An Open System Approach to Gravity

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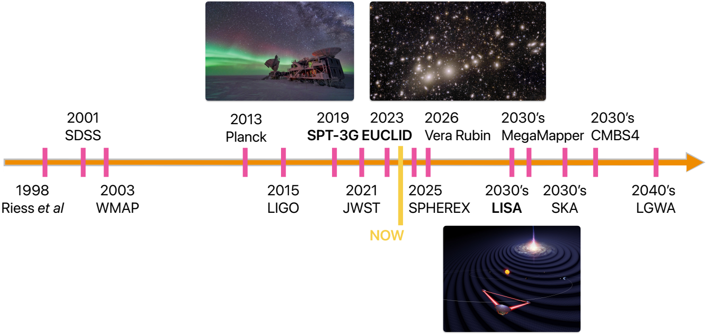
Aspirations

Learn about **fundamental physics**:

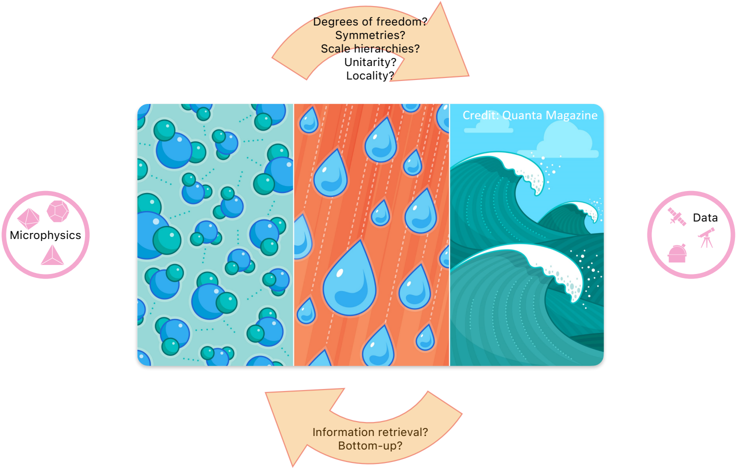
- New degrees of freedom;
- GR at high energy;
- QFT in curved spacetimes;
- Quantum gravity; $\cdots$

A defining moment for **cosmology**:

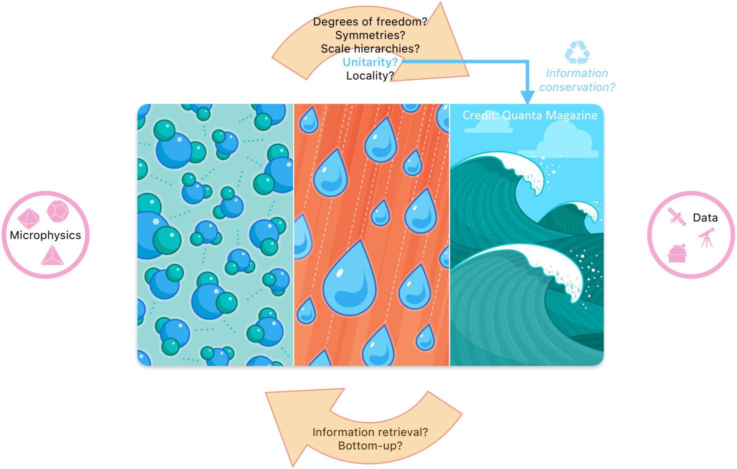


Need to organise dialogue between theory and observations

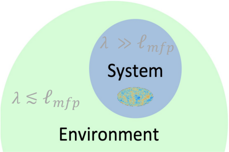
Effective Field Theories



Effective Field Theories



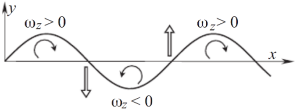
When (non)-unitarity matters



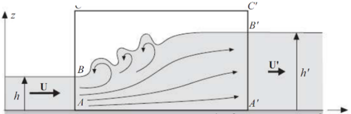
System interacts with environment:
$\Rightarrow$ non-unitary evolution

Dissipation & noise = energy & information losses

Perfect fluid: Wilsonian EFT



Imperfect fluid: non-equilibrium EFT



Credit: [Guyon *et al*, 2012]

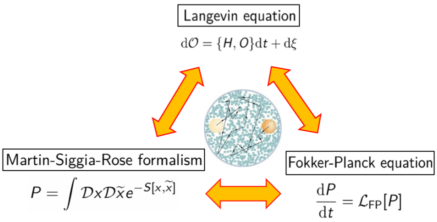
What about cosmology?

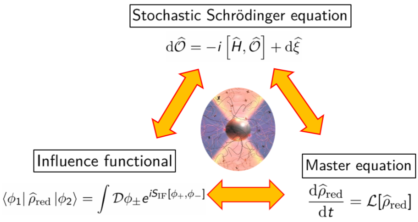
- **Observable** universe $\neq$ whole; 
- Continuously **evolving**; 
- Always a **medium**; 
- $\exists$ **fluxes** between sectors. 

$\Rightarrow$ effective dynamics often non-unitary [T.C., J. Grain, V. Vennin, 2212.09486]

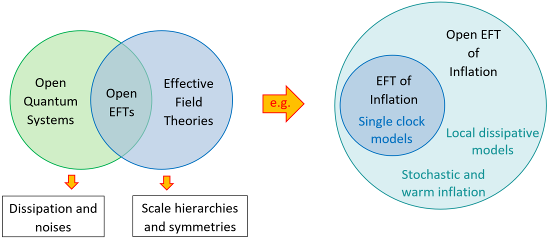
Goal: EFTs accounting for dissipation & noise in cosmology

Combining EFTs and Open Quantum Systems





Extend the **embedding power** of EFTs:



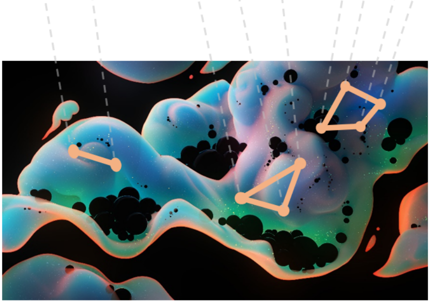
Outline

1. Open inflation
2. Open gravity
3. Applications: Primordial GWs & Dissipative DM

Outline

Cosmological correlators

$$\left\langle \prod_{i=1}^{n} \delta(\boldsymbol{k}_i) \right\rangle$$



$$\left\langle \prod_{i=1}^{n} \hat{\zeta}(\boldsymbol{k}_i, \eta_0) \right\rangle$$

Schwinger-Keldysh formalism

Consider some **observable**

$$\widehat{Q} \equiv \widehat{\zeta}(\boldsymbol{x}_1)\widehat{\zeta}(\boldsymbol{x}_2)\cdots\widehat{\zeta}(\boldsymbol{x}_n)$$

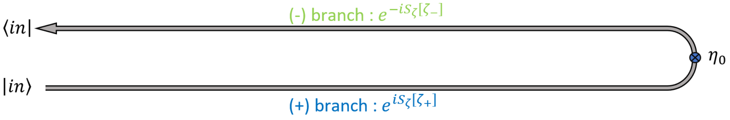
and some unitary **evolution operator** $\widehat{U}(\eta_0, \eta_\mathrm{i})$ so that

$$|\Psi(\eta_0)\rangle = \widehat{U}(\eta_0, \eta_\mathrm{i})\,|\mathrm{BD}\rangle \quad \text{with} \quad \langle\zeta|\,\widehat{U}(\eta_0, \eta_\mathrm{i})\,|\zeta_1\rangle = \int_{\zeta_1}^{\zeta} \mathcal{D}\left[\Phi\right] e^{iS[\Phi]}.$$

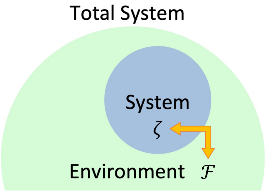
If $S\left[\Phi\right] = S_\zeta\left[\zeta\right]$, see [Donath & Pajer, 2402.05999]:

$$\left\langle \widehat{Q}(\eta_0)\right\rangle = \int \mathrm{d}\zeta\mathrm{d}\zeta_1\mathrm{d}\zeta_2\left[\zeta(\boldsymbol{x}_1)\cdots\zeta(\boldsymbol{x}_n)\right]\left[\langle\zeta|\,\widehat{U}(\eta_0,\eta_\mathrm{i})\,|\zeta_1\rangle\right]\left[\langle\zeta_1|\mathrm{BD}\rangle\langle\mathrm{BD}|\zeta_2\rangle\right]\left[\langle\zeta_2|\,\widehat{U}^\dagger(\eta_0,\eta_\mathrm{i})\,|\zeta\rangle\right]$$

$$= \int \mathrm{d}\zeta\mathrm{d}\zeta_1\mathrm{d}\zeta_2\left[\zeta(\boldsymbol{x}_1)\cdots\zeta(\boldsymbol{x}_n)\right]\int_{\zeta_1}^{\zeta}\mathcal{D}\left[\zeta_+\right]\int_{\zeta_2}^{\zeta}\mathcal{D}[\zeta_-]e^{iS_\zeta[\zeta_+]-iS_\zeta[\zeta_-]}\langle\zeta_1|\mathrm{BD}\rangle\langle\mathrm{BD}|\zeta_2\rangle$$



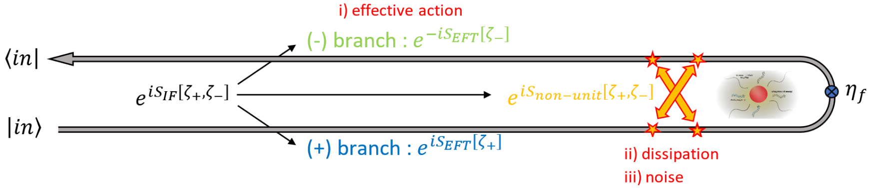
Integrating out an environment



- $S\left[\Phi\right] = S_{\zeta}\left[\zeta\right] + S_{\mathcal{F}}\left[\mathcal{F}\right] + S_{\mathrm{int}}\left[\zeta;\mathcal{F}\right]$ with $\mathcal{F}$ a **hidden sector**.
- *Goal:* tracing out $\mathcal{F}$, the environment being **unobservable**.

Effects of the environment captured by the **Influence Functional** (IF):

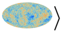
$$\left\langle \widehat{Q}(\eta) \right\rangle = \int \mathrm{d}\zeta \mathrm{d}\zeta_1 \mathrm{d}\zeta_2 \left[\zeta(\boldsymbol{x}_1)\cdots\zeta(\boldsymbol{x}_n)\right] \int_{\zeta_1}^{\zeta} \mathcal{D}\left[\zeta_+\right] \int_{\zeta_2}^{\zeta} \mathcal{D}[\zeta_-] e^{iS_\zeta[\zeta_+] - iS_\zeta[\zeta_-] + i\boldsymbol{S}_{\mathrm{IF}}[\boldsymbol{\zeta}_+;\boldsymbol{\zeta}_-]}$$



What are the rules obeyed by $S_{\mathrm{IF}}\left[\zeta_+;\zeta_-\right]$*?*

The Open EFT of Inflation [S.A. Agüí Salcedo, T.C. & E. Pajer, 2404.15416]

Early universe: one scalar degree of freedom $\pi(\boldsymbol{x}, t)$:

Observed ⟨  ⟩ $\Leftrightarrow$ $$\langle \hat{\pi}^n \rangle (t) = \int \mathrm{d}\pi \pi^n \mathrm{Prob}_\pi(t)\,.$$

- EFT of Inflation [Cheung *et al.*, 2008]: most generic wavefunction

$$\mathrm{Prob}_\pi(t) = |\Psi_\pi(t)|^2 = \left| \int_\Omega^\pi \mathcal{D}\pi \; e^{iS_{\mathrm{eff}}[\pi]} \right|^2$$

- **Dissipation** & **noise**: most generic density matrix

$$\mathrm{Prob}_\pi(t) = \rho_{\pi\pi}(t) = \int_\Omega^\pi \mathcal{D}\pi_+ \int_\Omega^\pi \mathcal{D}\pi_- \; e^{iS_{\mathrm{eff}}[\pi_+, \pi_-]}$$

Physical principles restrict $S_{\mathrm{eff}}[\pi_+, \pi_-]$:

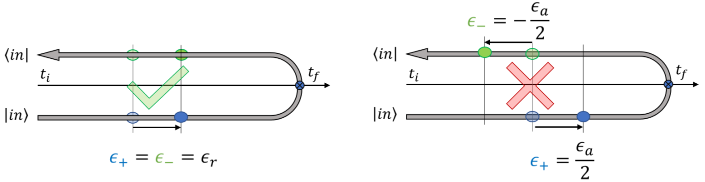
1. Unitarity: {Sys. + Env.} closed ⇒ non-equilibrium constraints; [Liu & Glorioso, 2018]
2. Symmetries: in-in coset construction; [Akyuz, Goon & Penco, 2023]
3. Locality: truncatable power counting scheme.

Dissipative shift symmetric scalar [Hongo *et al.*, 2018], [Akyuz, Goon & Penco, 2023]

$S_{\rm unit}\left[\pi_\pm\right]$ invariant under shift$_+$ $\times$ shift$_-$:

$$\pi_\pm(t) \to \pi'_\pm(t) = \pi_\pm(t+\epsilon_\pm) + \epsilon_\pm,$$

but $S_{\rm non-unit}\left[\pi_+;\pi_-\right]$ **is not**: **SSB** shift$_+$ $\times$ shift$_-$ $\to$ shift$_r$:



Retarded $\pi_r = (\pi_+ + \pi_-)/2$ and **advanced** $\pi_a = \pi_+ - \pi_-$ fields:

$$\pi_r(t) \to \pi'_r(t) = \pi_r(t+\epsilon_r) + \epsilon_r, \qquad \pi_a(t) \to \pi'_a(t) = \pi_a(t+\epsilon_r).$$

Building blocks:

$$\pi_a, \quad t+\pi_r, \quad \partial_\mu\pi_a, \quad \partial_\mu(t+\pi_r).$$

Effective functional

- *Quadratic order:* 1 $\rightarrow$ 5 EFT param (1 tadpole constraint):

$$S_{\rm eff}^{(2)} = \int \mathrm{d}^4x\sqrt{-g}\Big\{\overbrace{\dot{\pi}_r\dot{\pi}_a - c_s^2\partial_i\pi_r\partial^i\pi_a}^{\text{Kinetic term}}$$

$$\underbrace{-\gamma\dot{\pi}_r\pi_a}_{\text{Dissipation}} + i\,\underbrace{\left[\beta_1\pi_a^2 - (\beta_2-\beta_4)\,\dot{\pi}_a^2 + \beta_2\left(\partial_i\pi_a\right)^2\right]}_{\text{Noise}}\Big\}$$

- *Cubic order:* 1 $\rightarrow$ 13 EFT param: EFToI famous for relating operators at different orders because of **non-linearly realised boosts** [López Nacir *et al.*, 2011].

$$\begin{aligned} \text{EFToI}: &\quad \mathcal{L} \supset \left(c_s^2 - 1\right)\left[-2\dot{\pi}_r + \left(\partial_\mu\pi_r\right)^2\right]\dot{\pi}_a \\ \text{Dissipation}: &\quad \mathcal{L} \supset \gamma\left[-2\dot{\pi}_r + \left(\partial_\mu\pi_r\right)^2\right]\pi_a \\ \text{Noise}: &\quad \mathcal{L} \supset i\beta_4\left(-\dot{\pi}_a + \partial_\mu\pi_r\partial^\mu\pi_a\right)^2 \end{aligned}$$

Recover and extend EFToI construction.

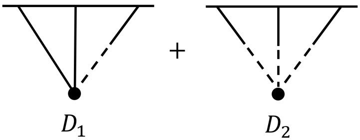
Standard observables

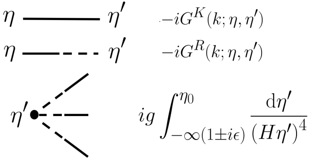
Symmetries ensure existence of nearly scale invariant power spectrum

$$\langle \zeta_{\boldsymbol{k}} \zeta_{\boldsymbol{k}'} \rangle = \frac{H^2}{f_\pi^4} \langle \pi^c_{\boldsymbol{k}} \pi^c_{\boldsymbol{k}'} \rangle \equiv (2\pi)^3 \delta(\boldsymbol{k} + \boldsymbol{k}) \frac{2\pi^2}{k^3} \Delta^2_\zeta(k).$$

$\Rightarrow \Delta^2_\zeta = 10^{-9}$ obtained by imposing hierarchies of scales.

Bispectrum computed in **perturbation theory** using standard in-in rules.

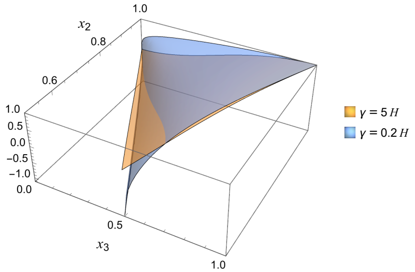




$$\langle \zeta_{\boldsymbol{k}_1} \zeta_{\boldsymbol{k}_2} \zeta_{\boldsymbol{k}_3} \rangle = -\frac{H^3}{f_\pi^6} \langle \pi^c_{\boldsymbol{k}_1} \pi^c_{\boldsymbol{k}_2} \pi^c_{\boldsymbol{k}_3} \rangle \equiv (2\pi)^3 \delta(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3) B(k_1, k_2, k_3).$$

$$S(x_2, x_3) \equiv (x_2 x_3)^2 \frac{B(k_1, x_2 k_1, x_3 k_1)}{B(k_1, k_1, k_1)}, \qquad f_{\mathrm{NL}}(k_1, k_2, k_3) \equiv \frac{5}{6} \frac{B(k_1, k_2, k_3)}{P(k_1)P(k_2) + 2 \text{ perms.}}.$$

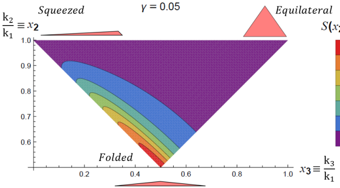
Bispectrum shapes

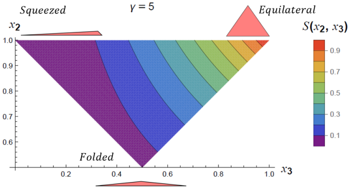


Main features:

- $\gamma \gg H$: equilateral;
- $\gamma \ll H$: folded;
- Consistency relations;
- Regularized divergence.

Consistent with **flat-space/sub-Hubble** analytic results:



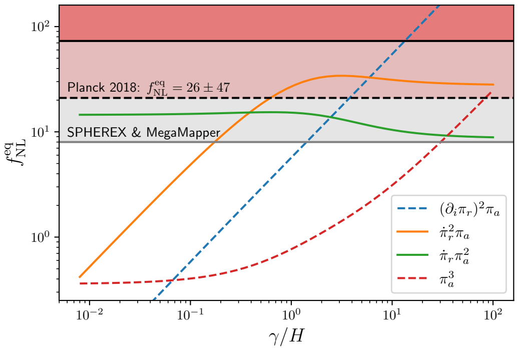


Matching and $f_{\rm NL}$ with [Creminelli *et al.*, 2305.07695]

UV completion: inflaton ϕ + massive scalar field χ with softly-broken $U(1)$:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm Pl}^2 R - \frac{1}{2}(\partial\phi)^2 - V(\phi) - |\partial\chi|^2 + M^2|\chi|^2 - \frac{\partial_\mu\phi}{f}\left(\chi\partial^\mu\chi^* - \chi^*\partial^\mu\chi\right) - \frac{1}{2}m^2\left(\chi^2 + \chi^{*2}\right)\right].$$

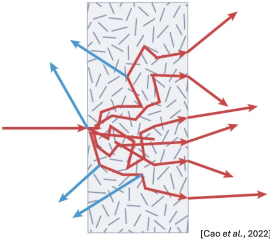
$\Rightarrow$ narrow **instability band** in sub-Hubble regime: *local* particle production.



Outline

Open gauge theories

[S.A. Agüí Salcedo, T.C. & E. Pajer, 2412.12299]: *Dissipative theory* for of light in a medium.



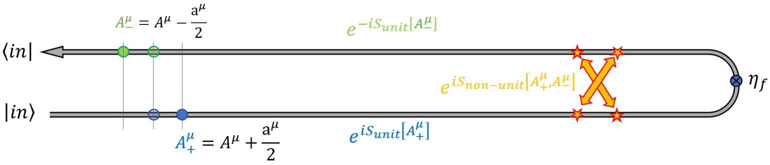
[Cao *et al.*, 2022]

- *dielectric material* (insulator):
  - 2 transverse d.o.f.
  - gauge invariance:

$$A^\mu_\pm \to A^\mu_\pm + \partial^\mu \epsilon_\pm$$

- *relax IR unitarity* = includes dissipation & noise.

Keldysh basis: retarded $A^\mu = \left(A^\mu_+ + A^\mu_-\right)/2$; advanced $a^\mu = A^\mu_+ - A^\mu_-$.



Retarded & advanced gauge transformation

Retarded gauge transformation $\epsilon_+ = \epsilon_- = \epsilon_r$:

$$A^\mu \to A^\mu + \partial^\mu \epsilon_r \,, \qquad\qquad a^\mu \to a^\mu \,.$$

Advanced gauge transformation $\epsilon_+ = -\epsilon_- = \epsilon_a$:

$$A^\mu \to A^\mu \,, \qquad\qquad a^\mu \to a^\mu + \partial^\mu \epsilon_a \,.$$

Principles: i) NEQ constraints, ii) locality and iii) retarded gauge invariance.

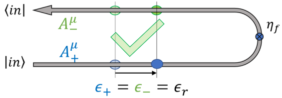
Effective functional constructed out of $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ and a^μ:

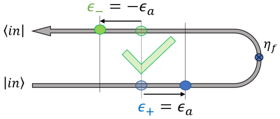
$$S_1 = \int_{\omega,\boldsymbol{k}} \left[a^0 i k_i F^{0i} + a_i \left(\gamma_2 F^{0i} + \gamma_3 i k_j F^{ij} + \gamma_4 \epsilon^i{}_{jl} F^{jl} \right) + a^\mu j_\mu \right]$$

$$S_2 = \int_{\omega,\boldsymbol{k}} i a^\mu N_{\mu\nu} a^\nu \,, \qquad S_{n\geq 3} = \cdots$$

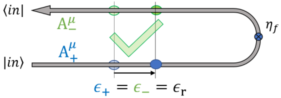
Summary

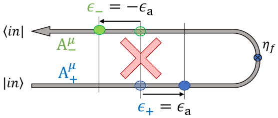
1. **Unitary**: $\Delta S^{\text{adv}}_{\text{eff}} = 0 \quad \Rightarrow \quad \partial^\mu j_\mu = 0$: Maxwell in medium;



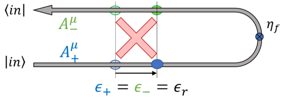


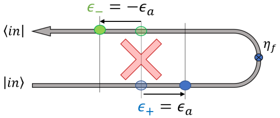
2. **Non-unitary**: $\Delta S^{\text{adv}}_{\text{eff}} \neq 0 \quad \Rightarrow \quad \partial^\mu j_\mu \neq 0$: modified charge conservation;





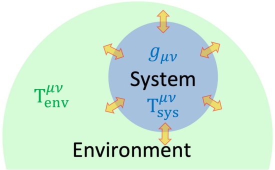
3. **Conductor**: $\Delta S^{\text{ret}}_{\text{eff}} \neq 0 \quad \Rightarrow$ new d.o.f.: dissipative Proca theory.





Open gravity

Dissipative theory for a **massless spin 2 graviton**: theory of gravity in a medium.



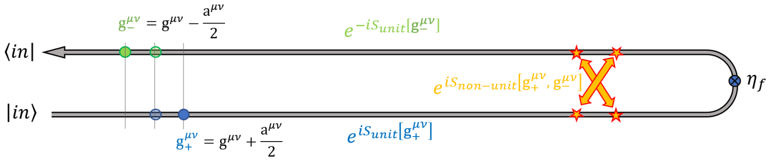
Diffeomorphisms invariance:

$$g^{\mu\nu}_{\pm}(x) \to \frac{\partial(x^\mu + \xi^\mu_\pm)}{\partial x^\alpha}\frac{\partial(x^\nu + \xi^\nu_\pm)}{\partial x^\beta} g^{\alpha\beta}_{\pm}(x)$$

for each branch of SK path integral contour.

Keldysh basis: **retarded** $g_{\mu\nu}$ and **advanced** $a^{\mu\nu}$ metric:

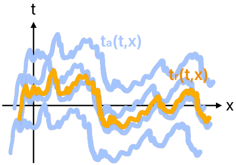
$$g = \frac{g_+ + g_-}{2} = \bar{g} + \delta g, \qquad \text{and} \qquad a = g_+ - g_- = \delta a$$



A tale of two clocks

Fluctuating clocks: $\phi_\pm(t,\mathbf{x})$, i.e.

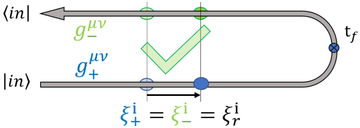
- $\phi_r(t,\mathbf{x}) \Leftrightarrow t_r(t,\mathbf{x})$: average clock;
- $\phi_a(t,\mathbf{x}) \Leftrightarrow t_a(t,\mathbf{x})$: stochasticity.

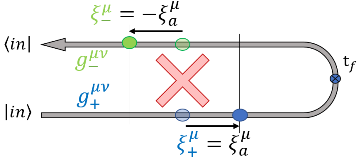


Gauge fixing. Work in *unitary gauges*: $\phi_+(t,\mathbf{x}) = \phi_-(t,\mathbf{x}) = \bar{\phi}(t)$:

$$\phi_r(t,\mathbf{x}) = \bar{\phi}(t) \quad \text{and} \quad \phi_a(t,\mathbf{x}) = 0 \qquad \text{i.e.} \qquad t_r = t \quad \text{and} \quad t_a = 0.$$

Following EFT of Inflation [Cheung *et al.*, 2008] and Dark Energy [Gubitosi, Piazza & Vernizzi, 2013]:





$$\text{4d-diff}_+ \times \text{4d-diff}_- \xrightarrow{\text{open}} \text{4d-diff}_r \xrightarrow{\text{clock}} \text{3d-diff}_r\,.$$

Unitary gauges

Most generic functional invariant under retarded spatial diffeomorphisms.

$$S_{\rm eff} = \int \mathrm{d}^4 x \sqrt{-g} \sum_{\ell=0} (g^{00}+1)^{\ell} \left[M_{\mu\nu,\ell} a^{\mu\nu} + i N_{\mu\nu\rho\sigma,\ell} a^{\mu\nu} a^{\rho\sigma} + \cdots \right]$$

with $M_{\mu\nu,\ell}$ and $N_{\mu\nu\rho\sigma,\ell}$ rank-2 and 4 cotensors under *retarded spatial diffs*.

$$M_{00,\ell} = \gamma^{tt}_{1,\ell} + \gamma^{tt}_{2,\ell} K + \gamma^{tt}_{3,\ell} K^2 + \gamma^{tt}_{4,\ell} K_{\alpha\beta} K^{\alpha\beta} + \gamma^{tt}_{5,\ell} \nabla^0 K + \gamma^{tt}_{6,\ell} R + \gamma^{tt}_{7,\ell} R^{00} \,;$$

$$M_{0\mu,\ell} = \gamma^{ts}_{1,\ell} R^{0}{}_{\mu} + \gamma^{ts}_{2,\ell} \nabla_{\mu} K + \gamma^{ts}_{3,\ell} \nabla_{\beta} K^{\beta}{}_{\mu} \,;$$

$$\begin{aligned} M_{\mu\nu,\ell} = {} & g_{\mu\nu} \left(\gamma^{ss}_{1,\ell} + \gamma^{ss}_{2,\ell} K + \gamma^{ss}_{3,\ell} K^2 + \gamma^{ss}_{4,\ell} K_{\alpha\beta} K^{\alpha\beta} + \gamma^{ss}_{5,\ell} \nabla^0 K + \gamma^{ss}_{6,\ell} R + \gamma^{ss}_{7,\ell} R^{00} \right) \\ & + \gamma^{ss}_{8,\ell} K_{\mu\nu} + \gamma^{ss}_{9,\ell} \nabla^0 K_{\mu\nu} + \gamma^{ss}_{10,\ell} K_{\mu\alpha} K^{\alpha}{}_{\nu} + \gamma^{ss}_{11,\ell} K K_{\mu\nu} + \gamma^{ss}_{12,\ell} R_{\mu\nu} + \gamma^{ss}_{13,\ell} R_{\mu}{}^{0}{}_{\nu}{}^{0} \\ & + \gamma^{\rm P.O.}_{1,\ell} \epsilon_{\mu}{}^{\alpha\beta 0} \nabla_{\alpha} K_{\beta\nu} + \gamma^{\rm P.O.}_{2,\ell} \epsilon_{\mu}{}^{\alpha\beta 0} R_{\alpha\beta}{}^{0}{}_{\nu} . \end{aligned}$$

and similarly for $N_{\mu\nu\rho\sigma,\ell} \Rightarrow$ can be studied **systematically**.

Background evolution

Modified Friedmann equations with α_i functions of EFT coefficients:

$$\text{Open:} \begin{cases} 3M_{\rm Pl}^2 H^2 = \alpha_1 + \alpha_2 H\,, \\ 2M_{\rm Pl}^2 \dot{H} = \alpha_3 + \alpha_4 H\,, \end{cases} \quad \text{vs} \quad \text{Closed:} \begin{cases} 3M_{\rm Pl}^2 H^2 = \rho_\phi\,, \\ 2M_{\rm Pl}^2 \dot{H} = -(\rho_\phi + P_\phi)\,, \end{cases}$$

1. *Bulk viscosity* [Weinberg, 1971]: $\alpha_4 = 3\zeta$

$$2M_{\rm Pl}^2 \dot{H} = -\left[\rho_\phi + (P_\phi - 3H\zeta)\right],$$

2. *Brane-world gravity* [Dvali, Gabadadze & Porrati, 2000]: $\alpha_2 = \pm 3M_{\rm Pl}^2/r_c$

$$3M_{\rm Pl}^2(H^2 \pm H/r_c) = \rho_\phi,$$

$\Rightarrow$ Modified continuity equation: $\dot{\alpha}_1 + \left(\dot{\alpha}_2 H + \alpha_2 \dot{H}\right) - 3H(\alpha_3 + \alpha_4 H) = 0$

3. *Interacting dark sector*: $Q = -(\dot{\alpha}_2 H + \alpha_2 \dot{H}) + 3\alpha_4 H^2$

$$\dot{\rho}_\phi + 3H(\rho_\phi + P_\phi) = Q.$$

Reintroducing the scalar

Restore covariance by *retarded* and *advanced* time diffeomorphism:

$$S_{\text{eff}}[g_{\mu\nu}, a_{\mu\nu}] \to S_{\text{eff}}[g_{\mu\nu}, a_{\mu\nu}, \pi_r, \pi_a]\,,$$

- **retarded Stückelberg trick**: $t_r \to t_r + \pi_r(x)$:

$$g^{00} \to g^{00} + 2g^{0\mu}\partial_\mu \pi_r + g^{\mu\nu}\partial_\mu\pi_r\partial_\nu\pi_r,$$
$$a^{00} \to a^{00} + 2a^{0\mu}\partial_\mu \pi_r + a^{\mu\nu}\partial_\mu\pi_r\partial_\nu\pi_r.$$

- **advanced Stückelberg trick**: $t_a \to t_a - \pi_a(x)$:

$$g_{\mu\nu} \to g_{\mu\nu},$$
$$a^{\mu\nu} \to a^{\mu\nu} + 2\nabla^{(\mu}\epsilon_a^{\nu)}.$$

Decoupling limit: slow-roll $\epsilon \ll 1$ (**even with dissipation & noise**):

$$\begin{aligned} -a^{00} + (g^{00}/2)g_{\mu\nu}a^{\mu\nu} &\to 2\dot{\pi}_a - 2\partial^\mu\pi_r\partial_\mu\pi_a\,, \\ g_{\mu\nu}a^{\mu\nu} &\to 2\dot{\pi}_a + 6H\pi_a\,, \\ 1 + g^{00} &\to -2\dot{\pi}_r + (\partial_\mu\pi_r)^2, \end{aligned}$$

$\Rightarrow$ *recover Open EFToI & more: primordial GWs and dissipative DM/DE*

Outline

Primordial gravitational waves

Transverse and traceless (TT) sector: $g_{ij} = a^2(t)\left(\delta_{ij} + h_{ij}\right), \; a^{ij} = a^{-2} h^a_{ij}$:

$$S^{(2)} = \int \mathrm{d}^4x \sqrt{-g}\, \frac{M_{\rm Pl}^2}{4c_T^2}\, h^a_{ij} \Big\{ \ddot{h}_{ij} - c_T^2 \frac{\nabla^2}{a^2} h_{ij} + \left(\Gamma_T + 3H\right) \dot{h}_{ij} + \frac{\chi_T}{a} \epsilon_{imn} \left(\partial_m \dot{h}_{nj} + 2H \partial_m h_{nj}\right) + i \frac{\beta_T}{M_{\rm Pl}^2} h^a_{ij} \Big\}.$$

1. Speed of propagation c_T^2;
2. Dissipation Γ_T;
3. Birefringence χ_T;
4. Noise β_T.

Effects correlated with **background** and **scalar sector** [Lau, Nishii & Noumi, 2024]:

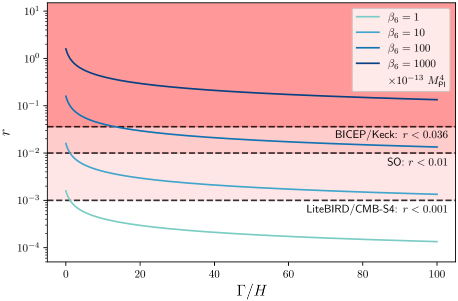
$$S_{\rm eff} \supset \int \mathrm{d}^4x \sqrt{-g} \gamma^{ss}_{8,\ell} K_{\mu\nu} a^{\mu\nu}.$$

- Background with bulk viscosity: $\zeta = 2\gamma^{ss}_{8,\ell}/3$.
- Dissipation in tensor sector: $\Gamma_T = 2\gamma^{ss}_{8,\ell}/M_{\rm Pl}^2$.

Tensor-to-scalar ratio

Without birefringence ($\chi_T = 0$): define $\nu_\Gamma \equiv 3/2 + \Gamma_T/(2H)$

$$\Delta_h^2 = \frac{\beta_T}{M_{\mathrm{P}l}^4} 2^{2\nu_\Gamma} \frac{\Gamma(\nu_\Gamma - 1)\Gamma(\nu_\Gamma)^2}{\Gamma(\nu_\Gamma - \frac{1}{2})\Gamma(2\nu_\Gamma - \frac{1}{2})} \propto \begin{cases} \frac{\beta_T}{M_{\mathrm{P}l}^4}, & \text{for} \quad \Gamma_T \ll H, \\ \frac{\beta_T}{M_{\mathrm{P}l}^4}\sqrt{\frac{H}{\Gamma_T}}\left[1 + \mathcal{O}\left(\frac{H}{\Gamma_T}\right)\right], & \text{for} \quad \Gamma_T \gg H. \end{cases}$$



⚠ In general, $\beta_T \propto f(\Gamma_T)$:

- Thermal equilibrium: $\beta_T \propto \Gamma_T T$ (FDR);
- What about out-of-equilibrium?

BICEP/Keck bound: $r < 0.036 \quad \Rightarrow \quad \beta_T \lesssim (0.002 M_{\mathrm{P}l})^4$

Late-universe: clockless example

Subset that preserves retarded time-diff: **no dynamical scalar**

$$S_{\text{eff}} = \int \mathrm{d}^4x\sqrt{-g}\,(f_1 R_{\mu\nu} + f_2 g_{\mu\nu} + f_3 T_{\mu\nu} + \text{h.d.})\, a^{\mu\nu}.$$

⇒ *Very few ingredients!* To lowest order, e.o.m. takes the form:

$$G_{\mu\nu} + \tilde{f}_1 R g_{\mu\nu} = \tilde{f}_2 T_{\mu\nu} + \tilde{f}_3 T g_{\mu\nu}.$$

w.l.o.g., separating **Standard Model**, **dark matter** and **cosmological constant**:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = G_{\mathrm{N}}\left[T^{\mathsf{SM}}_{\mu\nu} + T^{\mathsf{DM}}_{\mu\nu} + (\lambda^{\mathsf{SM}} T^{\mathsf{SM}} + \lambda^{\mathsf{DM}} T^{\mathsf{DM}}) g_{\mu\nu}\right].$$

If dark matter ≃ perfect fluid, absorb $\lambda^{\mathsf{DM}} T^{\mathsf{DM}} g_{\mu\nu}$ into equation of state w^{DM}:

Minimal Open GR: (≃ *Gravitational Aether* [Afshordi, 2008])

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = G_{\mathrm{N}}\left[T^{\mathsf{SM}}_{\mu\nu} + T^{\mathsf{DM}}_{\mu\nu} + \lambda^{\mathsf{SM}} T^{\mathsf{SM}} g_{\mu\nu}\right].$$

Minimal Open GR [with F. McCarthy, ACT & SO]

- Does not come from an action $\Rightarrow$ **evade [Lovelock, 1971] theorem**;
- Dissipative dark matter:

$$\nabla^\mu T^{\rm SM}_{\mu\nu} = 0\,, \qquad \nabla^\mu T^{\rm DM}_{\mu\nu} + \lambda^{\rm SM}\partial_\nu T^{\rm SM} = 0\,.$$

- To **UV complete**, need to find sector s.t. $\langle T^{\rm env}_{\mu\nu}\rangle = \lambda^{\rm SM} T^{\rm SM} g_{\mu\nu}$;
- One-parameter extension of Λ-CDM $\Rightarrow$ data-analysis CMB + BAO.

Background: change expansion history

$$3H^2 = \Lambda + G_{\rm N}\left[\rho^{\rm SM} + \rho^{\rm DM} + \lambda^{\rm SM}(\rho^{\rm SM} - 3p^{\rm SM})\right]$$

Perturbations: modify matter clustering

$$\dot{\delta}_c = -\theta_c + 3\dot{\phi} + \lambda^{\rm SM}\frac{\delta \dot{T}^{\rm SM}}{\rho_c}\,, \qquad \dot{\theta}_c = -H\theta_c + k^2\psi + \frac{\lambda^{\rm SM}k^2}{1+w}\frac{\delta T^{\rm SM}}{\rho_c}$$

Conclusion

Realistic environments are noisy and dissipative:
We need ***systematic modelling*** *for* ***upcoming data****.*

Open inflation:

1. An embedding for **local dissipative models** of inflation;
2. Smoking gun near **folded triangles** in **primordial non-Gaussianities**.

CMB constraints, cutoff & bounds

Open gravity:

1. Systematic EFT accounting for local **dissipation** and **noise**;
2. **Schwinger-Keldysh**: $\text{4d-diff}_+ \times \text{4d-diff}_- \xrightarrow{\text{open}} \text{4d-diff}_r \xrightarrow{\text{clock}} \text{3d-diff}_r$.

Dissipative Dark Sectors & Open Gravitational Waves

Rich phenomenology to explore, eventually **already constrained from data**.

Backup

Thomas Colas

Outline

Outline

Non-equilibrium constraints [Liu & Glorioso, 2018]

Requiring **Open QFT** originates from a unitary "closed" UV theory:

$$\text{i) } \mathrm{Tr}[\widehat{\rho}] = 1, \qquad \text{ii) } \widehat{\rho}^\dagger = \widehat{\rho} \qquad \text{and} \qquad \text{iii) } \widehat{\rho} \geq 0$$

implies constraints on $S_{\rm eff}[\pi_+, \pi_-] \equiv S_{\rm unit}[\pi_+] - S_{\rm unit}[\pi_-] + S_{\rm non-unit}[\pi_+, \pi_-]$:

i)	$S_{\rm eff}[\pi_+, \pi_+] = 0,$	$S_{\rm eff}[\pi_r, \pi_a = 0] = 0;$
ii)	$S_{\rm eff}[\pi_+, \pi_-] = -S^*_{\rm eff}[\pi_-, \pi_+],$	$S_{\rm eff}[\pi_r, \pi_a] = -S^*_{\rm eff}[\pi_r, -\pi_a];$
iii)	$\mathfrak{Im} S_{\rm eff}[\pi_+, \pi_-] \geq 0,$	$\mathfrak{Im} S_{\rm eff}[\pi_r, \pi_a] \geq 0,$

for $\pi_r = (\pi_+ + \pi_-)/2$ and $\pi_a = \pi_+ - \pi_-$. *Consequences*:

1. $S_{\rm eff}[\pi_r, \pi_a]$ starts **linear** in π_a;
2. **Odd** powers of π_a are purely real; **even** powers of π_a purely imaginary;
3. Positivity bounds on the **noise coefficients**.

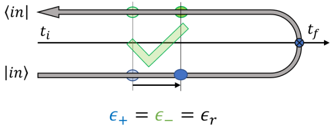
$\Rightarrow$ *Already reduce the scope of available Open EFTs*

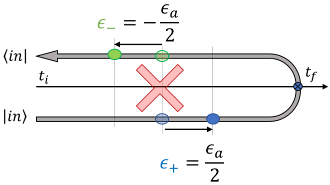
In-in coset construction [Hongo *et al.*, 2018], [Akyuz, Goon & Penco, 2023]

Two **simplifications** from [Cheung *et al.*, 2008]: [regime of validity? ⇒ later]

1. *Decoupling limit*: Mixing $\pi/\delta g$ small as long as $E \sim H \gg E_{\rm mix} \sim \epsilon^{1/2} H$

⇒ **enough** to construct theory of **dissipative shift symmetric scalar**:





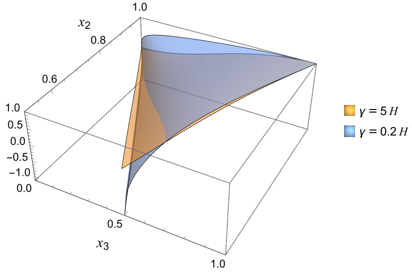
$S_{\rm eff}\left[\pi_r, \pi_a\right]$ invariant under *retarded time diffeomorphism*:

$$t \to t + \epsilon_r : \qquad \pi_r \to \pi_r - \epsilon_r, \qquad \pi_a \to \pi_a.$$

<u>*Building blocks:*</u> π_a, $t + \pi_r$, $\partial_\mu \pi_a$, $\partial_\mu(t+\pi_r)$.

2. *Derivative expansion*: locality and truncatable power counting scheme.

Primordial non-Gaussianities

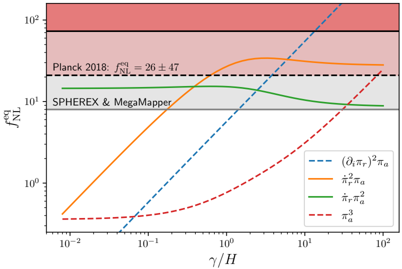


Main features:

- $\gamma \gg H$: equilateral shape;
- $\gamma \ll H$: **new** folded signal;
- Consistency relations hold.

Matching with [Creminelli *et al.*, 2305.07695]

- inflaton ϕ
- massive scalar field χ
- softly-broken $U(1)$



New challenge: dynamical gravity

Outline

Recovering electromagnetism in a medium

From S_{eff}, obtain modified Gauss and Ampère laws:

$$\frac{\delta S_{\text{eff}}}{\delta a^0} = 0 \qquad \Rightarrow \qquad \nabla . \mathbf{E} = j_0 + \xi_0,$$

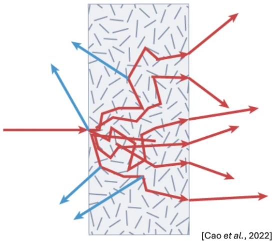
$$\frac{\delta S_{\text{eff}}}{\delta a^i} = 0 \qquad \Rightarrow \qquad \gamma_2 \mathbf{E} + \gamma_3 \nabla \times \mathbf{B} - 2\gamma_4 \mathbf{B} = \mathbf{j} + \xi,$$

and a noise constraint: *modified charge conservation in the system*

$$\partial^\mu (j_\mu + \xi_\mu) = \Gamma (j_0 + \xi_0).$$

Properties:

- Dispersive medium: $n = 1/\sqrt{-\gamma_3}$;
- Dissipative medium: $\gamma_2 = -i\omega + \Gamma$;
- Anisotropic medium: γ_4;
- Random medium: ξ^μ.

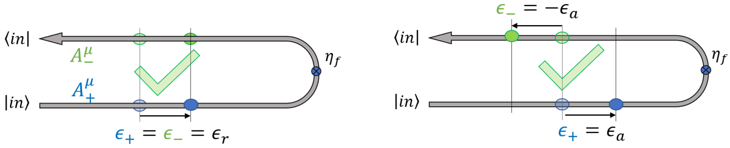


[Cao *et al.*, 2022]

Unitary limit

$$\Delta S_1^{\mathrm{ret}} = 0 \qquad \text{but} \qquad \Delta S_1^{\mathrm{adv}} = \int_{\omega,\boldsymbol{k}} (i\omega + \gamma_2)\epsilon_a i k_i F^{0i}.$$

- For general γ_2, $\Delta S_1^{\mathrm{adv}} \neq 0$;
- When $\gamma_2 = -i\omega$, $\Delta S_1^{\mathrm{adv}} = 0$.



Unitary theory with $S_1[A_+, A_-] = S[A_+] - S[A_-]$ and

$$S[A] = \frac{1}{4}\int \mathrm{d}^4x \left[F^{\mu\nu}F_{\mu\nu} + (c_s^2 - 1)F^{ij}F_{ij} + \theta F^{\mu\nu}\tilde{F}^{\mu\nu}\right]$$

Recover Maxwell in a medium.

Deformed advanced gauge

When $\gamma_2 \neq -i\omega$, does it exist ***physically equivalent*** *advanced configurations?*

1. Retarded gauge invariance: $M_{\mu\nu}k^\nu = 0$ where $k^\mu = (\omega, \mathbf{k})$.
2. $\exists$ "right kernel" $\Rightarrow \exists$ "left kernel" such that $v^\mu M_{\mu\nu} = 0$.
3. M is non-Hermitian $\Rightarrow$ different left and right kernels: $v^\mu = (i\gamma_2, \boldsymbol{k})$.

Conclusion: retarded gauge invariance generates advanced gauge invariance.

S_1 remains unchanged under

$$A^\mu \to A^\mu + \epsilon_r k^\mu, \qquad a^\mu \to a^\mu + \epsilon_a v^\mu.$$

Not broken but rather **deformed**! [Liu & Glorioso, 2018], [Akyuz, Goon & Penco, 2023].

Advanced gauge redundancy allows us to reduce number of advanced components.

Noise constraint

Add some noise: $N_{\mu\nu}$ positive semi-definite [Kamenev, 2011], [Breuer & Petruccione, 2007]

$$S = \int d^4x \left[a^\mu M_{\mu\nu} A^\nu + i a^\mu N_{\mu\nu} a^\nu\right],$$

Hubbard-Stratonovich trick:

$$\mathcal{Z} = \int [\mathcal{D}A^\mu] \int [\mathcal{D}a^\mu] \int [\mathcal{D}\xi_\mu] \exp\left[\int d^4x \; i \boldsymbol{a^\mu} \left(\boldsymbol{M_{\mu\nu} A^\nu} - j_\nu - \xi_\nu\right) - \frac{1}{4}\xi_\mu (N^{-1})^{\mu\nu} \xi_\nu\right]$$

Advanced gauge symmetry $\boldsymbol{a^\mu \to a^\mu + \epsilon_a v^\mu}$ induces noise constraint:

$$v^\mu (j_\mu + \xi_\mu) = 0.$$

Meaning: $\gamma_2 = \Gamma - i\omega$ leads to

$$\partial^\mu (j_\mu + \xi_\mu) = \Gamma (j_0 + \xi_0).$$

Conservation of the total current ⇔ ***Non-conservation of the system's current***

Advanced Stueckelberg [Lau, Nishii & Noumi, 2412.21136]

Stueckelberg trick: $X_a \to X_a - \epsilon_a$ s.t. $\mathcal{A}_a^\mu = a^\mu + \partial^\mu X_a \to \mathcal{A}_a^\mu$ invariant

$$S_{\text{eff}} = \int_{\omega,\boldsymbol{k}} \left[\mathcal{A}_{a0} ik_i F^{0i} + \mathcal{A}_{ai}\left(\gamma_2 F^{0i} + \gamma_3 ik_j F^{ij} + \gamma_4 \epsilon^i_{\ jl} F^{jl}\right) - \mathcal{A}_{a\mu}\left(j^\mu + \xi^\mu\right)\right],$$

$$= S_{\text{eff}}^{\text{old}} + \int_{\omega,\boldsymbol{k}} X_a \left[(i\omega + \gamma_2) ik_i F^{0i} + \partial_\mu\left(j^\mu + \xi^\mu\right)\right].$$

X_a e.o.m. recovers noise constraint: for $\gamma_2 = \Gamma - i\omega$,

$$\frac{\delta S_{\text{eff}}}{\delta X_a} = 0 \quad \Rightarrow \quad \partial^\mu(j_\mu + \xi_\mu) = \Gamma(j_0 + \xi_0).$$

using on-shell relation $\delta S_{\text{eff}}/\delta a_0 = 0$ that is $ik_i F^{0i} = -\left(j^0 + \xi^0\right)$.

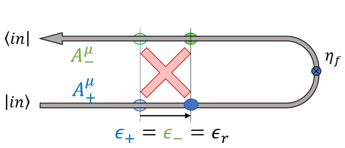
Lessons:

- Manifest advanced gauge invariance at the price of X_a;
- If $\partial^\mu(j_\mu + \xi_\mu) = 0$, only solution is $\gamma_2 = -i\omega$, *i.e.* no dissipation $\Gamma = 0$.

Proca mass term

Adding a mass term breaks **both** retarded and advanced gauge symmetries:

$$S_{\text{eff}} = \int_{\omega,\boldsymbol{k}} ik_i a_0 F^{0i} - i\omega a_i F^{0i} - m^2 a_\mu A^\mu$$



Stueckelberg trick:

- $X_a \to X_a - \epsilon_a$ s.t. $\mathcal{A}_a^\mu = a^\mu + \partial^\mu X_a \to \mathcal{A}_a^\mu$ invariant;
- $X_r \to X_r - \epsilon_r$ s.t. $\mathcal{A}_r^\mu = A^\mu + \partial^\mu X_r \to \mathcal{A}_r^\mu$ invariant

$$S_{\text{eff}} = S_{\text{eff}}^{\text{old}} - m^2 \int \mathrm{d}^4 x \left[\partial_\mu X_a \partial^\mu X_r - a_\mu \partial^\mu X_r - \partial_\mu X_a A^\mu\right]$$

Decoupling: when $E \gg m$, mixing between X and A negligeable.

Breaking retarded gauge invariance triggers new d.o.f.

Dispersion relations

Gauge fixing

- retarded Coulomb gauge: $\partial_i A^i = 0$

$$\exists\ \epsilon_r \quad \text{s.t.} \quad k_i A'^i = 0, \quad \text{where} \quad A'^\mu = A^\mu + \epsilon_r k^\mu .$$

- advanced Coulomb gauge: $\partial_i a^i = 0$

$$\exists\ \epsilon_a \quad \text{s.t.} \quad k_i a'^i = 0, \quad \text{where} \quad a'^\mu = a^\mu + \epsilon_a v^\mu .$$

Eigenvalues of the kinetic matrix: 1 constrained dof, 2 propagating dof

$$\left(k^2\,, i\gamma_2\omega + \gamma_3 k^2 + 2\gamma_4 k, i\gamma_2\omega + \gamma_3 k^2 - 2\gamma_4 k\right).$$

Introduce $\gamma_2 = \Gamma - i\omega$, $\gamma_3 = -c_s^2$:

$$\omega^2 + i\Gamma\omega - c_s^2 k^2 \pm 2\gamma_4 k = 0 \quad \Rightarrow \quad \omega = -i\frac{\Gamma}{2} \pm \sqrt{c_s^2 k^2 - (\Gamma/2)^2 \mp 2\gamma_4 k}\,.$$

Outline

A glimpse on what to expect

- Dissipative and stochastic Einstein Equations: $G_{\mu\nu} + \Gamma \mathcal{D}_{\mu\nu} = T_{\mu\nu} + \xi_{\mu\nu}$
- Non-conserved stress-energy tensor: $\nabla_\mu T^{\mu\nu} \neq 0$

Phenomenology:

- **Background**: Viscous cosmology & Interacting dark sectors

$$\dot{\rho}_{DE} + 3H(\rho_{DE} + p_{DE}) = \Gamma \quad \text{and} \quad \dot{\rho}_m + 3H(\rho_m + p_m) = -\Gamma$$

- **Clustering** $\Rightarrow$ redshift space distortion (RSD) and weak lensing (WL)

$$k^2 \langle \psi \rangle = -4\pi G \mu(a,k) a^2 \rho_m \langle \delta \rangle, \qquad k^2 \frac{\langle \psi + \phi \rangle}{2} = -4\pi G \Sigma(a,k) a^2 \rho_m \langle \delta \rangle$$

- **Gravitational waves** $\Rightarrow$ GW production, propagation and dissipation

$$\ddot{h}_{ij} + \Gamma \dot{h}_{ij} + c_T^2 h_{ij} + \chi \epsilon_{ilm} k_m h_{jl} = T_{ij} + \xi_{ij}$$

Rich phenomenology to explore,
eventually ***already constrained from data***.

Including birefringence

$$\left[\partial_\eta^2 - \left(\frac{2+\frac{\Gamma_T}{H}}{\eta} \mp \chi_T k\right)\partial_\eta + k^2\right] G^R_{\times,+}(k;\eta,\eta') = H^2\eta^2\delta(\eta-\eta').$$

Analytic expression in terms of Appell F_2 function: $\kappa_\pm \equiv \mp\frac{i\chi_T}{\sqrt{1-\chi_T^2}}(\nu_T - 1/2)$

$$\Delta^2_{\times,+} = \frac{\beta_T/M_{\mathrm{Pl}}^4}{2\pi^2\nu_\Gamma^2\chi_T^3} F_2\left(3; \tfrac{1}{2}+\nu_\Gamma-\kappa_\pm, \tfrac{1}{2}+\nu_\Gamma+\kappa_\pm; 2\nu_\Gamma+1, 2\nu_\Gamma+1; \mp\sqrt{1-\frac{1}{\chi_T^2}}, \pm\sqrt{1-\frac{1}{\chi_T^2}}\right)$$

Recover **instability band** at low dissipation: