Entanglement in solvable quantum many-body systems

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Physics (and) Statistics
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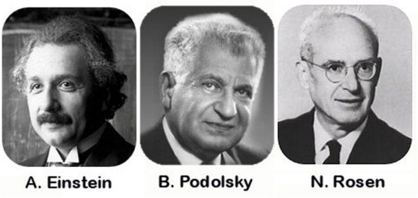
Entanglement



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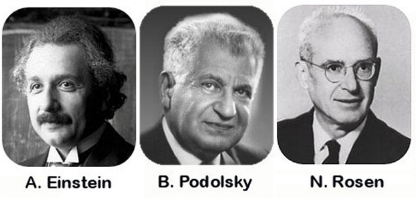

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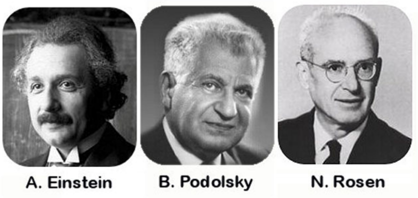
Entanglement



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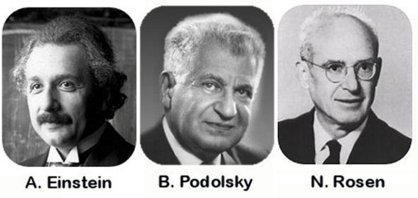

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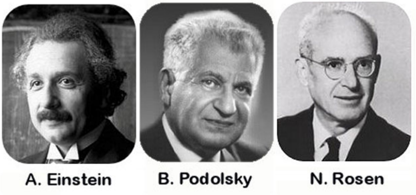

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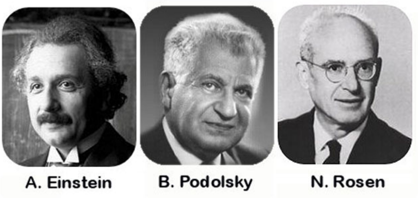

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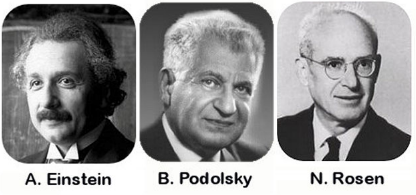

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Quantum system in the state $|\psi^{\mathbf{AB}}\rangle$ with subsystems $\boldsymbol{A}$ and $\boldsymbol{B}$

Entanglement $\Leftrightarrow$ $|\psi^{\mathbf{AB}}\rangle \neq |\phi^{\mathbf{A}}\rangle \otimes |\varphi^{\mathbf{B}}\rangle$

No entanglement $\Leftrightarrow$ $|\psi^{\mathbf{AB}}\rangle = |\phi^{\mathbf{A}}\rangle \otimes |\varphi^{\mathbf{B}}\rangle$

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What is the difference between these two states?

- $|\psi^1\rangle = \frac{1}{\sqrt{2}}(|{\uparrow\downarrow}\rangle - |{\downarrow\uparrow}\rangle)$
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Answer:

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- $|\psi^2\rangle$: measure in **A** does not affect **B**: **No entanglement**

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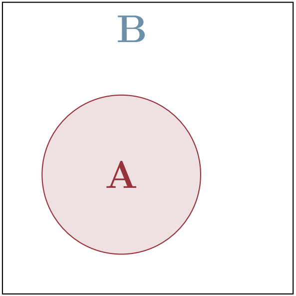
  - Relations to orthogonal polynomials, asymptotic analysis, ...

Entanglement in many-body systems



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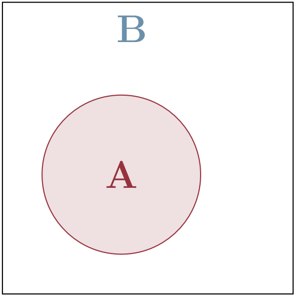

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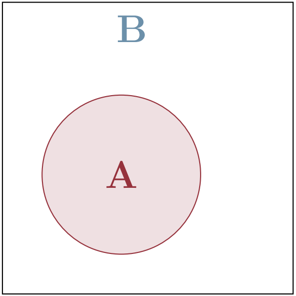

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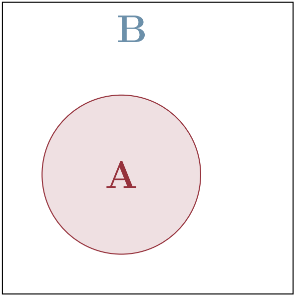

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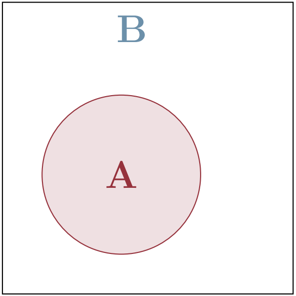

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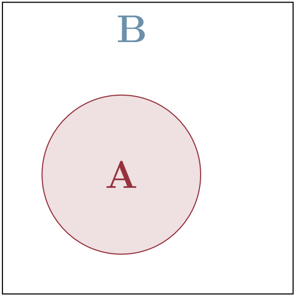

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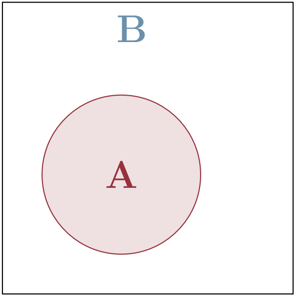

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Separability

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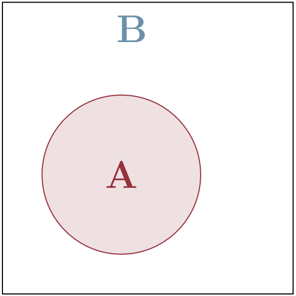

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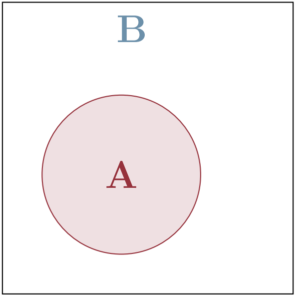

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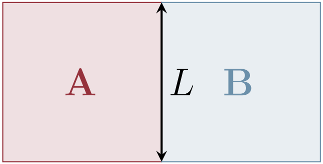
A state is **entangled** if it is **not separable**: $S_{\mathbf{A}} > 0$

Equilibrium and quantum phase transitions

Entanglement and criticality



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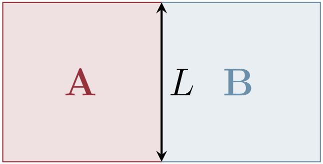

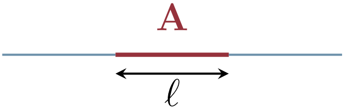

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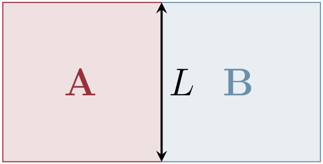

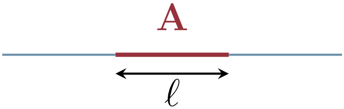

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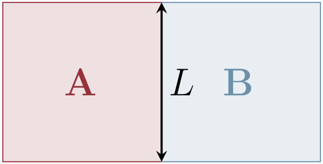

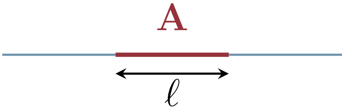

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- Area law for gapped systems

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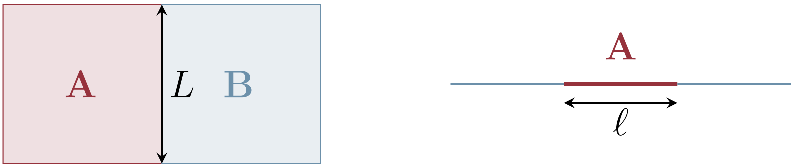

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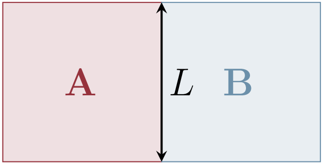

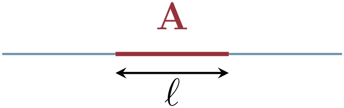

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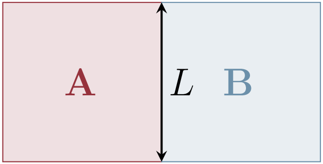

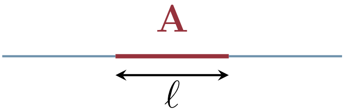

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- Logarithmic violation for $1 + 1d$ CFT

$$S_{\mathbf{A}} = \frac{c}{3} \log \ell + \dots$$

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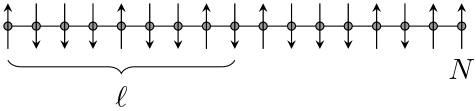

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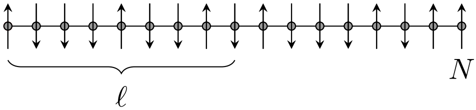

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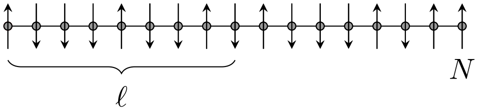

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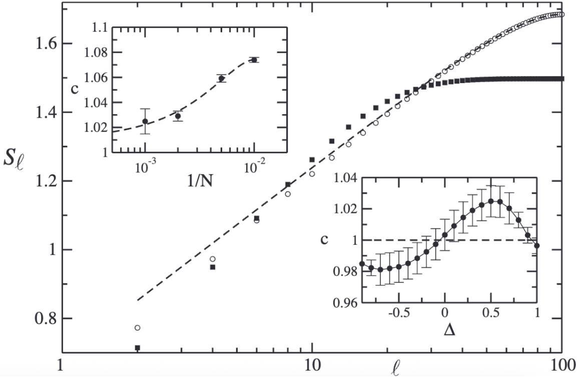

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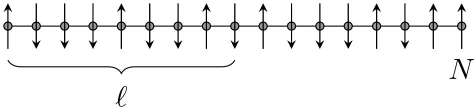

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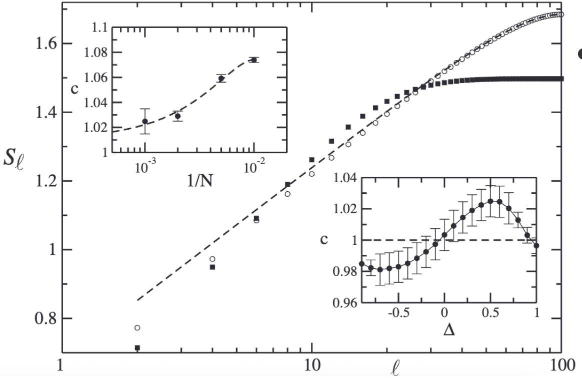

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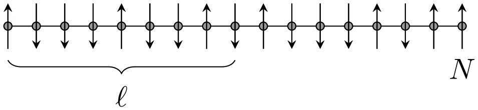

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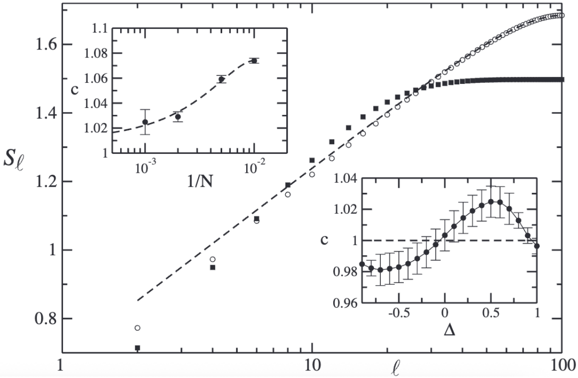

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$$S_\ell = \frac{c}{6} \log\left[\frac{N}{\pi} \sin\left(\frac{\pi \ell}{N}\right)\right] + a$$

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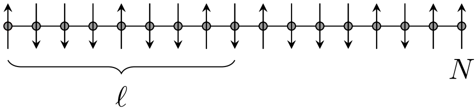
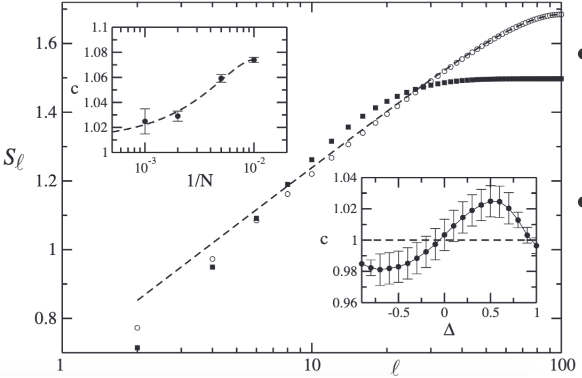
[De Chiara, Montangero, Calabrese, Fazio, 2006]



$$H = \sum_{i=1}^{N-1} \left(\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + \Delta \sigma_i^z \sigma_{i+1}^z \right)$$



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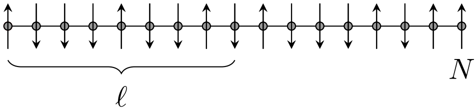

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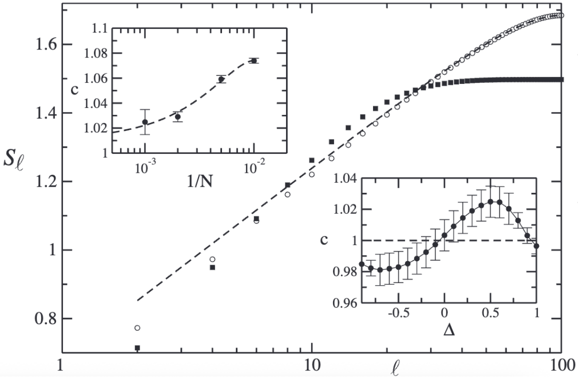

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Out of equilibrium and quantum quench

What is a quantum quench?

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Quantum quench

Protocol to study an **isolated quantum system** out of equilibrium

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quantum mechanics → statistical mechanics

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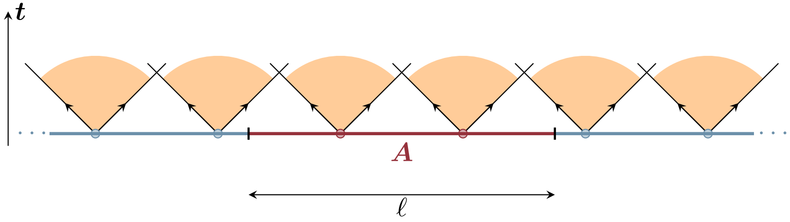
quantum mechanics $\rightarrow$ **statistical mechanics**

Question: what happens between between $t=0$ and $t=\infty$?

Quasiparticle picture: Single interval

[Calabrese, Cardy, 2005], [Alba, Calabrese, 2017-2018]



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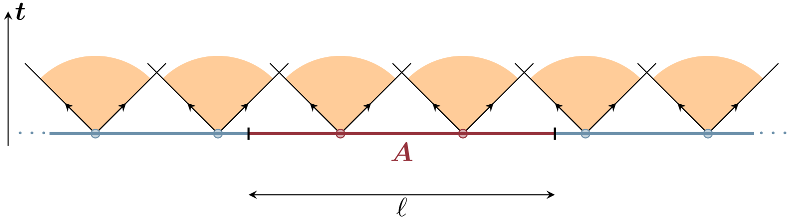

$$S_{\mathbf{A}}(t) = \underbrace{2t \int_{2tv_k<\ell} \mathrm{d}k\ s(k) v_k}_{\text{linear growth}} + \underbrace{\ell \int_{2tv_k>\ell} \mathrm{d}k\ s(k)}_{\text{saturation}}$$

$$= \int_{-\pi}^{\pi} \mathrm{d}k\ s(k) \min(\ell, 2v_k t)$$

Tests of the QPP

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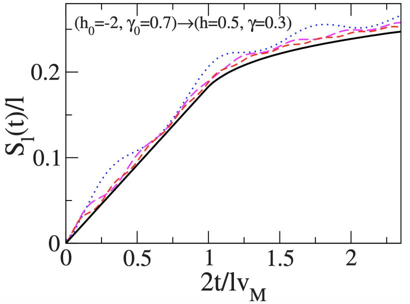
Exact results for the XY spin chain
[Fagotti, Calabrese, 2008]



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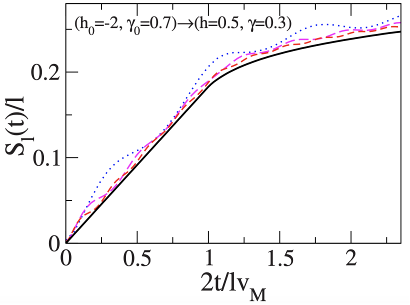
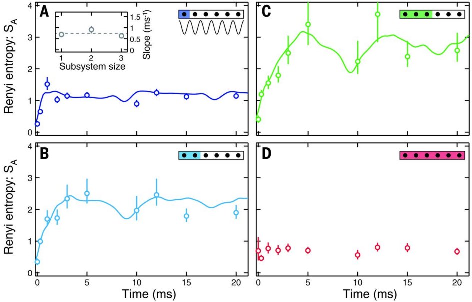


Experimental verification
[Kaufman *et al.*, 2016]



Multipartite setting

Multipartite separability: mixed state

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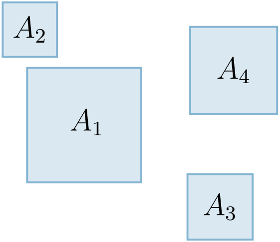

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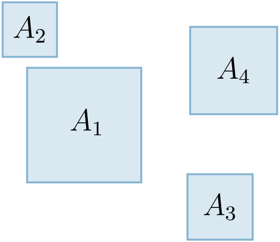

Physical states ρ satisfy

- $\mathrm{Tr}\rho = 1$

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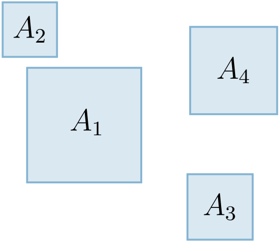

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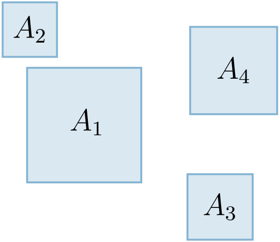

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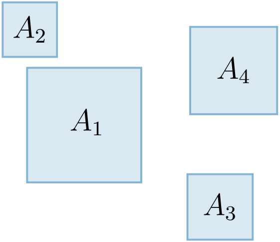

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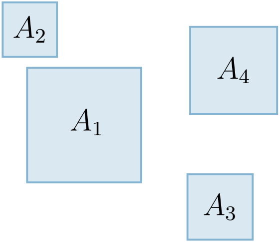

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The most classical state for $A_1 \cup A_2 \cup \cdots \cup A_m$ is a product,

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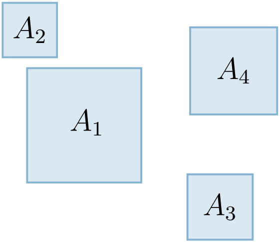

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Separable states are convex combinations of product states,

$$\rho_{\text{sep}} = \sum_k p_k \; \rho^{(k)}_{A_1} \otimes \rho^{(k)}_{A_2} \otimes \cdots \otimes \rho^{(k)}_{A_m}$$

with $p_k \geqslant 0$ and $\sum_k p_k = 1$

Genuine multipartite entanglement

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Let us consider the 4-spin state

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Multipartite entanglement **structure** in many-body systems is non-trivial. It is important to go **beyond the bipartite case!**

Fate of entanglement

[GP, Witczak-Krempa, 2024]

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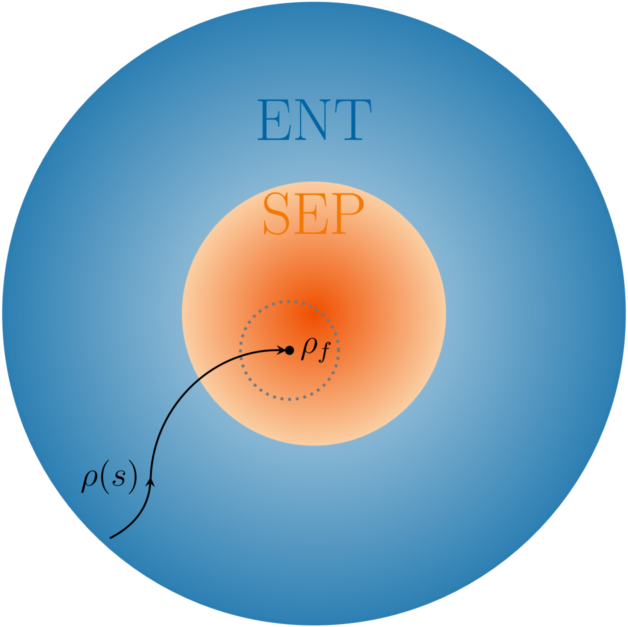
What is the **fate of entanglement** during such evolutions?

Separable continent



Applications and entanglement sudden death

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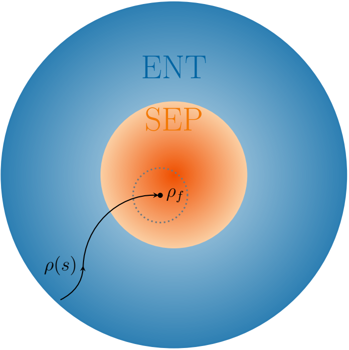

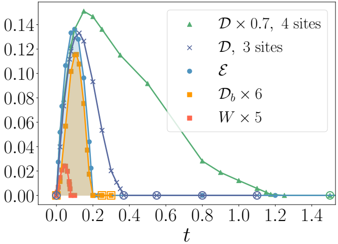

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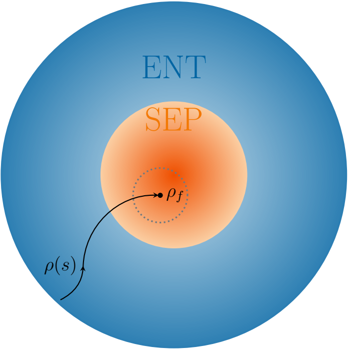
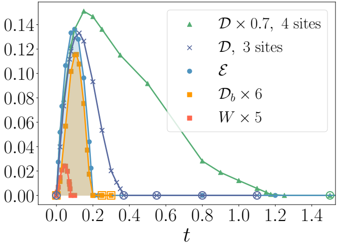
Applications and entanglement sudden death





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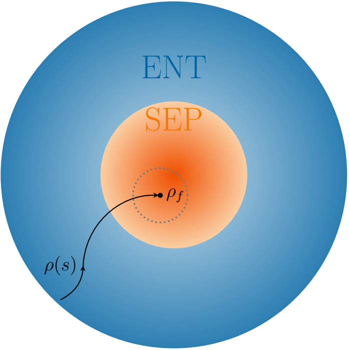
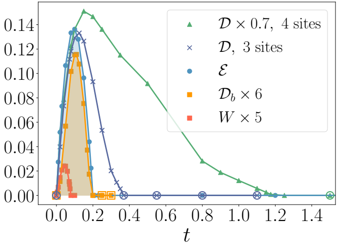
Applications and entanglement sudden death





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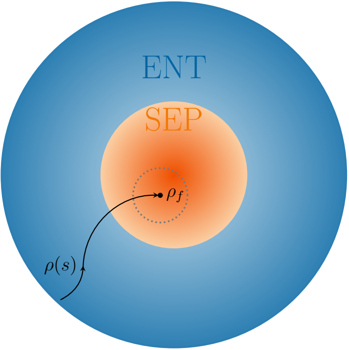

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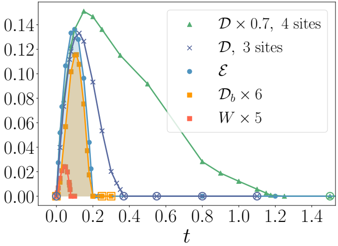
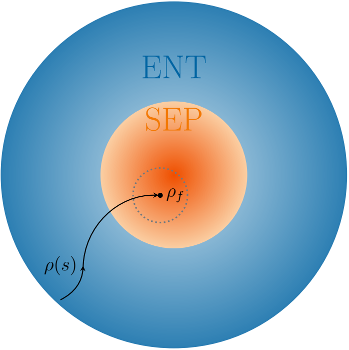
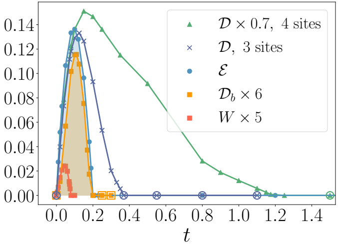


Applications and entanglement sudden death





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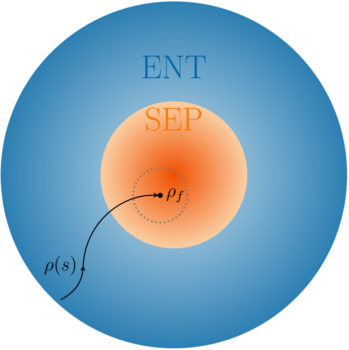
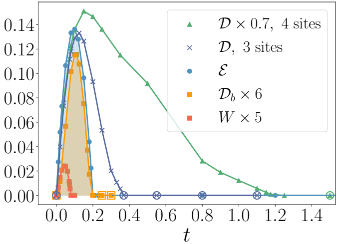
Applications and entanglement sudden death





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- Time t:
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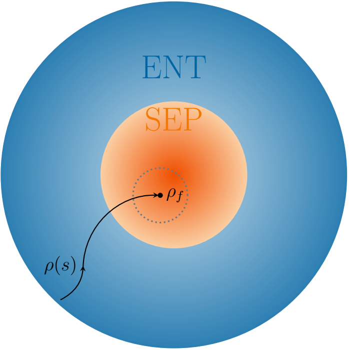
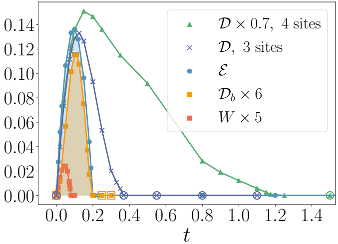
Applications and entanglement sudden death





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  - $\rho(T) \propto \mathsf{e}^{-H/T}$; $\rho_f \propto \mathbb{I}$
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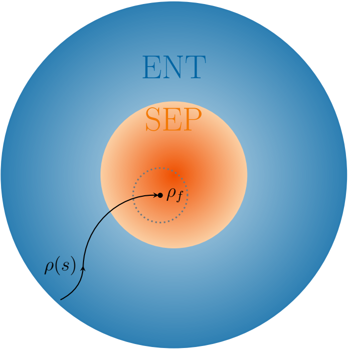

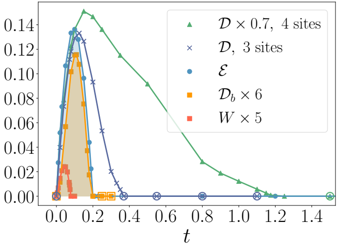

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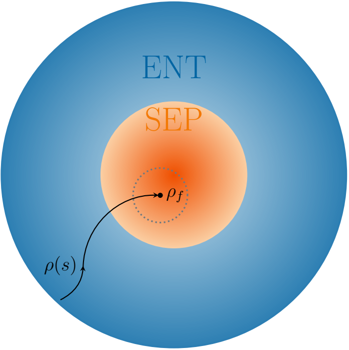

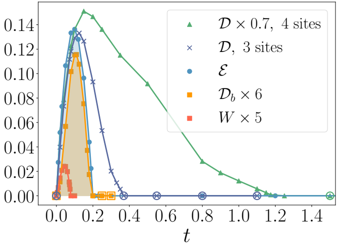

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Sudden death with distance: Quantum criticality

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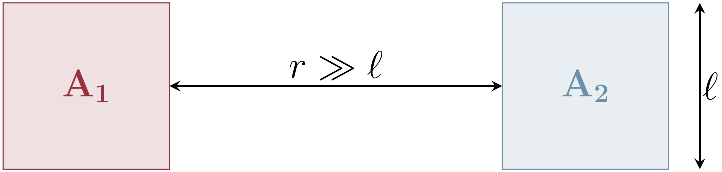

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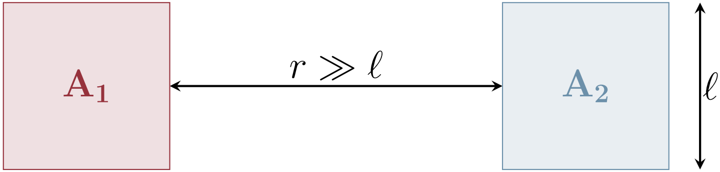

CFTs in arbitrary dimensions:

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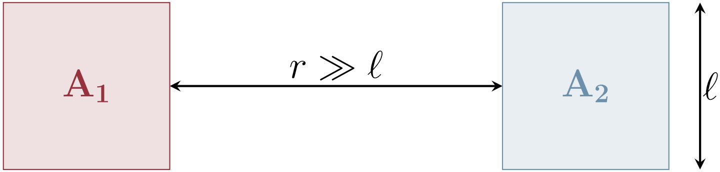

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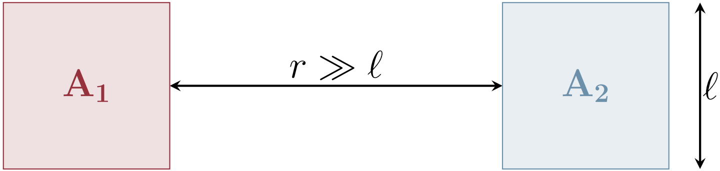

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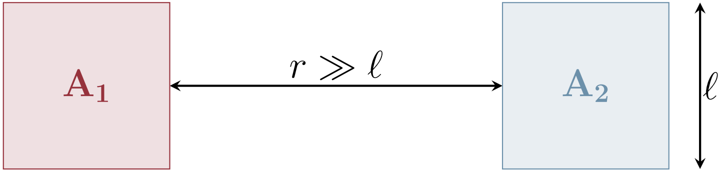

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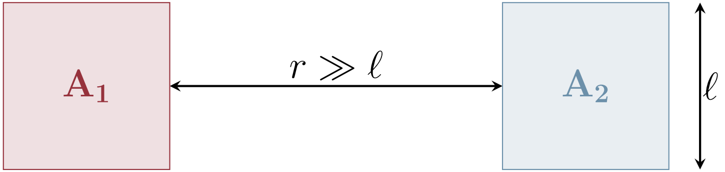

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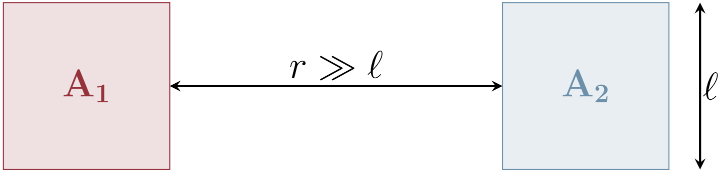

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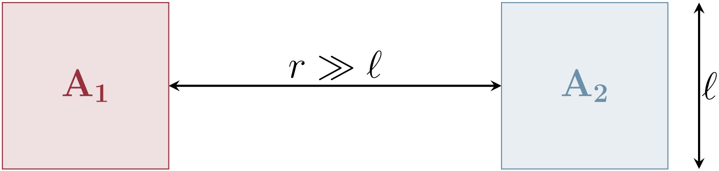

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Long-range correlations **do not imply** long-range entanglement!

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States built from your favourite statistical model [Rokhsar, Kivelson, 1988]

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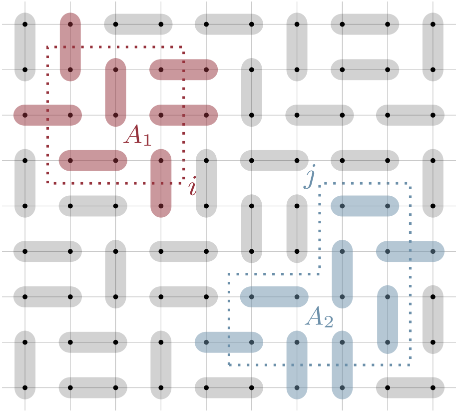

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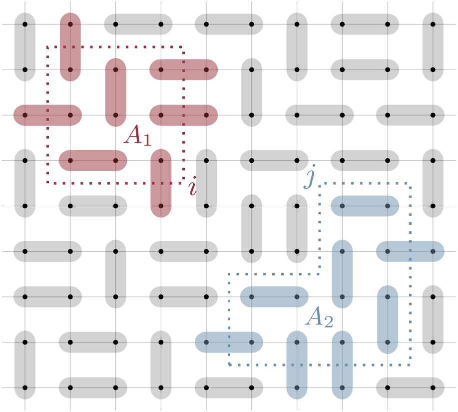

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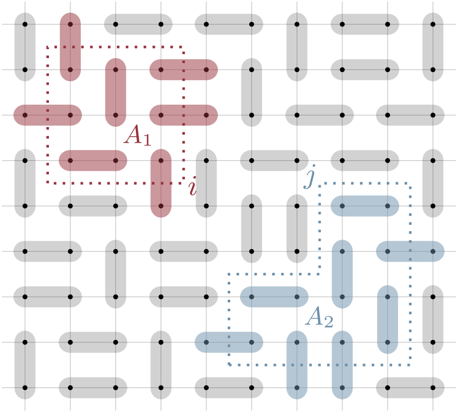

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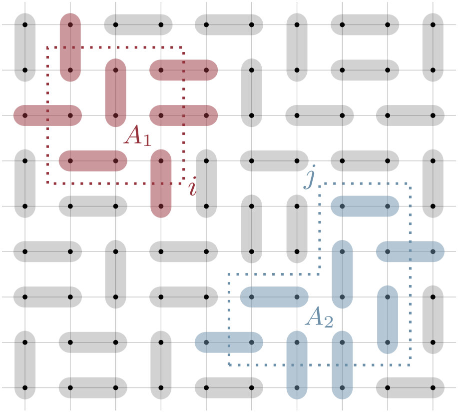

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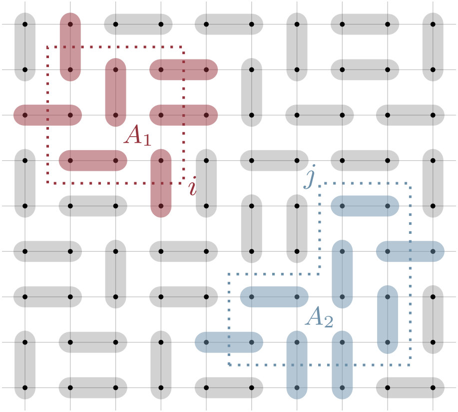

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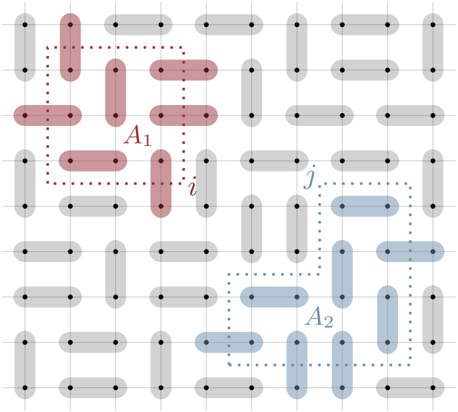

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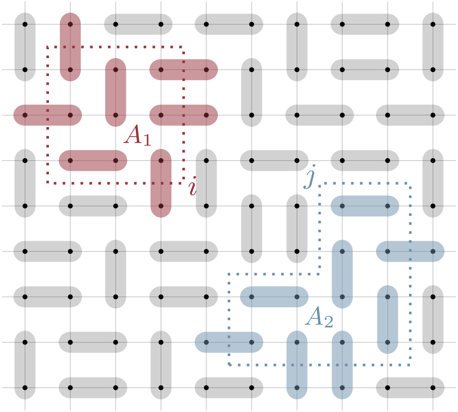

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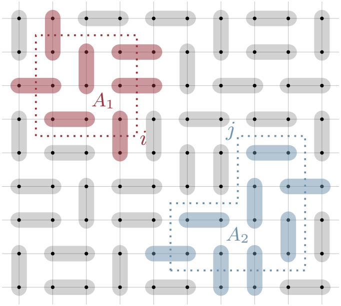
In the following, we focus on dimer RK states

$$E(c) = 0, \quad Z = \#\text{dimer configurations}$$

Separability for RK states on arbitrary lattice

[GP, Berthiere, Witczak-Krempa, 2023]



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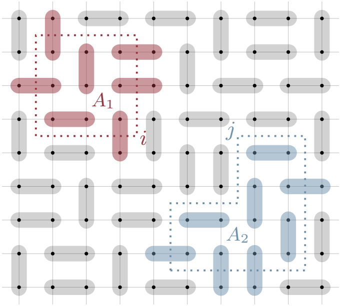

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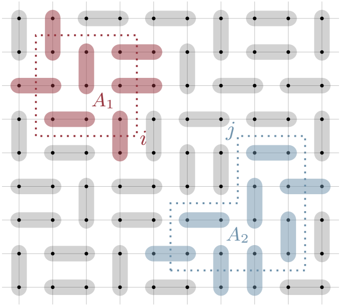

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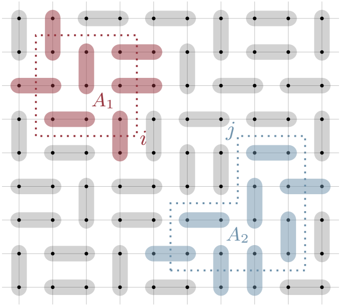

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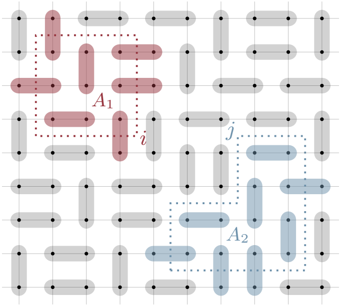

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The matrix $\boldsymbol{\rho_{A_1 \cup A_2}}$ is separable!

(for disjoints $\mathbf{A_1}$ and $\mathbf{A_2}$)

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