Bayesian inverse problem with scattering transform : application to instrumental decontamination

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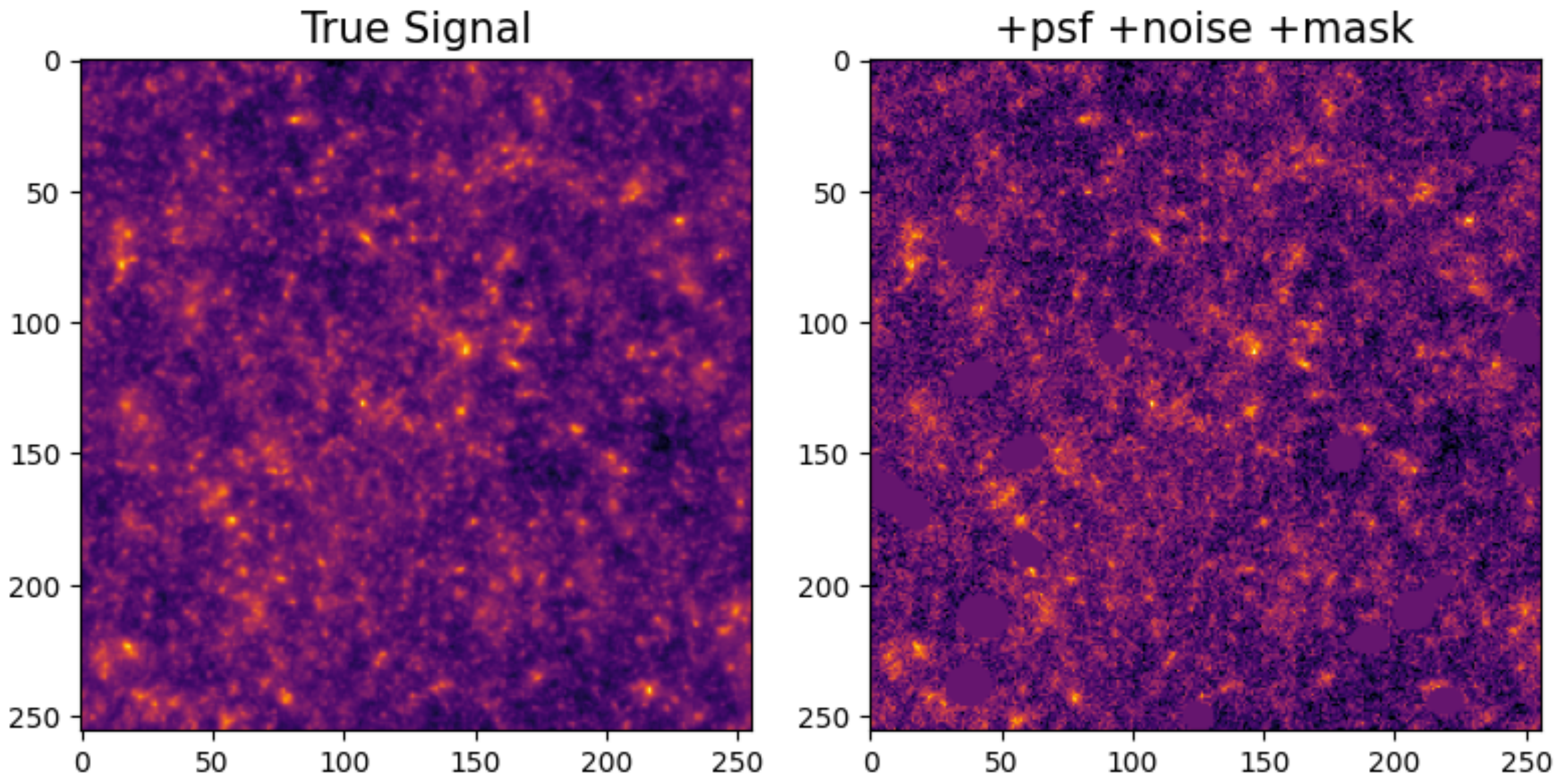
Bayesian inverse problems

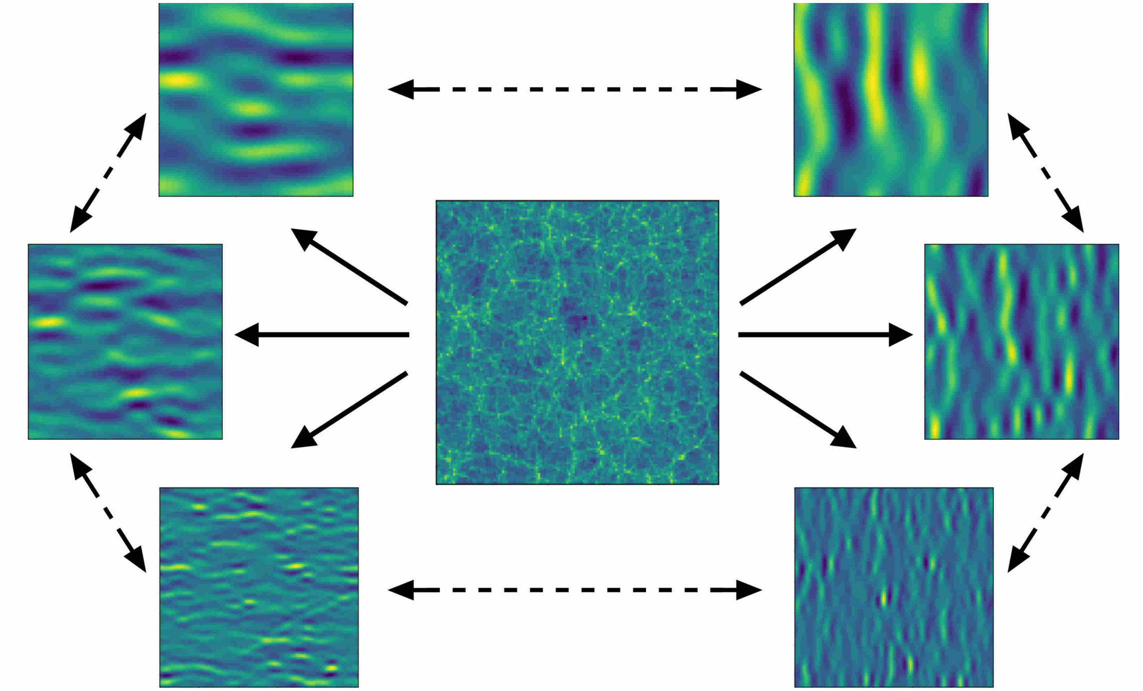
- We observe a single data $d_0 = f(s_0)$ with s_0 signal of interest
- f probabilistic forward model, assumed known. No external prior model for s



Ill posed problem : need for a probabilistic formulation $\rightarrow p(s \mid d_{obs})$

Scattering transform statistics

- Scattering transform (ST) statistics $\phi(s)$: non-Gaussian and multi-scale statistics Allys+20, Mallat+12
- Can be efficiently estimated from a single image

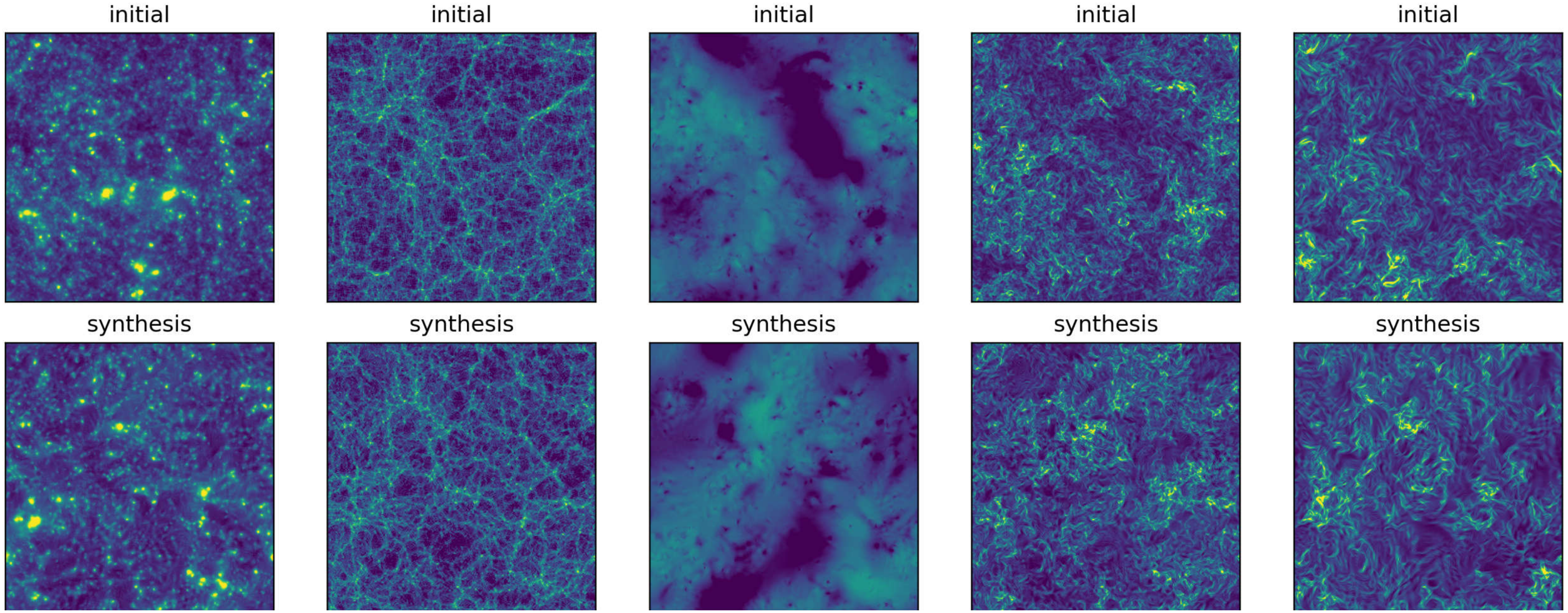


- Wavelet filters separating the different scales
- Interaction between scales with non-linearities and covariances

Generative models from ST statistics

- Ability to construct and sample maximum entropy models from ST statistics
- Parametrised by $\mu_S = \mathbb{E}_{s \sim p(s)}[\phi(s)]$

$$p(s) \rightarrow s \xrightarrow{\text{estimation}} \phi(s) \simeq \mu_S \rightarrow p_{\mu_S}^{m.e.} \xrightarrow{\text{sample}} \tilde{s}$$



Cheng+24

Realistic non-Gaussian models from $O(10^2)$ coefficients

Bayesian formulation

- Framework :
  - Target μ_{S_0} instead of s_0
  - Also describe d_0 by its ST statistics $\phi(d_0)$
  - Parameter space : μ_S — data space : $\phi(d)$
- Bayesian Formulation :
  - $p(\mu_S \mid \phi(d)) \propto p(\phi(d) \mid \mu_S)p(\mu_S)$
  - No other a priori information in ST space —> flat prior

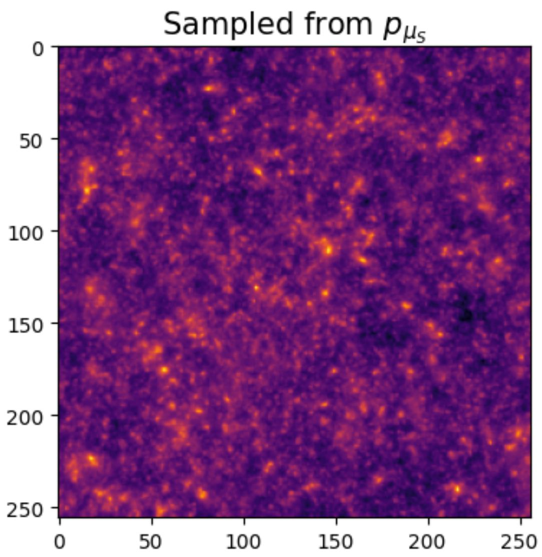
How can we get the likelihood ?

ST Forward Model

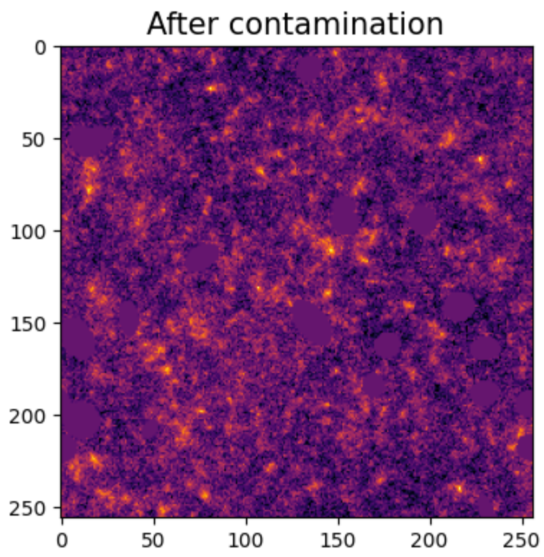
$$\mathscr{F} : \mu_s \to p_{\mu_s} \to s \to f(s) = d \to \phi(d)$$

$$\mu_S = \begin{bmatrix} \mu_{S_1} \\ \mu_{S_2} \\ \mu_{S_3} \\ \vdots \\ \mu_{S_n} \end{bmatrix}$$

Synthesis $\longrightarrow$



Contamination $\longrightarrow$



Compression $\longrightarrow$

$$\begin{bmatrix} \phi_1(d) \\ \phi_2(d) \\ \phi_3(d) \\ \vdots \\ \phi_n(d) \end{bmatrix} = \phi(d)$$

No simple or analytical way to evaluate the likelihood

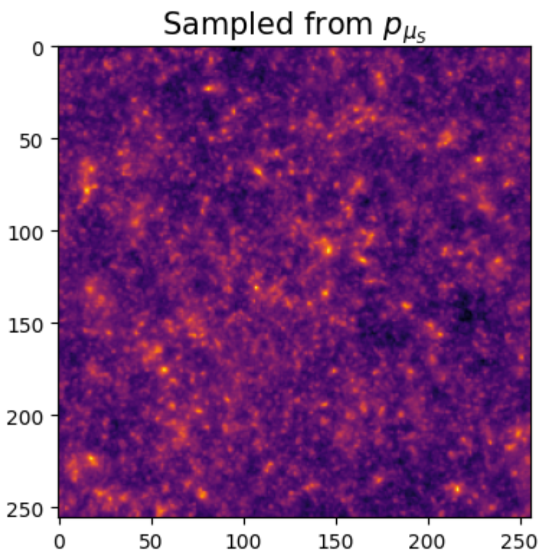
Possible to generate pairs of $(\mu_S, \phi(d))$

Estimating the likelihood $p(\phi(d) \mid \mu_S)$

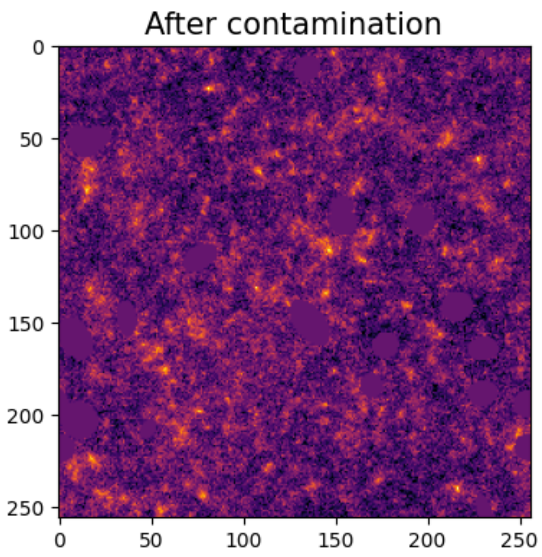
- Taylor expansion of the ST forward model : $p(\phi(d) \mid \mu_S) \simeq \mathcal{N}(A\mu_S + b \mid \Sigma)$
- Approximate A, b, Σ with samples $(\mu_S, \phi(d))$ of our ST generative model

$$\mu_S = \begin{bmatrix} \mu_{S_1} \\ \mu_{S_2} \\ \mu_{S_3} \\ \vdots \\ \mu_{S_n} \end{bmatrix}$$

Synthesis →



Contamination →



Compression →

$$\begin{bmatrix} \phi_1(d) \\ \phi_2(d) \\ \phi_3(d) \\ \vdots \\ \phi_n(d) \end{bmatrix} = \phi(d)$$

- Iterative algorithm based on adaptive proposal to approximate the posterior $p(\mu_S \mid \phi(d))$

Results

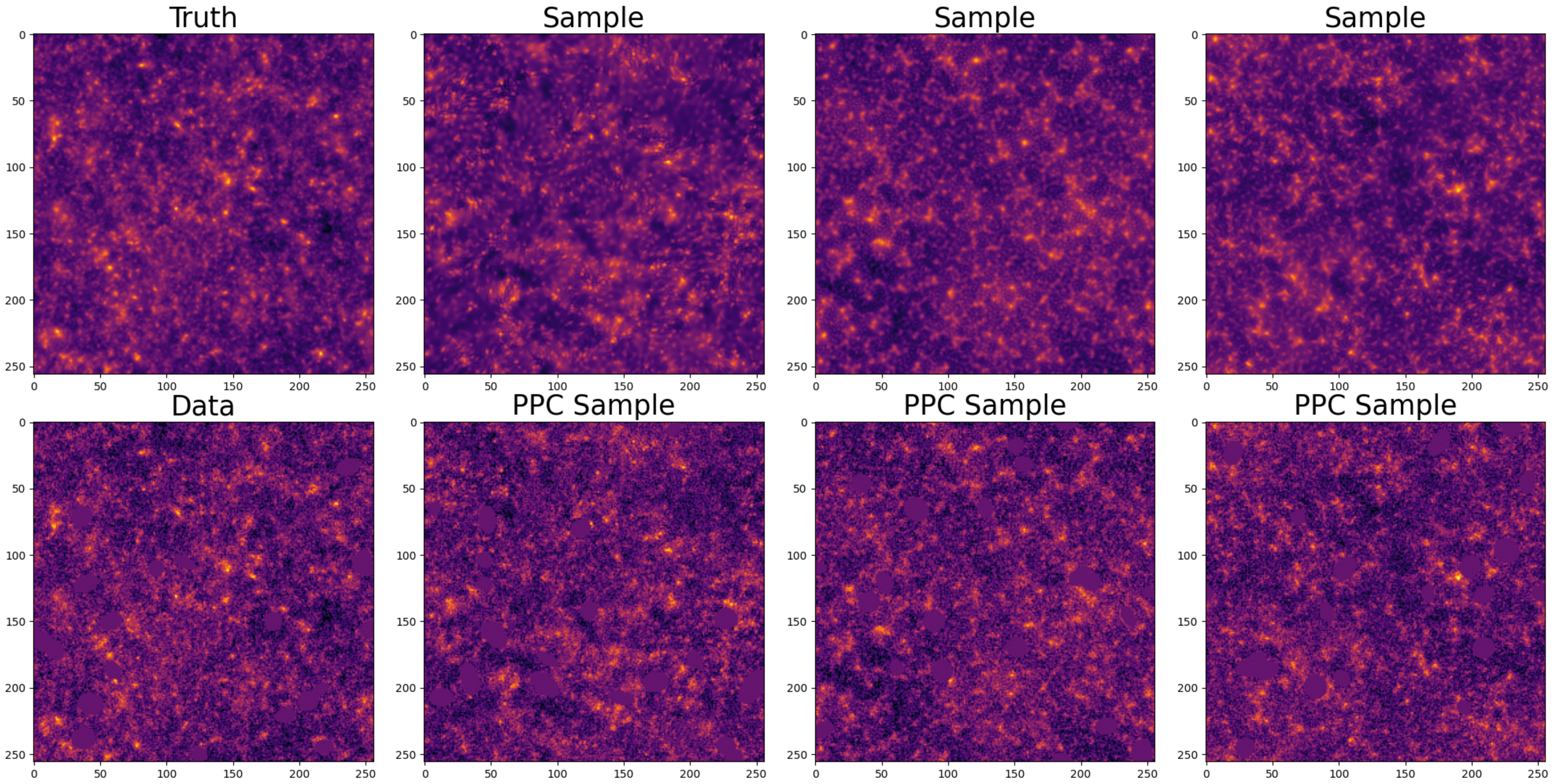
- Setting :
  - Only one observed data d_0
  - No external prior model for s
  - Know the pixel space forward model f
- Modelling hypothesis :
  - s well described by a ST-based model
  - Taylor expansion on the ST forward model $p(\phi(d) \mid \mu_S) \simeq \mathcal{N}(A\mu_S + b \mid \Sigma)$

$\longrightarrow$ **Posterior distribution $p(\mu_S \mid \phi(d_0))$ of scattering statistics of s_0**

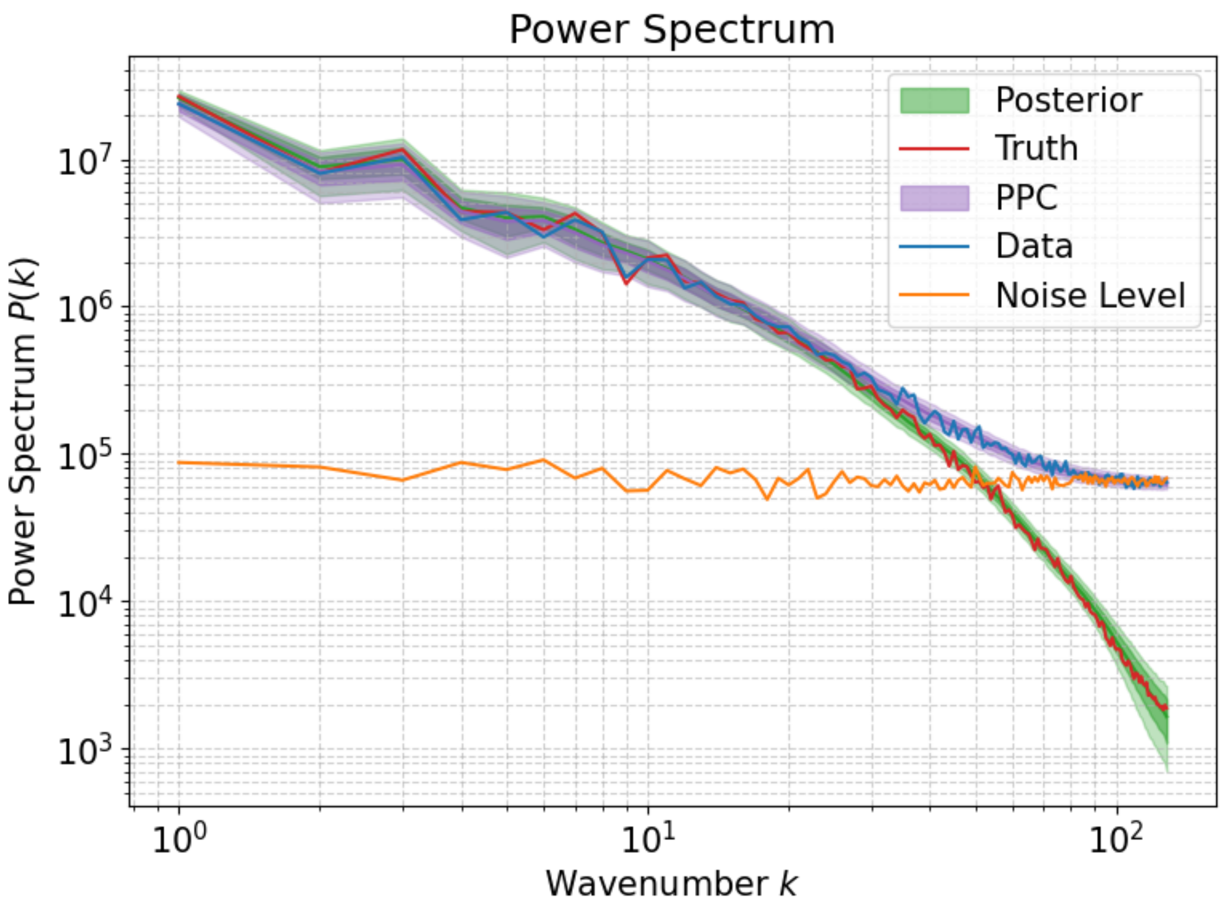
And of any other statistics by sampling s from ST-based generative model

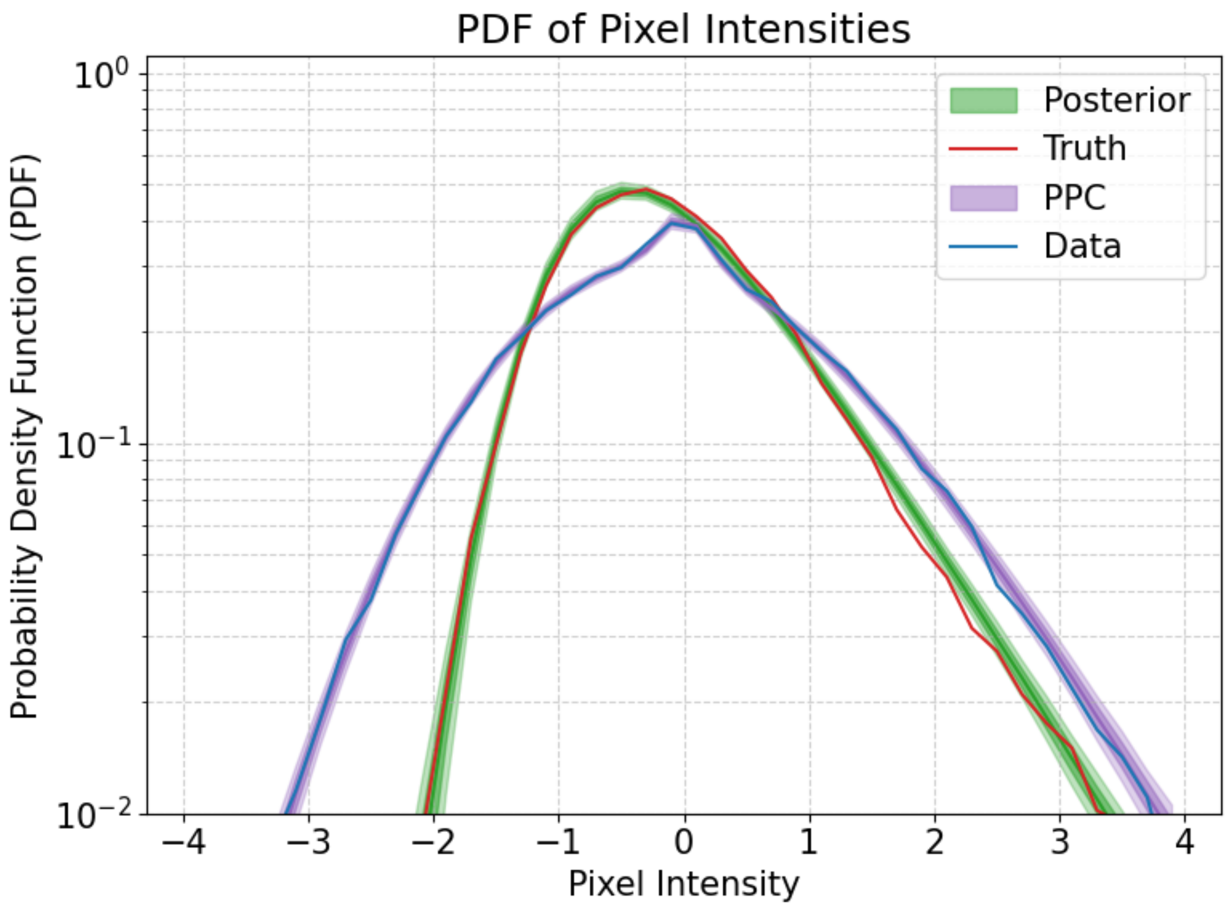
PRELIMINARY

Results



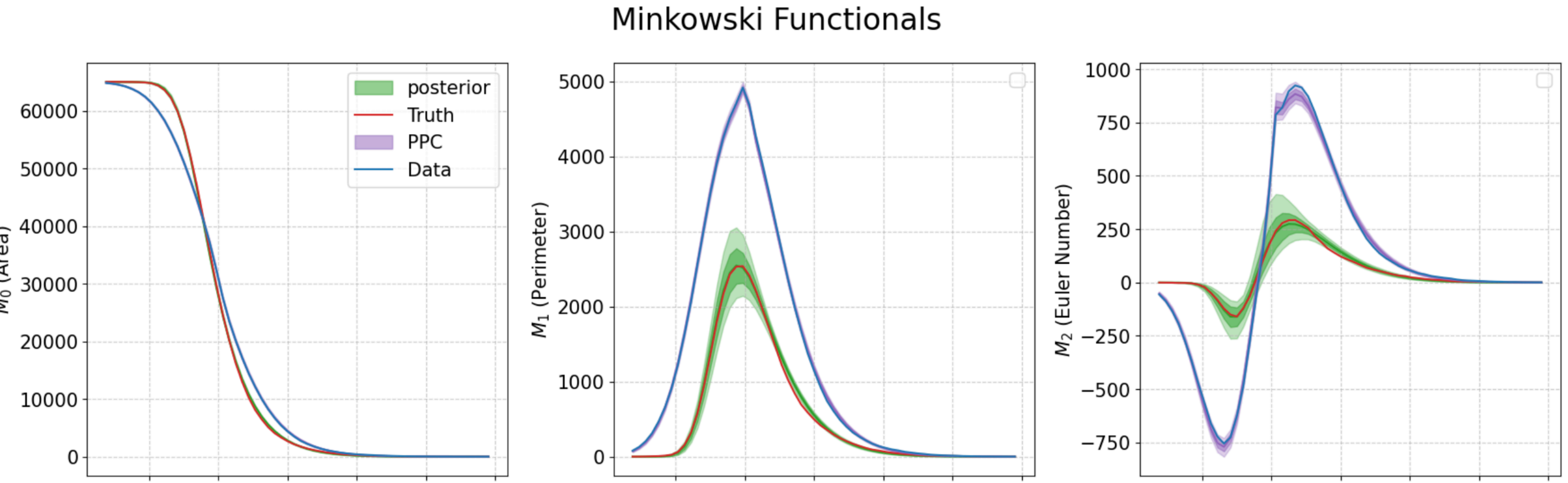
Results





Power spectrum and PDF recovered

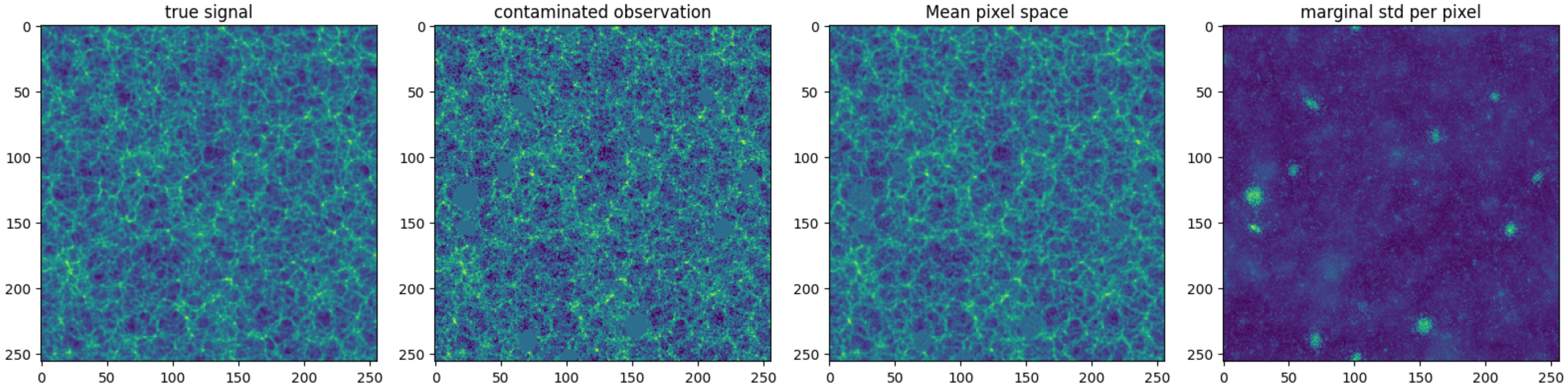
Results



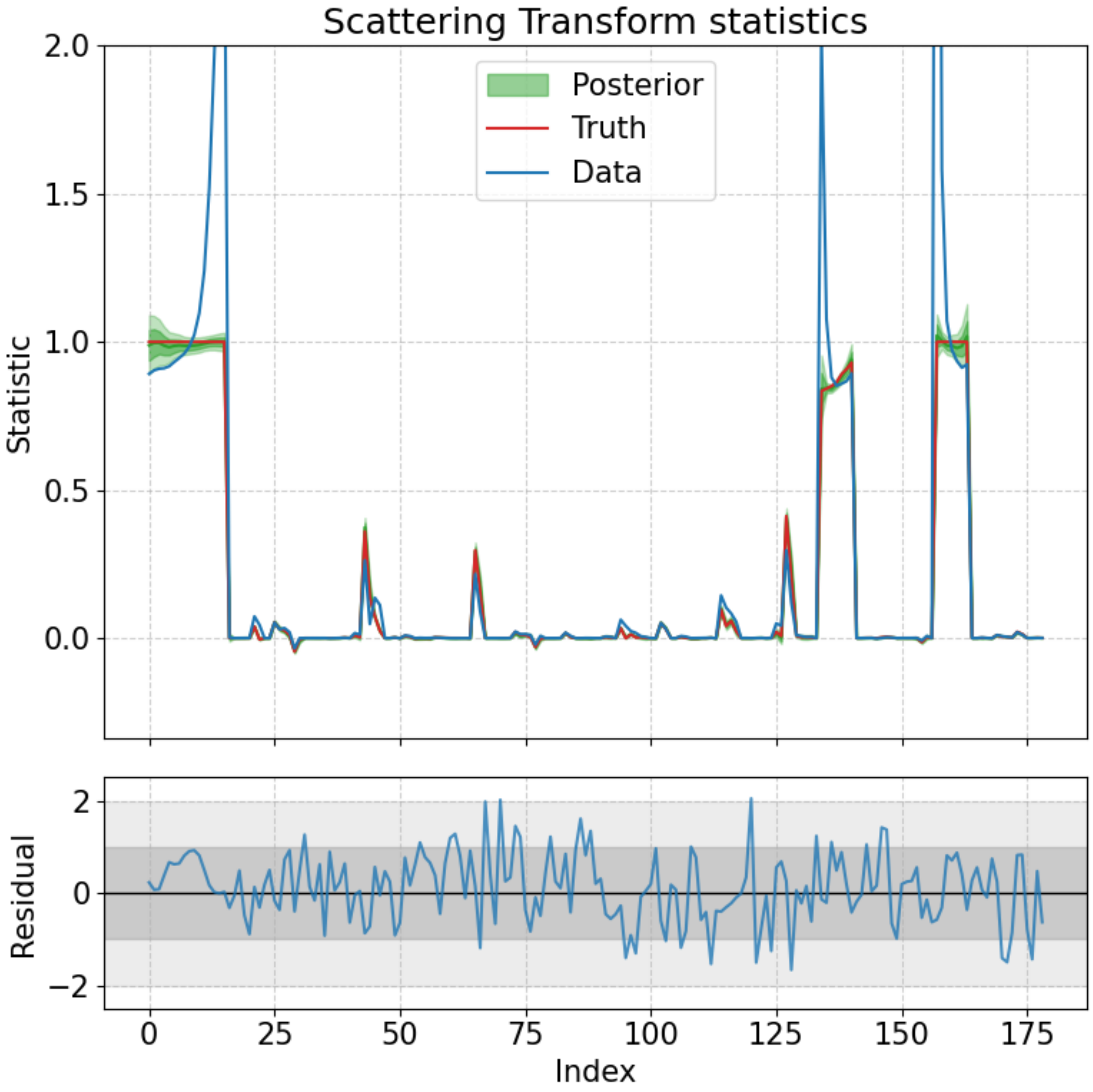
Minkowski functionals recovered

Conclusion

- Challenging setting : one data and no external prior model for s
- Recover a posterior of ST statistics of s and other usual astrophysical statistics
- Proof of concept before going to more complete cases
- Possibility to have a pixel space reconstruction once a ST-based model is known Jeffrey+22



Results


Scattering Transform statistics
Posterior
Truth
Data
Statistic
2.0
1.5
1.0
0.5
0.0
Residual
2
0
−2
0
25
50
75
100
125
150
175
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