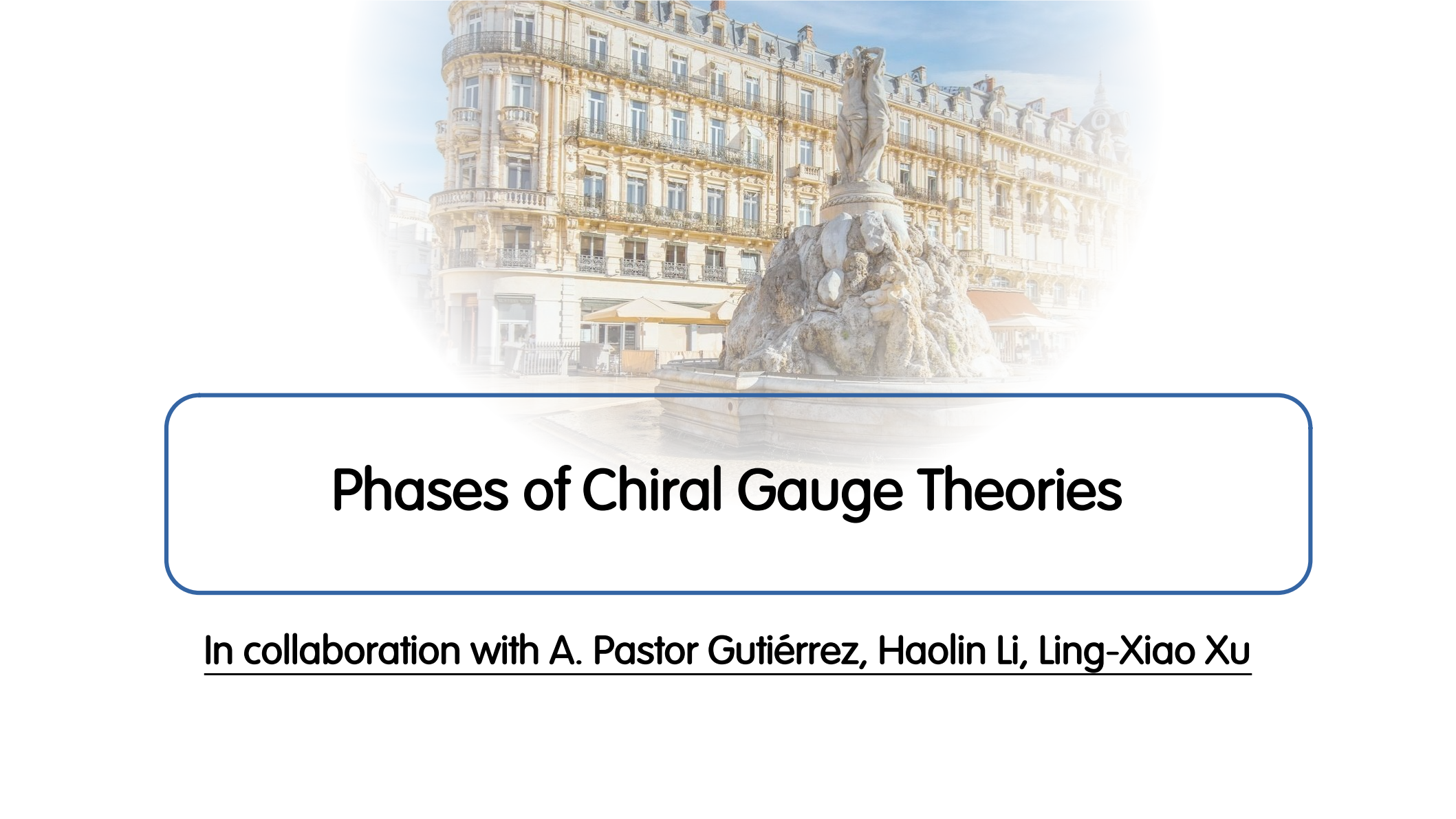




Phases of Chiral Gauge Theories

In collaboration with A. Pastor Gutiérrez, Haolin Li, Ling-Xiao Xu

Motivations

Motivations

- Compelling mechanism for the EWSB



- $\text{Mass NP} \gg \text{Higgs Mass}$

Motivations

- Compelling mechanism for the EWSB



- Mass NP $\gg$ Higgs Mass

- QFT has still not shown us all of its tricks ?



- Chiral Gauge Dynamics ?

Motivations

- Compelling mechanism for the EWSB

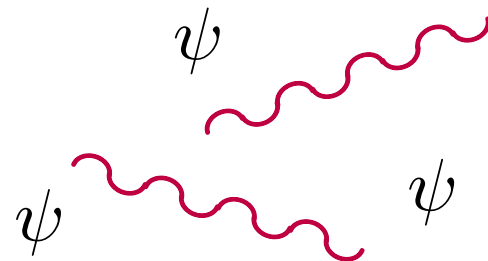


- Mass NP $\gg$ Higgs Mass

- QFT has still not shown us all of its tricks ?

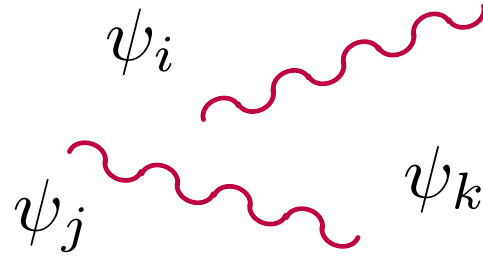


- Chiral Gauge Dynamics ?



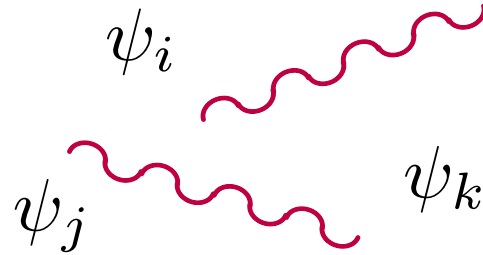
Motivations

Chiral Gauge Dynamics :



Motivations

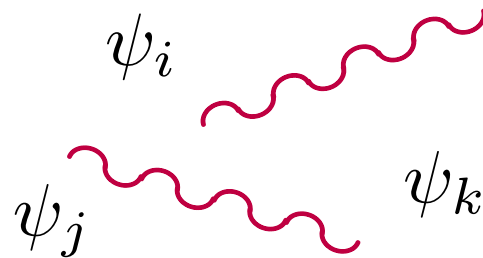
Chiral Gauge Dynamics :



- 4D Gauge Theory

Motivations

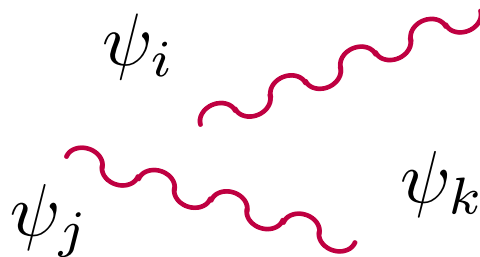
Chiral Gauge Dynamics :



- 4D Gauge Theory $SU(N_c)$, $Sp(N_c)$, $SO(N_c) \dots$

Motivations

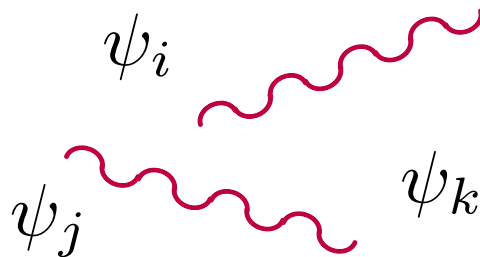
Chiral Gauge Dynamics :



- 4D Gauge Theory $SU(N_c)$, $Sp(N_c)$, $SO(N_c) \dots$
- The fermions cannot be arranged as Dirac

Motivations

Chiral Gauge Dynamics :

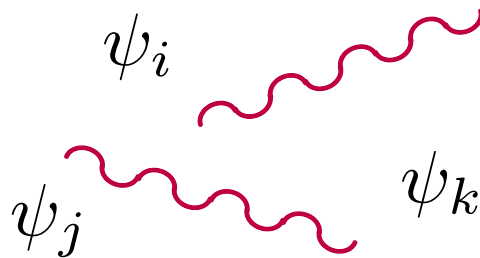


- 4D Gauge Theory $SU(N_c)$, $Sp(N_c)$, $SO(N_c) \dots$

- The fermions cannot be arranged as Dirac $\begin{pmatrix} \psi_i \\ \psi_j \end{pmatrix} \sim \begin{pmatrix} L \\ R \end{pmatrix}$

Motivations

Chiral Gauge Dynamics :



- 4D Gauge Theory $SU(N_c), Sp(N_c), SO(N_c) \dots$

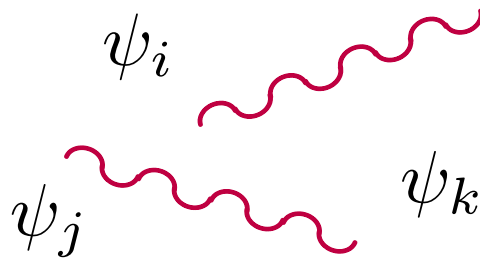
- The fermions cannot be arranged as Dirac

$$\begin{pmatrix} \psi_i \\ \psi_j \end{pmatrix} \sim \begin{pmatrix} L \\ R \end{pmatrix}$$

- Asymptotically Free setup

Motivations

Chiral Gauge Dynamics :



- 4D Gauge Theory $SU(N_c), Sp(N_c), SO(N_c) \dots$

- The fermions cannot be arranged as Dirac $\begin{pmatrix} \psi_i \\ \psi_j \end{pmatrix} \sim \begin{pmatrix} L \\ R \end{pmatrix}$

- Asymptotically Free setup
... Strong Coupling...

- 1) QCD VS Chiral Dynamic

- 2) Functional Methods

- 3) Results !

QCD VS Chiral Dynamic

QCD !



QCD !



- Vafa-Witten Theorems (Weingarten inequalities)

QCD !



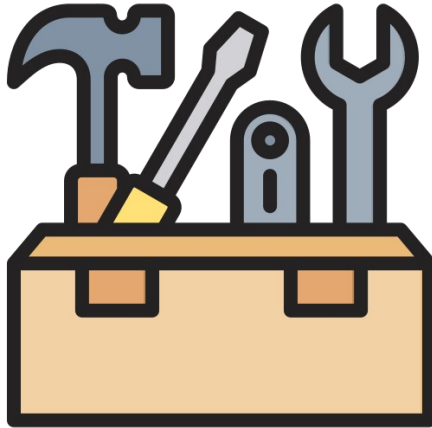
- Vafa-Witten Theorems (Weingarten inequalities)
- t'Hooft Anomaly Matching

QCD !



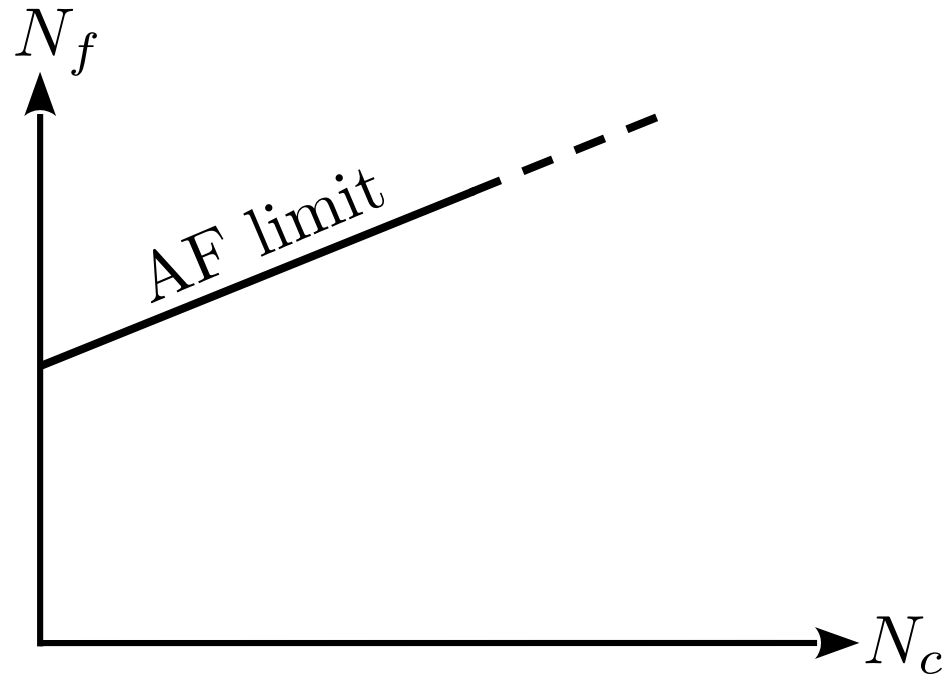
- Vafa-Witten Theorems (Weingarten inequalities)
- t'Hooft Anomaly Matching
- Large N

QCD !

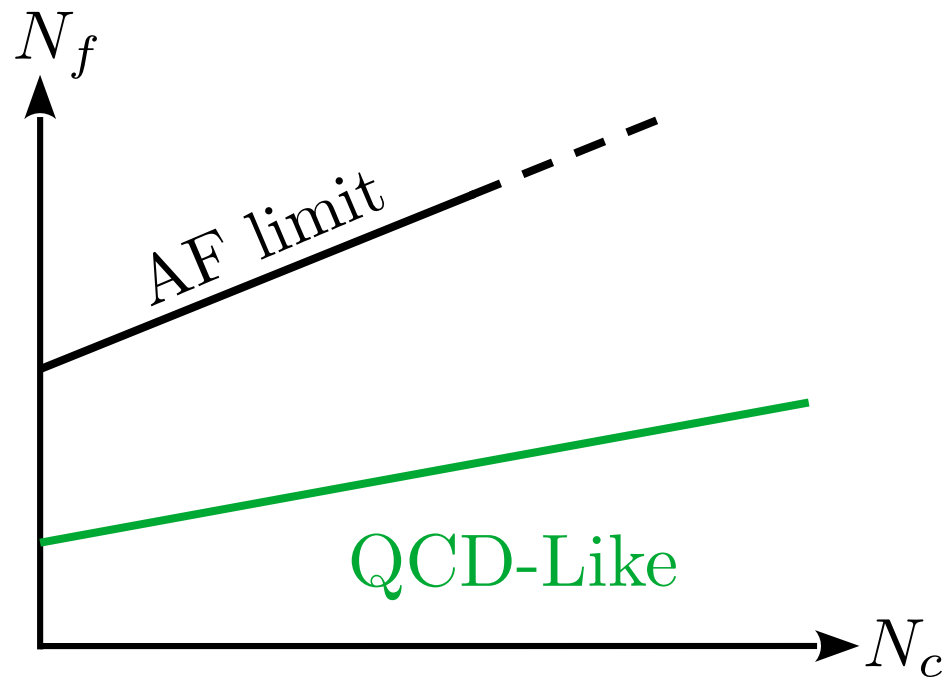


- Vafa-Witten Theorems (Weingarten inequalities)
- t'Hooft Anomaly Matching
- Large N
- Lattice

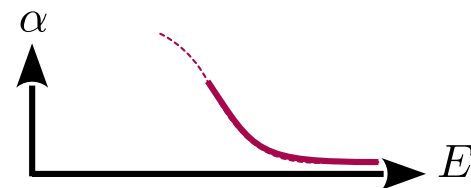
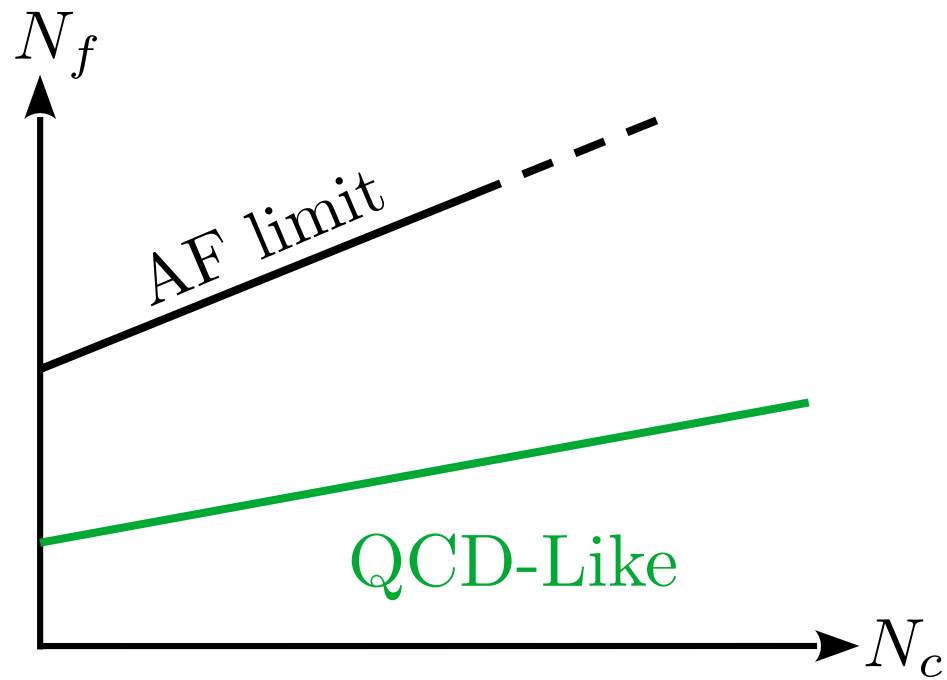
QCD !



QCD !



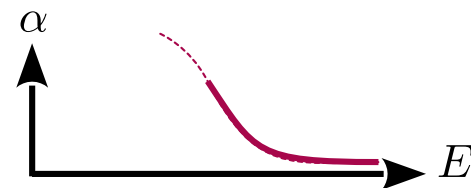
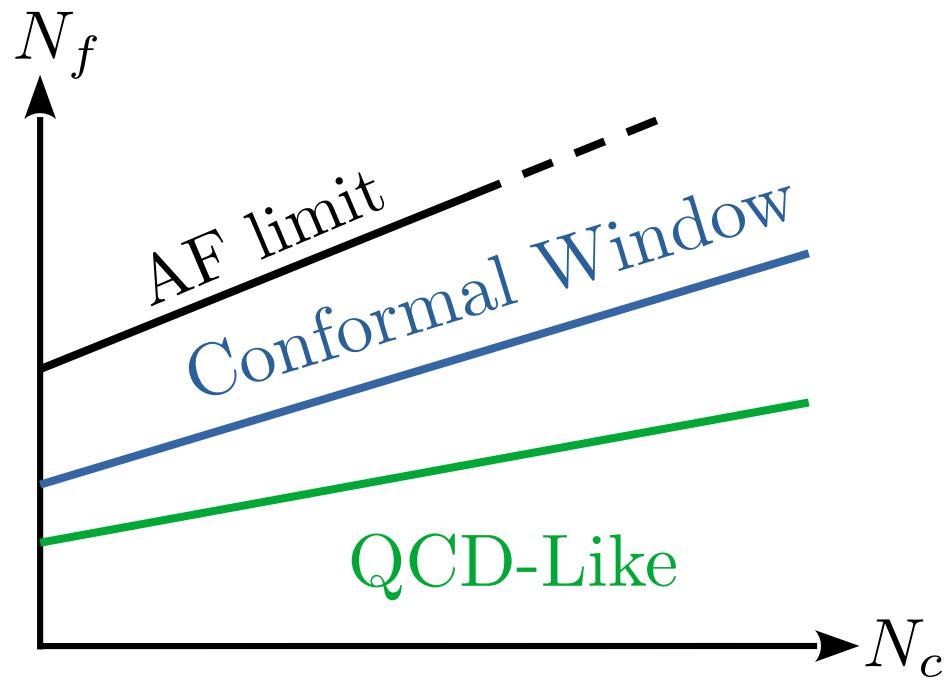
QCD !



Confinement & Condensation

$$\langle \bar{q}q \rangle \neq 0$$

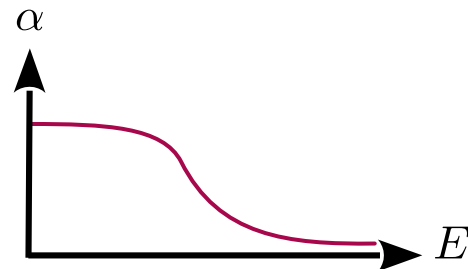
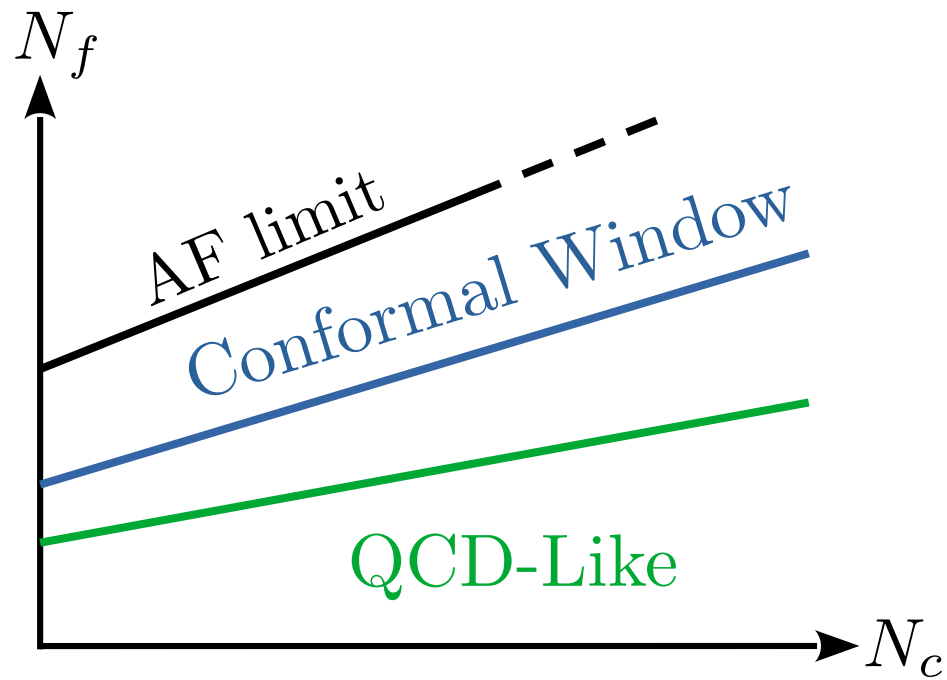
QCD !



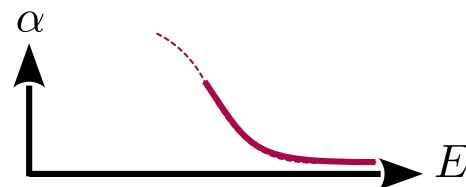
Confinement & Condensation

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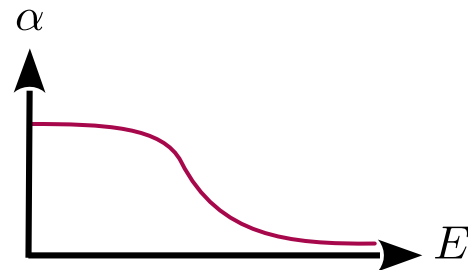
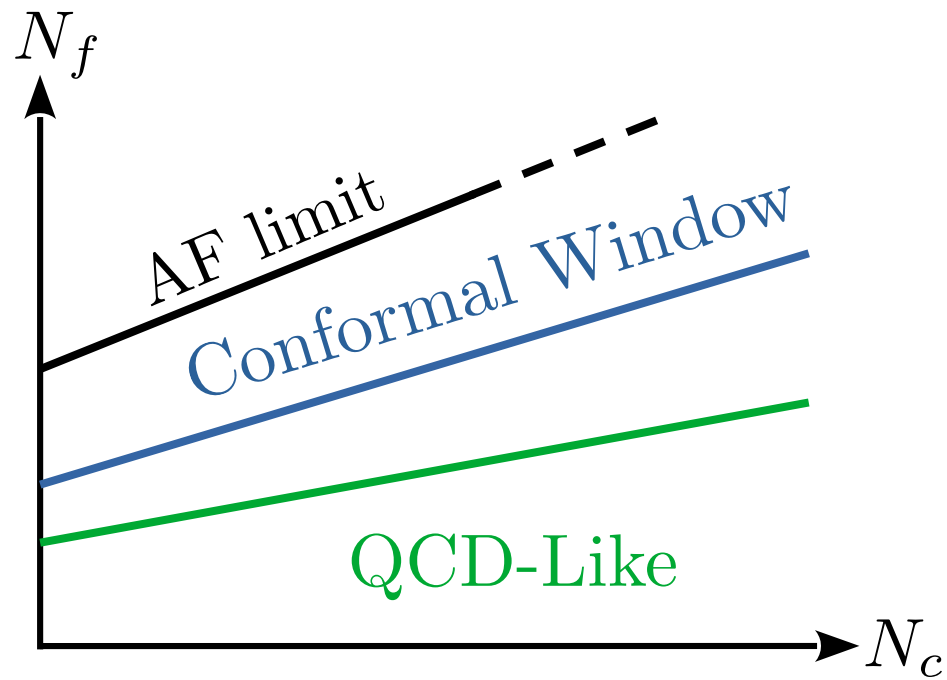
No Confinement & No Condensation



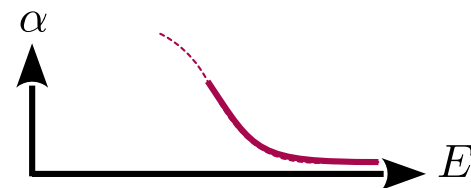
Confinement & Condensation

$$\langle \bar{q}q \rangle \neq 0$$

QCD !



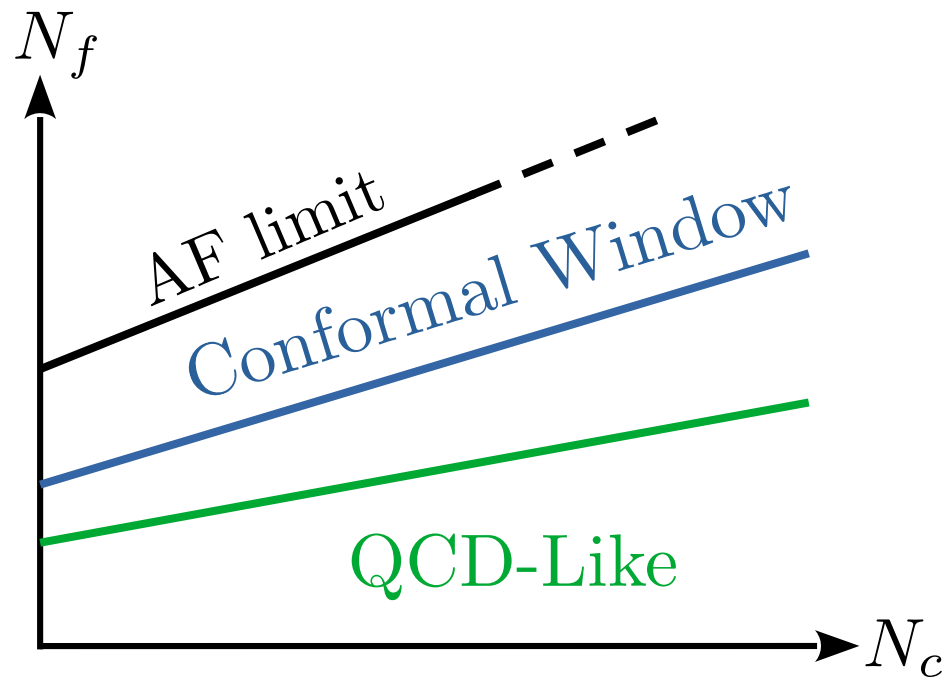
No Confinement & No Condensation
But a Conformal phase



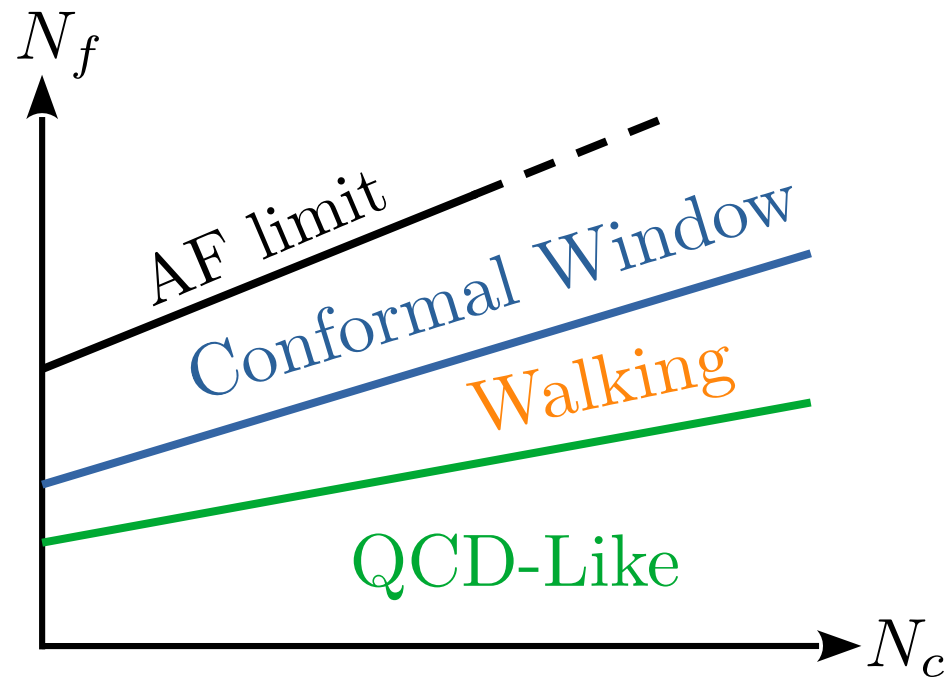
Confinement & Condensation

$$\langle \bar{q}q \rangle \neq 0$$

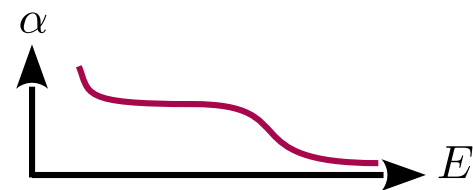
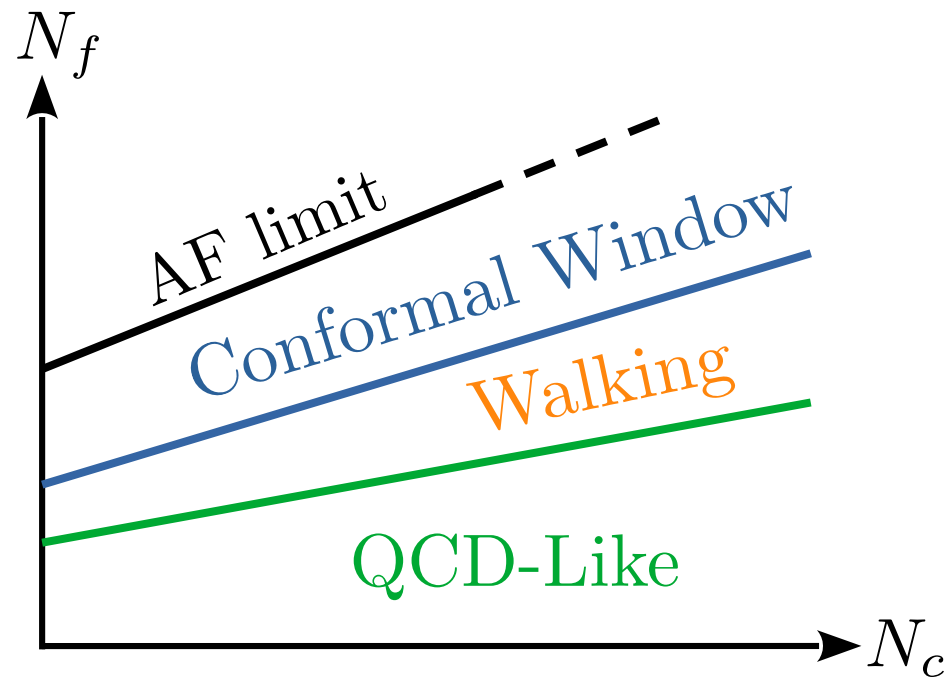
QCD !



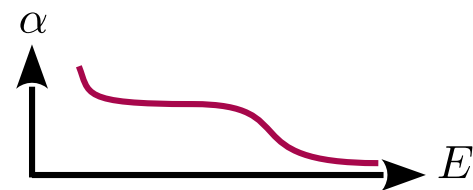
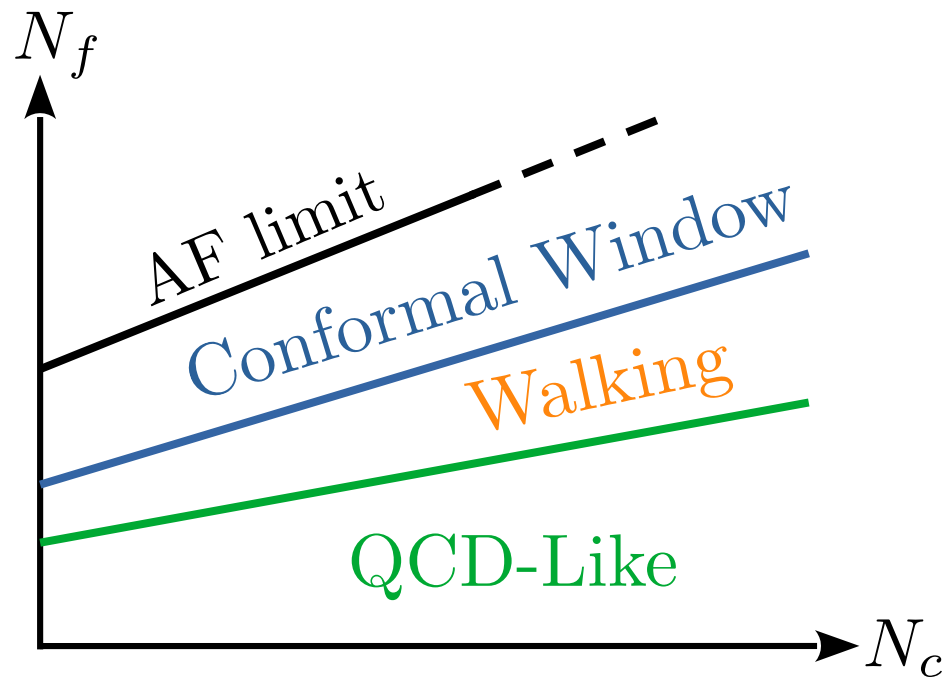
QCD !



QCD !

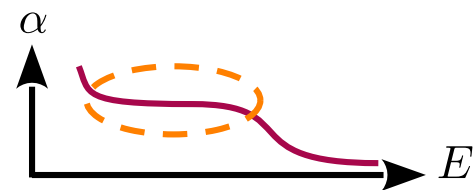
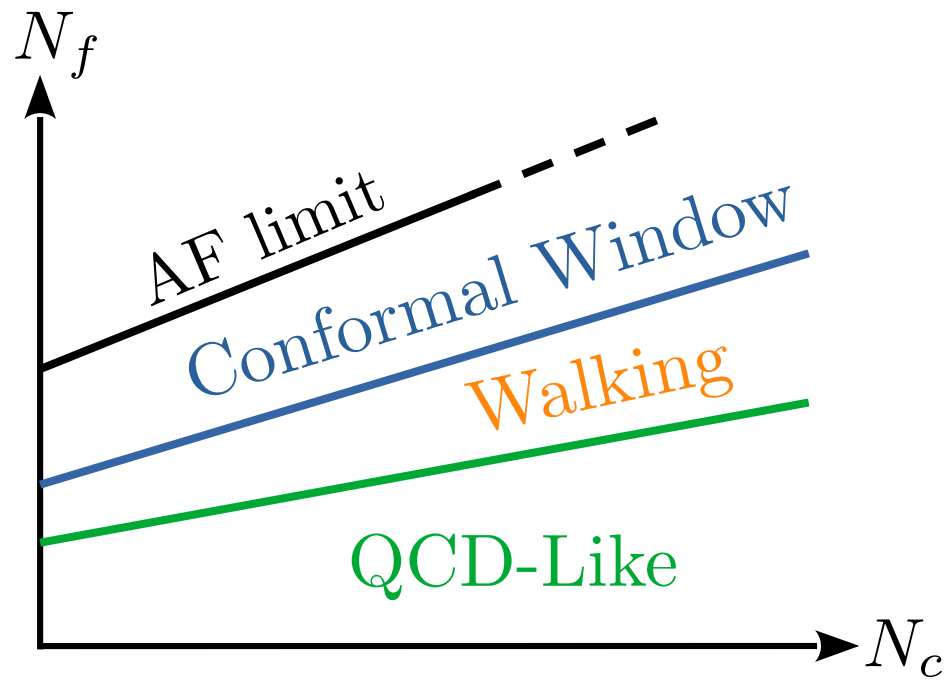


QCD !



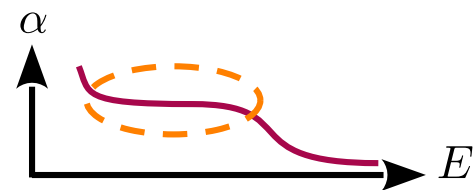
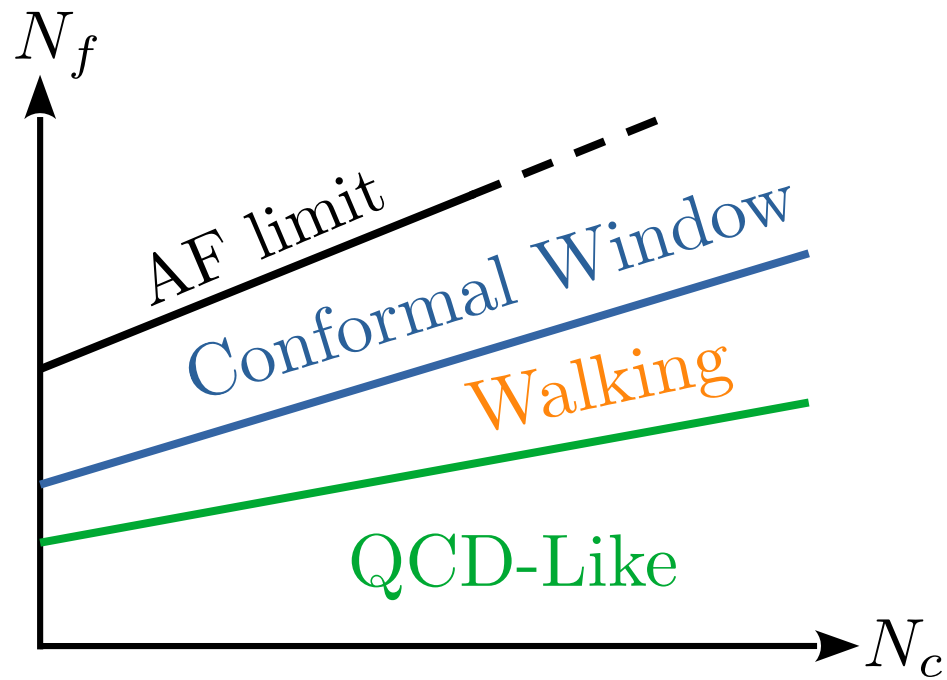
Confinement & Condensation

QCD !



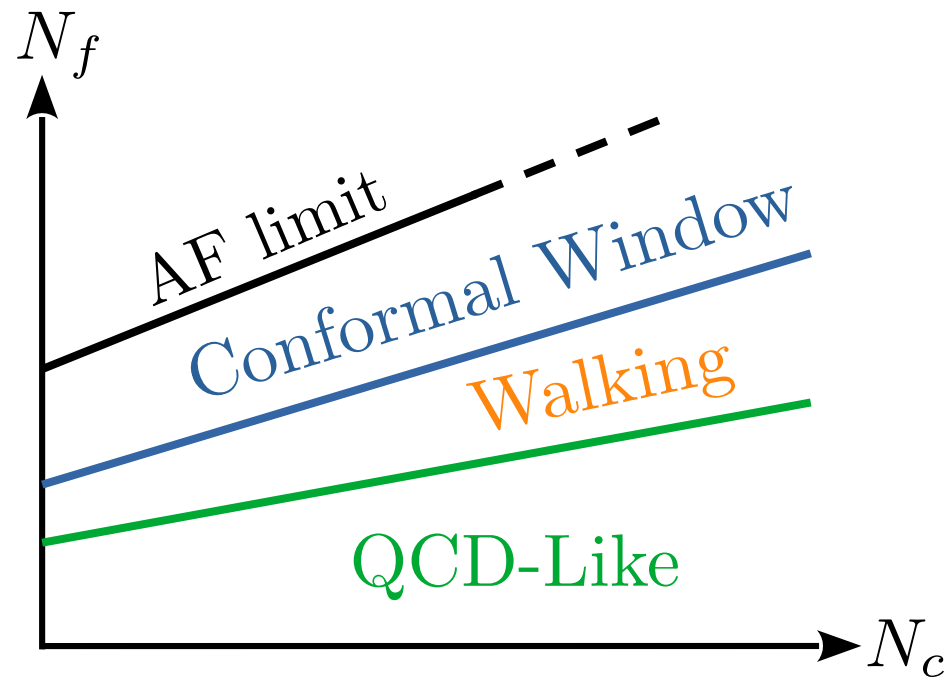
Confinement & Condensation

QCD !

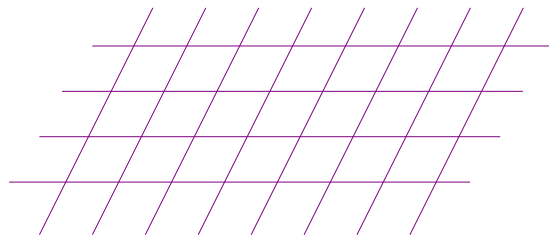
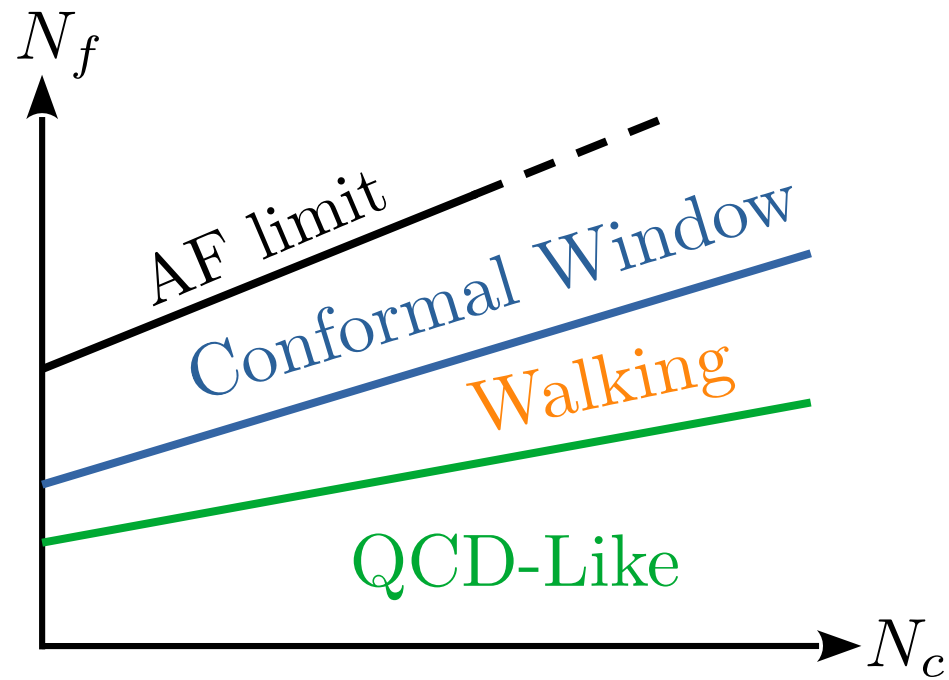


Confinement & Condensation
And near Conformal phase !

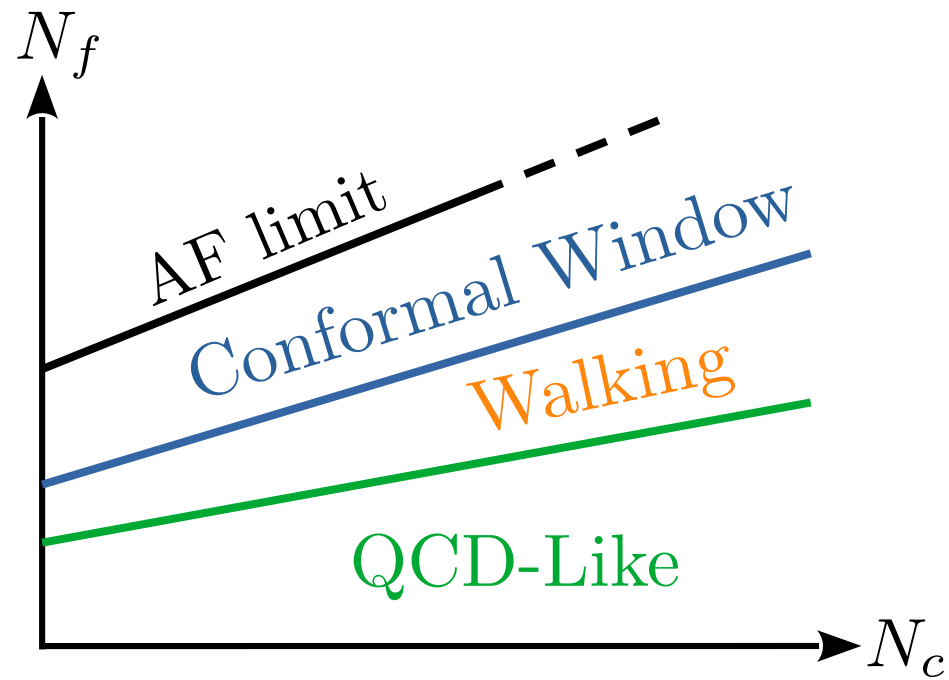
QCD !



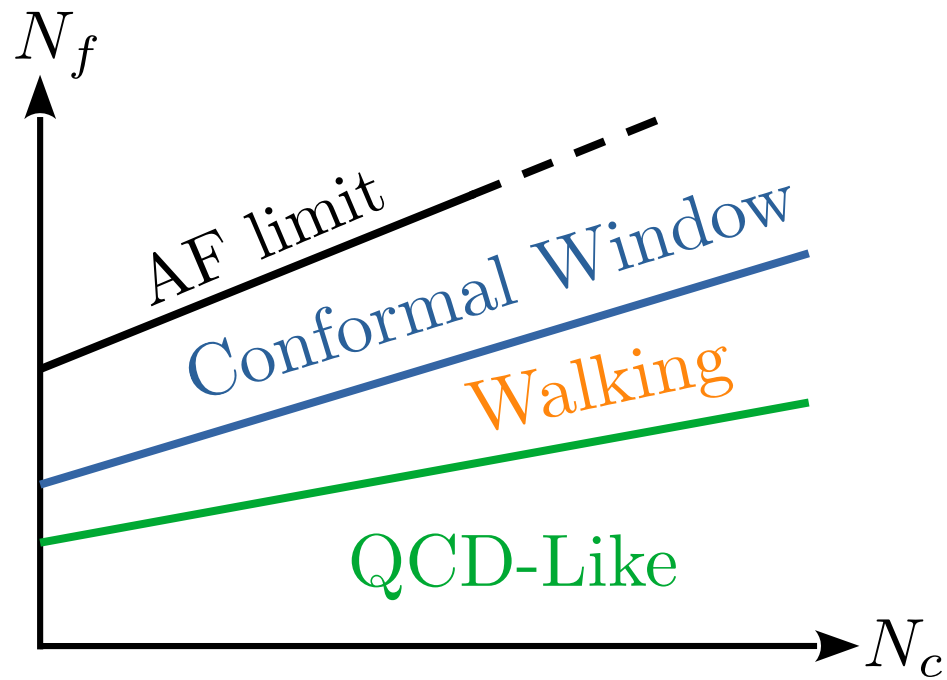
QCD !



QCD !

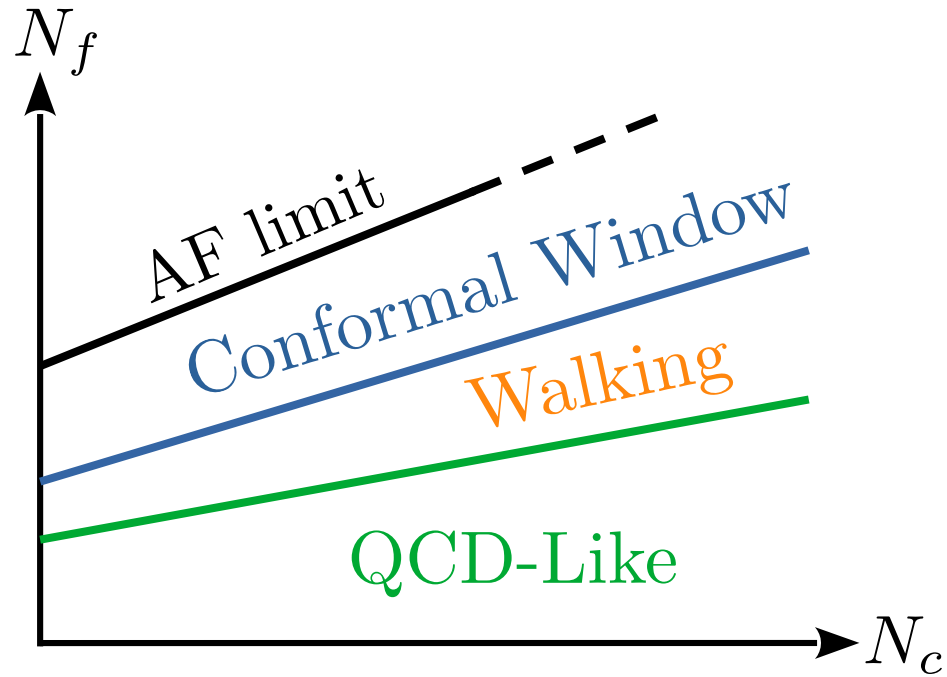


QCD !



The Landscape is not fully explored

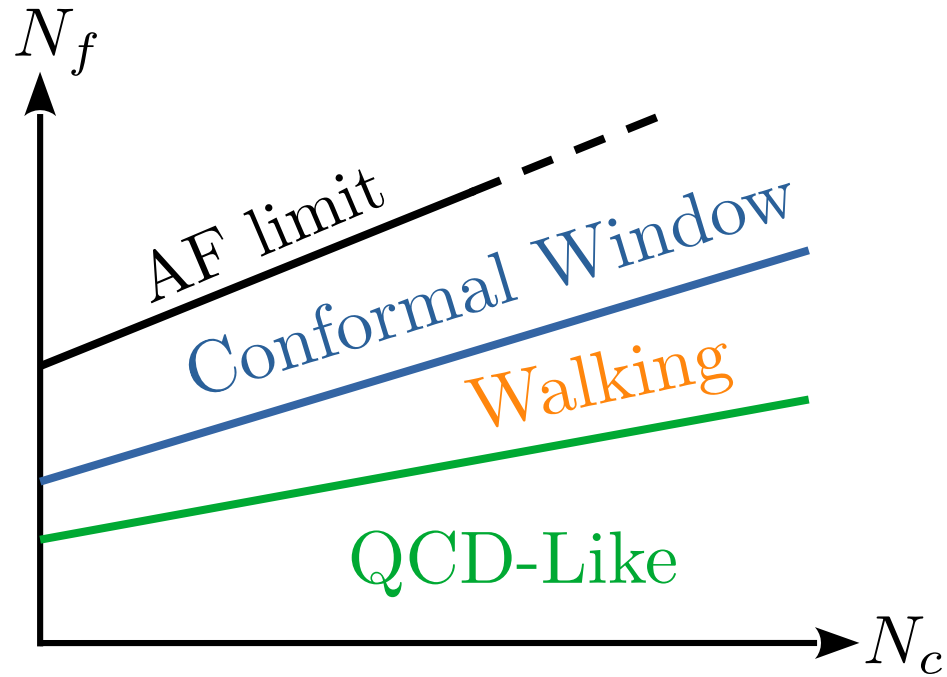
QCD !



The Landscape is not fully explored

- m_{proton} f_{π} f_{ρ} ?

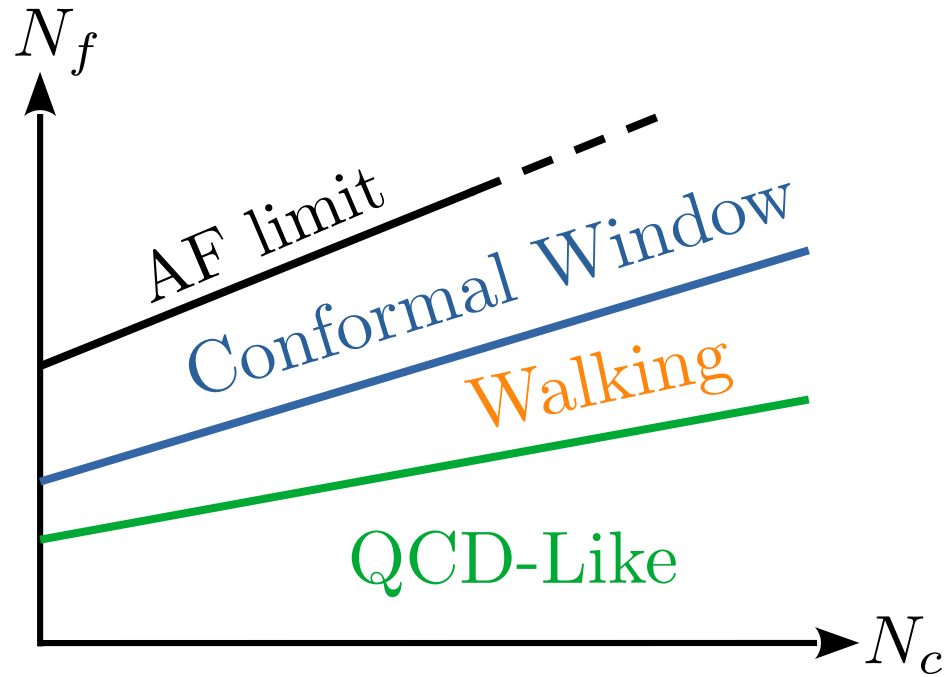
QCD !



The Landscape is not fully explored

- m_{proton} f_π f_ρ ?
- EWSB from $\langle \bar{\psi}\psi \rangle \neq 0$

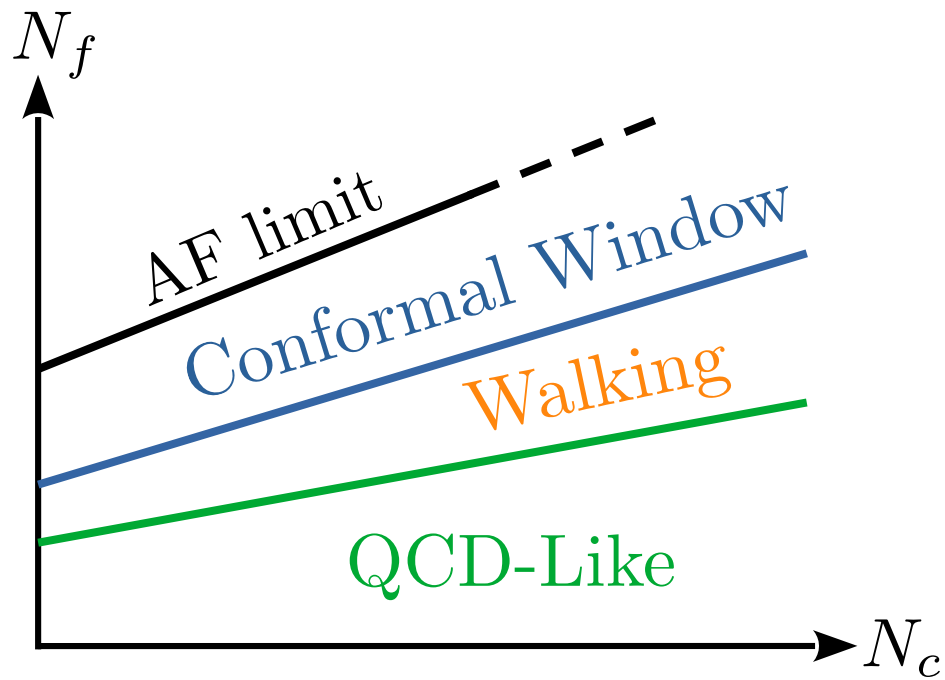
QCD !



The Landscape is not fully explored

- m_{proton} f_π f_ρ ?
- EWSB from $\langle \bar{\psi}\psi \rangle \neq 0$
- $H \sim \pi$

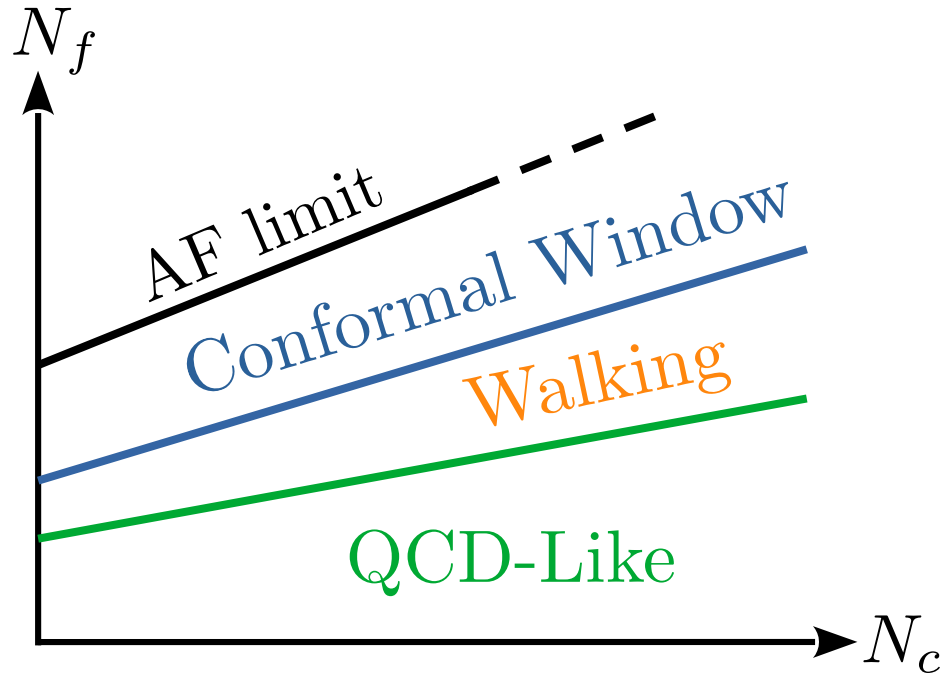
QCD !



The Landscape is not fully explored

- m_{proton} f_π f_ρ ?
- EWSB from $\langle \bar{\psi}\psi \rangle \neq 0$
- $H \sim \pi$ $\rho \sim Z'$

QCD !



The Landscape is not fully explored

- m_{proton} f_π f_ρ ?
- EWSB from $\langle \bar{\psi}\psi \rangle \neq 0$
- g_ρ $[\pi \partial_\mu \pi]$ ρ^μ ?

Chiral Dynamic

Chiral Dynamic

- Vafa-Witten Theorems

Chiral Dynamic

-  Vafa-Witten Theorems

Chiral Dynamic

-  Vafa-Witten Theorems

- t'Hooft Anomaly

Chiral Dynamic

-  Vafa-Witten Theorems

-  t'Hooft Anomaly

Chiral Dynamic

-  Vafa-Witten Theorems

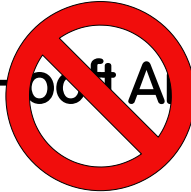
- Large N

-  t'Hooft Anomaly

Chiral Dynamic

-  Vafa-Witten Theorems

-  Large N

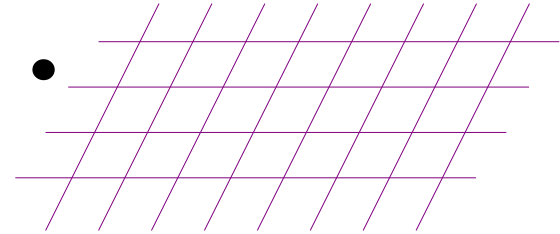
-  t'Hooft Anomaly

Chiral Dynamic

- ~~Vafa-Witten Theorems~~

- ~~t'Hooft Anomaly~~

- ~~Large N~~

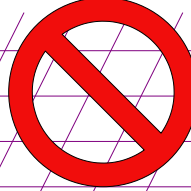


Chiral Dynamic

-  Vafa-Witten Theorems

-  t'Hooft Anomaly

-  Large N

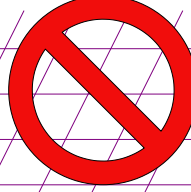
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Chiral Dynamic

-  Vafa-Witten Theorems

-  Large N

-  t'Hooft Anomaly

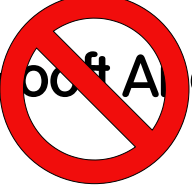
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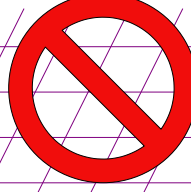
Spectrum ?

Chiral Dynamic

-  Vafa-Witten Theorems

-  Large N

-  t'Hooft Anomaly

- 

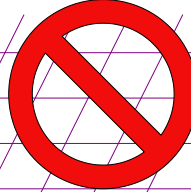
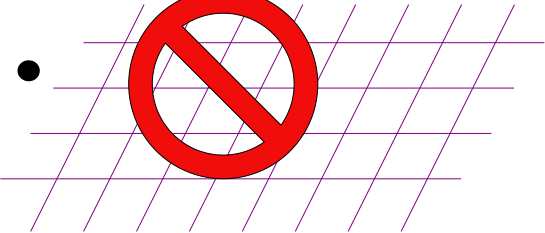
Spectrum ? Coupling ?

Chiral Dynamic

-  Vafa-Witten Theorems

-  Large N

-  t'Hooft Anomaly

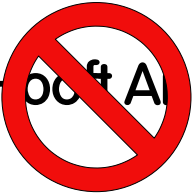
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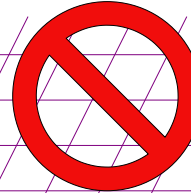
Spectrum ? Coupling ? Conformality ?

Chiral Dynamic

-  Vafa-Witten Theorems

-  Large N

-  t'Hooft Anomaly

- 

Spectrum ? Coupling ? Conformality ? Breaking Pattern ?

Chiral Dynamic

Chiral Dynamic

Georgi-Glashow model (GG) :

Chiral Dynamic

Georgi-Glashow model (GG) :	$SU(N_c)$	Flavor
	A	
	$\bar{F}$	

Chiral Dynamic

Georgi-Glashow model (GG) :	$SU(N_c)$	Flavor
	A	1
	$\bar{F}$	

Chiral Dynamic

Georgi-Glashow model (GG) :	$SU(N_c)$	Flavor
	A	1
	$\bar{F}$	$N_c - 4$

Chiral Dynamic

Georgi-Glashow model (GG) :	$SU(N_c)$	Flavor
	A	1
	$\bar{F}$	$N_c - 4$

→ Global Symmetry : $SU(N_c - 4) \times U(1)$

Chiral Dynamic

Georgi-Glashow model (GG) :	$SU(N_c)$	Flavor
	A	1
	$\bar{F}$	$N_c - 4$

→ Global Symmetry : $SU(N_c - 4) \times U(1)$

→ $\mathcal{B} = A \{ \bar{F} \bar{F} \}$

Chiral Dynamic

Georgi-Glashow model (GG) :	$SU(N_c)$	Flavor
	A	1
	$\bar{F}$	$N_c - 4$

→ Global Symmetry : $SU(N_c - 4) \times U(1)$

→ $\mathcal{B} = A \{ \bar{F} \bar{F} \}$ $m_{\mathcal{B}} = 0 ?$

Chiral Dynamic

Georgi-Glashow model (GG) :	$SU(N_c)$	Flavor
	A	1
	$\bar{F}$	$N_c - 4$

→ Global Symmetry : $SU(N_c - 4) \times U(1)$

What is the IR ?

Chiral Dynamic

Georgi-Glashow model (GG) :	$SU(N_c)$	Flavor
	A	1
	$\bar{F}$	$N_c - 4$

→ Global Symmetry : $SU(N_c - 4) \times U(1)$

-
- Generalized Symmetries (1-form 't Hooft Anomaly)

Chiral Dynamic

Georgi-Glashow model (GG) :	$SU(N_c)$	Flavor
	A	1
	$\bar{F}$	$N_c - 4$

→ Global Symmetry : $SU(N_c - 4) \times U(1)$

-
- Generalized Symmetries (1-form 't Hooft Anomaly) → $\langle A\bar{F} \rangle \neq 0$

Chiral Dynamic

Georgi-Glashow model (GG) :	$SU(N_c)$	Flavor
	A	1
	$\bar{F}$	$N_c - 4$

→ Global Symmetry : $SU(N_c - 4) \times U(1)$

- Generalized Symmetries (1-form 't Hooft Anomaly) → $\langle A\bar{F} \rangle \neq 0$
- Anomaly Mediated SuSy Breaking

Chiral Dynamic

Georgi-Glashow model (GG) :	$SU(N_c)$	Flavor
	A	1
	$\bar{F}$	$N_c - 4$

→ Global Symmetry : $SU(N_c - 4) \times U(1)$

- Generalized Symmetries (1-form 't Hooft Anomaly) → $\langle A\bar{F} \rangle \neq 0$
- Anomaly Mediated SuSy Breaking → $\langle A\bar{F} \rangle \neq 0$, $\langle \bar{F}\bar{F} \rangle \neq 0$

Toy Model

Georgi-Glashow (GG):

$SU(N_c)$	Flavor
A	1
$\bar{F}$	$N_c - 4$

Toy Model

General Georgi-Glashow (GG) :	$SU(N_c)$	Flavor
	A	1
	$\bar{F}$	$N_c - 4$

Toy Model

General Georgi-Glashow (GG) :

→ $N_g \times$ Georgi-Glashow

$SU(N_c)$	Flavor
A	1
$\bar{F}$	$N_c - 4$

Toy Model

General Georgi-Glashow (GG) :

→ $N_g \times$ Georgi-Glashow

$SU(N_c)$	Flavor
A	N_g
$\bar{F}$	$N_g (N_c - 4)$

Toy Model

General Georgi-Glashow (GG) :

→ $N_g \times$ Georgi-Glashow

$SU(N_c)$	Flavor
A	N_g
$\bar{F}$	$N_g (N_c - 4)$

→ Global Symmetry : $SU(N_g) \times SU(N_g(N_c - 4)) \times U(1)$

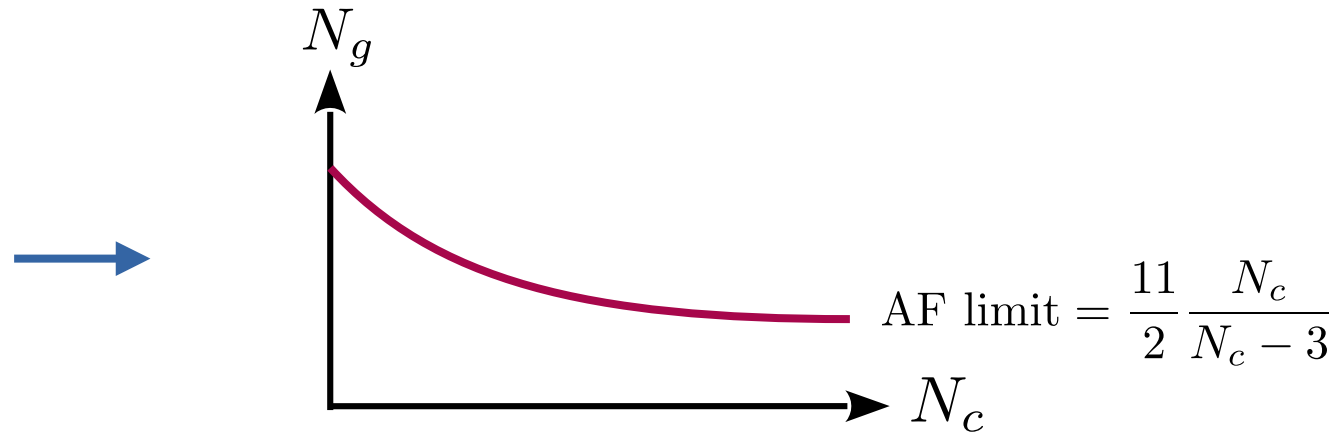
Toy Model

General Georgi-Glashow (GG) :

$SU(N_c)$	Flavor
A	N_g
$\bar{F}$	$N_g (N_c - 4)$

→ $N_g \times$ Georgi-Glashow

→ Global Symmetry : $SU(N_g) \times SU(N_g(N_c - 4)) \times U(1)$



Functional Methods

How To Diagnose χ SB?

How To Diagnose χ SB?

→ $\mathcal{Z}[J] = \int \mathcal{D}\varphi \, e^{-S[\varphi] + J\varphi}$

How To Diagnose χ SB?

→ $\mathcal{Z}[J] = \int \mathcal{D}\varphi \ e^{-S[\varphi] + J\varphi}$

→ $\mathcal{W}[J] = \ln \mathcal{Z}[J]$

How To Diagnose χ SB?

$$\rightarrow \mathcal{Z}[J] = \int \mathcal{D}\varphi \, e^{-S[\varphi] + J\varphi}$$

$$\rightarrow \mathcal{W}[J] = \ln \mathcal{Z}[J] \qquad \langle \phi(x_1) \dots \phi(x_n) \rangle_c = \frac{\partial^n}{\partial J(x_1) \dots \partial J(x_n)} \mathcal{W}[J]$$

How To Diagnose χ SB?

→ $\mathcal{Z}[J] = \int \mathcal{D}\varphi \ e^{-S[\varphi] + J\varphi}$

→ $\mathcal{W}[J] = \ln \mathcal{Z}[J]$ $\langle \phi(x_1) \dots \phi(x_n) \rangle_c = \frac{\partial^n}{\partial J(x_1) \dots \partial J(x_n)} \mathcal{W}[J]$

→ **1-PI Effective Action :**

How To Diagnose χ SB?

→ $\mathcal{Z}[J] = \int \mathcal{D}\varphi \ e^{-S[\varphi] + J\varphi}$

→ $\mathcal{W}[J] = \ln \mathcal{Z}[J]$ $\langle \phi(x_1) \dots \phi(x_n) \rangle_c = \frac{\partial^n}{\partial J(x_1) \dots \partial J(x_n)} \mathcal{W}[J]$

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How To Diagnose χ SB?

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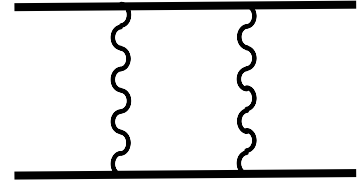
→ $\Gamma [\bar{\psi}, \psi, A_\mu]$

How To Diagnose χ SB?

$$\rightarrow \Gamma [\bar{\psi}, \psi, A_\mu] \supset \int \lambda_{4F} \bar{\psi} \psi \bar{\psi} \psi$$

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Divergent 4F flow signal χ SB

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Divergent 4F flow signal χ SB

$$(\psi_1 \psi_2) (\psi_1 \psi_2) \implies \langle \psi_1 \psi_2 \rangle \neq 0$$

How To Diagnose χ SB?

$$e^{-\Gamma[\phi]} = \int \mathcal{D}\varphi \ e^{-S[\phi+\varphi] + \frac{\partial \Gamma}{\partial \phi} \varphi}$$

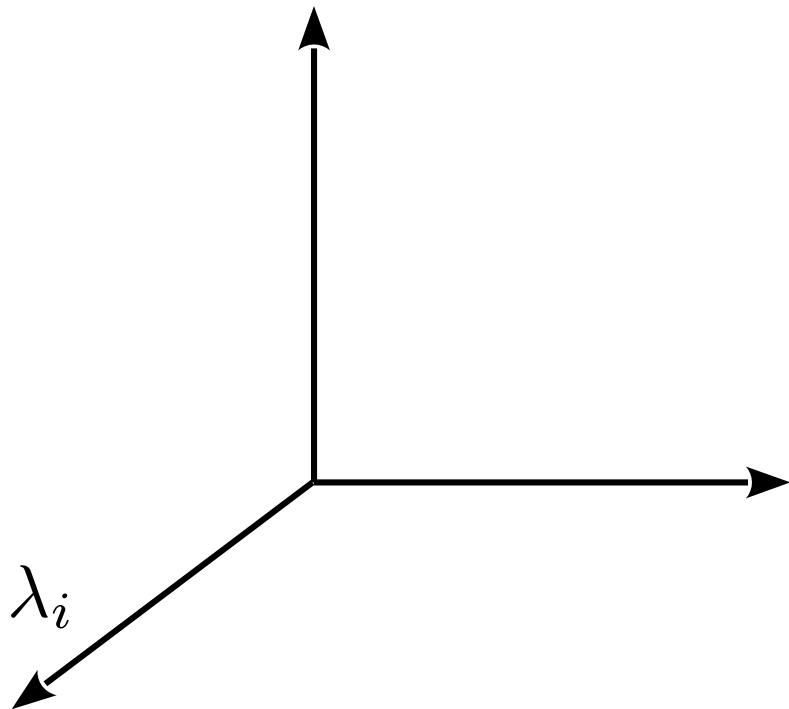
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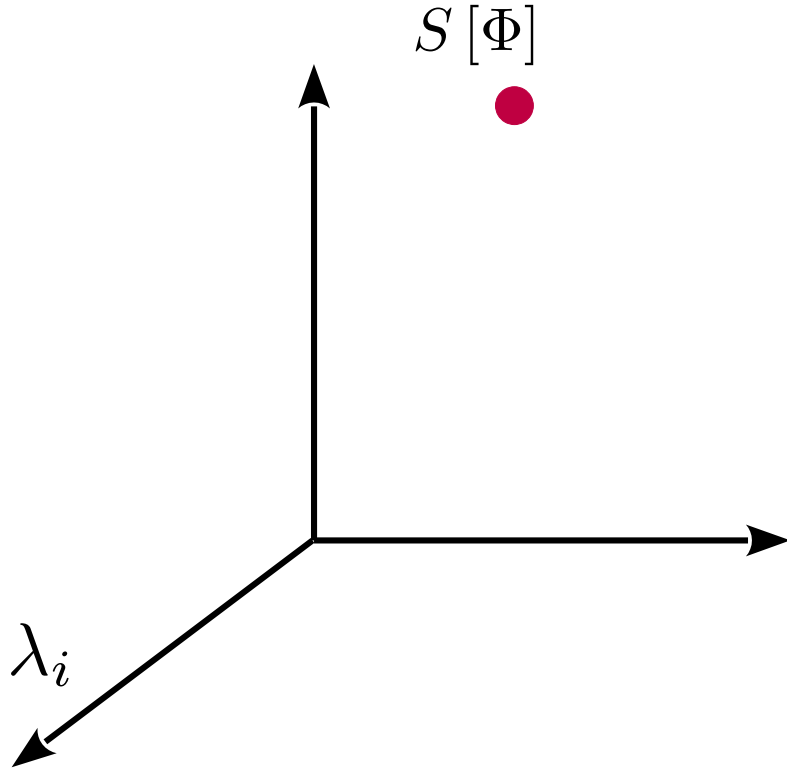
Integro-Differential Equation

HARD

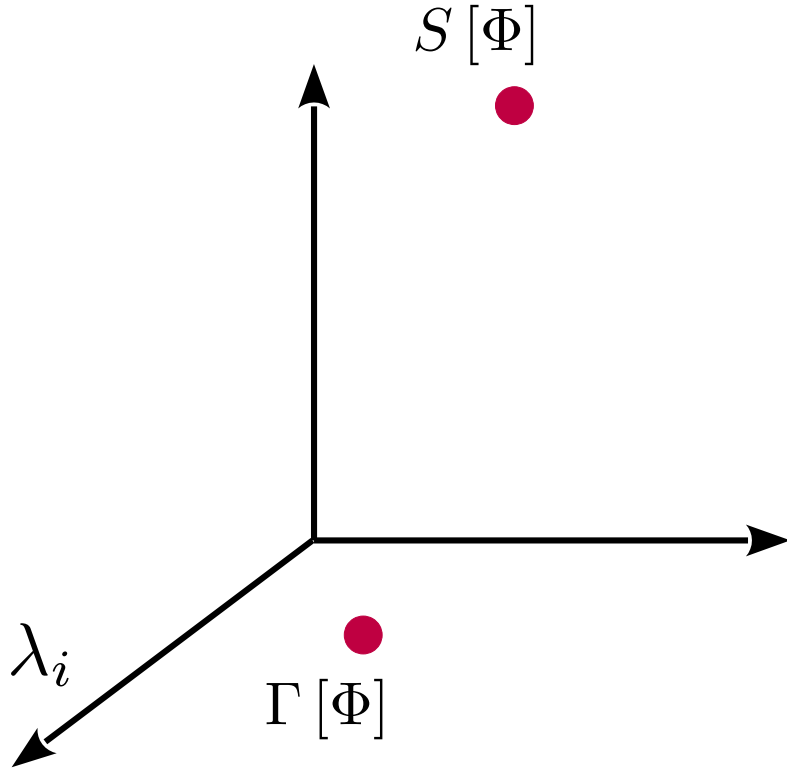
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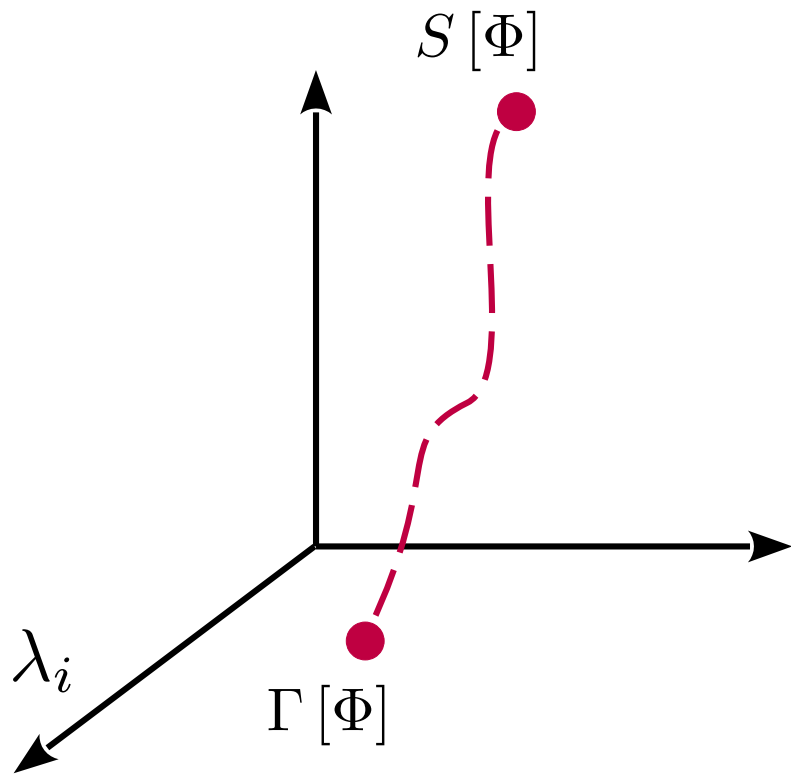
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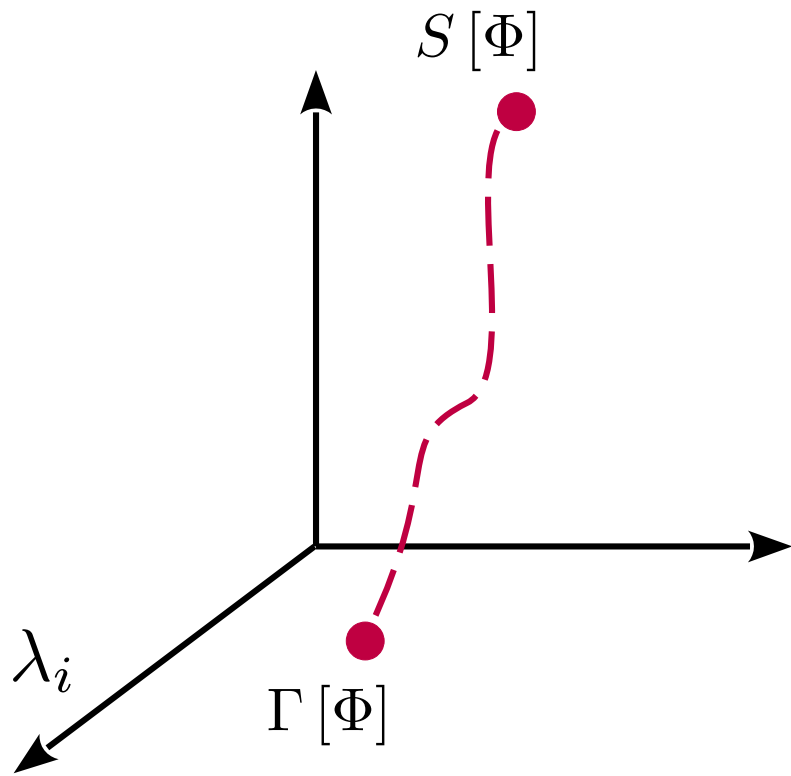
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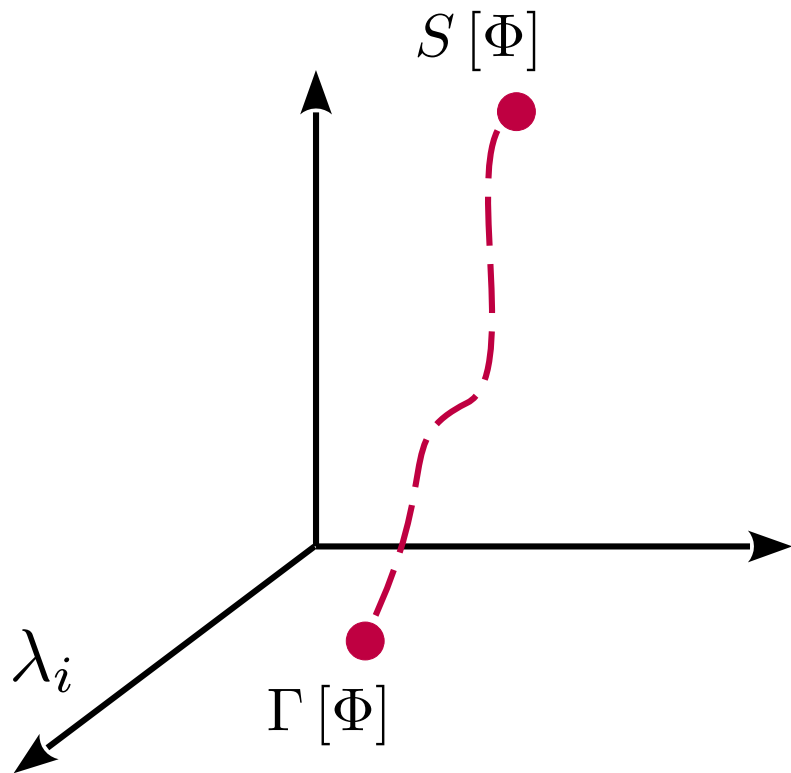


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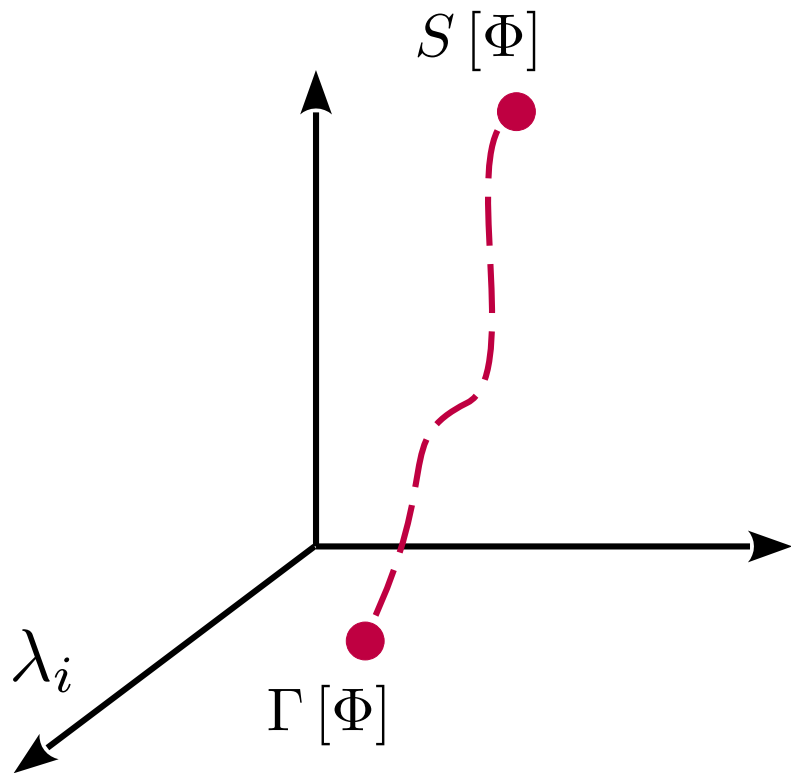
$$\mathcal{Z}_k[J] = \int_{\varphi(p) > k} \mathcal{D}\varphi \, e^{S+J\varphi}$$

How To Diagnose χ SB?



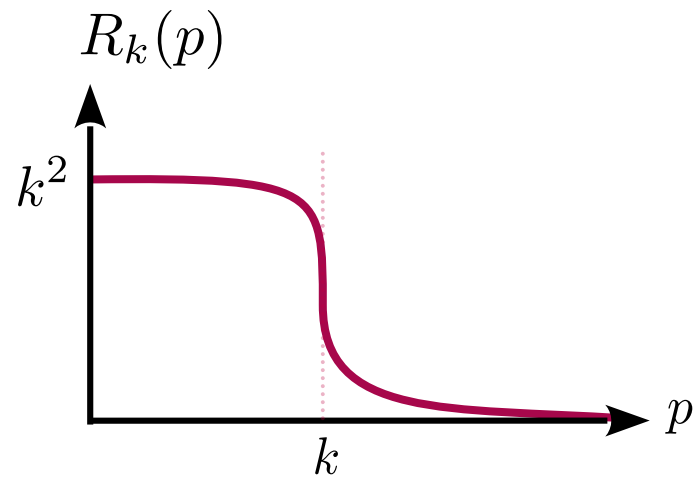
$$\mathcal{Z}_k[J] = \int_{\varphi(p) > k} \mathcal{D}\varphi e^{S+J\varphi}$$
$$\underbrace{\int \mathcal{D}\varphi e^{-\frac{1}{2} \phi(p) R_k(p) \phi(-p)}}$$

How To Diagnose χ SB?

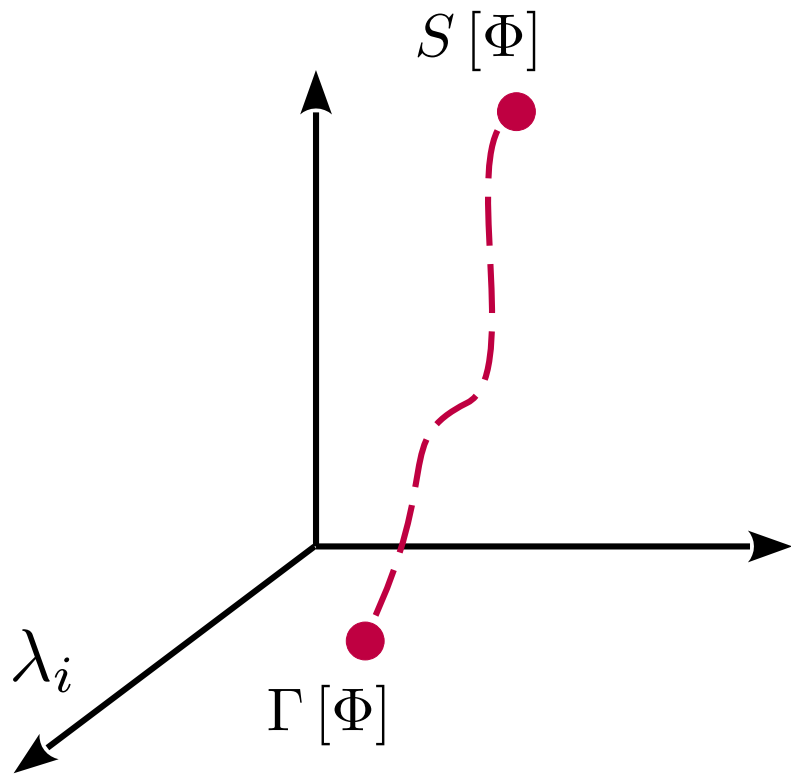


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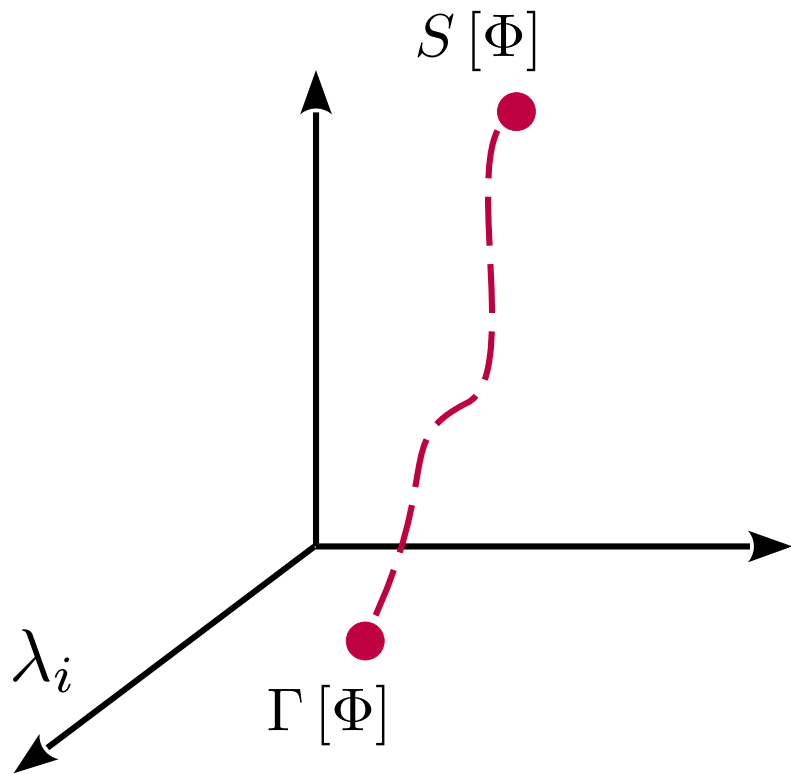


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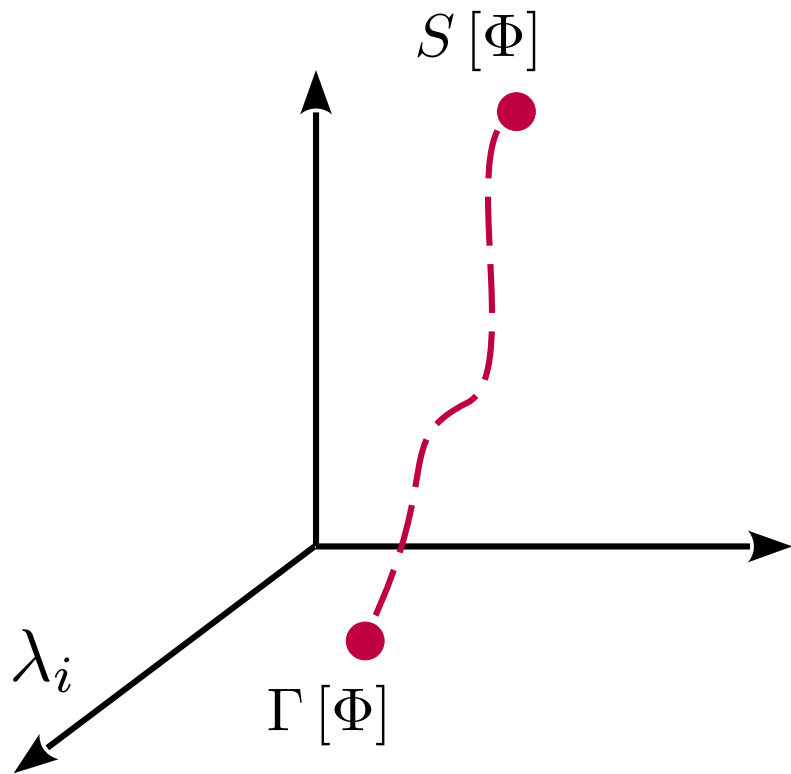


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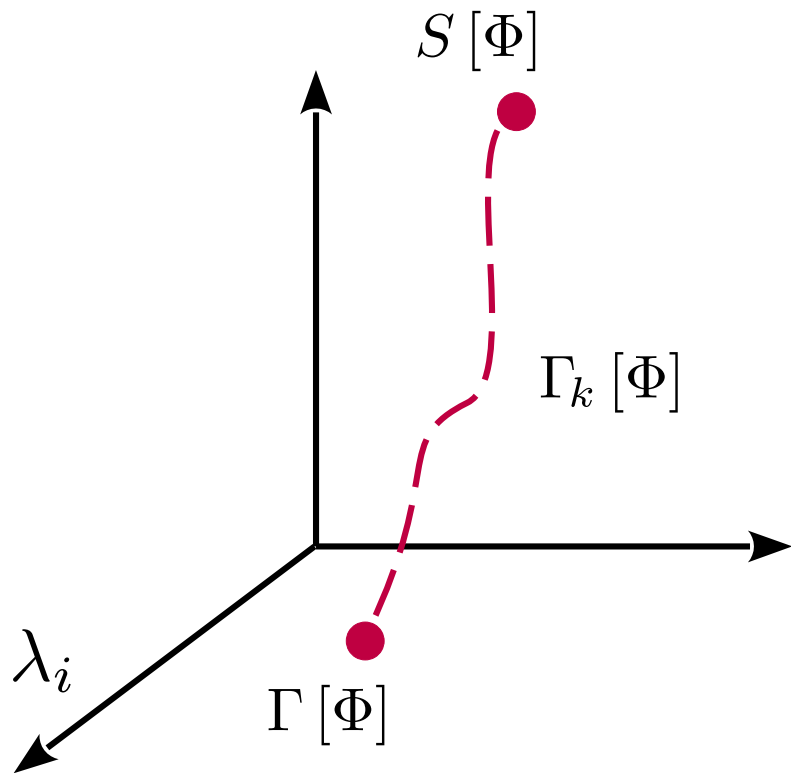


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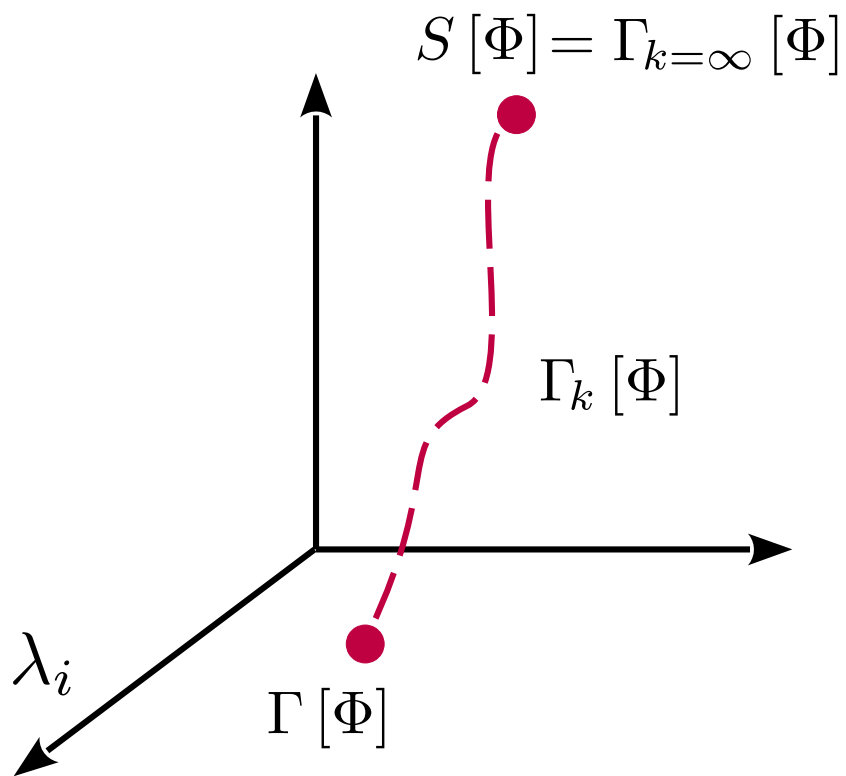


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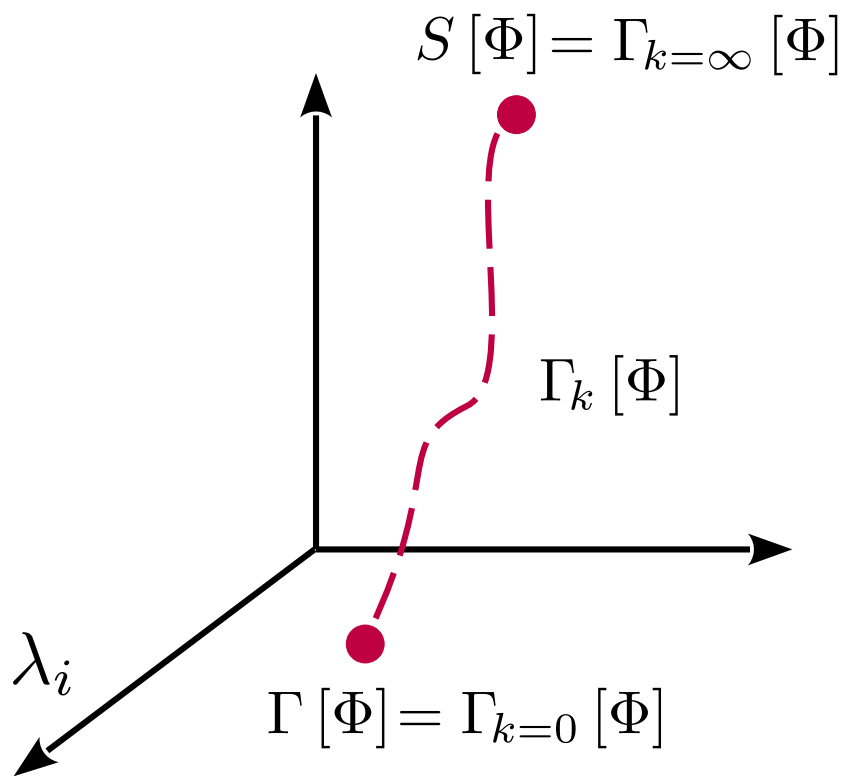
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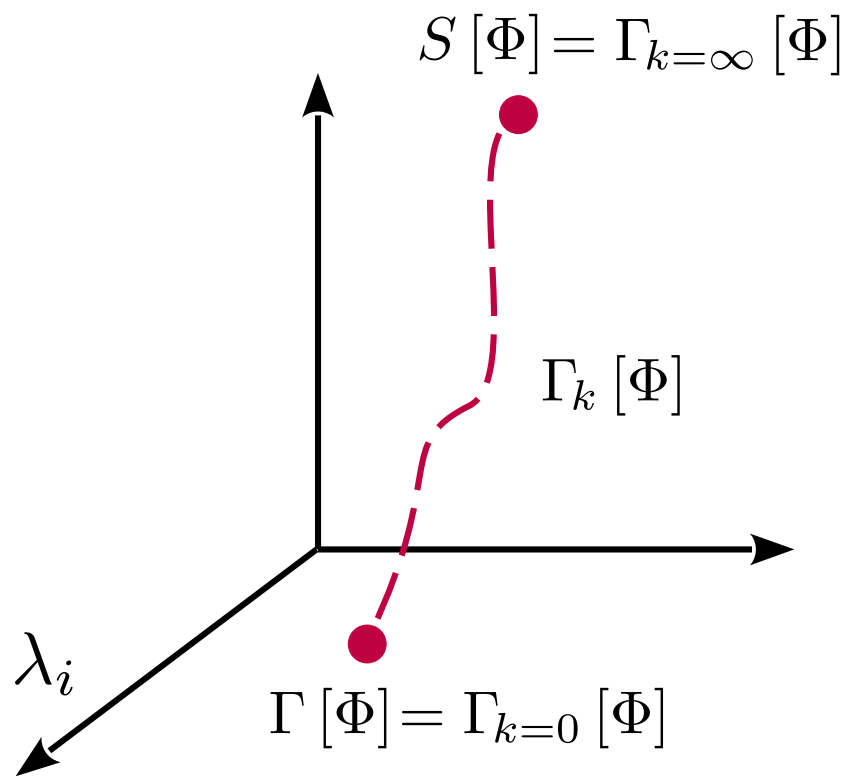


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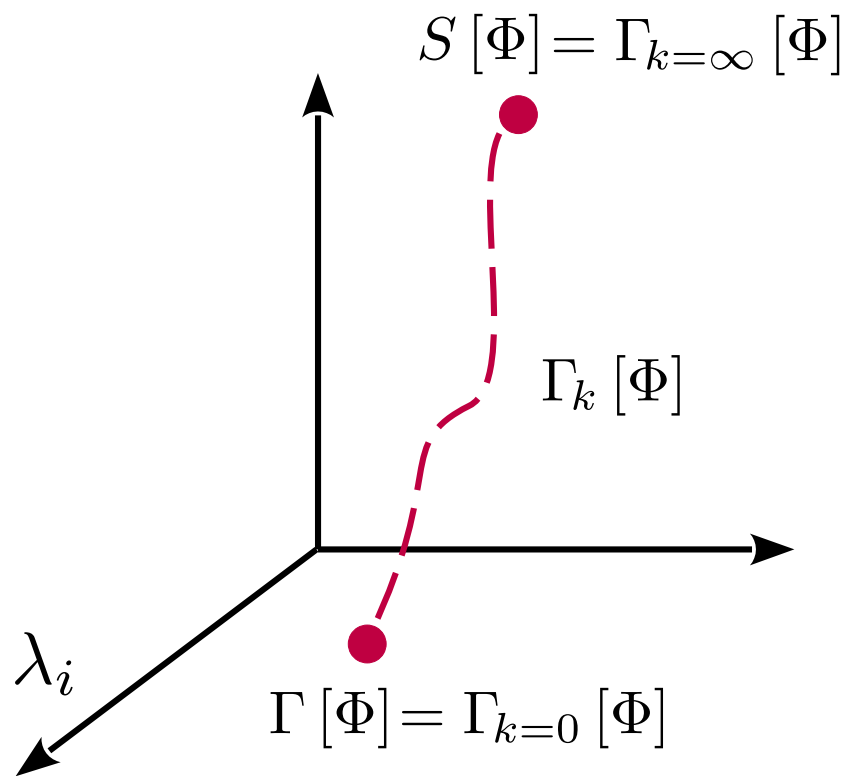


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$$t = \ln \frac{k}{\Lambda_{\text{UV}}}$$

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« Simple » Differential Equation

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Infinite tower of differential equations !

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Infinite tower of differential equations ! In practice, $\Gamma^{(n)} = 0, \quad \forall n > N_{\max}$

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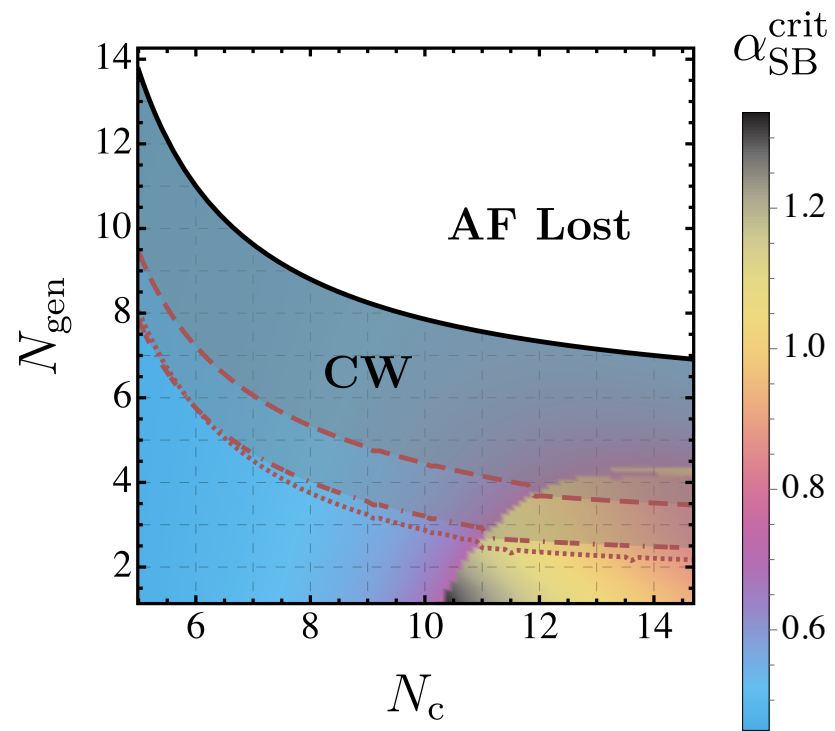
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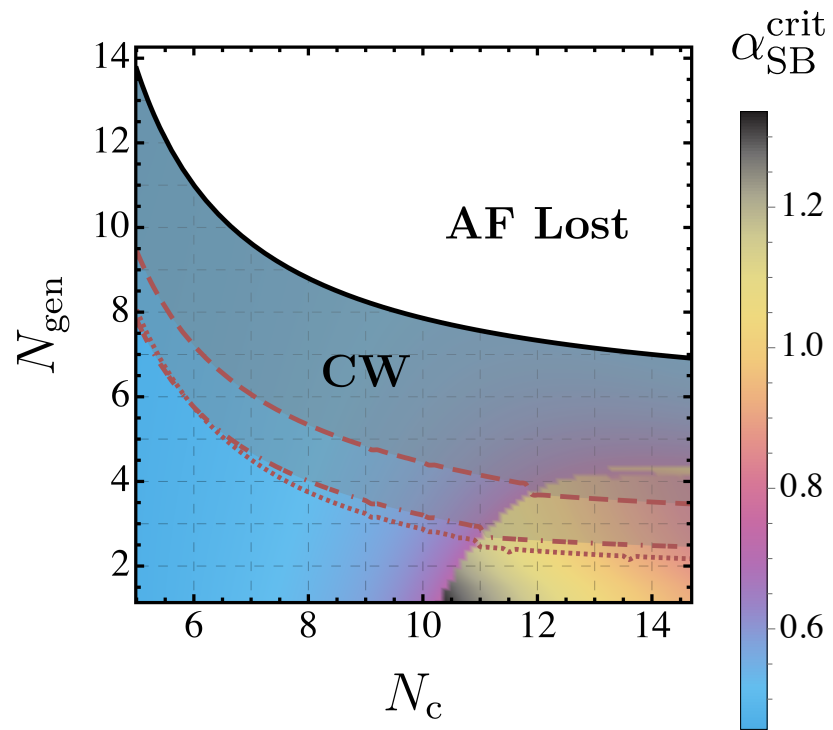
→ Does one of the λ_i diverge ?

Results

Results

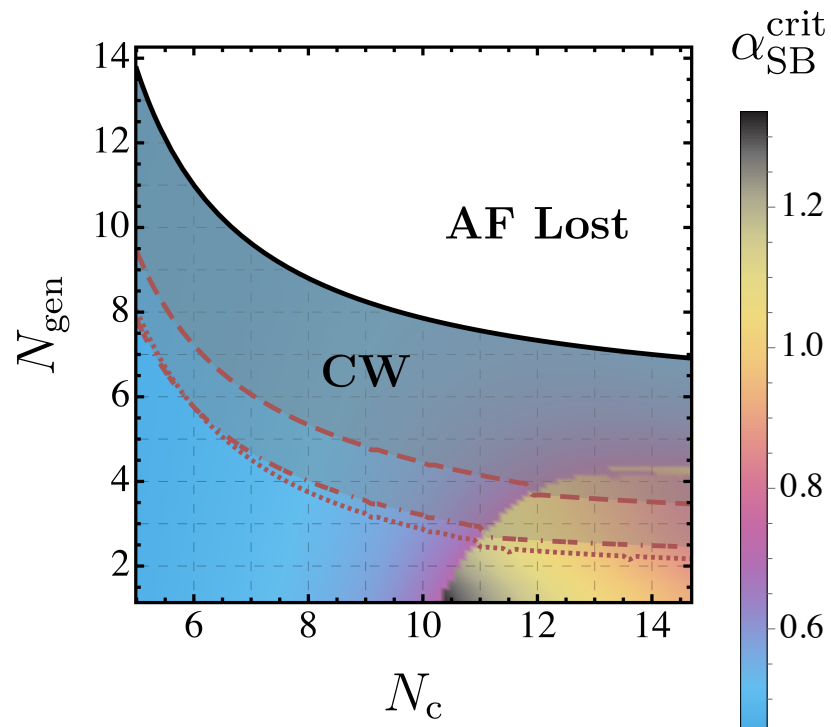


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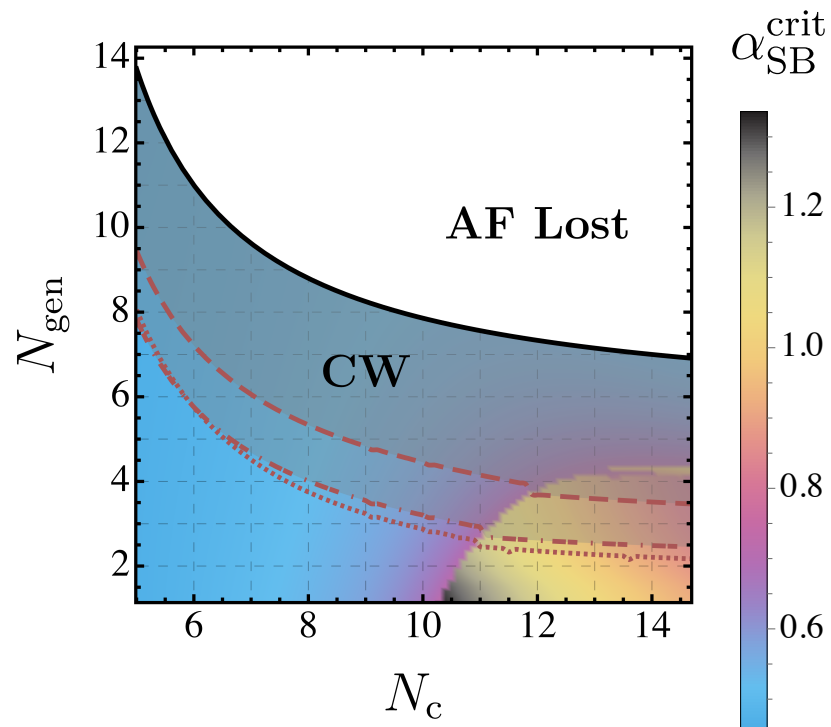
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Results



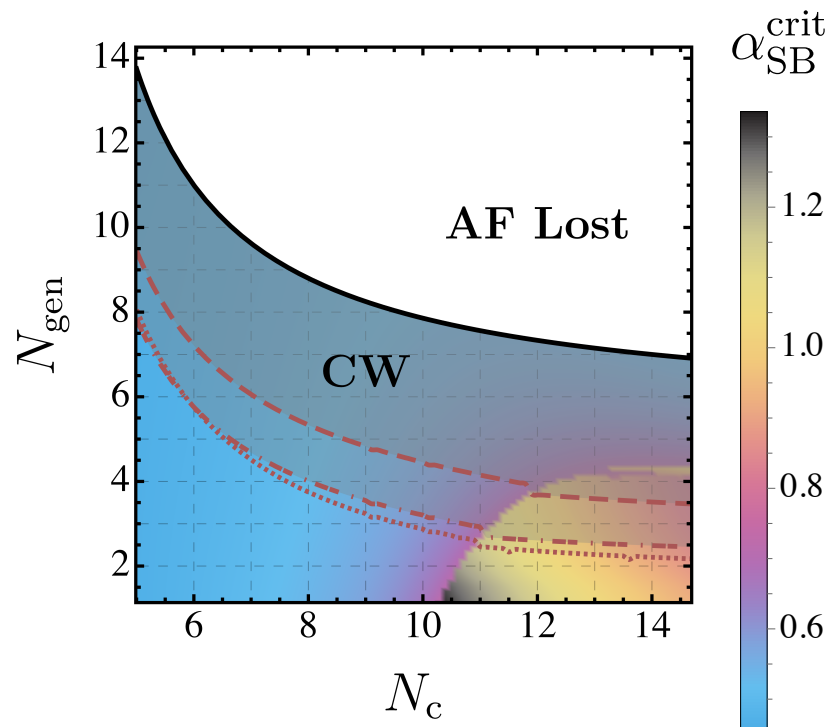
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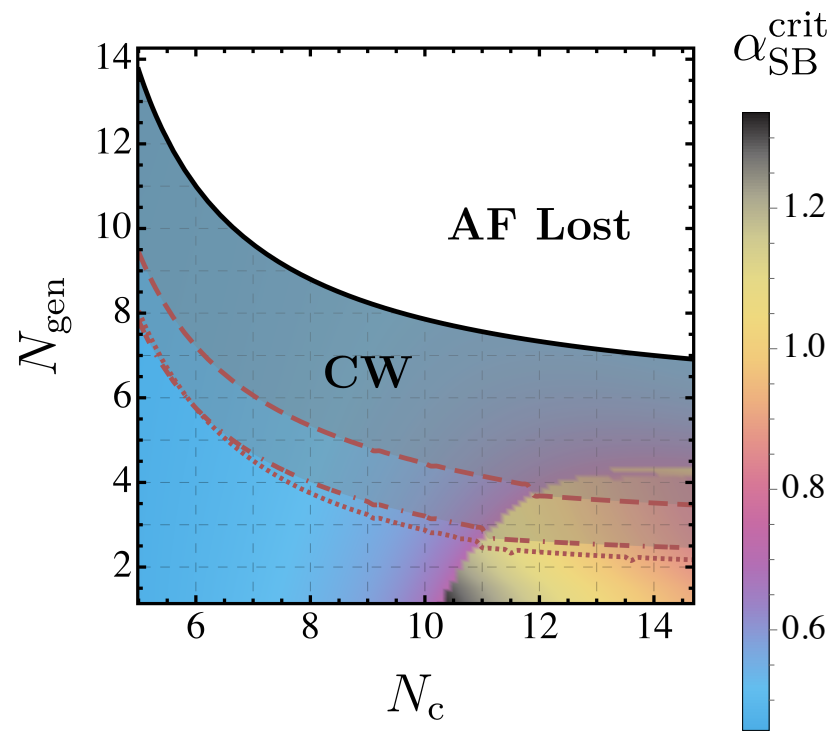
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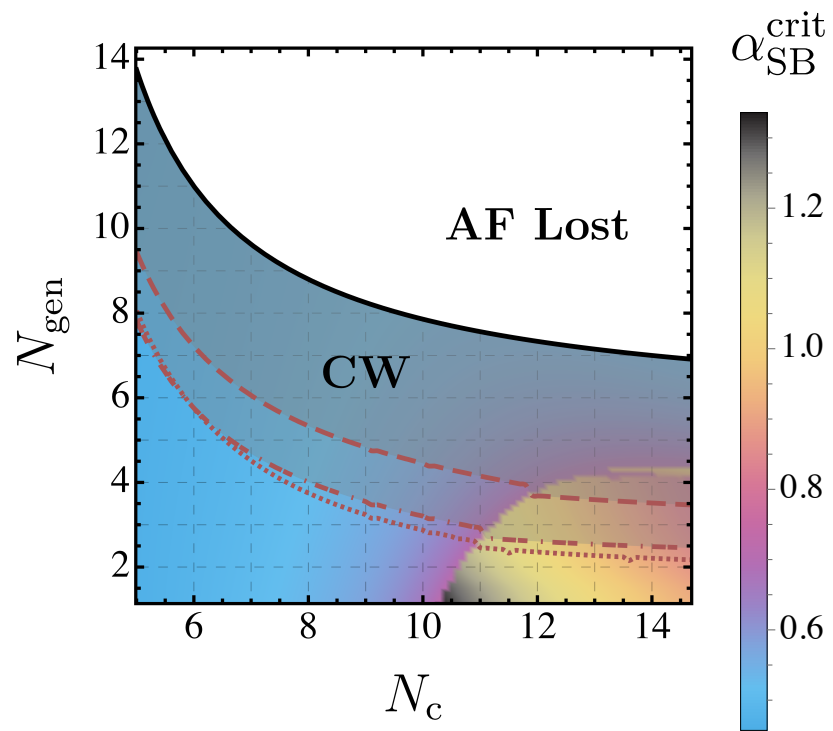


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- We used the 2-, 3- and 4-loop β

Results

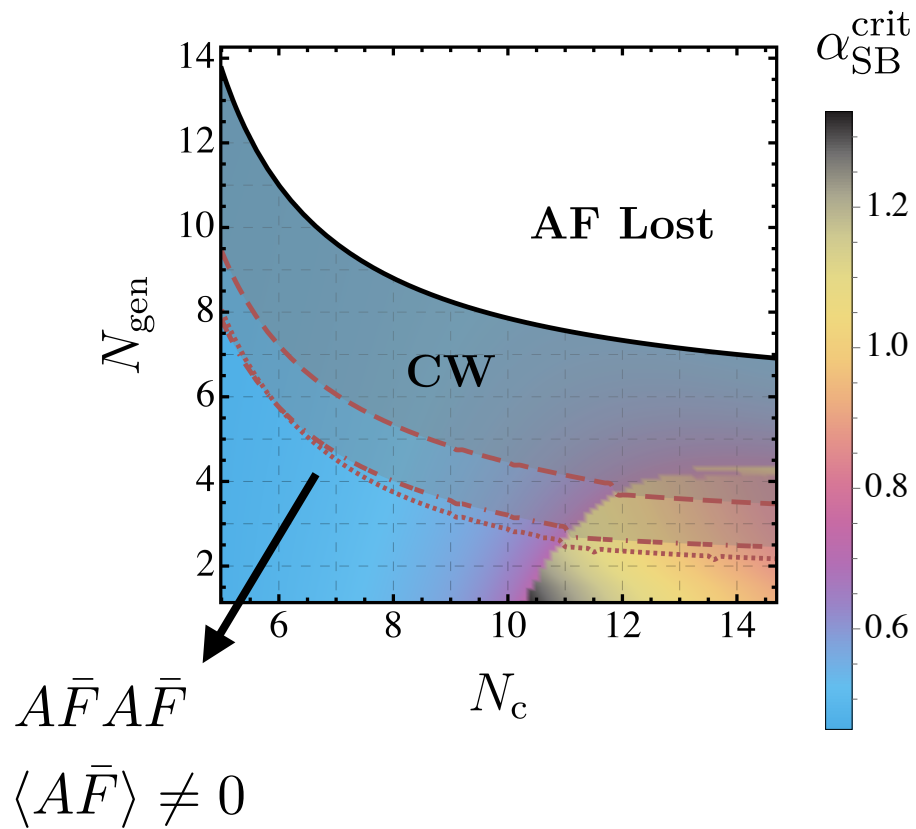


Results



→ IR Conformal !

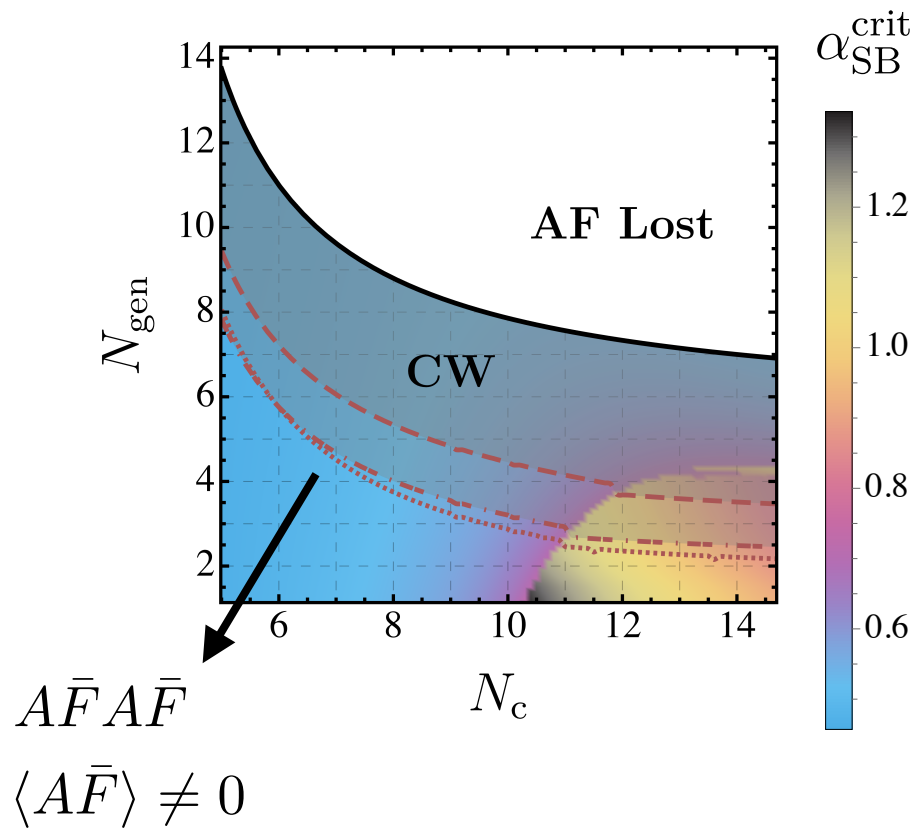
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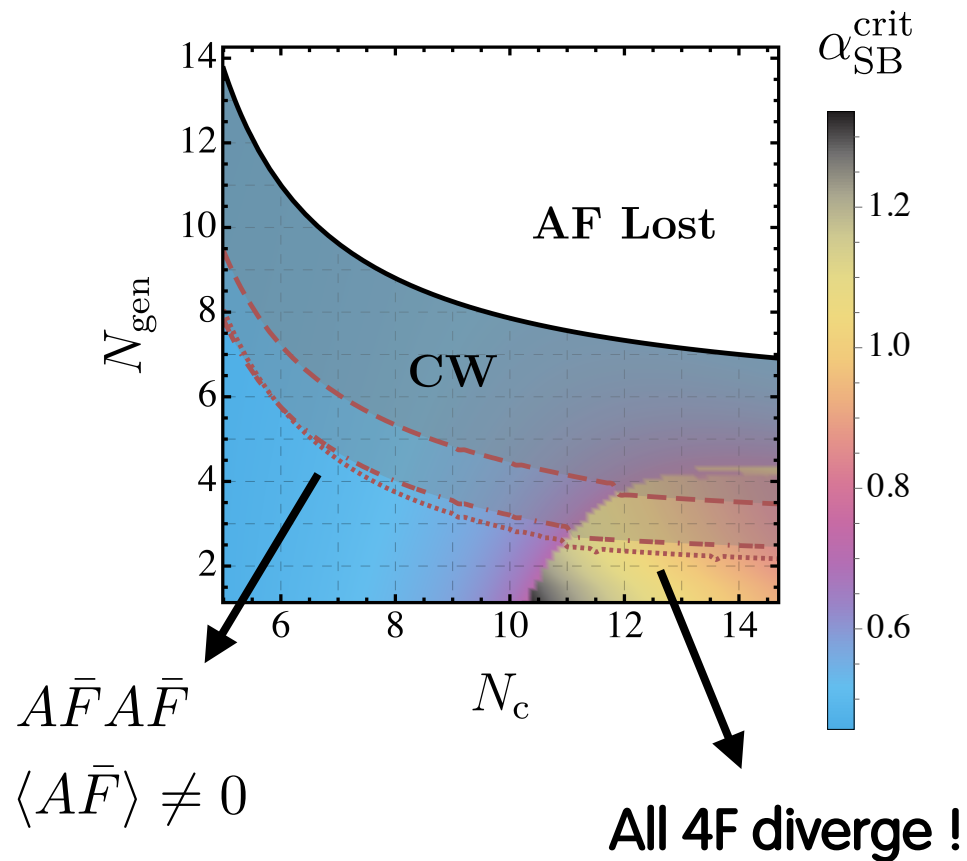
→ Clear signal for χSB

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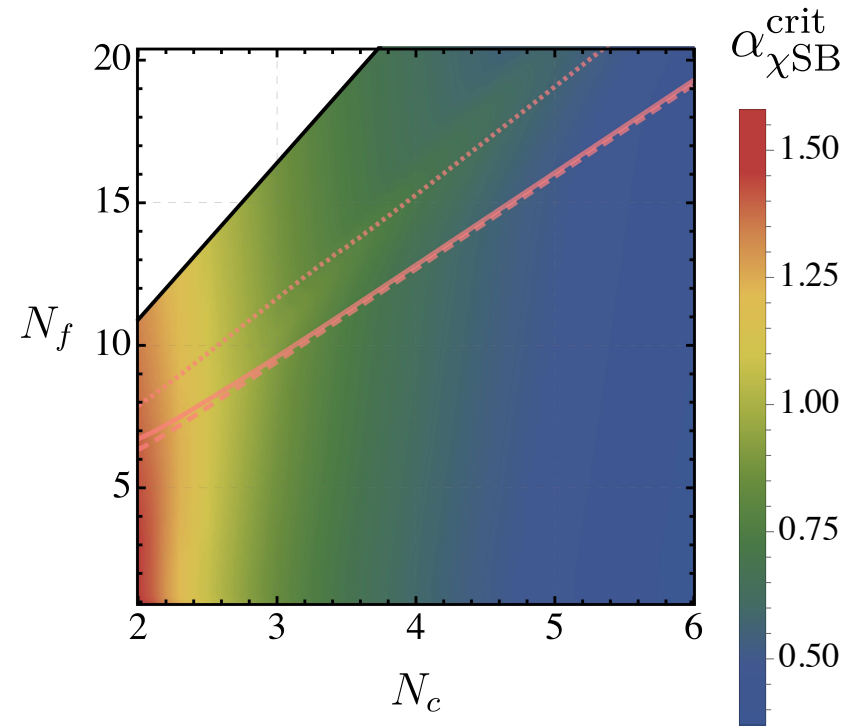
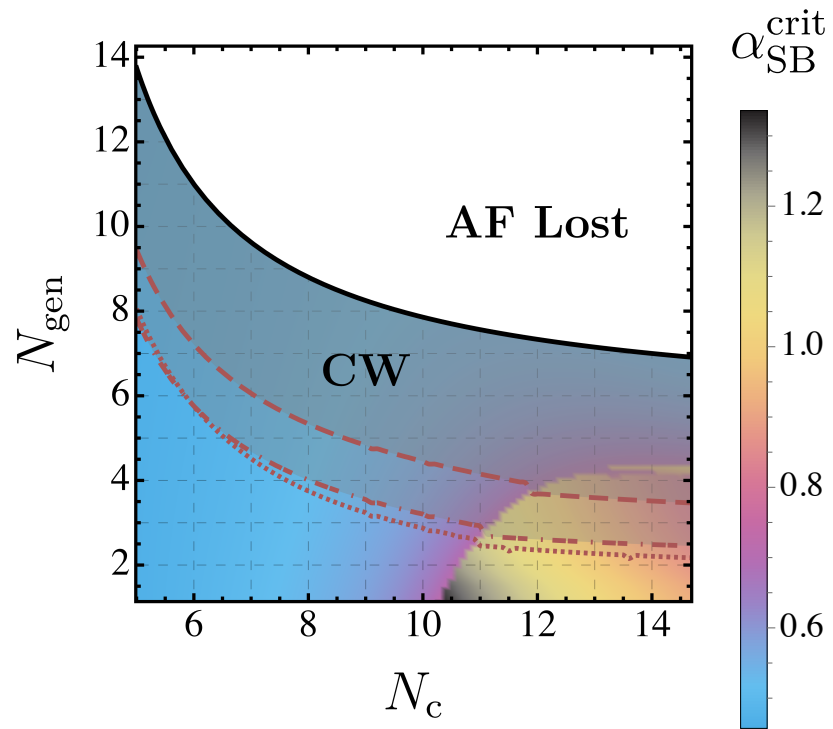
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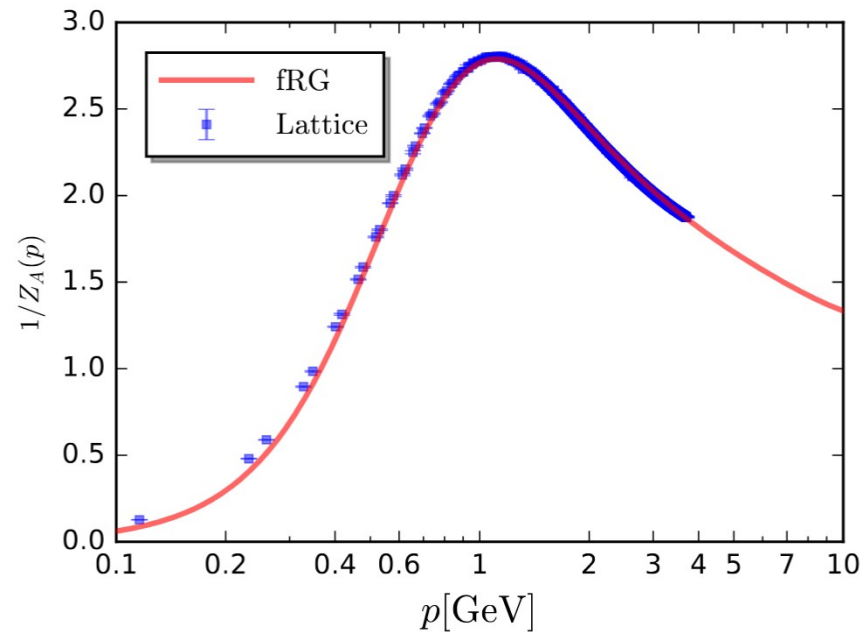
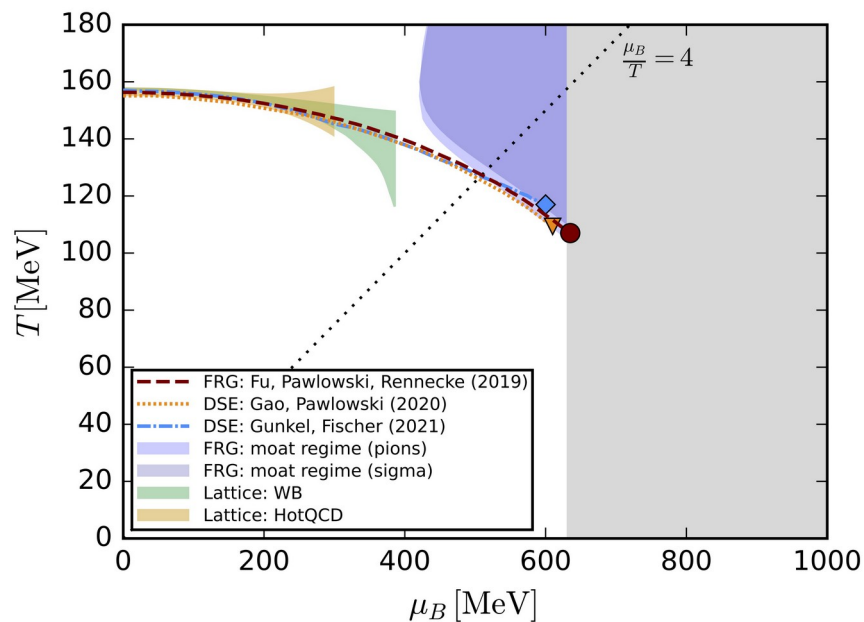
Results



Conclusion

- Strong evidence for conformal phase
- Color-dependent IR
- Deep IR exploration (Confinement, Spectrum, ...)

Power of Functional Methods



Results

$$\mathcal{O}^1 = \left(\bar{\psi}^{f_1 i_1} \gamma^\mu P_L \psi_{f_1 i_1} \right) \left(\bar{\psi}^{f_2 i_2} \gamma^\mu P_L \psi_{f_2 i_2} \right),$$

$$\mathcal{O}^2 = \left(\bar{\psi}^{f_1 i_1} \gamma^\mu P_L \psi_{f_2 i_1} \right) \left(\bar{\psi}^{f_2 i_2} \gamma^\mu P_L \psi_{f_1 i_2} \right),$$

$$\mathcal{O}^3 = \left(\bar{\chi}^{f_1 a_1} \gamma^\mu P_L \chi_{f_1 a_2} \right) \left(\bar{\chi}^{f_2 a_2} \gamma^\mu P_L \chi_{f_2 a_1} \right),$$

$$\mathcal{O}^4 = \left(\bar{\chi}^{f_1 a_1} \gamma^\mu P_L \chi_{f_1 a_1} \right) \left(\bar{\chi}^{f_2 a_2} \gamma^\mu P_L \chi_{f_2 a_2} \right),$$

$$\mathcal{O}^5 = \left(\bar{\chi}^{f_1 a_1} \gamma^\mu P_L T_{a_1}^{a_2} \chi_{f_1 a_2} \right) \left(\bar{\chi}^{f_2 a_3} \gamma^\mu P_L T_{a_3}^{a_4} \chi_{f_2 a_4} \right),$$

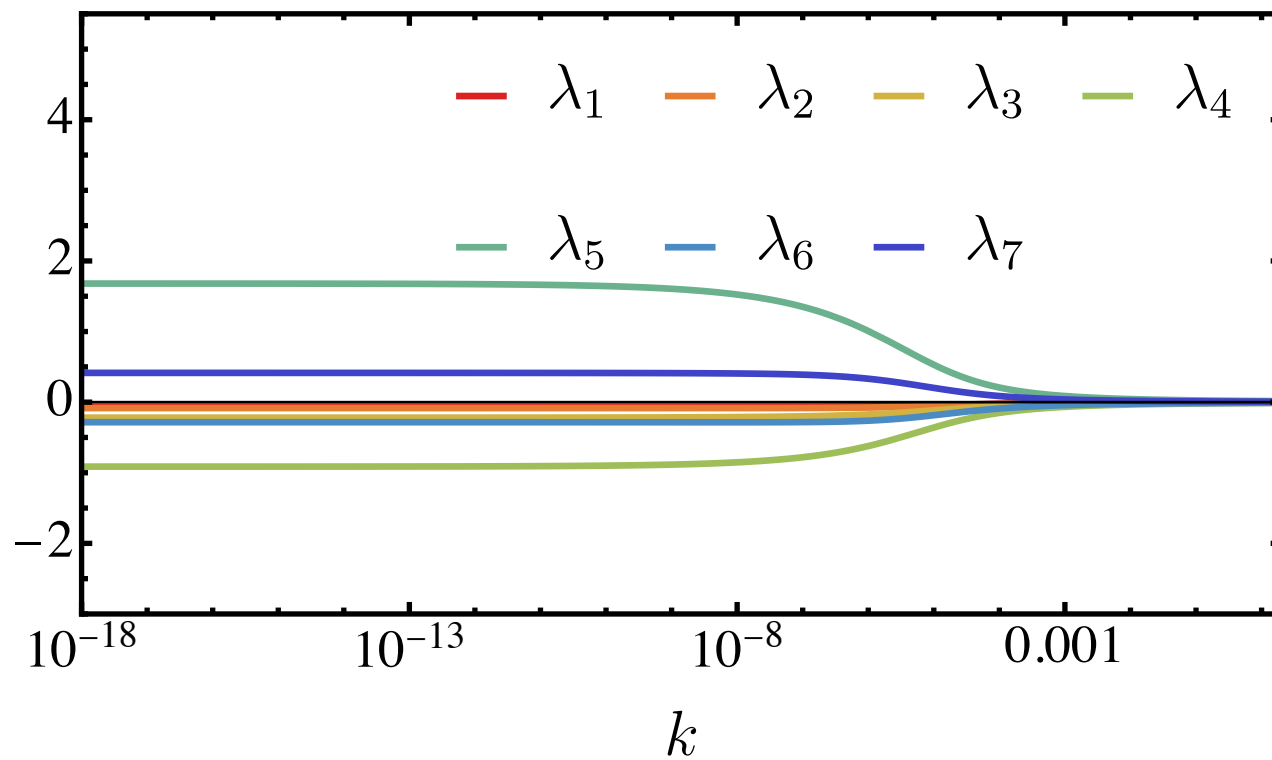
$$\mathcal{O}^6 = \left(\bar{\psi}^{f_1 i_1} \gamma^\mu P_L \psi_{f_1 i_1} \right) \left(\bar{\chi}^{f_2 a_2} \gamma^\mu P_L \chi_{f_2 a_2} \right),$$

$$\mathcal{O}^7 = \left(\bar{\psi}^{f_1 i_1} \gamma^\mu P_L T_{i_1}^{i_2} \psi_{f_1 i_2} \right) \left(\bar{\chi}^{f_2 a_3} \gamma^\mu P_L T_{a_3}^{a_4} \chi_{f_2 a_4} \right).$$

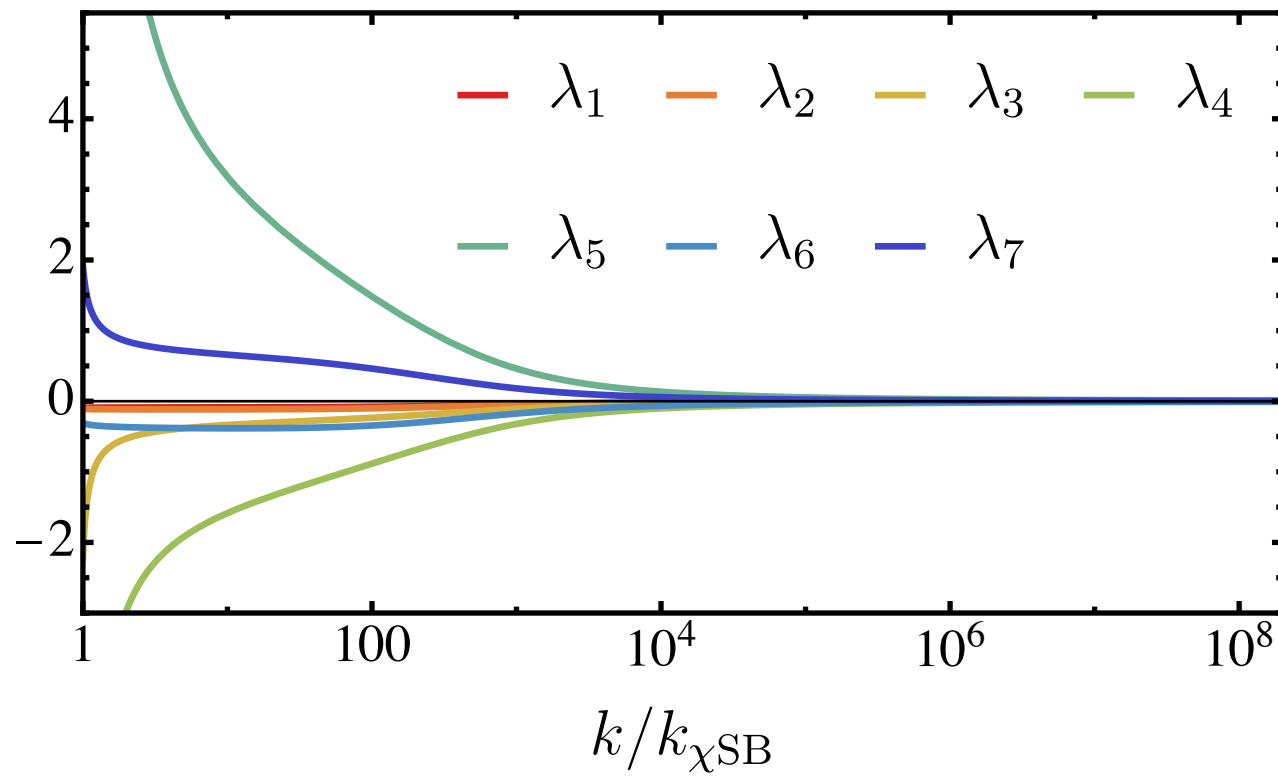
$$\psi \longleftrightarrow \bar{F}$$

$$\chi \longleftrightarrow A$$

Results



Results



How To Diagnose χ SB?

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→ Nambu-Jona-Lasino Model

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$$S = \int \bar{\psi} \not{\partial} \psi - \frac{\lambda}{2} \left((\bar{\psi} \psi)^2 - (\bar{\psi} \gamma^5 \psi)^2 \right)$$

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→
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$$\rightarrow \sigma \rightarrow \sigma + y \frac{\bar{\psi}\psi}{\sqrt{2}m^2} \quad \pi \rightarrow \pi + i y \frac{\bar{\psi}\gamma^5\psi}{\sqrt{2}m^2}$$

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$$\text{with } \lambda \equiv \frac{y^2}{2m^2}$$

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with $\lambda \equiv \frac{y^2}{2m^2}$

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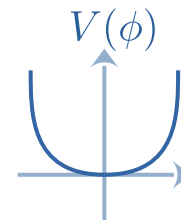
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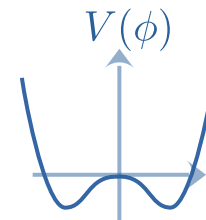
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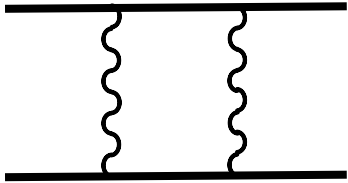
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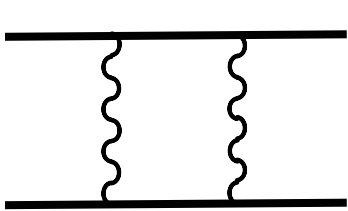
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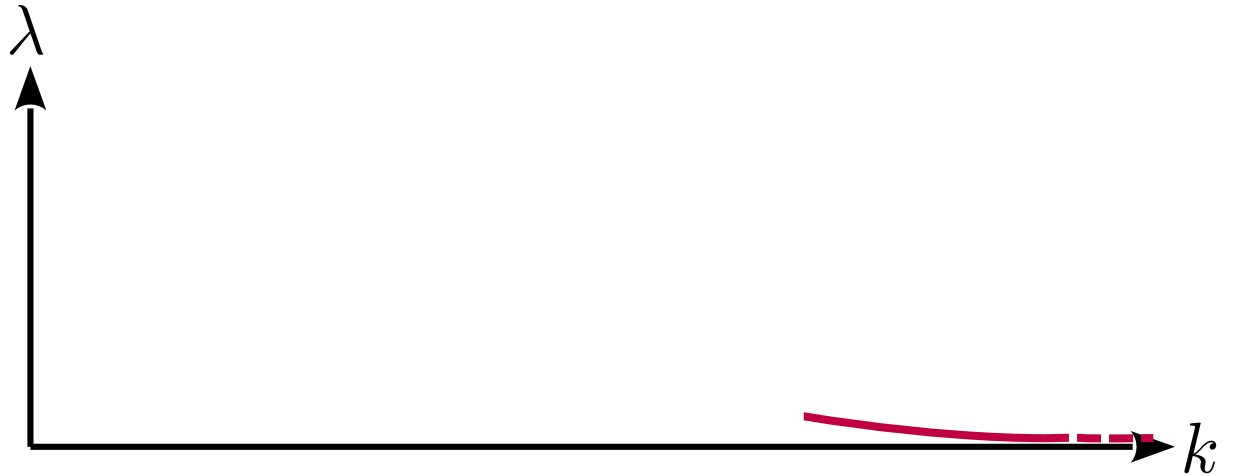
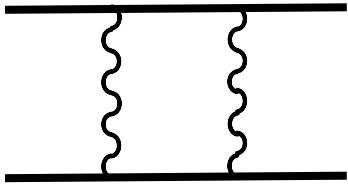
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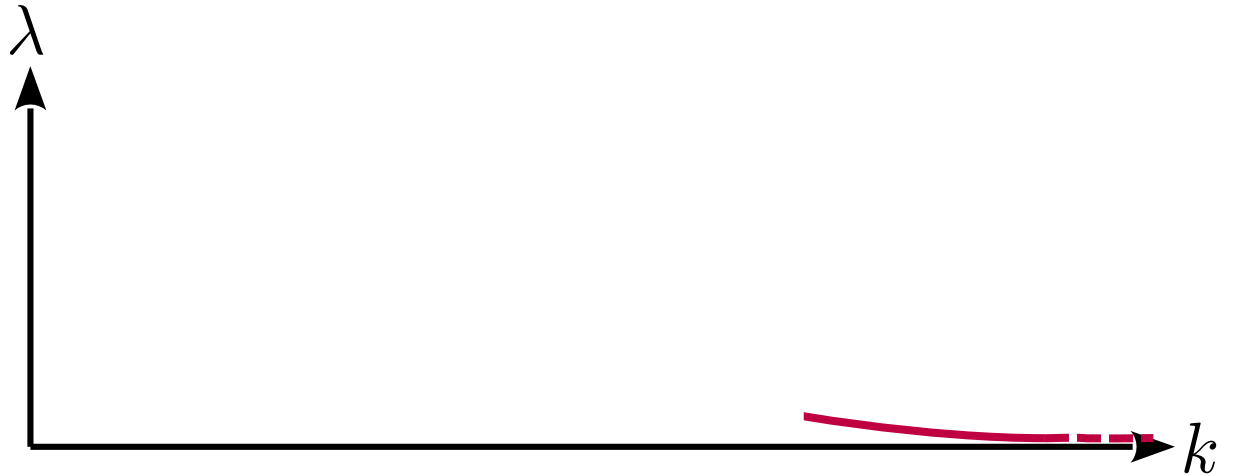


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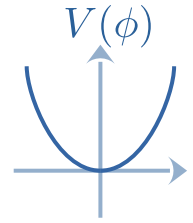
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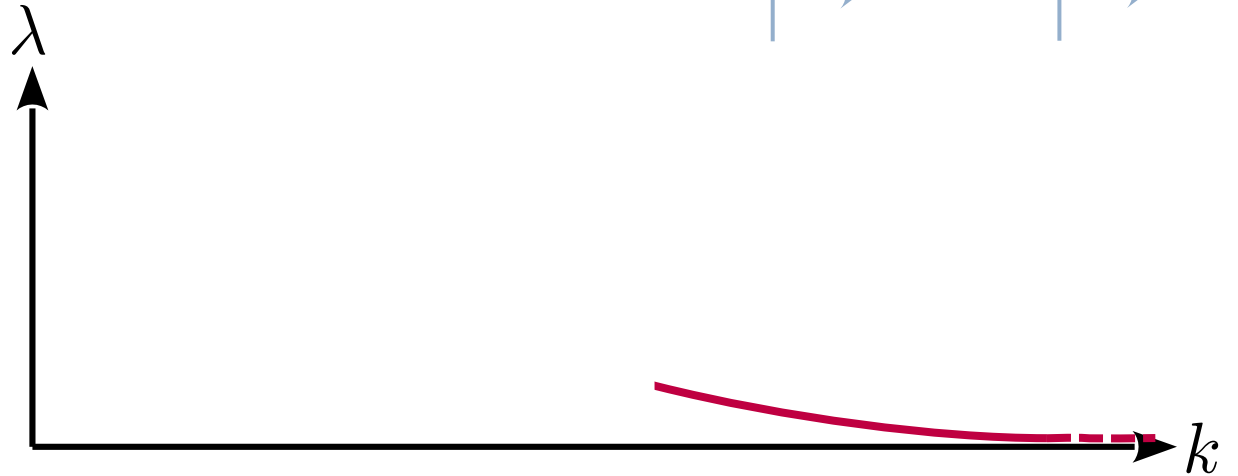
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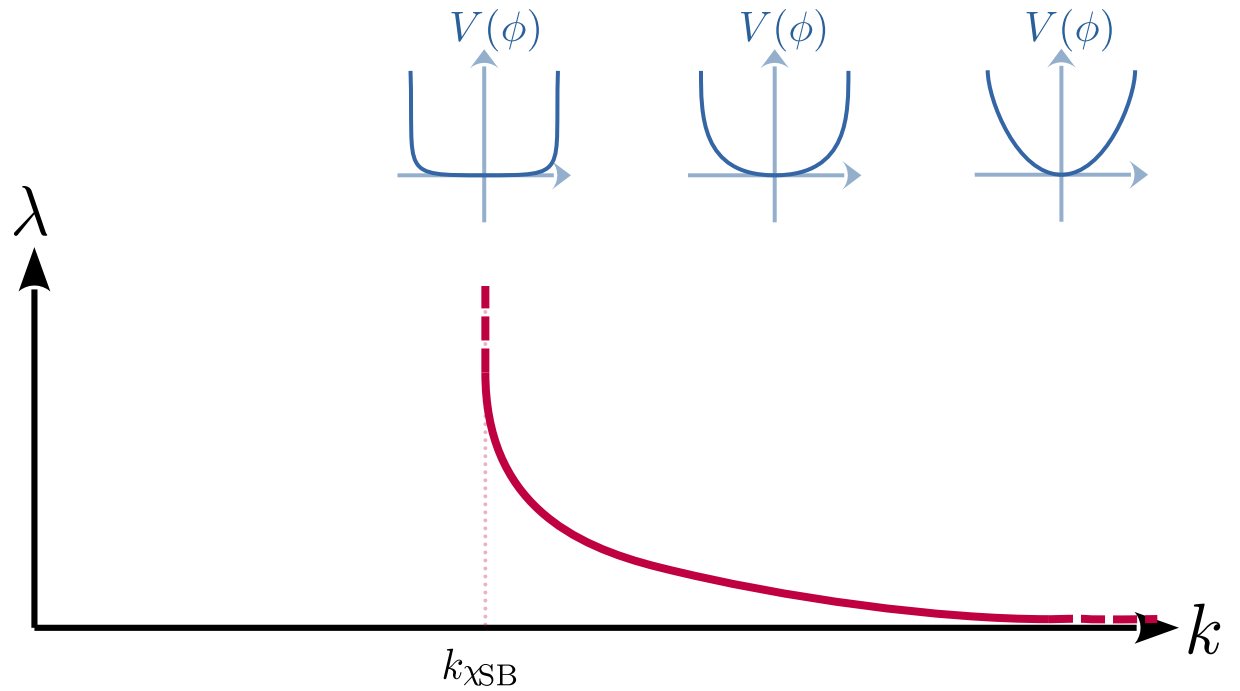
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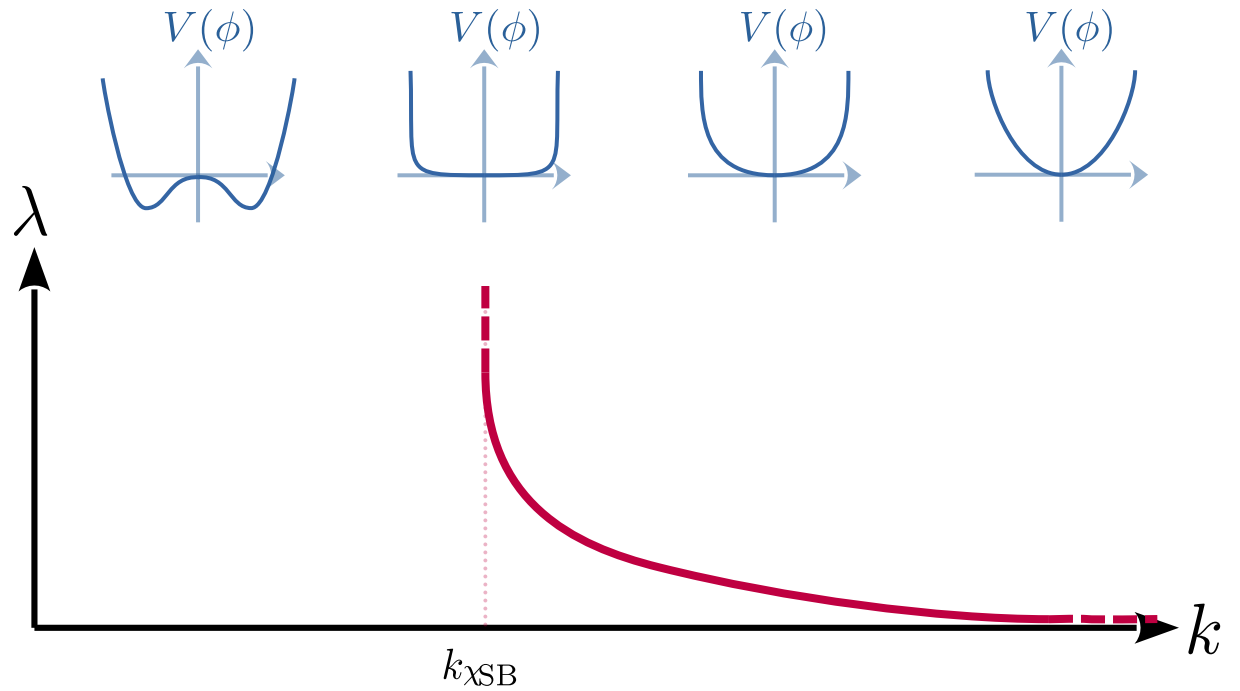
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How To Diagnose χ SB?

$$\text{Diagram 1} = - \text{Diagram 2}$$

Diagram 1: A circle labeled $\mathcal{W}^{(3)}$ with three external lines (one horizontal, two diagonal).

Diagram 2: A circle labeled $\Gamma^{(3)}$ with three external lines, each passing through a smaller circle labeled $\frac{1}{\Gamma^{(2)}}$.

$$\text{Diagram 3} = \text{Diagram 4}$$

Diagram 3: A circle labeled $\mathcal{W}^{(2)}$ with two external lines (one horizontal, one diagonal).

Diagram 4: A circle labeled $\frac{1}{\Gamma^{(2)}}$ with two external lines (one horizontal, one diagonal).

$$\text{Diagram 5} = 3 \times \text{Diagram 6} - \text{Diagram 7}$$

Diagram 5: A circle labeled $\mathcal{W}^{(4)}$ with four external lines (two horizontal, two diagonal).

Diagram 6: A chain of two $\Gamma^{(3)}$ circles connected by a $\frac{1}{\Gamma^{(2)}}$ circle, with each $\Gamma^{(3)}$ having two external lines passing through $\frac{1}{\Gamma^{(2)}}$ circles.

Diagram 7: A circle labeled $\Gamma^{(4)}$ with four external lines, each passing through a smaller circle labeled $\frac{1}{\Gamma^{(2)}}$.

Chiral Dynamic

- Vacuum $\Rightarrow G/H$

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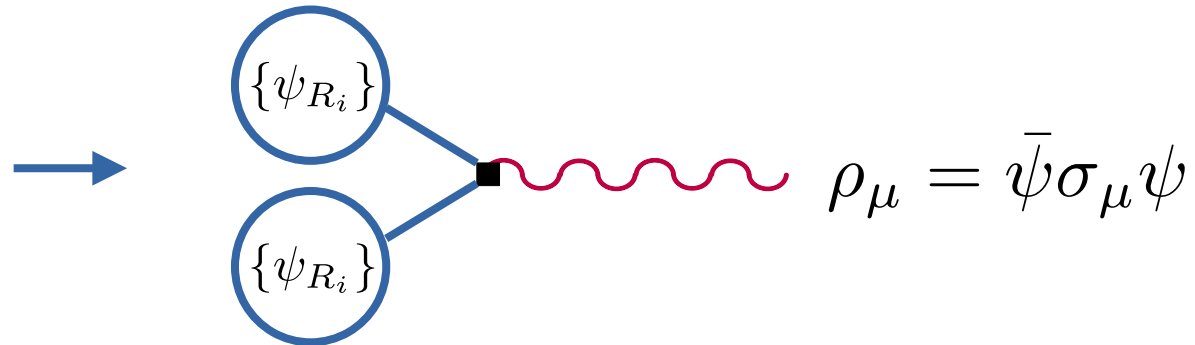
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$$\longrightarrow \mathcal{B} = \bigcirc \{\psi_{R_i}\} \quad m_{\mathcal{B}} = 0$$

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