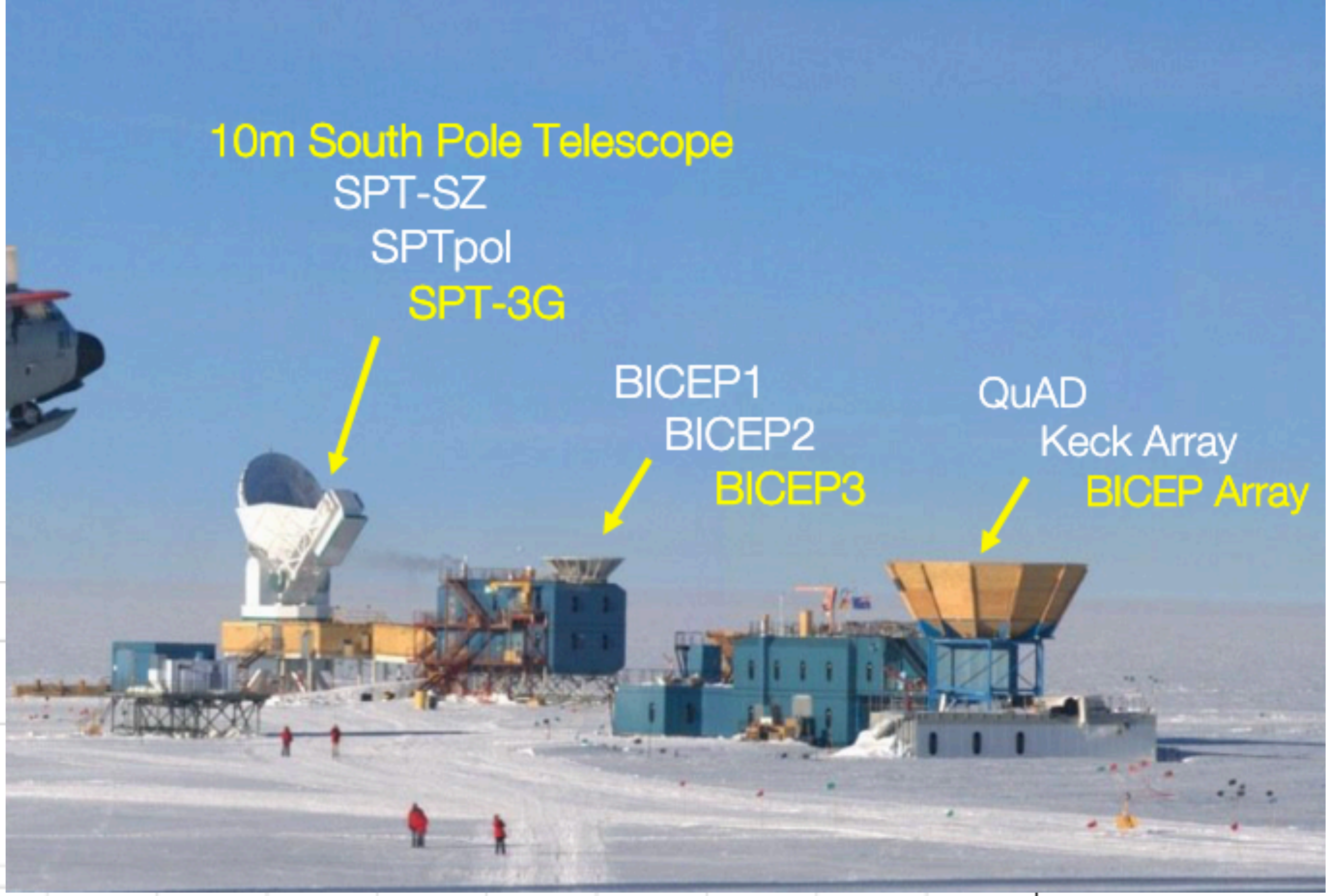


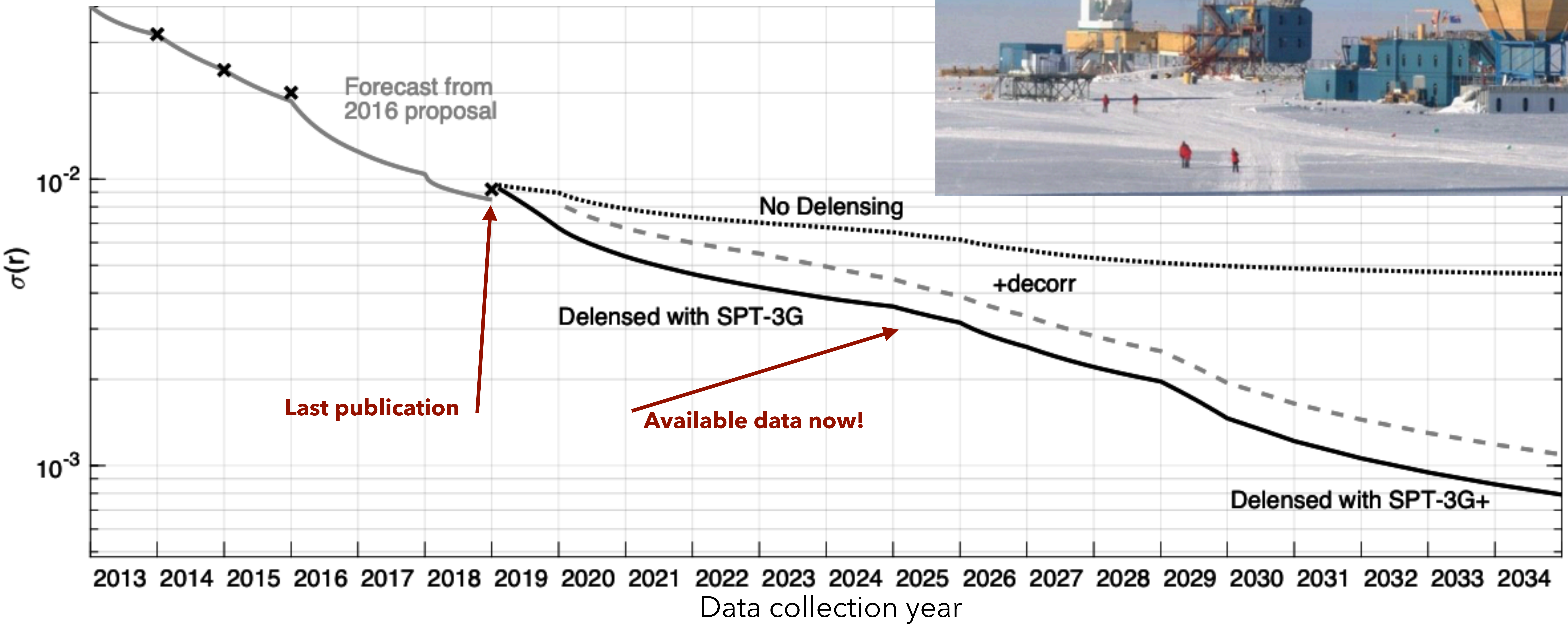
BK18 : $\sigma(r_{0.05}) = 0.009, \quad r_{0.05} < 0.036$ (95 % c.l.), The South Pole Observatory **(SPO)**

$\sigma(r) \sim 0.009$ published (BK only)

$\sigma(r) \sim 0.003$ achievable with current data with SPT-3G

$\sigma(r) \sim 0.001$ by 2034





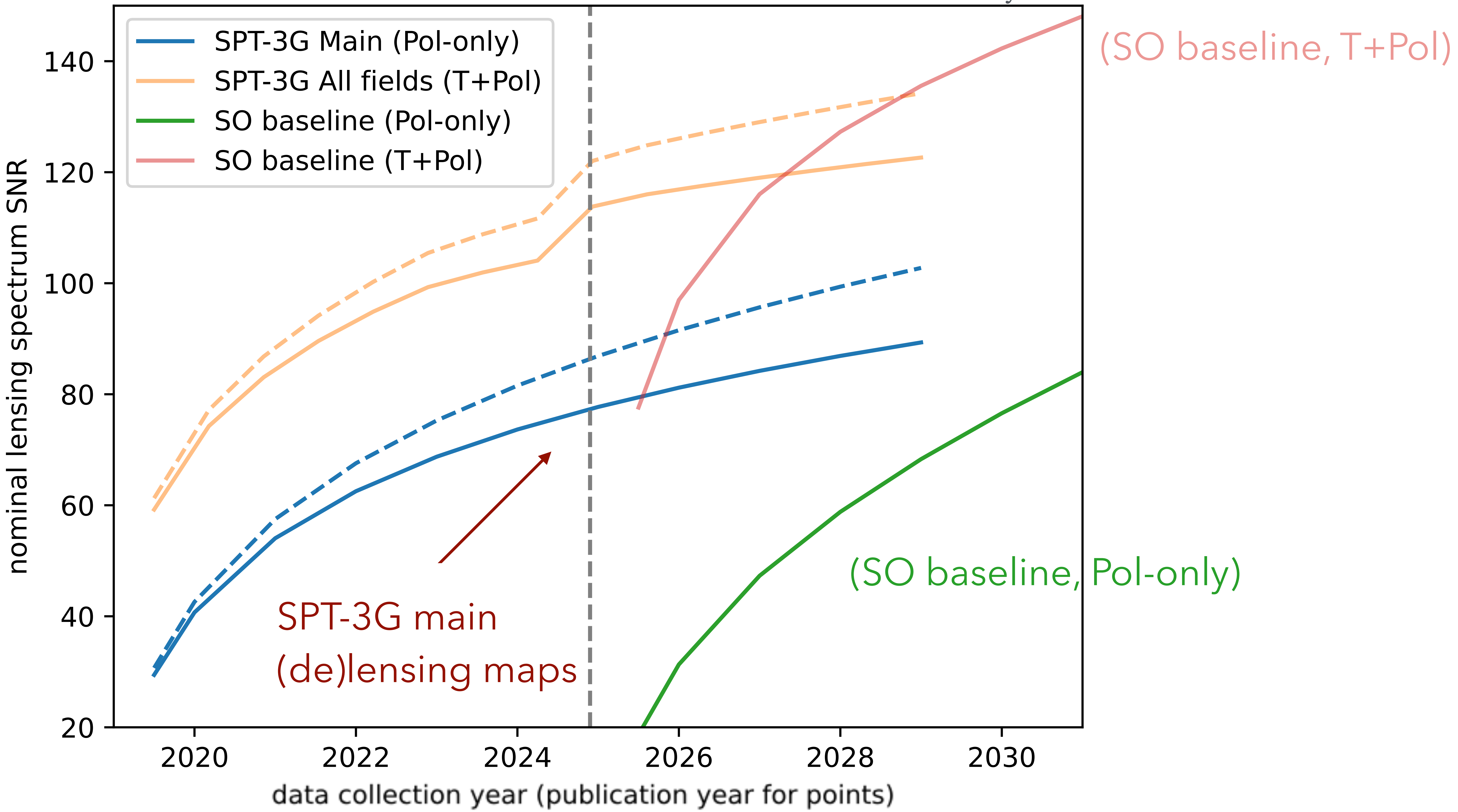
BICEP

Lensing spectrum SNR in the near future

(Not official in any sort of ways)

SO: Simons Observatory f_{sky} SO $= 40\,\%$

f_{sky} SPT-3G Main $= 3.6\,\%$



SPT-3G 5yrs Main field lensing

- Main field $\hat{\kappa}^{EB}$ is in principle the theory-land superstar estimator.

  It is the most powerful for deep data, and often sold as the most robust (since only sensitive to shear, it is immune to magnification-alike systematics). It is also the easiest in practical terms.

- Improves significantly with more Main field data, since reconstruction noise not dominated by primordial fluctuations like TT

- EB QE eventually gets limited by B lensing power, but this can be de-lensed (Beyond-QE techniques). 30% to 20% improvements in reconstruction noise for SPT-3G 5 yrs data.

- 5yrs nominal lensing spectrum SNR pred (for beyond-QE, EB-only!) is ~66 ($\geq$ Planck + ACT + SPT-3G lensing just out 2504.20038)

2. Fast general Spherical Harmonic Transforms on small sky fractions

JC & Reinecke, in prep.

(Meaning to or from arbitrary points)

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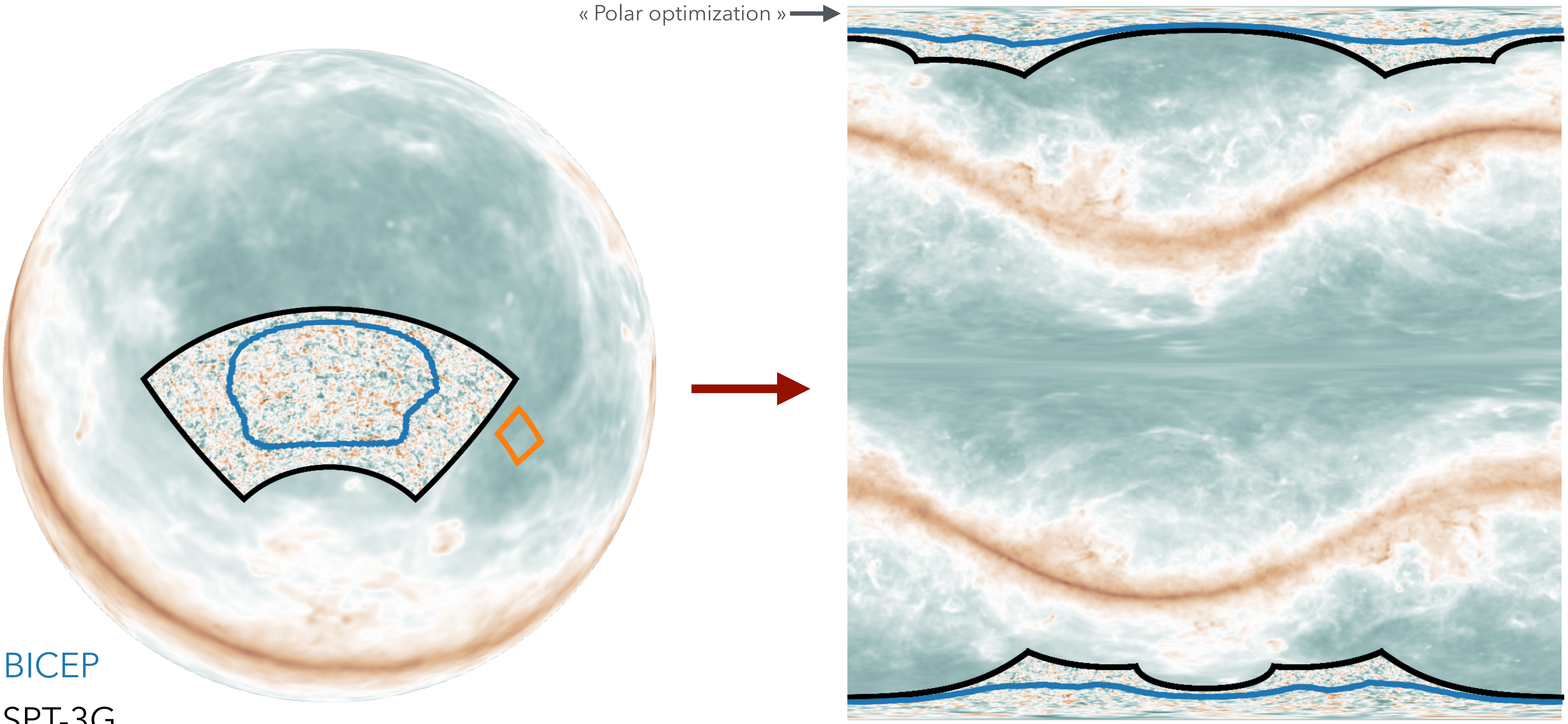
Motivations

- CMB-(de-)lensing work has loads of high-resolution SHTs to perform.
- In the coming years, CMB r-constraints will come from small sky fractions.
- So…. let's speed-up the SHTs. Generic purpose code.

Step 1: Doubled Fourier Sphere (Reinecke, Belkner et Carron 2023, Basak et al 2009) + « Polar optimization »

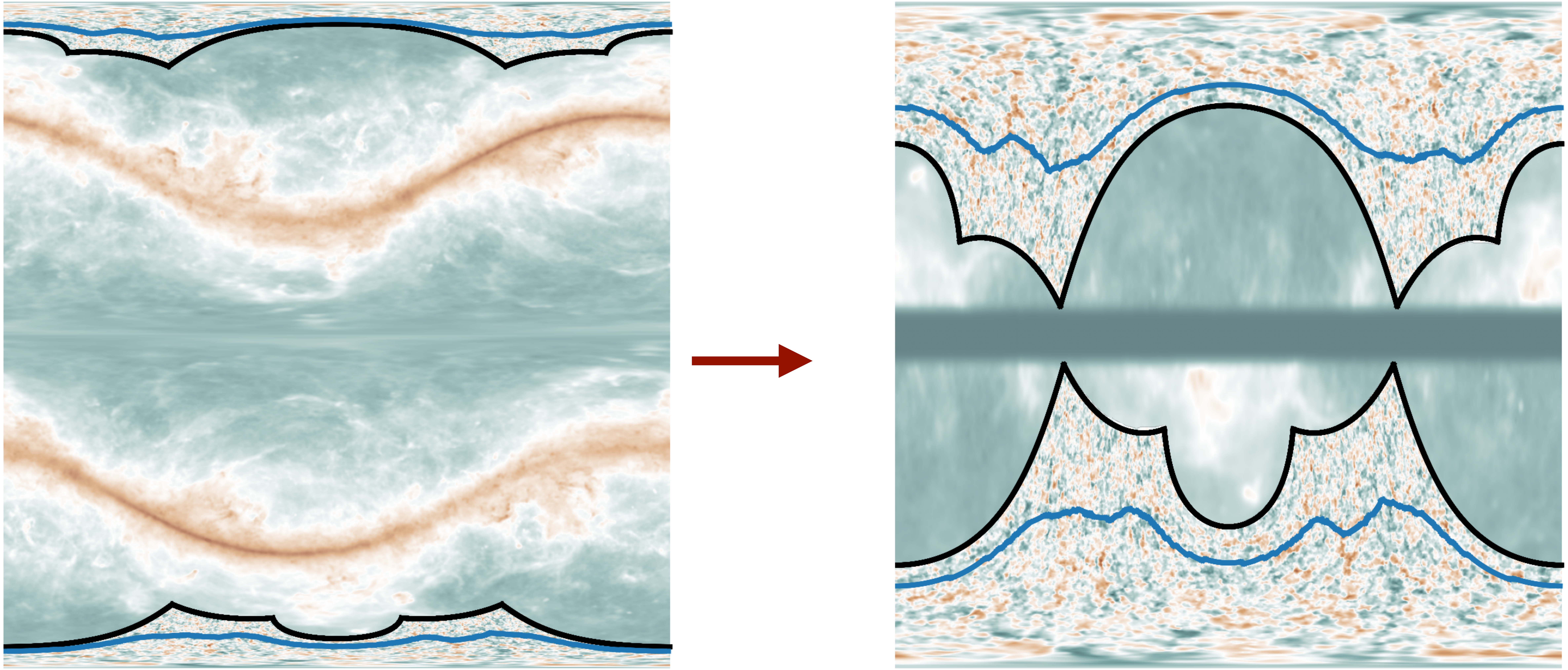


BICEP

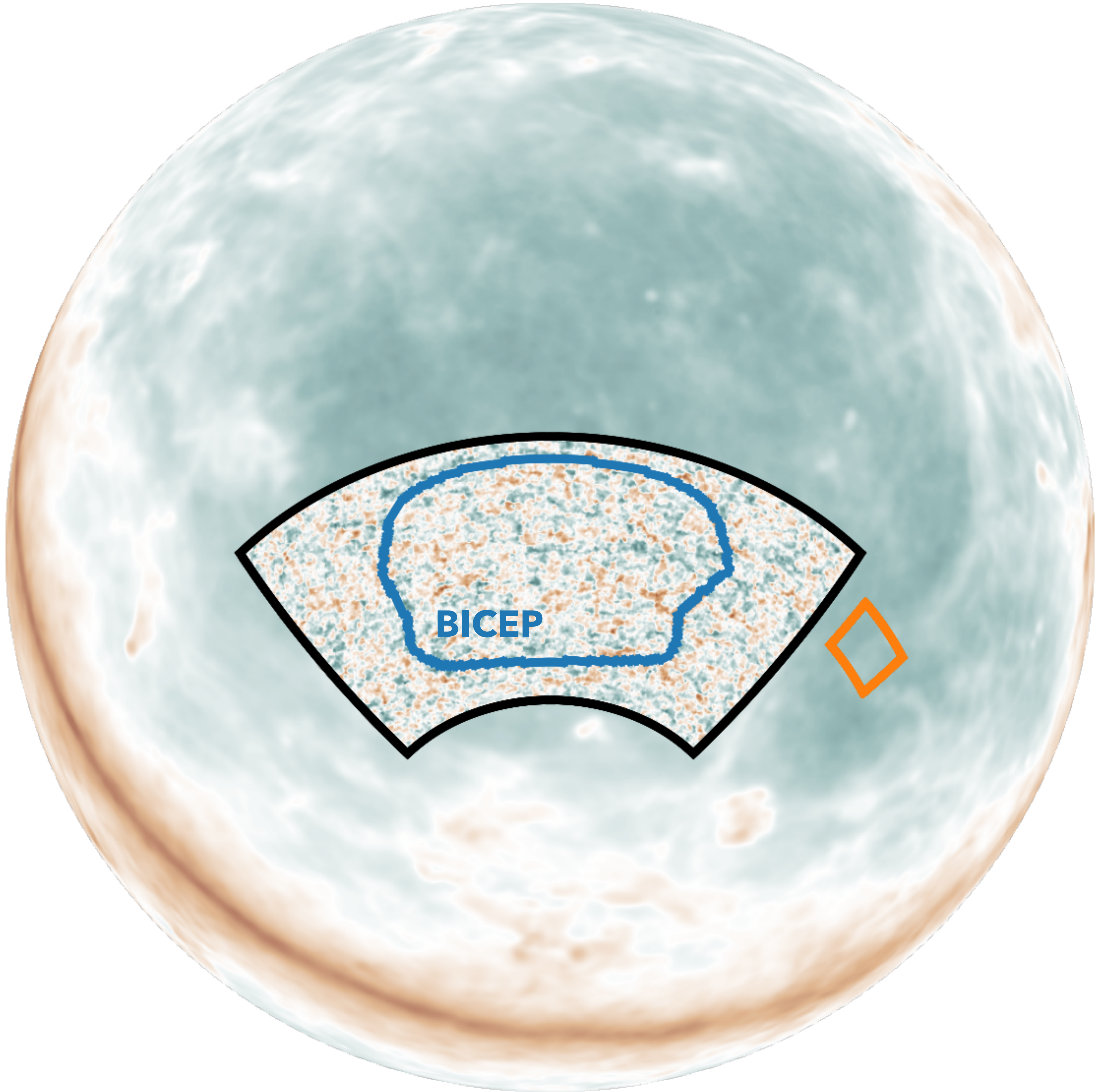
SPT-3G

EUCLID deep field

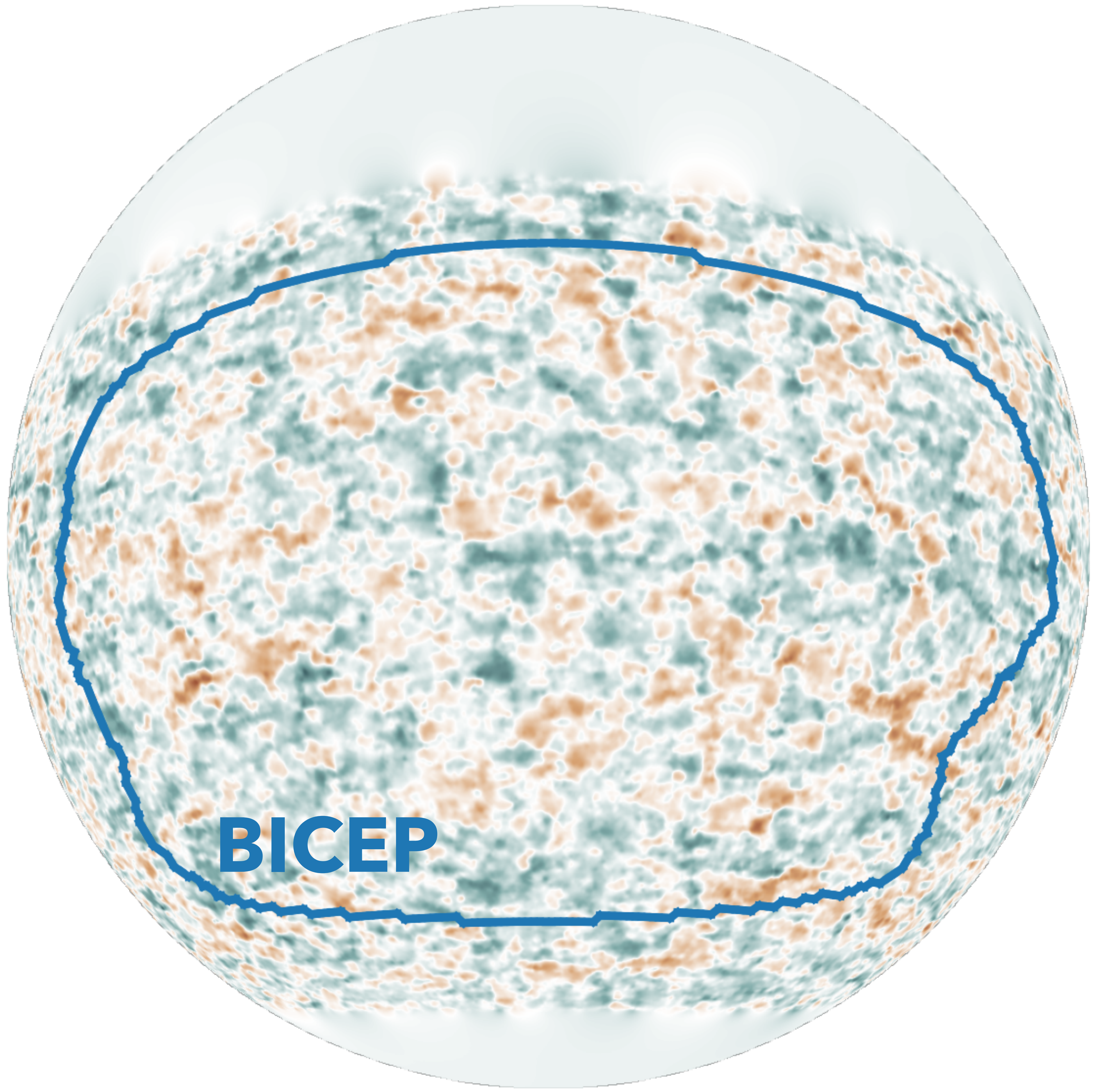
Step 2: Magnifying and cross-fading/weighting



So, for practical purposes, …



$\approx$



SHT numerical cost typically dominated by the « effective number of required Legendre transforms » (rings).

For a band-limit $\ell_{\max}$, this number is $\simeq \ell_{\max}$, and this is brought down now to $\underline{\ell_{\max}\left(\frac{\theta^*}{\pi}\right)(1+\epsilon)^2}$

speed-up factor comes from this, and polar optimization

3. Extending the QE toolbox to beyond-QE

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Motivations

- « Optimal » beyond-QE CMB lensing estimation is important for best r -constraints
- Quadratic estimation is however much better understood and has lots of redundancy / robustness variations
- So…. let's transpose all these variations to beyond-QE In a simple way

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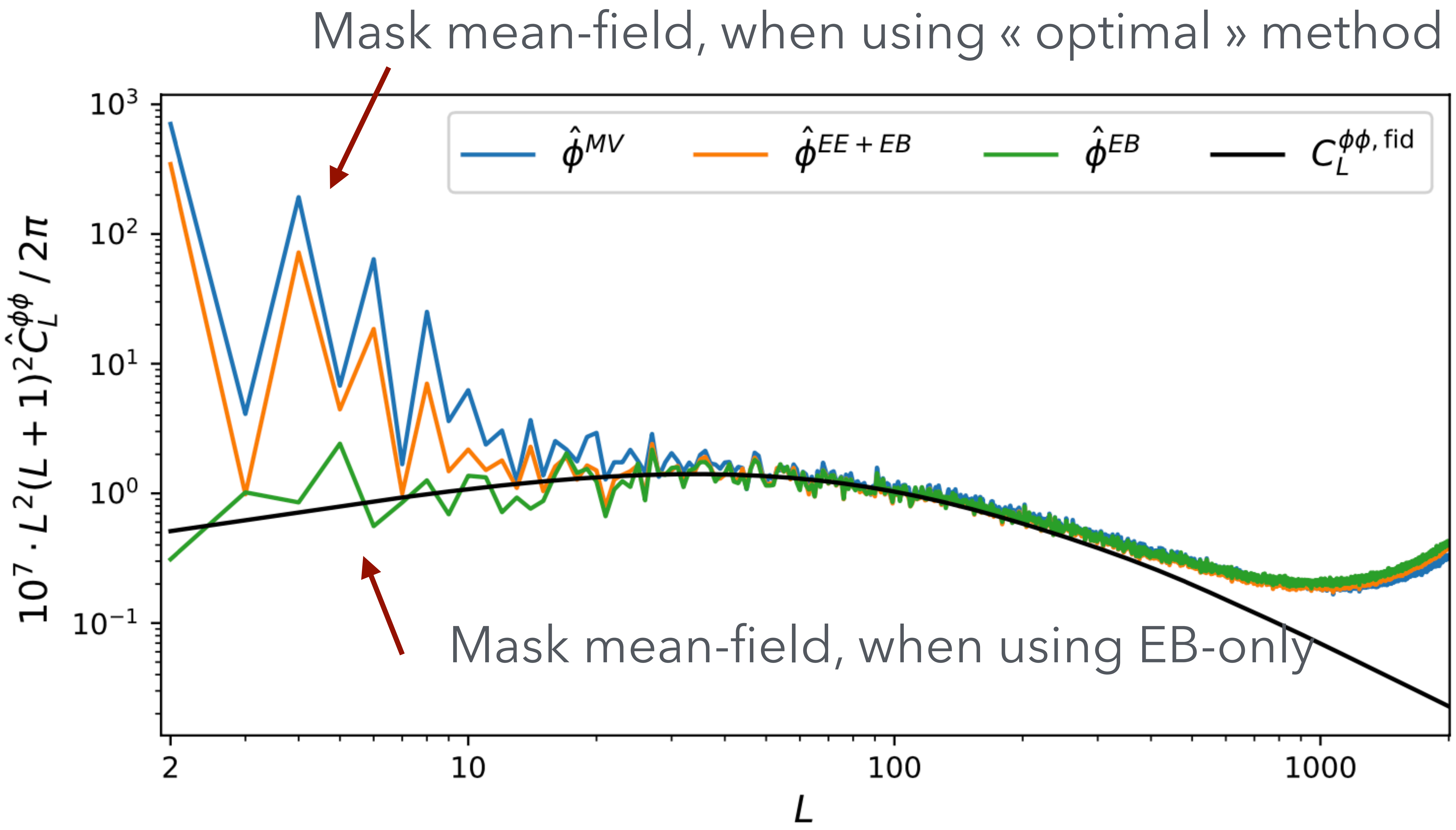

Simplest example: EB-lensing is « shear-only »

CMB lensing likelihood gradients

$$-\ln p(X^{\rm dat}\,|\,\phi) = \frac{1}{2} X^{\rm dat} \underline{\mathrm{Cov}_\phi^{-1}} X^{\rm dat} + \frac{1}{2}\ln\det \mathrm{Cov}_\phi$$

(Hirata & Seljak 2003, JC & Lewis 2017)

Anisotropic CMB spectra

Likelihood gradients w.r.t. to ϕ_{LM}:

$$g_\phi - \left\langle g_\phi \right\rangle$$

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Likelihood gradients w.r.t. to ϕ_{LM}:

$$g_\phi - \left\langle g_\phi \right\rangle$$

Massage these equations a bit, to find:

$$g_\phi(\hat{n}) = \underbrace{|A(\hat{n})|}_{\substack{\text{Magnification}\\ \text{(coordinate distortions}\\ \text{from delensing)}}} \underbrace{(\text{Optimal QE on }\phi\text{-delensed CMB})(\hat{n} + \nabla\phi(\hat{n}))}_{\text{Deflected QE on maps delensed by }\phi}$$

Beyond-QE EB lensing reconstruction

- « Optimal » EB-estimator of residual lensing

Weighted B-residuals
(~ data - prediction from delensed E,
« template delensing »)

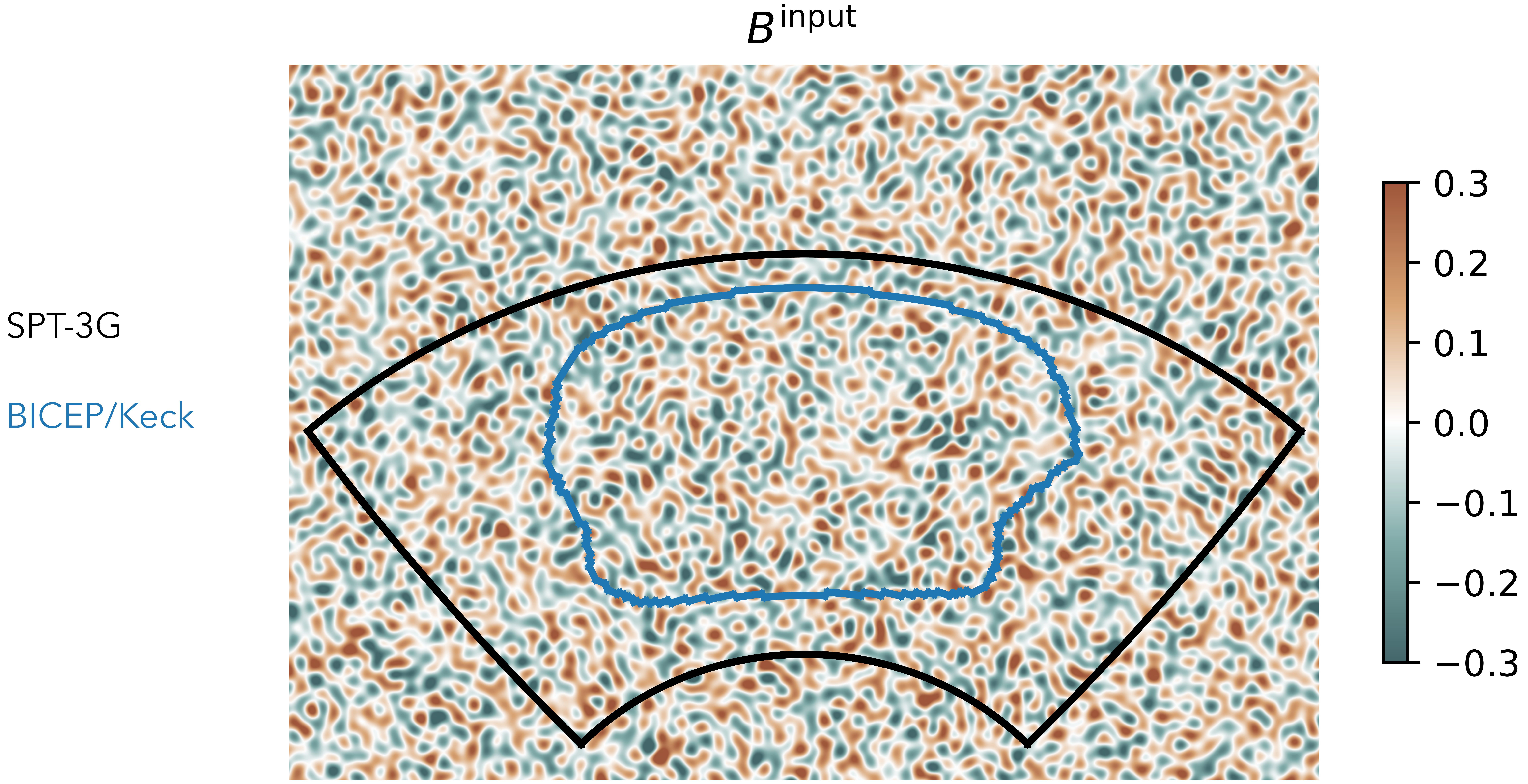
Stokes gradient maps from rec. E

$$\delta \nabla \hat{\phi} \propto |A(\hat{n})| \left[\bar{B}(\hat{n} + \nabla \phi(\hat{n}))\, \nabla E^{\mathrm{del}}(\hat{n} + \nabla \phi(\hat{n}))\right]$$

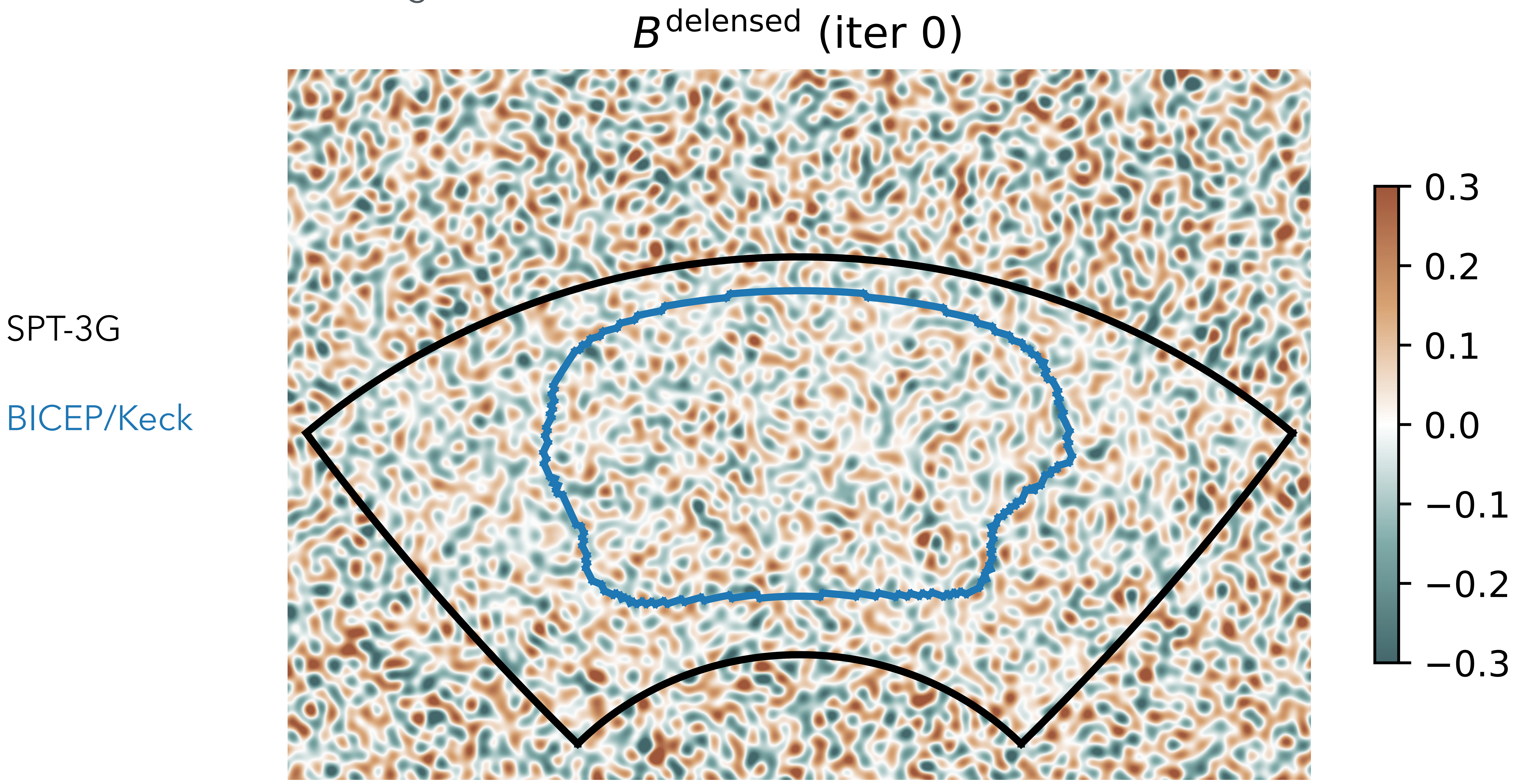
Magnification
(coordinate distortions
from delensing)

Deflected QE on maps delensed by ϕ

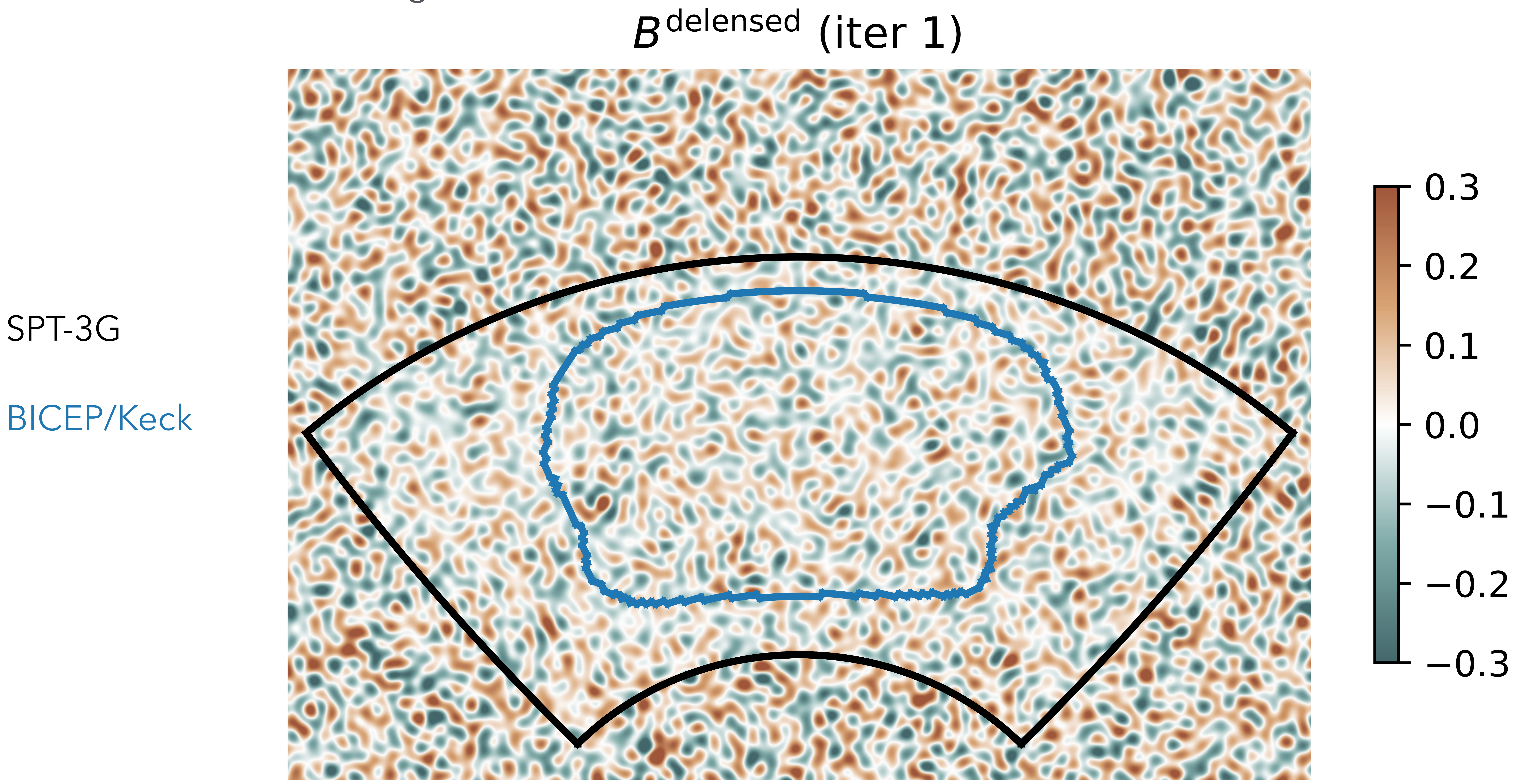
Simulated BK24-delensing



Simulated BK24-delensing



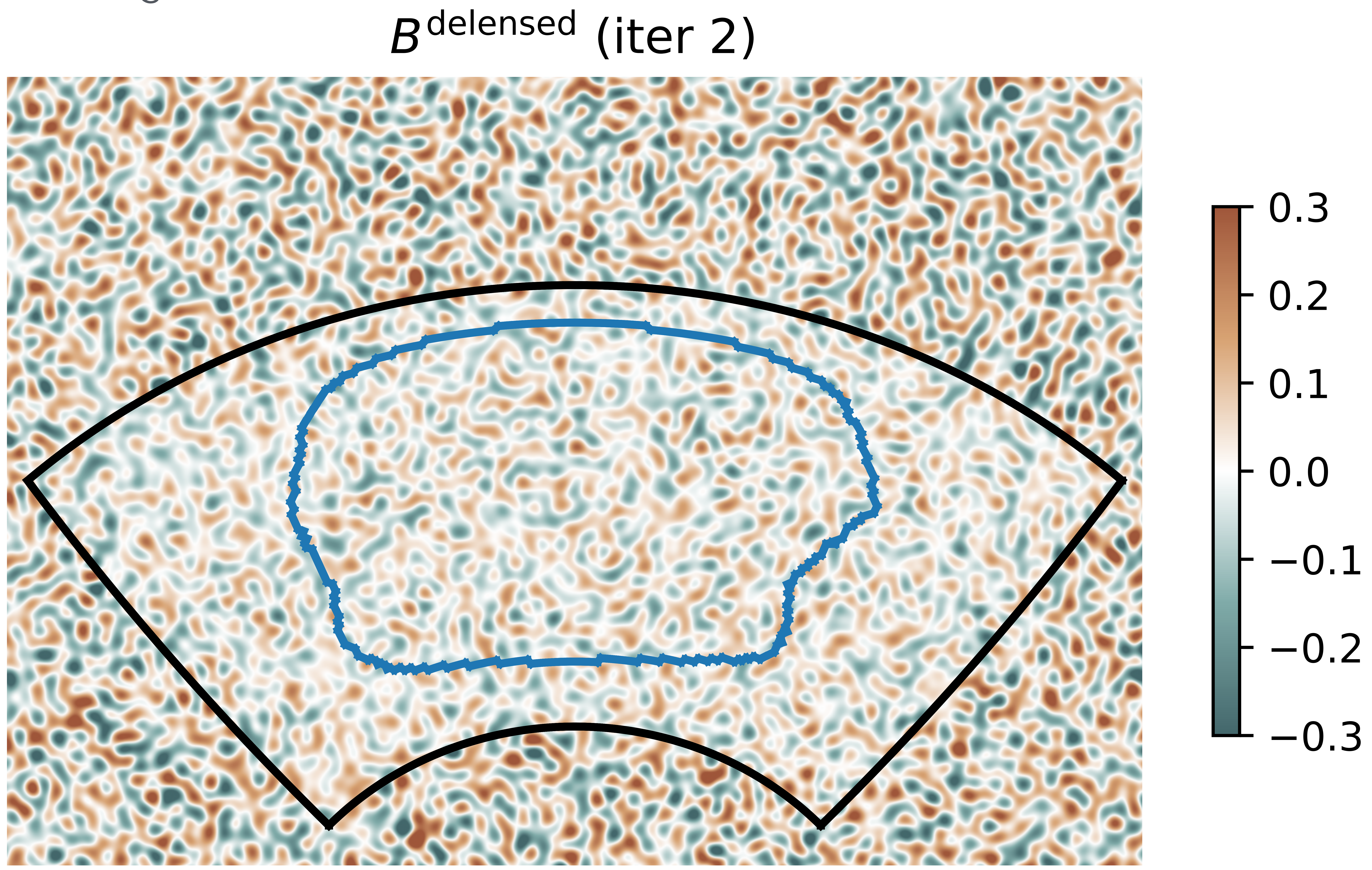
Simulated BK24-delensing



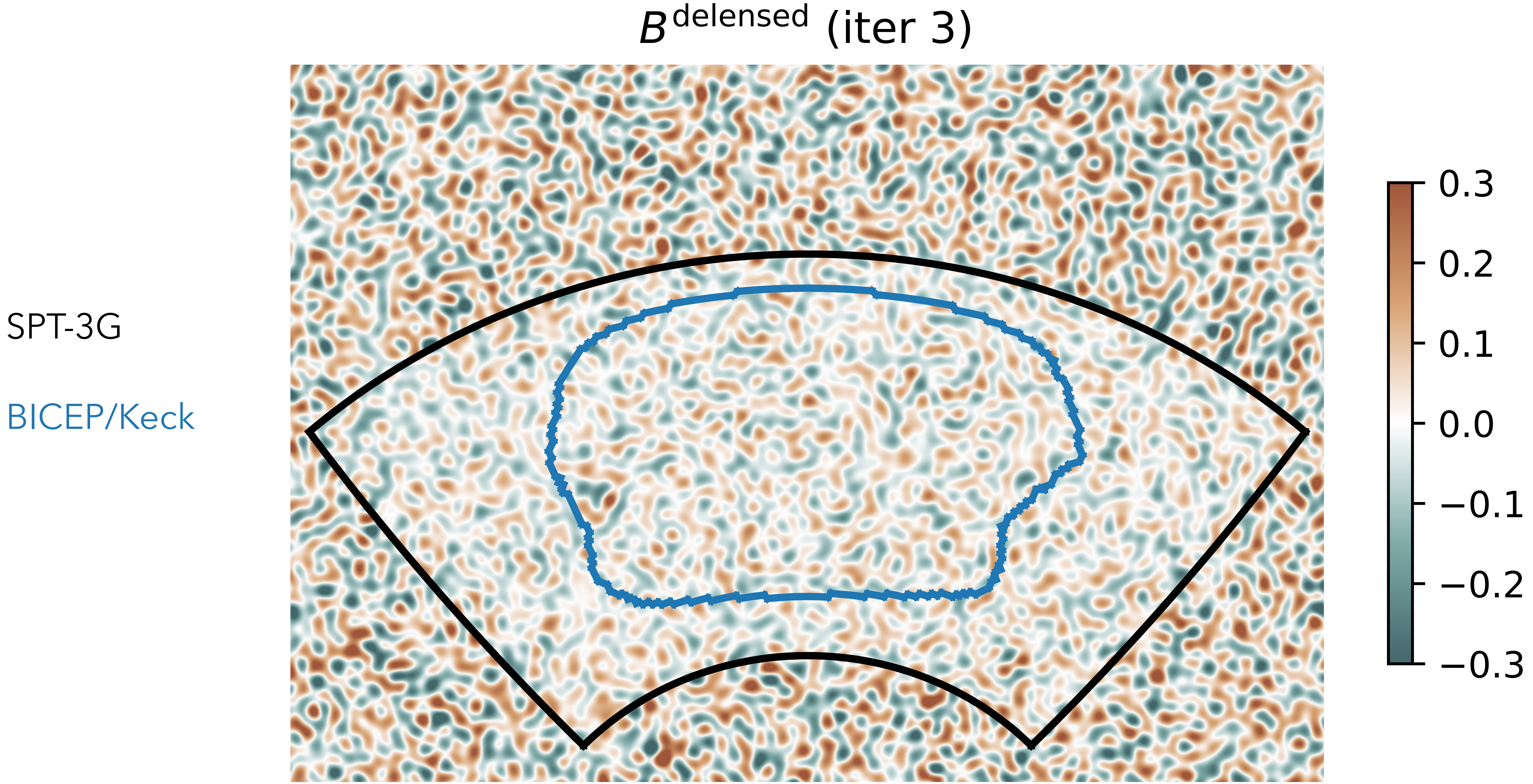
Simulated BK24-delensing

SPT-3G

BICEP/Keck



Simulated BK24-delensing



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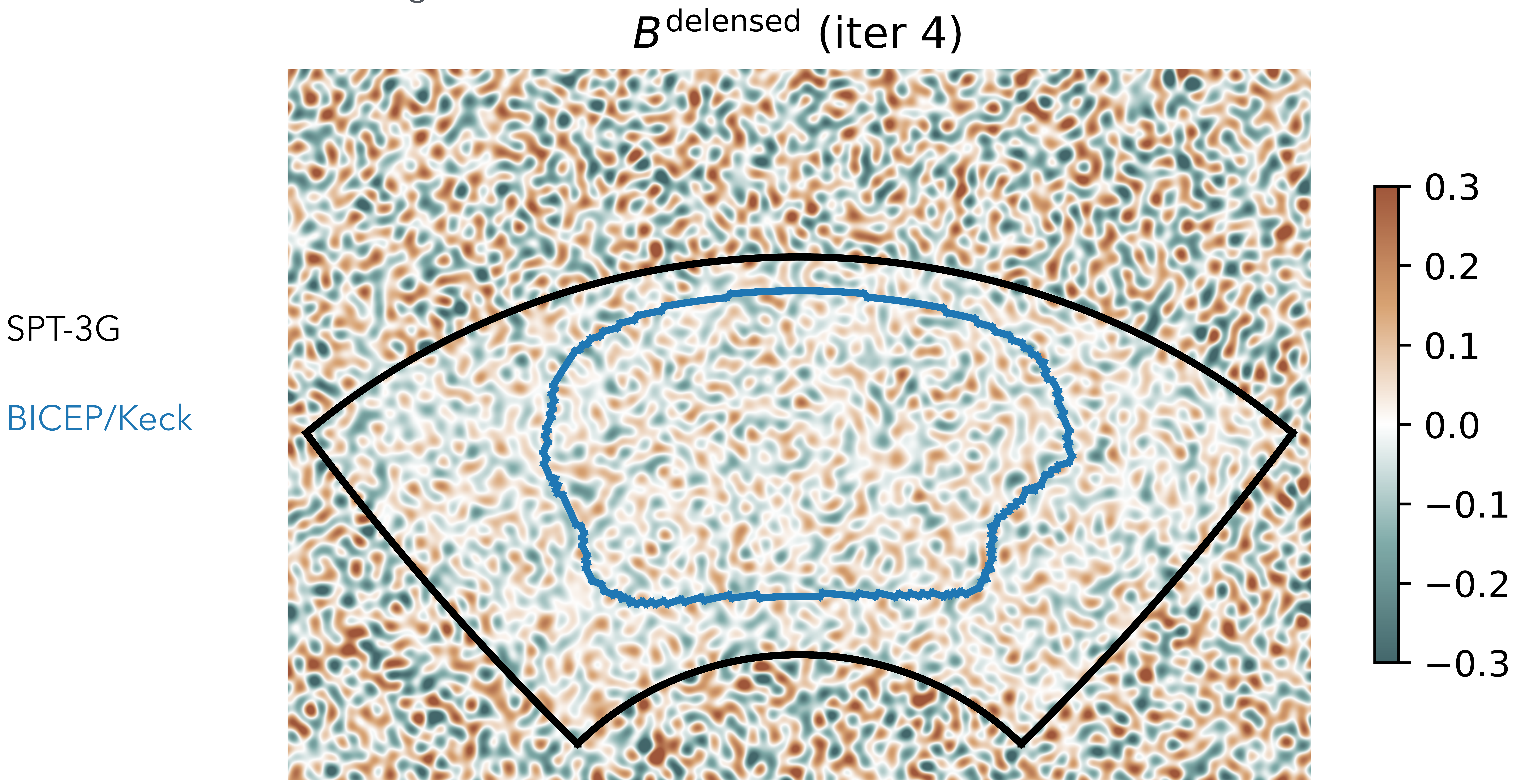
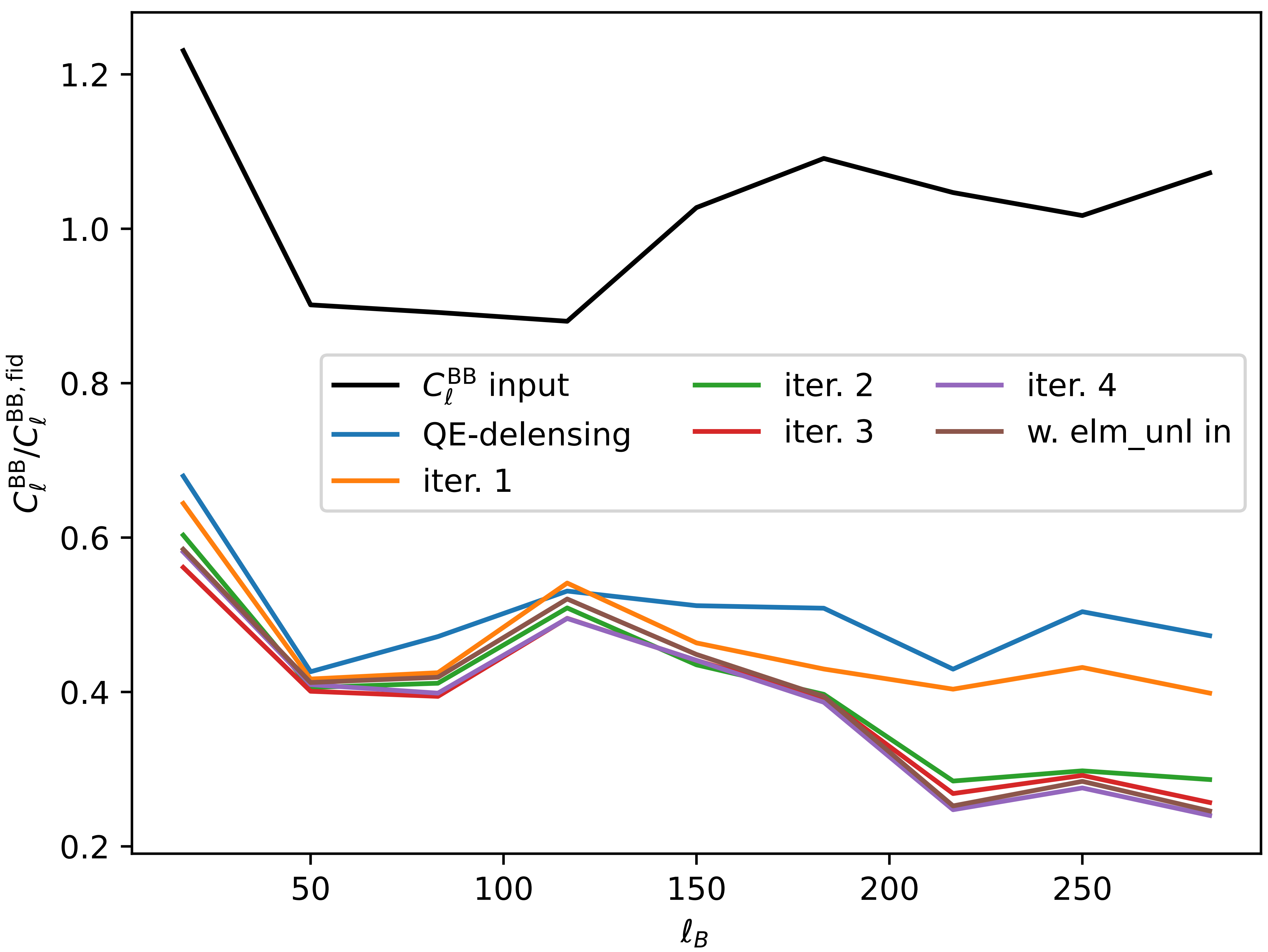
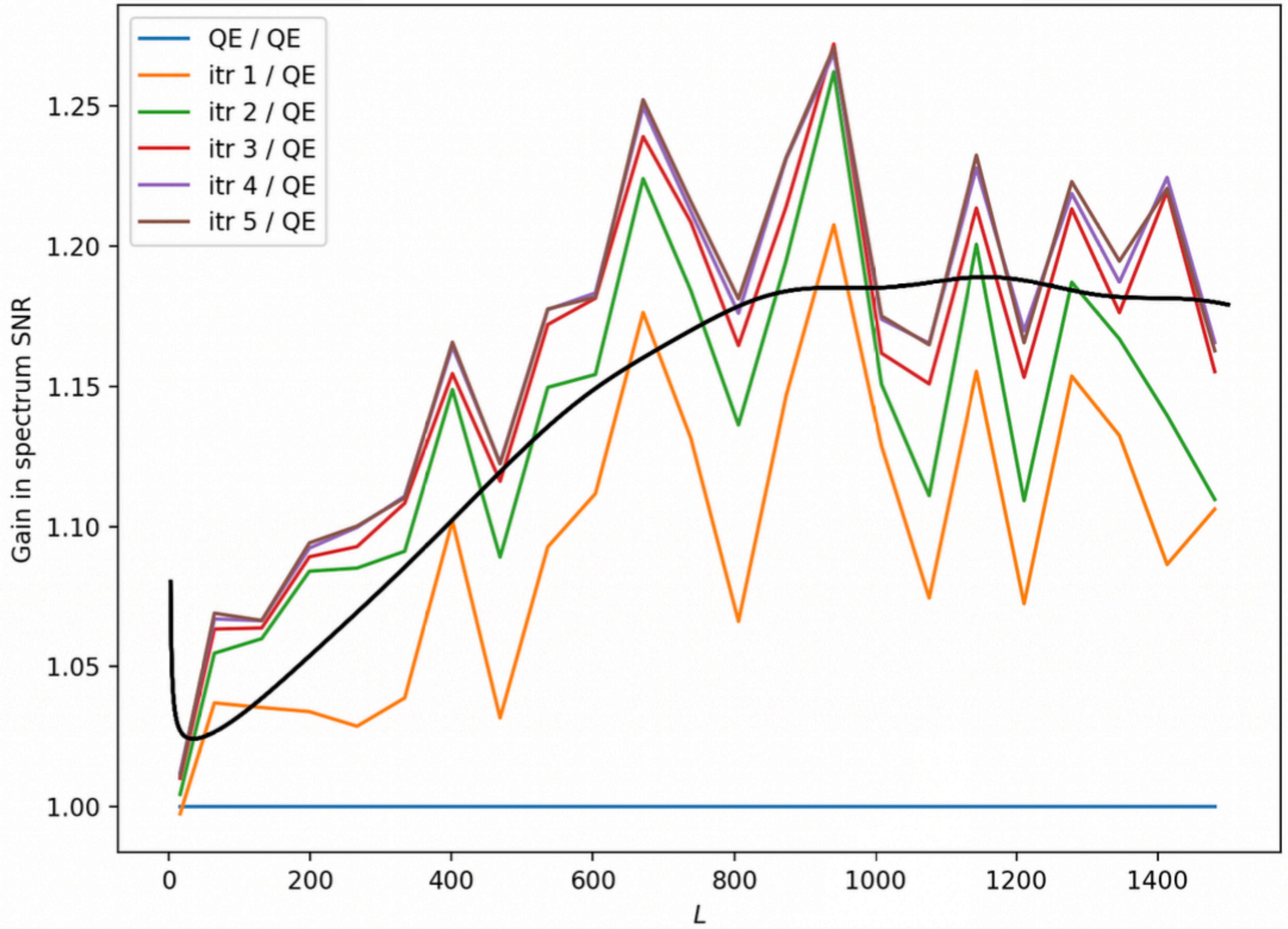


Reduction in degree-scale B-power



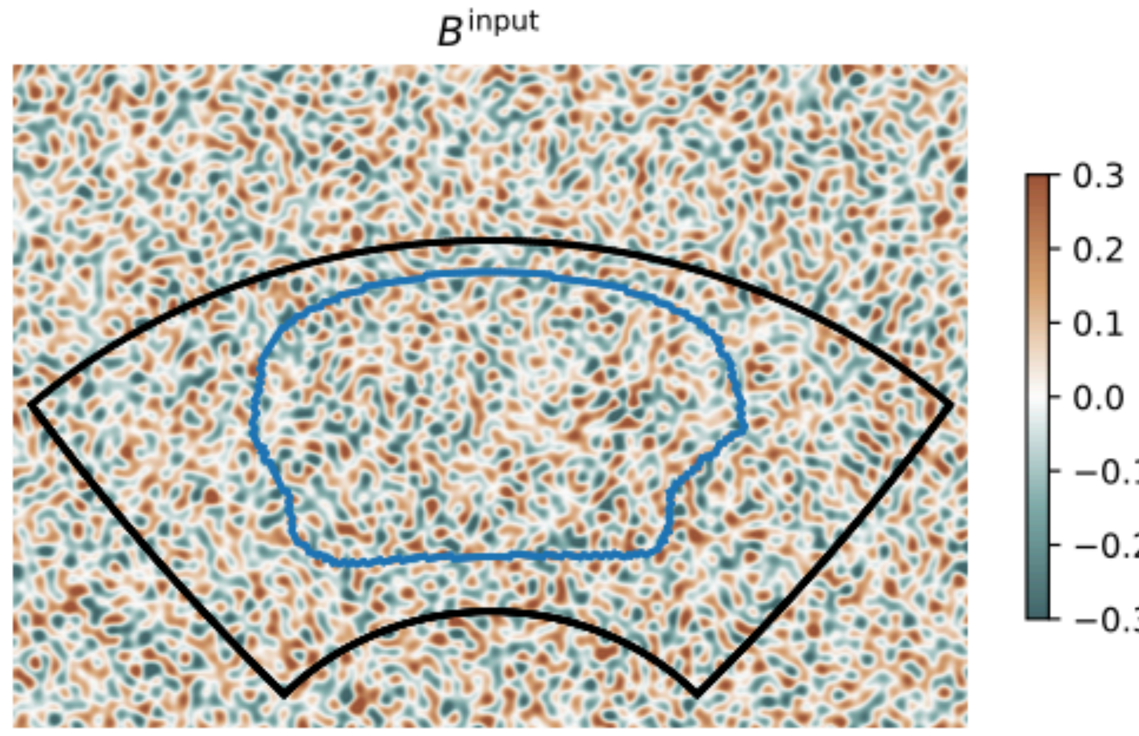
Improvement in lensing spectrum SNR

(Includes cosmic variance)

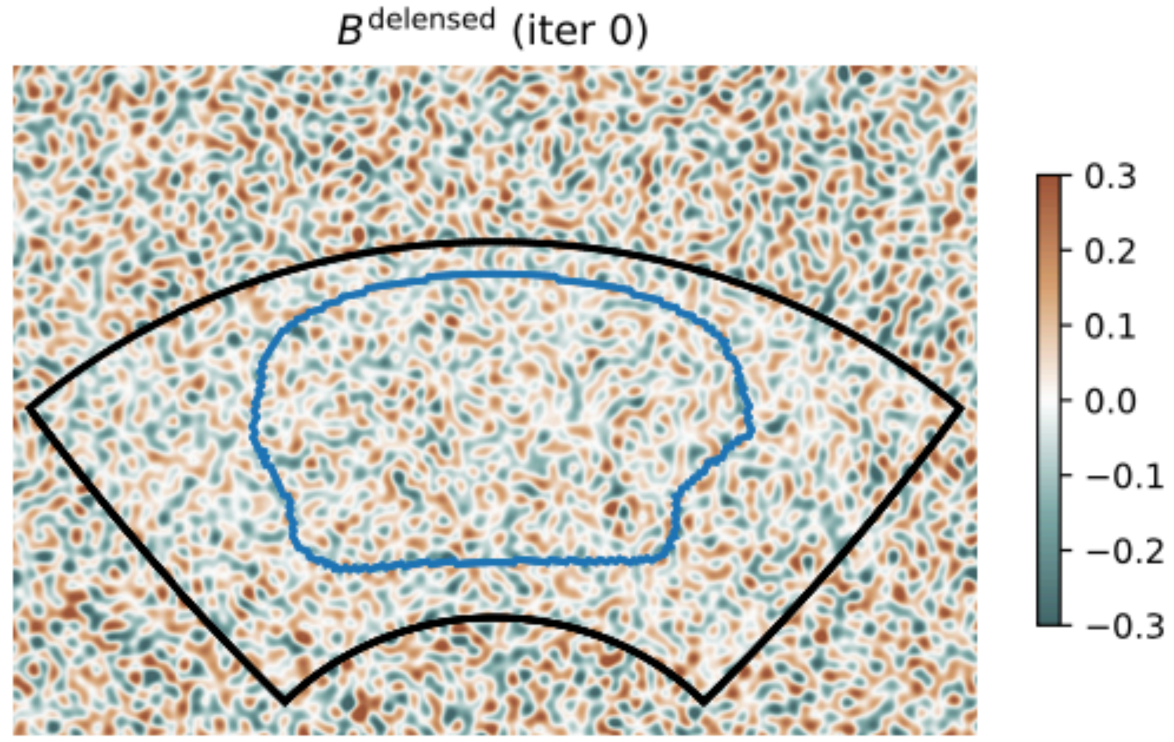


EB nominal SNR ~ 66 (Prabhu et al 2024)

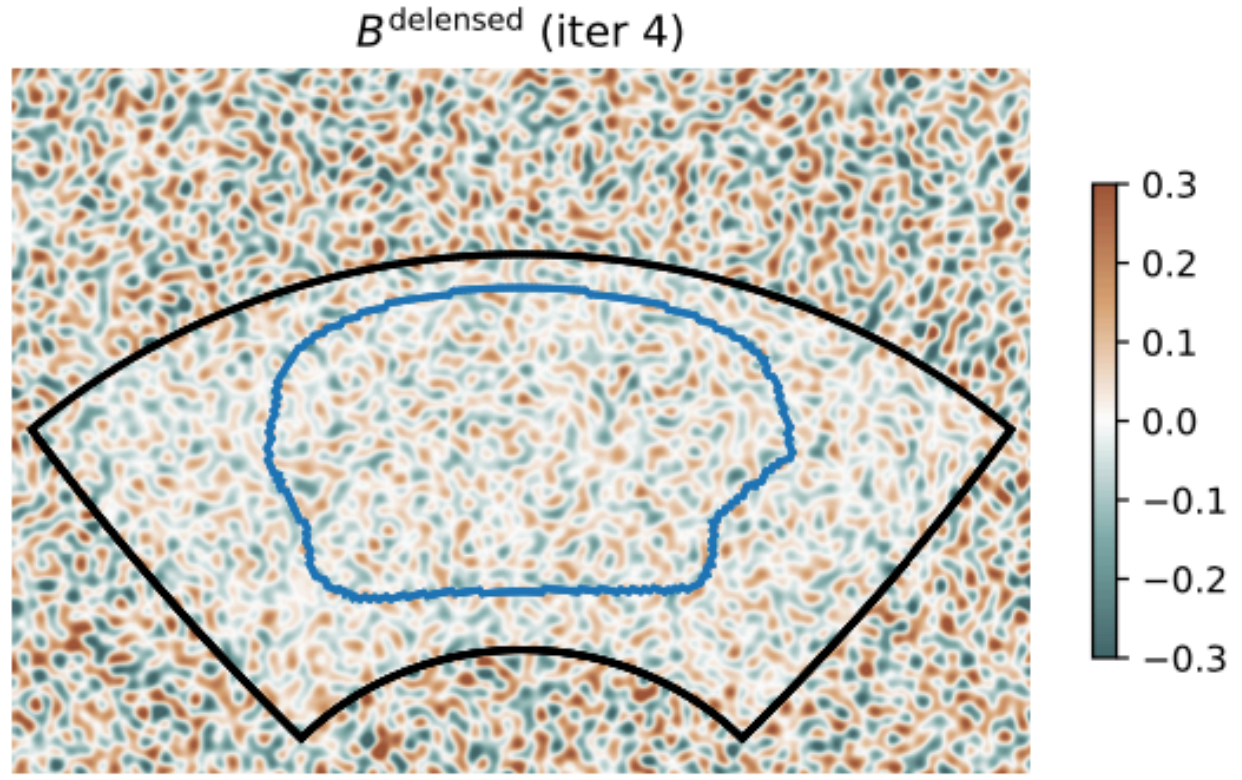
Input



QE



bQE



Conclusions

- The top constraints on tensor to scalar ratio are now limited by gravitational lensing effects on the CMB.

- BICEP/Keck + SPT-3G now available data has potential for $\sigma(r) \sim 0.003$ (3x better than BK18), and delensing is essential to achieve this.
  We are working on it, and building brand new robust tools to deal with it.

- Fast SHTs might be useful for other purposes

Thank you

How does this approach connect to MUSE:

Say you have a good statistic.
To infer parameters from it you can

Build a likelihood for what you see.
That's the standard, QE-alike way of doing lensing
(Faster, but one must understand what is seen!)

Repeat the same for various parameters values until data and sims match. =MUSE
(Slower, but one needs not understand what is seen!)

- MUSE can also be (exactly) described as beyond-QE iterative lensing estimation, probing $C_L^{\phi\phi} + N_L^{(1)} + N_L^{3/2} + \cdots$
- But MUSE also uses the Gaussian reconstruction noise as potential source of lensing info. (No $N_L^{(0)}$-subtraction. hence matching the iterative version of $C_L^{\phi\phi} + N_L^{(0)} + N_L^{(1)} + N_L^{3/2} \cdots$ instead)