Renormalization of Asymptotic Grand Unified Theories

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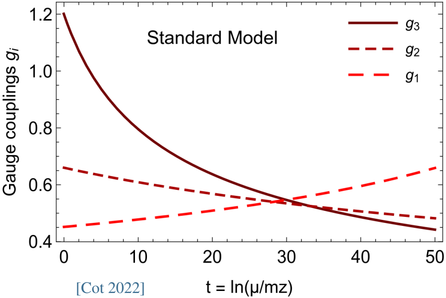
Introduction: Grand Unified Theories (GUTs)

- Standard Model: $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$
- The gauge couplings depends on the energy $\rightarrow$ running , RGE
- At high energy, the gauge couplings seem to merge: $\Lambda_{\mathrm{GUT}} \sim 10^{16}$ GeV



[Cot 2022]

Unique gauge group at high energy?

- Example: $\mathrm{SU}(5)$

$$\psi_{\bar{5}} = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e^- \\ -\nu_e \end{pmatrix} \quad \phi_{\mathbf{5}} = \begin{pmatrix} H_1 \\ H_2 \\ H_3 \\ \varphi_+ \\ -\varphi_0 \end{pmatrix}$$

$$\psi_{\mathbf{10}} = \begin{pmatrix} 0 & u_3^c & -u_2^c & -u_1 & -d_1 \\ -u_3^c & 0 & u_1^c & -u_2 & -d_2 \\ u_2^c & -u_1^c & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & -e^c \\ d_1 & d_2 & d_3 & e^c & 0 \end{pmatrix}$$

[Georgi, Glashow 1974]

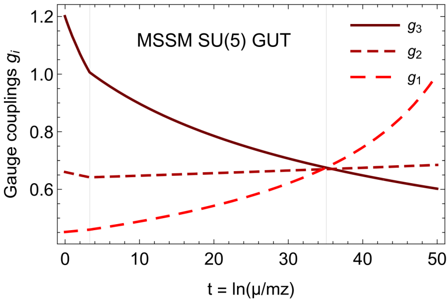
Introduction: Limits of traditional GUTs

- Assume more gauge bosons → **proton decay**
  Example: SU(5), $\mathbf{24} = G(\mathbf{8},\mathbf{1})_0 \oplus W(\mathbf{1},\mathbf{3})_0 \oplus B(\mathbf{1},\mathbf{1})_0 \oplus X(\mathbf{3},\mathbf{2})_{-5/6} \oplus Y(\mathbf{3},\mathbf{2})_{5/6}$, X and Y allow $p \to e^+ + \pi^0$

- Doublet-triplet splitting: mechanisms to make H heavier than φ

- Large matter representations to break the GUT gauge group → Landau pole

- Need extra scalar/SUSY to have exact unification:



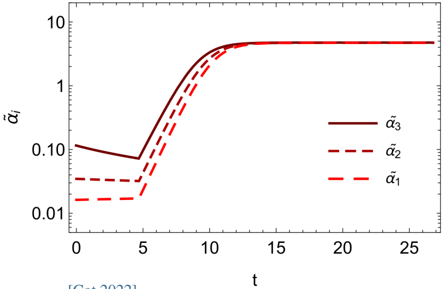
[Cot 2022]

Introduction: Asymptotic Unification?

Asymptotic Unification

- New paradigm: No exact unification, gauge couplings tend to the same UV fixed point
  [Cacciapaglia, Cornell, Cot, Deandrea 2020] [Abdalgabar, Khojali, Cornell, Cacciapaglia, Deandrea 2017]

- Realized in asymptotic safe theories or **extra dimension** (power law running)



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- Lower effective unification scale, no proton decay, natural doublet-triplet splitting

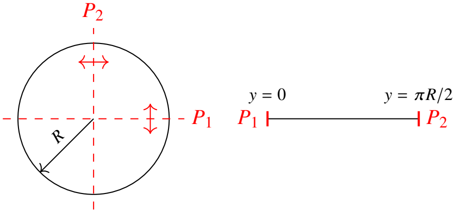
Content

I. Construction of Asymptotic GUTs

II. Renormalization Group Equations (RGE) in 5D

III. One-Loop renormalization of 5D gauge theories

- Higher dimensional QFT:
  - Gauge group $\mathcal{G}$
  - $\mathcal{M} = \mathbb{R}^4 \times C$, coordinates $\{x^\mu, y\}$
- Extra dimension compactified on a circle/interval, radius R
- Define a parity on each boundary: P_1 and P_2



- Action of the parity on the fields Φ (boundary conditions):

$$P_1 : \ \Phi(x^\mu, y) \sim P_1 \, \Phi(x^\mu, -y), \qquad P_2 : \ \Phi(x^\mu, y') \sim P_2 \, \Phi(x^\mu, -y')$$

- Choose diagonal basis for P_1 and P_2: $\quad P_{1,2} \, \Phi(x^\mu, -y) = \pm\Phi(x^\mu, y)$
- Classify fields according to their eigenvalues $(P_1, P_2) = (\pm, \pm)$

I. Construction of Asymptotic GUTs

Parities and Symmetry Breaking

- Kaluza Klein (KK) Decomposition:

$$\phi_{5D}(x,y) = \sum_{n=0}^{+\infty} \phi_{4D}^{(n)}(x)\, f_n(y) \quad \text{with} \quad f_n(y) = \begin{cases} (++) & \frac{1}{\sqrt{2\pi R}} + \frac{1}{\sqrt{\pi R}} \cos\left(\frac{ny}{R}\right) & \longrightarrow m_n = \frac{n}{R} \\ (+-) & \frac{1}{\sqrt{\pi R}} \cos\left(\frac{(n+1/2)y}{R}\right) & \longrightarrow m_n = \frac{n+1/2}{R} \\ (-+) & \frac{1}{\sqrt{\pi R}} \sin\left(\frac{(n+1/2)y}{R}\right) & \longrightarrow m_n = \frac{n+1/2}{R} \\ (--) & \frac{1}{\sqrt{\pi R}} \sin\left(\frac{(n+1)y}{R}\right) & \longrightarrow m_n = \frac{n+1}{R} \end{cases}$$

- Low energy effective theory: KK towers integrated out, only zero modes coming from $(++)$ states remain

- Apply it to gauge fields: each parity breaks the GUT gauge group $\mathcal{G}$ at low energies:

[Cacciapaglia 2023]

$$\left.\begin{array}{l} P_1 : \mathcal{G} \to \mathcal{H}_1 \\ P_2 : \mathcal{G} \to \mathcal{H}_2 \end{array}\right\} (P_1, P_2) : \mathcal{G} \to \mathcal{H}_1 \cap \mathcal{H}_2 = \mathcal{H} \supset \mathcal{G}_{SM}$$

- Allow construction of models with only the Standard Model fields as zero modes

II. RGE in 5D

Gauge couplings

- Include contributions coming from the KK modes:

$$2\pi \frac{\mathrm{d}\alpha_g}{\mathrm{d}\ln\mu} = b_{\mathrm{SM}}\alpha_g^2 + (S(\mu) - 1)b_5\alpha_g^2$$

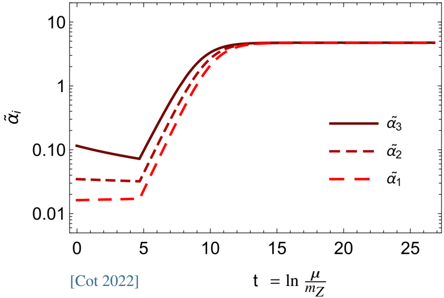
$$S(\mu) = \begin{cases} \mu R, & \text{for } \mu \geq 1/R \\ 1, & \text{otherwise} \end{cases}$$

- t'Hooft coupling (adimensional): $\tilde{\alpha}_g = \mu R \alpha_g$

- At high energy ($\mu R b_5 \gg b_{\mathrm{SM}}$):

$$2\pi \frac{\mathrm{d}\tilde{\alpha}_g}{\mathrm{d}\ln\mu} = 2\pi\tilde{\alpha}_g + b_5\tilde{\alpha}_g^2$$

- Fixed point: $\tilde{\alpha}_g^* = -\frac{2\pi}{b_5} \longrightarrow$ exist for $b_5 < 0$



[Cot 2022]

- Example: $SU(5)$, $b_5 = -\frac{4}{3}$

II. RGE in 5D

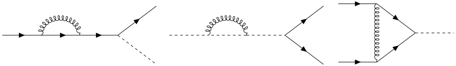
Yukawa couplings

- For Yukawa couplings:

$$16\pi^2 \frac{\mathrm{d}y}{\mathrm{d}\ln\mu} = \beta_y^{\mathrm{SM}} + (S(\mu) - 1)\beta_y^5$$

$$\beta_y^5 = \sum_{y'} c_{yy'}\, y y'^2 - d_y y g^2$$

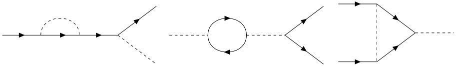
$d_y \sim$



- Similarly for $\alpha_y = y^2/4\pi$ at high energy:

$$2\pi \frac{\mathrm{d}\tilde{\alpha}_y}{\mathrm{d}\ln\mu} = 2\pi\,\tilde{\alpha}_y + \sum_{y'} c_{yy'}\,\tilde{\alpha}_y\tilde{\alpha}_{y'} - d_y\,\tilde{\alpha}_g\,\tilde{\alpha}_y$$

$c_{yy'} \sim$



- Fixed point: $\tilde{\alpha}_y^* = \sum_{y'} c_{yy'}^{-1}\,(d_y\,\tilde{\alpha}_g^* - 2\pi)$

- Repulsive fixed point: $\sum_{y'} c_{yy'} > 0$, then also need $d_y\,\tilde{\alpha}_g > 2\pi$

II. RGE in 5D

Scalar couplings

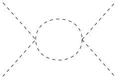
- Quartic coupling: $\frac{\lambda}{4!}\phi^4$
- RGE at high energy for $\alpha_\lambda = \lambda/4\pi$:

$$4\pi \frac{\mathrm{d}\tilde{\alpha}_\lambda}{\mathrm{d}\ln(\mu)} = 4\pi\,\tilde{\alpha}_\lambda + a_\lambda \tilde{\alpha}_\lambda^2 - b_\lambda \tilde{\alpha}_y^2 + d_\lambda \tilde{\alpha}_g^2 - c_\lambda \tilde{\alpha}_g \tilde{\alpha}_\lambda + e_\lambda \tilde{\alpha}_\lambda \tilde{\alpha}_y\,.$$

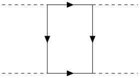
- Fixed point $\tilde{\alpha}_\lambda^*$ can be obtained by solving:

$$a_\lambda^* \tilde{\alpha}_\lambda^2 + (4\pi - c_\lambda \tilde{\alpha}_g^* + e_\lambda \tilde{\alpha}_y^*)\,\tilde{\alpha}_\lambda^* - b_\lambda \tilde{\alpha}_y^{*2} + d_\lambda \tilde{\alpha}_g^{*2} = 0\,.$$

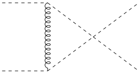
$a_\lambda \sim$



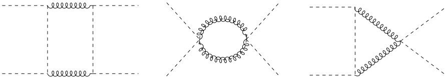
$b_\lambda \sim$



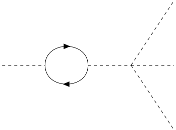
$c_\lambda \sim$



$d_\lambda \sim$



$e_\lambda \sim$



III. 1 Loop Renormalization

Non Renormalizability of 5D gauge theories

- Consider $d = 5$ Lagrangian with renormalizable interactions in 4D:

$$\mathcal{L}_5 = -\frac{1}{4g_d^2}\, F^{a,MN} F^a_{MN} - \frac{1}{2\xi g_d^2}\left(\partial_M A^{a,M}\right)^2 + i\overline{\Psi}\Gamma^M \mathcal{D}_M \Psi + (\mathcal{D}_M \Phi^\dagger)(\mathcal{D}^M \Phi)$$
$$- y_d\, \overline{\Psi}_1 \Psi_2 \Phi - \lambda_d\, (\Phi^\dagger \Phi)^2$$

- Mass dimensions of fields and couplings in $d = 5$:

$$[A_M] = m \quad [\Psi] = m^2 \quad [\Phi] = m^{3/2} \quad [g_d] = [y_d] = m^{-1/2} \quad [\lambda_d] = m^{-1}$$

- Negative mass dimension $\rightarrow$ Sign of non-renormalizability

Question: Can we have a perturbative control of the 5D theory at one-loop?

III. 1 Loop Renormalization

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< 0

- Negative mass dimension → Sign of non-renormalizability

Question: If a fixed point exists, can we have a perturbative control of the 5D theory at one-loop?

III. 1 Loop Renormalization

Bulk Divergences

- Dimensional Analysis: general loop in d dimensions

Number of propagators

$$I_d \sim \int \frac{\mathrm{d}k^d}{(2\pi)^d} \frac{f(k^M, k^2)}{\Delta(k^2)} \qquad \Delta(k^2) \sim k^{2n} \qquad f \sim k^{2m}$$

$n-m$	$d=$ 4	5	6	7	Examples
1	Λ^2	Λ^3	Λ^4	Λ^5	Scalar masses
2	log	Λ	Λ^2	Λ^3	Coupling renormalization
3	finite	finite	log	Λ	Four-fermions, $(\Phi^\dagger\Phi)^3$
≥ 4	finite	finite	finite	finite	

- $d=4$ and $d=5$ share the same divergent integrals

- In $d=5$, counter-terms absorbed **linear** divergences instead of **logarithmic**

[Georgi, Grant, Hailu 2001]
[Gersdoff, Irges, Quiros 2002]
[Cheng, Matchev, Schmaltz 2002]

- Orbifold constrains modify the 5D propagators ($\xi = 1$):

$$G^A_{MN}(p,p_5) = \frac{-ig_{MN}}{2(p^2 - p_5^2)}\left(\delta_{p_5,p'_5} + \overset{\pm 1}{\alpha^M}\delta_{-p_5,p'_5}\right) \qquad G^{\pm}_{\Phi}(p,p_5) = \frac{i}{2(p^2 - p_5^2)}\left(\delta_{p_5,p'_5} \pm \delta_{-p_5,p'_5}\right)$$

$$G^{\pm}_{\Psi}(p,p_5) = \frac{i}{2(\not{p} + i\gamma_5 p_5)}\left(\delta_{p_5,p'_5} \pm \delta_{-p_5,p'_5}\gamma_5\right)$$

- Amplitude:

$$i\Sigma = \underbrace{\boxed{p_5 \ \text{conserving}}}_{\text{BULK}} + \underbrace{\boxed{p_5 \ \text{non conserving}}}_{\text{LOCALIZED}}$$

- Leads to divergences localized on the boundaries

$$S_5 = \int \mathrm{d}^5x \left\{ \mathcal{L}_5 + \left[\mathcal{L}_4 \frac{\delta(y) + \delta(y - \pi R)}{2\pi R}\right]\right\}$$

- 5D Renormalizable $\rightarrow$ $\mathcal{L}_4$ also 4D Renormalizable

III. 1 Loop Renormalization

Localized Divergences in Yang-Mills

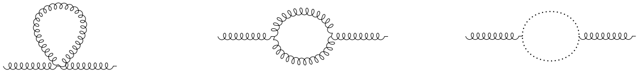
- Example with Yang-Mills: dimensional regularization, $\overline{MS}$ scheme

- 2 point function:

Group Factor

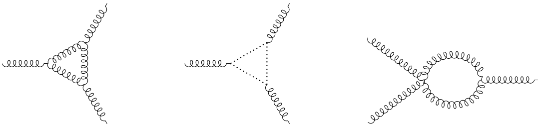
$\sim (\partial_5^2 A_\mu^A) A^{A,\mu} + A_\mu^A (\partial_5^2 A^{A,\mu})$

$$\Pi^{\text{loc, gauge}}_{\mu\nu} = \frac{g^2}{64\pi^2} \ln\left(\frac{\Lambda^2}{\mu^2}\right) \delta^{AA'} C \left[(g_{\mu\nu} p^2 - p_\mu p_\nu) \left(\frac{11}{3} - (\xi - 1)\right) + g_{\mu\nu} \frac{p_5^2 + p_5'^2}{2} (4 + (\xi - 1)) \right]$$



- 3 point function:

$$\Pi^{\text{loc}}_{\mu\nu\rho} = \frac{g^3 f^{ABC}}{64\pi^2} C \ln\left(\frac{\Lambda^2}{\mu^2}\right) O_{\mu\nu\rho} \left[\frac{14}{3} + \frac{3}{4}(1 - \xi)\right]$$



- As in 4D, the coefficients match in the magic gauge $\xi = -3$ $\quad\rightarrow\quad$ unique counterterm to $F^{A,\mu\nu} F^A_{\mu\nu}$

III. 1 Loop Renormalization

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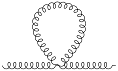
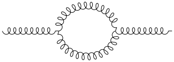


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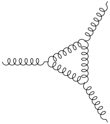
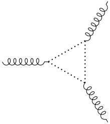
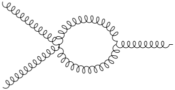

- As in 4D, the coefficients match in the magic gauge $\xi = -3$ $\quad\longrightarrow\quad$ unique counterterm to $F^{A,\mu\nu} F^A_{\mu\nu}$

III. 1 Loop Renormalization

Full Localized Renormalized Lagrangian

- Add counterterms coming from fermionic and scalar couplings

- Localized Renormalized Lagrangian ($\xi = -3$):

$$\begin{aligned}\mathcal{L}_4 \quad = \quad & -\frac{A}{4g^2} F_{\mathcal{H}}^{\mu\nu} F_{\mathcal{H},\mu\nu} + B_1 \, i\bar{\psi} \mathcal{D} \frac{1 \pm \gamma_5}{2} \psi + B_2 \left[(\partial_5 \bar{\psi}) \frac{1 \pm \gamma_5}{2} \psi - \bar{\psi} \frac{1 \mp \gamma_5}{2} (\partial_5 \psi) \right] + \\ & + C_1 \, (\mathcal{D}_\mu \phi_{\mathrm{e}})^\dagger (\mathcal{D}^\mu \phi_{\mathrm{e}}) + C_2 \left[\underbrace{(\partial_5^2 \phi_{\mathrm{e}}^\dagger) \phi_{\mathrm{e}} + \phi_{\mathrm{e}}^\dagger (\partial_5^2 \phi_{\mathrm{e}})}_{\text{Even scalar}} \right] + C_3 \, \underbrace{(\partial_5 \phi_{\mathrm{o}})^\dagger (\partial_5 \phi_{\mathrm{o}})}_{\text{Odd scalar}} + \\ & - m_{\mathrm{loc}}^2 (\phi_{\mathrm{e}}^\dagger \phi_{\mathrm{e}}) - D \, (\phi_{\mathrm{e}}^\dagger \phi_{\mathrm{e}})^2 \end{aligned}$$

- Finite number of counterterms at one-loop, no higher order operators

Conclusion

- 5D $\longrightarrow$ Modification of the RGE $\longrightarrow$ Possibility of UV fixed points
- 5D theories can be computed at one-loop, finite number of counterterms
- Going beyond one-loop? Naive dimensional analysis of the N-loop integral:

$$I_N \sim \left(\Pi_{i=1}^{N} \int d^d k_i\right) \frac{f(k_i^2, k_j \cdot k_l) \sim k^{2m}}{\left(\Pi_{i=1}^{N} (k_i^2)^{n_i}\right) \left(\Pi_{l>j}((k_l - k_j)^2)^{n_{lj}}\right)} \qquad n_I = \sum_i n_i + \sum_{l>j} n_{lj} - m$$

- Suggests for $2n_I = 5N$ new **logarithmic** divergences in $d = 5$ in the bulk
- 5D theory not valid to arbitrary scales but could still break down much higher than the compactification scale in the presence of fixed points
- Need more detailed analysis for higher loops, using Zimmermann's forest equations