Charting the Hidden Valley

Exploring the various phases of dark sector theories

Joshua Lockyer
joshua.lockyer@uni-graz.at

In collaboration with: Suchita Kulkarni, Simon Plätzer, Matthew Strassler & Wei Liu

Based on
arXiv:2502.18566
JHEP **2025**, 150 (2025)
and arXiv:2505.03058
PRD **2025**, 112 (2025)

Confining Hidden Valley models

- Hidden Valley (HV) models extend the SM with a new dark sector uncharged under the SM gauge group, instead connecting to the SM through a heavy mediator, here we use a $U(1)$ Z_D / Z'.

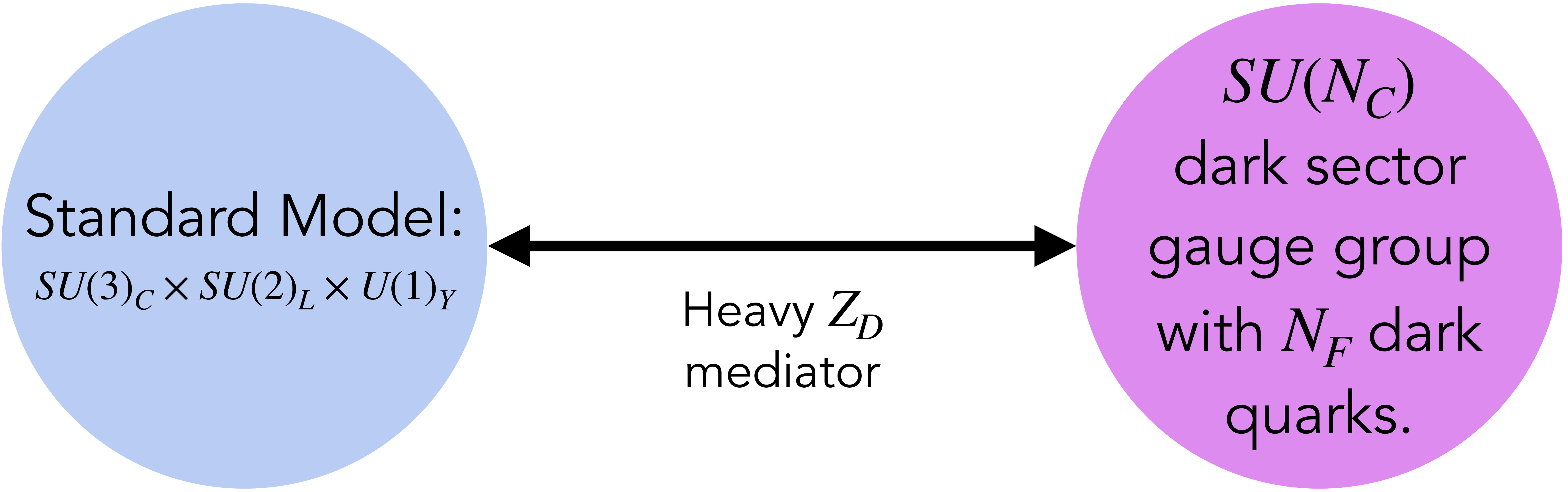
Theory

arXiv:0604261, M.J. Strassler et al.

arXiv:1502.05409, P. Schwaller et al.

arXiv:1503.00009, T. Cohen et al.

arXiv:0712.2041, T. Han et al.



Experimental searches:

arXiv:2112.11125 (CMS)

arXiv:2305.18037 (ATLAS)

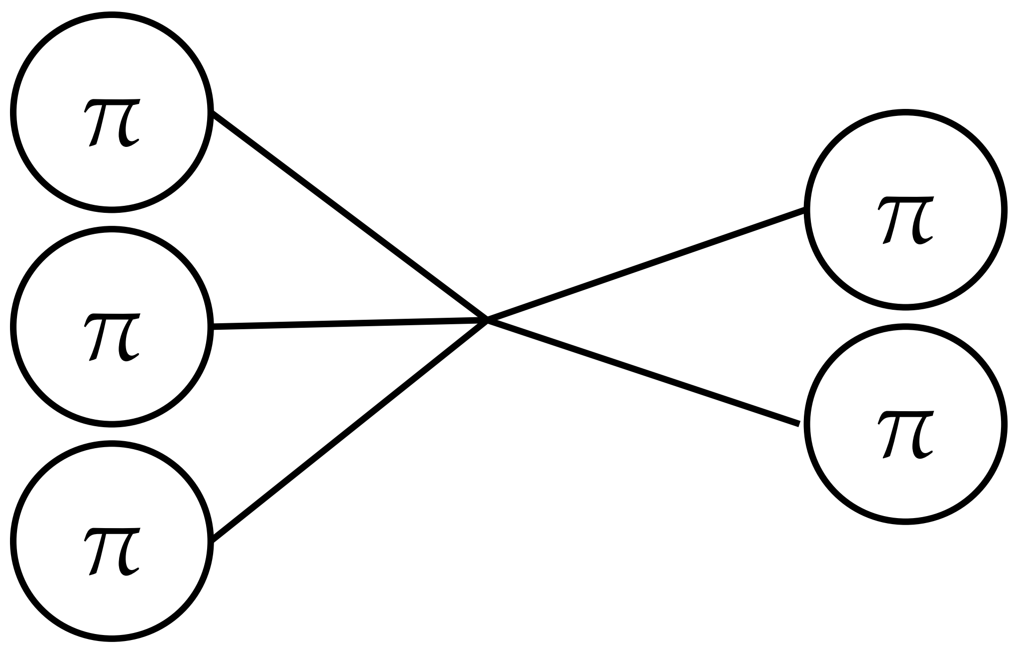
arXiv:2505.02429 (ATLAS)

arXiv:2102.10874 (ATLAS)

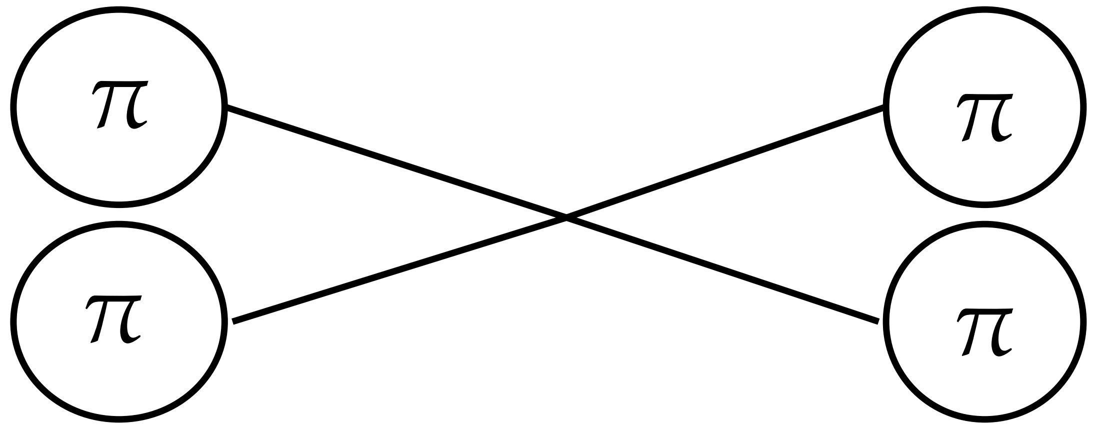
arXiv:2403.01556 (CMS)

- HVs are rich dark sectors (DS), we study DS with a $SU(N_C)$ gauge group with N_F flavours of degenerate fundamental Dirac fermions (dark quarks). Such sectors are characterised by four parameters; N_C , N_F , Λ and m_π/Λ. Confinement ensures the formation of bound states; in our case dark mesons - typically dark pions or dark rhos.
- Certain classes of HV models resembling QCD present novel collider signatures and exciting opportunities for new physics discovery. Many searches already exist at colliders for the signatures of QCD-like dark sectors.

Why Hidden Valleys?



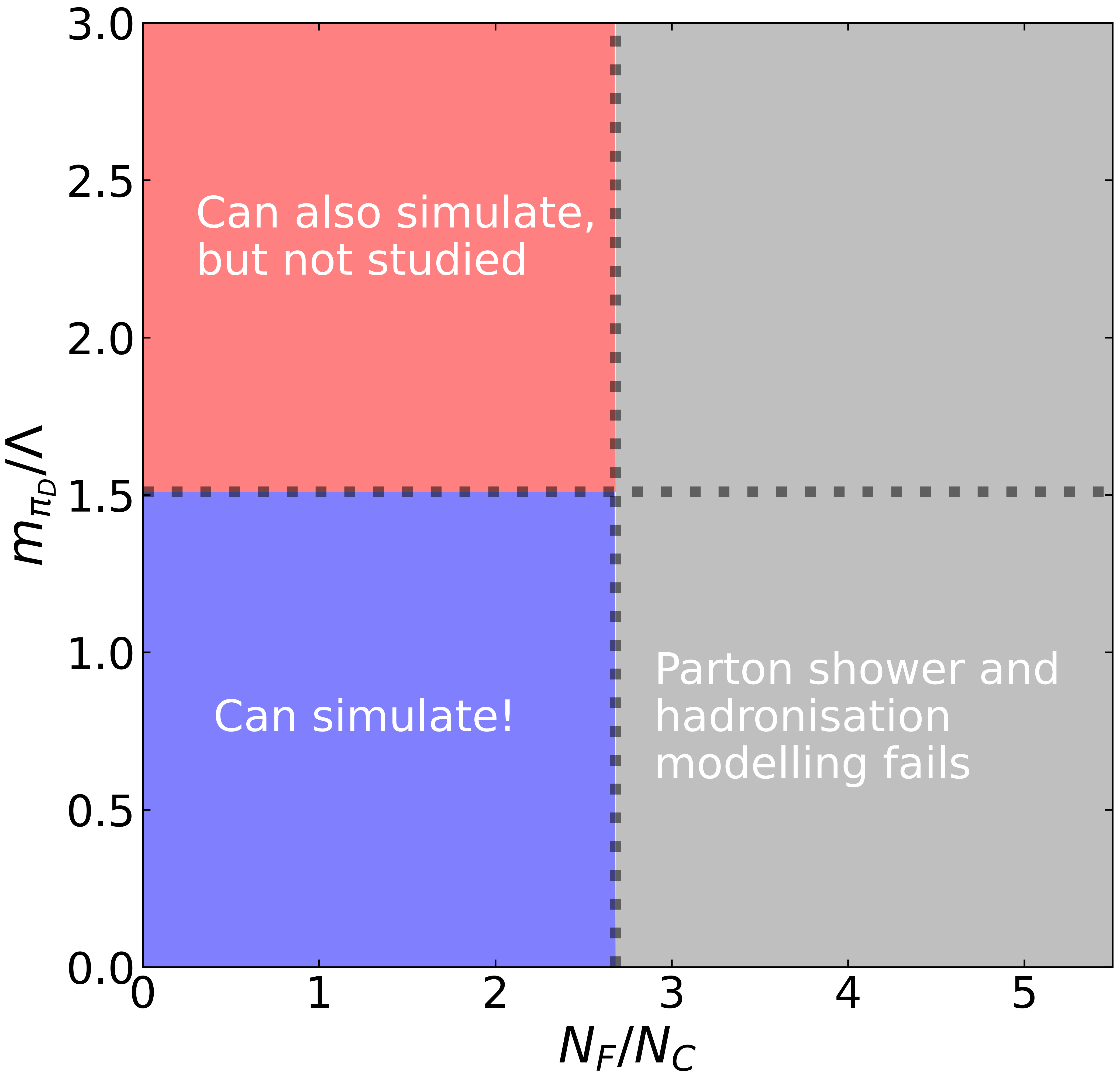
- Many models propose the dark pion being a thermal relic of $3 \to 2$ self-interactions - the SIMP mechanism. Occurs naturally in QCD-like sectors with dynamic χ symmetry breaking.
  arXiV:1402.5143,1411.3727,1910.10724, 2108.10314, 2401.12283, 2404.07601



- Small-scale structure formation problems can be explained by self-interacting dark matter arising from sizeable $2 \to 2$ interactions.
  arXiV:1402.5143,2108.10314

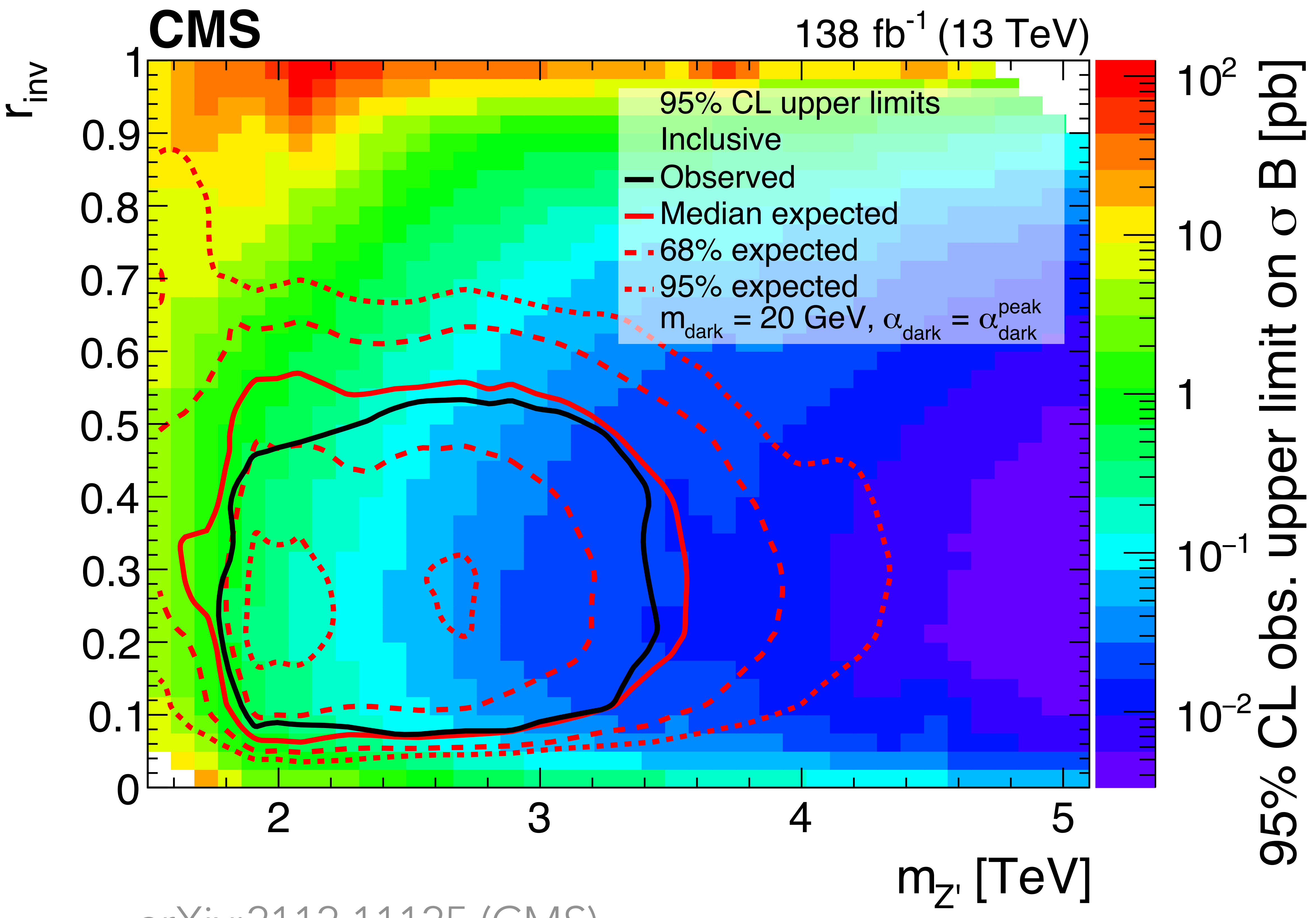
- Confining phase transition can produce gravitational wave signals. arXiv:2211.08877
- QCD-like dark sectors from "Twin Higgs" scenarios can help alleviate the "little hierarchy problem" by cancelling quantum corrections to the EW scale, so-called "Neutral Naturalness" models.
  arXiV:1304.7006,1310.4423,1501.05310,2110.10691,2507.15935

Theory space



- It's important to map out the theory space completely - there's a possibility of missing out on a chance of discovery if we don't, especially given upcoming HL-LHC run.
- To fully encompass parameter space, need to go beyond safety of QCD-like theories in 2-body phase; no strong theory expectation that dark sector should look like QCD.
- For $m_\pi/\Lambda \gtrsim 1.5$, $\rho_D^{0/\pm} \to \pi_D^{0/\pm}\pi_D^{\mp}$ decay channels are closed since $m_{\rho_D} < 2m_{\pi_D}$. This leads to a different phenomenology where the ρ_D's decay off-shell through a Z' - not yet studied.
- In current Pythia, simulations of $N_F/N_C \gtrsim 2.7$ are not robust at parton shower and hadronisation levels. Could have new distinct signatures from unique running coupling behaviour.

Searching for Hidden Valleys



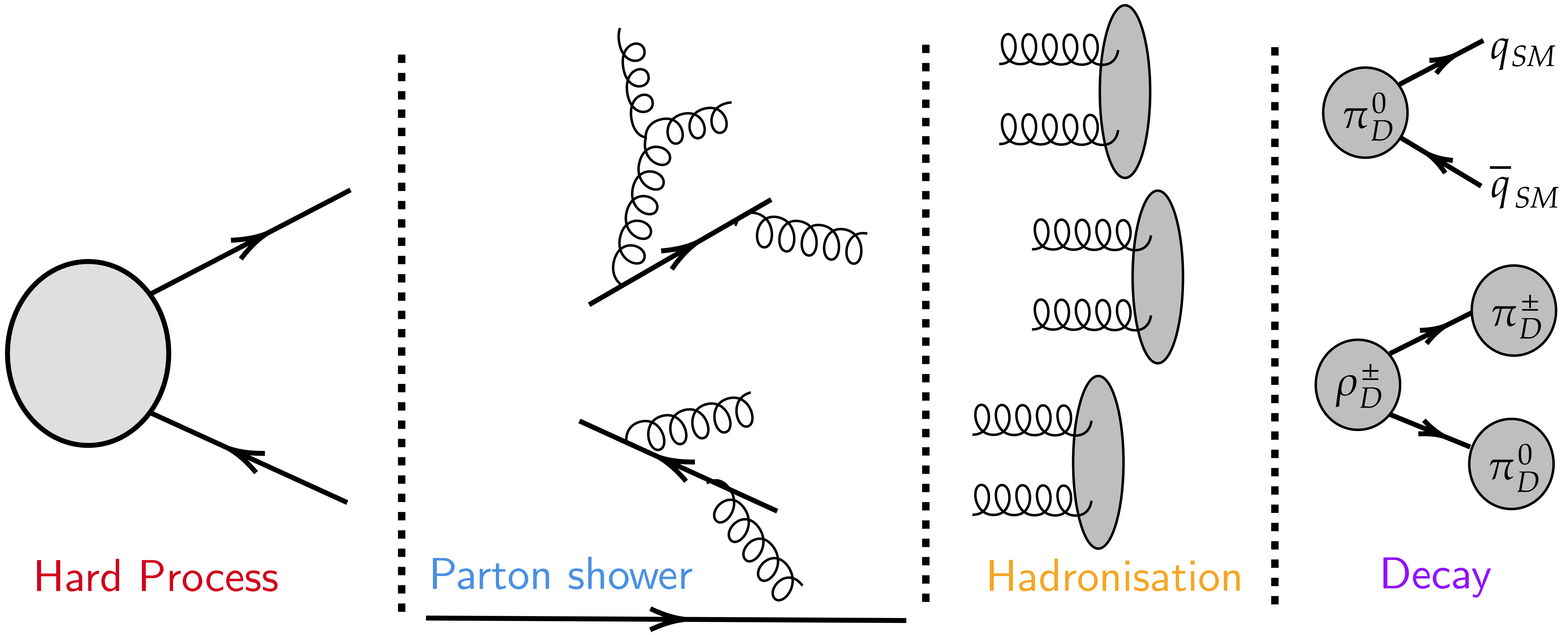
arXiv:2112.11125 (CMS)

- Extensive searches exist for DS in the 2-body parameter space. Could there be DS areas of parameter space that are evading our current searches?
- While signatures such as "semi-visible" jets are well-known, confining HVs are a relatively understudied area.
- Good time to think about the theory-signature space connections to see if we're missing any smoking guns. Need to be agnostic to internal DS parameters.

arXiv:2112.11125 (CMS)
arXiv:2305.18037 (ATLAS)
arXiv:1910.08447 (ATLAS)
arXiv:2102.10874 (ATLAS)
arXiv:1810.10069 (CMS)

Anomalous jet signatures

arXiv:1503.00009 - T. Cohen et al.



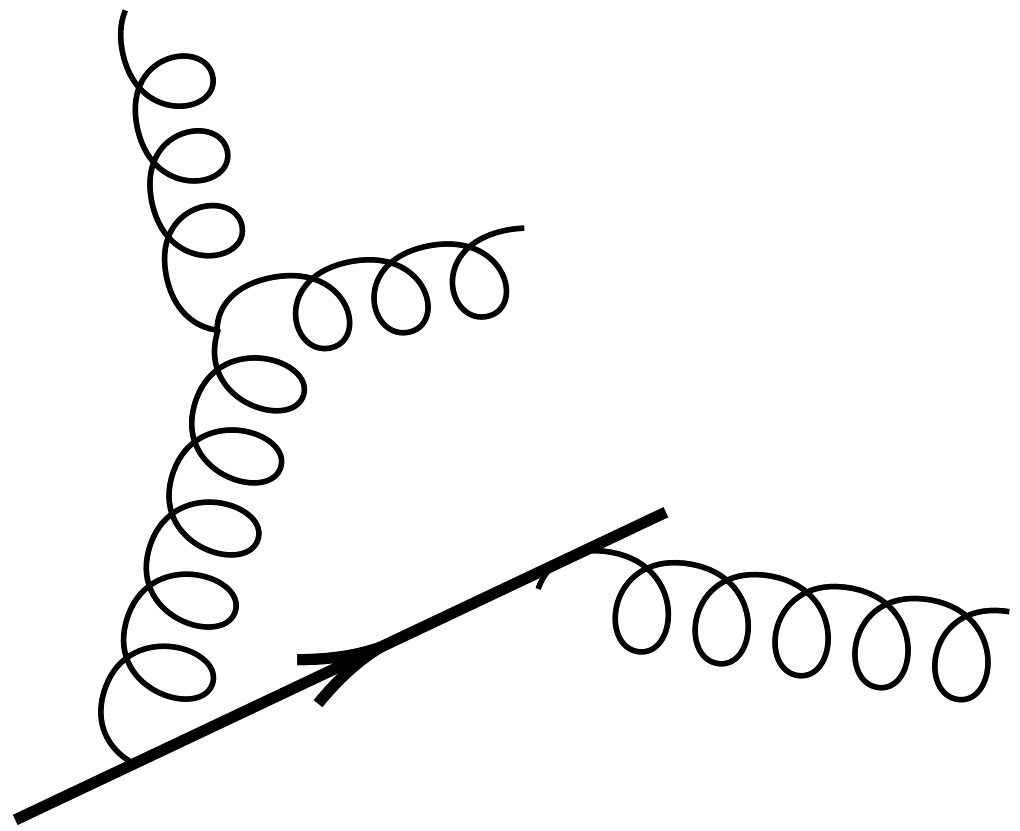
α runs, controlled by $N_F\,/\,N_C$ and Λ

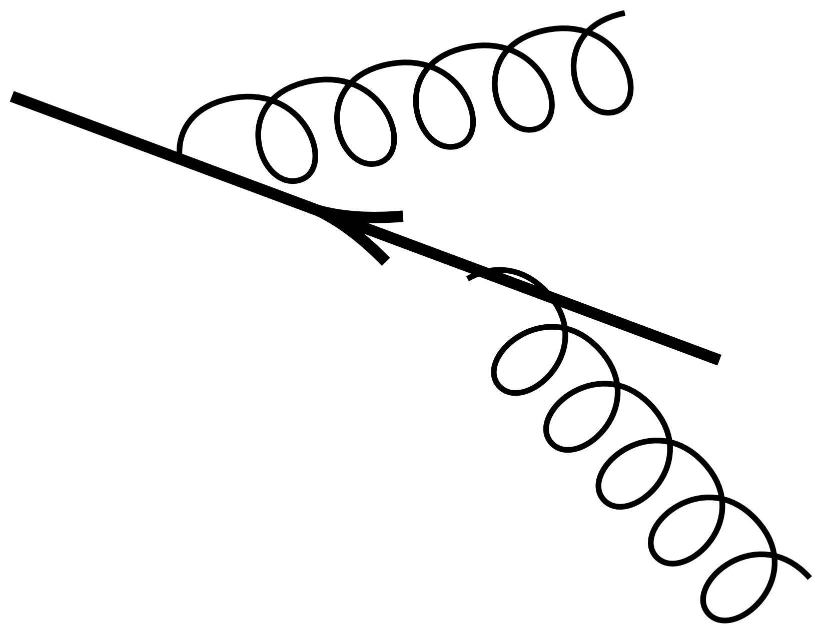
- Production of initial dark partons initiated through a hard process at a collider, via the mediator.
- The initial dark partons undergo parton showering eventually reach close to a characteristic energy scale where the shower stops and hadronisation occurs.

- A portion of dark mesons will decay to SM particles through the mediator, resulting in a jet with a mixture of stable dark hadrons and SM decay products.
- Typically these exotic jet signatures are known as "dark showers" and generically give rise to high multiplicity signatures which can have displaced vertices e.g. emerging, semi-visible jets or soft-bombs/SUEP.

Dark parton showering

Based on arXiv:2502.18566, Kulkarni, **JL**, Strassler, JHEP **2025**, 150 (2025)





- The 't Hooft gauge coupling, $\lambda = \alpha N_C$, in part controls parton showering behaviour, where α is governed by the Renormalisation Group Equations (RGE),

$$\mu^2 \frac{d\alpha}{d\mu^2} = \beta(\alpha) = -\alpha^2 \left(\beta_0 + \beta_1 \alpha\right) \qquad \text{(at 2-loop)}$$

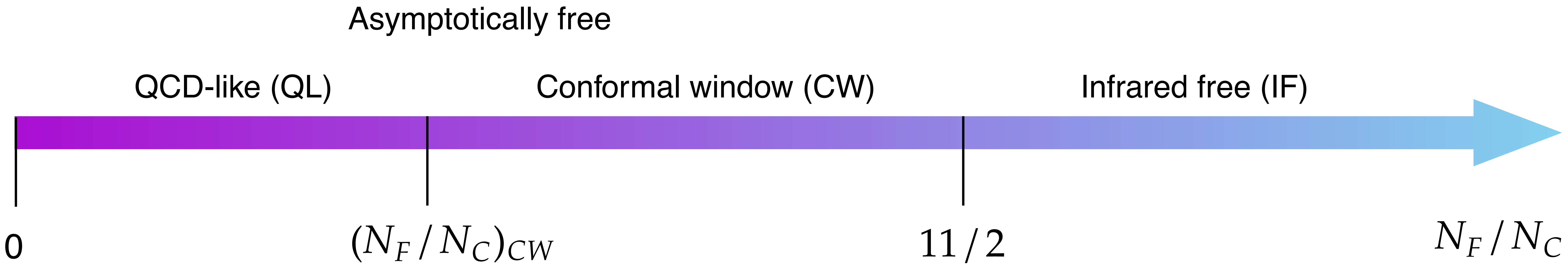
- Parton shower ends near scale Λ, which characterises breakdown of perturbative expansion of α. To a good approximation, the 't Hooft gauge coupling is governed solely by N_F/N_C and μ/Λ.

$$N_C\,\alpha = f(N_F/N_C, \mu/\Lambda) \; + \mathcal{O}(N_F/N_C^2) \text{ corrections}$$

- At two-loop, for $N_F/N_C \gtrsim 2.7$, α flows to a non-trivial infra-red fixed point (IRFP); α 'slows down' as $N_F/N_C \to 5.5$. New procedures are needed to understand parton showering within this region. T. Banks., A. Zaks, Nucl.Phys.B 196 ('82)

Non-trivial fixed point: $\alpha_* = -\dfrac{\beta_0}{\beta_1}$; > 0 for $N_F/N_C \gtrsim 2.7$

Near-conformal dark sector models



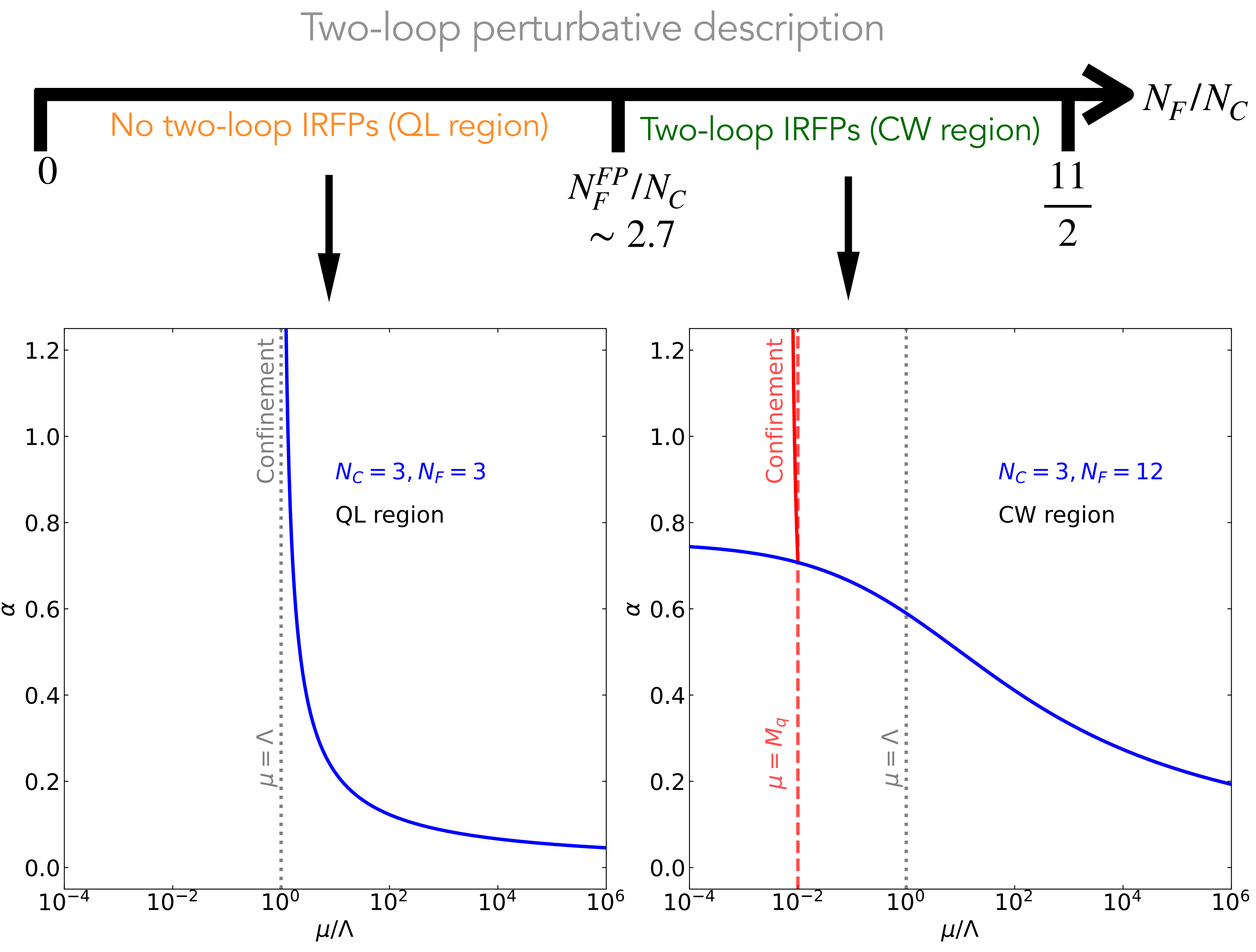
- For $N_F/N_C \geq \left(N_F/N_C\right)_{CW}$, the dark sector enters a "conformal window" (CW) - for a massless theory that is weakly coupled in the UV, the theory flows to an IR fixed point (IRFP). The theory is only strictly conformal for $\alpha(\mu_0) = \alpha_*$. Near the onset of the CW, the β function is hypothesised to become small over a large range of α - a "walking theory"

- Lots of work already done on the non-perturbative structure and the low-energy EFT descriptions of large N_F/N_C theories. Exact value of $\left(N_F/N_C\right)_{CW}$ still a matter of debate.

arXiv:2306.07236, A. Hasenfratz et al.
arXiv:2008.12223, J.W. Lee
arXiv:0902.3494, T. Appelquist et al.
arXiv:2312.08332, A. Pomarol et al.
arXiv:2312.13761, R. Zwicky
arXiv:0902.3494, F. Sannino

- Meson spectrum close to $\left(N_F/N_C\right)_{CW}$ very different from regular QCD. The 0^{++} σ meson is parametrically as light as the π's - different regime of the theory, σ acts like a Goldstone of conformal symmetry. Pythia hadronisation messy as is in HV module, adding σ quite a way off. arXiv:2306.07236

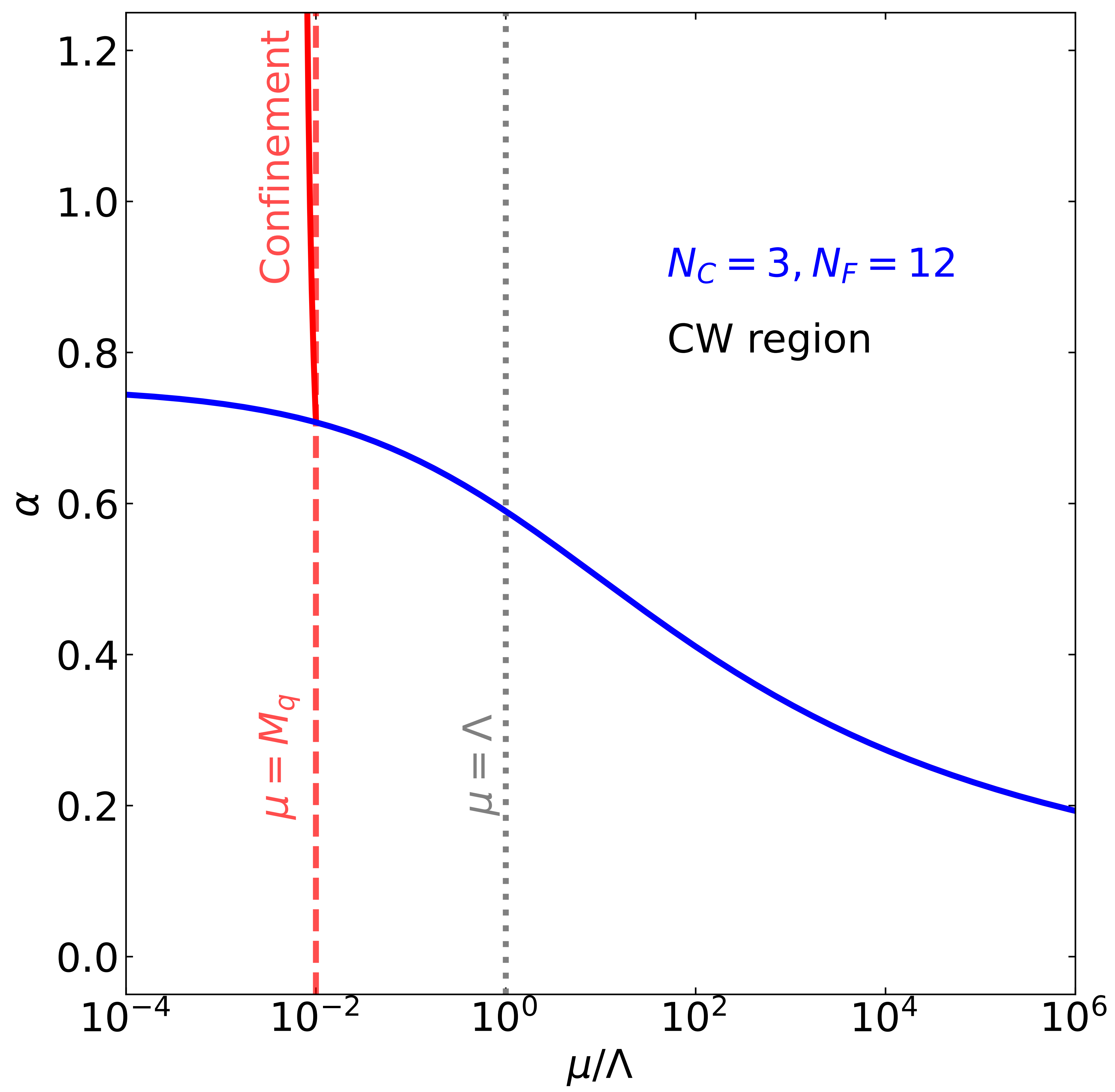
Near-conformal parton showering



"Confinement" for illustrative purposes only

- Choose to do parton showering with two-loop α - the first order IRFPs appear. New procedures are needed to understand parton showering within this region. T. Banks., A. Zaks, Nucl.Phys.B 196 ('82)
- Interesting phenomenology could occur for $M_q \neq 0$, $M_q \ll \Lambda$. If the number of massive quarks is enough to push the IR theory out of the conformal window then exotic hadronisation can occur around $\mu \sim M_q$.
- Can get around hadronisation problems by returning to a QL spectrum with conformal characteristics.
- Two-loop is merely illustrative - where confinement actually occurs is an open question. This requires more theory and lattice input.

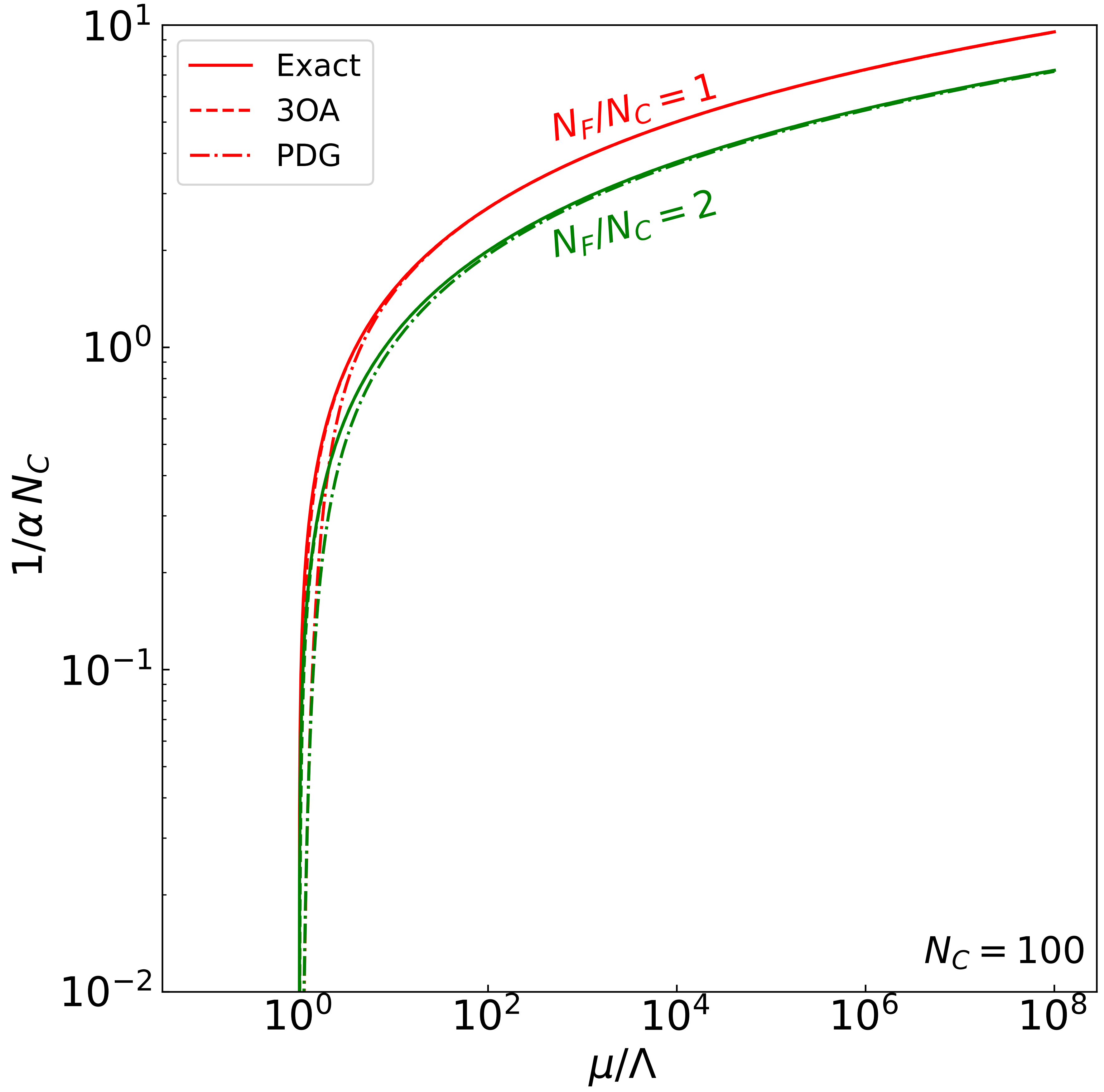
Near-conformal dark sectors



- Don't expect these scenarios to look like Soft Unclustered Energy Packets (SUEPs) BUT they can if you shower over a large IRFP ($\gg 1$) over a wide energy range - spherical event.
- For $\lambda \sim \mathcal{O}(1)$ over a large range of energies, expect a high-multiplicity soft event, but not spherically shaped events, just large ΔR. For $\lambda \ll 1$, expect low multiplicity and small ΔR - pencil-like jets.
- Can understand theories perturbatively when $\lambda_* = \alpha_* N_C \ll 1$ - Banks-Zaks (BZ) regime. BZ theories can appear as UV completions to a dark CFT sector. T. Banks., A. Zaks, Nucl.Phys.B 196 ('82) , arXiv:2207.10093
- A CFT dark sector is nothing new, often found as an interesting UV completion for dark sector or composite/Twin Higgs models. Interesting connections to the hierarchy problem. arXiv:1103.2571,1304.7006, 2007.14396,2308.16219, 2506.21659

Improving upon the current procedure

- QL parton showering is parameterised by a scale Λ. For SM QCD, Λ_{QCD} is a useful proxy for the scale of the theory. As such, to a good approximation, $\lambda(\mu) = \alpha(\mu)N_C$ is governed solely by N_F/N_C and μ/Λ.



$$N_C\,\alpha = f(N_F/N_C, \mu/\Lambda) \; + \mathcal{O}(N_F/N_C^2) \text{ corrections}$$

- Due to the presence of the IRFP, this definition of Λ no longer works beyond $N_F/N_C \gtrsim 2.7$ and thus existing approximations within event generators (the PDG formula) are insufficient to describe CW behaviour.

$$\alpha(\mu) = \frac{1}{\beta_0 \ln(\mu^2/\Lambda^2)}\left[1 + \frac{1}{\alpha_*}\frac{\ln[\ln(\mu^2/\Lambda^2)]}{\beta_0 \ln(\mu^2/\Lambda^2)}\right]$$

- α has a power-law form at μ/Λ in the CW region. Λ is not a proxy for the confinement scale, but rather characterises crossover between power-law and logarithmic running behaviours in the CW region.

$$\alpha - \alpha_* \sim \left(\frac{\mu^2}{\Lambda^2}\right)^{\gamma} \quad ; \quad \gamma = \left.\frac{\partial\beta}{\partial\alpha}\right|_{\alpha=\alpha_*} = \beta_0\alpha_* \qquad \text{(at 2-loop)}$$

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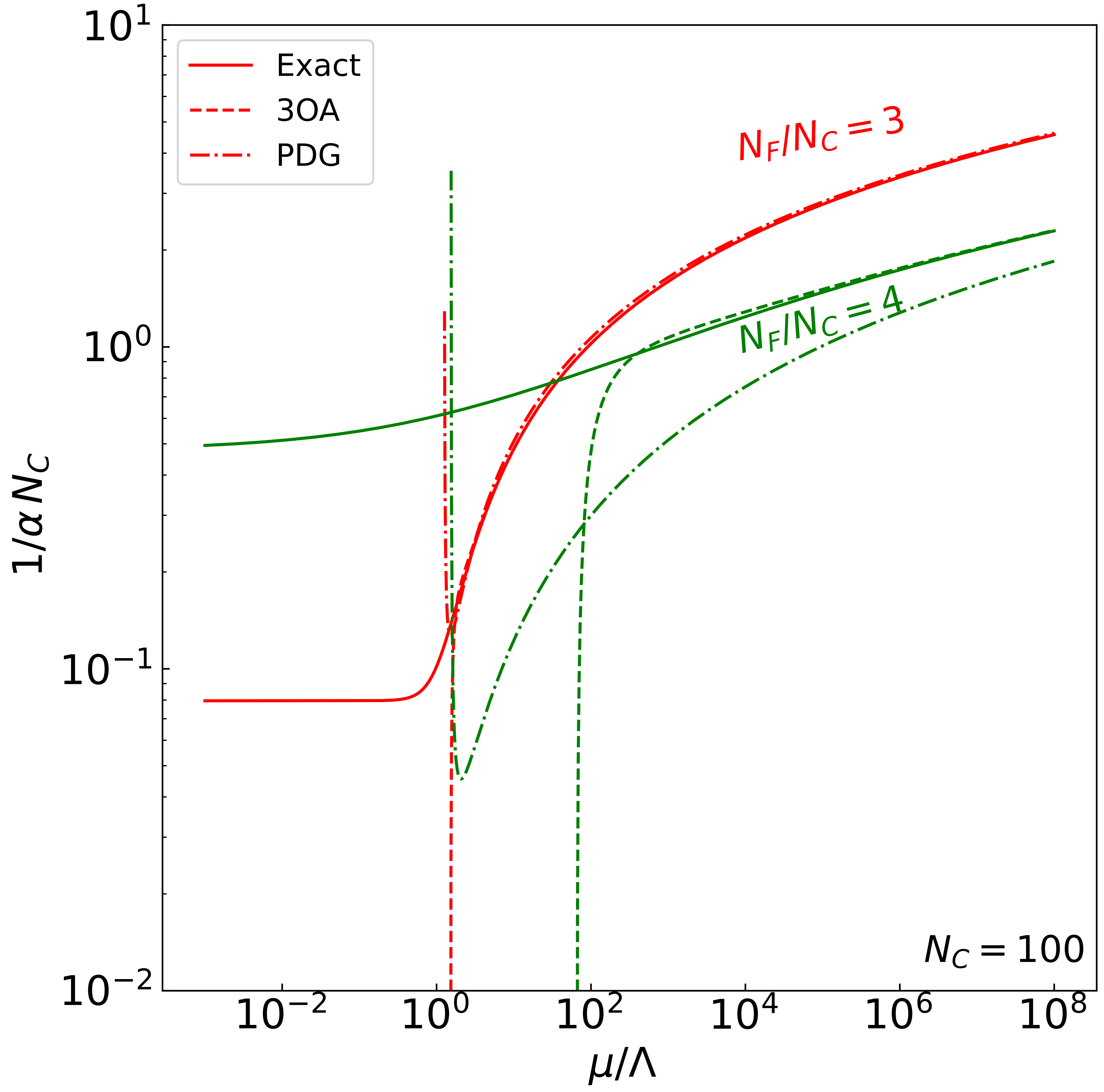

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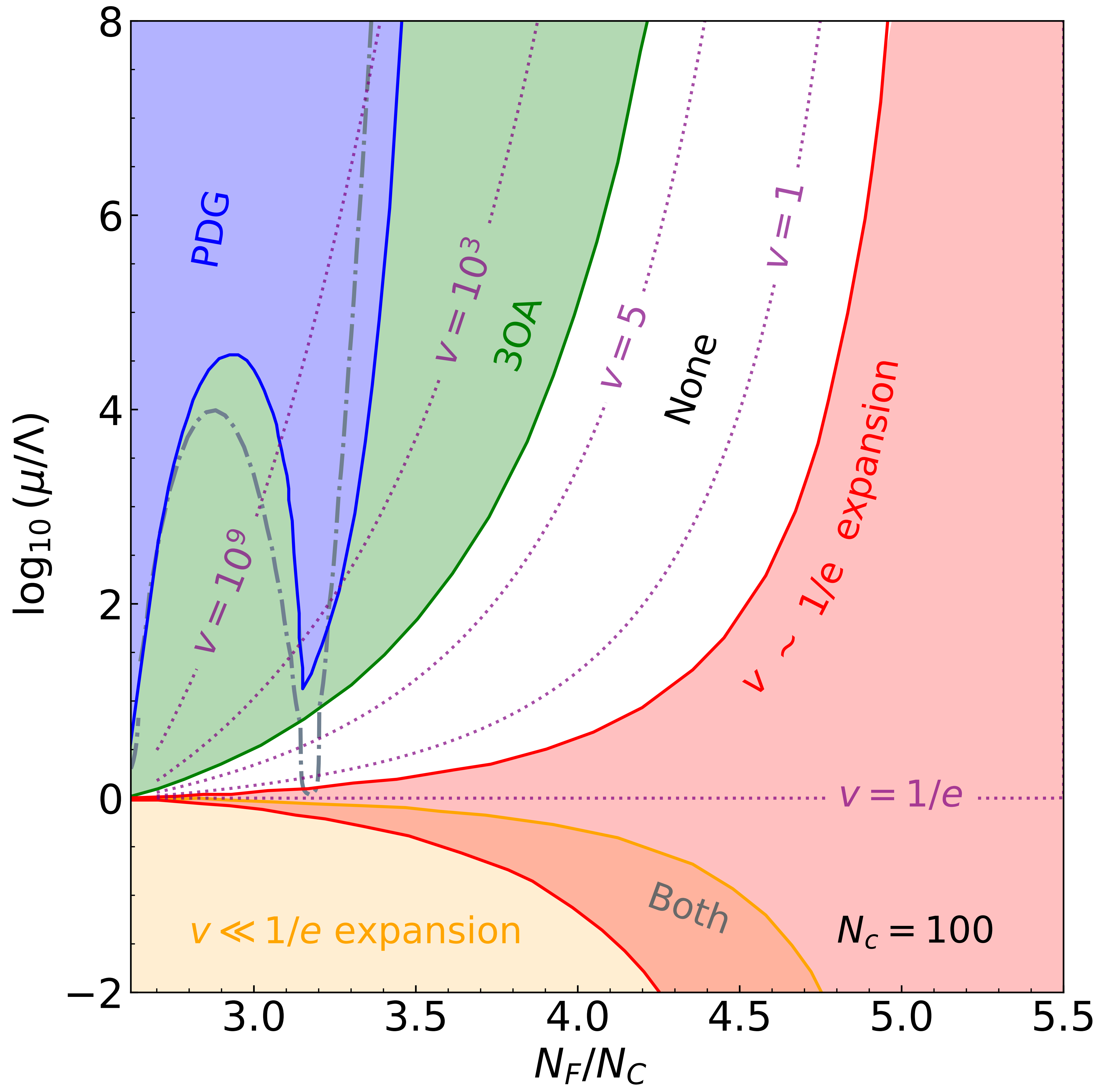
Monte Carlo implementation



- By taking IRFP into account, we get two RGE solutions describing α in both regions. Can find the explicit forms in both regions in terms of the two real branches of the LambertW function,

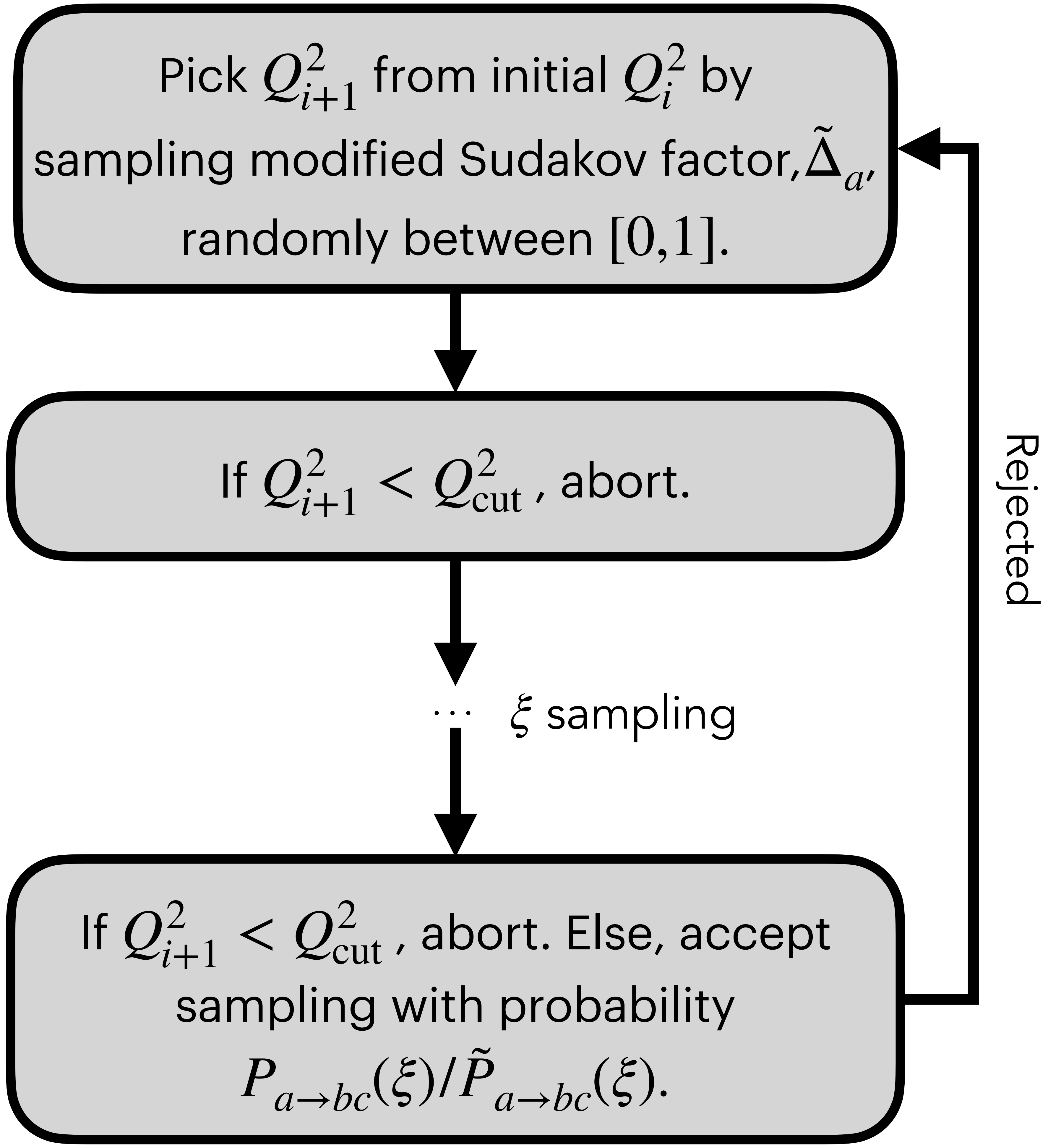
$$\alpha = \alpha_* \left[W_{-1}(-z) + 1\right]^{-1} \quad ; \quad \alpha = \alpha_* \left[W_0(z) + 1\right]^{-1} \quad ; \quad z = \frac{1}{e}\left(\frac{\mu^2}{\Lambda^2}\right)^{\beta_0 \alpha_*}$$

QL-region CW-region

- For large μ/Λ, use an asymptotic expansion at third order (3OA). For small μ/Λ, use Taylor expansions around $z = 0$ and $z = 1/e$. Validity (deviation ≥ 2%) reveals a large area of parameter space covered by no expansions.

- 3OA fails in CW region as $N_F/N_C \to 5.5$ since it expands in $1/|\alpha_*|$, valid in all QL but fails where $\alpha \sim \text{const}$. Natural to use 3OA where applicable and linearly interpolate in regions where it fails.

Two-loop veto algorithm in PYTHIA



- Δ_a is the Sudakov factor, the probability of no parton emissions between Q_i^2 and Q_{i+1}^2 and serves to interface α and parton emission effects.

$$\Delta_a = \exp\left(- \int_{Q_i^2}^{Q_{i+1}^2} \frac{dQ'^2}{Q'^2} \int_{\xi_{min}(Q'^2)}^{\xi_{max}(Q'^2)} \frac{\alpha}{2\pi} P_{a\to bc}(\xi') d\xi' \right)$$

arXiv:0603175 - T. Sjöstrand et al.
arXiv:1102.2126 - W. Giele et al.
arXiv:1101.2599 - A. Buckley et al.
arXiv:1211.7204 - L. Lonnblad et al.
R. Ellis, W. Stirling, B. Webber

- To generate a scale Q_{i+1}^2 given an initial scale Q_i^2, event generators pick a random number R_1 and solve for $\Delta_a\left(Q_{i+1}^2, Q_i^2\right) = R_1$.

- Difficult to invert Δ, so usually invert a much simpler $\tilde{\Delta}_a$, formed by overestimating $P_{a\to bc}$ with $\tilde{P}_{a\to bc}(\xi')$ and $\tilde{\xi}_{max}(Q_i^2) > \xi_{max}({Q_{i+1}}^2)$ and $\tilde{\xi}_{min}(Q_i^2) < \xi_{min}({Q_{i+1}}^2)$. Corrected for through Sudakov veto algorithm.

Two-loop veto algorithm in PYTHIA

Pick Q_{i+1}^2 from initial Q_i^2 by sampling modified Sudakov factor, $\tilde{\Delta}_{a'}$, randomly between $[0,1]$.

↓

For two-loop, overestimate $\alpha_{2-\text{loop}}$ with $\alpha_{1-\text{loop}}$ and accept sampling with probability $\alpha_{2-\text{loop}}/\alpha_{1-\text{loop}}$.

↓

If $Q_{i+1}^2 < Q_{cutoff}^2$, abort.

- At one-loop, we can now relate the inverted Q_{i+1}, given a modified Sudakov factor $\tilde{\Delta}_a$ and initial Q_i in closed form,

$$Q_{i+1}^2 = \Lambda^2 \left(\frac{Q_i^2}{\Lambda^2}\right)^{\tilde{\Delta}_a^{2\pi\beta_0/\epsilon}} \quad ; \quad \epsilon_a = \int_{\tilde{\xi}_{min}}^{\tilde{\xi}_{max}} \sum_{b,c} \tilde{P}_{a\to bc}(\xi')d\xi';$$

ϵ_a is the "emission coefficient"

- At two-loop, it gets trickier; no implementation of the LambertW function in HV module and Pythia uses the PDG form of α - hard to invert like at one-loop.

- Instead, Pythia uses the one-loop equation above, and assumes that two-loop corrections are small. Corrects for this using a mini-veto algorithm with probability $\alpha_{2-\text{loop}}/\alpha_{1-\text{loop}}$.

arXiv:0603175 - T. Sjöstrand et al.
arXiv:1101.2599 - A. Buckley et al.
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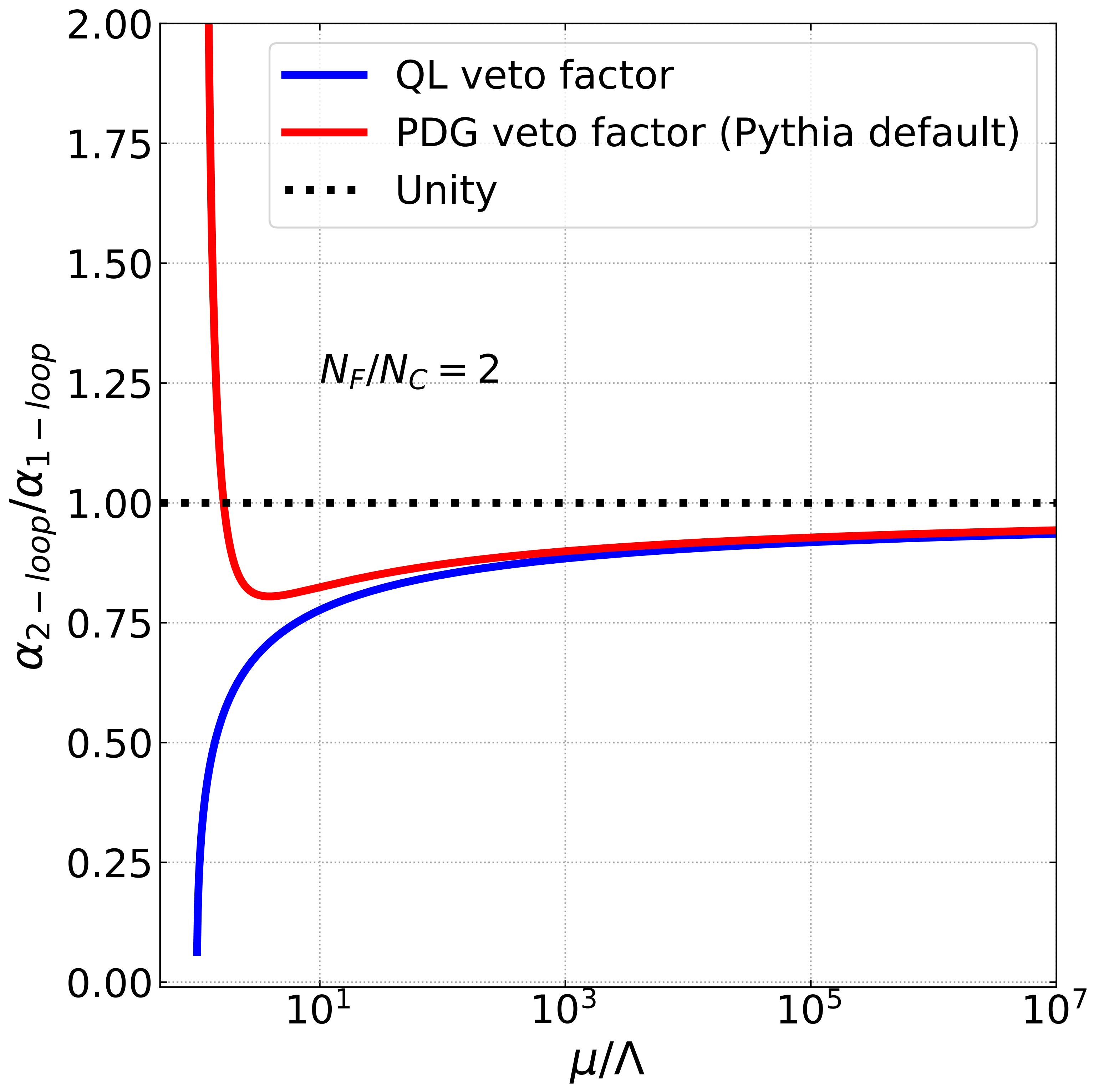
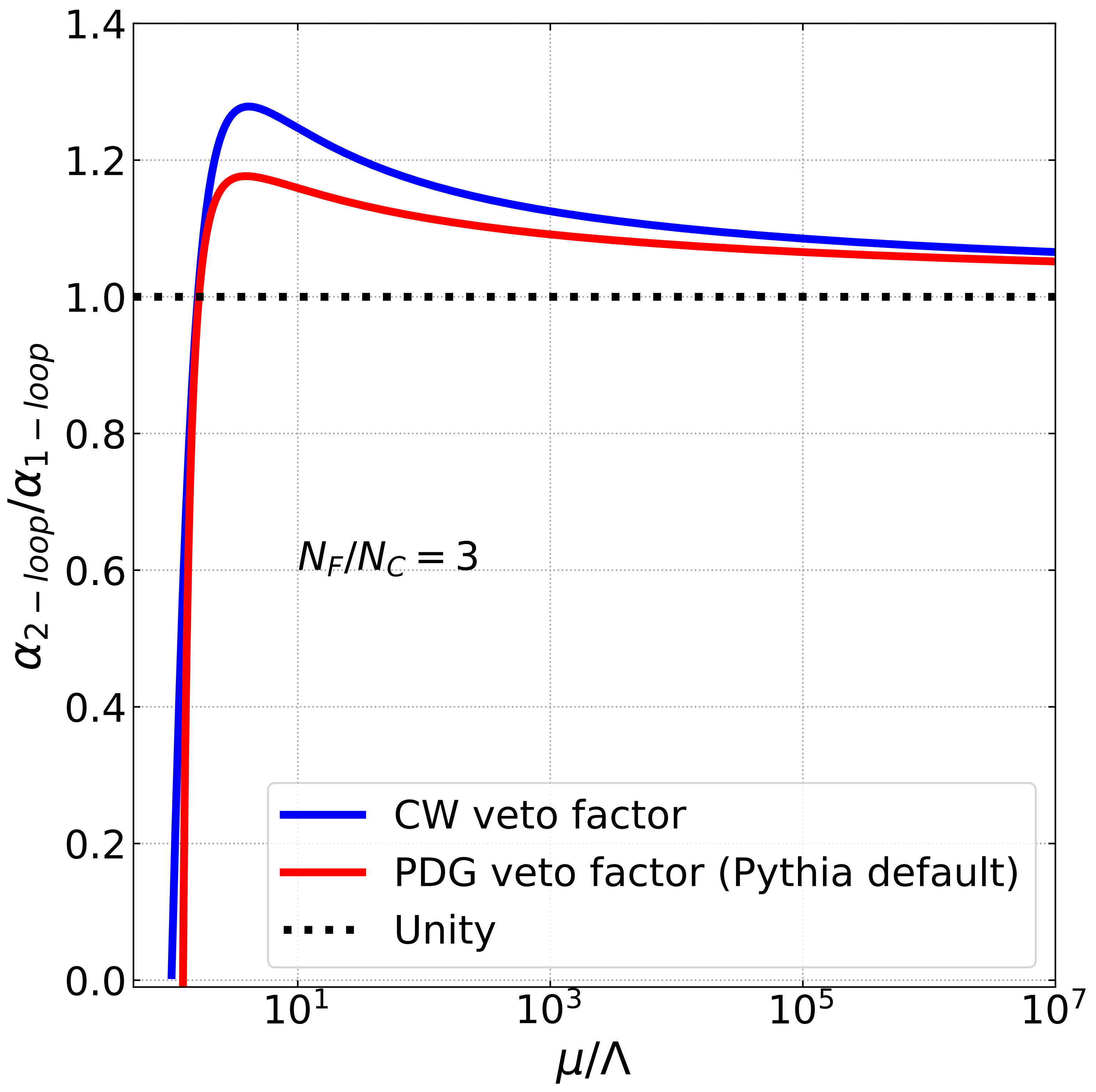




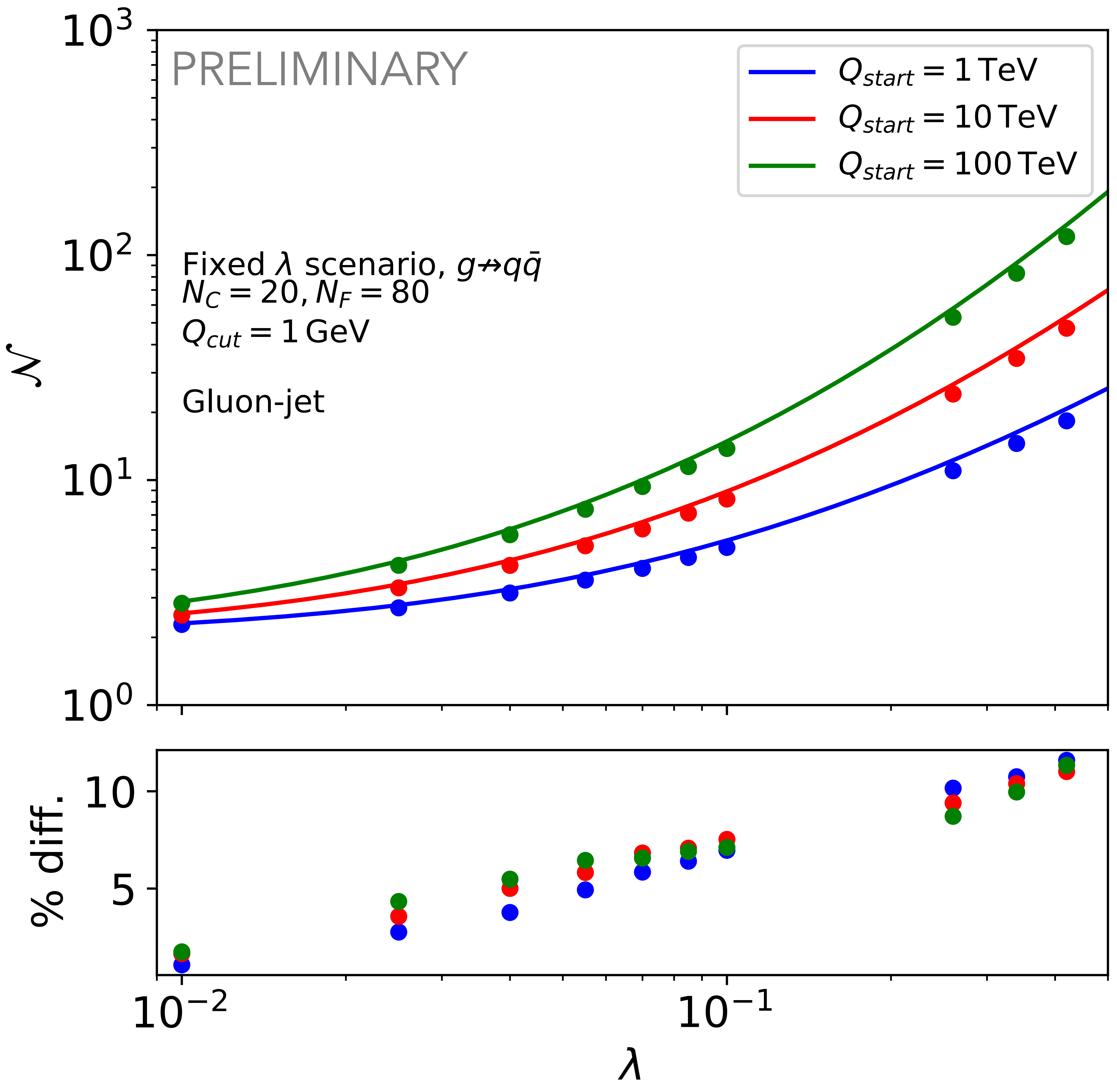
- Two-loop α veto algorithm is not applicable for the entire $N_F/N_C - \mu/\Lambda$ space, especially the CW region where we know two-loop corrections are large. Works well in QL region.
- Can calculate an inverse at two-loop using overestimated splitting functions, no need for any $\alpha_{2-\mathrm{loop}}$ veto algorithm.
- Here's where the LambertW function enters Pythia. As before, can use 3OA in QL and combination of 3OA + interpolation in CW.

QL region
$$Q_{i+1}^2 = \Lambda^2 \left(\frac{Q_i^2}{\Lambda^2}\right)^{\tilde{\Delta}_a^{2\pi\beta_0/\epsilon}} \left[\tilde{\Delta}_a^{2\pi\beta_0/\epsilon} \left(-eW_{-1}(-z_i)\right)^{1-\tilde{\Delta}_a^{2\pi\beta_0/\epsilon}}\right]^{1/\gamma}$$

CW region
$$Q_{i+1}^2 = \Lambda^2 \left(\frac{Q_i^2}{\Lambda^2}\right)^{\tilde{\Delta}_a^{2\pi\beta_0/\epsilon}} \left[\tilde{\Delta}_a^{2\pi\beta_0/\epsilon} \left(eW_0(z_i)\right)^{1-\tilde{\Delta}_a^{2\pi\beta_0/\epsilon}}\right]^{1/\gamma}$$

$$z_i = z\Big|_{Q=Q_i}$$

Average dark parton multiplicity



Nucl.Phys.B 377 (1992) 445-460 (Catani et al.), arXiV: hep-ph/9709246, 1310.8534

*Suchita Kulkarni, **JL**, Simon Plätzer, Matt Strassler, work ongoing*

- First Pythia HV simulations for two-loop running using Sudakov/LambertW implementations; not quantitative without parton multiplicity. Toy-model - neglect $P_{g\to q\bar{q}}$ and CMW scheme. S. Catani, B. R. Webber, G. Marchesini, Nucl. Phys. B 349 ('91)
- Two-loop complex, better to evaluate in const $\lambda = \alpha N_C$ scenario. Occurs when; a) $Q_{\rm start}, Q_{\rm cut} \geq \Lambda$, b) the BZ scenario, c) $Q_{\rm start}, Q_{\rm cut} \leq \Lambda$ and d) if $\lambda(Q_{\rm start}) = \lambda_*$.
- Can solve differential equation at $\mathcal{O}(\alpha^{1/2})$ for $N_{\rm gluon-jet}$ (in the approximation $g \nrightarrow q\bar{q}$). $N_{\rm gluon-jet}$ increases with λ and $Q_{\rm start}/Q_{\rm cut}$. Analytic results and simulation differ as λ increases and boundary conditions a source of concern.

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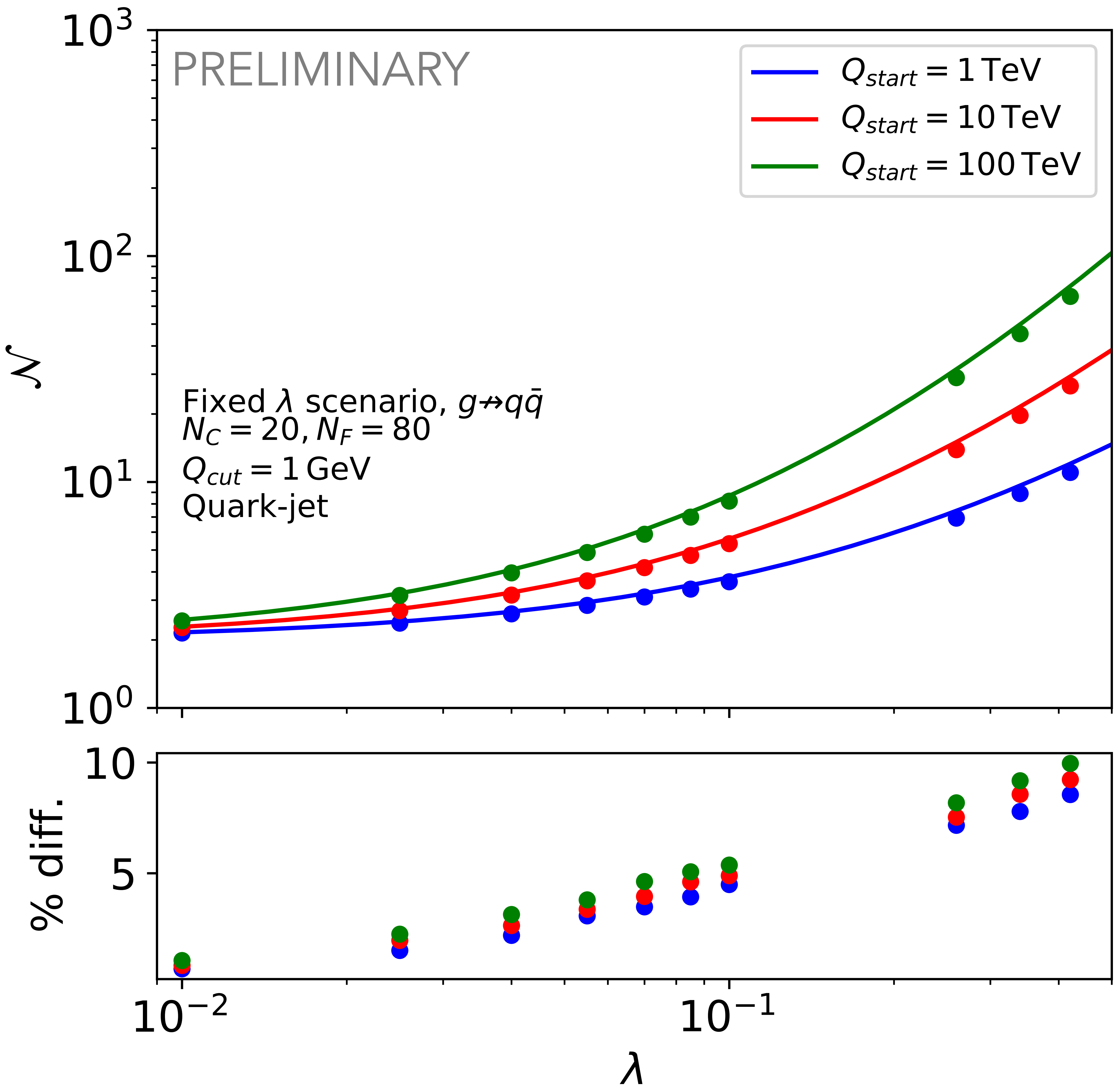

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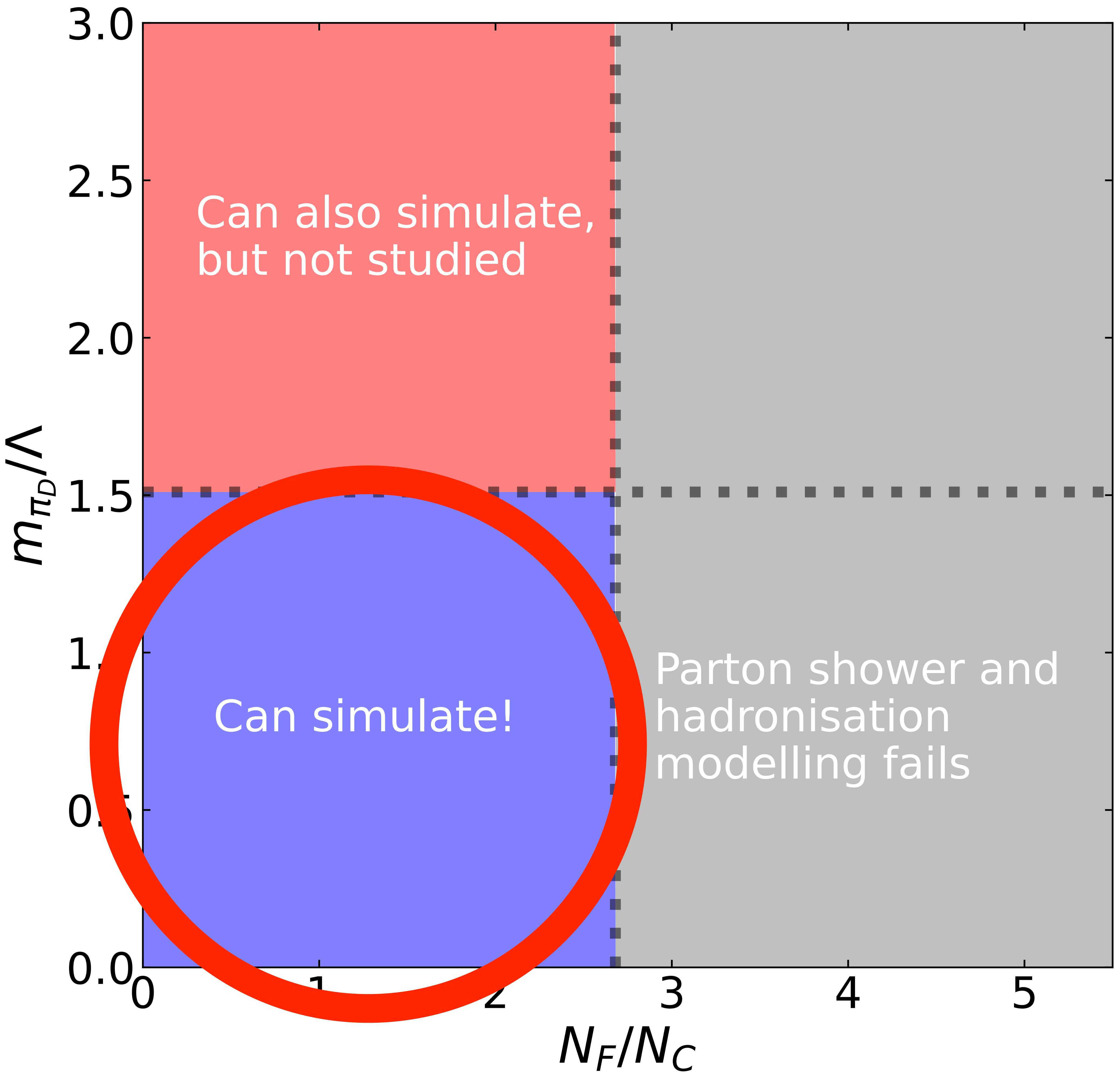
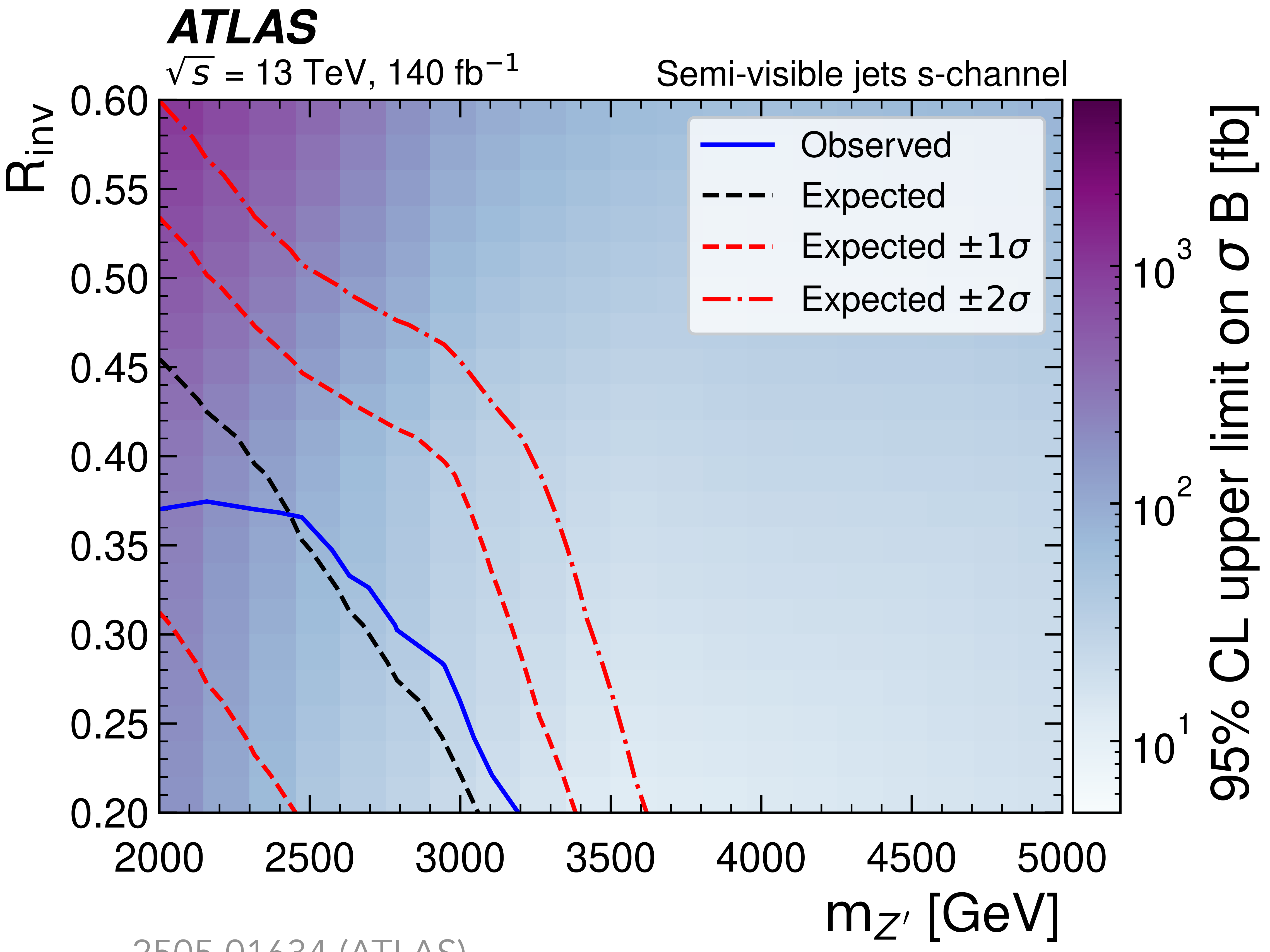
Presenting Hidden Valley results

Based on arXiv:2505.03058, Wei Liu, JL, Suchita Kulkarni PRD, 2025 (accepted)



- Simulations of near-conformal HV signatures partially complete, once confirmed need to focus on hadronisation and understand the decay of the bound states. How do we search for these theories?
- Many different phases/phenomenologies within HV theory space - must design search strategies that accommodate all of these signatures topologies and are agnostic to dark-sector content.
- Let's return to the safety of QL dark-sectors (below the conformal window) within $\rho^{0/\pm} \rightarrow \pi^{0/\pm}\pi^{\mp}$ channel. Here, standard search results presented in $R_{\text{inv}} — m_{Z'}$ plane - convenient but it can only tell you so much. arXiv: 1503.00009, 2403.01556 (CMS), 2502.11237, 2505.01634 (ATLAS)

Presenting Hidden Valley results



2505.01634 (ATLAS)

$$R_{\rm inv} = \left\langle \frac{\text{number of stable hadrons}}{\text{number of total hadrons}} \right\rangle$$

- It's like hiking with a road map; useful for generic properties, can't describe detailed terrain.
- Dark-showers properties dependence $m_{Z'}$ is well-known. Properties also a function of Λ, m_π and N_F/N_C - $R_{\rm inv}$ can't encompass this multi-dimensional space without losing info and features.
- Good idea to rethink $R_{\rm inv}$ for HL-LHC; not applicable to the many dark sector phases. Instead, let's think about the multi-dimensional DS; benchmark the dark sector, not the mediator!

Presenting Hidden Valley results

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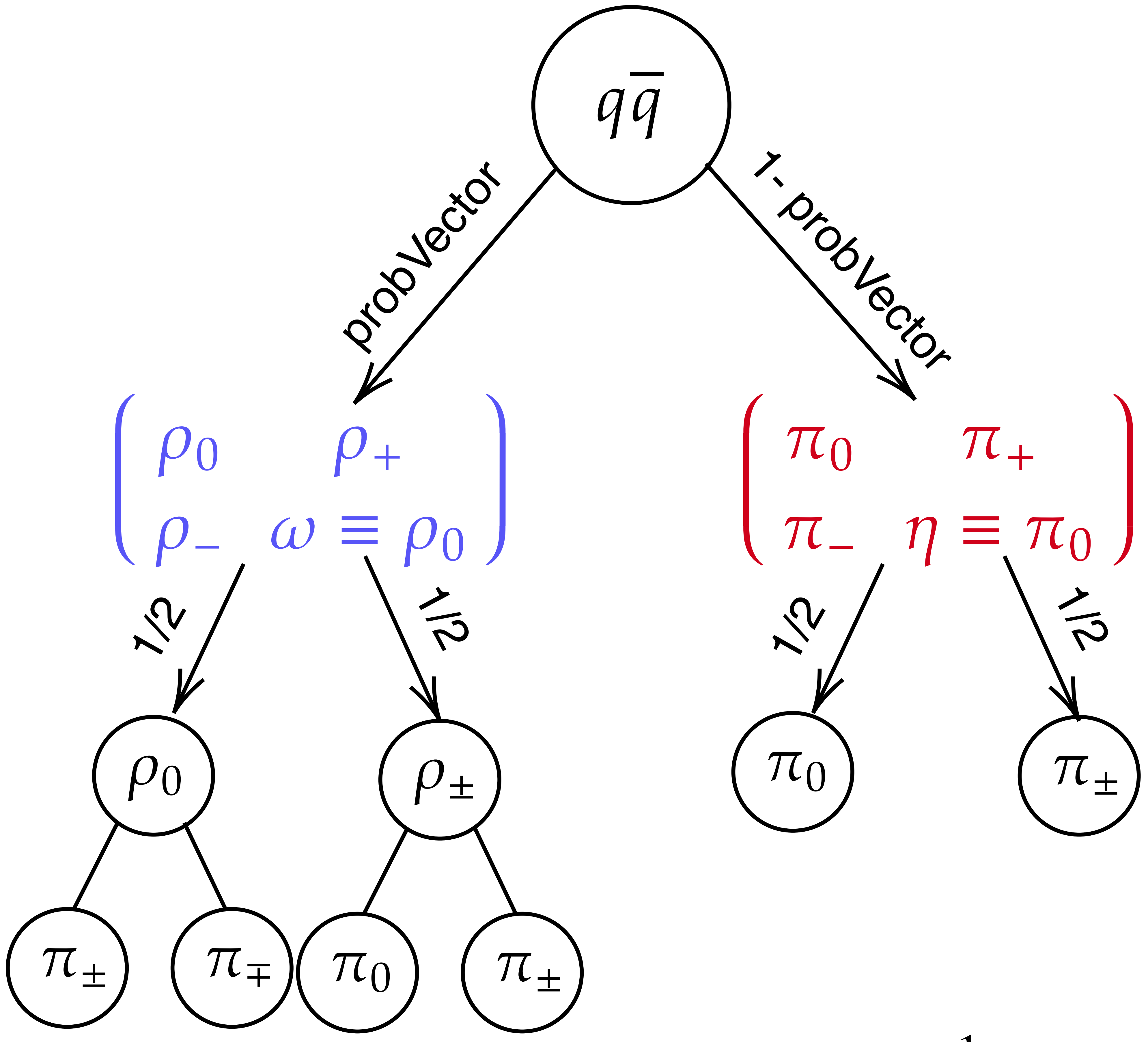
Tarvisio, Italy (©Tabacco)

$$R_{\text{inv}} = \left\langle \frac{\text{number of stable hadrons}}{\text{number of total hadrons}} \right\rangle$$

- It's like hiking with a road map; useful for generic properties, can't describe detailed terrain.
- Dark-showers properties dependence $m_{Z'}$ is well-known. Properties also a function of Λ, m_π and N_F/N_C - R_{inv} can't encompass this multi-dimensional space without losing info and features.
- Good idea to rethink R_{inv} for HL-LHC; not applicable to the many dark sector phases. Instead, let's think about the multi-dimensional DS; benchmark the dark sector, not the mediator!

The complex dark sector

- Will study $\rho^{0/\pm} \to \pi^{0/\pm}\pi^{\mp}$ open channel ($m_\pi/\Lambda \lesssim 1.5$, $m_\rho > 2m_\pi$) and $N_F/N_C \lesssim 2.7$ (to avoid conformal window). $\pi^\pm$ stable, π^0 are the LLPs and only decay via $\pi^0 \to q_{SM}\bar{q}_{SM}$.
- We will reintrepret the search for these hadronic decays of LLPs in the CMS muon endcap detector for DS. arXiv: 2107.04838 (CMS)
- LLP multiplicity is a complicated function of N_F/N_C, probVector (determines spin-0/1 counting) and Λ for fixed Z′ properties.
- Assuming a degenerate η', in general, we have N_F^2 π with $1/N_F$ diagonal π^0 and $1 - 1/N_F$ off-diagonal $\pi^\pm$. Similarly situation occurs for the ρ with a degenerate ω.



$$\langle N_{\pi_0} \rangle = \frac{1}{2} \langle N^0_{\text{init}} \rangle$$

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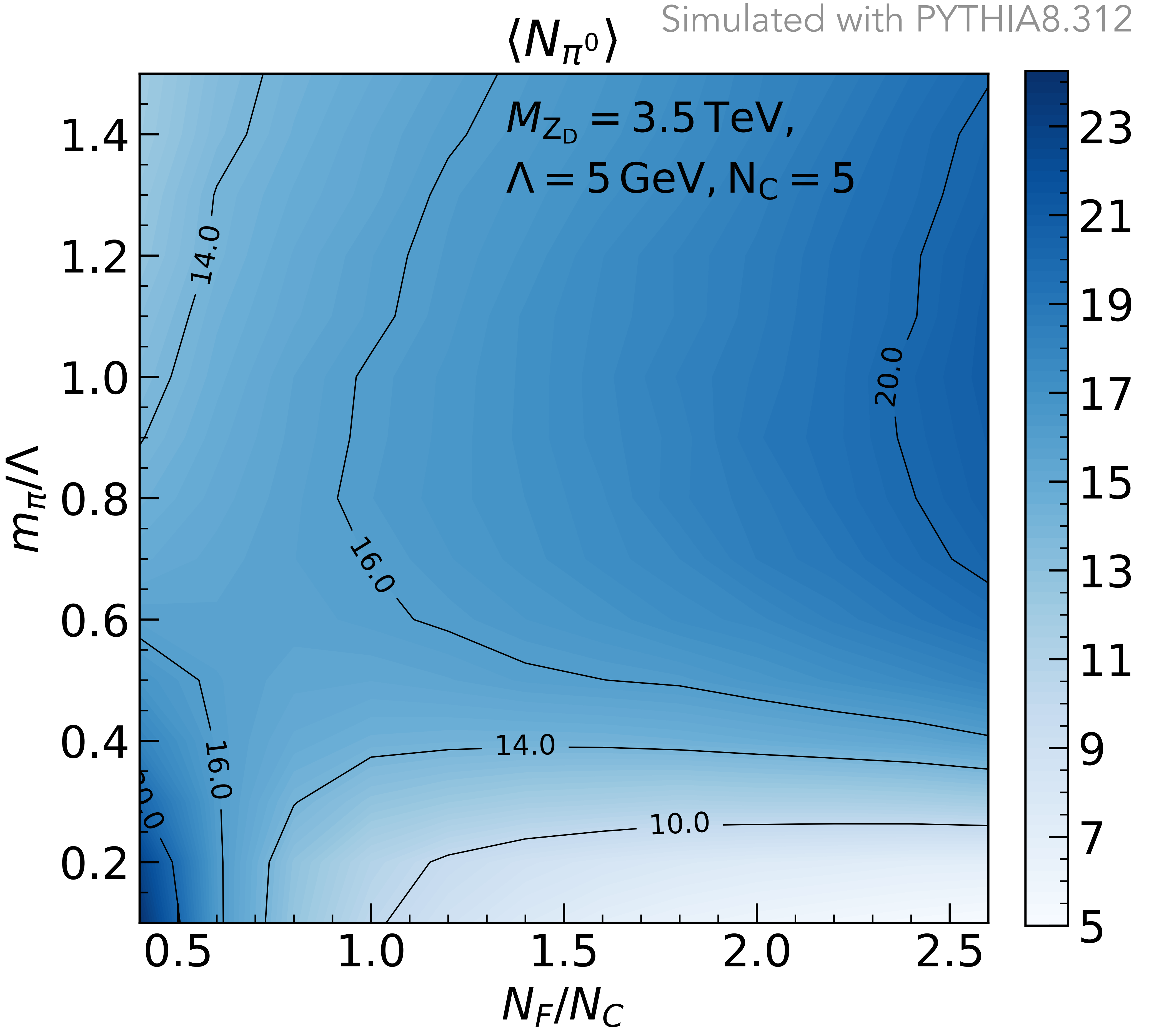
Simulated with PYTHIA8.312



$$\langle N_{\pi_0}\rangle = \left(\frac{1}{N_F} + \mathrm{probVector}\left(1 - \frac{2}{N_F}\right)\right)\langle N^0_{\mathrm{init}}\rangle$$

The complex dark sector

- Pythia uses the Lund Model: breaks strings between colour sources into hadrons through a probability distribution.
- Is determined by three parameters: a, $\hat{b} = b_{\text{Lund}}/m_Q$ - which determine the momentum fraction of hadron, and $\hat{\sigma} = \sigma m_Q^2$ which determines the p_T of every hadron. $M_Q \approx m_q + \Lambda$ - constituent quark mass.
- The fragmentation function for string breaking is given as

$$f(z) = z^{-1}(1 - z)^a \exp(-\hat{b} m_T^2 / m_Q^2 z)$$

- a is related to $\hat{b}$ through the string tension (default is Monash tune)

$$\langle \kappa\tau \rangle / m_Q^2 = (1 + a)/\hat{b}$$

- Modelling of the $\mathrm{probVector}$, $\hat{b}$ and $\hat{\sigma} = \sigma m_Q^2$ are all sources of significant hadronisation uncertainty.

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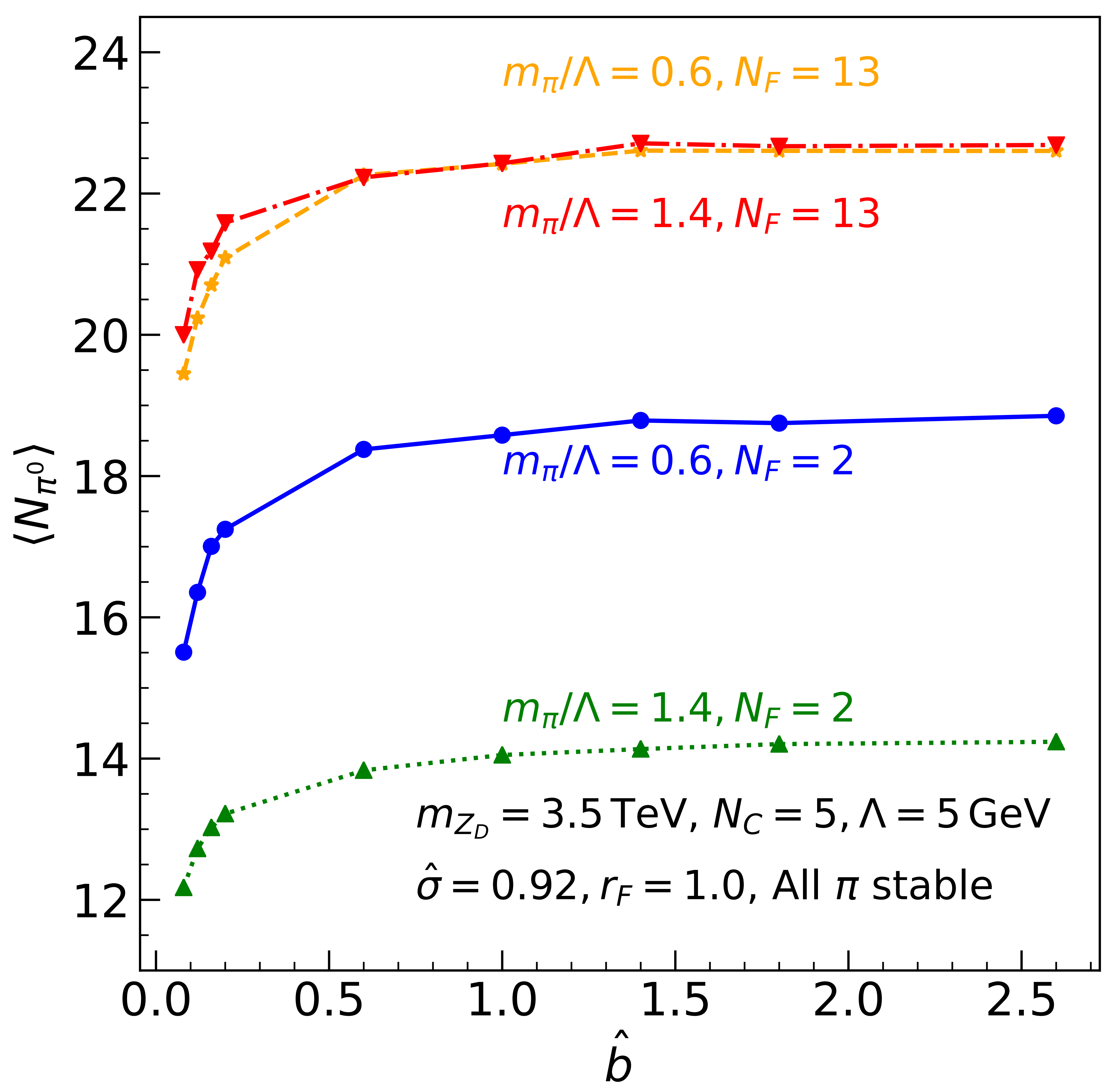

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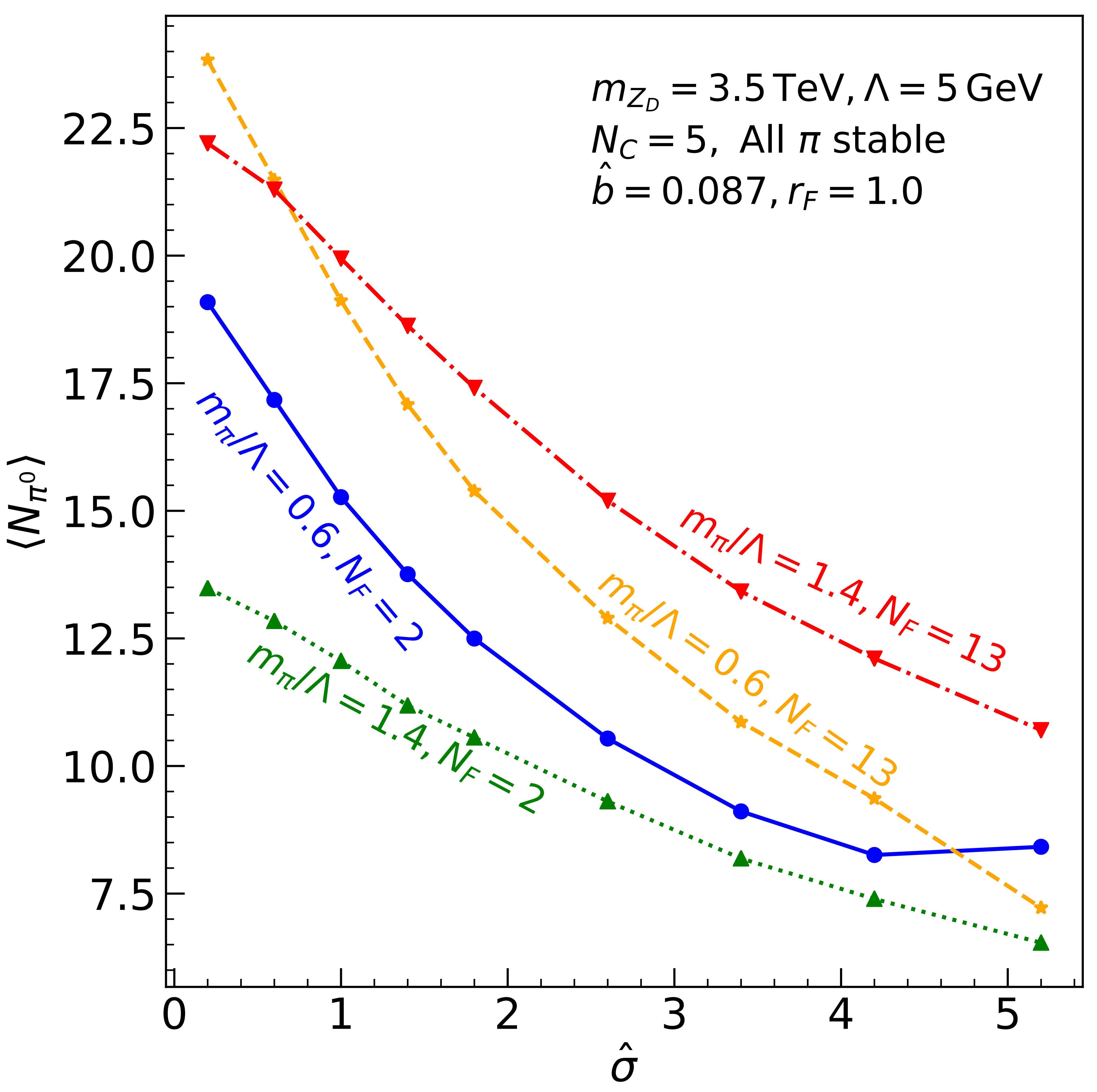
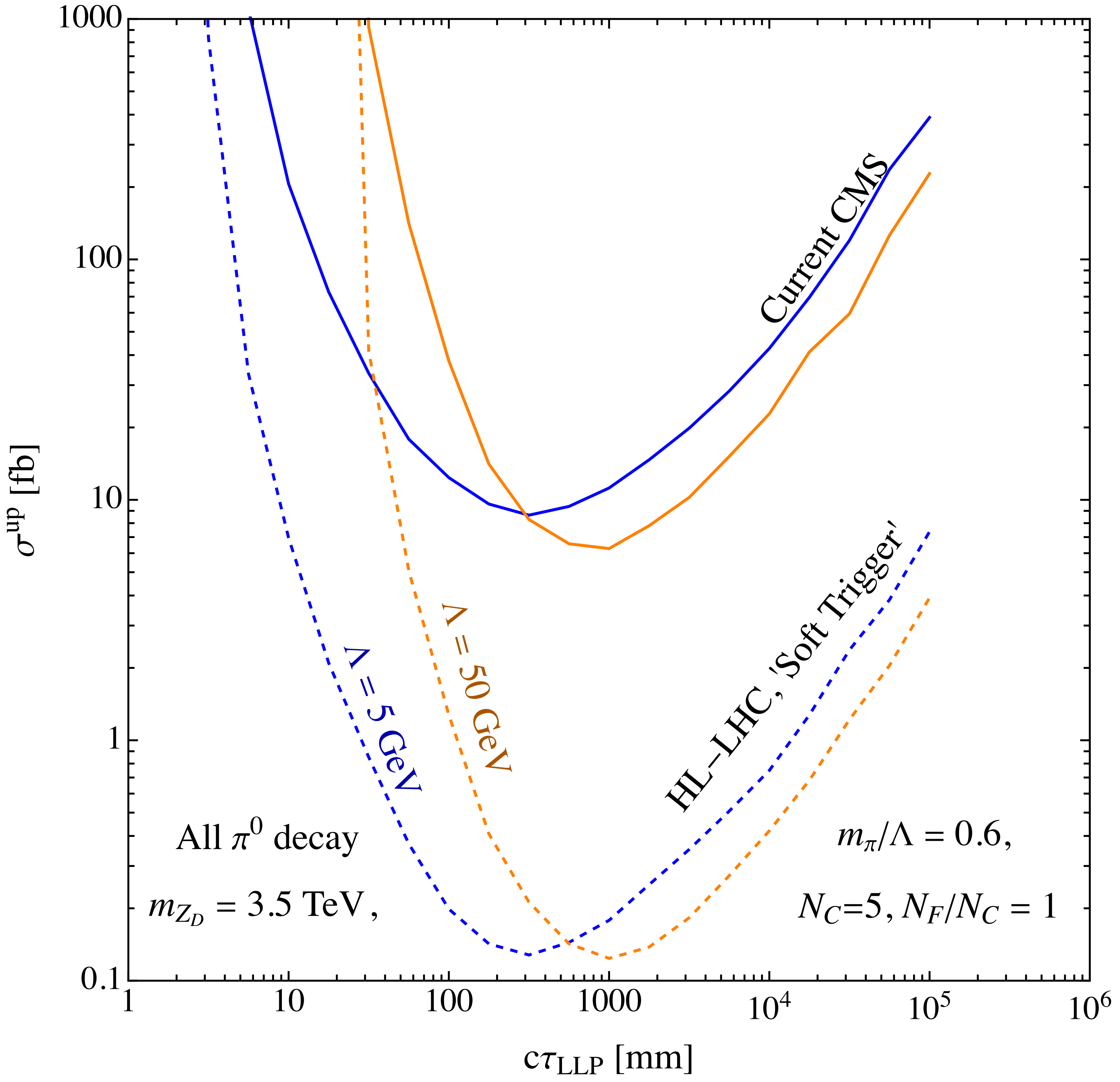
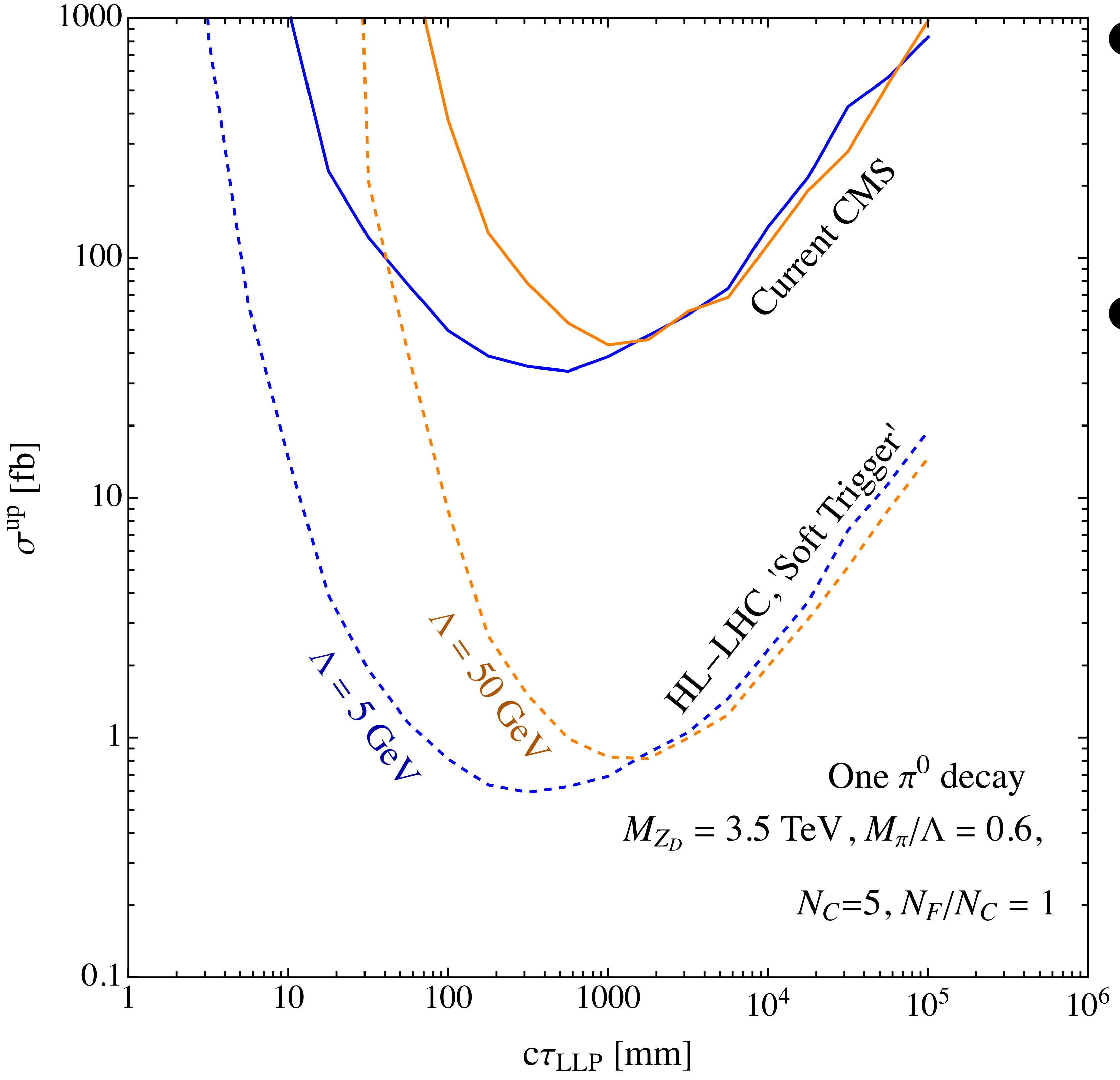


Dark sector parameters: lifetime sensitivity





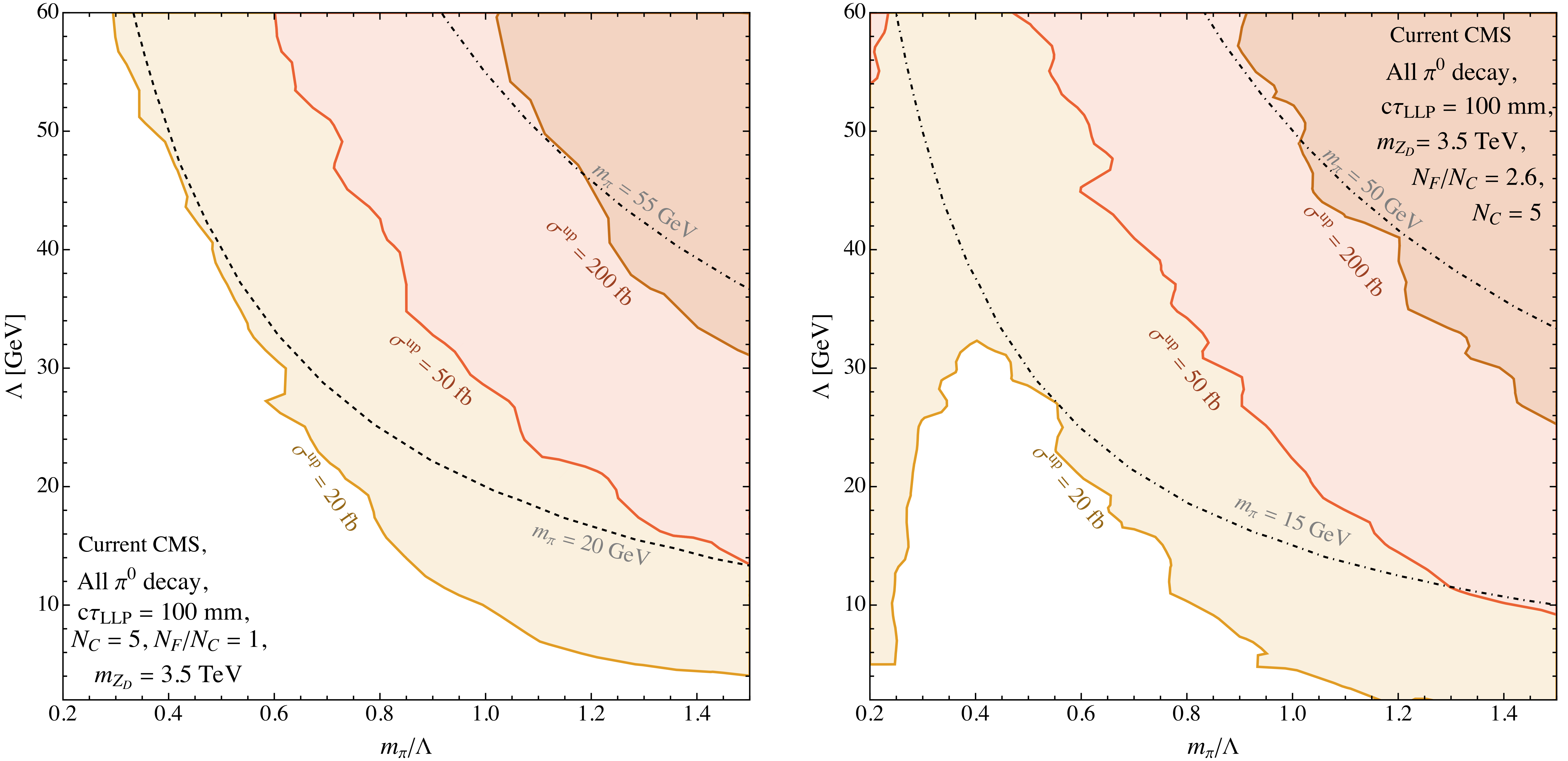
- $\pi^0 \rightarrow q_{SM}\bar{q}_{SM}$ appears as a high multiplicity cluster of hits within the CSC (Cathode Strip Chambers).
- Can derive model-independent cross section upper-limits through

$$\sigma^{up} = \frac{N^{up}}{\epsilon_{tot} \times \mathscr{L}}$$

- Current CMS run-2 has $\mathscr{L}$ = 137 fb $^{-1}$, $N^{up} \approx 6$, where HL-LHC run has $\mathscr{L}$ = 3000 fb^{-1}, $N^{up} \approx 3$.

- Cross-section upper-limit (σ^{up}) improved by $\mathscr{O}(10^2)$ at HL-LHC due to increased luminosity and acceptance. Cross section upper limits are the best in the all-π^0 decay scenario by up to a factor of $1/N_F$. $c\tau_{LLP} \sim \mathscr{O}$(100mm) displays the best sensitivity.
- Focus on all π_0 decay scenario, with $c\tau_{LLP} = 100$mm and $M_{Z_D} = 3.5$ TeV as these give us the best upper limits.

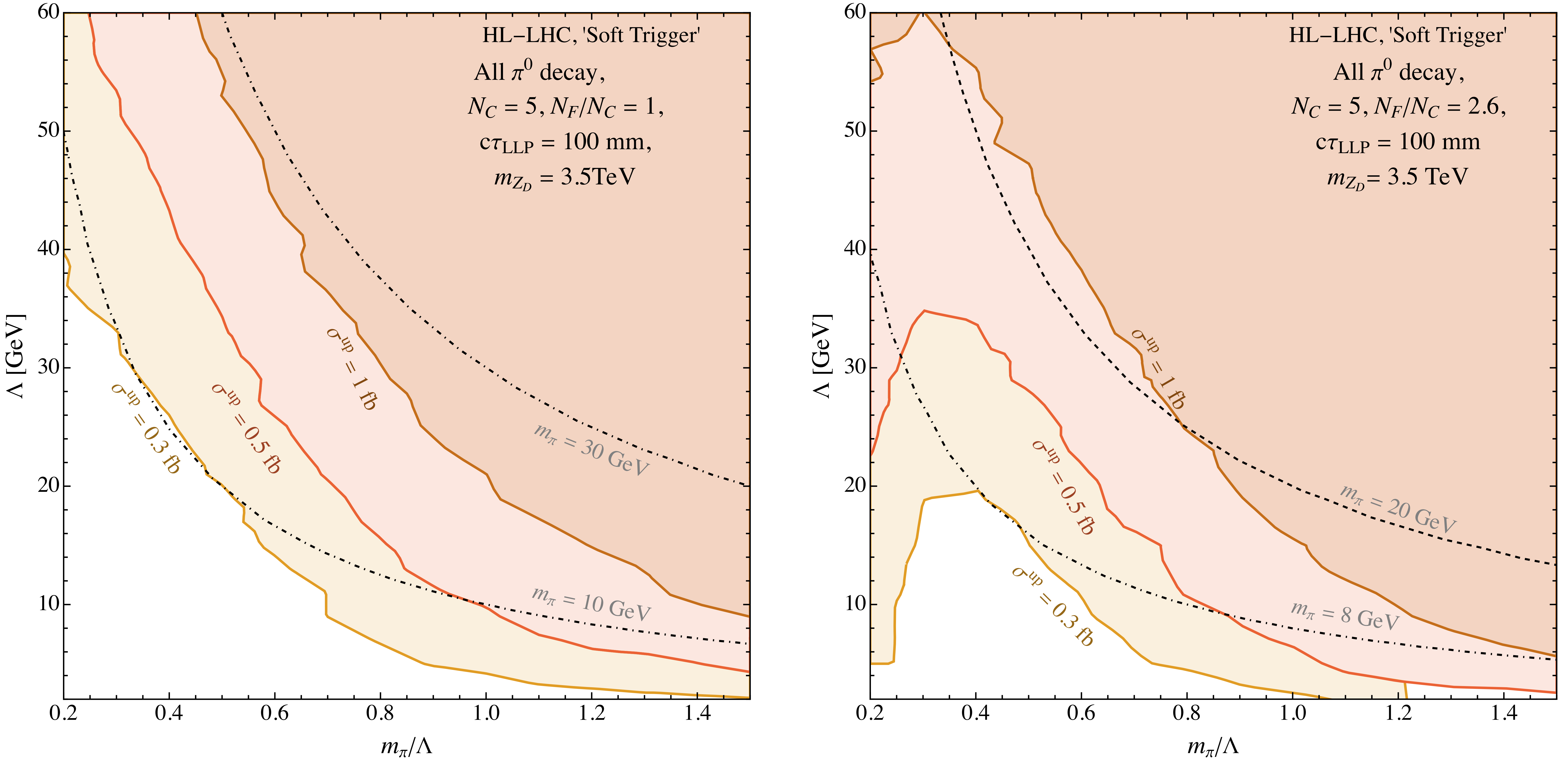
Dark sector parameters: current sensitivity



- Upper limits are strongest for small Λ, m_π/Λ (avoiding the chiral limit) regardless of our chosen $m_{Z'}$.
- Heavier pions - need a larger $c\tau_{\rm LLP}$ to reach the endcap detector.
- Upper limits strongest for lower N_F/N_C.

- First demonstration of the capacity for existing searches to constrain internal DS parameters.
- Given a specific model, can set constraints from these upper limits. Benchmarking the dark sector gives lots of info about their structure that we would otherwise miss; a viable strategy for presenting future searches?

Dark sector parameters: HL-LHC sensitivity



- Upper limits are strongest for small Λ, m_π/Λ (avoiding the chiral limit) regardless of our chosen $m_{Z'}$.
- Heavier pions - need a larger $c\tau_{LLP}$ to reach the endcap detector.
- Upper limits strongest for lower N_F/N_C.

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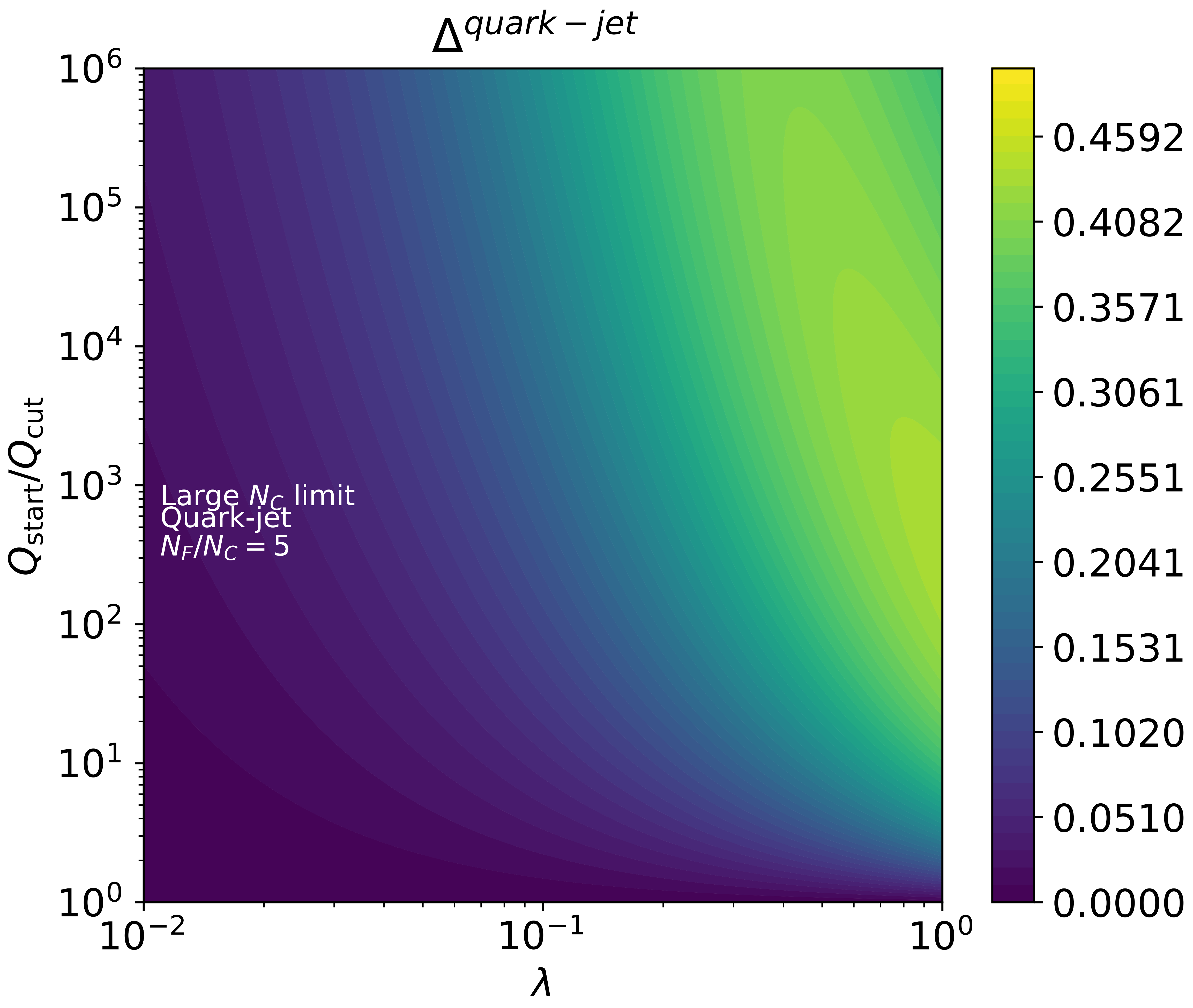
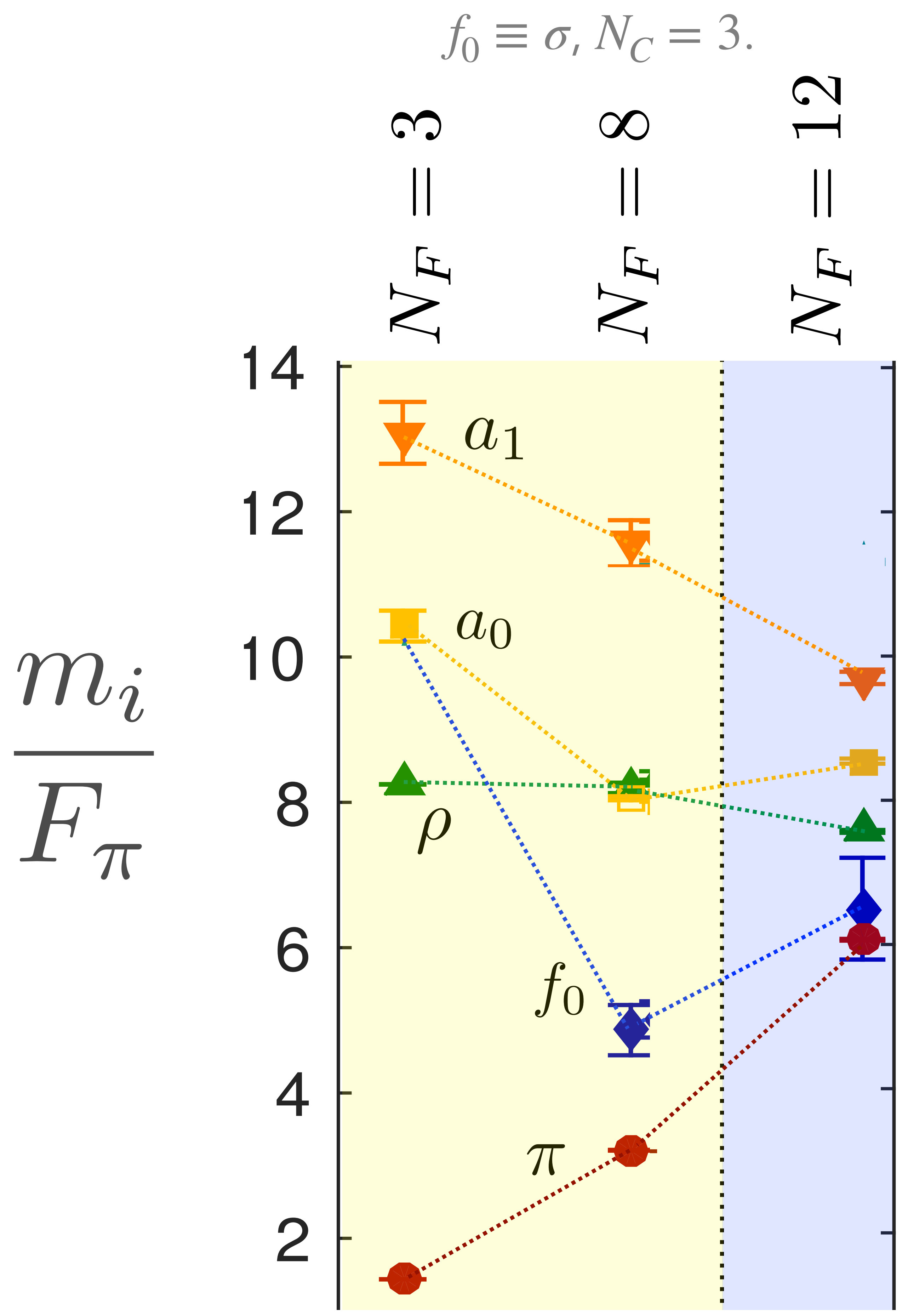
Outlook - simulation side



- Hadronisation into some dark meson spectrum is also an open question. Depending how far we knock theory out of CW in IR, we could make use of existing glueball hadronisation efforts arXiv:2310.13731.
- Dark hadronisation unreliable at large N_F/N_C; circumnavigate by making some flavours heavy ($M_q \gg \Lambda$). This comes with additional problems such as threshold effects + mass corrections.
- Need to add in $P_{G_D \to q_D \bar{q}_D}$ branching, in certain regimes of our theory it can be a sizeable effect on results. Additionally, we need to assess the magnitude of adding in the CMW scheme.

Outlook

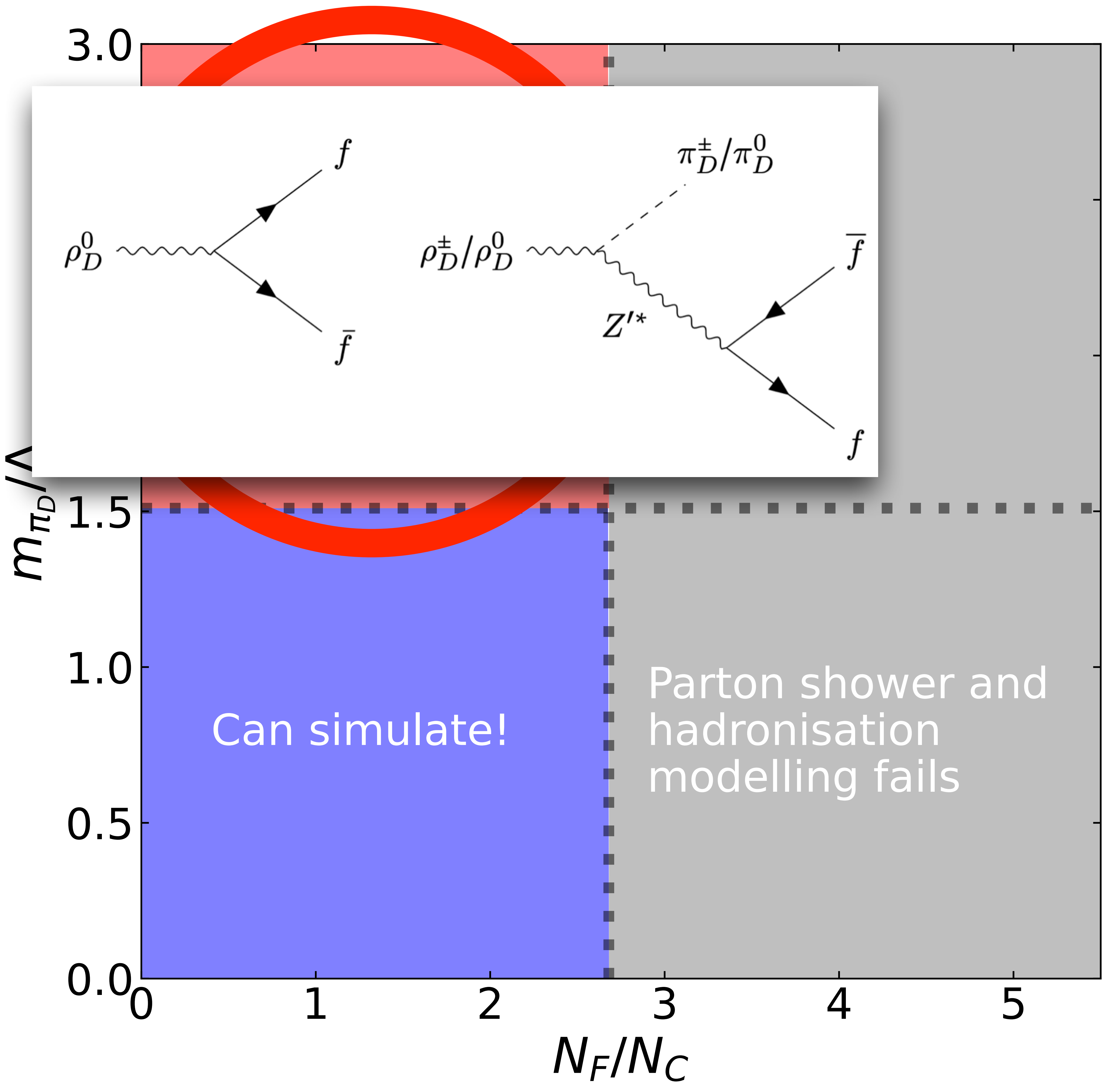


Lattice results for change of spectrum in high N_F/N_C theories.

arXiv: 2312.08332, 1611.07427

- Advancement of simulation tools allows us to probe even more exotic dark-shower signatures.
- Each regime of theory space has own EFT; need to investigate to properly map theory and experiment. Can look at Banks-Zaks regime, regimes close to IRFP, QCD-like confinement with CW-like running.
- Accommodating dilaton in Pythia opens a new regime large IRFPs theories. Need to work closely with non-perturbative theorists; spectrum of large N_F/N_C theories, string tension, etc. New techniques: AdS/CFT?
- Exciting model-building opportunities!

Outlook



- As we go beyond $m_\pi/\Lambda \gtrsim 1.5$, $m_\rho < 2m_\pi$ and the two-body decays are forbidden. Instead, ρ's decay through off-shell decays via the Z_D. Can apply what we learnt from 2-body to more complicated signal topologies.

- η is not degenerate to the π's; axial anomaly. $m_\eta = f(m_\pi/\Lambda, N_F/N_C)$ which could have interesting consequences on theory space. Effect greatest at $N_F = 2$. The correct modelling of how it enters Pythia simulations is an interesting question. arXiv:2509.04892

- Hadronisation uncertainties; modelling of the meson masses and its uncertainties, Lund uncertainties; it a showstopper or can we handle it?

Conclusion

- Very interesting theories with unique signatures; must explore all of dark sector parameter space to ensure we don't miss anything. In regions we know well, adopting a model agnostic approach seems to be the best way to go, we demonstrated the first sensitivity to underlying DS parameters. Gives us some nice ways to present future searches, let's do more of this!

- In the regions we didn't know so well, we've made good progress. The current approaches in Pythia are insufficient to describe two-loop α for high N_F/N_C since it neglects IRFP. By deriving RGE solutions and sampling the two-loop Sudakov factor correctly allows for the first simulations of near-conformal dark parton showers. High N_F/N_C theories can be tackled now!

- Motivates further investigation into model-building and hadronisation; this can potentially play an even larger role in dark shower phenomenology. Lots more needs to be done, but the road forward is clearer now that we have the right tools!

Thanks for listening!
Questions?

Back-up

Multiplicity (2 body)

- Initial state multiplicity of the individual species can be found as a function of probVector.

$$\langle N^0_{\pi_0}\rangle = \frac{1-\mathrm{probVector}}{N_F}\left\langle N^0_{\mathrm{init}}\right\rangle \qquad \langle N^0_{\pi_\pm}\rangle = (1-\mathrm{probVector})\left(1-\frac{1}{\mathrm{N_F}}\right)\left\langle \mathrm{N}^0_{\mathrm{init}}\right\rangle$$

$$\langle N^0_{\rho_0}\rangle = \frac{\mathrm{probVector}}{N_F}\left\langle N^0_{\mathrm{init}}\right\rangle \qquad \langle N^0_{\rho_\pm}\rangle = \mathrm{probVector}\left(1-\frac{1}{\mathrm{N_F}}\right)\left\langle \mathrm{N}^0_{\mathrm{init}}\right\rangle$$

- Final state multiplicity of the individual species can be deduced from the decays: $\rho_{0/\pm} \to \pi_\pm \pi_{\mp/0}$.

$$\langle N_{\pi_0}\rangle = \left(\frac{1}{N_F} + \mathrm{probVector}\left(1-\frac{2}{N_F}\right)\right)\left\langle N^0_{\mathrm{init}}\right\rangle \qquad \langle N_{\pi_\pm}\rangle = \left(\left(1-\frac{1}{N_F}\right) + \frac{2\,\mathrm{probVector}}{N_F}\right)\left\langle N^0_{\mathrm{init}}\right\rangle$$

$$\langle N_{\pi}\rangle = \left(1+\mathrm{probVector}\right)\left\langle N^0_{\mathrm{init}}\right\rangle$$

R_{inv} calculations

- R_{inv} is calculated from: $$R_{\text{inv}} = \left\langle \frac{\text{number of stable hadrons}}{\text{number of total hadrons}} \right\rangle$$

- Some proportion of diagonal dark pions decay to SM via $\pi_0 \to q_{\text{SM}}\bar{q}_{\text{SM}}$. We denote the proportion as "R_{decay}" and is obviously bounded between 0 and 1. For two-body systems:

$$R_{\text{inv}} = \frac{\langle N_{\pi_\pm} \rangle + \left(1 - R_{\text{decay}}\right)\langle N_{\pi_0} \rangle}{\langle N_\pi \rangle} = 1 - \frac{R_{\text{decay}}}{N_F}\left(\frac{1 + \text{probVector}\left(N_F - 2\right)}{1 + \text{probVector}}\right) \qquad 0.5 \le R_{\text{inv}} \le 1$$

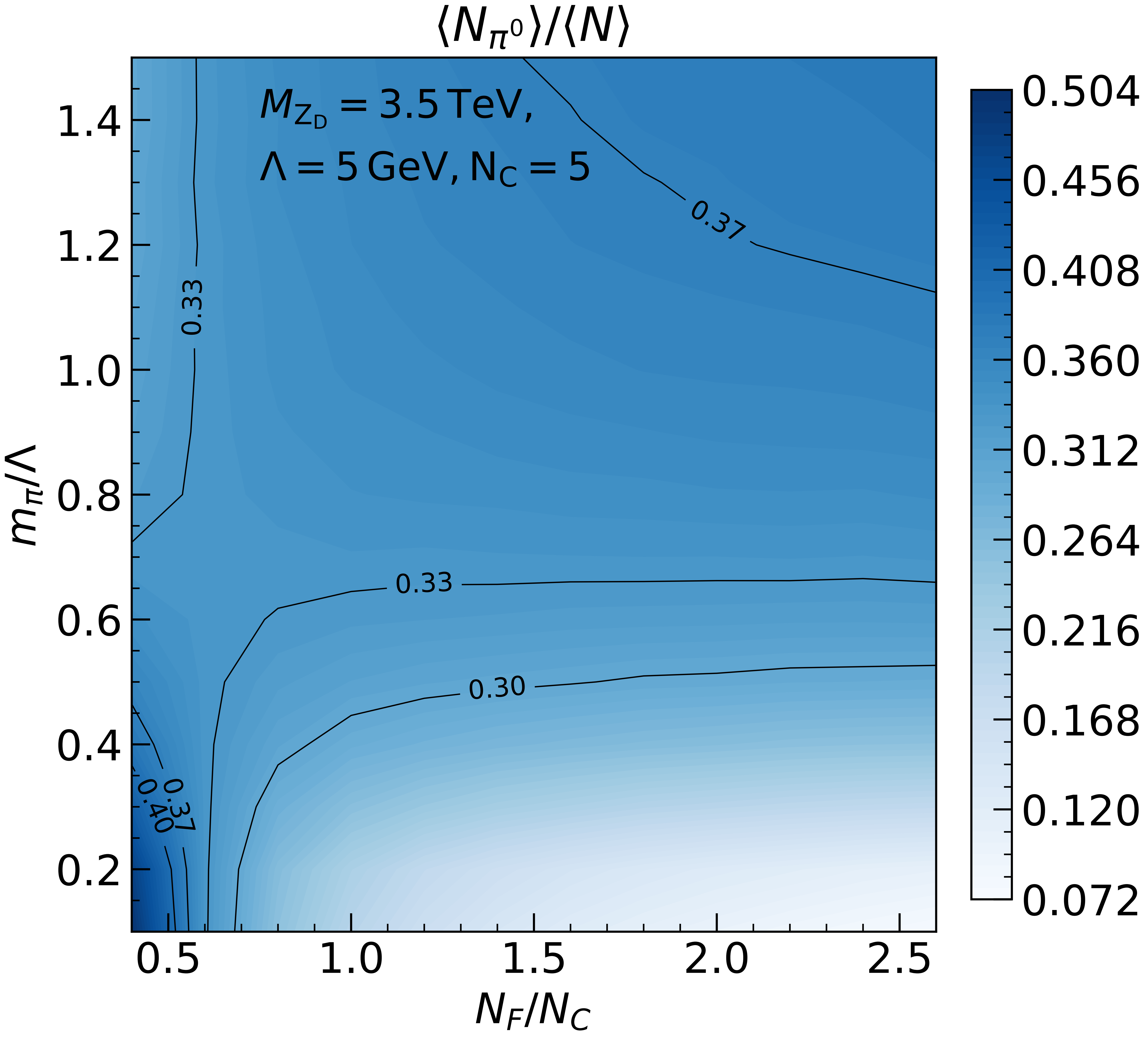
- For three-body systems:

$$R_{\text{inv}} = \frac{\langle N_{\pi_\pm} \rangle + \left(1 - R_{\text{decay}}\right)\langle N_{\pi_0} \rangle}{\langle N_\pi \rangle} = 1 - R_{\text{decay}}\left(\frac{N_F - 1}{N_F - \text{probVector}}\right) \qquad 0 \le R_{\text{inv}} \le 1$$

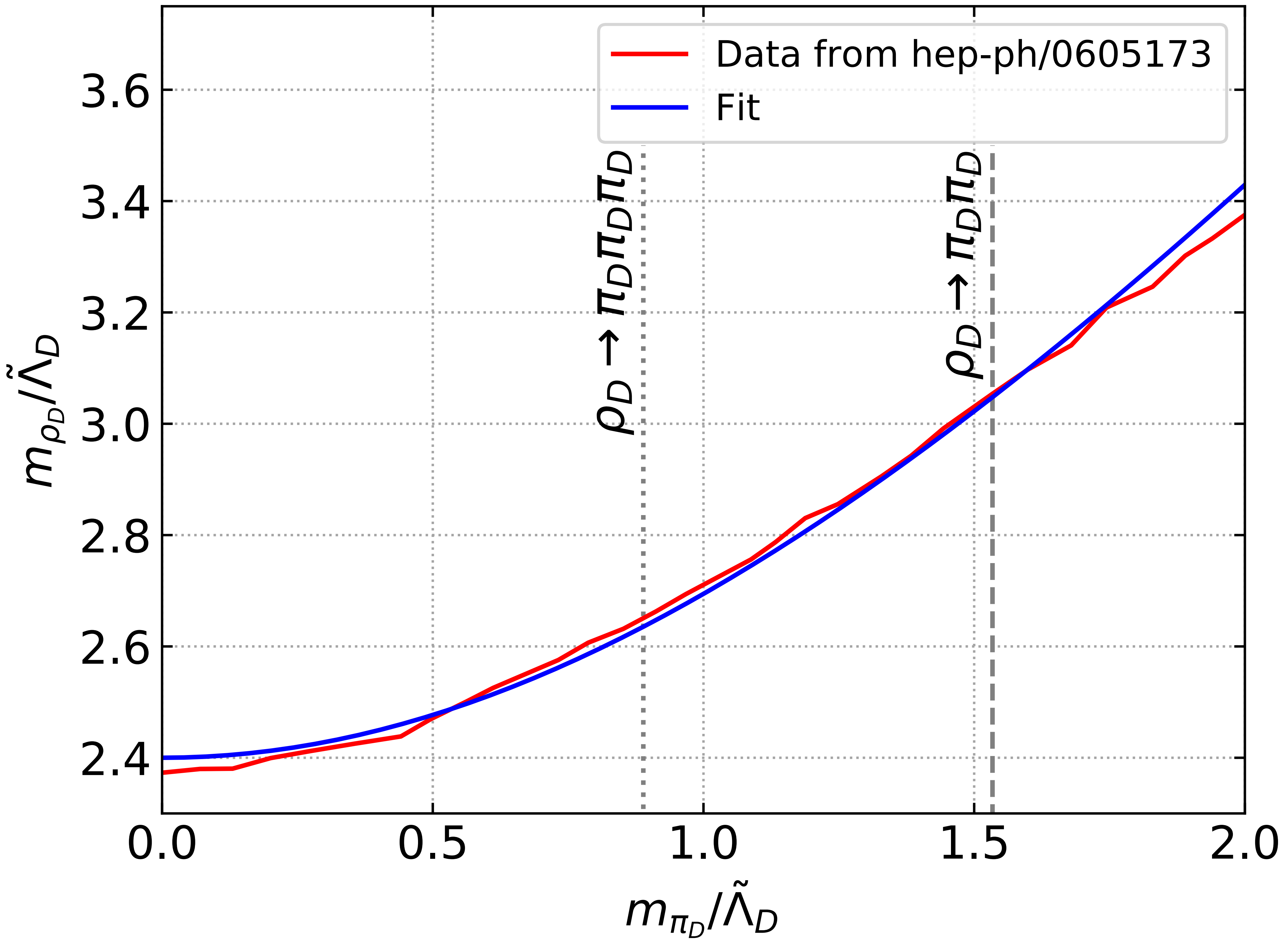
- When $R_{\text{decay}} = 0$, $R_{\text{inv}} = 1$, but $N_{q_{\text{SM}}\bar{q}_{\text{SM}}} \neq 0$. R_{inv} not a good parameter in three-body due to off-shell decays.

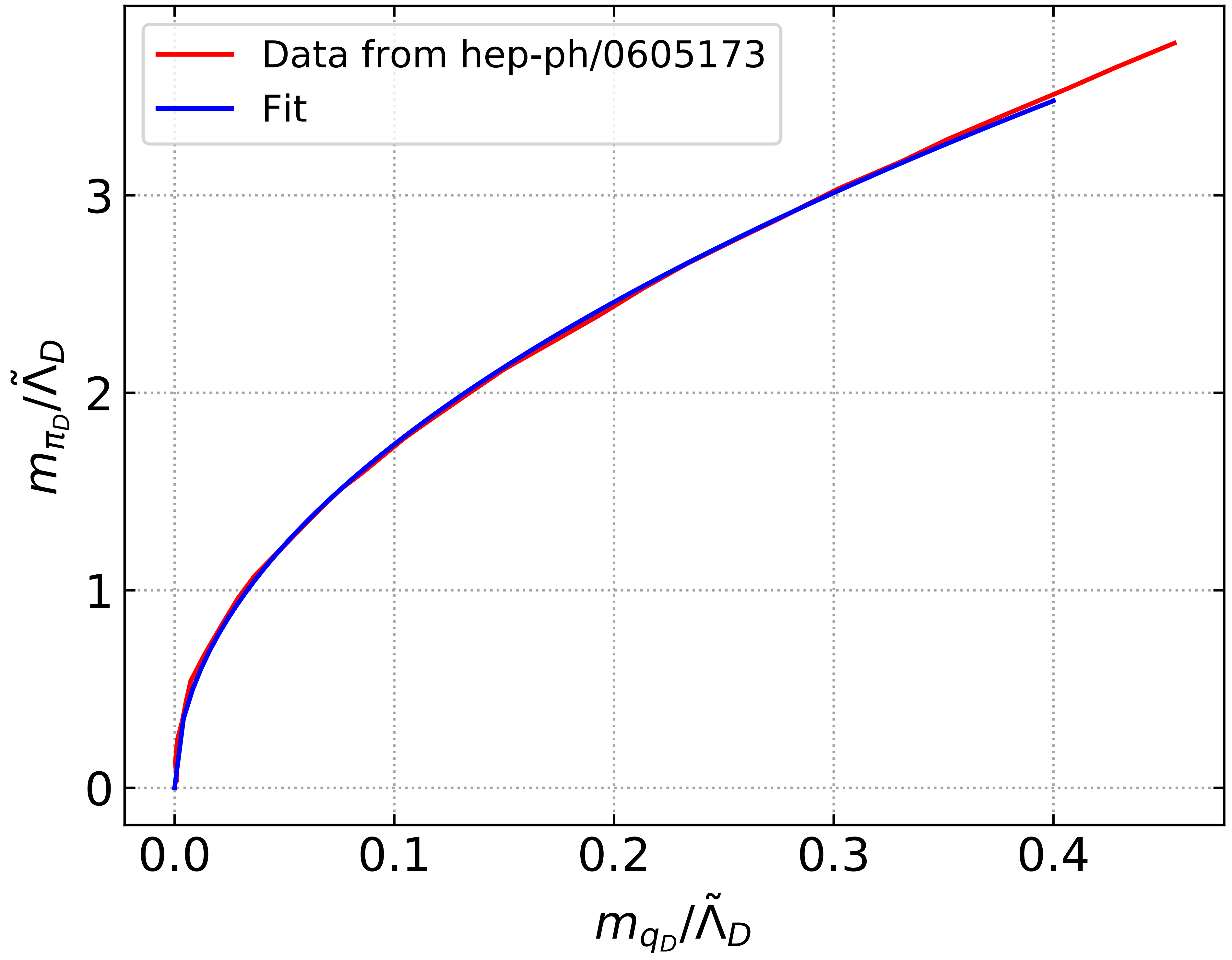
$$\langle N_{q_{\text{SM}}\bar{q}_{\text{SM}}} \rangle = \left(R_{\text{decay}}\frac{1 - \text{probVector}}{N_F} + \text{probVector}\right)\langle N^0_{\text{init}} \rangle$$

Amount of LLP ratio



Dark sector model parameters



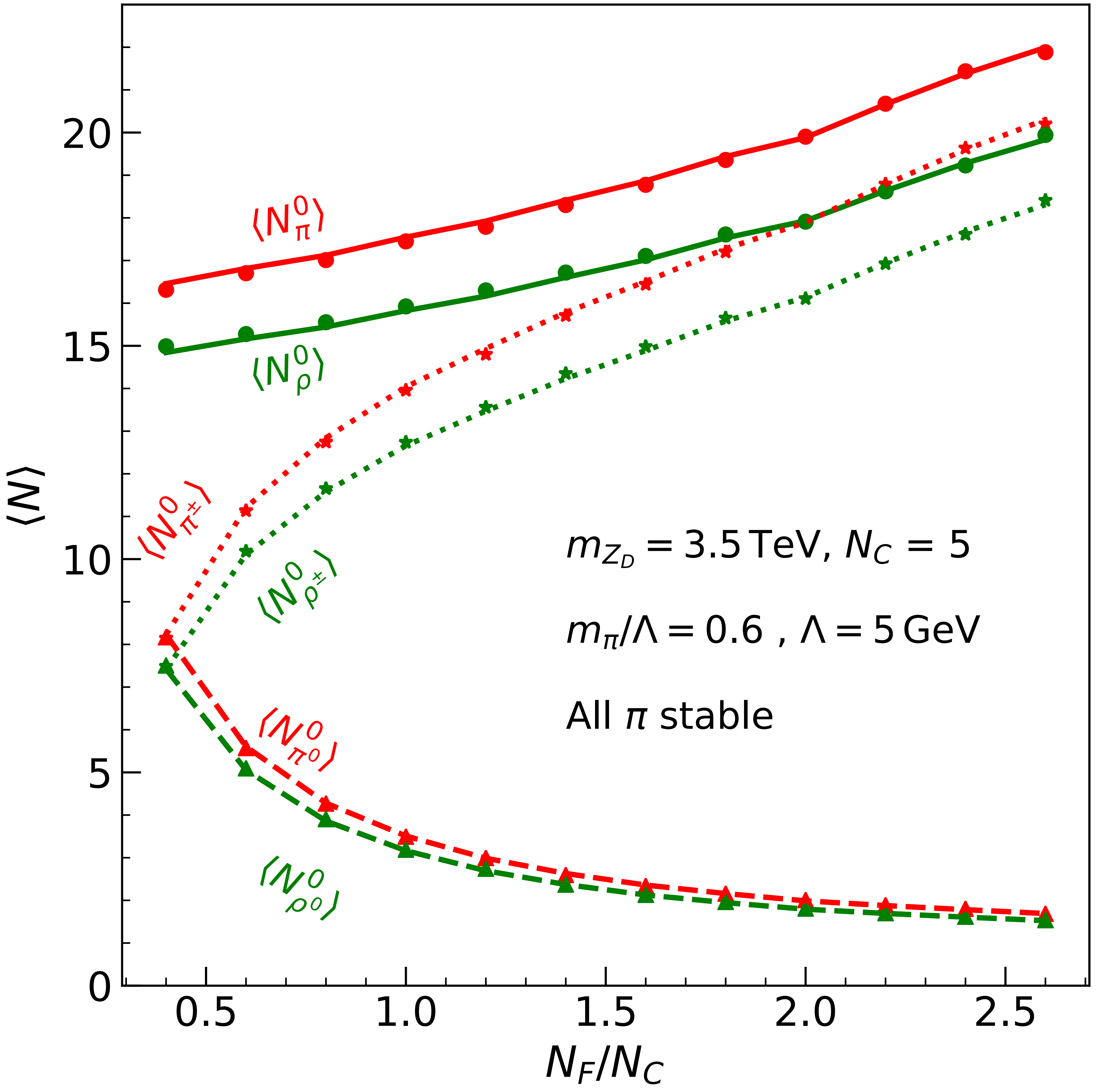


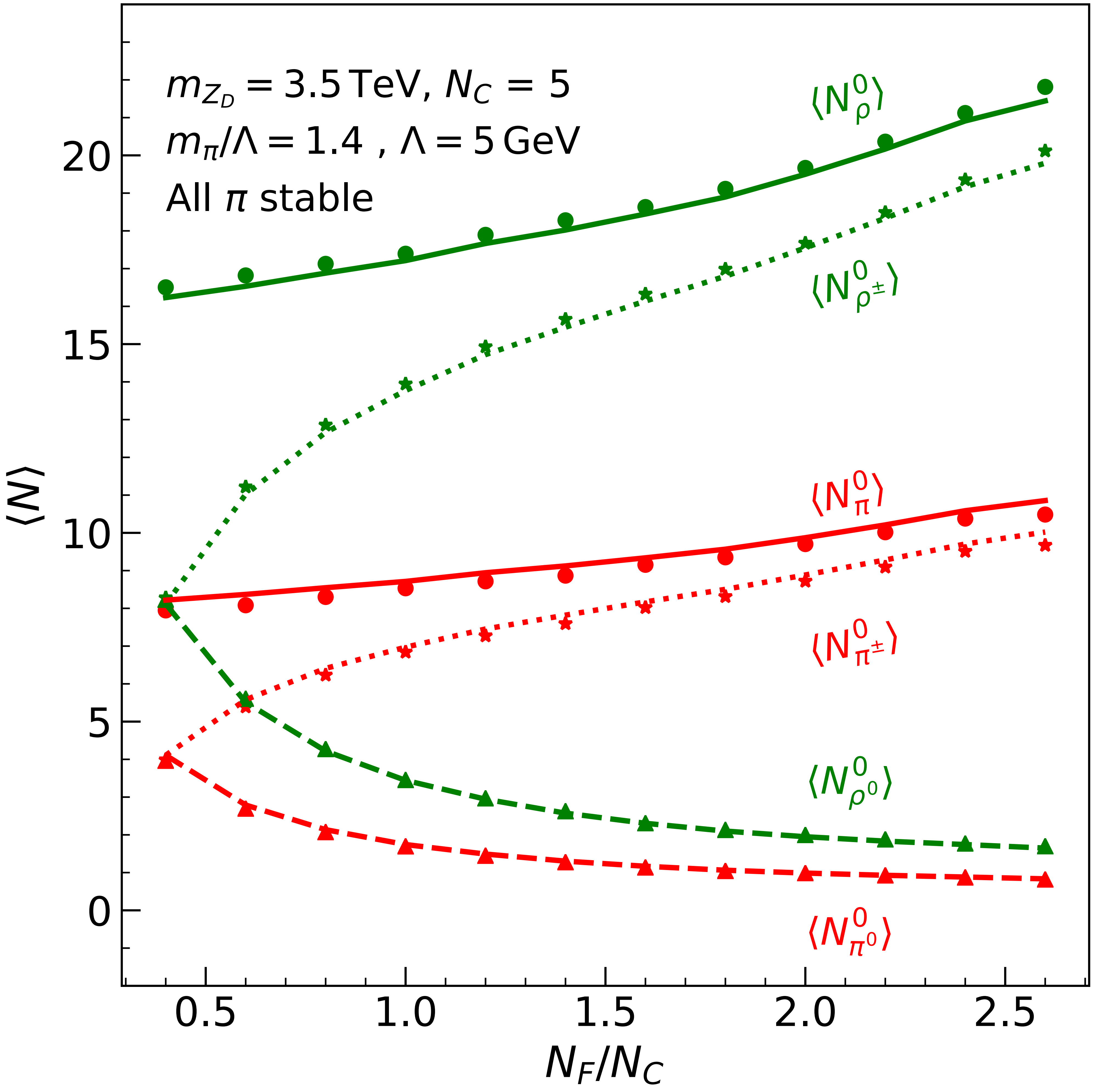
- For $m_{\pi_D}/\tilde{\Lambda} \lesssim 2$, the meson and current quark masses set by arXiv:0605173,2203.09503. We will assume $\tilde{\Lambda} = \Lambda_{\text{one-loop}} = \Lambda$.

$$\frac{m_{\pi_D}}{\tilde{\Lambda}} = 5.5\sqrt{\frac{m_{q_D}}{\tilde{\Lambda}}}, \qquad \frac{m_{\rho_D}}{\tilde{\Lambda}} = \sqrt{5.76 + 1.5\frac{m_{\pi_D}^2}{\tilde{\Lambda}^2}}$$

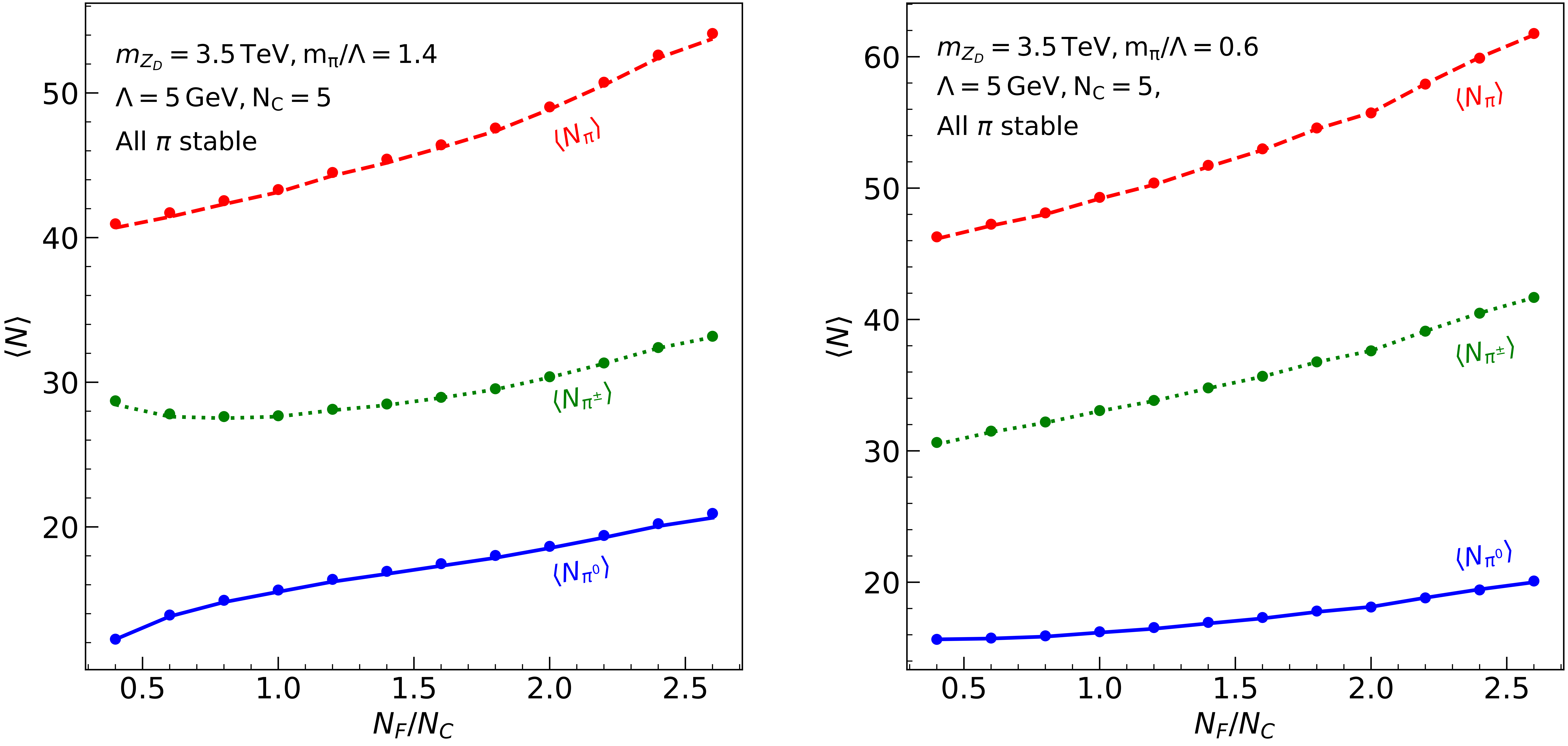
- Valid for $m_{\pi_D}/\tilde{\Lambda} \lesssim 2$. Fitting assumed to not depend on N_C, N_F.

Initial meson multiplicity

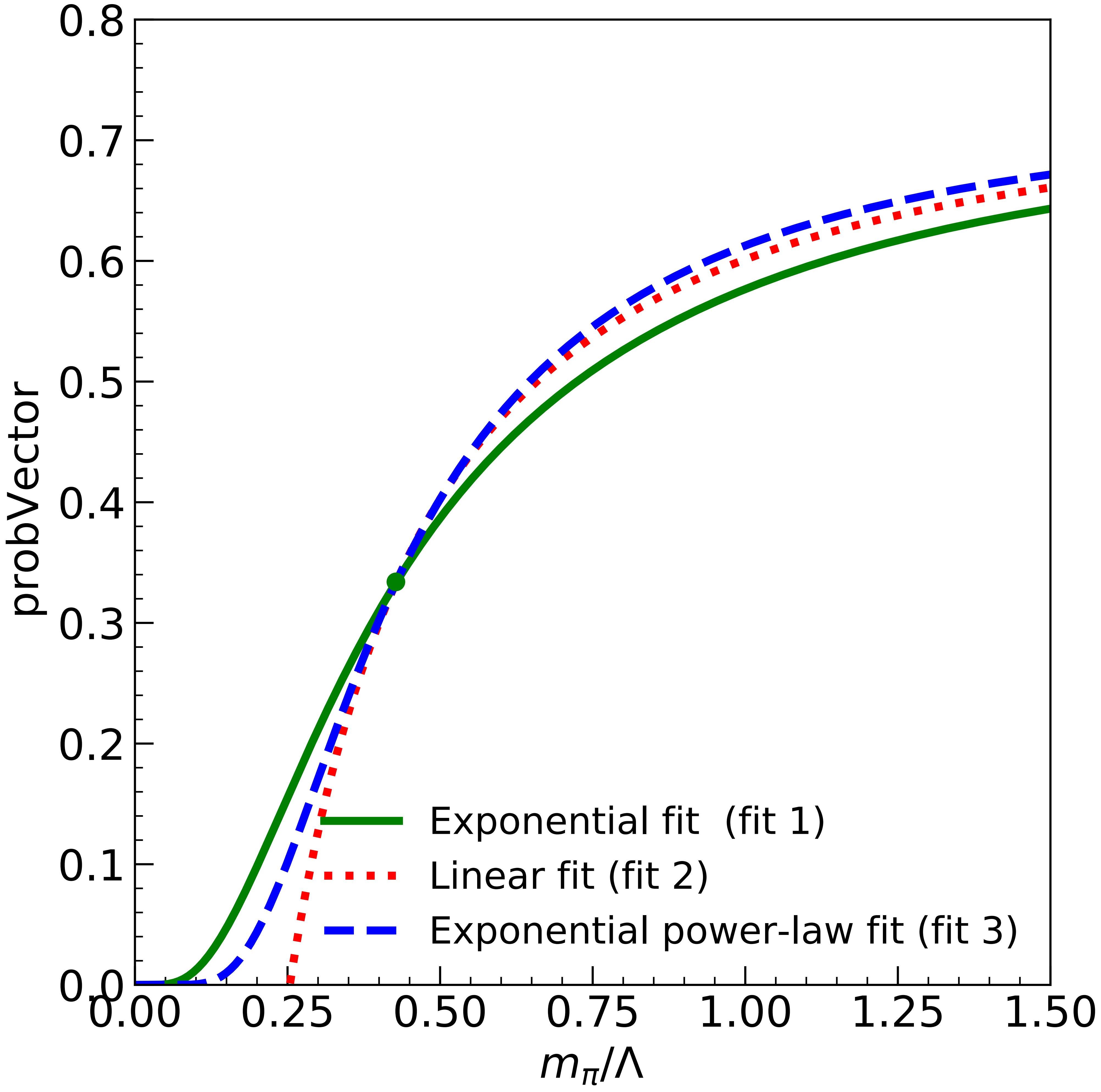




Final meson multiplicity



ProbVector fitting

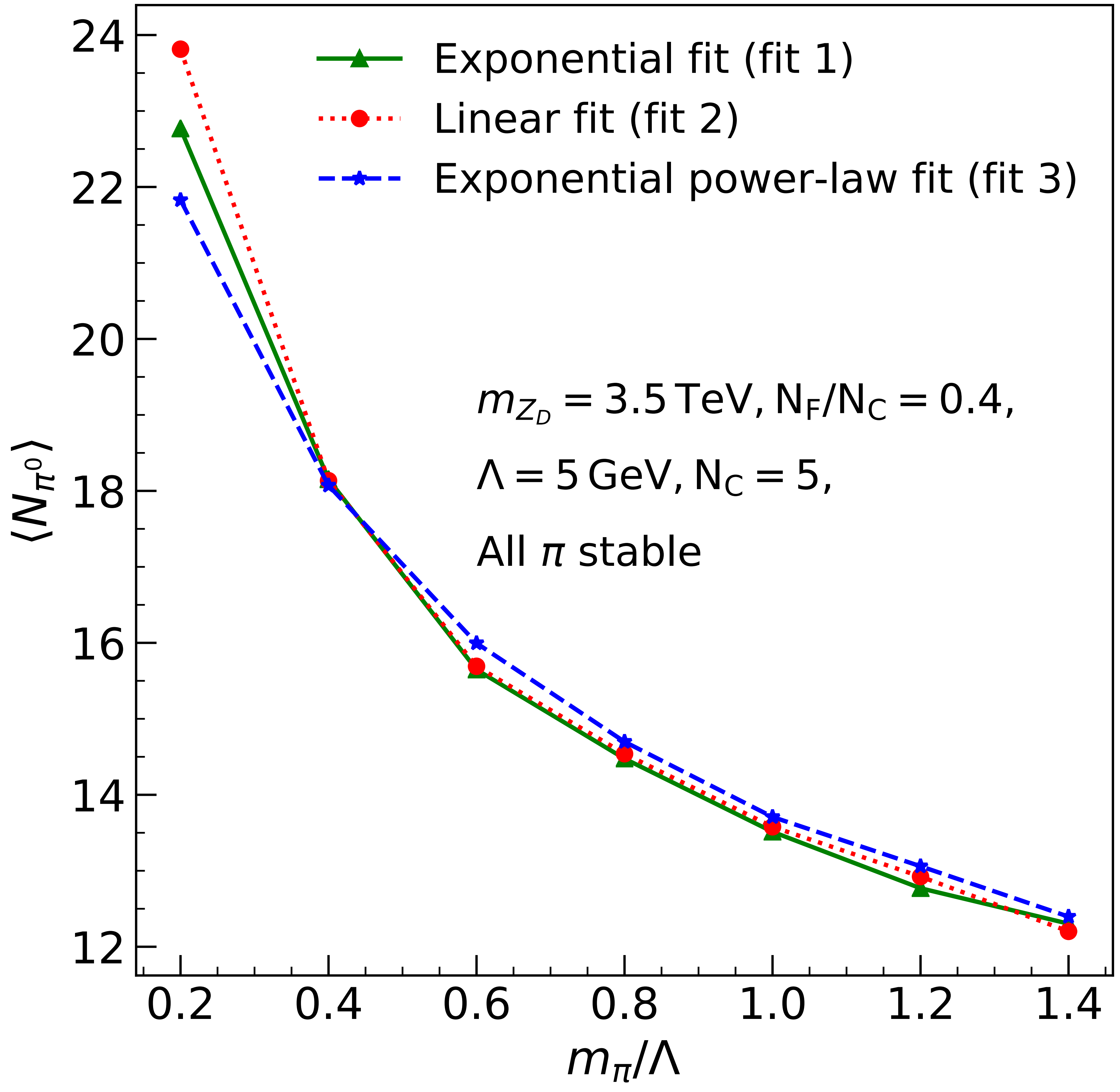


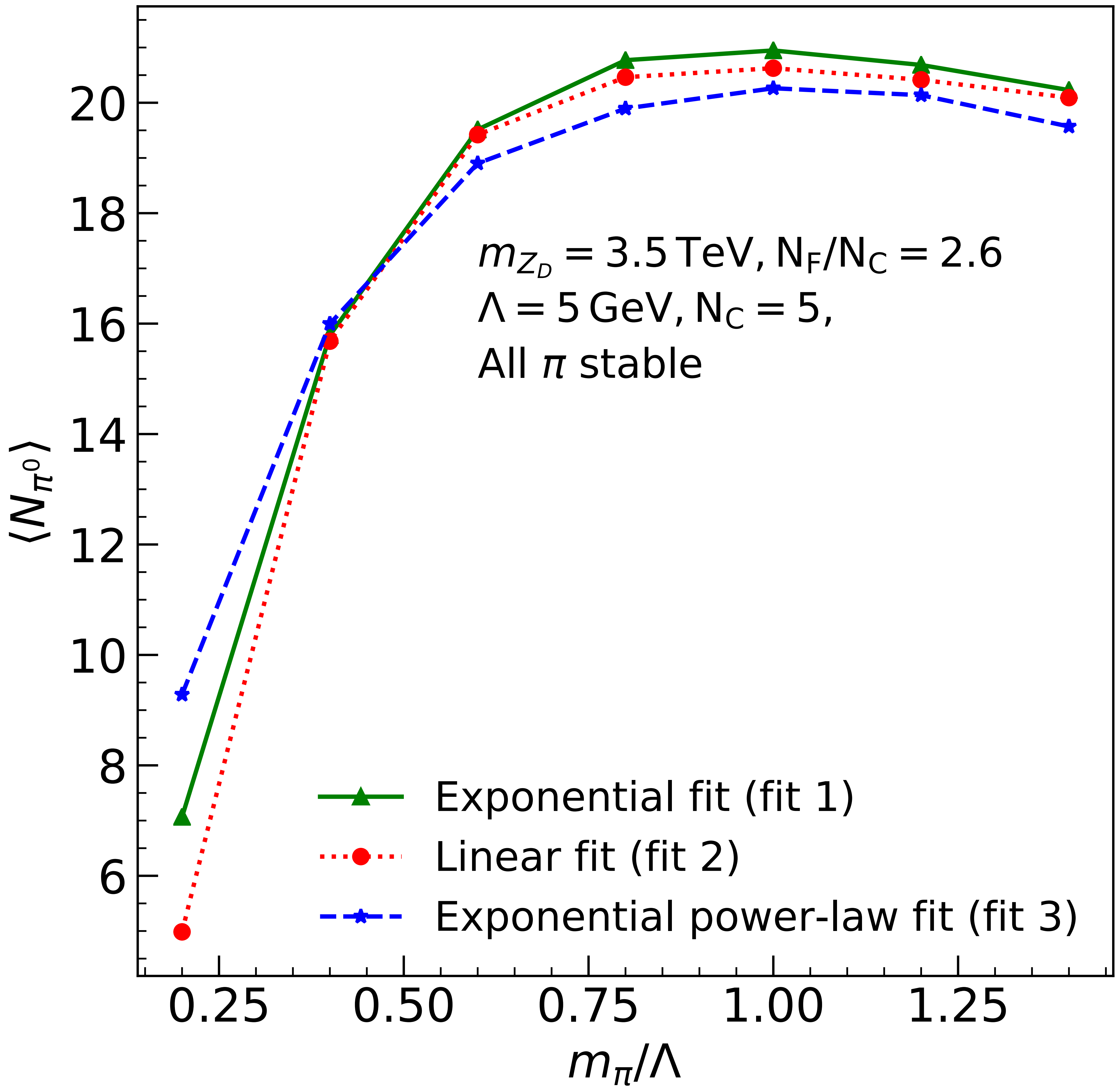
- The value *probVector* is the probability of producing a ρ_D after hadronisation. In the SM for $m_{\pi_D}/\Lambda \sim 0.428$, *probVector = 0.33*. For degenerate π_D/ρ_D it is *probVector = 0.75* from spin counting arguments. In the chiral limit, it tends to *probVector = 0.*

$$\text{probVector} = \frac{3\exp(-\omega(m_{\rho_D}/m_{\pi_D} - 1))}{1 + 3\exp(-\omega(m_{\rho_D}/m_{\pi_D} - 1))}$$

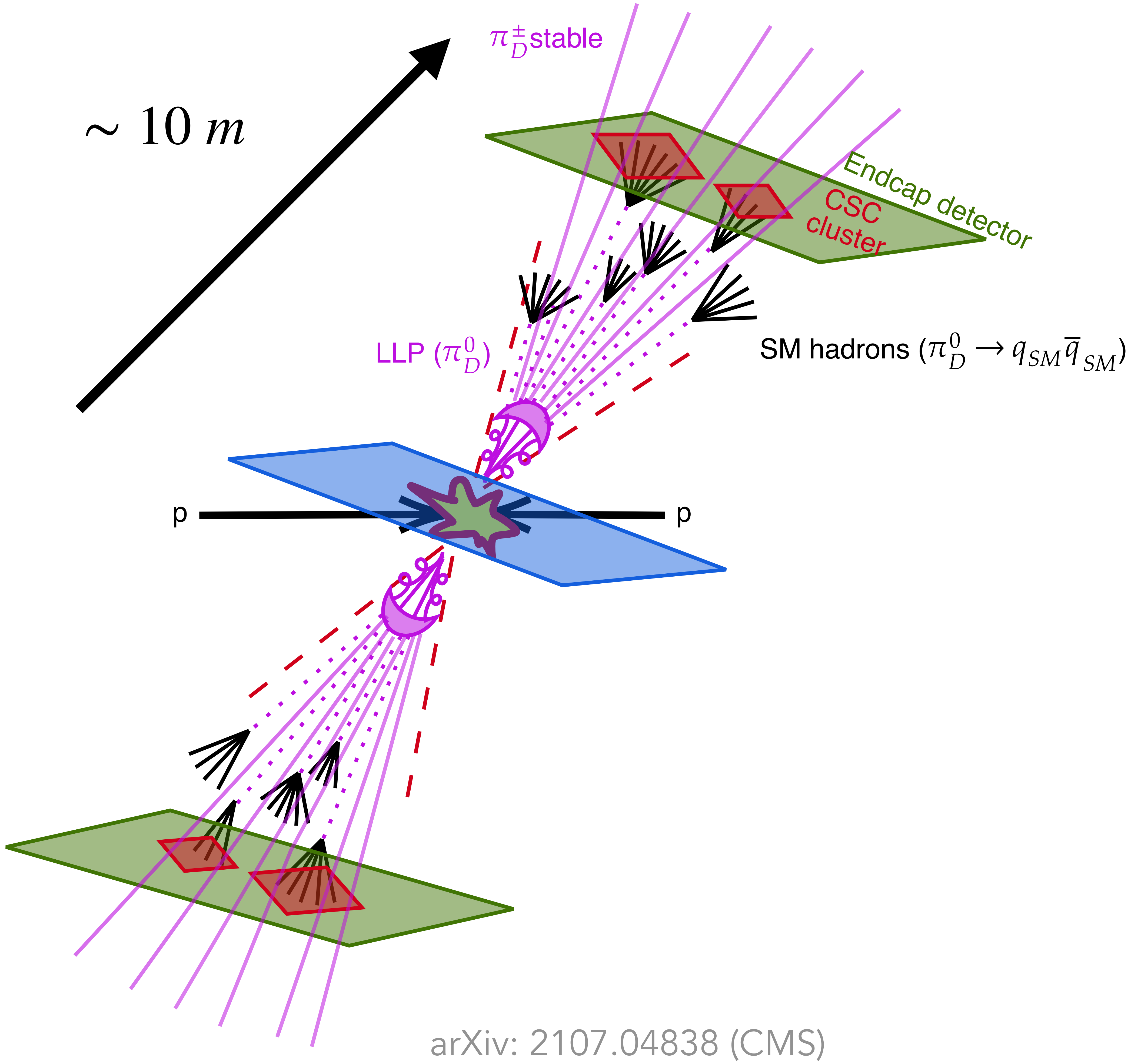
- We use a fit inspired by the expected Boltzmann distribution of probVector, where $\omega = 0.378$. We endeavour to explore better and more motivated fit methods in the future (perhaps from string tension).

ProbVector fitting





Search description



arXiv: 2107.04838 (CMS)

- Whether LLPs decay inside the muon calorimeter is not just a function of proper lifetime, but additionally a function of geometry and of mediator and HV theory parameters.
- SM shower associated with a signal requires $N_{hits} > 130(370)$ and $\Delta\phi_c > 0.75$. Search subject to various selection cuts - MET > 200GeV at current run, MET > 50 GeV at HL-LHC.
- Set upper limits on the cross-section through,

$$N_{signal} = \sigma_{pp\to q_D\bar{q}_D} \times \epsilon_{\text{tot}} \times \mathscr{L}$$

- Where ϵ_{tot} is the efficiency and for $N_{LLP} = N_{\pi_D^0}$ is given as,

$$\epsilon_{\text{tot}} = N_{\text{LLP}}^{\text{CSC}} \times \epsilon_{\text{reco}}\left(E_{\text{had}}^{\text{CSC}}\right) \times \epsilon_{\text{cut}}(\text{MET}),$$

- Where $N_{\text{LLP}}^{\text{CSC}} = N_{\text{LLP}} \times \epsilon_{\text{geo}}(\beta\gamma c\tau_{\text{LLP}})$. Complex since $E_{\text{had}}^{\text{CSC}}$, $\beta\gamma$ and $N_{LLP} \equiv N_{\pi_0}$ are all functions of the HV theory parameters.

Search description

The search requires the following selection criteria:

1. The *missing transverse energy*, defined as the negative vector sum of visible p_T from particles identified in the tracker and calorimeter, is MET > 200 GeV, as trigger requirement. For models under consideration, there are two sources of missing energy. First is the initial-state-radiation and a second more subtle source is related to the presence of multiple LLPs as shown in section 2. Given the large LLP multiplicity, it is possible that some LLPs decay within the tracker or the calorimeter, which contributes to the overall MET. It is expected that this effect will be more important for intermediate lifetimes, with high LLP multiplicities, than for extremely large lifetimes or low LLP multiplicities. It will also depend on the number of LLPs present in the signal. We also consider HL-LHC sensitivity and use an optimistic trigger of MET > 50 GeV for that purpose [64].

2. *No electron (muon)* with transverse momentum $p_T > 35$ (25) GeV and pseudorapidity $|\eta| <$ 2.5 (2.4), to remove W and top background.

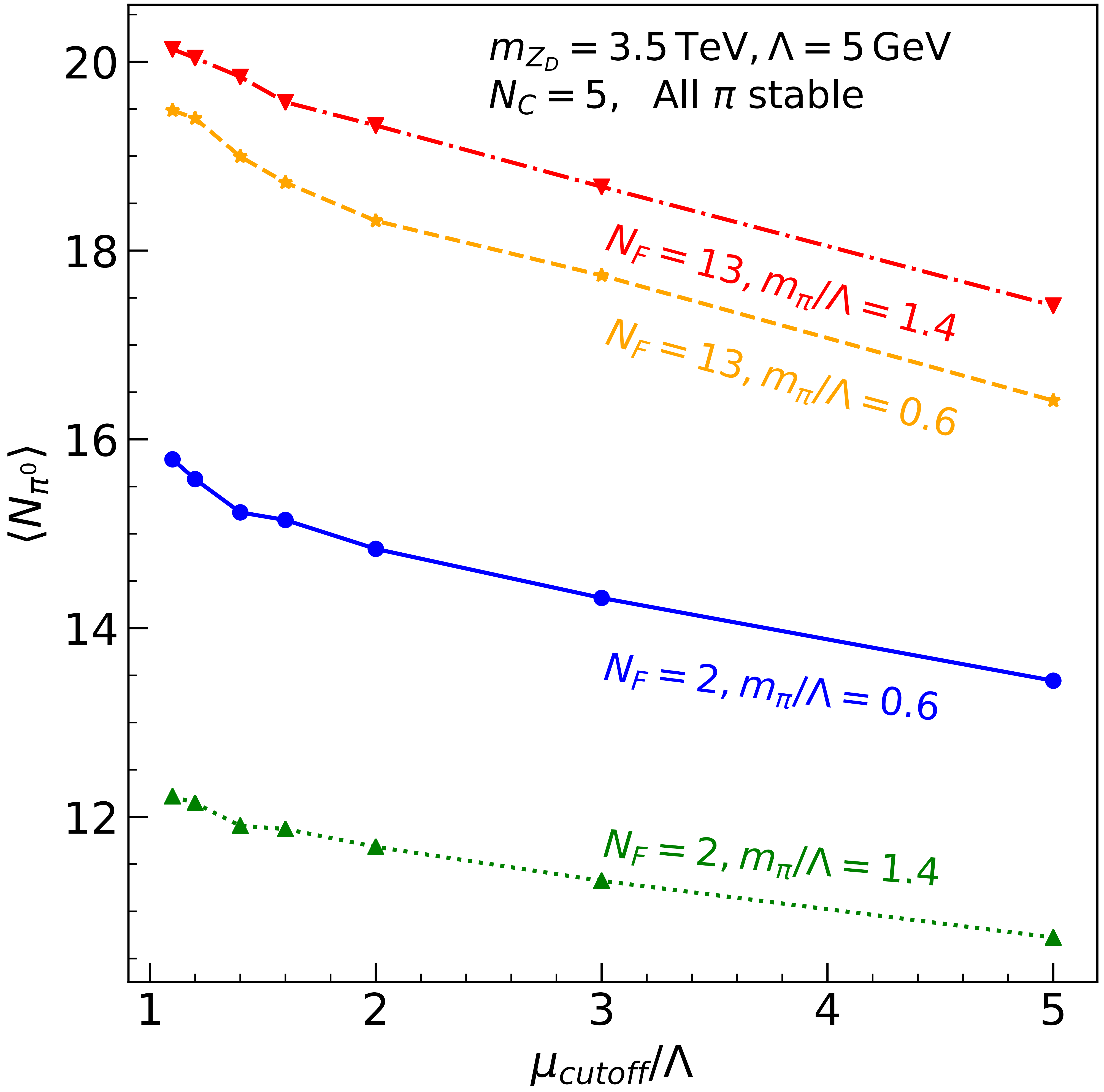
3. *At least one CSC cluster* with $|\Delta\phi_c| < 0.75$ to ensure that it originated from the LLP decay. Here, $\Delta\phi(\mathbf{x}_{\mathrm{CSC}}, \mathrm{MET})$ is defined as the azimuthal angle between the missing transverse momentum and the cluster location from the IP. The LLP is then close to the missing energy, which points opposite to the vector sum of the visible p_T.

Search description

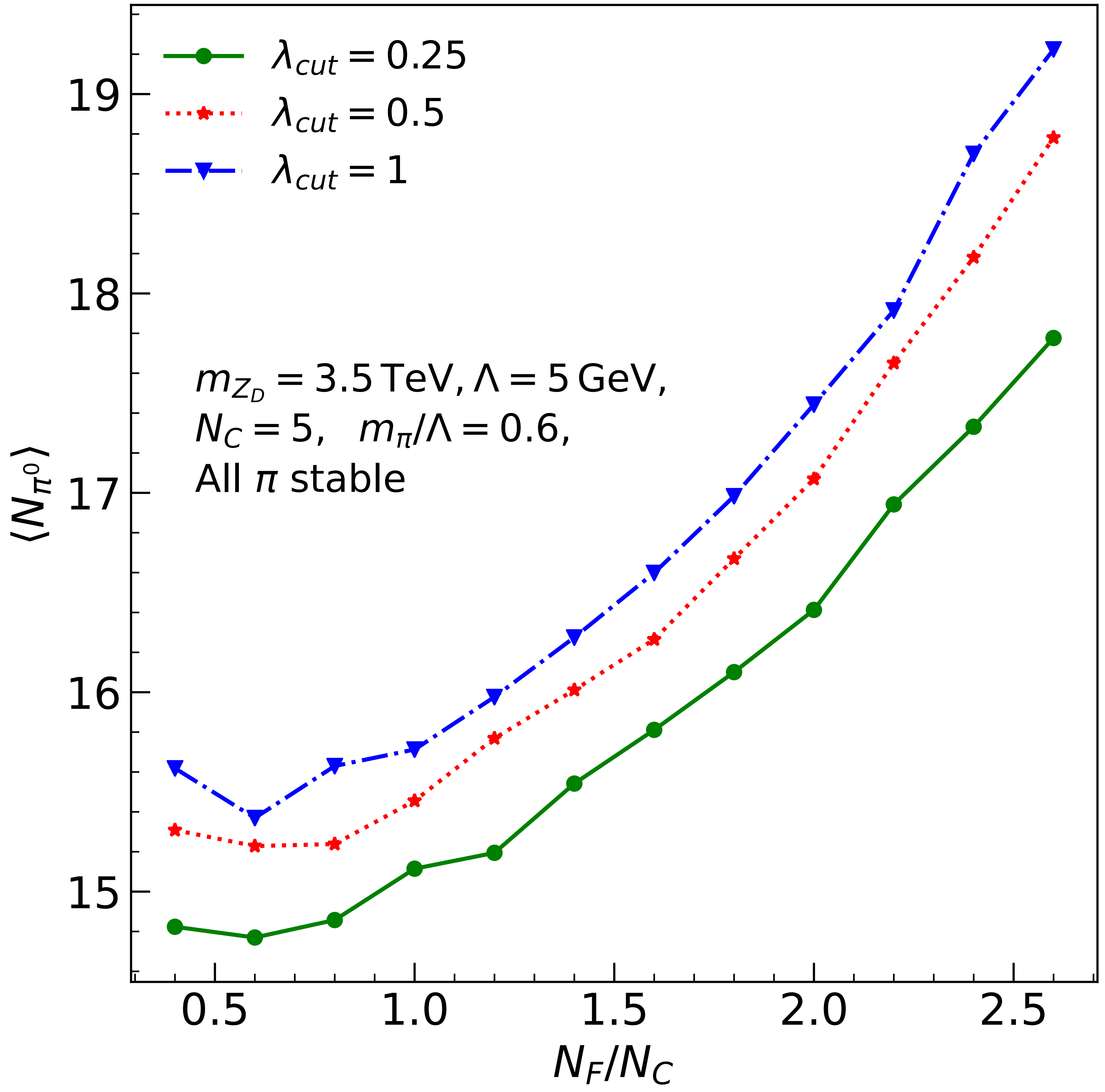
4. Events with *clusters too close to a jet (muon) are removed*, for $\Delta R = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2} < 0.4$. If $\Delta R(\text{jet/muon}, \text{cluster}) < 0.4$, the CSC cluster is matched to so-called punch-through jets/muons with $p_T > 10$ (20) GeV and it is assumed to be created by LLPs inside the jet, e.g. K_L, or muon bremsstrahlung. This requirement may affect our signal as we expect some of the LLPs to decay within the calorimeter thus resembling a SM jet. Such events containing prompt plus displaced collimated LLPs will be vetoed by $\Delta R(\text{jet/muon}, \text{cluster}) < 0.4$.

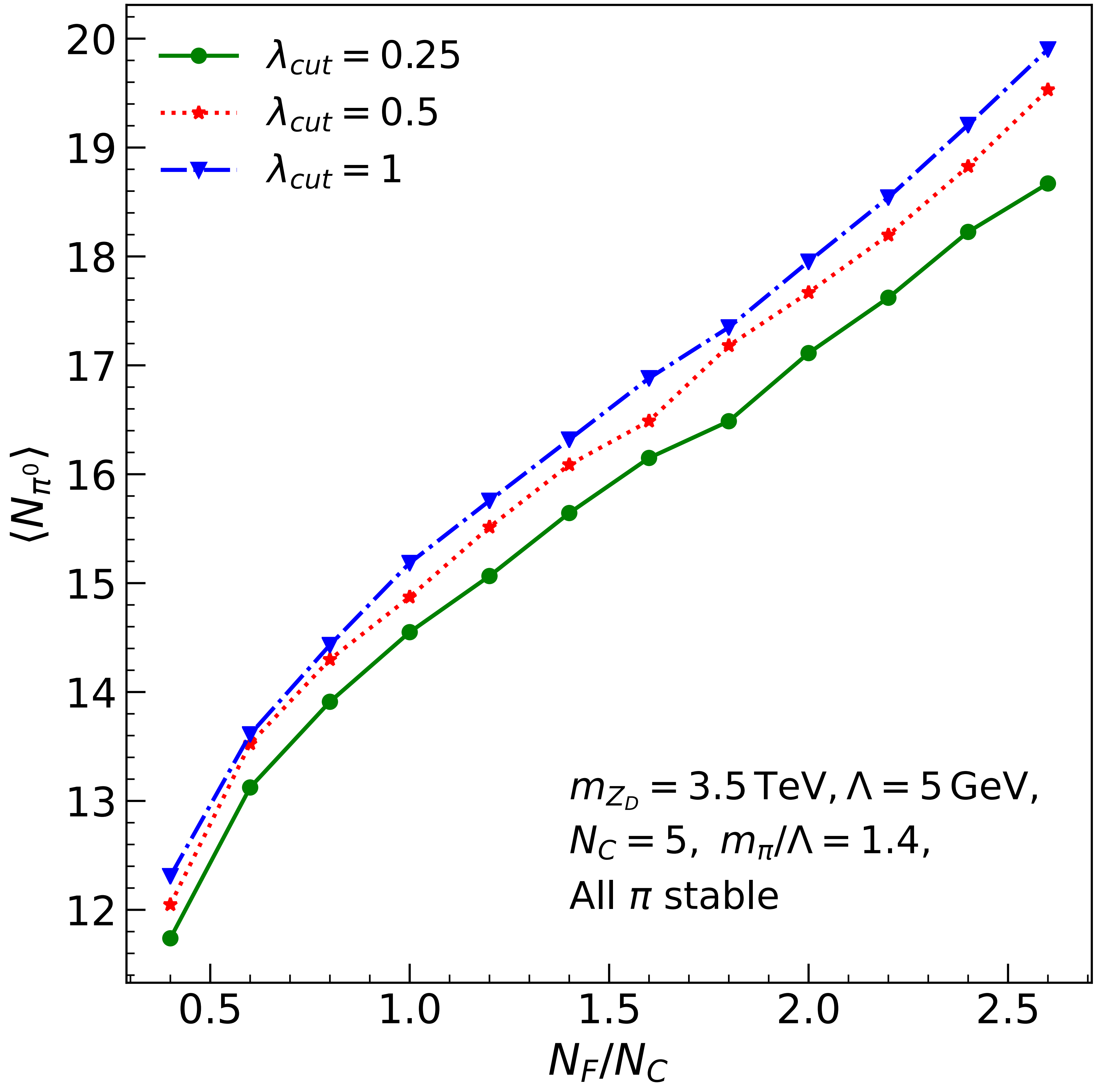
5. The *average time of detector hits* in the CSC cluster, relative to the collision is -5 ns $< \langle \Delta t_{\text{CSC}} \rangle < 12.5$ ns, to reject pileup clusters.

Cutoff dependence



't Hooft cutoff dependence

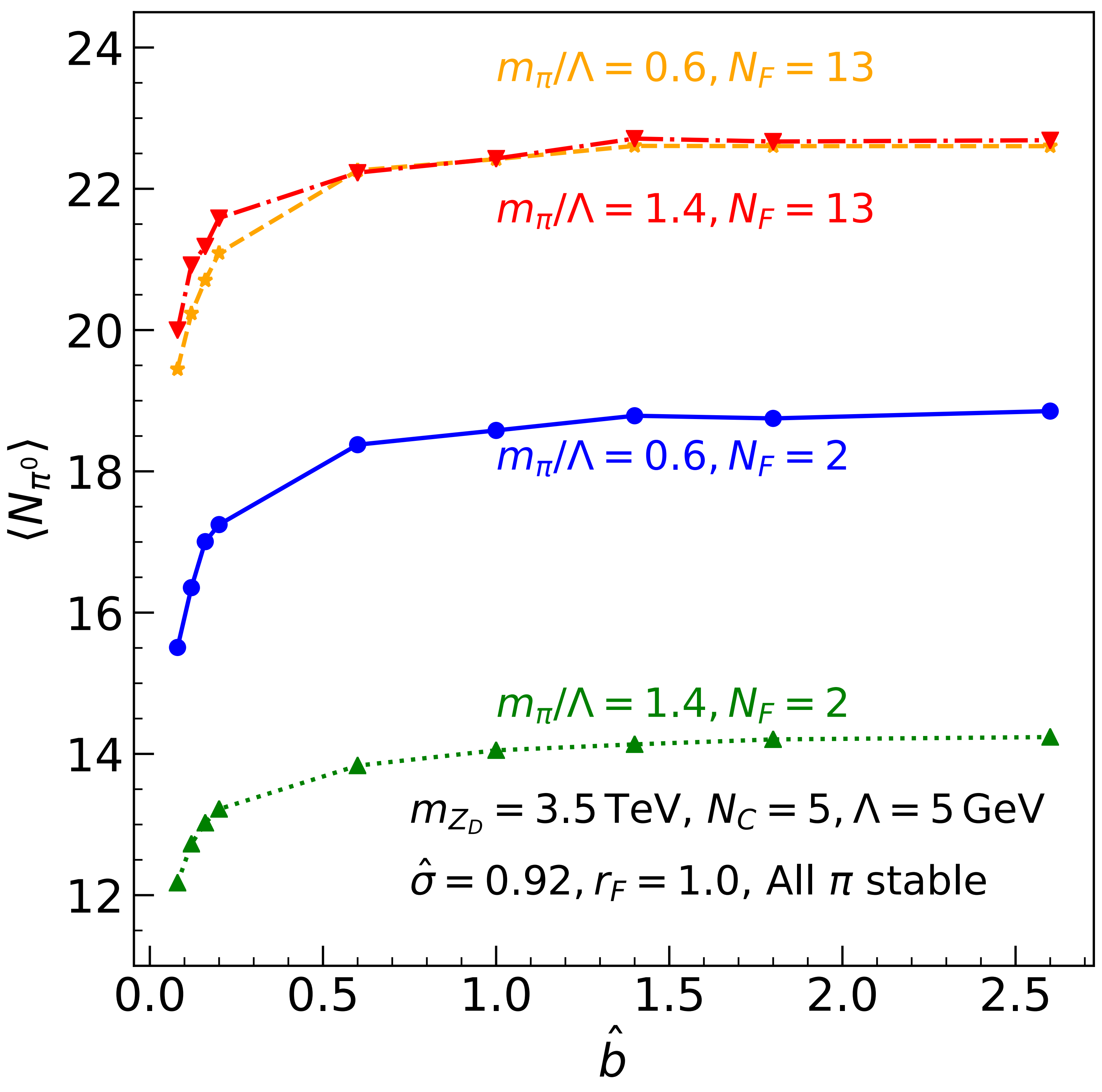


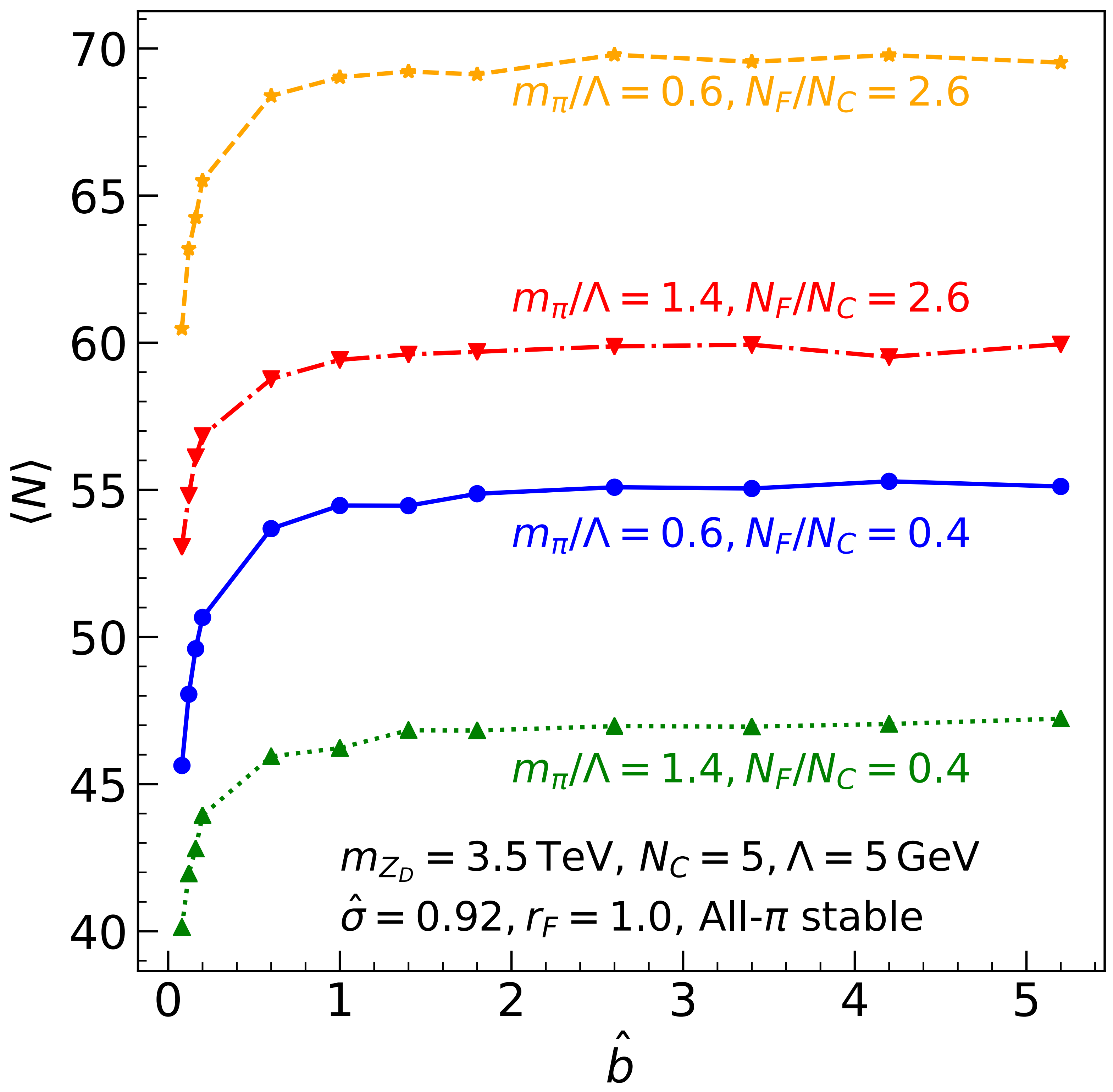


- $\mu_{\text{cutoff}}/\Lambda$ determined by λ_{cutoff} through the equation,

$$N_C \alpha(\mu_{\text{cutoff}}/\Lambda) = \lambda_{\text{cutoff}}$$

$\hat{b} = b_{\text{Lund}}/m_Q$ dependence





- PYTHIA Hidden Valley module performs hadronisation using the Lund string fragmentation function to determine the probability for a meson to be produced with momentum fraction z. Given as,

$$f(z) = z^{-1}(1-z)^a \exp(-\hat{b} m_T^2 / m_Q^2 z)$$

- The average break-up time relates a change in a to a change in $\hat{b}$ as

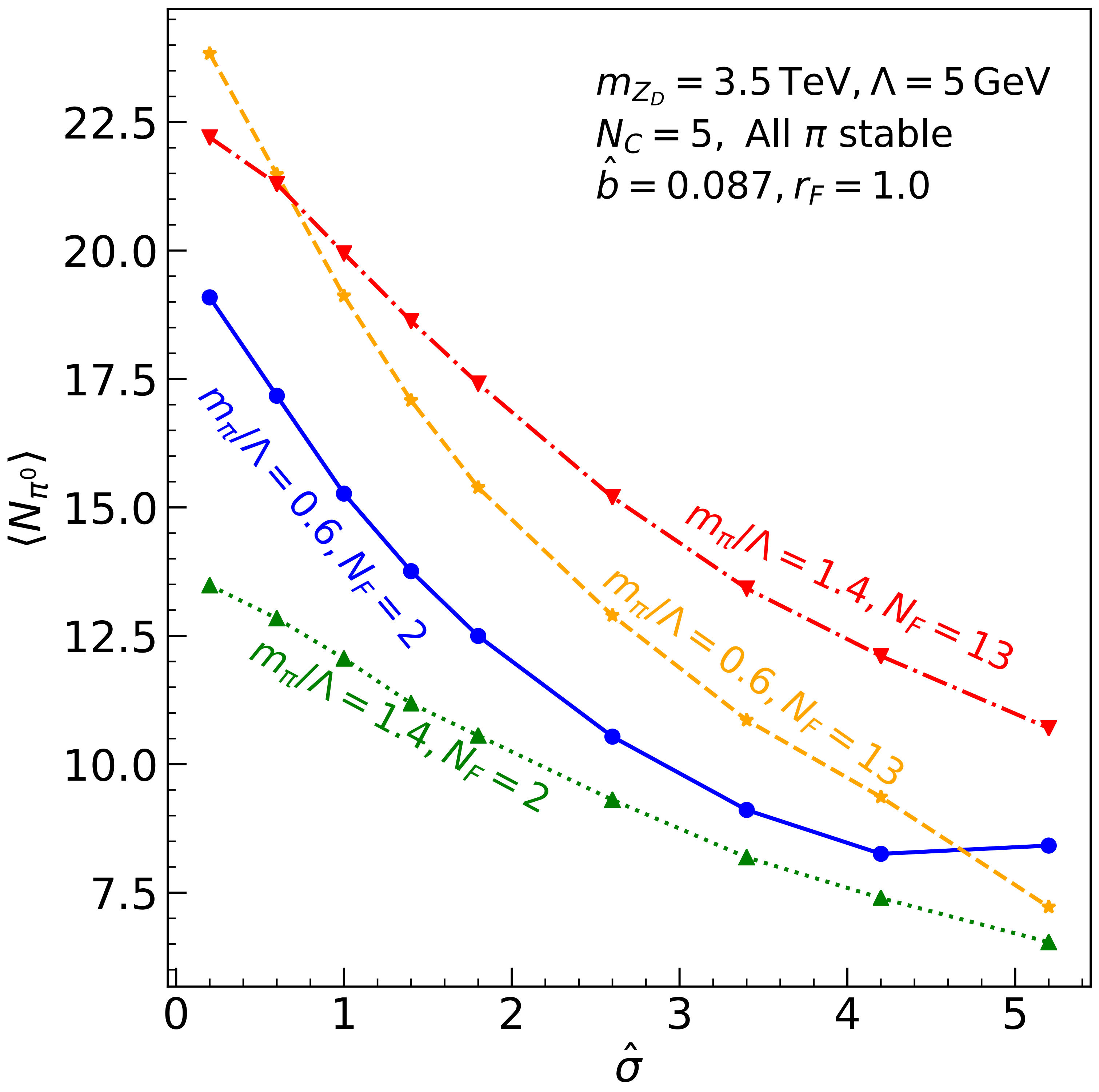
$$\langle \kappa\tau \rangle / m_Q^2 = (1+a)/\hat{b}$$

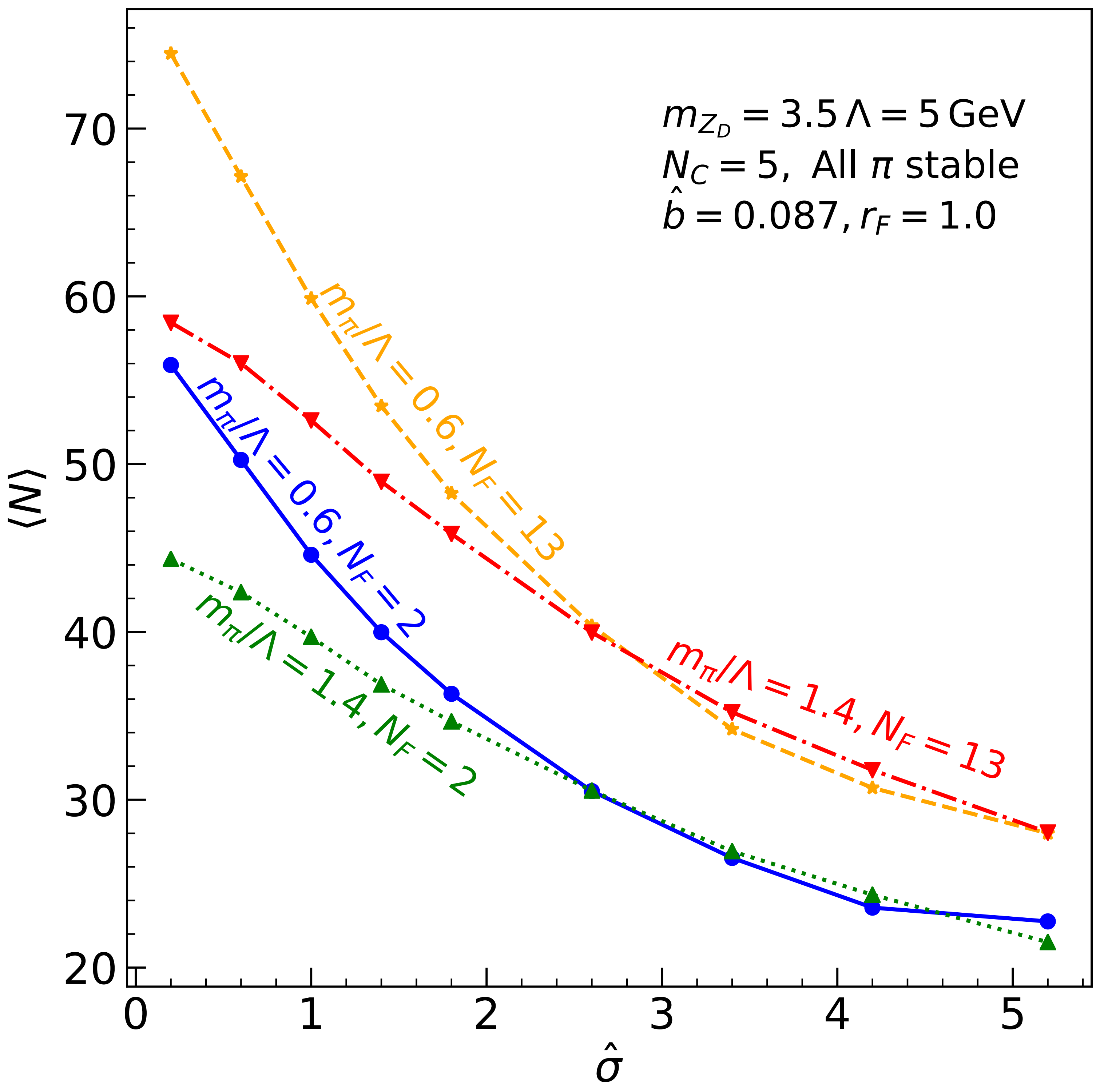
- Parameters set through the Monash tune - some new a' is related to the tuned a through,

$$a' = (\hat{b}(1+a)/\hat{b}') - 1$$

- $M_Q \approx m_q + \Lambda$ - constituent quark mass.

$\hat{\sigma} = \sigma_{p_T}/m_Q$ dependence





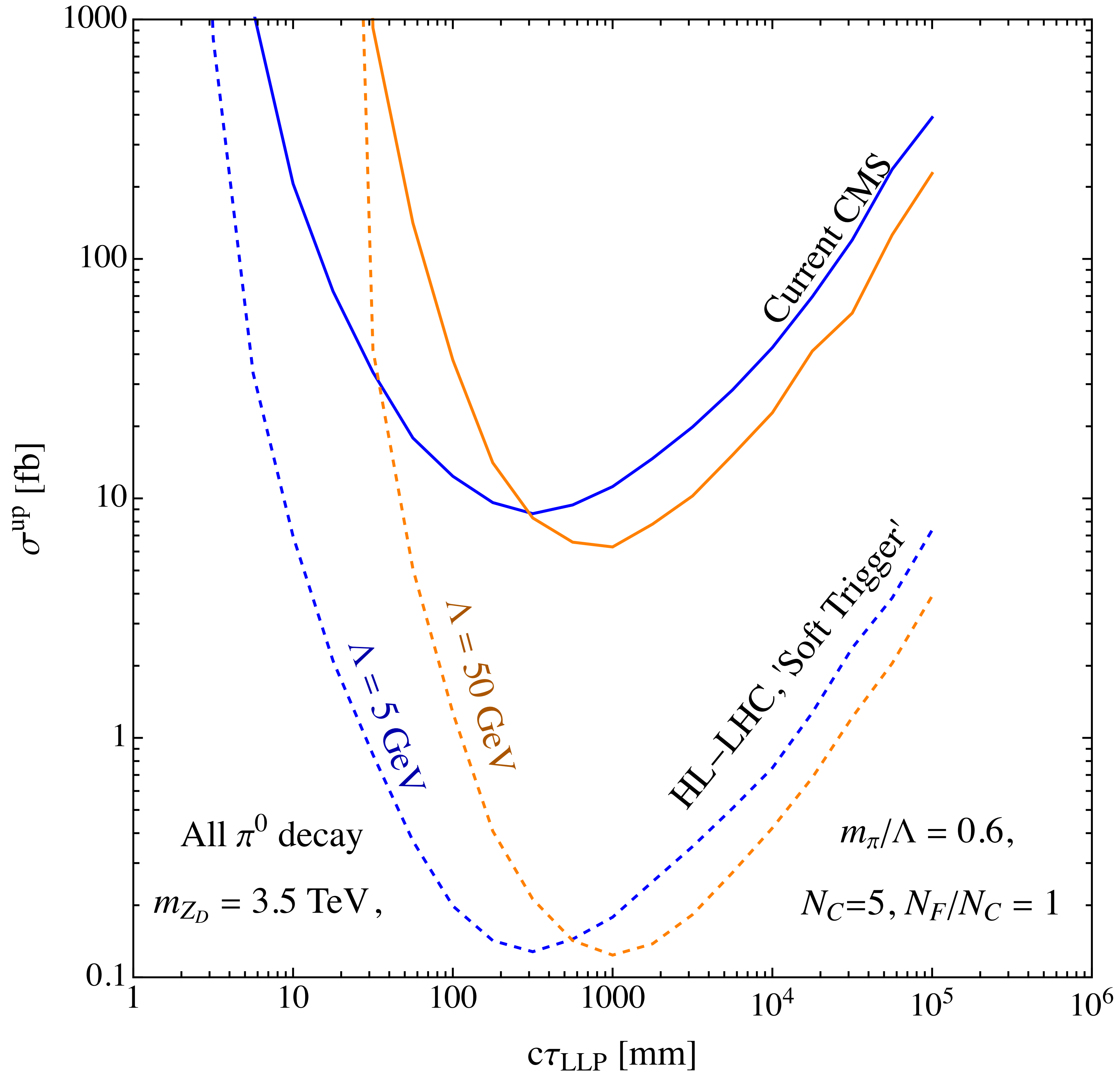
- The p_T of every hadron is obtained from the transverse momentum of the $q_D\bar{q}_D$ pair which is sampled from a Gaussian distribution. This Gaussian comes from the the tunnelling probability of a $q_D\bar{q}_D$ in the WKB approximation.

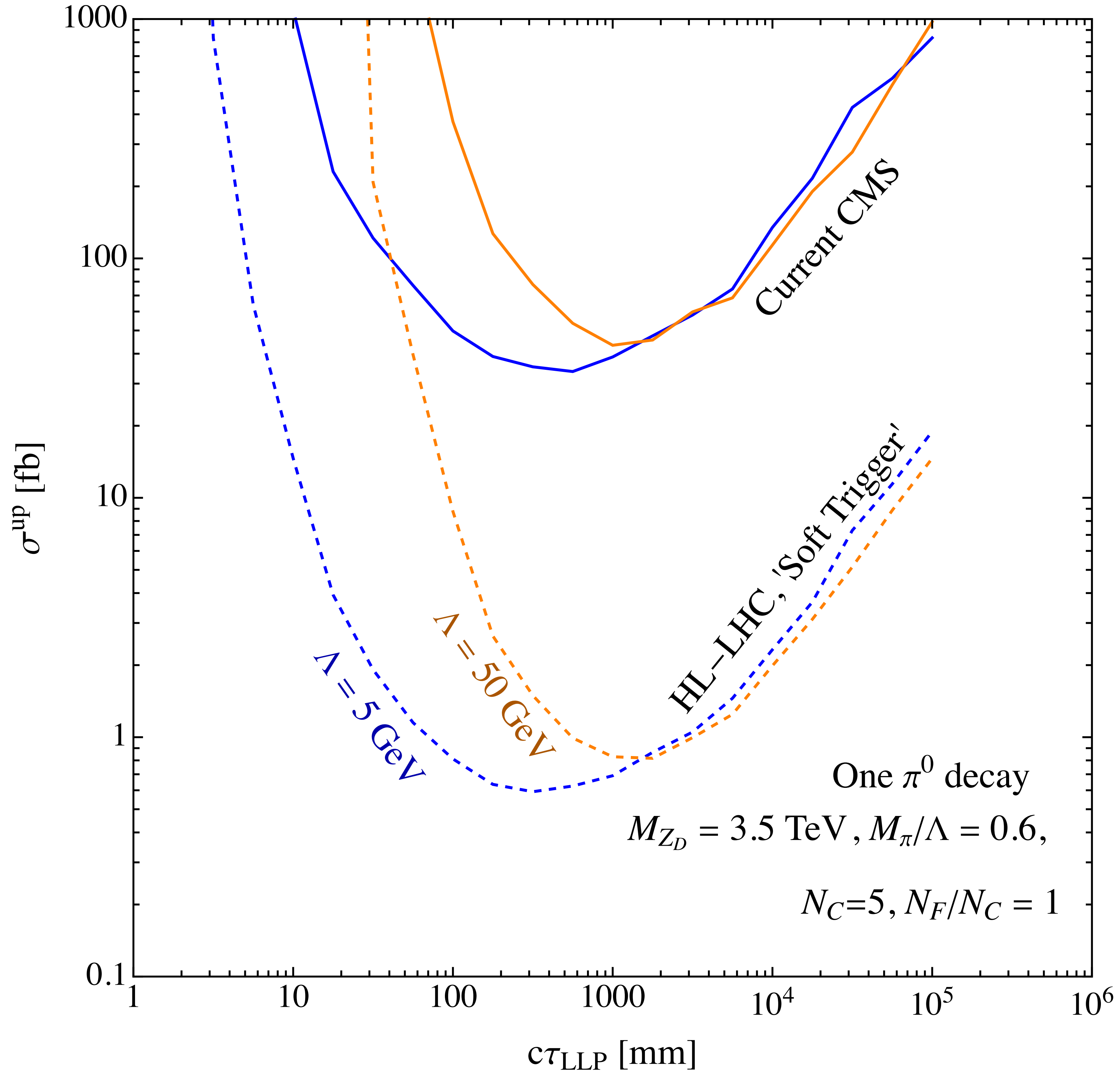
$$d\mathcal{P}/dp_T^2 \propto \exp(-\pi p_T^2/\kappa)\exp(-\pi m_Q^2/\kappa)$$

- Naively, one would expect the sampling of hadron pT to be given with a variance of $\sigma_{p_T}/m_Q = 1/m_Q\sqrt{\kappa/\pi} \approx 0.744$. This produces too soft of a hadron spectrum so $\hat{\sigma} = \sigma_{p_T}/m_Q$ is left as a free parameter in the PYTHIA Hidden Valley module

- $M_Q \approx m_q + \Lambda$ - constituent quark mass.

Dark sector parameters: lifetime sensitivity

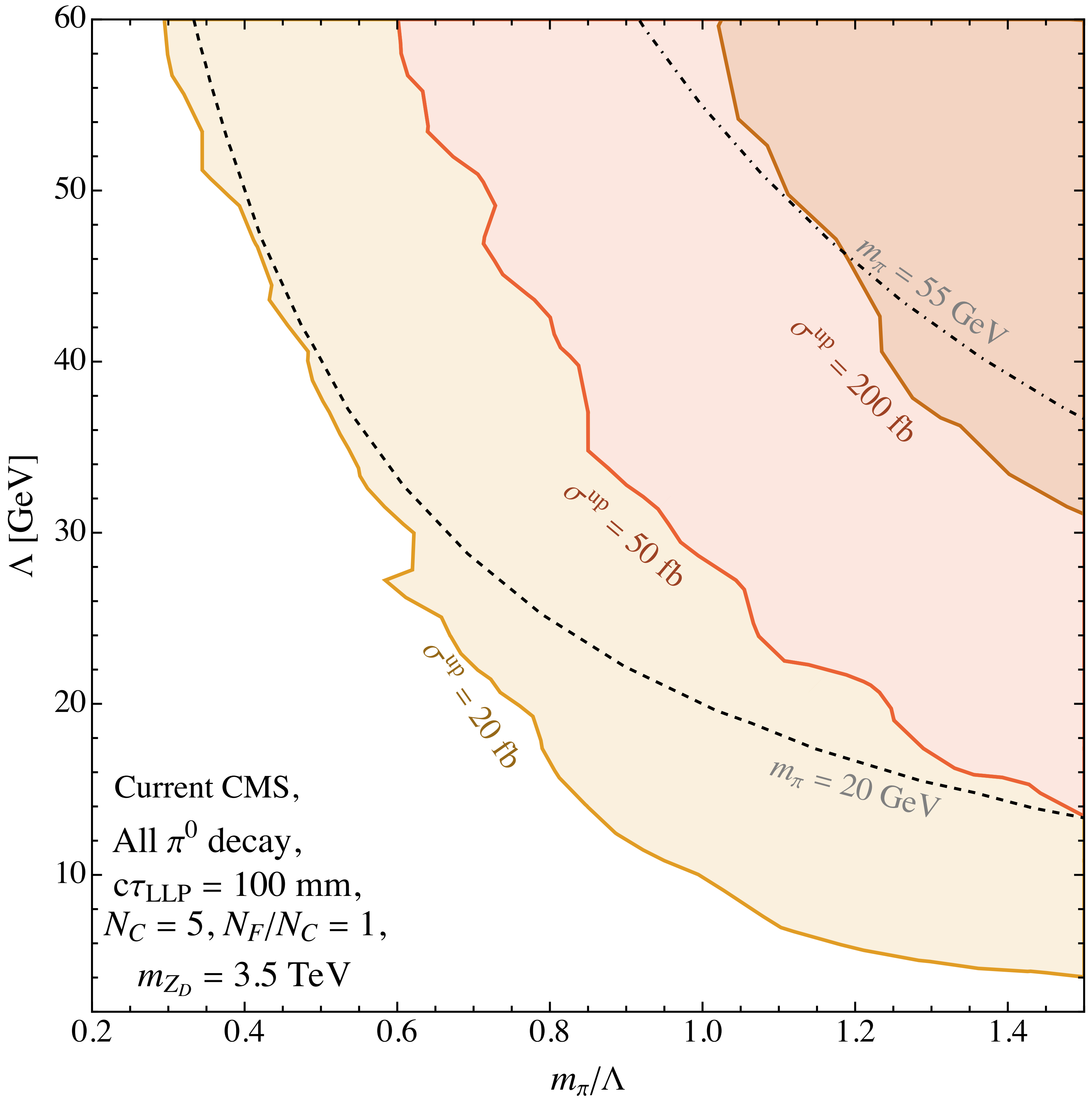


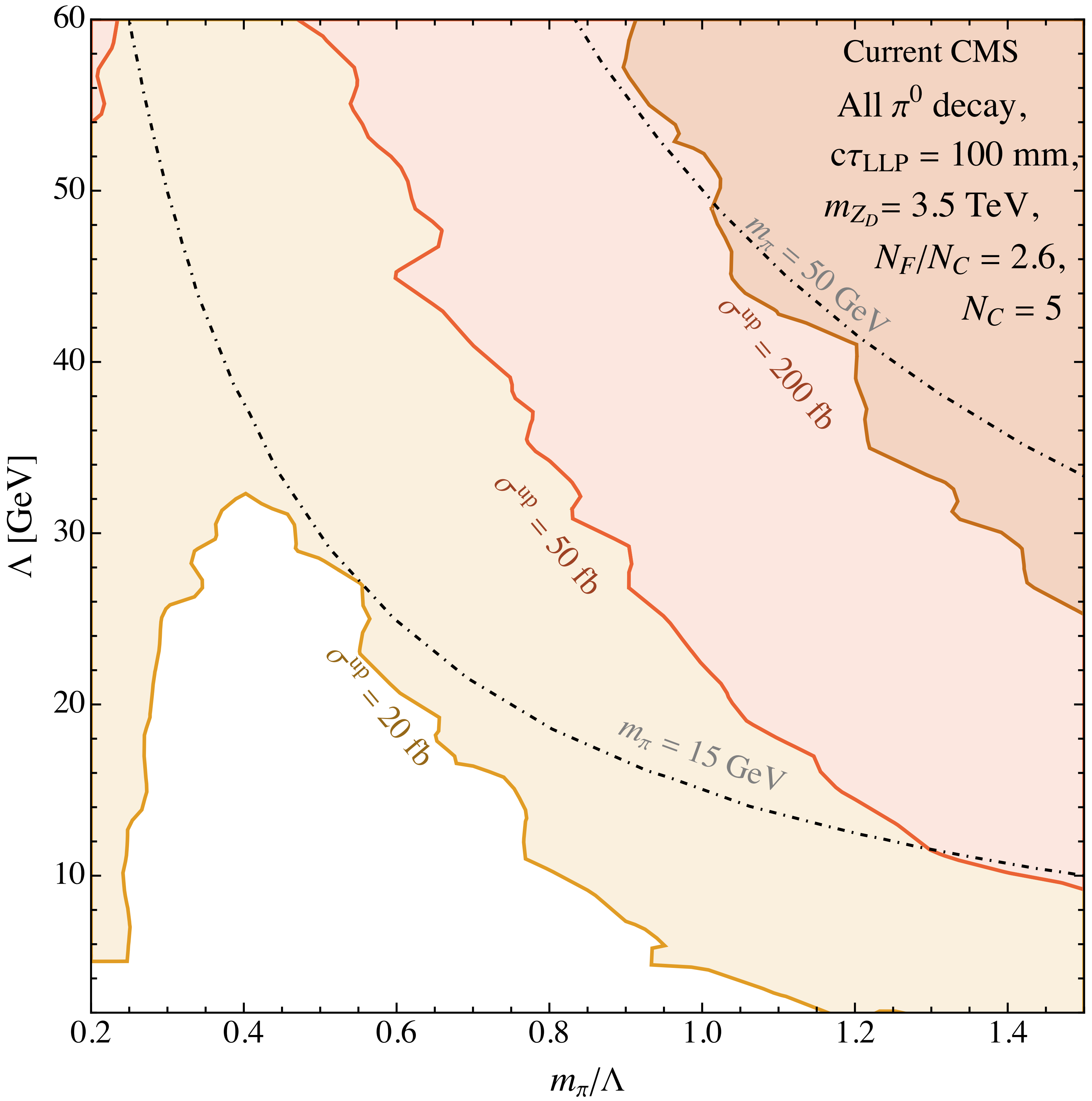


- Can derive model-independent cross section upper-limits through
$$\sigma^{up} = \frac{N^{up}}{\epsilon_{tot} \times \mathscr{L}}.$$
- Current CMS run-2 has $\mathscr{L}$ = 137 fb^{-1}, $N^{up} \approx 6$, whereas the High Luminosity LHC run has $\mathscr{L}$ = 3000 fb^{-1}, $N^{up} \approx 3$.
- Lower MET trigger (due to Level-1 + High Level Triggers at Run-3), mean lower ϵ_{cut} and thus σ^{up} improved by up to $\mathcal{O}(10^2)$ at HL-LHC with this "soft trigger".

- Analyse current and HL-LHC projected for one-π^0 and all-π^0 decay scenarios for two values of $\Lambda = 5(50)$ GeV. All-π^0 decay case display better sensitivity up to a factor of $1/N_F$. For one-π^0 scenario, best limits for $\Lambda = 5(50)$ GeV set around $600(2000)$ mm for run-2 and $300(1750)$ mm at HL-LHC. Heavier pions - need a larger $c\tau_{LLP}$ to reach the endcap detector.

Dark sector parameters: current sensitivity

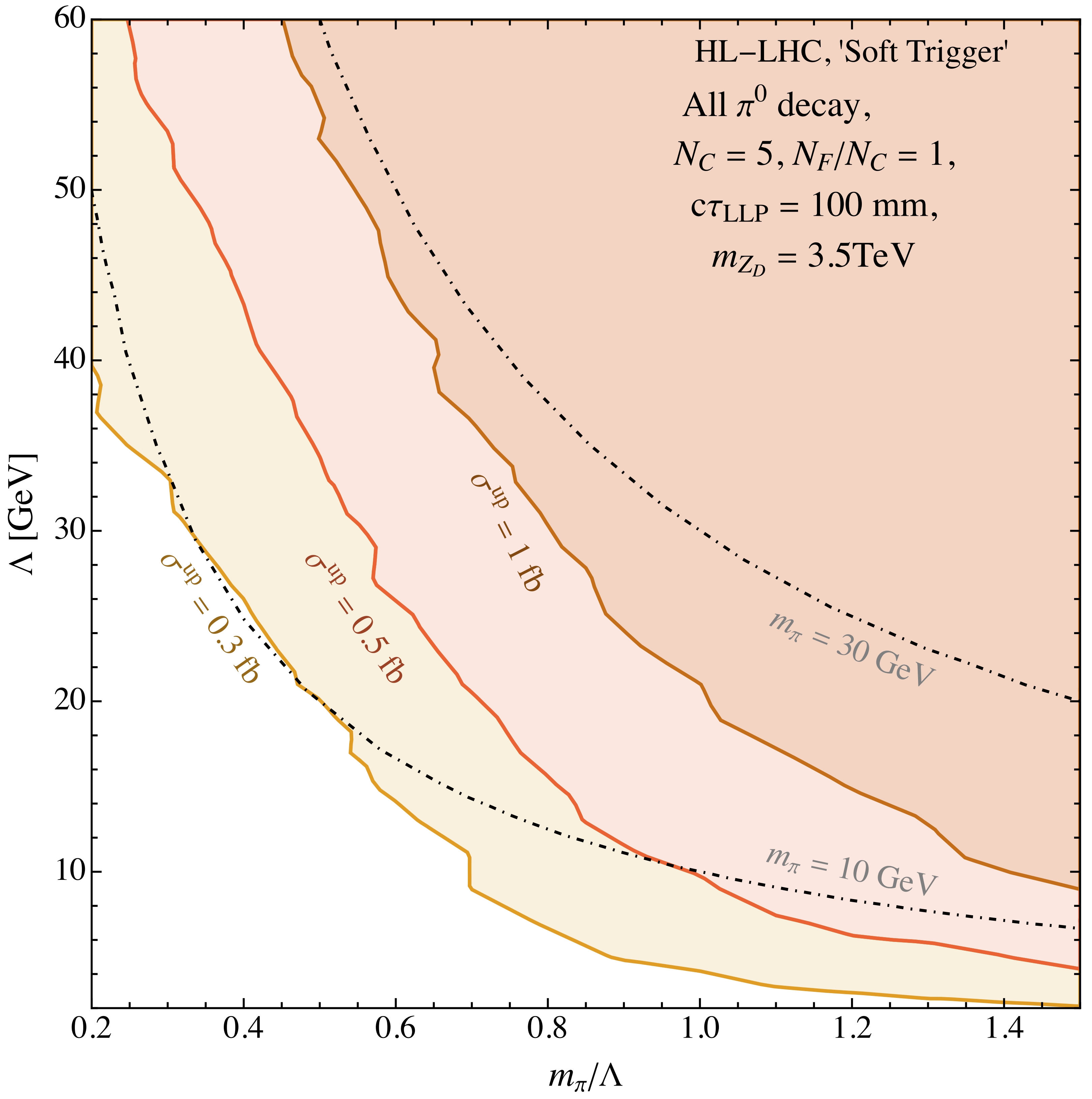


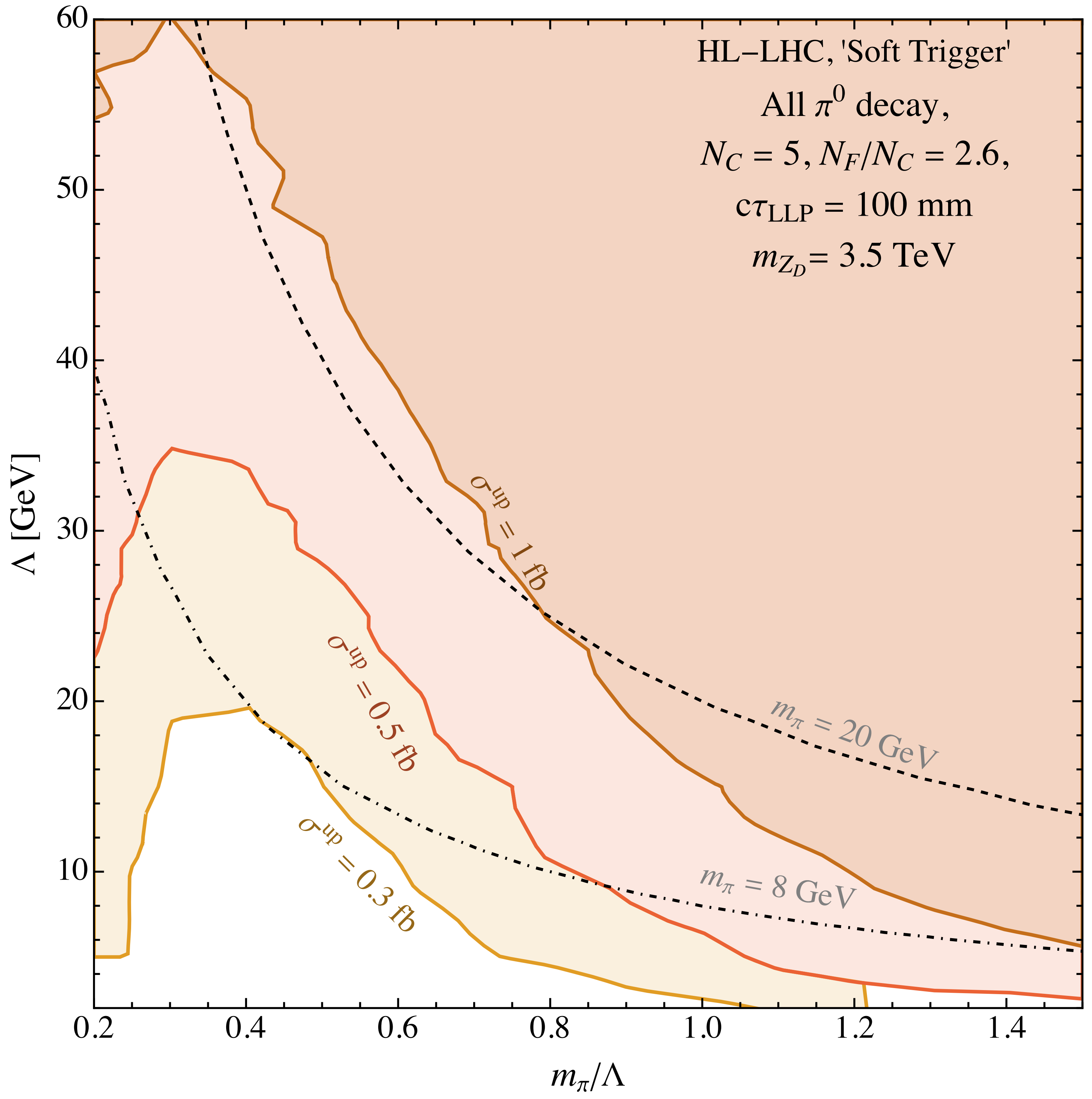


- Best sensitivity achieved around $\mathcal{O}(10^2)$ mm, let's fix $c\tau_{LLP}$ here and discuss sensitivity to HV parameters.
- As expected from efficiency, the upper limits of σ^{up} are strongest for small Λ and m_π/Λ (avoiding the chiral limit!!).
- Choose two mediator benchmarks: $M_{Z_D} = 2(3.5)$ TeV. Better upper limits for higher M_{Z_D}.

- Upper limits strongest for lower N_F/N_C. The upper limits only display significant N_F/N_C sensitivity in the one-π^0 decay case but have worse upper limits. Shark-fin shape in $N_F/N_C = 2.6$ case due to change in multiplicity trend.
- Demonstrates the first sensitivity of an LLP search to the internal HV parameters rather than a typical mediator search. Given a specific model - lifetime and breaking pattern - can set constraints from these upper limits.

Dark sector parameters: HL-LHC sensitivity





- Best sensitivity achieved around $\mathcal{O}(10^2)$ mm, let's fix $c\tau_{LLP}$ here and discuss sensitivity to HV parameters.
- As expected from efficiency, the upper limits of σ^{up} are strongest for small Λ and m_π/Λ (avoiding the chiral limit!!).
- Choose two mediator benchmarks: $M_{Z_D} = 2(3.5)$ TeV. Better upper limits for higher M_{Z_D}.

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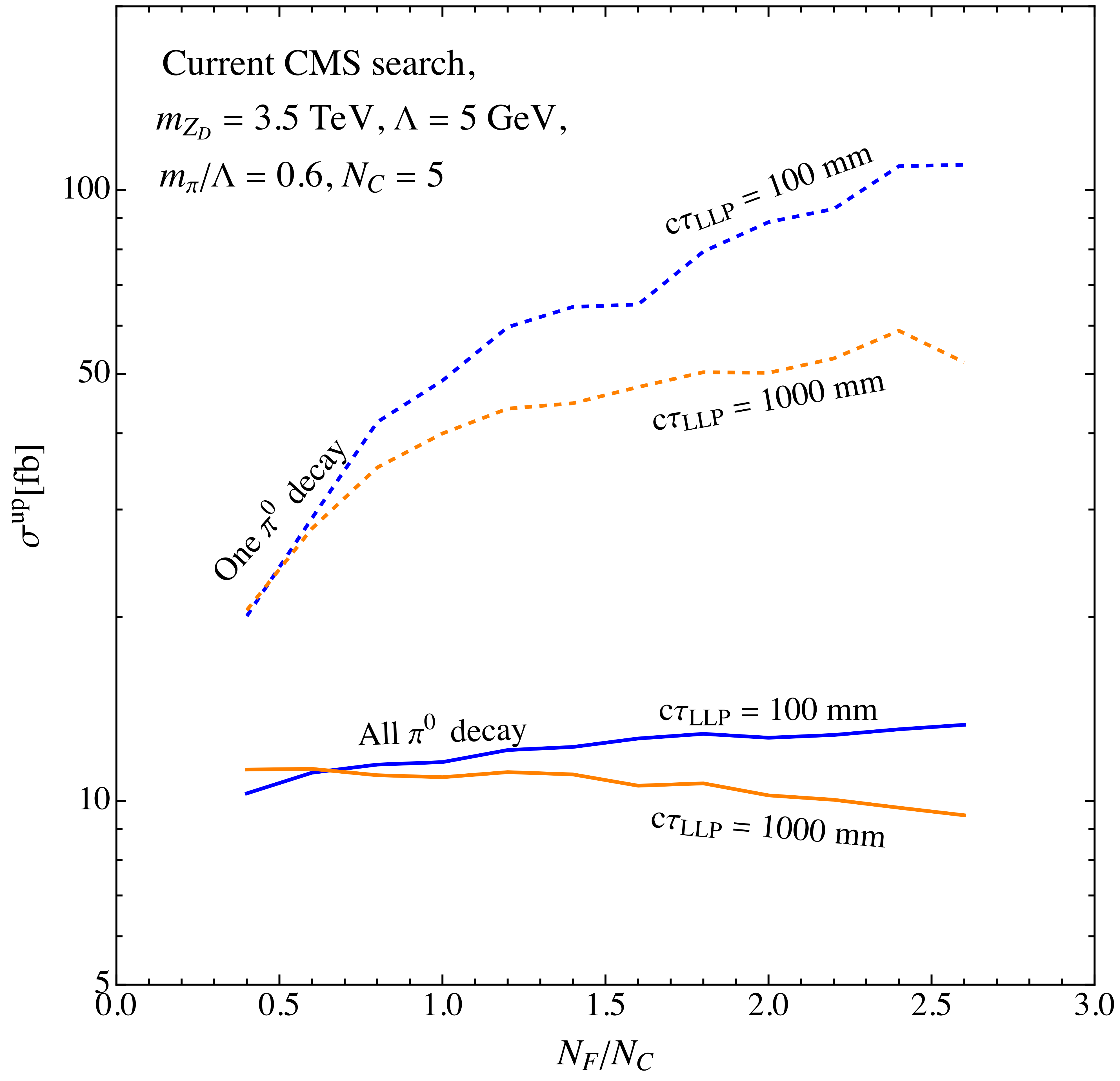
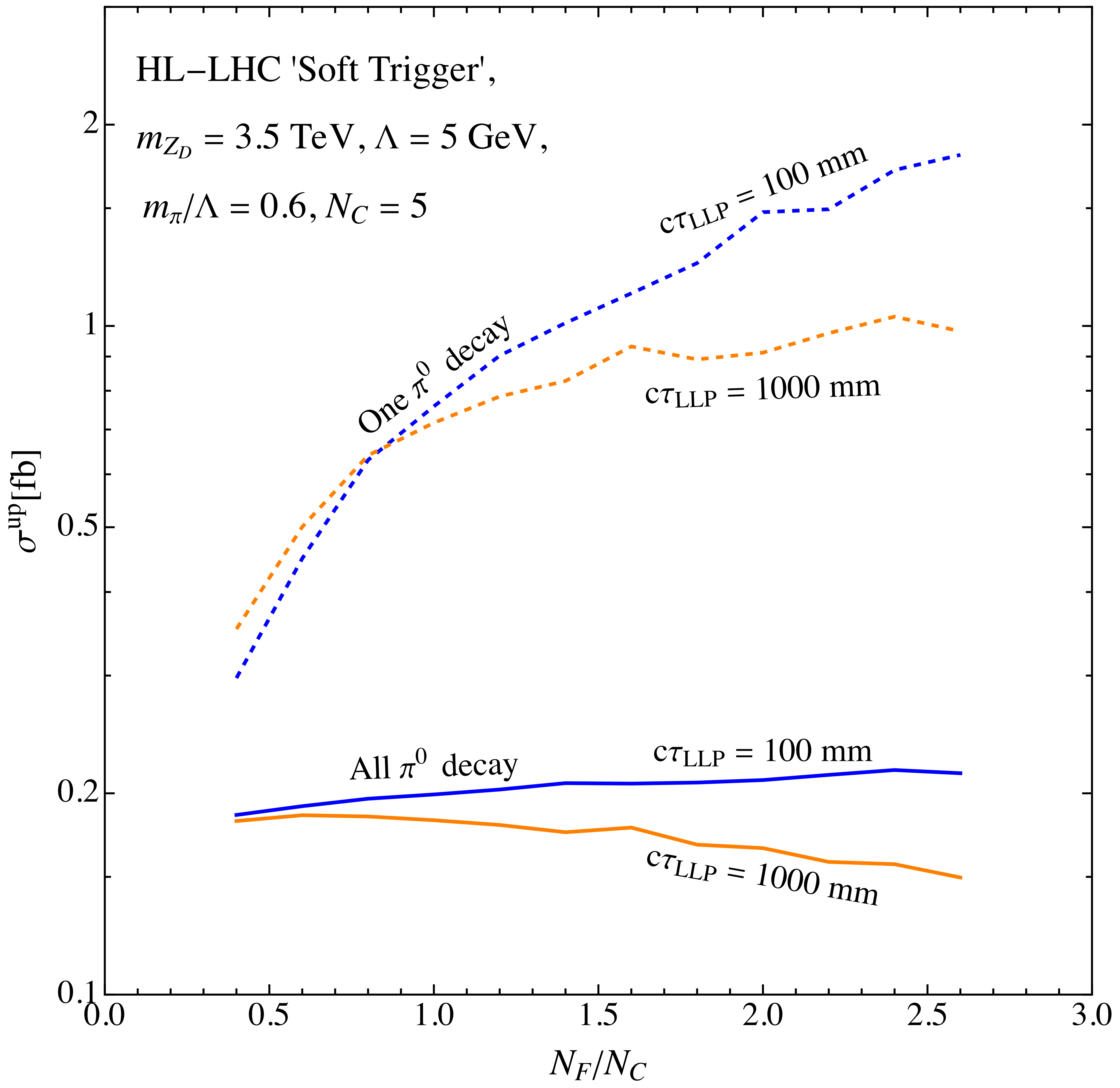
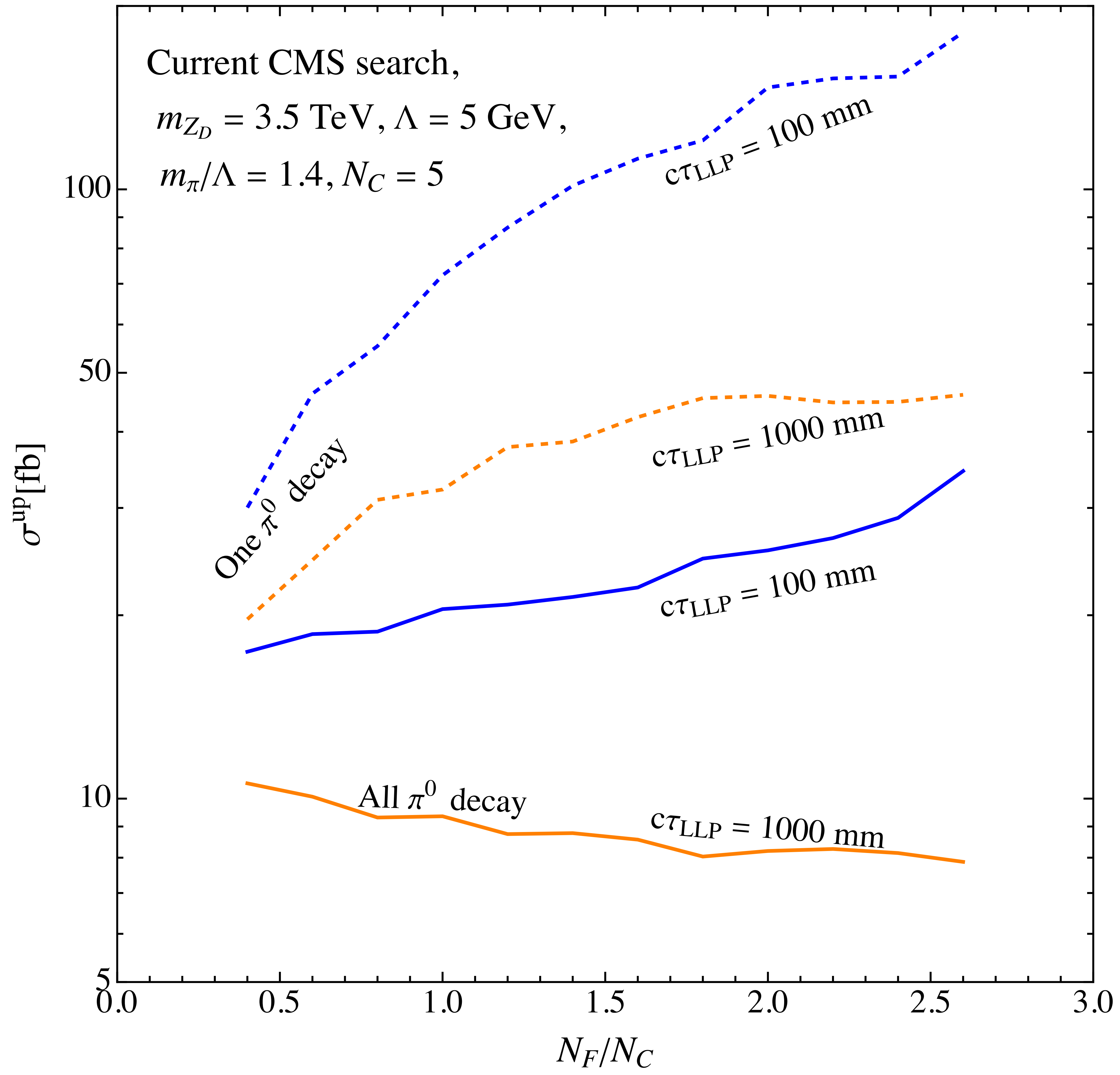
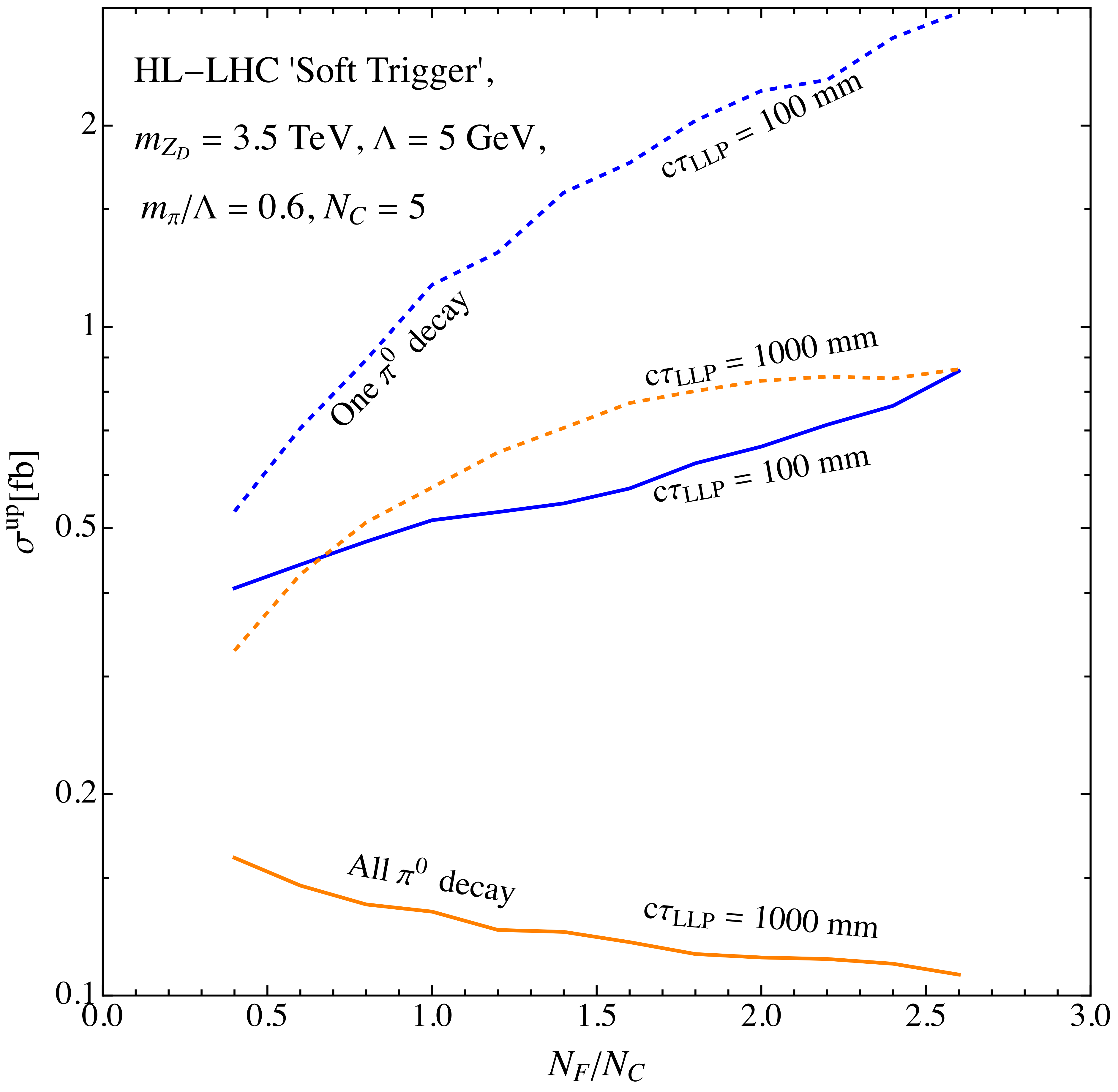
Dark sector parameters: N_F/N_C sensitivity



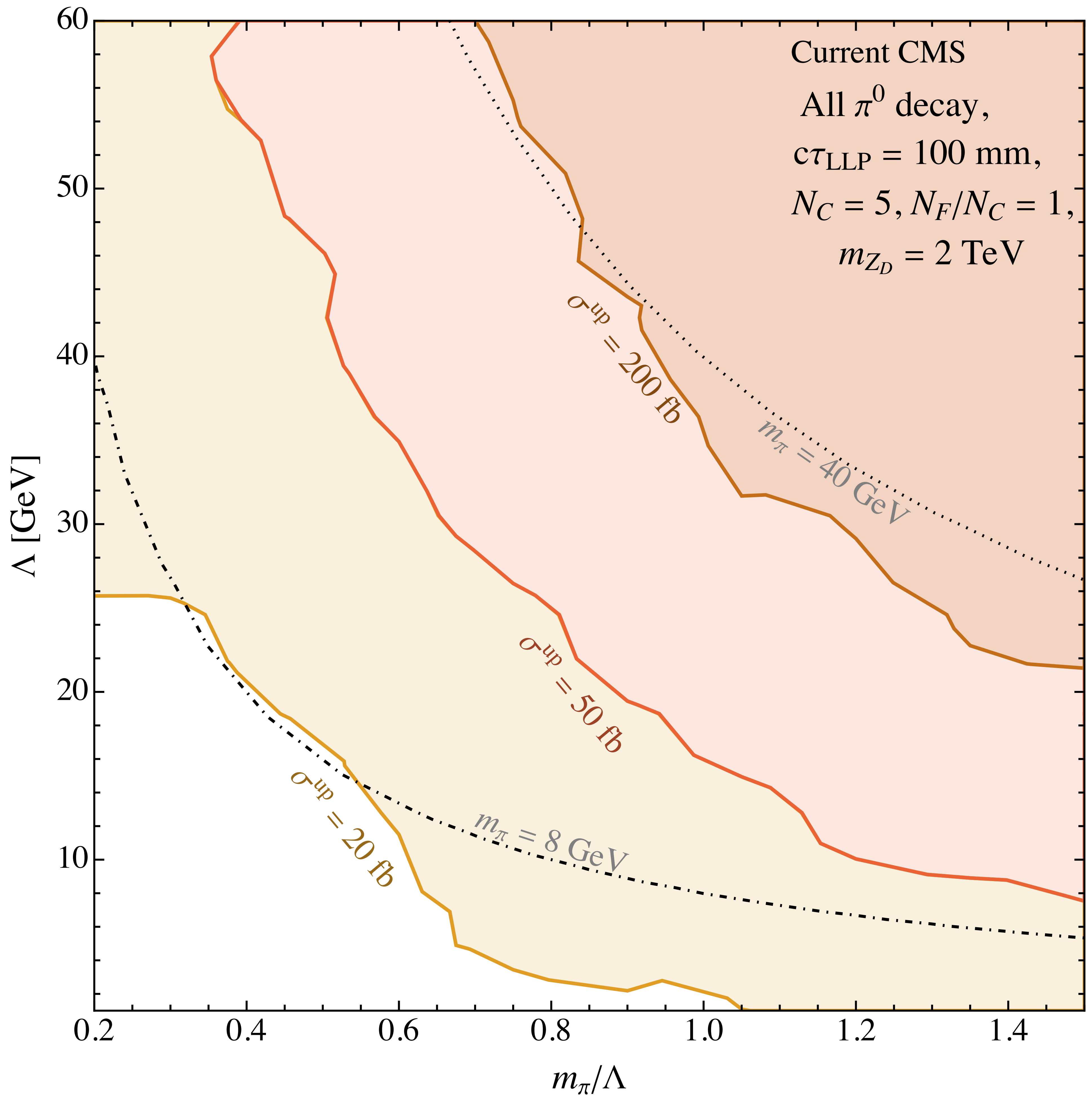


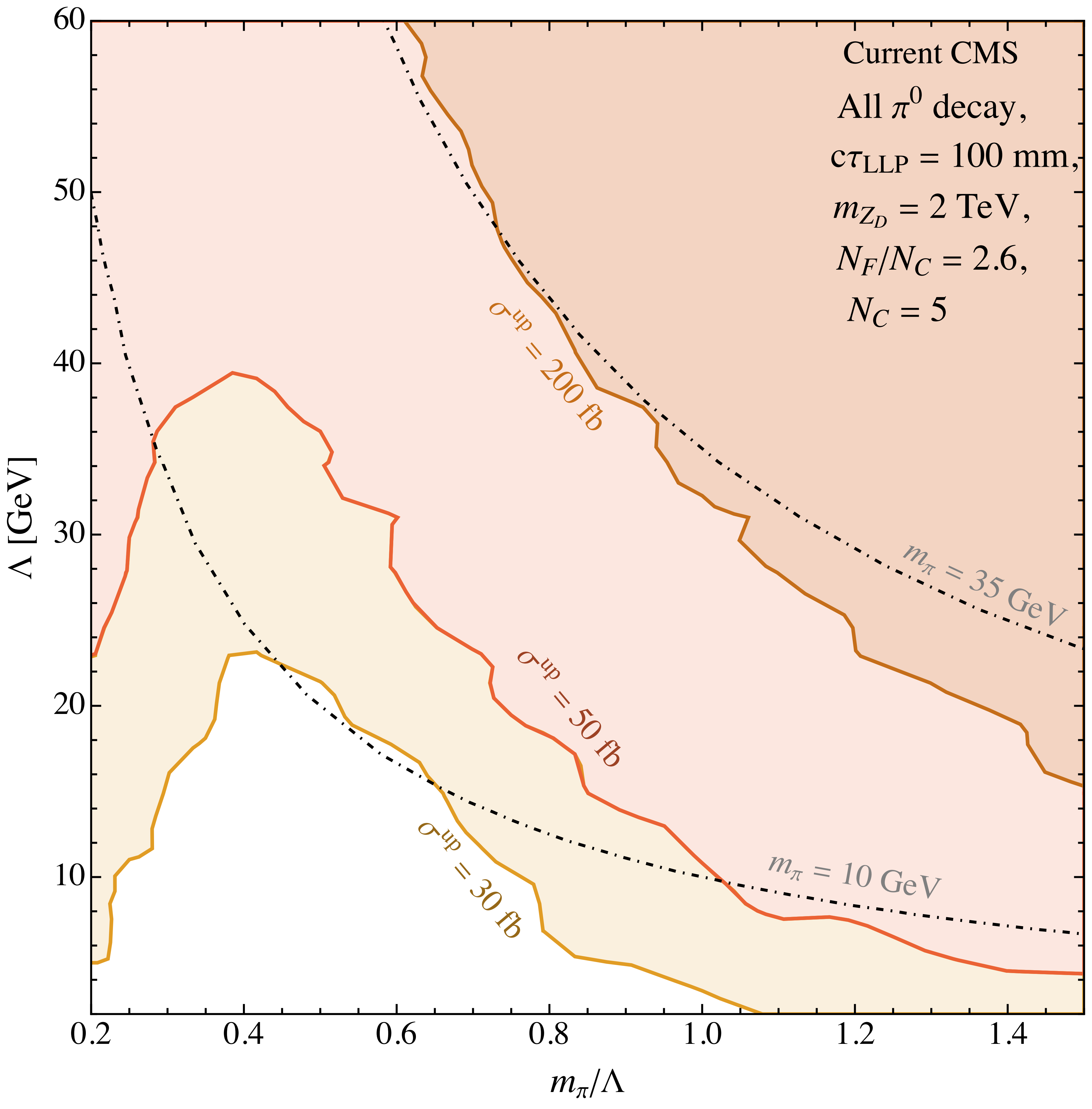
Dark sector parameters: N_F/N_C sensitivity



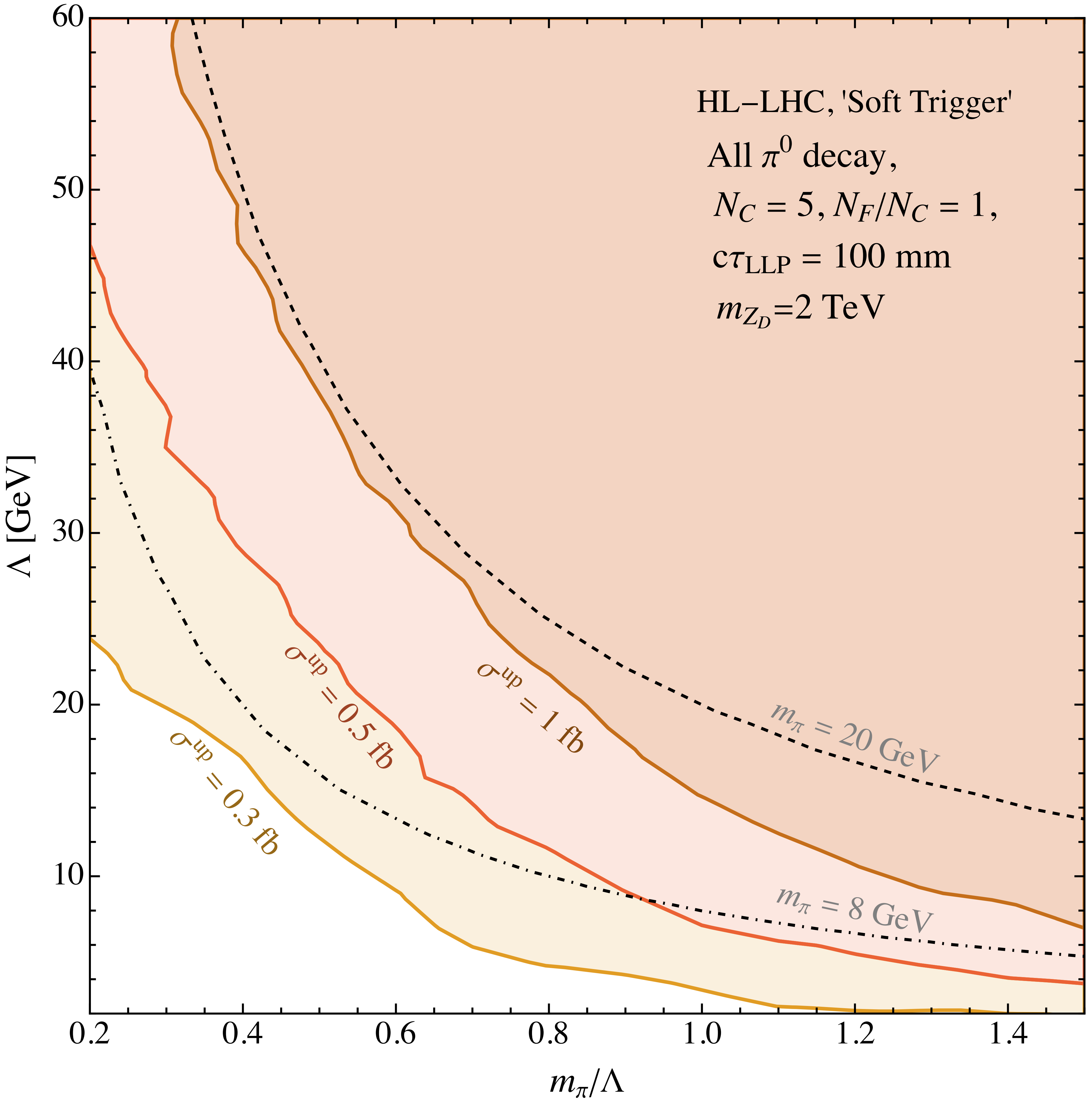


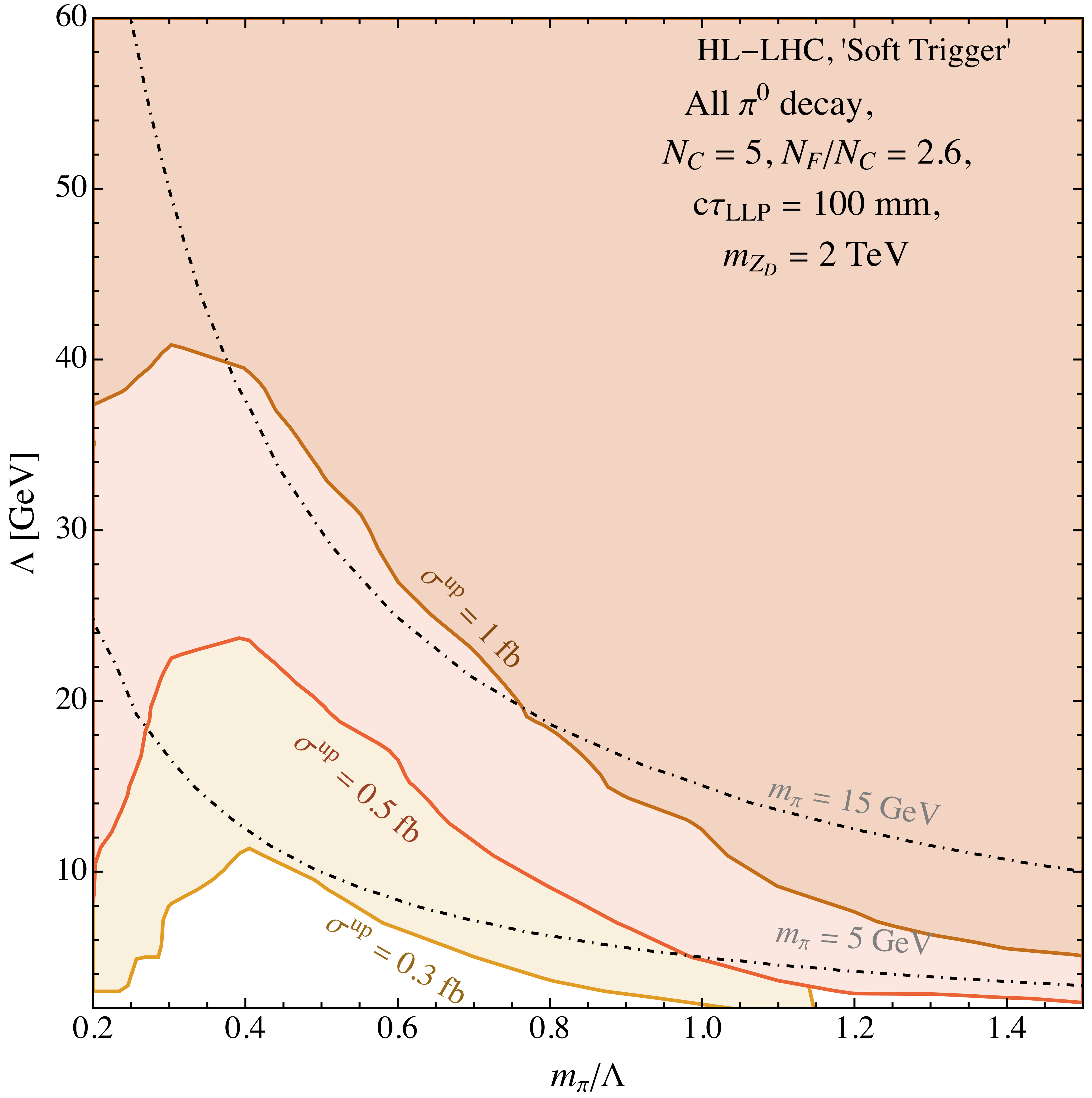
Dark sector parameters: Λ and m_π/Λ sensitivity





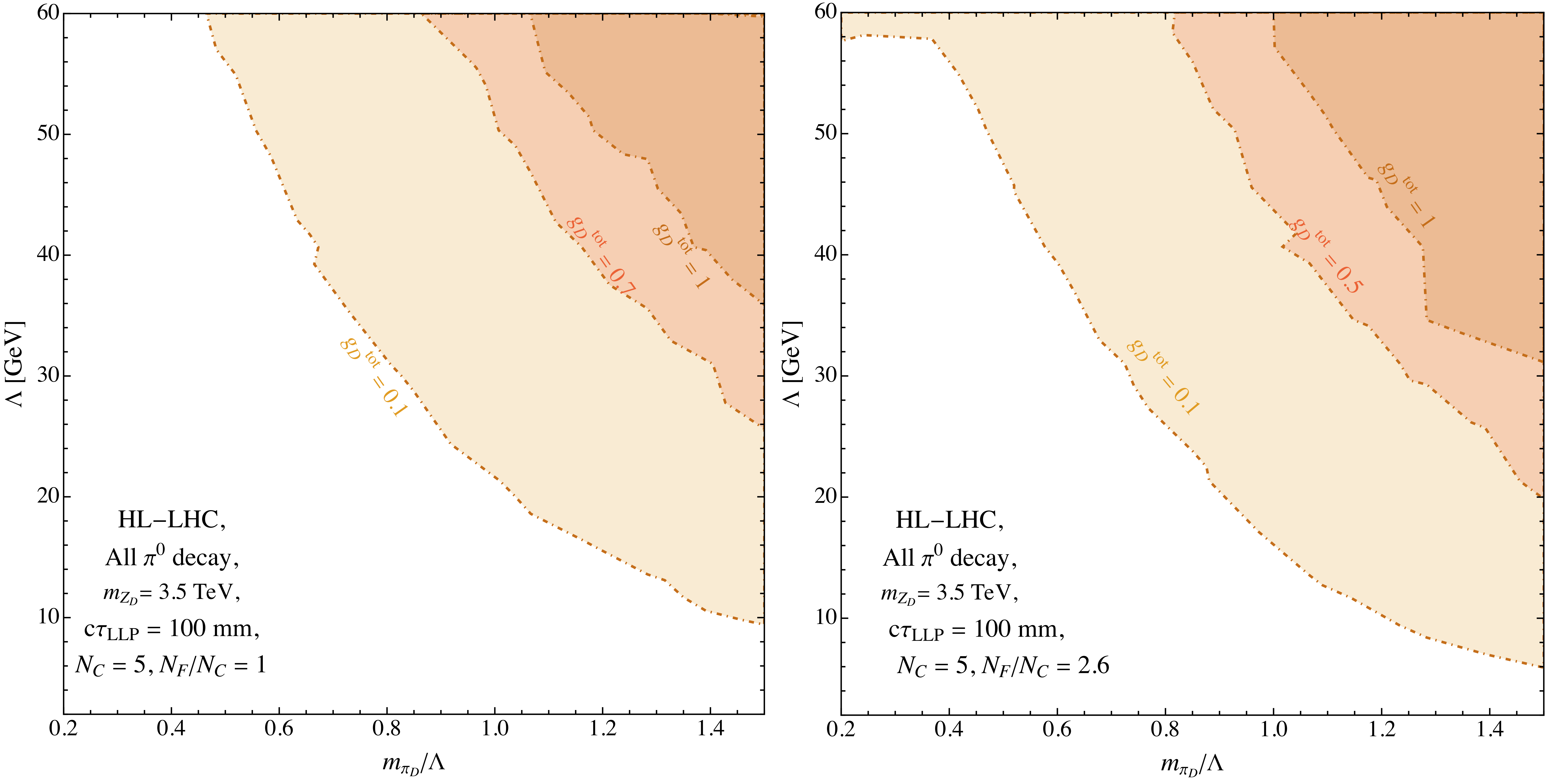
Dark sector parameters: Λ and m_π/Λ sensitivity





Model dependent exclusions (HL-LHC)

arXiv:2404.15930

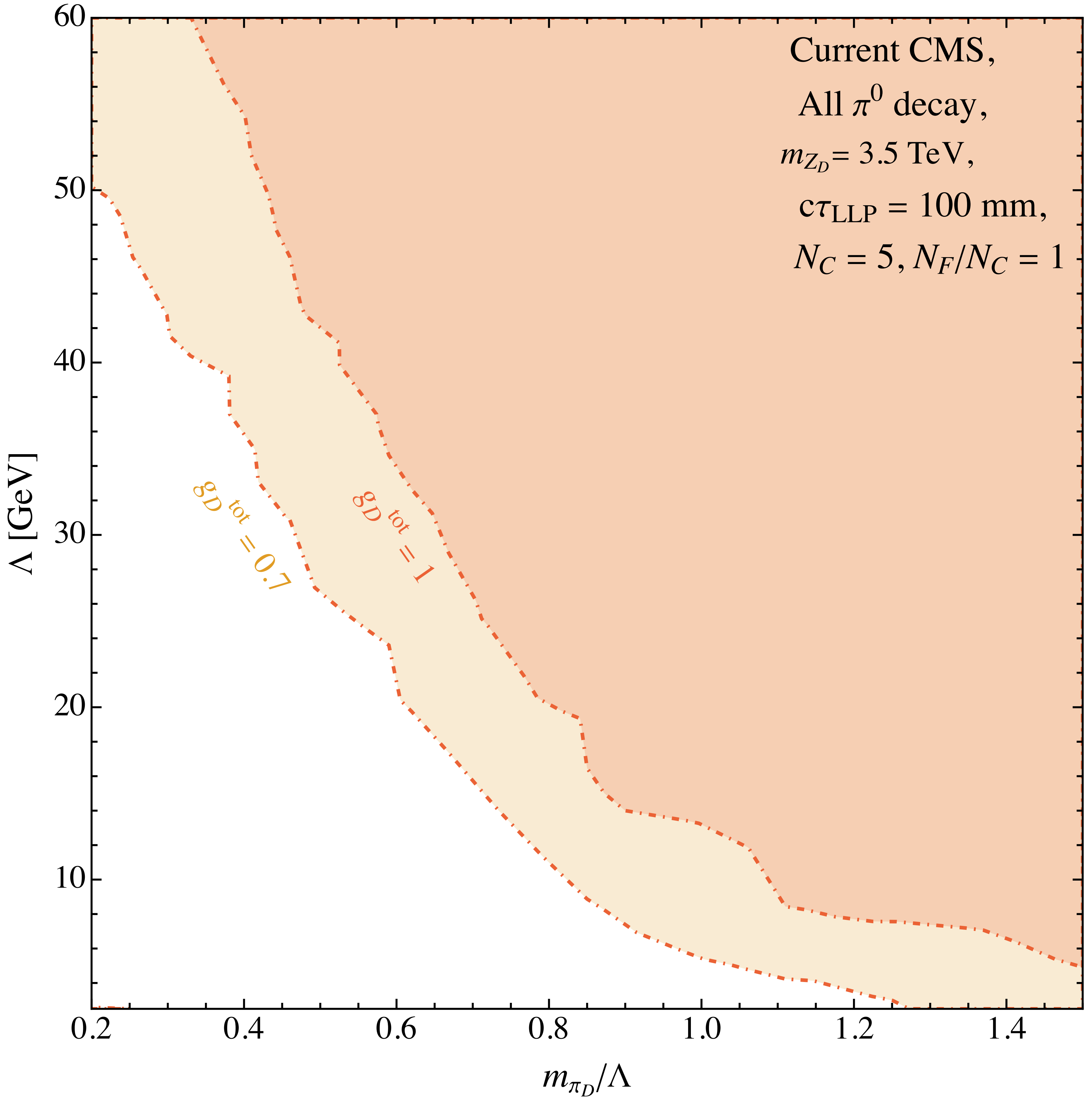


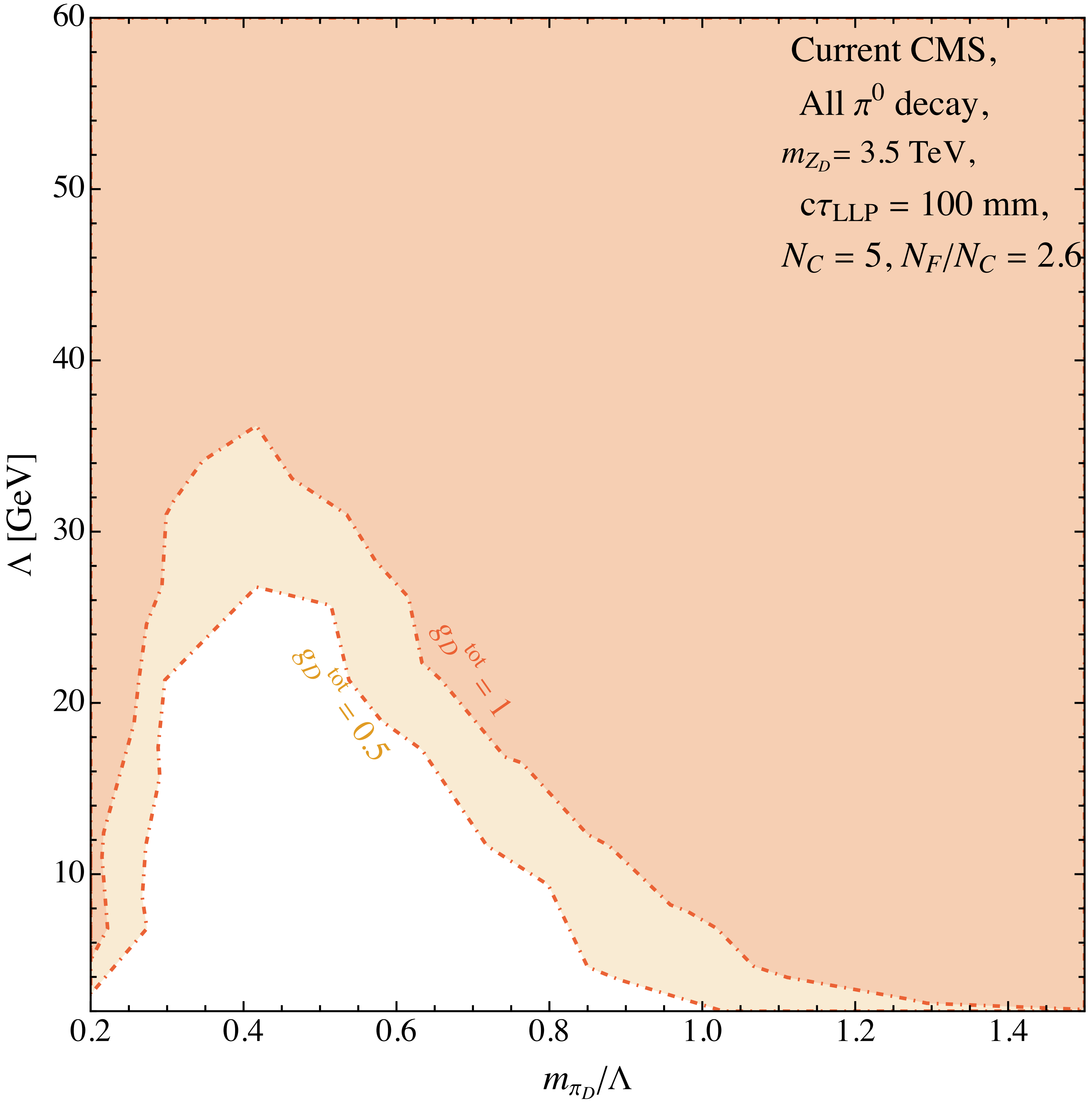
- Can derive model-dependent exclusions: the max. $Z_D - q_D$ coupling (g_D^{tot}) using σ^{up}, existing dijet resonance searches and solving for a specific lifetime.
- A desired lifetime can always be obtained by appropriate charge differences. Keep $g_D^{tot} \leq 1$.

$$\Gamma_\pi \propto \mathrm{Tr}[(\mathcal{Q}_A^D) \cdot T_i]^2 (\mathcal{Q}_A^{SM})^2$$

- Large N_F/N_C relatively unconstrained, proper investigation of the details like meson spectrum could improve exclusion limits. arXiv:2502.18566

Model dependent exclusions (current run)





arXiv:2404.15930

- Can derive model-dependent exclusions: the max. $Z_D - q_D$ coupling (g_D^{tot}) using σ^{up}, existing dijet resonance searches and solving for a specific lifetime.
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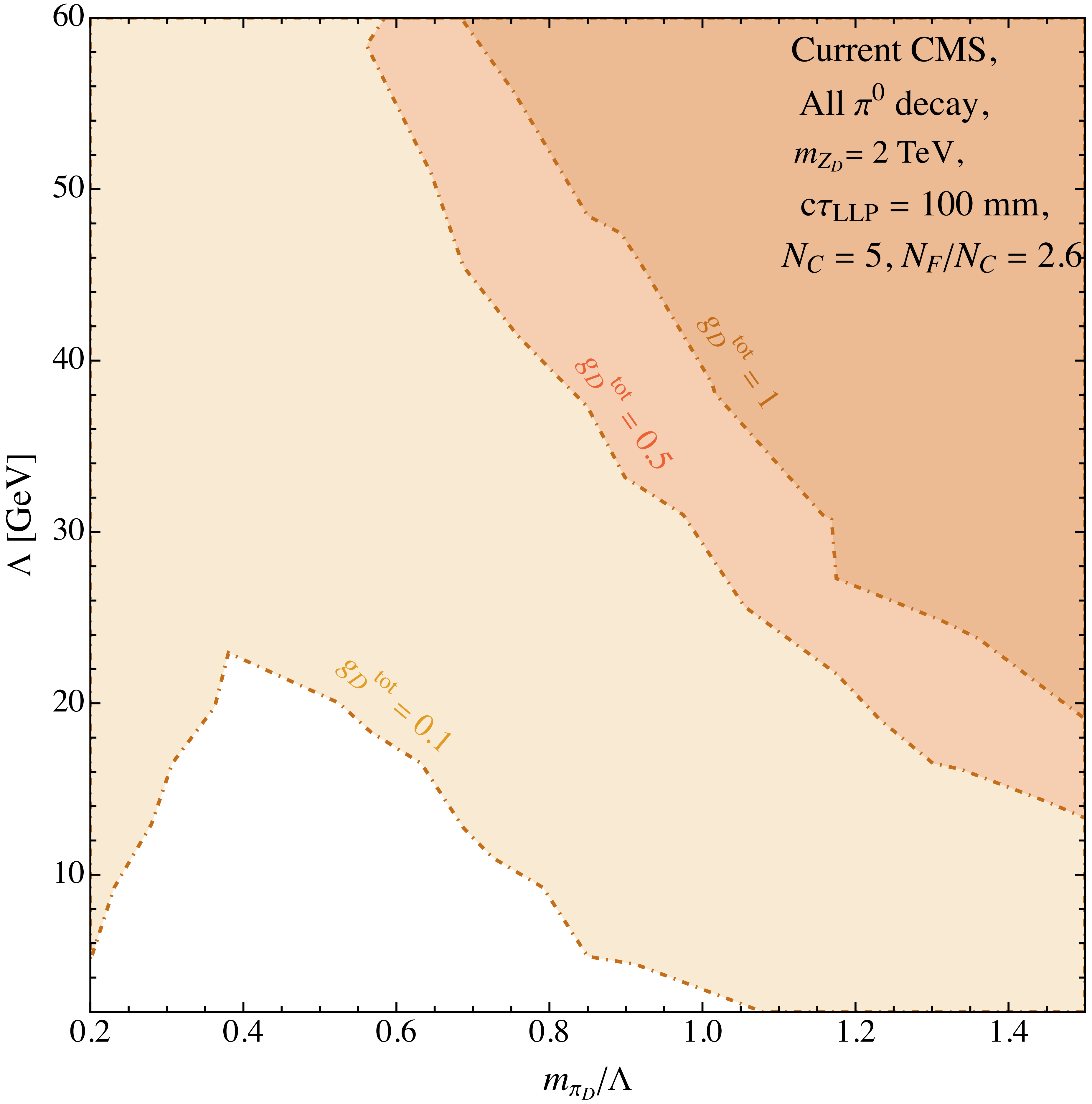
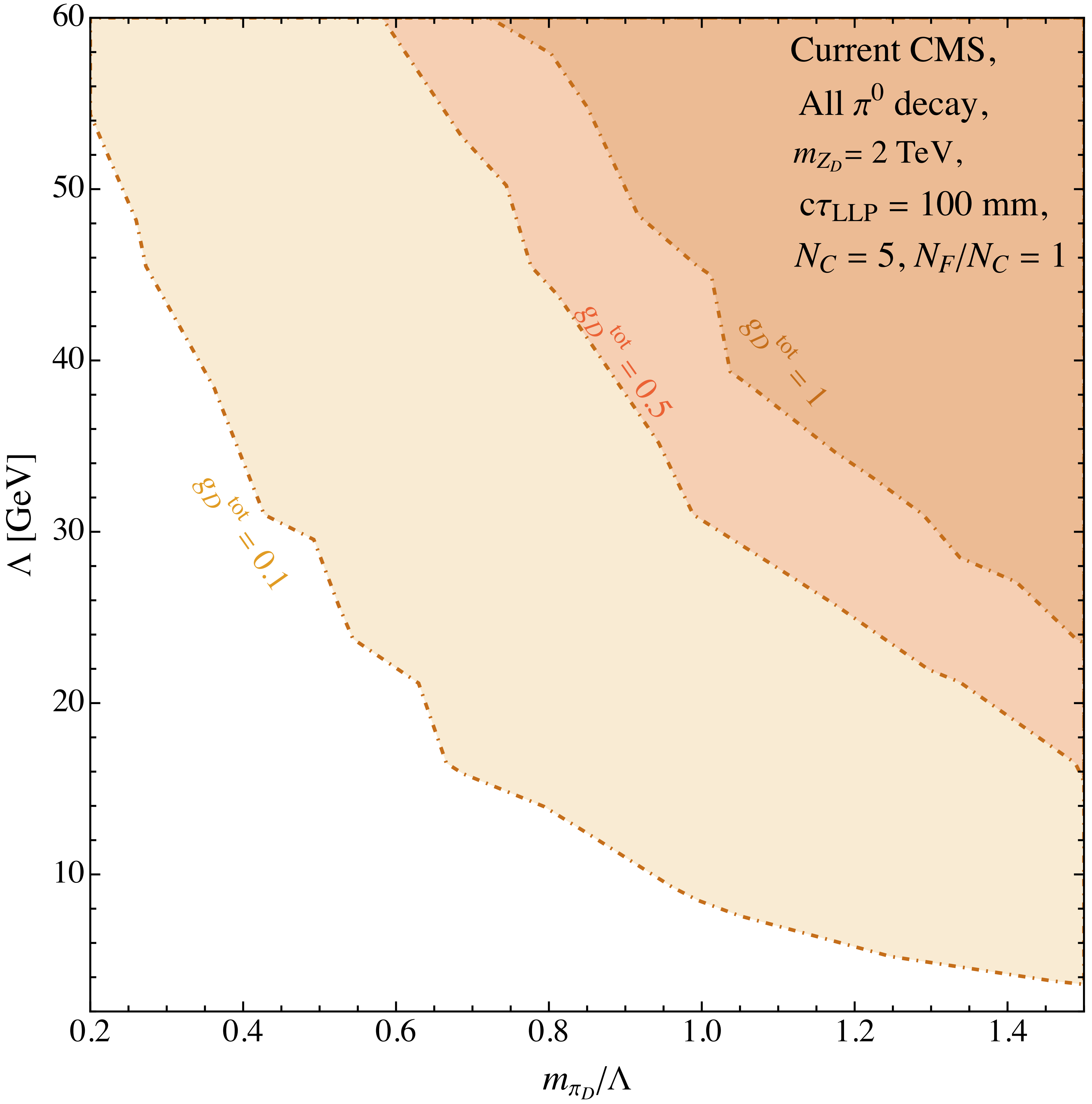
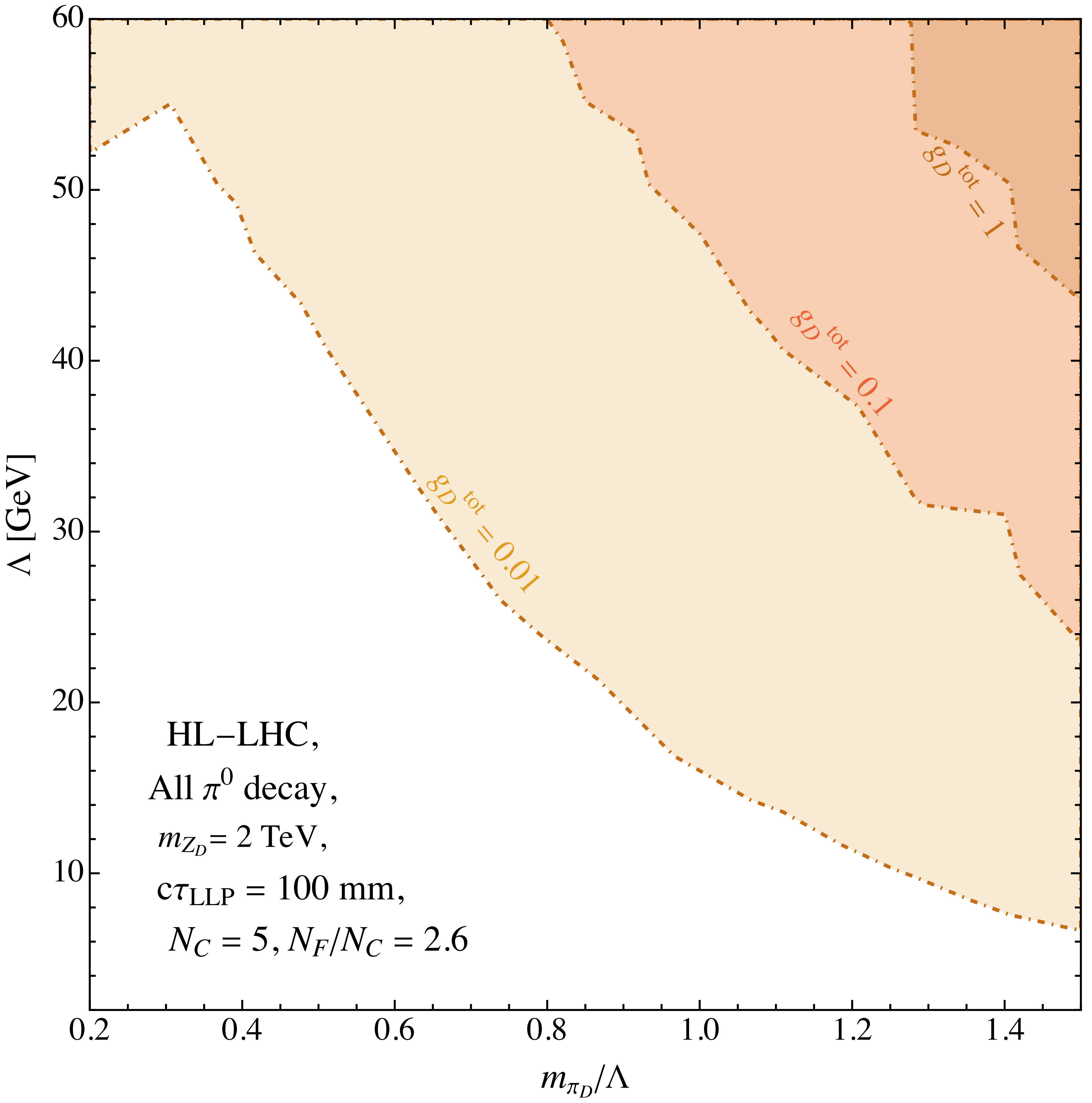
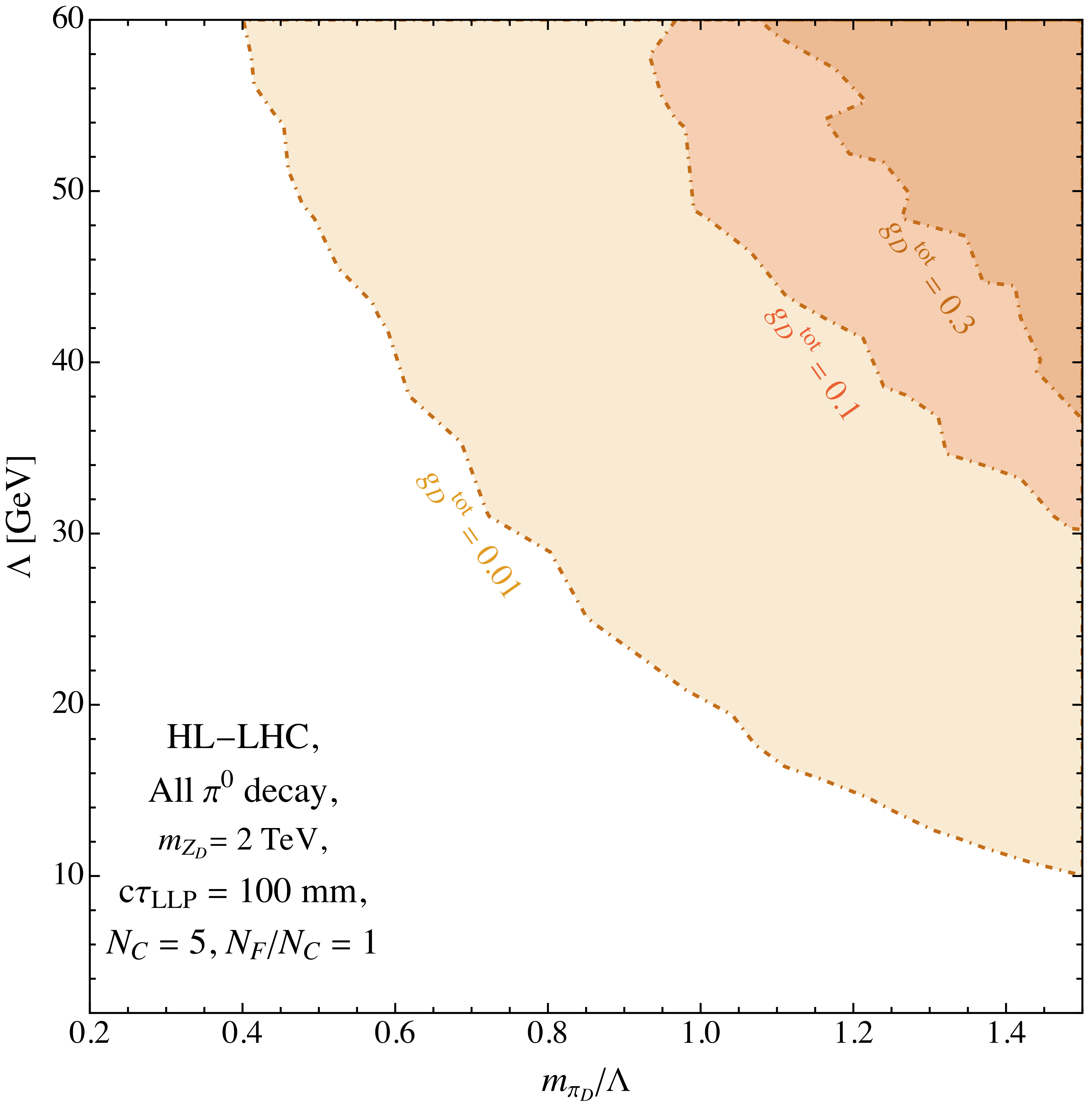
Dark sector parameters: Λ and m_π/Λ exclusions





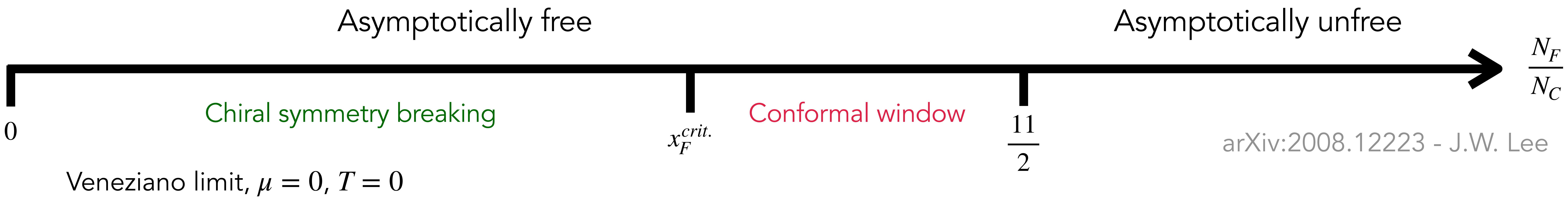
Dark sector parameters: Λ and m_π/Λ exclusions





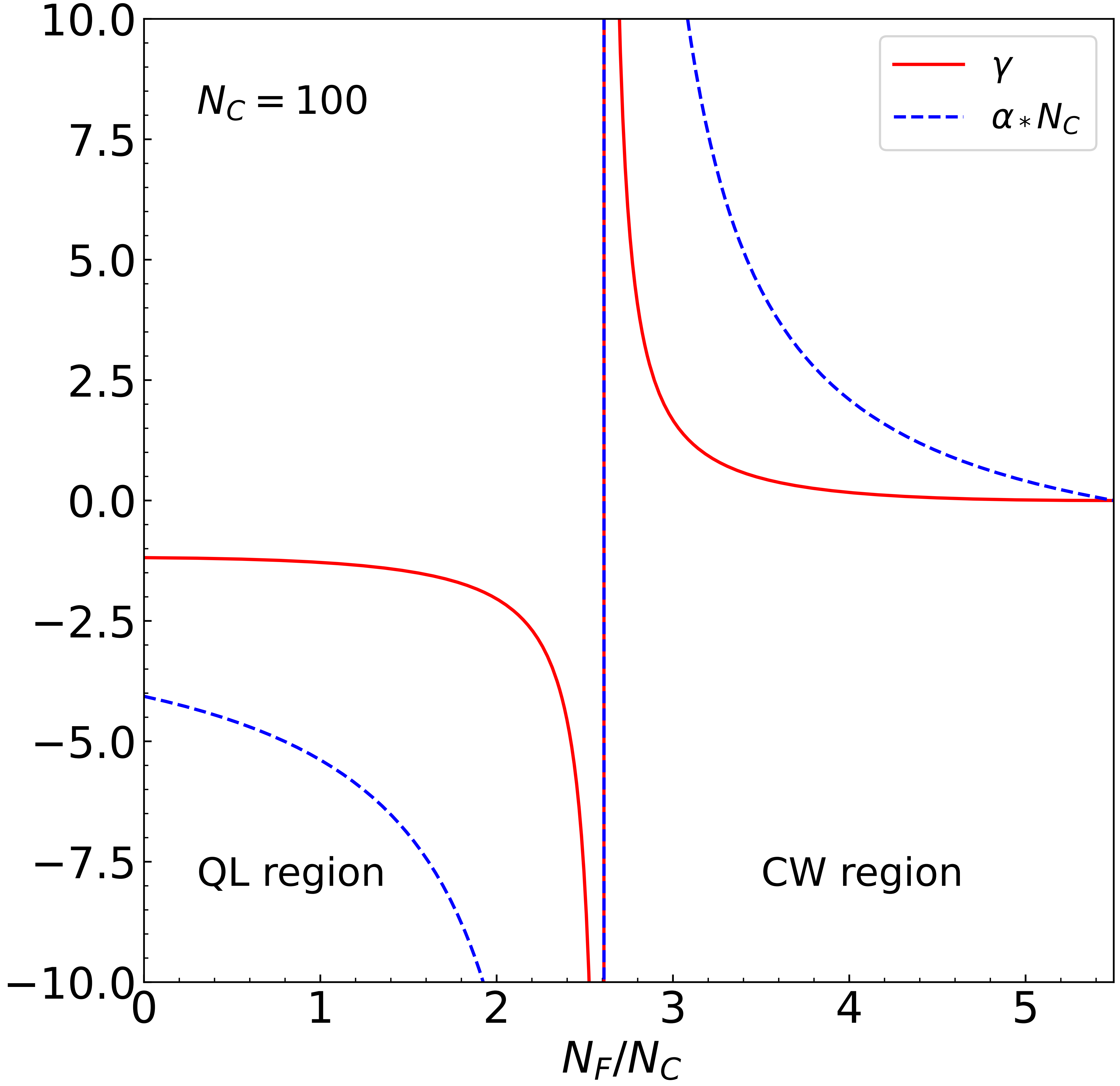
The conformal window?

- At some critical value $N_F/N_C = x_F^{crit.}$, chiral symmetry is restored and the running coupling of such massless conformal theories will flow toward an IRFP.



- Non-perturbative calculations place this critical number anywhere between $\frac{N_f}{N_c}$ = 3 - 4.
- Two-loop running coupling with IRFPs, which occur at $\frac{N_F}{N_C} \gtrsim 2.7$, provide a perturbative approximation of behaviour near the conformal window.

Fixed points and critical exponents



New procedures for IRFPs

- Starting from the two-loop exact solution for the running coupling,

$$\alpha = \alpha_* \left[W_{-1}\left(-\frac{1}{e}\left(\frac{\mu^2}{\Lambda^2}\right)^{\beta_0 \alpha_*}\right) + 1 \right]^{-1}$$

- Can do large μ/Λ as before under the assumption that $\dfrac{1}{\beta_0 \ln(\mu^2/\Lambda^2)} \ll |\alpha_*|$, giving PDG approximation,

$$\alpha(\mu^2) = \frac{1}{\beta_0 \ln(\mu^2/\Lambda^2)}\left(1 - \frac{\beta_1}{\beta_0^2}\frac{\ln(\ln(\mu^2/\Lambda^2))}{\ln(\mu^2/\Lambda^2)}\right)$$

- Since this neglects effects of the IRFP and is an expansion in large μ/Λ, it clearly cannot capture the effects of the IRFP.

- In the CW region, Λ is not the confinement scale, but rather characterises the crossover between power-law and logarithmic running behaviours. The exact scale below which the power-law dominates can be found to be,

$$\beta_0 \ln\left(\frac{\Lambda^2}{\mu_0^2}\right) = -\frac{1}{\alpha_0} - \frac{1}{\alpha_*}\ln\left(\frac{\alpha_*}{\alpha_0} - 1\right)$$

arxiv:9602385,
arxiv:9806409 - T. Appelquist et al.
arxiv:9810192 - E. Gardi et al.

New procedures for IRFPs

- By expanding for large μ/Λ, we obtain closed-form UV expansions of our two solutions. This gives the following third order expansion in the QL region of,

$$\frac{1}{\alpha} = \beta_0 \ln\left(\frac{\mu^2}{\Lambda^2}\right) - \frac{1}{\alpha_*}\ln\left(1 - \beta_0\alpha_*\ln\left(\frac{\mu^2}{\Lambda^2}\right)\right) + \frac{1}{\alpha_*}\frac{\ln\left(1 - \beta_0\alpha_*\ln\left(\frac{\mu^2}{\Lambda^2}\right)\right)}{\beta_0\alpha_*\ln\left(\frac{\mu^2}{\Lambda^2}\right) - 1}$$

QL approximation

- And in the CW region of,

$$\frac{1}{\alpha} = \beta_0 \ln\left(\frac{\mu^2}{\Lambda^2}\right) - \frac{1}{\alpha_*}\ln\left(\beta_0\alpha_*\ln\left(\frac{\mu^2}{\Lambda^2}\right) - 1\right) + \frac{1}{\alpha_*}\frac{\ln\left(\beta_0\alpha_*\ln\left(\frac{\mu^2}{\Lambda^2}\right) - 1\right)}{\beta_0\alpha_*\ln\left(\frac{\mu^2}{\Lambda^2}\right) - 1}$$

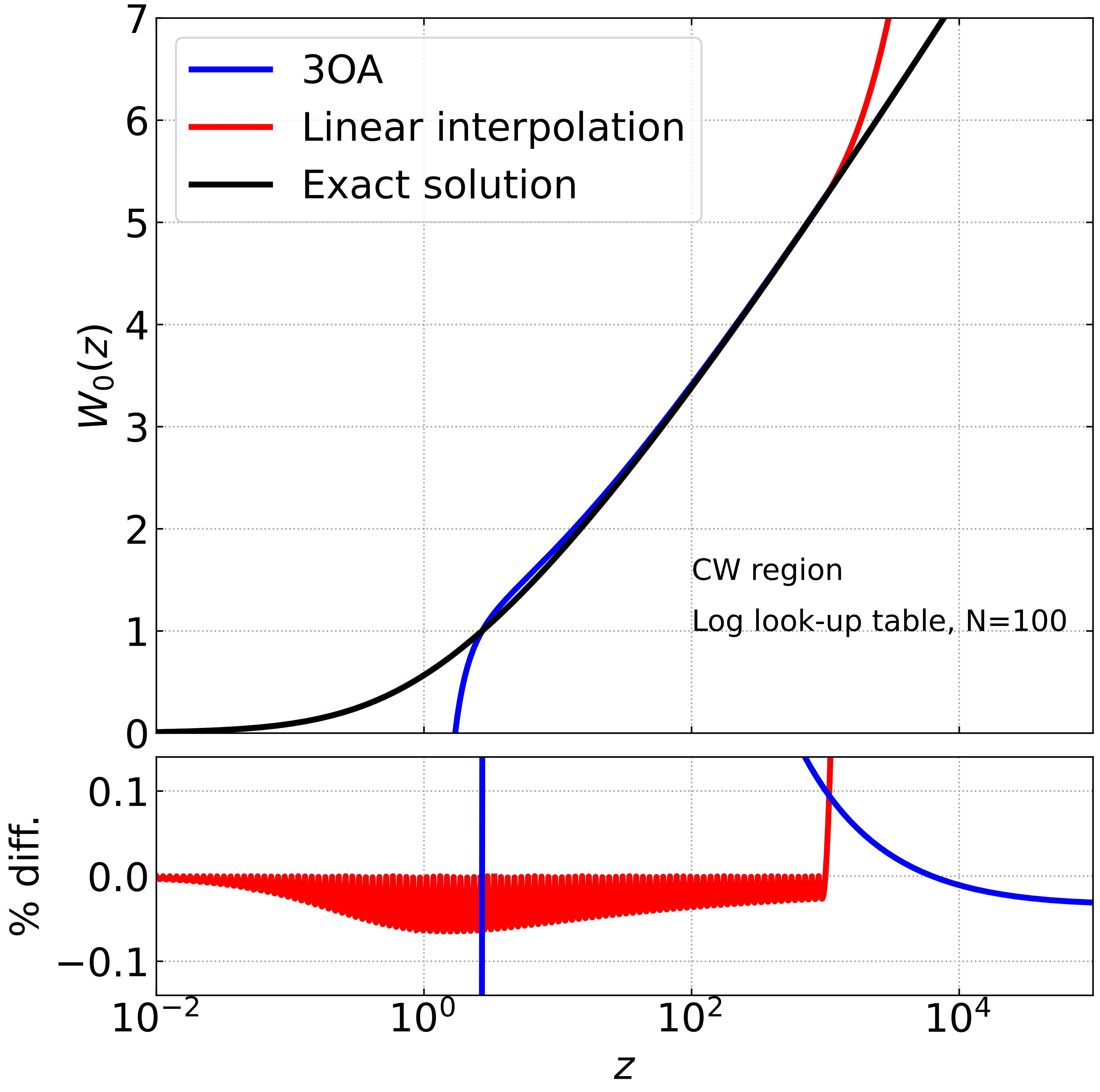
CW approximation

- This formula is valid to within the hadronisation cutoff of Pythia, unlike the PDG approximation. From,

$$W(x) = L_1 - L_2 + \frac{L_2}{L_1} + \mathcal{O}\left(\left[\frac{L_2}{L_1}\right]^2\right)$$

- Where $L_1 = \ln(z)$, $L_2 = \ln(\ln(z))$ for $W_0(z)$ and $L_1 = \ln(z)$, $L_2 = \ln(-\ln(z))$ for $W_{-1}(-z)$,

Interpolation procedure



- Within the QL region, the implementation of the running coupling suffices with a large μ/Λ approximation (3OA) for all $\mu/\Lambda > 1$.
- The 3OA fails in the CW region when the running coupling is slow - an area that lends itself nicely to interpolation. The best solution was to use a 3OA where applicable and linearly interpolate the regions where it fails.
- It is convenient to interpolate in z space; taken over z between a range of 10^{-2} and 10^{3} using 100 data points. The upper boundary of this interpolation is determined by when the large μ/Λ approximation deviates from the exact solution by 0.1%.

Implementing running coupling

- Where ξ is the fraction of energy given to b; it governs the longitudinal evolution of the parton shower. $P_{a\to bc}$ are the Altarelli-Parisi splitting functions. Parton shower splitting functions can be expressed in terms of Casimir invariants and ξ,

$$P_{G_D\to G_DG_D} = C_A\frac{1+\xi^3}{1-\xi}; \quad P_{q_D\to q_DG_D} = \frac{1}{2}C_F\frac{1+\xi^2}{1-\xi}; \quad P_{G_D\to q_D\bar{q}_D} = T_R\left(\xi^2+(1-\xi)^2\right)$$

- Inverting the Sudakov factor is not always possible. Instead we overestimate the tree-level splitting functions by some $\tilde{P}_{a\to bc}(\xi')$.

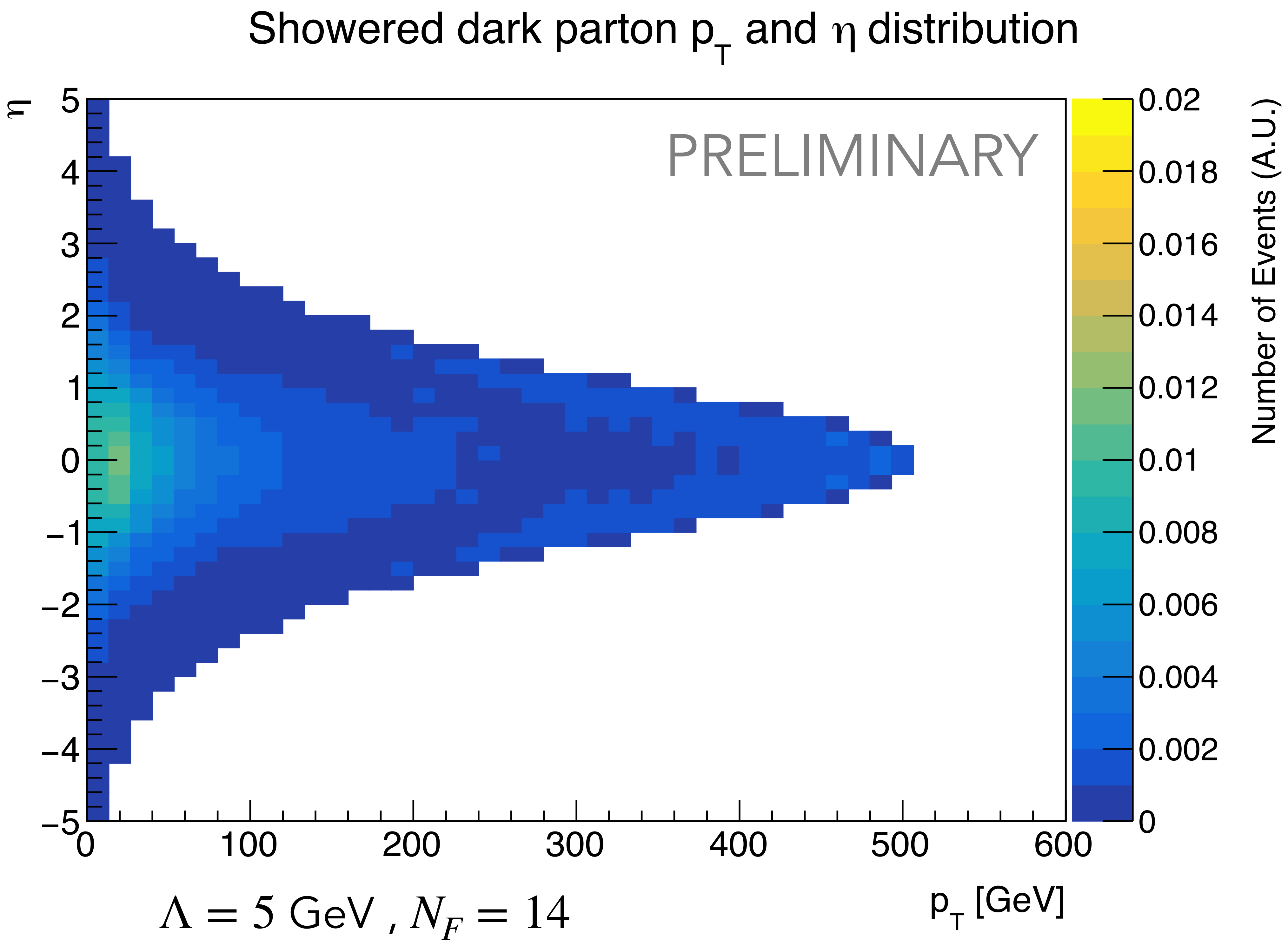
$$\tilde{P}_{G_D\to G_DG_D} = 2C_A\frac{1}{1-\xi}; \quad \tilde{P}_{q_D\to q_DG_D} = C_F\frac{1}{1-\xi}; \quad \tilde{P}_{G_D\to q_D\bar{q}_D} = T_R$$

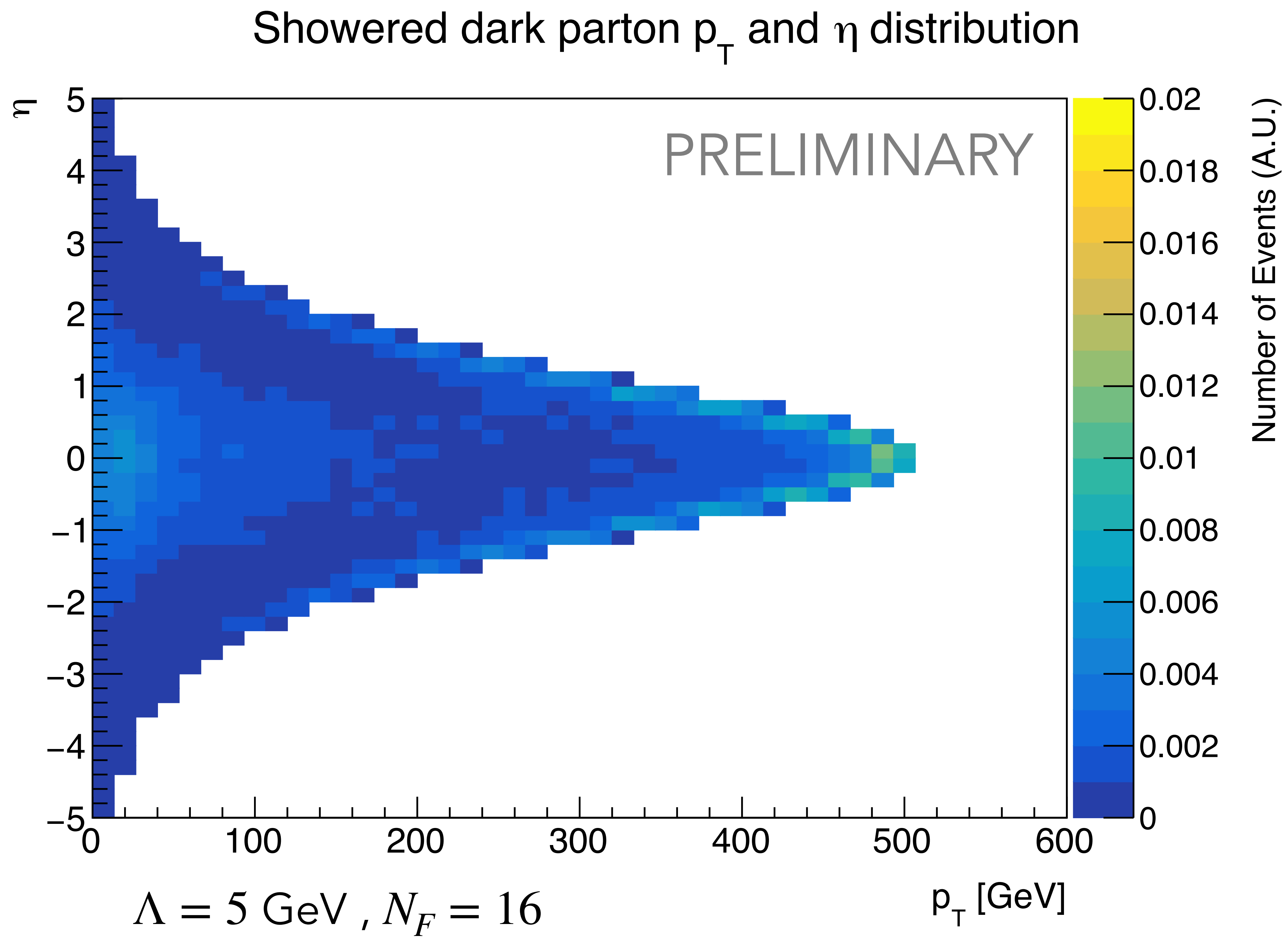
- We also overestimate the integration region with boundaries independent of Q^2 : $\tilde{\xi}_{max}(Q_0{}^2) > \xi_{max}(Q'^2)$ and $\tilde{\xi}_{min}(Q_0{}^2) < \xi_{min}(Q'^2)$.
- At one-loop, we can now write a closed-form expression in terms of the modified Sudakov factor, $\tilde{\Delta}_a$.

$$\ln(Q_2^2/\Lambda^2) = \tilde{\Delta}_a^{2\pi\beta_0/\epsilon_a}\ln(Q_1^2/\Lambda^2); \quad \epsilon_a = \int_{\tilde{\xi}_{min}}^{\tilde{\xi}_{max}}\sum_{b,c}\tilde{P}_{a\to bc}(\xi')d\xi';$$

ϵ_a is the "emission coefficient"

Simulation of dark parton showers

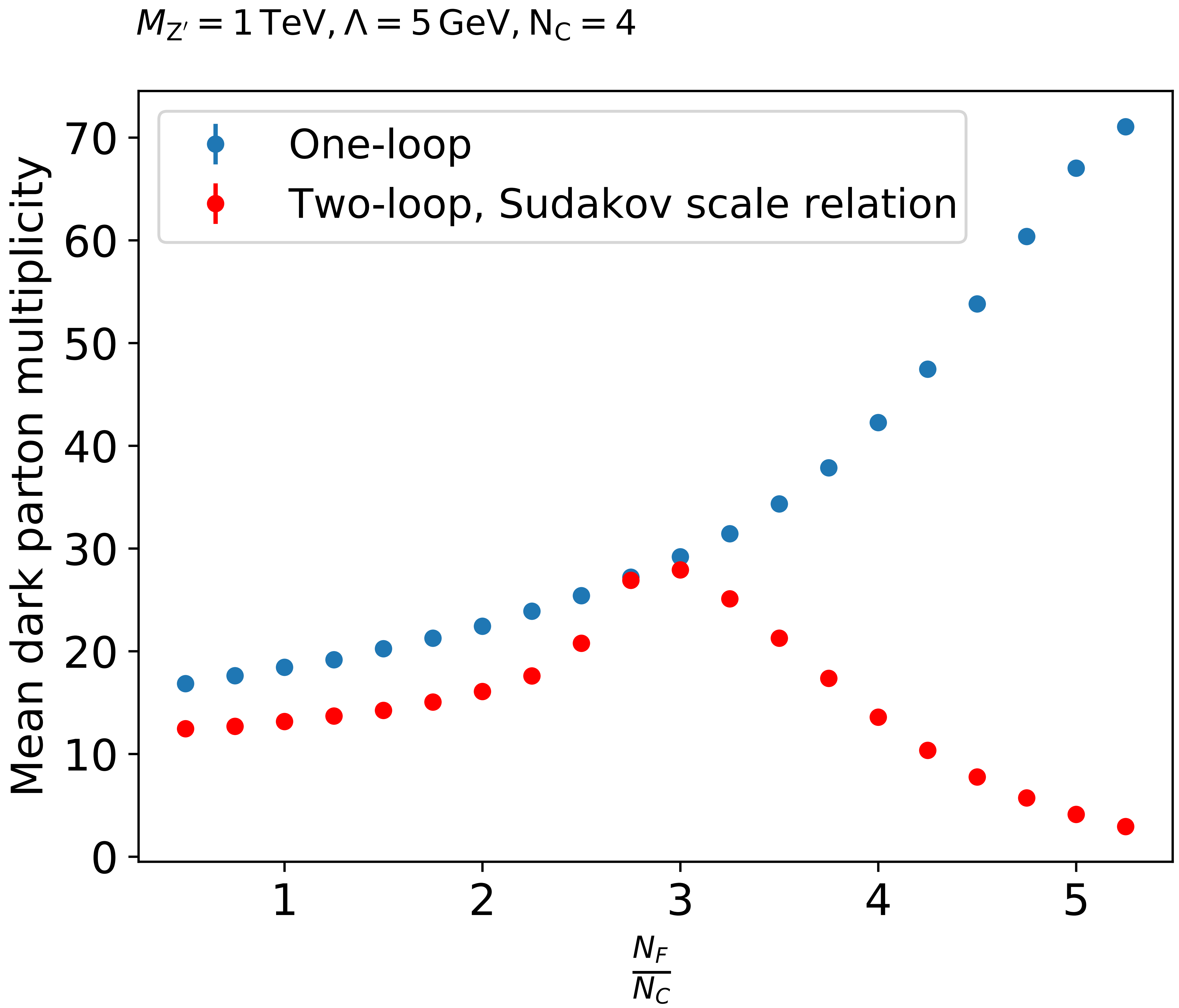




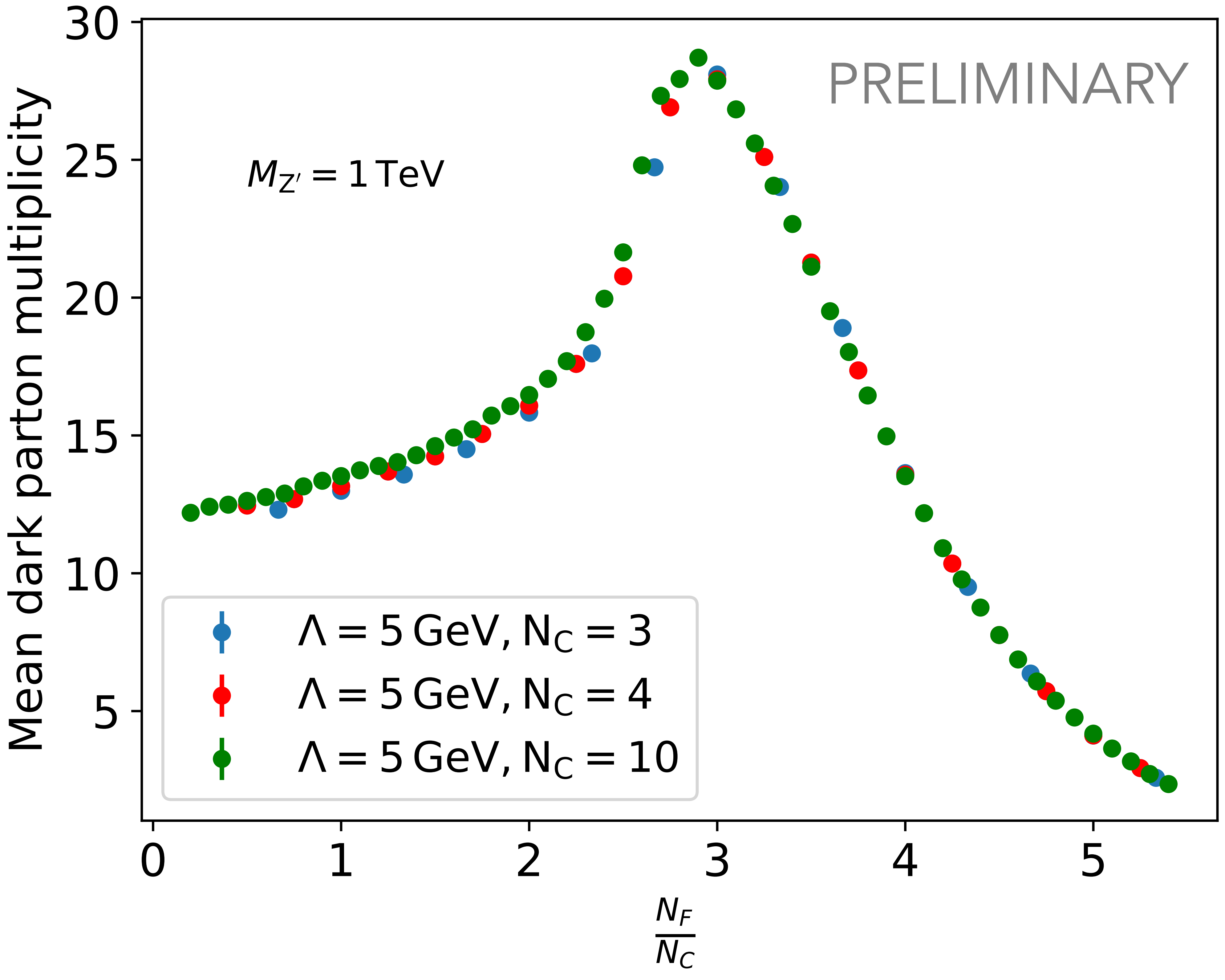
Simulated with a custom Pythia 8.307 with benchmark: $e^+e^- \to Z' \to q_D\overline{q_D}$, $\sqrt{s} = 1.1 M_{Z'} = 1.1$ TeV, hadronisation off , $\Lambda = 5$ GeV , $N_C = 3$. Cutoff at $Q = 1.1\Lambda$.

- At around N_F/N_C ~5, there is a transition in the p_T — η plane from the majority of dark partons being soft to a majority being hard - the majority of dark partons are initial dark quarks.
- For every dark parton splitting, the two resulting dark partons share the transverse momentum p_T meaning the more splittings, the softer the final state dark partons. In the IRFP region, the average $\eta \to 0$ as $N_F/N_C \to 5.5$, more events are back-to-back with respect to the beam line.

One-loop vs two-loop



Average dark parton multiplicity



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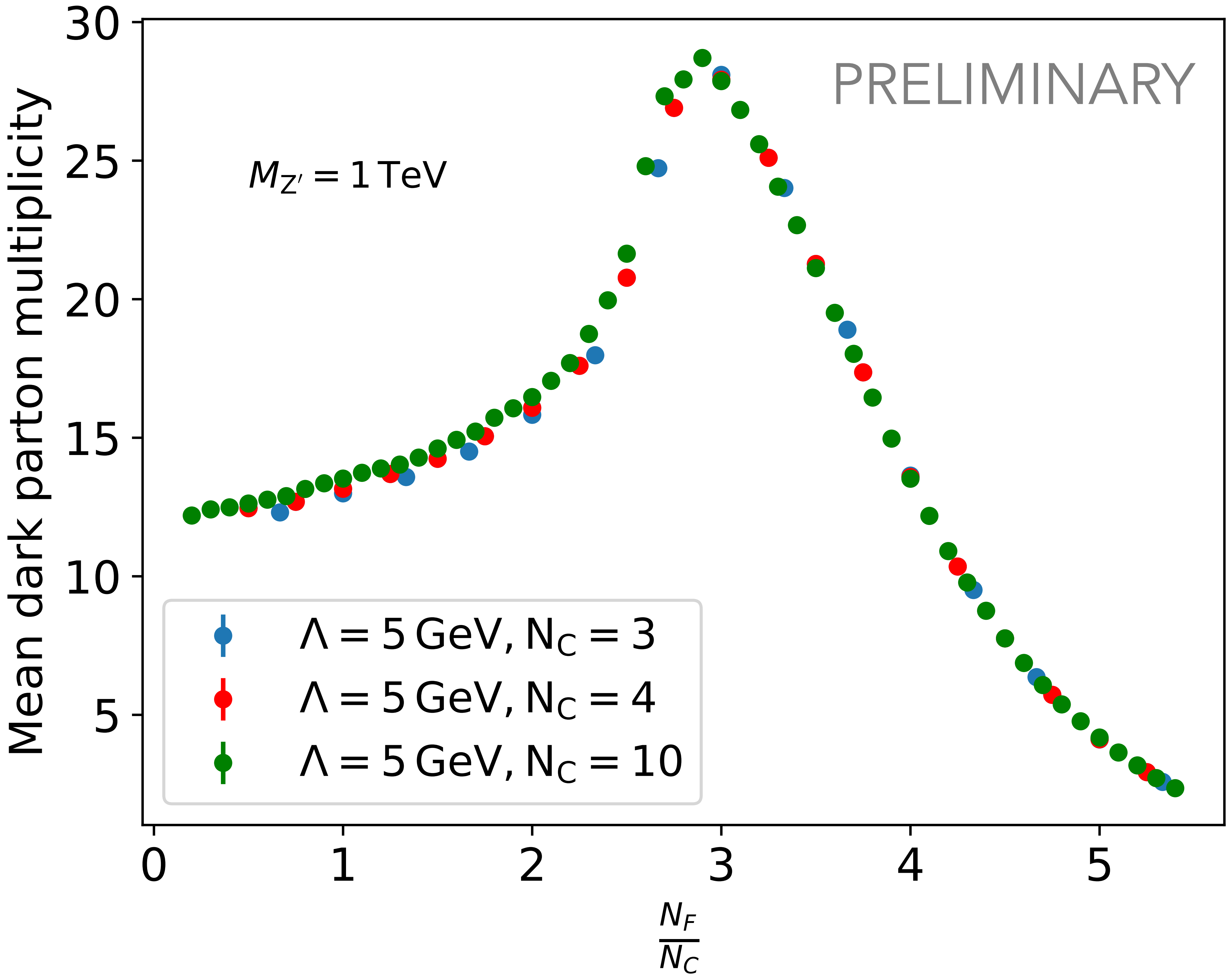
- Simulated using a custom version of Pythia 8.307; treat this implementation as a toy-model of near-conformal dark sectors. NOTE: we neglect the $P_{G_D \to q_D \bar{q}_D}$ branching - plan to add in future.

- Within the QL region, dark parton multiplicity increases with N_F/N_C. R. K. Ellis, W. J. Stirling and B. R. Webber, QCD and Collider Physics

- Theories with large IRFPs (around N_F/N_C ~3-4) have a large average multiplicity, which starts to decrease as $N_F/N_C \to 5.5$. Naively expect fat jets for large IRFPs and narrow (pencil-like) jets for N_F/N_C close to 5.5.

- The original PDG veto algorithm within Pythia can not predict this indicative rise and decreasing behaviour.

Average dark parton multiplicity



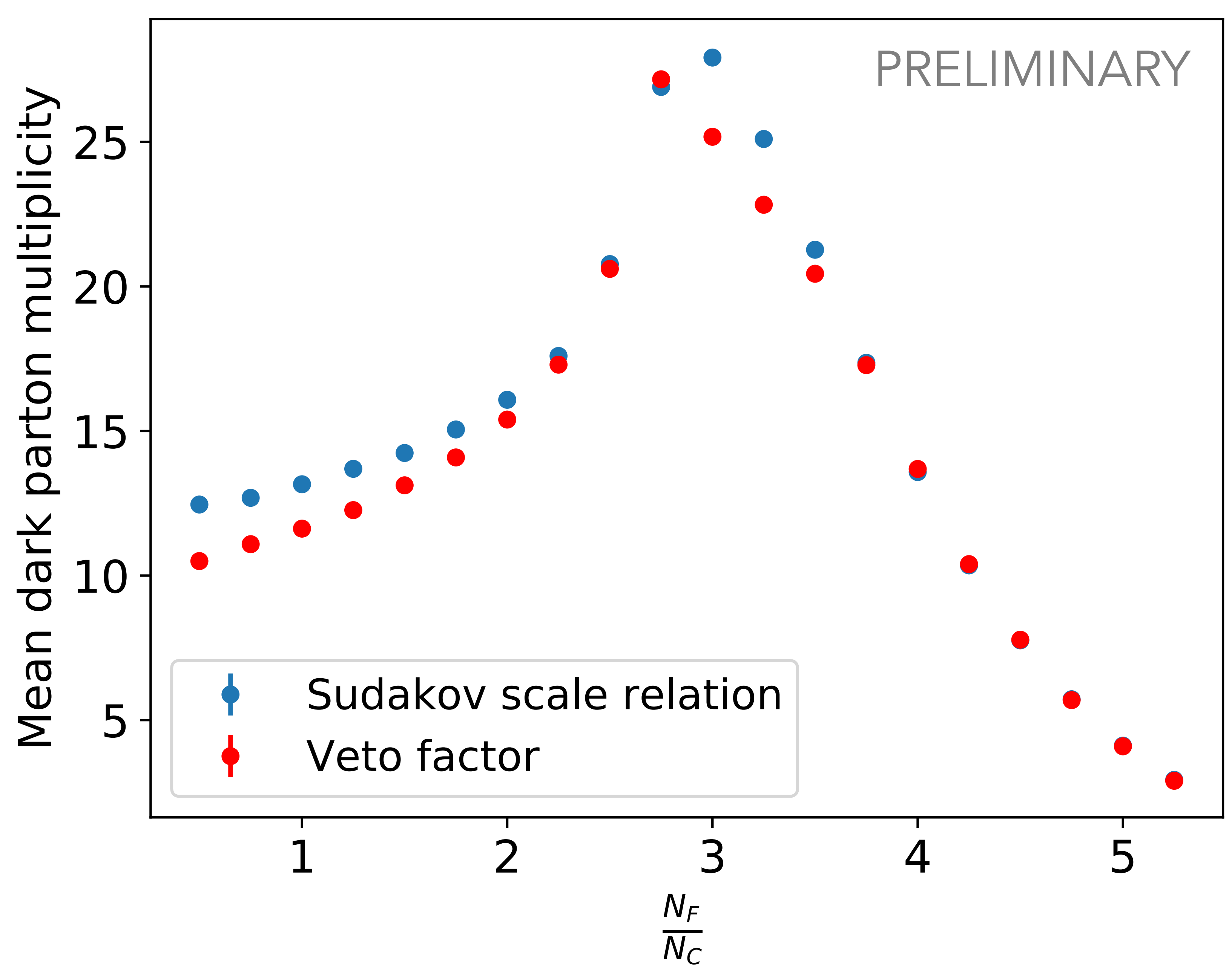
Simulated with a custom Pythia 8.307 with benchmark: $e^+e^- \to Z' \to q_D\overline{q_D}$, $\sqrt{s} = 1.1 M_{Z'} = 1.1$ TeV, hadronisation off , $\Lambda = 5$ GeV , $N_C = 3$. Cutoff at $Q = 1.1\Lambda$.

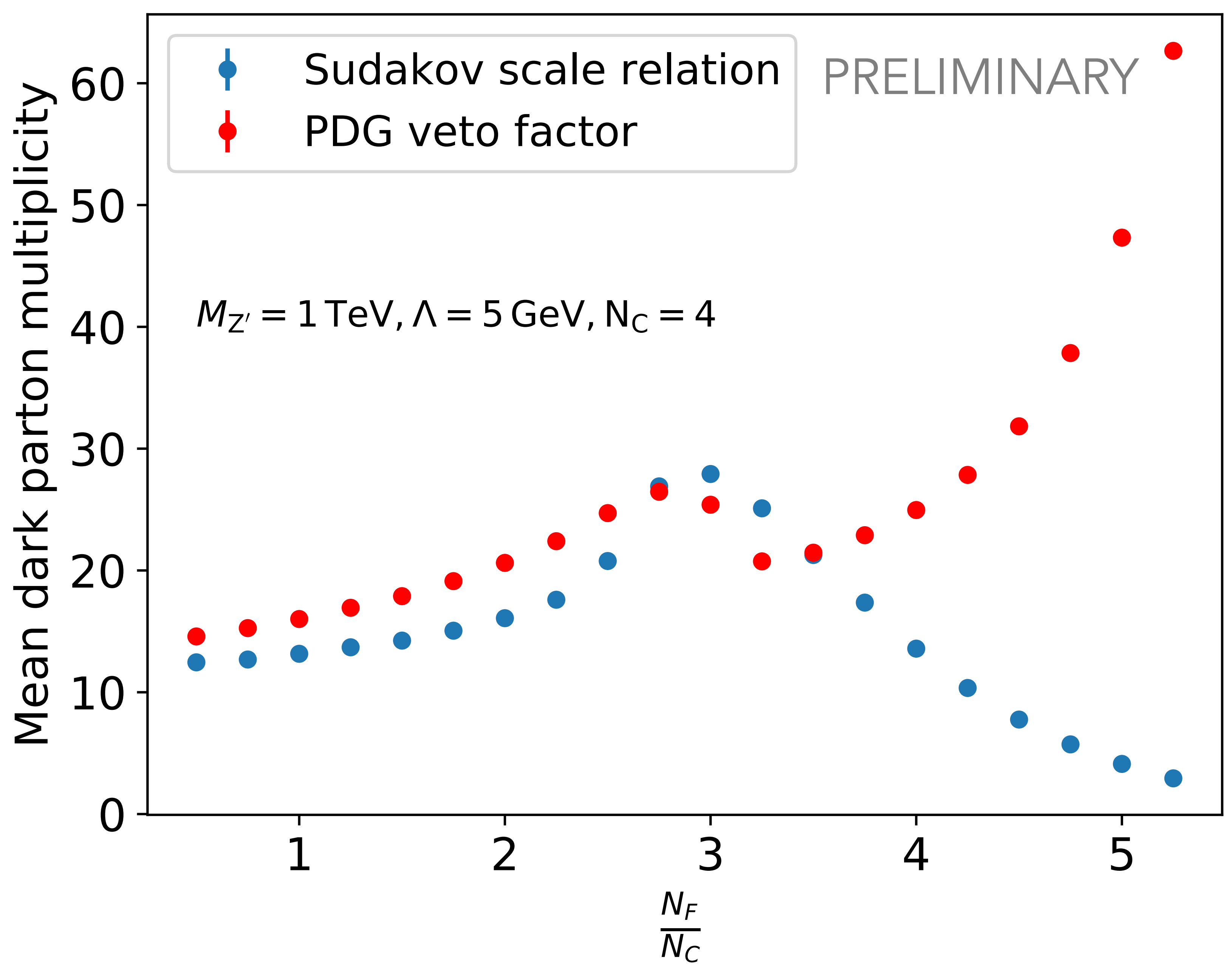
- Parton splitting probability is proportional to α and vanishes as $N_F/N_C \to 5.5$. Parton splitting is unlikely at $N_F/N_C \sim 5$ and the average multiplicity tends to 2 - the 2 initial dark partons.

$$d\mathscr{P}_a\left(\xi, Q^2\right) = \frac{dQ^2}{Q^2}\frac{\alpha(Q^2)}{2\pi}\sum_{b,c} P_{a\to bc}(\xi)d\xi$$

- Need to validate this implementation by comparing jet multiplicities initiated by different partons i.e. $\langle N\rangle_{\mathrm{gluon-jet}}$, $\langle N\rangle_{\mathrm{quark-jet}}$. Then can move onto more complicated objects such as jet observables.

Comparison with PDG and IRFP veto algorithm





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- The IRFP veto algorithm underestimates estimates the parton multiplicity in the QL region. In the CW region, for large IRFPs, it misses the correct maximum whilst for small IRFPs, the veto algorithm works and results match the Sudakov implementation.
- The original two-loop veto algorithm that used the PDG running coupling overestimates the parton multiplicity in the QL region. In the CW region, the PDG multiplicity curve replicates the IRFP veto algorithm for large IRFPs, but completely fails for small IRFPs since the effects of the IRFPs become significant in this region.

Multiplicity calculations

- Fragmentation functions (the single-particle distribution) of $a \to b, c$ is given by complicated integral equation. Taking Mellin moments of this integral equation simplifies to a differential equation.

$$\frac{\partial}{\partial Y} D(\omega, Y) = P(\Omega) \frac{\alpha(Y)}{2\pi} D(\omega, Y) \quad ; \quad P(\Omega) = \begin{pmatrix} P_{q\to qg}(\Omega) & 2N_F P_{g\to q\bar{q}}(\Omega) \\ P_{q\to gq}(\Omega) & P_{g\to gg}(\Omega) \end{pmatrix} ; \quad D(\omega, Y) = \begin{pmatrix} D_q(\omega, Y) \\ D_g(\omega, Y) \end{pmatrix}$$

- Where ω is the order of the Mellin moment, $\Omega = \omega + \frac{\partial}{\partial Y}$, $Y = \ln(Q_{\max}/Q_{\min})$, and $P(\Omega)$. $D(\omega, Y)$ is a vector of the Mellin moments of the quark and gluon respectively. $P(\Omega)$ is a matrix of Mellin transformed DGLAP splitting functions.

- Although solvable, we still have a coupled differential equation. We can get around this by diagonalising the matrix $P(\Omega)$. Its eigenvalues are given as,

$$\nu_{\pm}(\Omega) = \frac{1}{2}\left(P_{q\to qg}(\Omega) + P_{g\to gg}(\Omega) \pm \sqrt{\left(P_{q\to qg}(\Omega) - P_{g\to gg}(\Omega)\right)^2 + 8N_F P_{q\to gq}(\Omega) P_{g\to q\bar{q}}(\Omega)} \right)$$

Multiplicity calculations

- We therefore get two differential equations, each corresponding to the positive and negative eigenvalue.

$$\frac{\partial}{\partial Y} D_+(\omega, Y) = \nu_+(\Omega) \frac{\alpha(Y)}{2\pi} D_+(\omega, Y) \quad ; \quad \frac{\partial}{\partial Y} D_-(\omega, Y) = \nu_-(\Omega) \frac{\alpha(Y)}{2\pi} D_-(\omega, Y)$$

- Expanding $\nu_\pm$ to $\mathcal{O}(\Omega^0)$ gives the Modified Leading Log Approximation and one obtains (where $\delta = 0{,}1$ without/with the $g \to q\bar{q}$ splitting enabled). $\mathcal{O}(\Omega^{-1})$ yields the Double Logarithmic Approximation.

$$\nu_- = -\delta \frac{4T_R N_F}{3} \frac{2C_F}{C_A} \quad ; \quad \nu_+ = 4C_A/\Omega - \left(\frac{11C_A}{3} + \delta \frac{4T_R N_F}{3} \left(1 - \frac{2C_F}{C_A} \right) \right) = 4C_A/\Omega - a$$

- We are of course only interested in the $\omega = 0$ moment as this is just the the integral of the single-particle distribution - the multiplicity! This yields two corresponding differential equations.

$$\frac{\partial^2 D_+}{\partial Y^2} = \frac{4C_A \alpha(Y)}{2\pi} D_+ - a \frac{\partial}{\partial Y} \left(\alpha(Y) D_+ \right) \quad ; \quad \frac{\partial D_-}{\partial Y} = -\delta \frac{4T_R N_F}{3} \frac{2C_F}{C_A} \frac{\alpha(Y)}{2\pi} D_- \quad \text{(Constant for } \delta = 0!\text{)}$$

Multiplicity calculations

- Can be solved at constant coupling and one-loop running coupling, numerically possible for two-loop. Both solved with the following boundary condition: $D_{\pm}(Y=0)=1$ (or simply the initial number of partons). However D_{+} requires a second boundary condition - some literature has incorrect/unphysical boundary conditions - this is an open question for us.

- Given our diagonalization of $P(\Omega)$, we can construct a diagonalising matrix whose elements correspond to the colour factors. The number of gluons/quarks inside a jet initiated by an initial gluon/quark is given by the colour factors plus the initial jet conditions. For colour factors at $\mathcal{O}(\Omega^0)$ we have:

Gluon jet $(D_q(Y=0)=0\ ,\ D_g(Y=0)=1)$

$$D_q = 0 \qquad ; \qquad D_g = D_+$$

Quark jet $(D_q(Y=0)=0\ ,\ D_g(Y=0)=1)$

$$D_q = N_- \qquad ; \qquad D_g = \frac{C_F}{C_A}\left(D_+ - D_-\right)$$

Average dark parton multiplicity

Nucl.Phys.B 377 (1992) 445-460 (Catani et al.), arXiV: hep-ph/9709246, 1310.8534

- Mean parton multiplicity defined as 1st Mellin moment of the fragmentation functions. Gives coupled differential equation, but by diagonalising we can solve the two differential equations of the eigenvalues $\mathcal{N}_+, \mathcal{N}_-$ at $\mathcal{O}(\alpha^{1/2})$,

$$N_+ = \left[\cosh(\bar{\gamma}_0 Y) + \frac{\eta}{\bar{\gamma}_0}\sinh(\bar{\gamma}_0 Y)\right]\exp(-\eta Y),$$

$$N_- = \exp(-\sigma Y),$$

  - Where $Y = \ln(Q_{\text{start}}/Q_{\text{cut}})$, $\bar{\gamma}_0{}^2 = \gamma_0^2 + \eta^2$, $\gamma_0 = \sqrt{2\lambda/\pi}$, $\eta = a_1\lambda/4\pi C_A$. Subject to the boundary conditions of $N_\pm|_{Y=0} = 1$, $\partial N_+/\partial Y|_{Y=0} = 0$. a_1 is given by,

$$a_1 = \left(\frac{11C_A}{3} + \delta\frac{4T_R N_F}{3}\left(1 - 2\frac{C_F}{C_A}\right)\right)$$

- Where $\sigma = 2\delta\frac{4T_R N_F}{3}\frac{C_F}{C_A}\frac{\alpha}{2\pi}$. δ is 0 if $g \nrightarrow q\bar{q}$ and δ is 1 if $g \to q\bar{q}$.

Average dark parton multiplicity

Nucl.Phys.B 377 (1992) 445-460 (Catani et al.), arXiV: hep-ph/9709246, 1310.8534

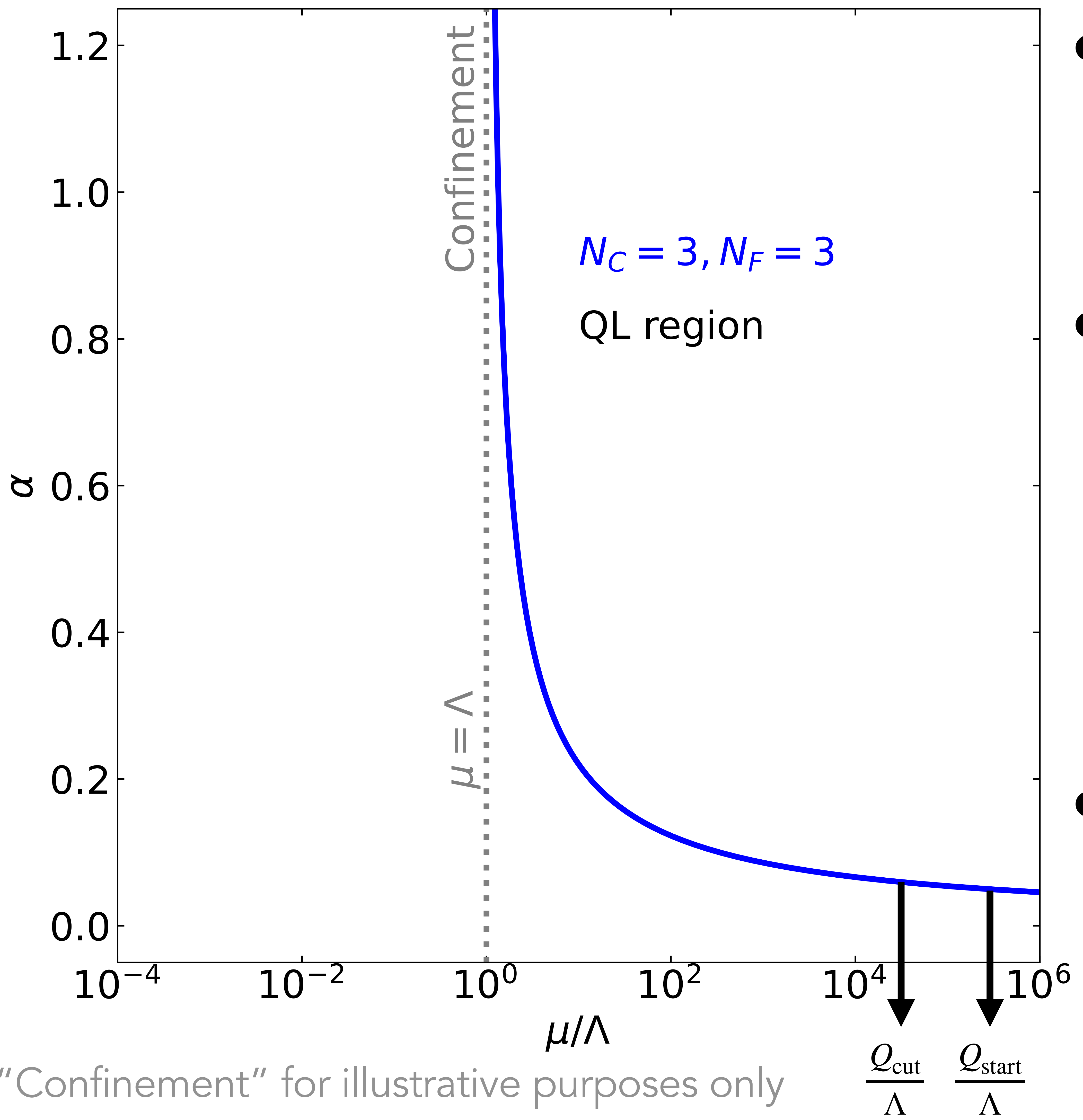
- Rediagonalising back to the gluon-jet basis, we obtain the following equation for mean gluon-jet multiplicity,

$$N_{\text{gluon-jet}} = N_+ + \delta(2T_R)\left(1 - \frac{C_F}{C_A}\right)\frac{N_F}{3C_A}\frac{\partial N_+}{\partial Y}$$

- Rediagonalising back to the quark-jet basis, we obtain the following equation for mean quark-jet multiplicity,

$$N_{\text{quark-jet}} = N_-\left(1 - \frac{C_F}{C_A}\right) + \frac{C_F}{C_A}N_+ + \frac{C_F}{C_A}\left(\frac{a_1 - 3C_A}{4C_A} + \delta(2T_R)\frac{N_F}{3C_A}\left(1 - \frac{C_F}{C_A}\right)\right)\frac{\partial N_+}{\partial Y}$$

Average dark parton multiplicity



"Confinement" for illustrative purposes only

- Simulated within a custom version of Pythia 8.307; a toy-model of near-conformal dark parton showers, qualitative and not quantitative. We neglect $P_{G_D \to q_D \bar{q}_D}$ branching and CMW scheme change. S. Catani, B. R. Webber, G. Marchesini, Nucl. Phys. B 349 ('91)
- Validate simulation with analytic results for mean parton multiplicity, which one expects to increase with α from parton splitting probability. Expect semi-clustered jets for large α and narrow (pencil-like) jets for small α.

$$d\mathscr{P}_a\left(\xi, Q^2\right) = \frac{dQ^2}{Q^2}\frac{\alpha(Q^2)}{2\pi}\sum_{b,c} P_{a\to bc}(\xi)d\xi$$

- Two-loop complex, better to evaluate in const $\lambda = \alpha N_C$ scenario. Occurs in many two-loop limits; a) $Q_{\text{start}}, Q_{\text{cut}} \geq \Lambda$, b) in the Banks-Zaks scenario, c) $Q_{\text{start}}, Q_{\text{cut}} \leq \Lambda$ and d) if $\lambda(Q_{\text{start}}) = \lambda_*$.

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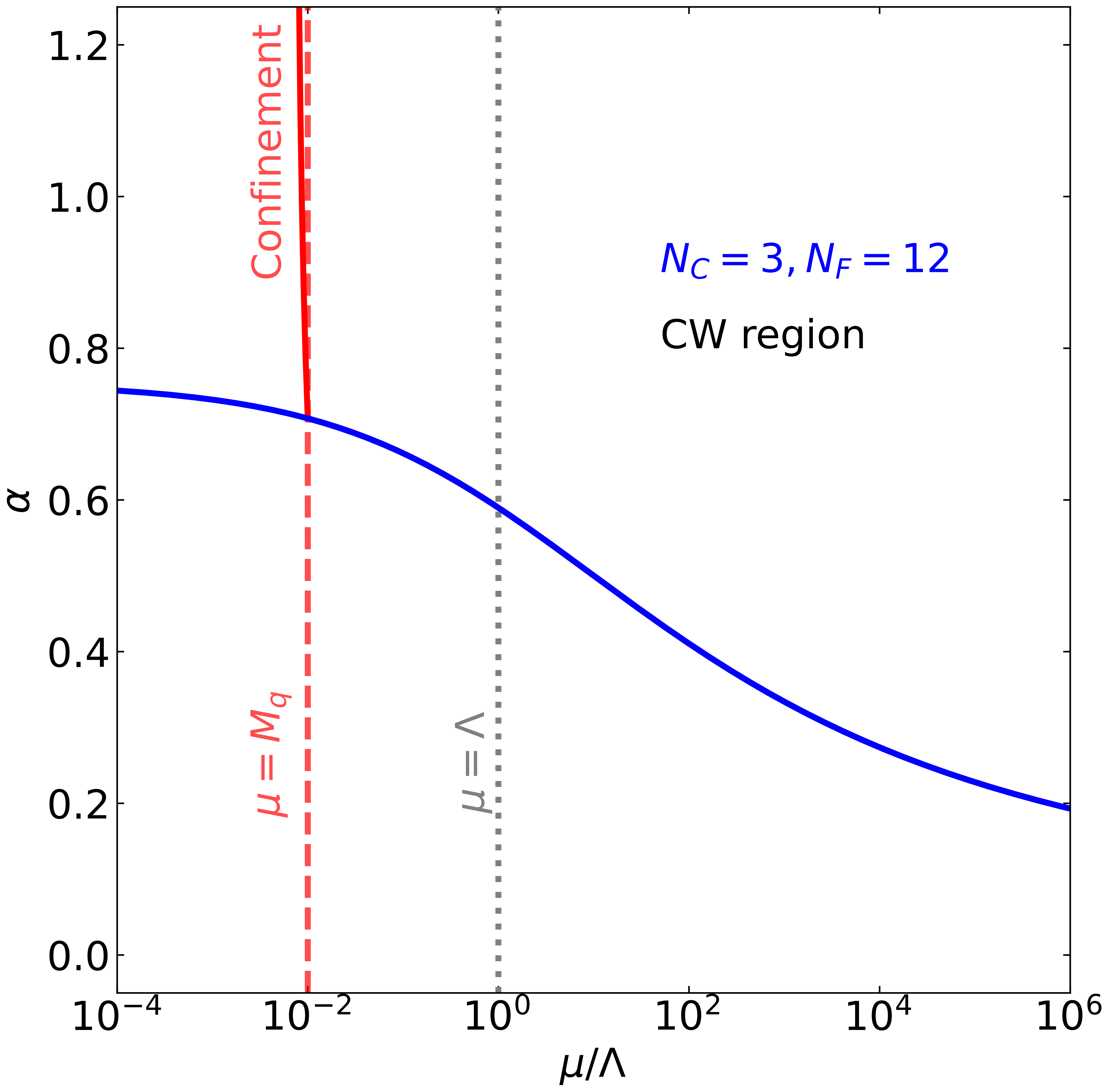

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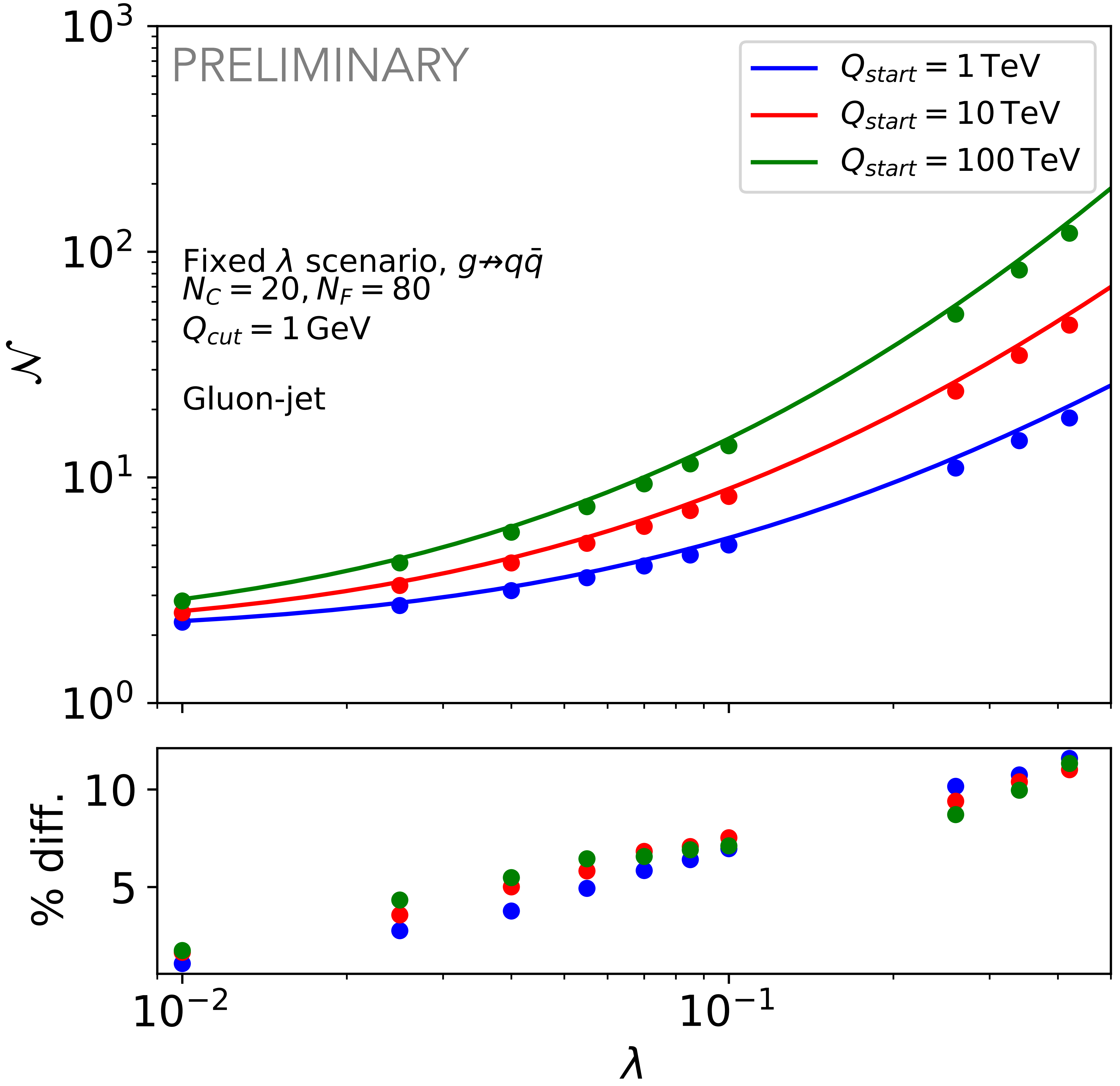

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- Mean parton multiplicity is the 1st Mellin moment of fragmentation functions. Gives coupled differential equation, diagonalising gives us two differential equations of the eigenvalues N_+, N_- at $\mathcal{O}(\alpha^{1/2})$,

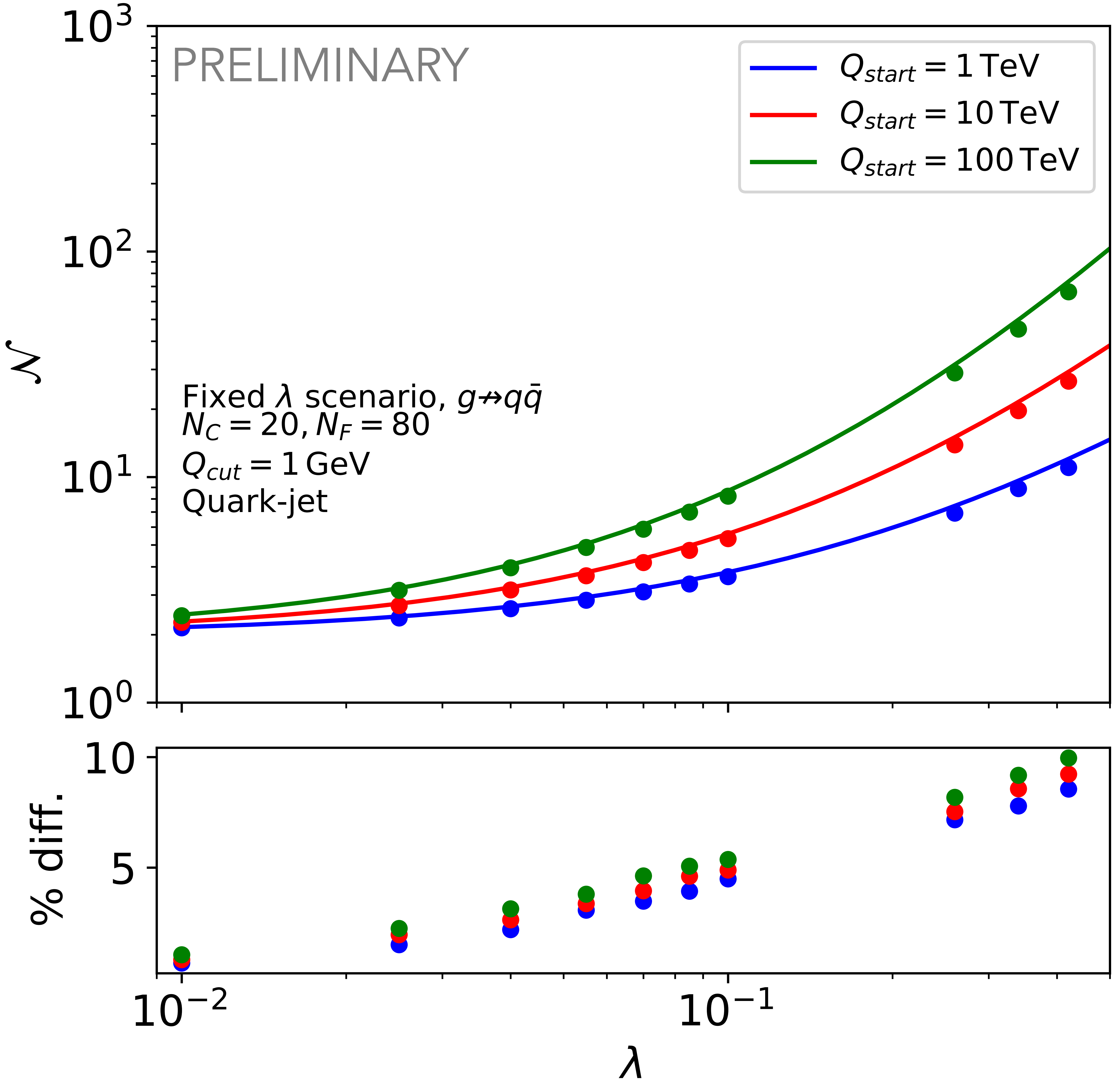
$$N_+ = \left[\cosh(\bar{\gamma}_0 Y) + \frac{\eta}{\bar{\gamma}_0}\sinh(\bar{\gamma}_0 Y)\right]\exp(-\eta Y), \quad ; \quad N_- = 1$$

- Solved in the approximation $g \nrightarrow q\bar{q}$, where $Y = \ln(Q_{\text{start}}/Q_{\text{cut}})$ and $\bar{\gamma}_0, \gamma_0, \eta = f(\lambda)$. B.c.s are $N_\pm|_{Y=0} = 1$, $\partial N_+/\partial Y|_{Y=0} = 0$.
- Rediagonalising back to the gluon-jet basis, we obtain the following equation for mean gluon-jet multiplicity,

$$N_{\text{gluon-jet}} = N_+$$

- $N_{\text{gluon-jet}}$ increases with λ and $Q_{\text{start}}/Q_{\text{cut}}$. $\mathcal{O}(\alpha^{1/2})$ truncation means analytic results and simulation differ as λ increases.

Average dark parton multiplicity



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- Rediagonalising back to the quark-jet basis, we obtain the following equation for mean quark-jet multiplicity,

$$N_{\text{quark-jet}} = \frac{C_F}{C_A}N_+ + \left(1 - \frac{C_F}{C_A}\right) + \frac{1}{6}\frac{C_F}{C_A}\frac{\partial N_+}{\partial Y}$$

- $N_{\text{quark-jet}}$ increases with λ and $Q_{\text{start}}/Q_{\text{cut}}$. $\mathcal{O}(\alpha^{1/2})$ truncation means analytic results and simulation differ as λ increases.