TENSOR NETWORKS FOR CLASSICAL FRUSTRATED SPIN SYSTEMS

Jeanne Colbois | Institut Néel

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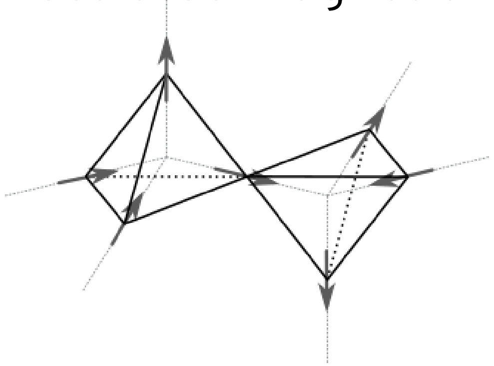
Frustrated magnetism



Classical statistical mechanics

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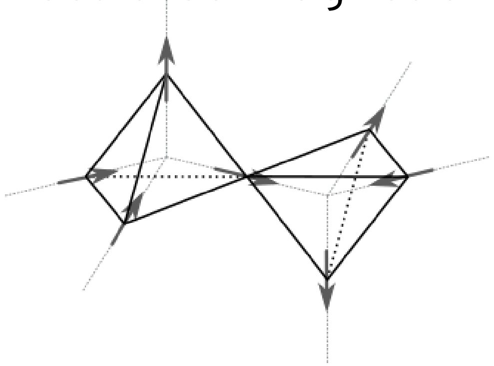

Classical statistical mechanics

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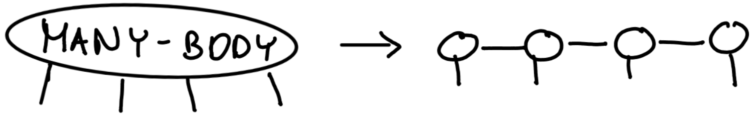

ACKNOWLEDGEMENTS

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EPFL | Switzerland

PRR **3**.013041 (arXiv:2006.14341)

PRB **106**.174403 (arXiv:2206.11788)

arXiv:2505.05889

SCOPE

1. **Classical** frustrated **magnetism**?
2. Tensor networks for classical statistical mechanics
3. Frustrated magnetism : Ising models as **weighted counting problems**
4. Some applications & perspectives

FRUSTRATED MAGNETISM

What is it?

Why study it?

Why is it difficult?

ISING MODELS OF FRUSTRATED MAGNETISM

$$H = J \sum_{\langle i,j \rangle} \sigma_i \sigma_j \qquad \sigma_i = \pm 1$$

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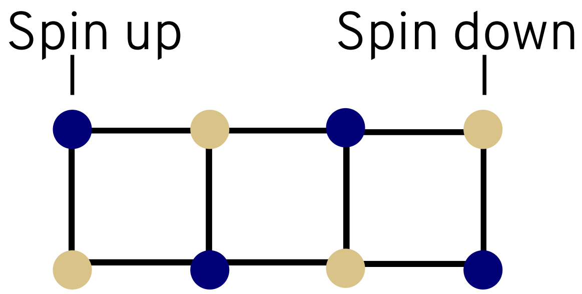

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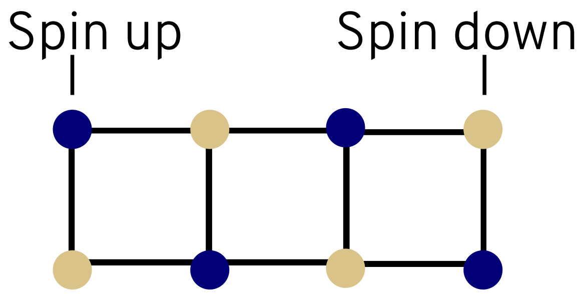

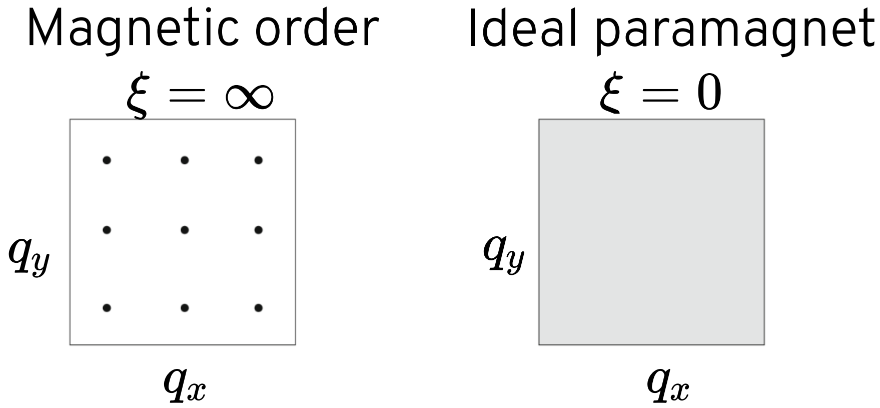

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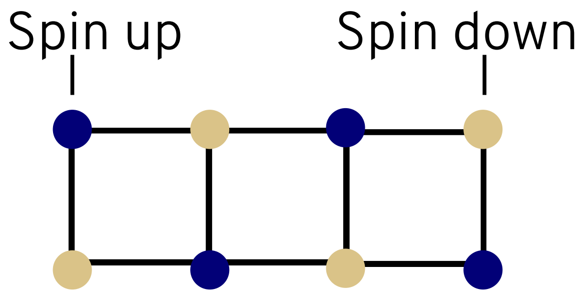
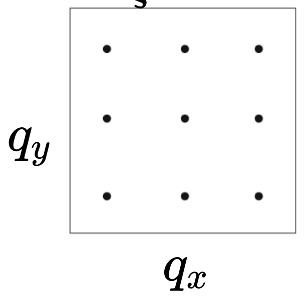


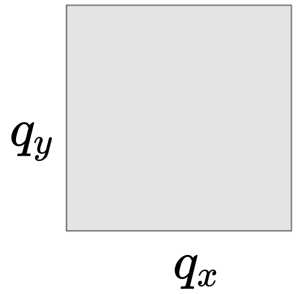
Magnetic order

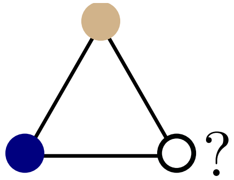
$\xi = \infty$

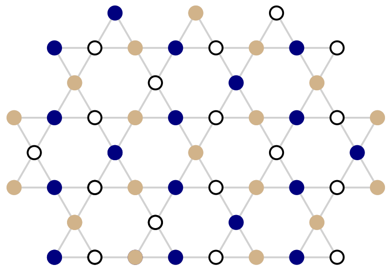


Ideal paramagnet

$\xi = 0$







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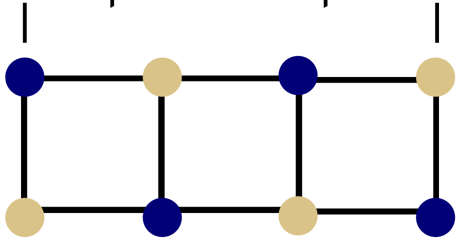
Spin up Spin down



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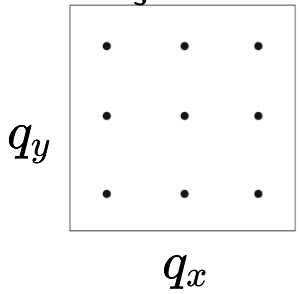

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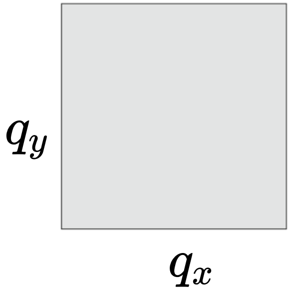

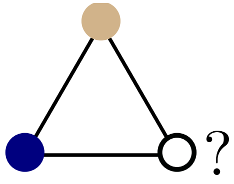

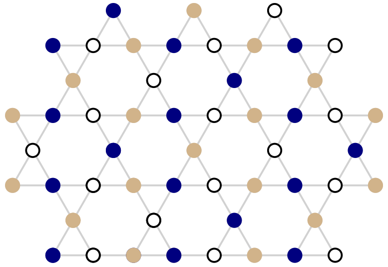

$$W_{G.S.} \gtrsim 2^{N/3}$$

Competing interactions

2100 sites : 2^{700} ground states!

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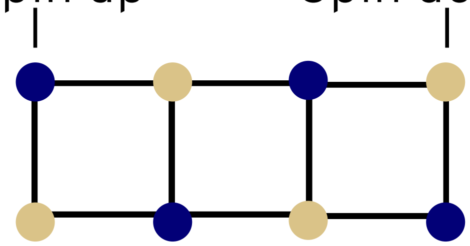

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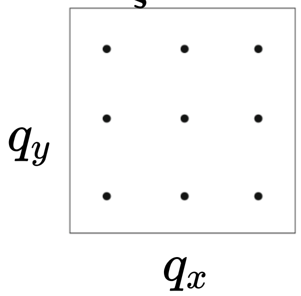

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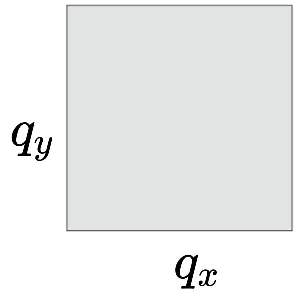

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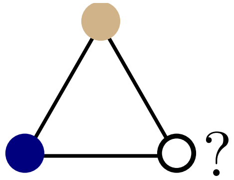

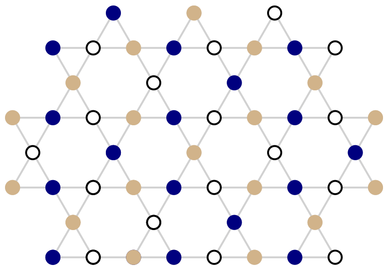

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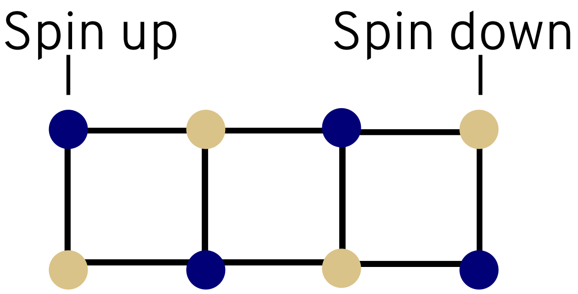

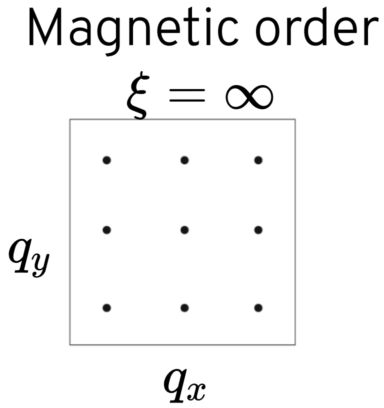

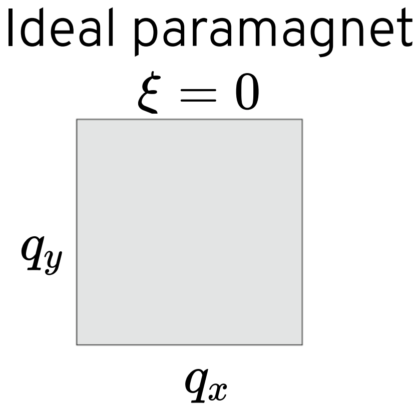

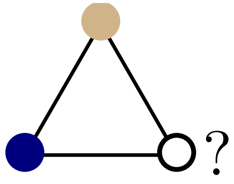

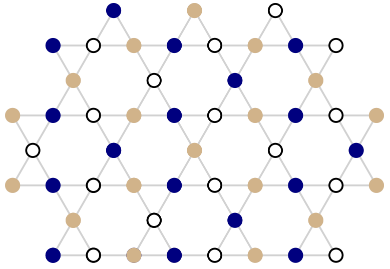

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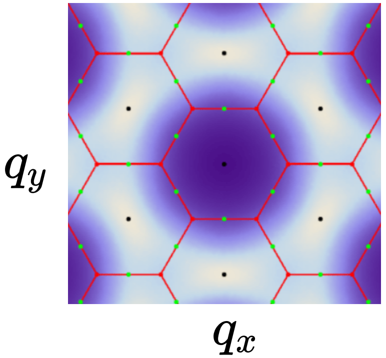
Spin liquid

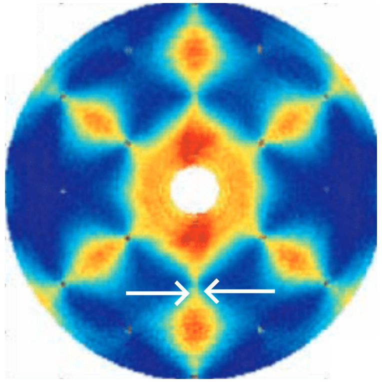


$$\xi = 1.2506...$$

A. Sütö, Z. Phys. B **44**, (1981)
W. Apel, H.-U. Everts, J. Stat. Mech, (2011)

EMERGENT PHENOMENA FROM THE LOCAL CONSTRAINTS

$\xi = \infty$



Fennell et al. (2009)
Neutron scattering on Ho2Ti2O7

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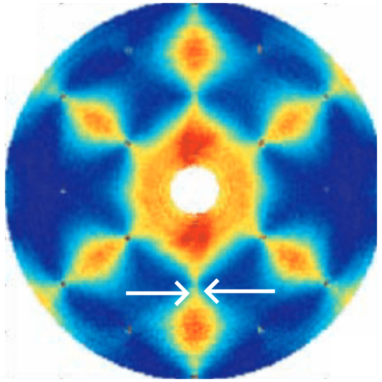

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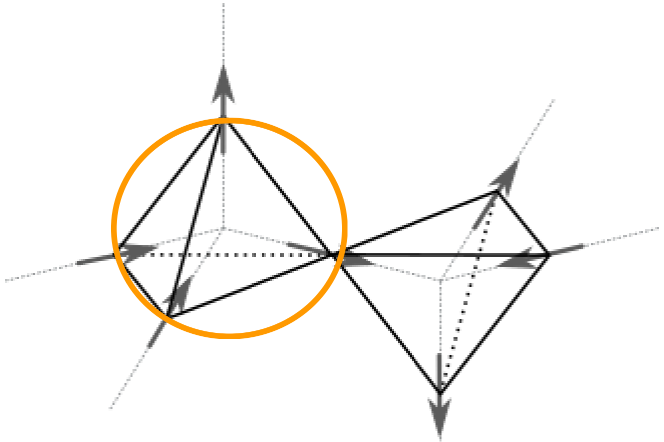

Gauss law $\nabla \cdot \mathbf{B} = 0$

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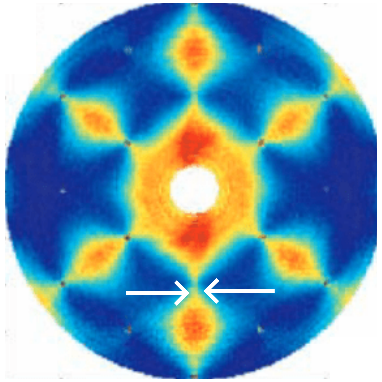

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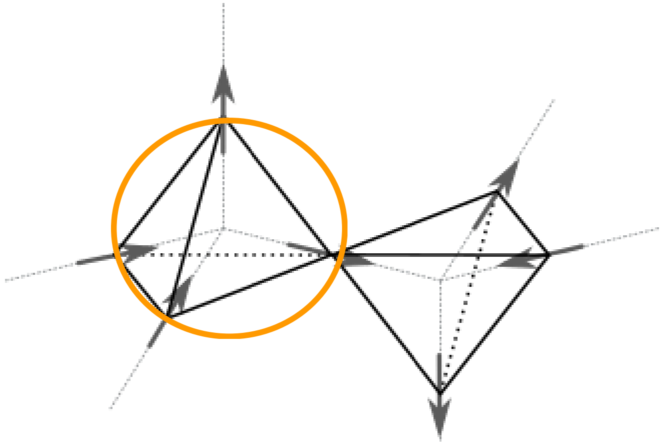

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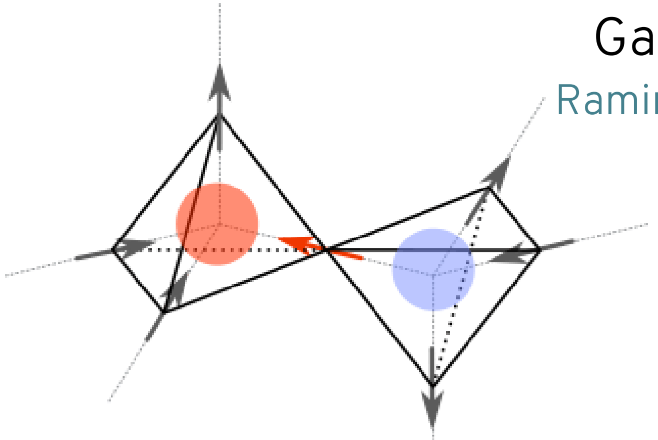

Effective Coulomb interaction $|F| \propto q^2/r^2$

Henley (2005, 2010), Castelnovo etal(2008)

Classical / quantum electrodynamics

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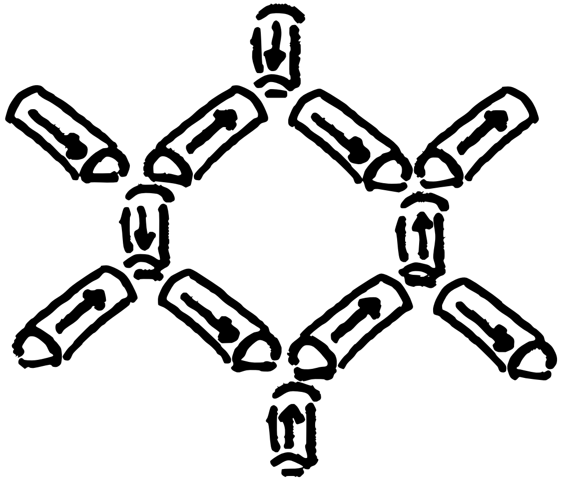

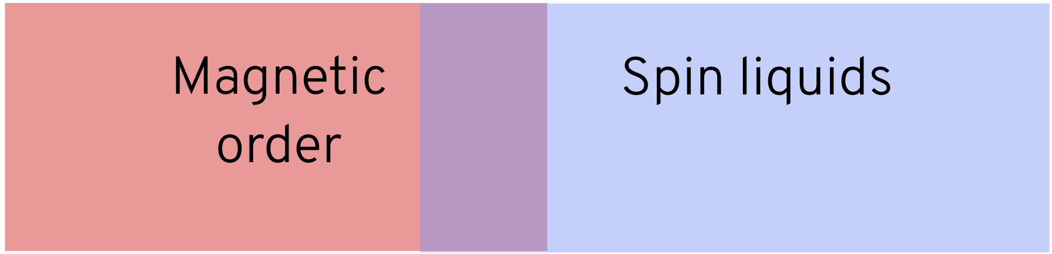

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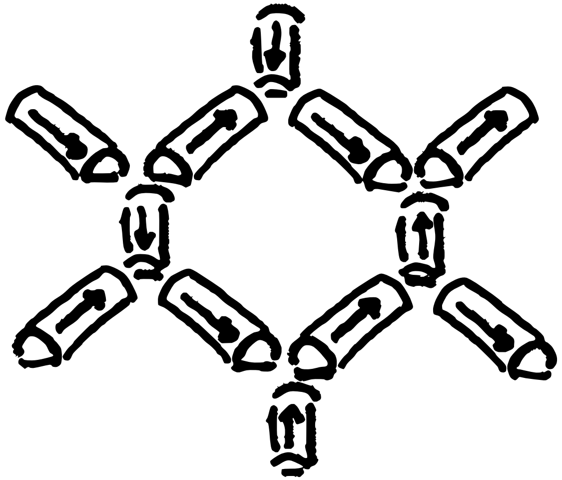

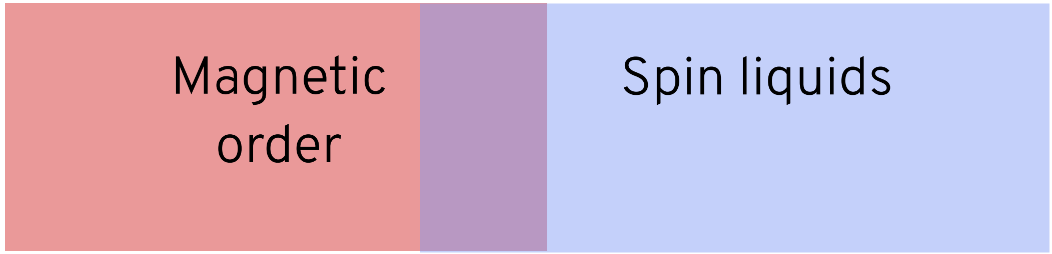

Each spin participates to both phases!

Brooks-Bartlett et al (2014), Canals et al (2016), Petit et al (2016)

Magnetic moment fragmentation

SOLVING ISING MODELS IS HARD

1D Ising

Solved exactly 1925

Ising

Planar Ising models

Solved exactly 1944 / 1960

Onsager

Non-planar Ising models :

no closed forms

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2^Nx2^N

A VERSION OF THE N-BODY PROBLEM

Counting the number of ground states in spin glasses

#P-complete Barahona (1982)

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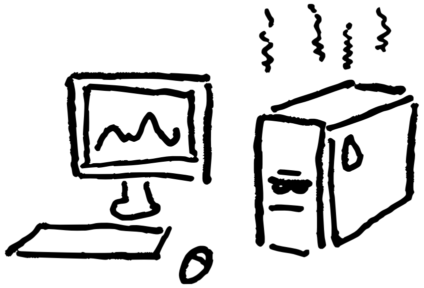
#P-complete Barahona (1982)

Methods:

Monte Carlo, series expansion, RG & CFT

Ergodicity issues

Limited to some regimes



TENSOR NETWORKS FOR STATISTICAL MECHANICS

What *is* a tensor network ?

Tensor networks as compression schemes

Ising models as tensor networks

Physique théorique

Réseaux de tenseurs : les ordinateurs classiques contre-attaquent

Antoine Tilloy

28 août 2024 | POUR LA SCIENCE N° 563 | Temps de lecture : 26 mn

Physique théorique

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2D quantum and more

Topological order

Two-dimensional t-J model

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Simulation of quantum circuits
Challenging quantum supremacy
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Vidal (2003),
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Vidal (2003),
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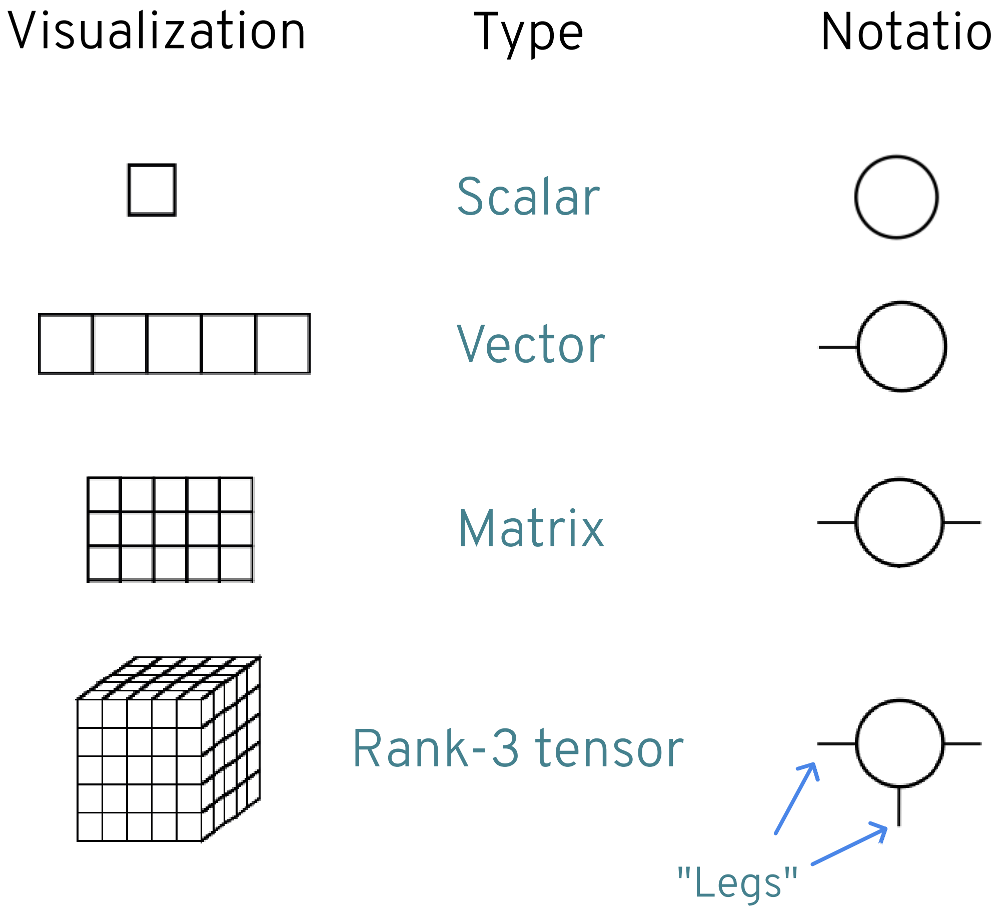
Chemistry, machine learning, mathematics...

TENSOR NETWORKS NOTATION

Visualization Type Notation

Tensor notation: **R. Penrose, in *Combinatorial Mathematics*** *and its applications*, (1971)

TENSOR NETWORKS NOTATION



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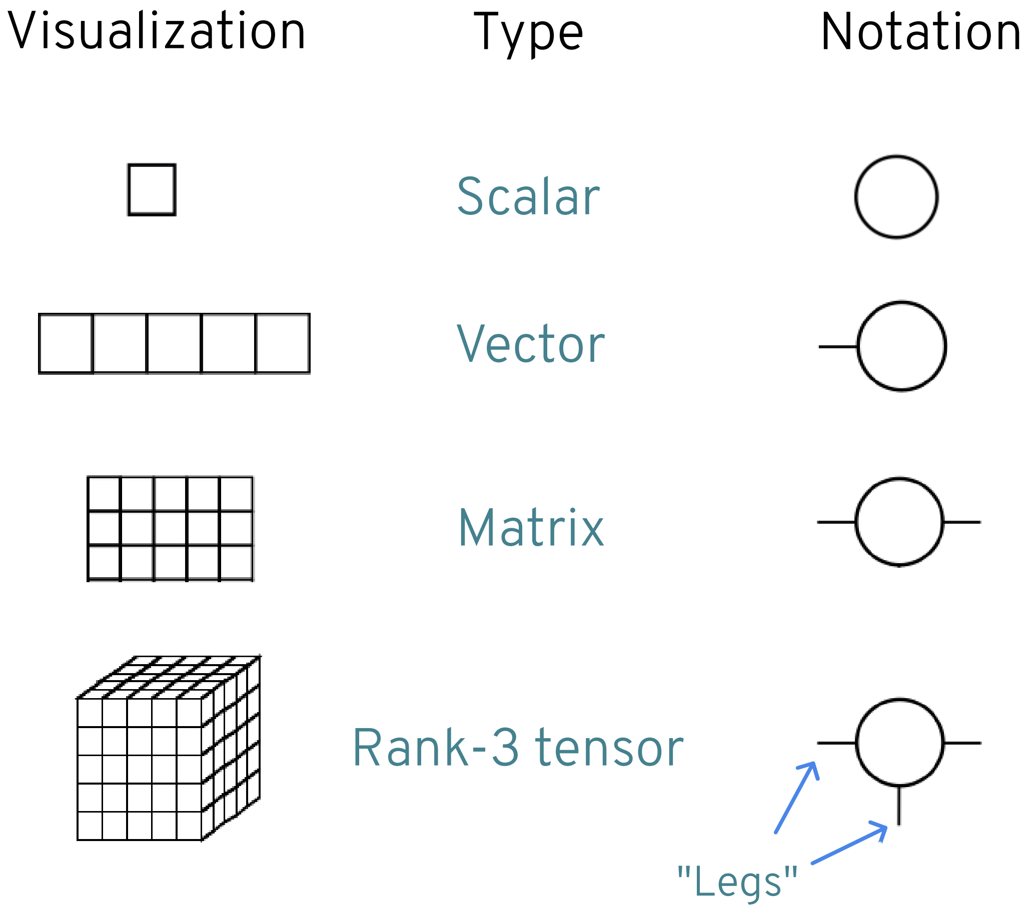

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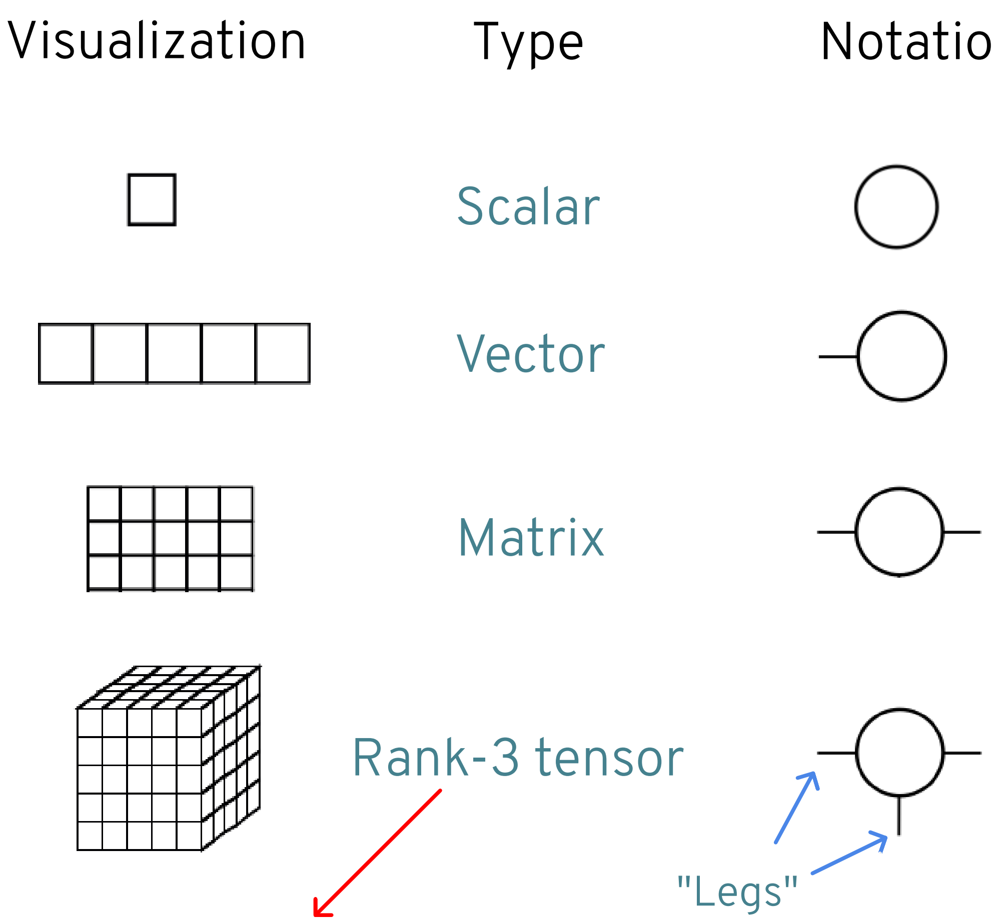

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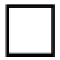
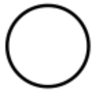
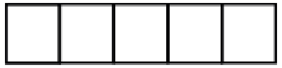
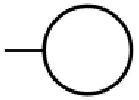
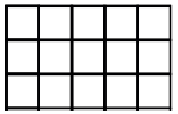
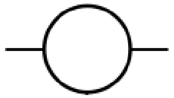
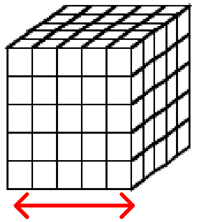
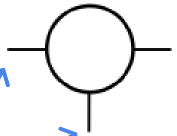
	Scalar	
	Vector	
	Matrix	
	Rank-3 tensor	

"Legs"

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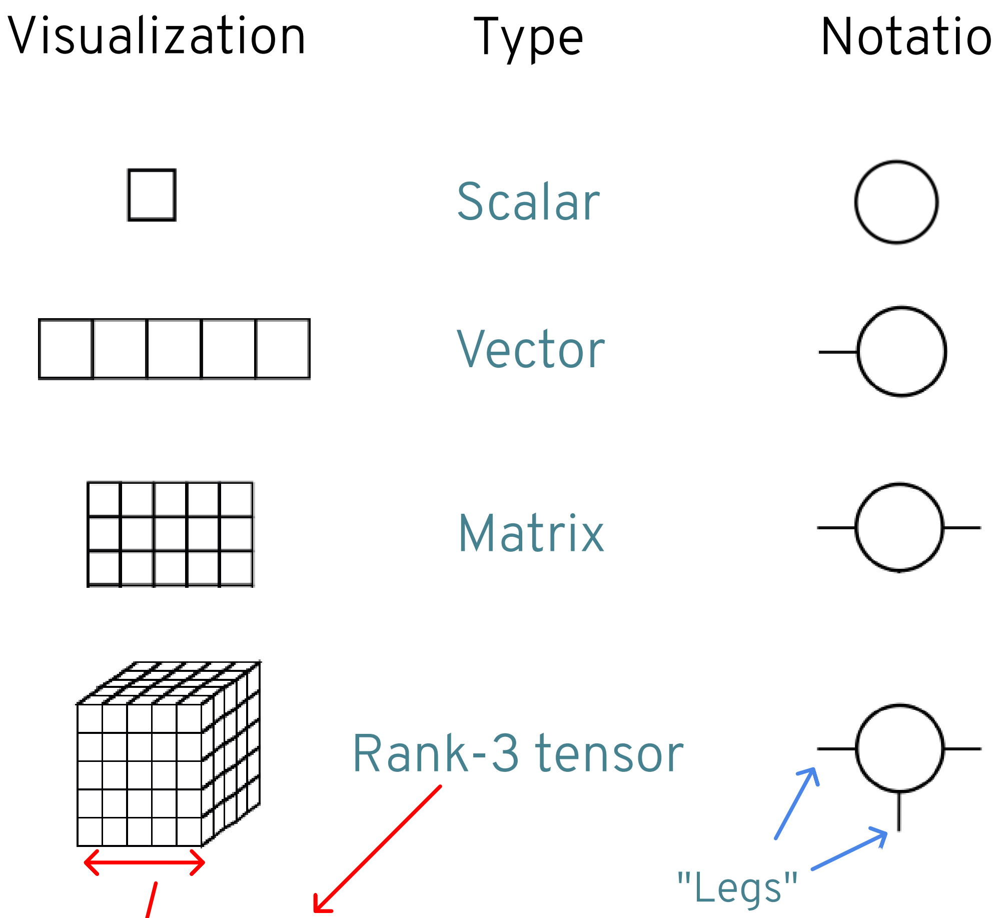

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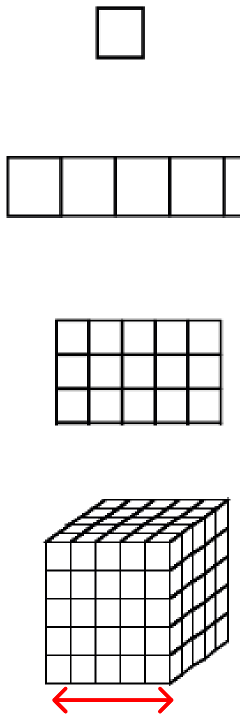
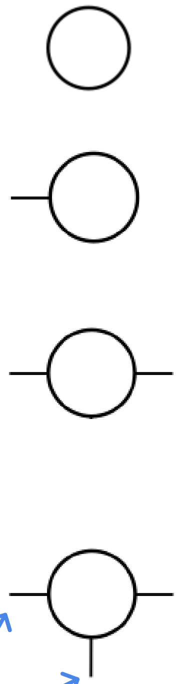
Connecting legs = make the product

"CONTRACTION"

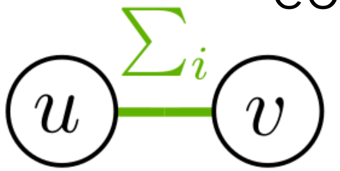
TENSOR NETWORKS NOTATION

Visualization	Type	Notation
	Scalar Vector Matrix Rank-3 tensor	

"Legs"

Connecting legs = make the product

"CONTRACTION"



Scalar product

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Size of the index = bond dimension = χ or D

Tensor notation: **R. Penrose, in *Combinatorial Mathematics*** *and its applications*, (1971)

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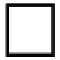
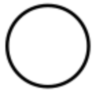
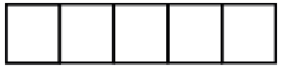
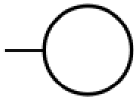
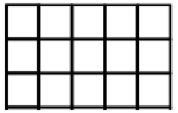
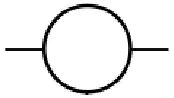
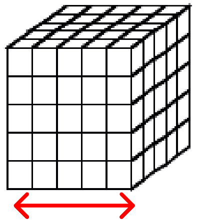
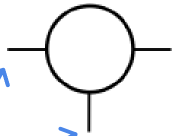
	Scalar	
	Vector	
	Matrix	
	Rank-3 tensor	

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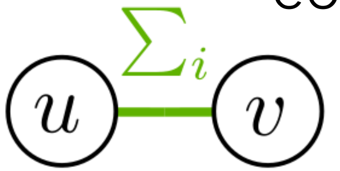
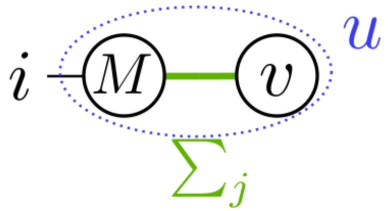


Scalar product



Matrix-vector product

Tensor notation: **R. Penrose, in *Combinatorial Mathematics*** *and its applications*, (1971)

TENSOR NETWORKS NOTATION

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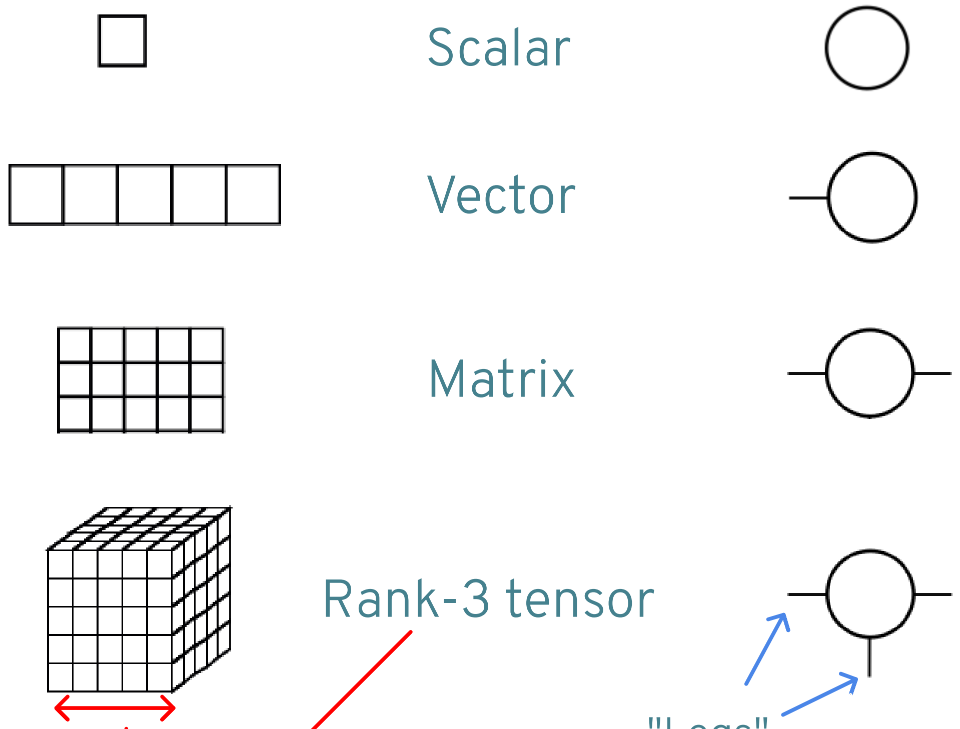

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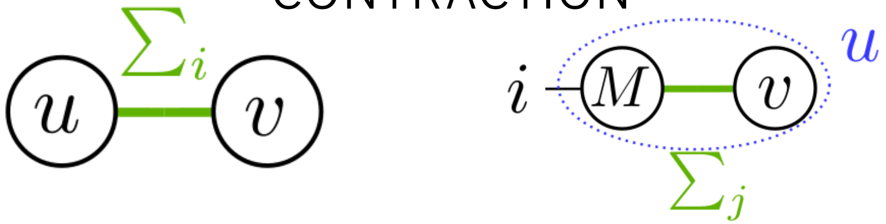
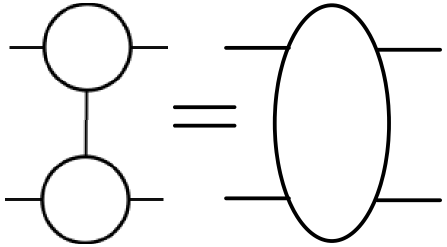


Scalar product   Matrix-vector product

You can **group** indices:



$\chi \times \chi \times \chi \times \chi$

tensor

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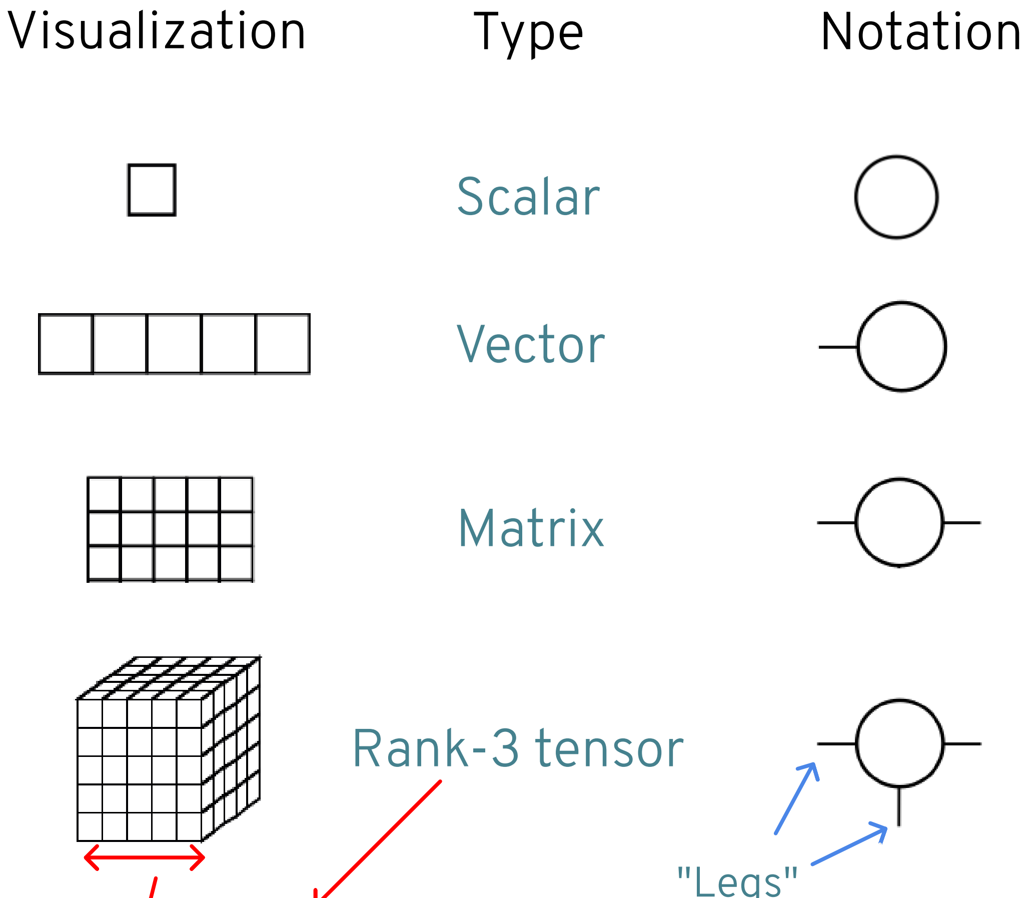
TENSOR NETWORKS NOTATION



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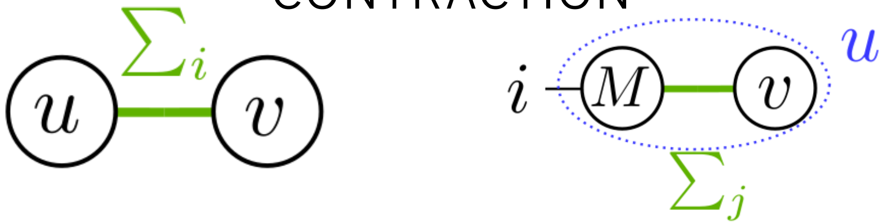

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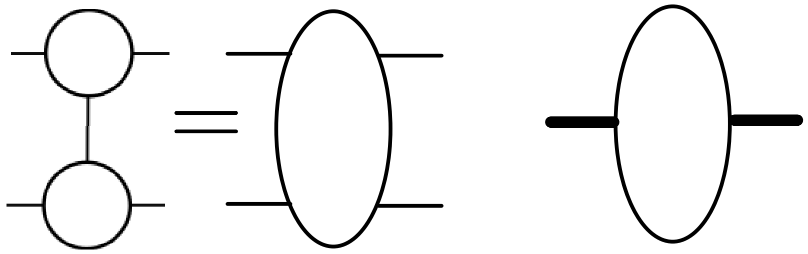

$\chi \times \chi \times \chi \times \chi$ tensor | $\chi^2 \times \chi^2$ matrix

Tensor notation: **R. Penrose, in *Combinatorial Mathematics*** *and its applications*, (1971)

COMPRESSING MANY-BODY WAVEFUNCTIONS

Many-body wavefunction

$$|\phi\rangle = \sum_{\{s_j\}} \phi_{\dots,s_{j-1},s_j,s_{j+1},\dots} |\dots, s_{j-1}, s_j, s_{j+1}, \dots\rangle$$

High number of parameters (2^L)

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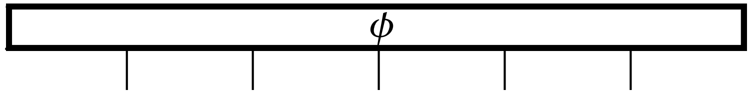

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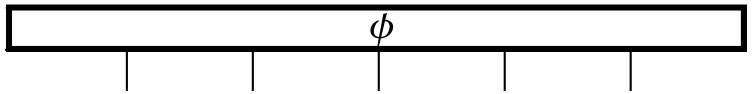

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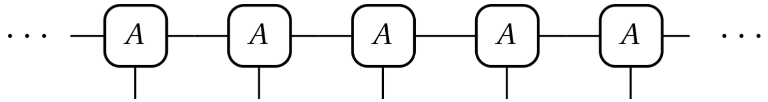
We want to "factorize" or compress it:



Much smaller number

$(\chi \times 2 \times \chi)$

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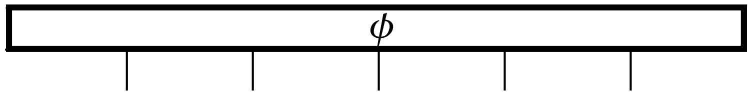

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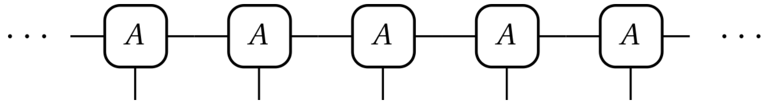

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WHY **(WHEN) DO WE EXPECT THE BOND DIMENSION TO BE LIMITED?**

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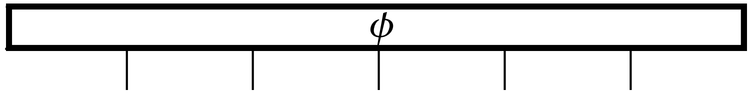

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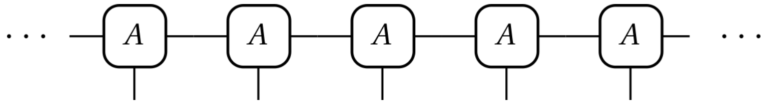

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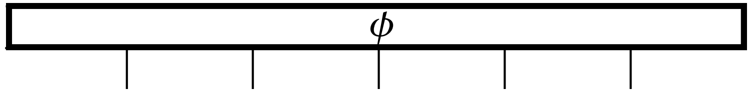

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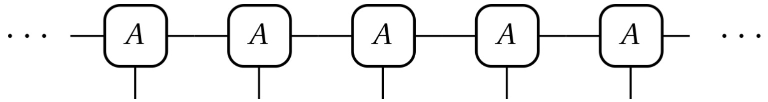

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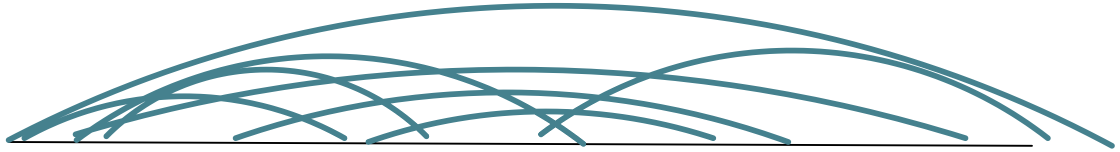

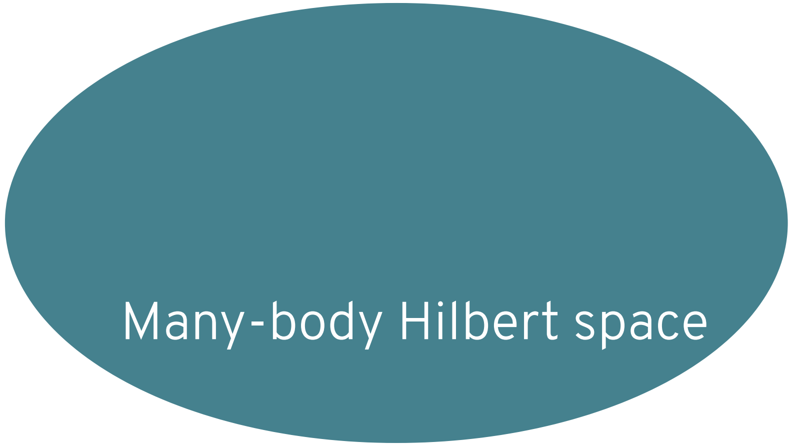

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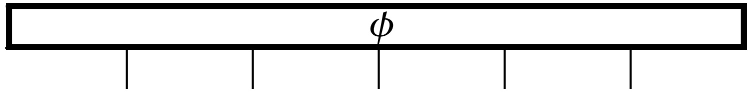

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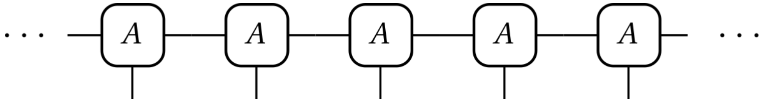

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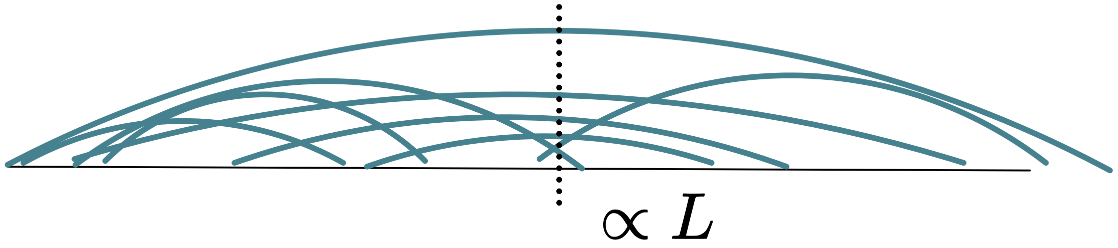

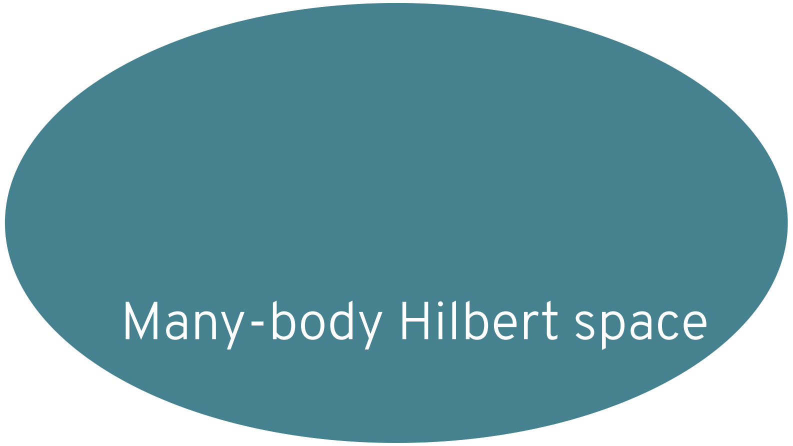

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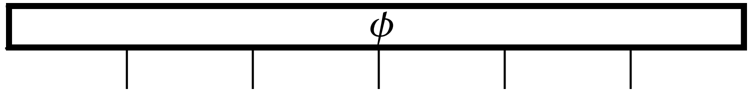

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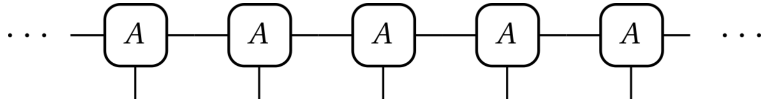

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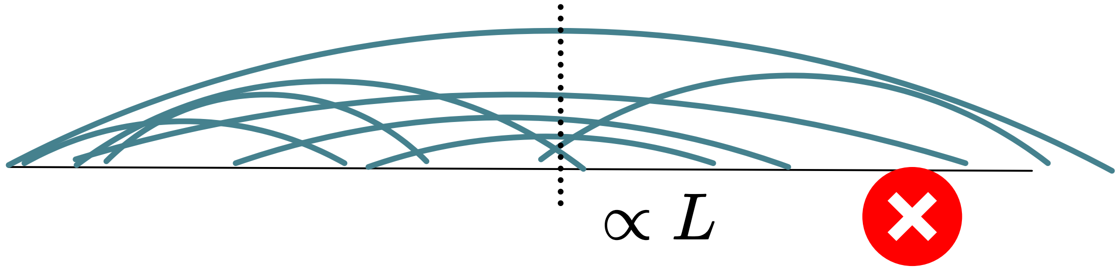

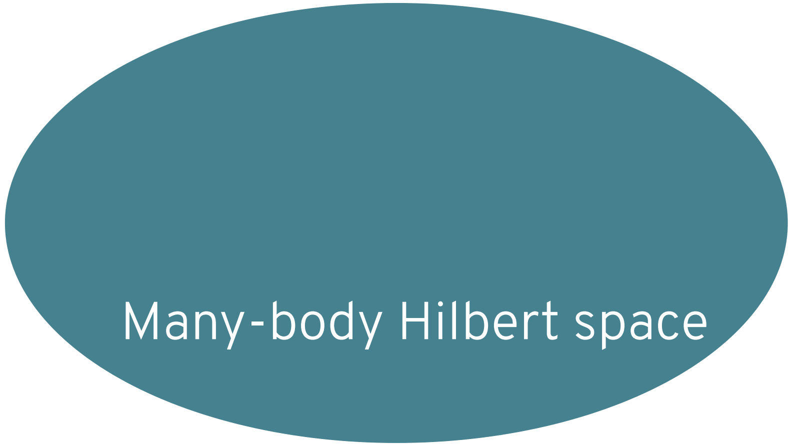

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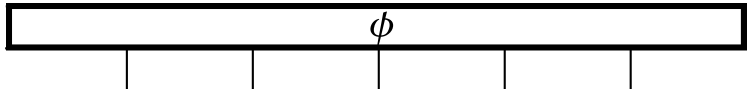

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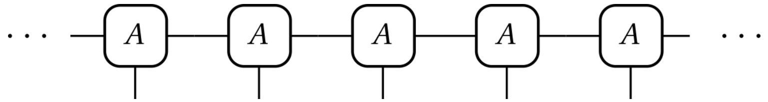

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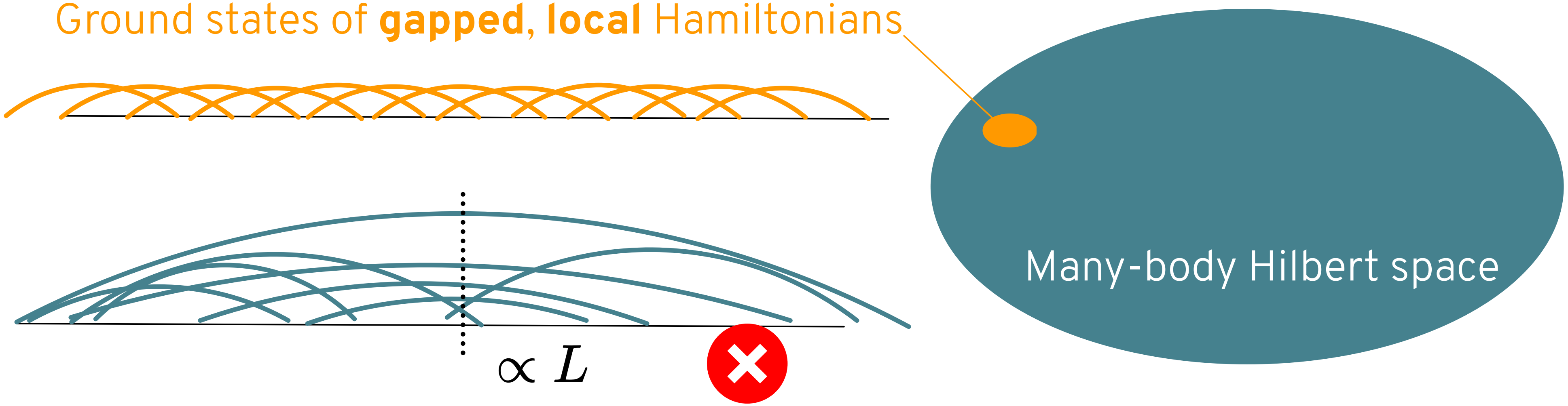
Ground states of **gapped**, **local** Hamiltonians



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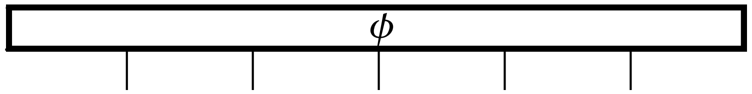

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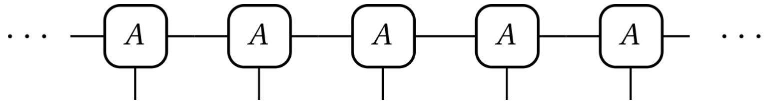

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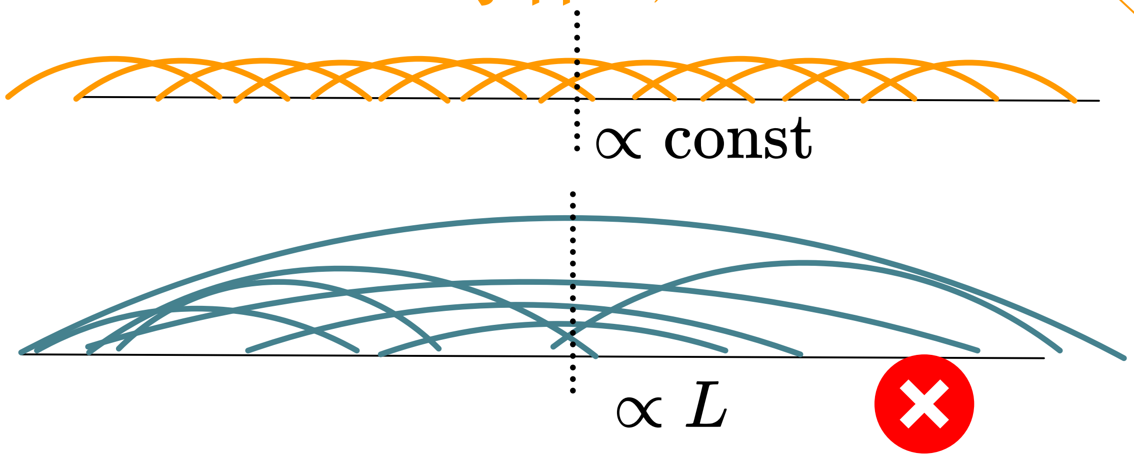

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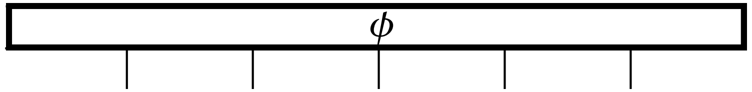

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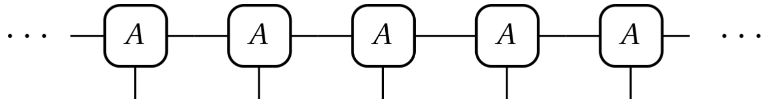

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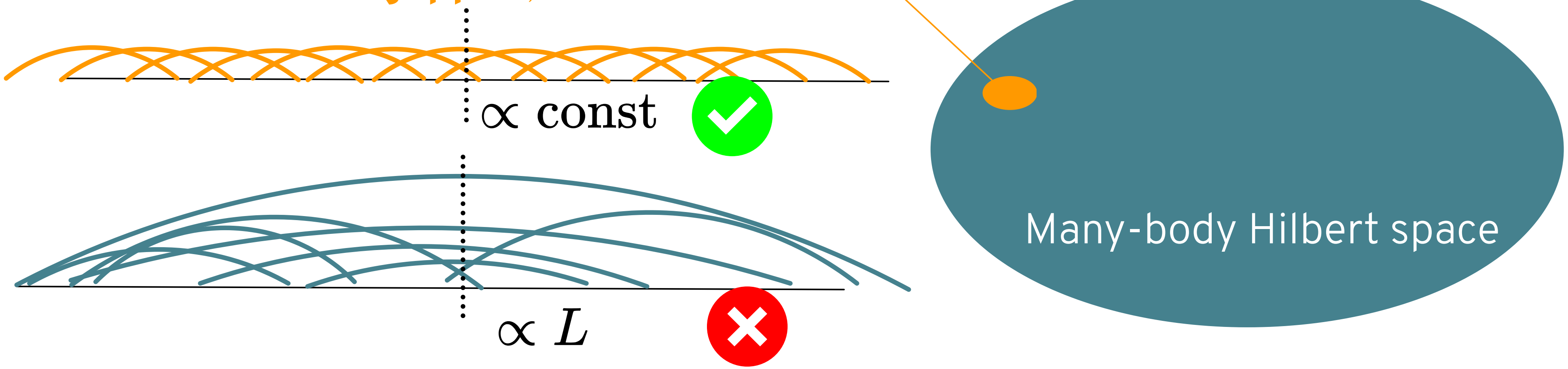

THE 1D ISING MODEL AS A TENSOR NETWORK

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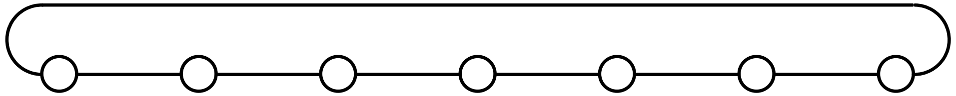

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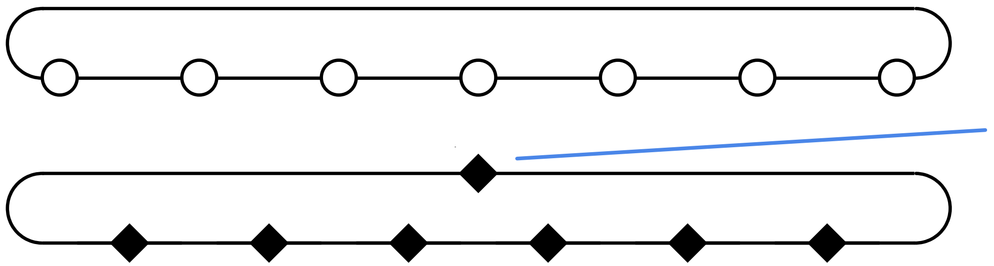

$$T = \begin{pmatrix} e^{\beta J} & e^{-\beta J} \\ e^{-\beta J} & e^{\beta J} \end{pmatrix}$$

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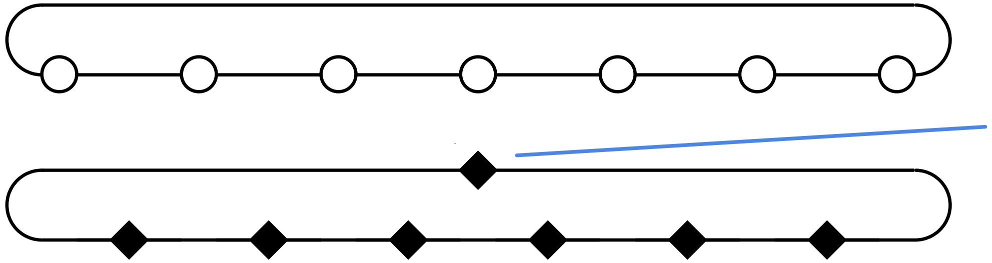

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The partition function is just the exponentiation of a 2x2 matrix!

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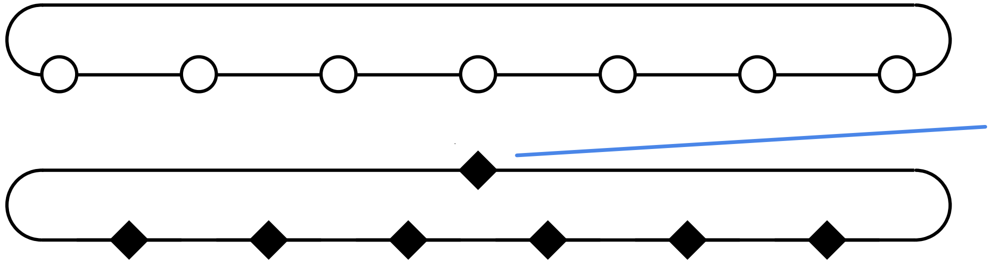

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$$T^L = \left(P^{-1} \Lambda P\right)^L = P^{-1} \Lambda^L P$$

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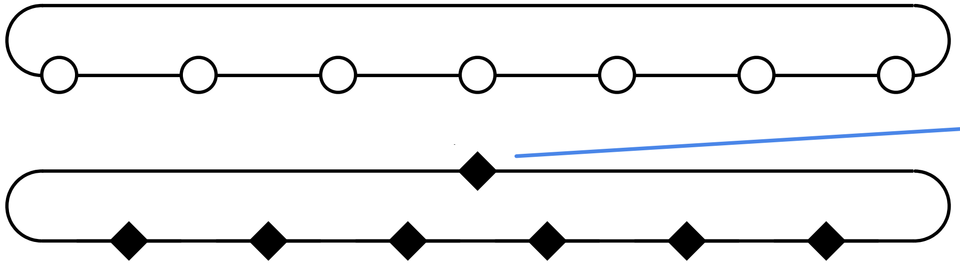

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$$T^L = \left(P^{-1}\Lambda P\right)^L = P^{-1}\Lambda^L P$$

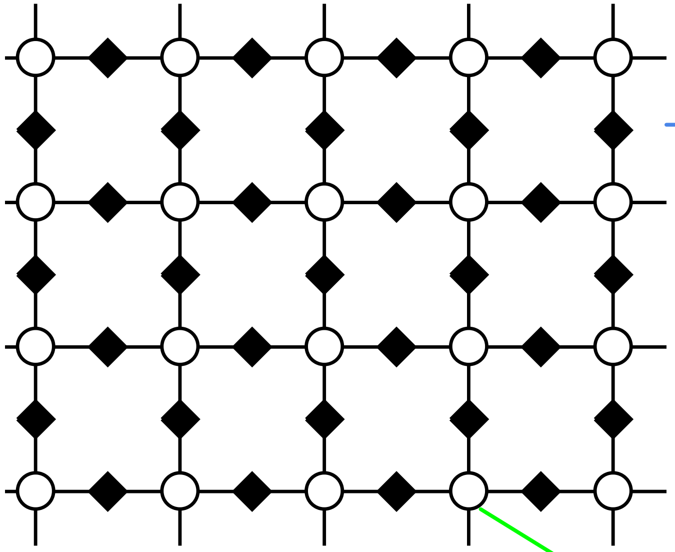
2. Compute

$$\Lambda^L = \lambda_+^L \begin{pmatrix} 1 & 0 \\ 0 & \left(\frac{\lambda_-}{\lambda_+}\right)^L \end{pmatrix} \xrightarrow[L\to\infty]{} \lambda_+^L \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

Leading eigenvalue!

THE 2D ISING MODEL AS A TENSOR NETWORK

$$\mathcal{Z} =$$



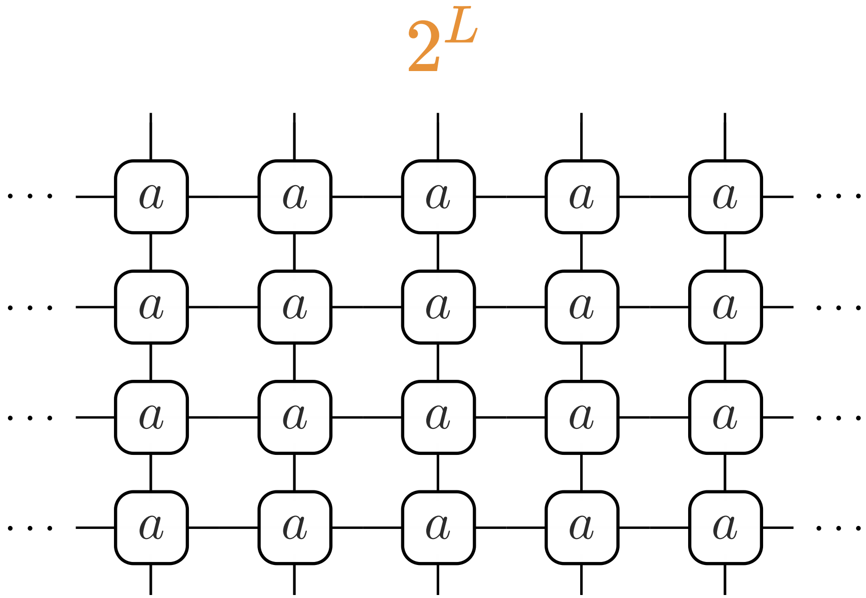
$$T = \begin{pmatrix} e^{\beta J} & e^{-\beta J} \\ e^{-\beta J} & e^{\beta J} \end{pmatrix}$$

Generalized kronecker
δ tensor

THE 2D ISING MODEL AS A TENSOR NETWORK

Decomposition & reshaping:

$\mathcal{Z} =$



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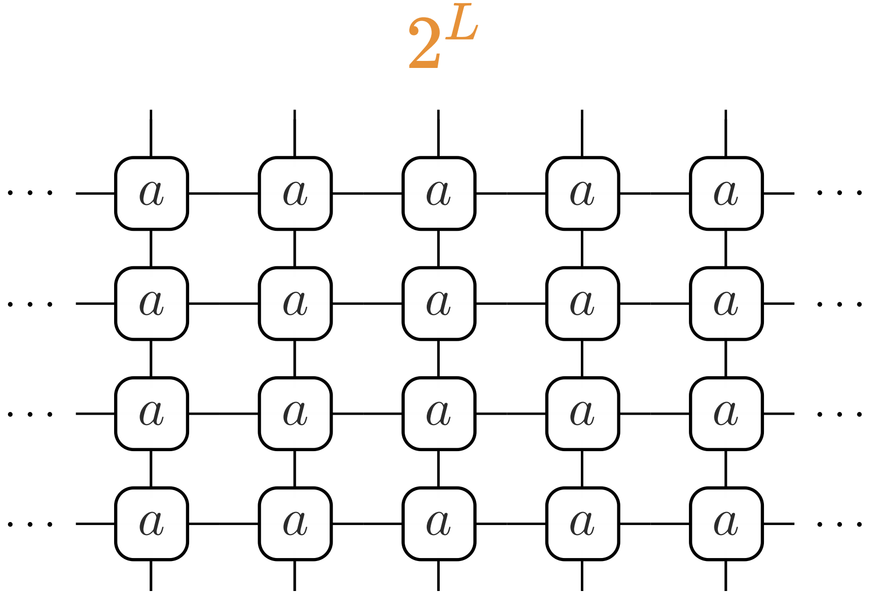
$\mathcal{Z} =$ 

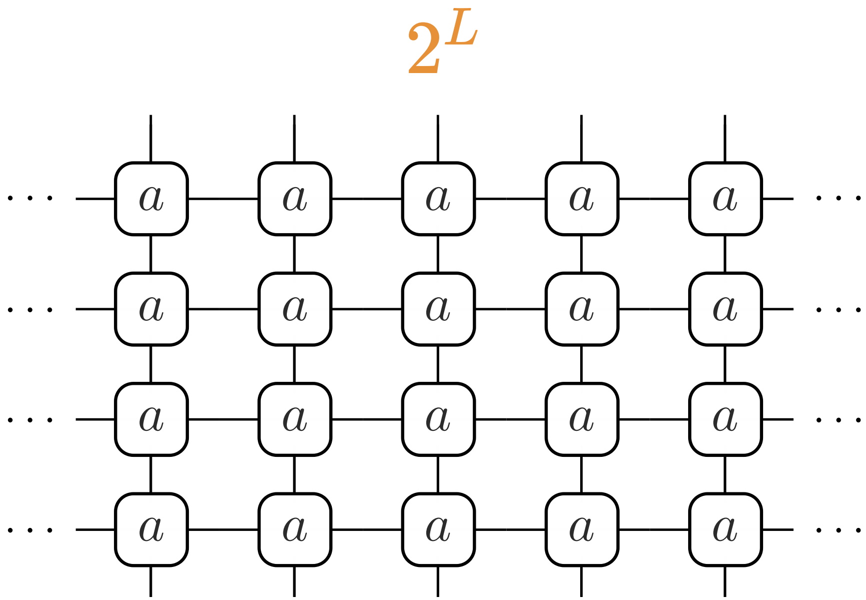
EVALUATE THE PARTITION FUNCTION ?

Use the right level of description to catch the phenomena of interest. Don't model bull-dozers with quarks.

Goldenfeld & Kadanoff, Science, **284** (1999)

THREE MAIN SCHEMES

$\mathcal{Z} =$



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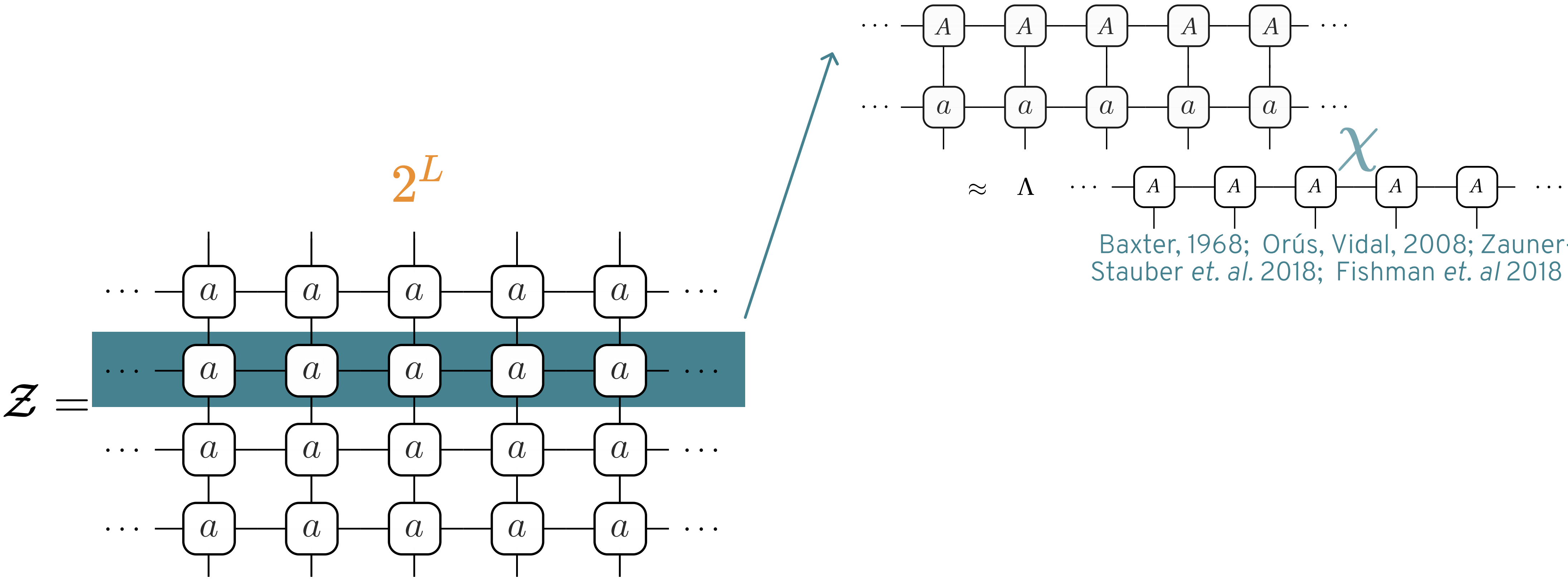

Baxter, 1968; Orús, Vidal, 2008; Zauner-Stauber *et. al.* 2018; Fishman *et. al* 2018

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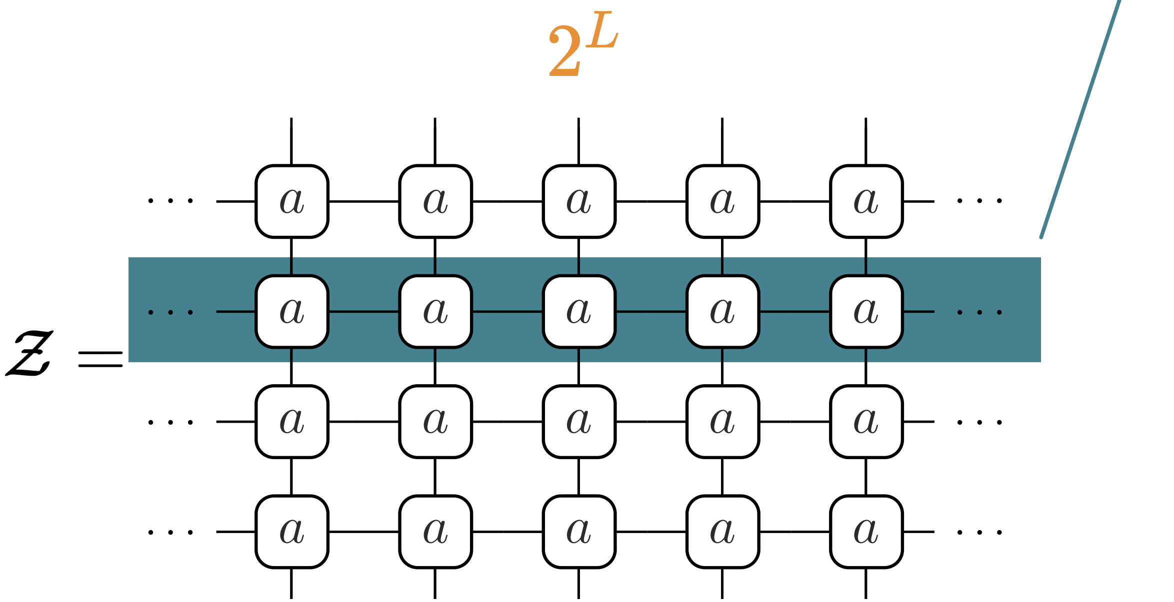
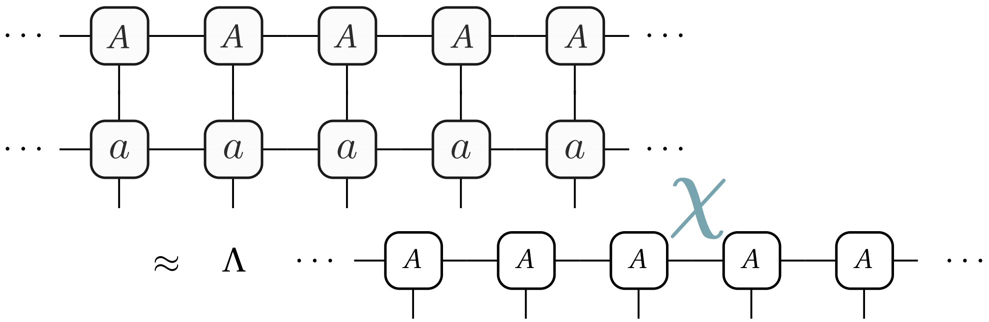
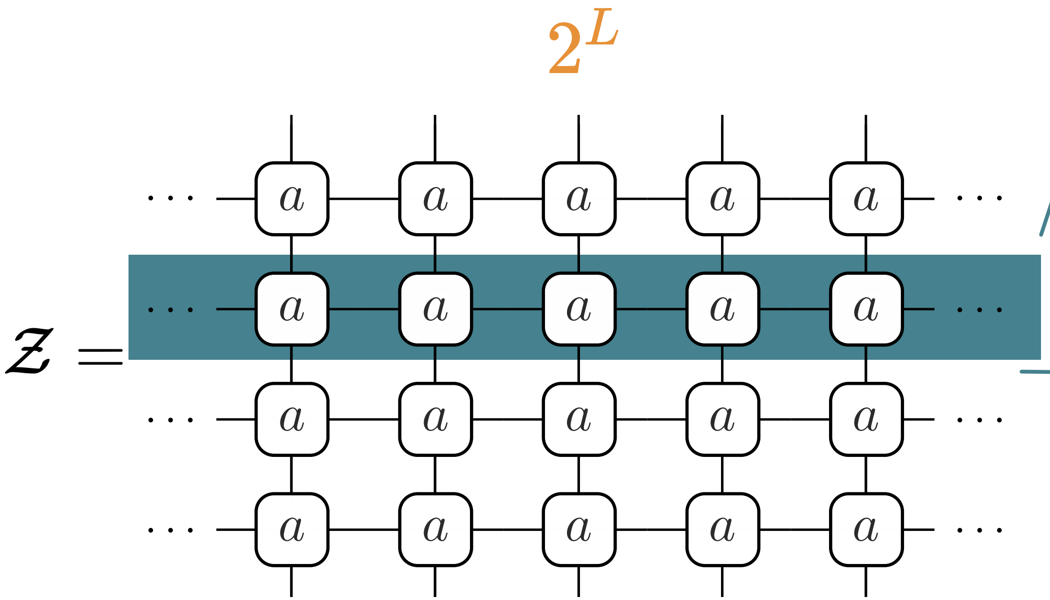


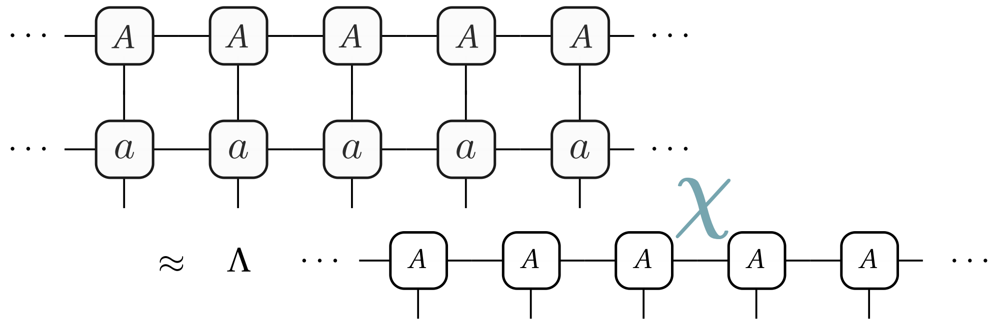


Baxter, 1968; Orús, Vidal, 2008; Zauner-Stauber *et. al.* 2018; Fishman *et. al* 2018

2D classical is "like" 1D quantum

THREE MAIN SCHEMES





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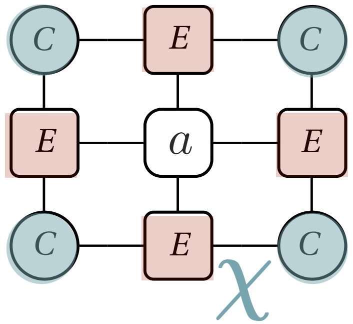
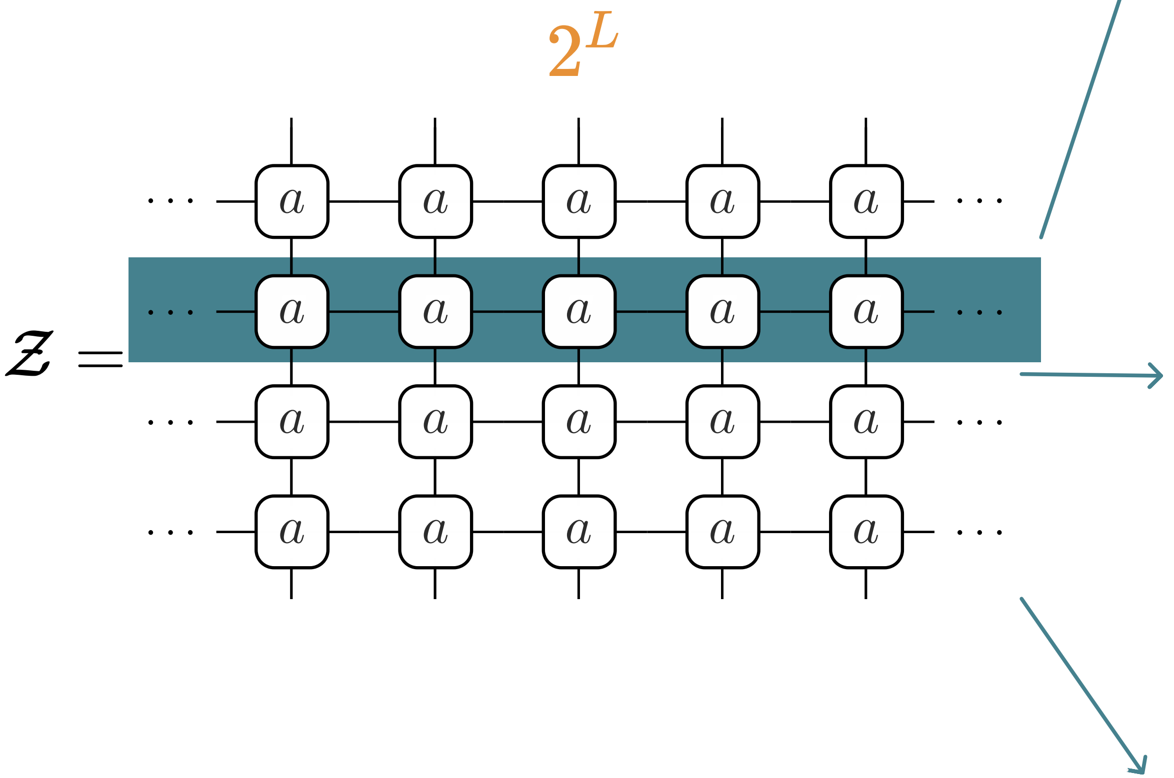
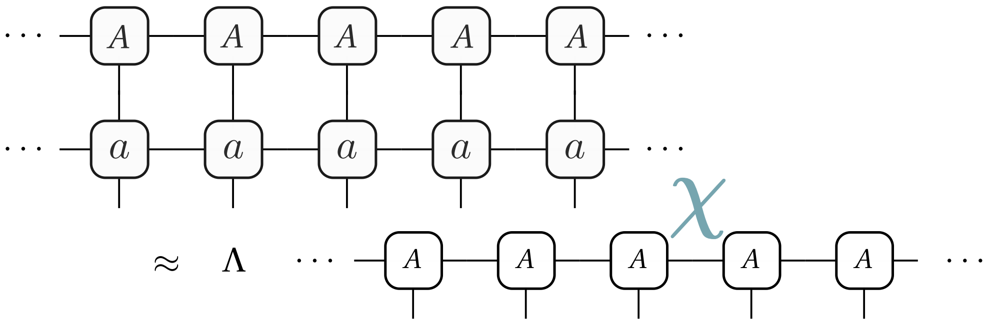


R. J. Baxter, 1968; T. Nishino, K. Okunishi, 1996;
Corboz et al (2014), ...

Building block for quantum problems :
algorithms are already optimized

THREE MAIN SCHEMES





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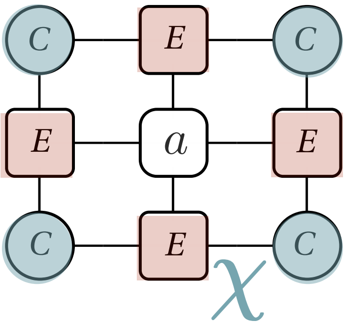

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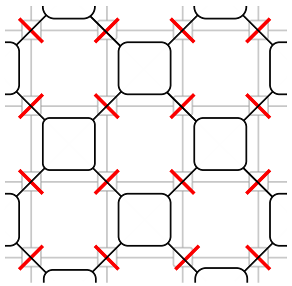

Levin & Nave, 2007; Evenbly & Vidal (2014); Ebel, Kennedy, Rychkov (2025)....

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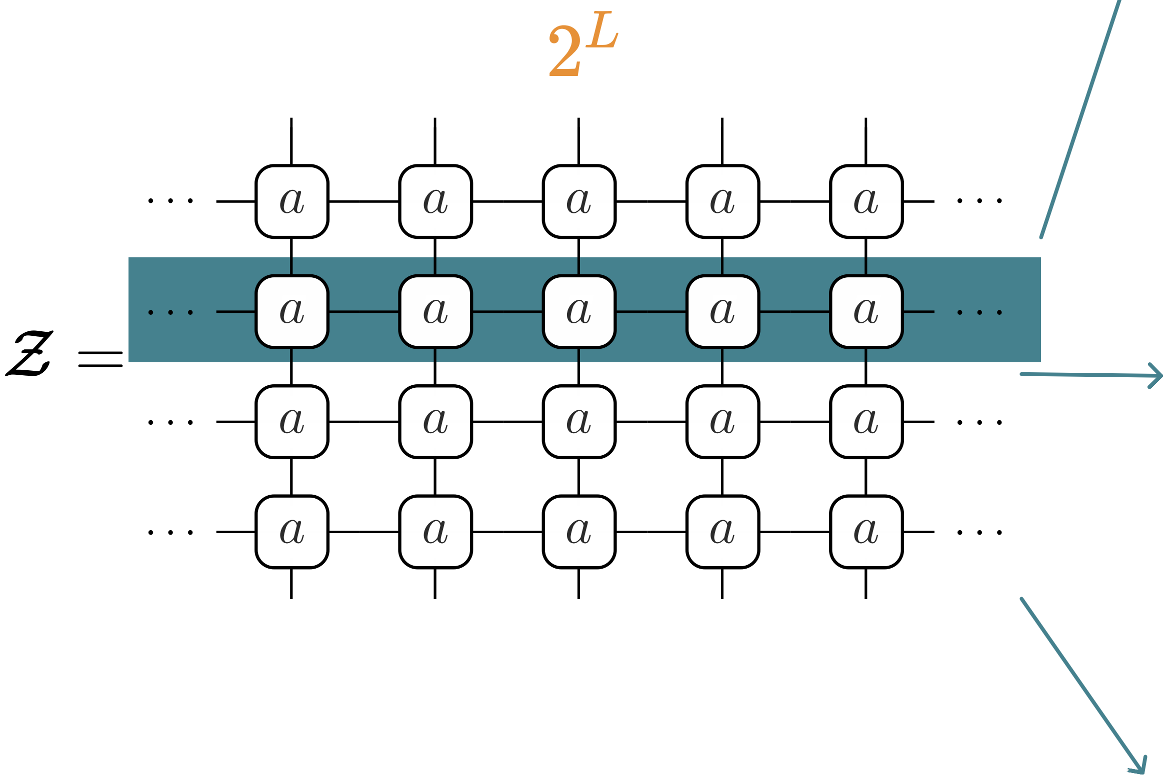

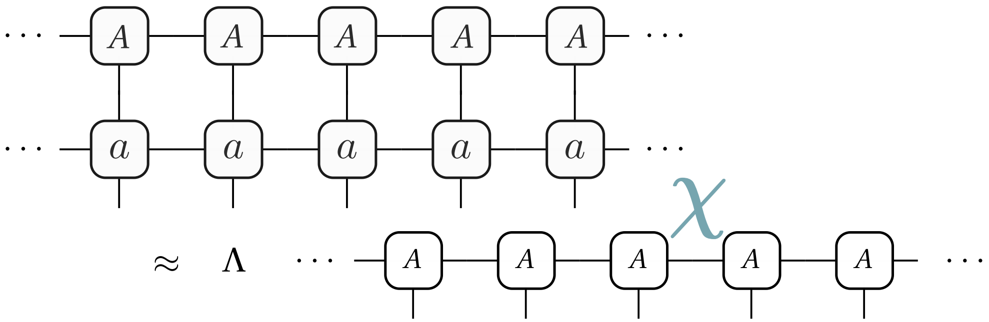

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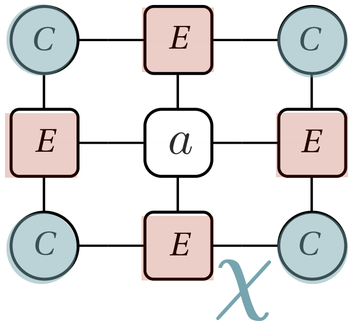

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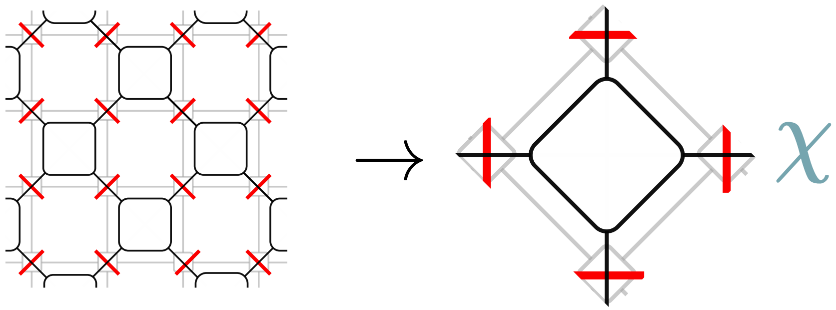

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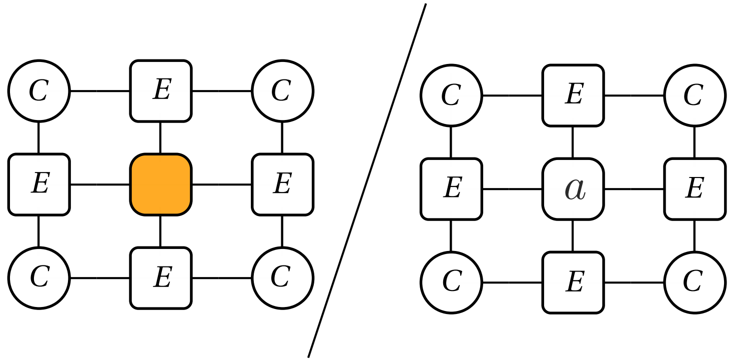
OBSERVABLES

Local observable:

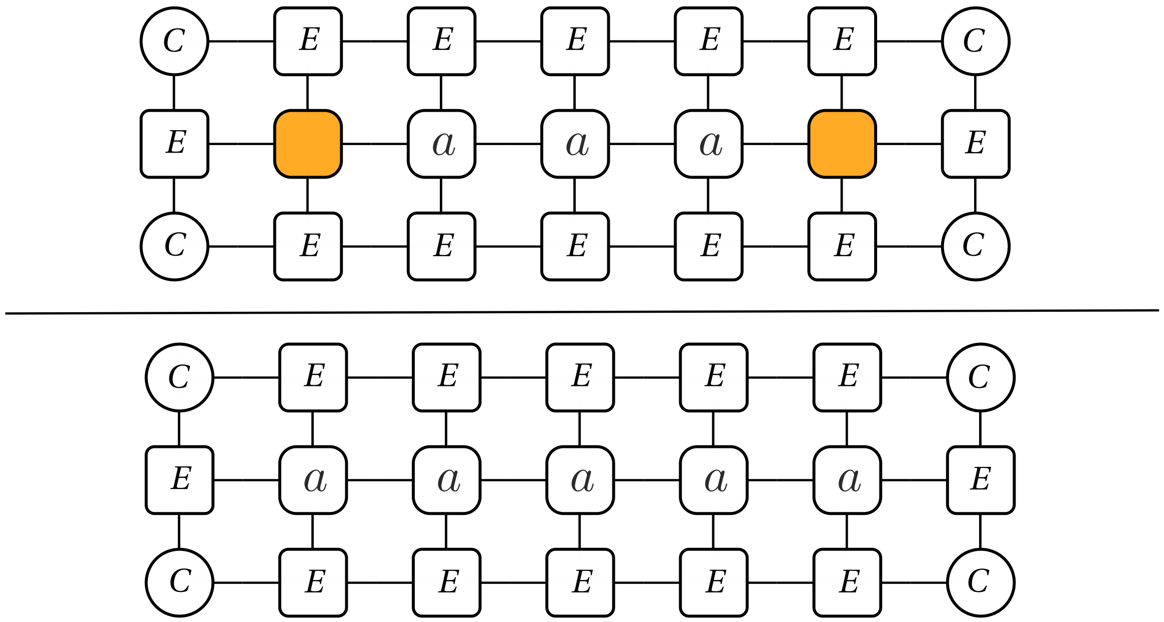
$$\langle O \rangle =$$



$\mathcal{Z}$

Correlations:

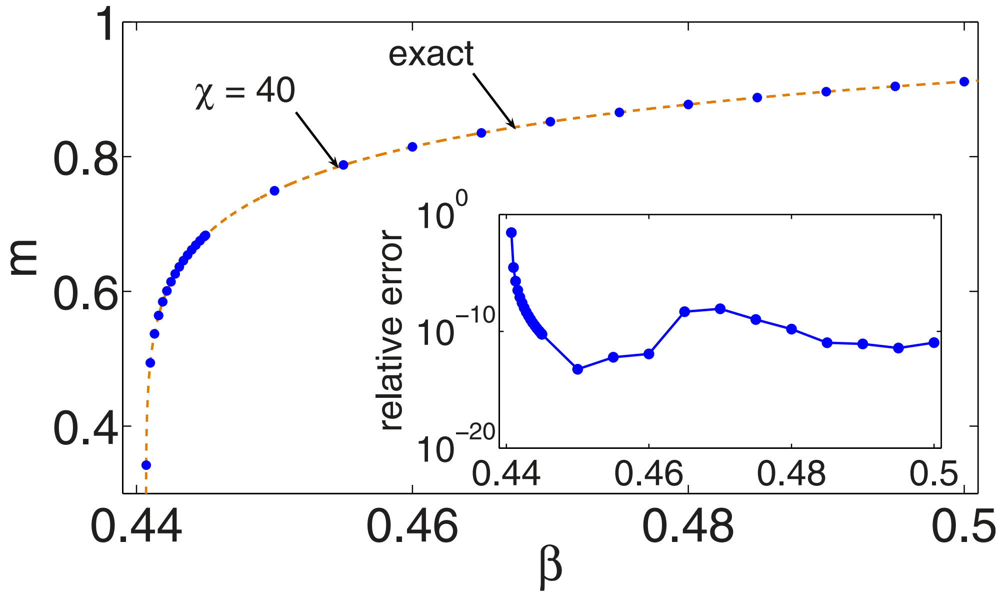
$$\langle O_i O_j \rangle =$$



Correlation length:

$$\xi = -\ln \frac{\lambda_2}{\lambda_1}$$

APPLICATIONS : DISCRETE SPINS



Orús, Vidal, PRB **78**, 2008

2D square lattice Ising model

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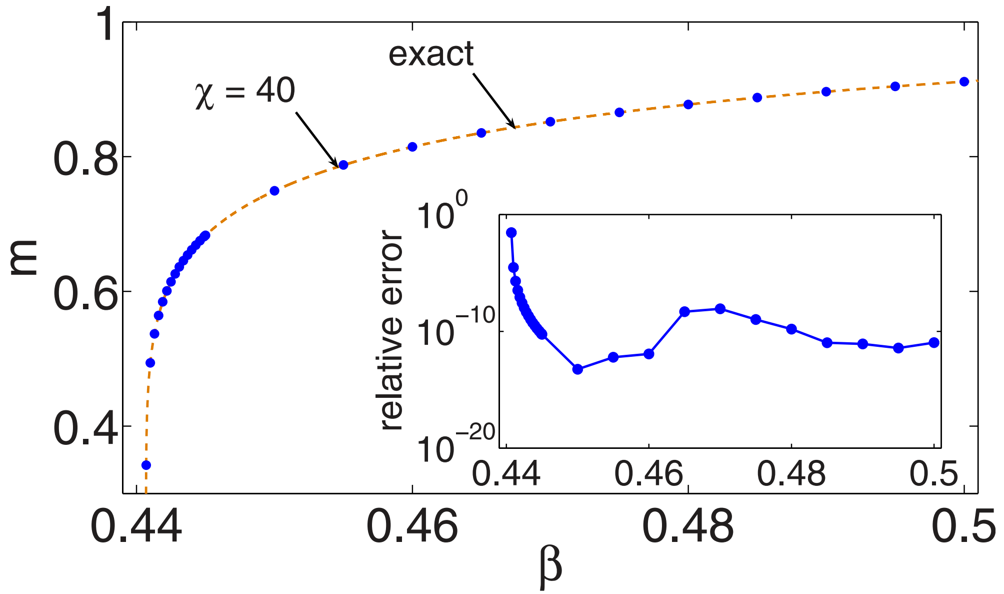

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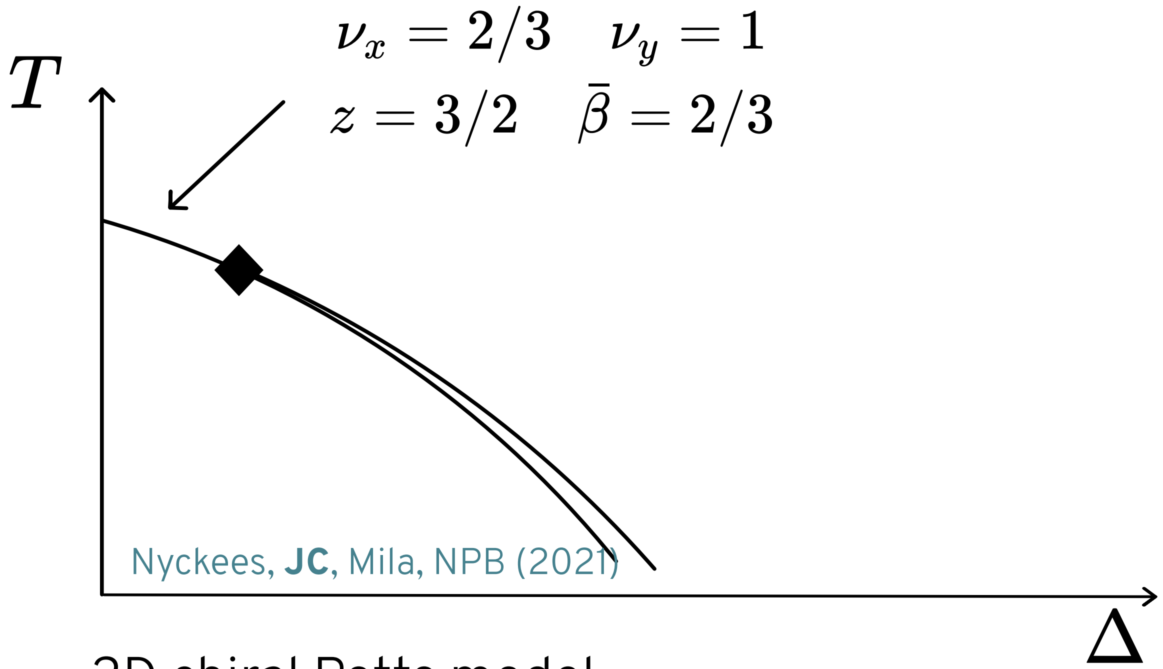
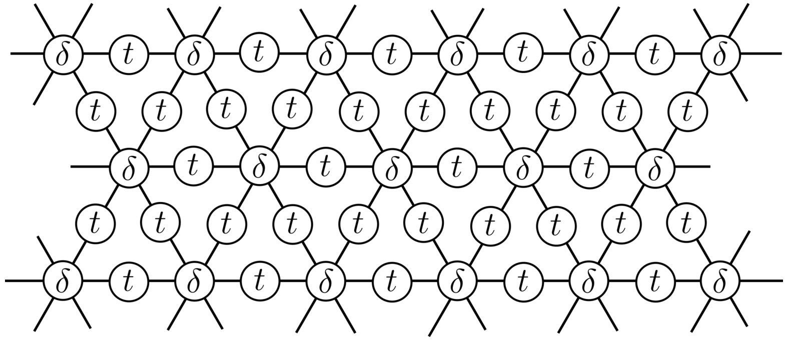


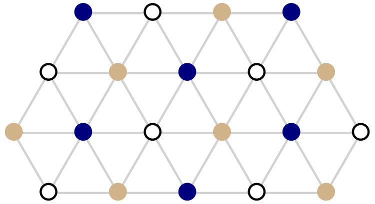
2D chiral Potts model

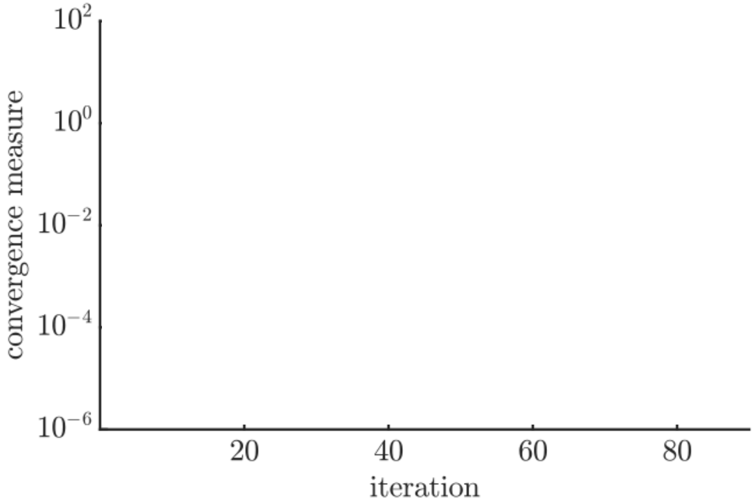
$$H = -\sum_{\vec{r}} \cos\left(\frac{2\pi}{3}(n_{\vec{r}+\vec{x}} - n_{\vec{r}} + \Delta)\right) - \sum_{\vec{r}} \cos\left(\frac{2\pi}{3}(n_{\vec{r}+\vec{y}} - n_{\vec{r}})\right)$$

FRUSTRATED MAGNETS

A SIMPLE CASE







Vanhecke, **JC** et al (2021)

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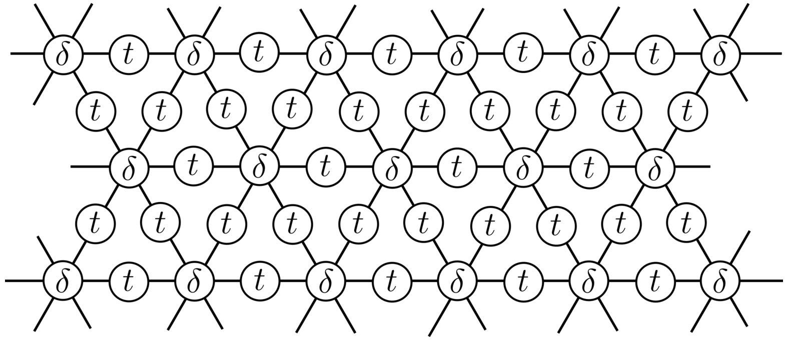

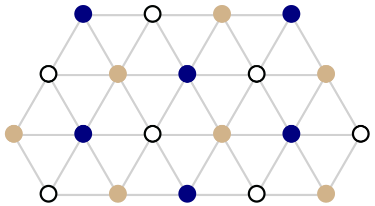

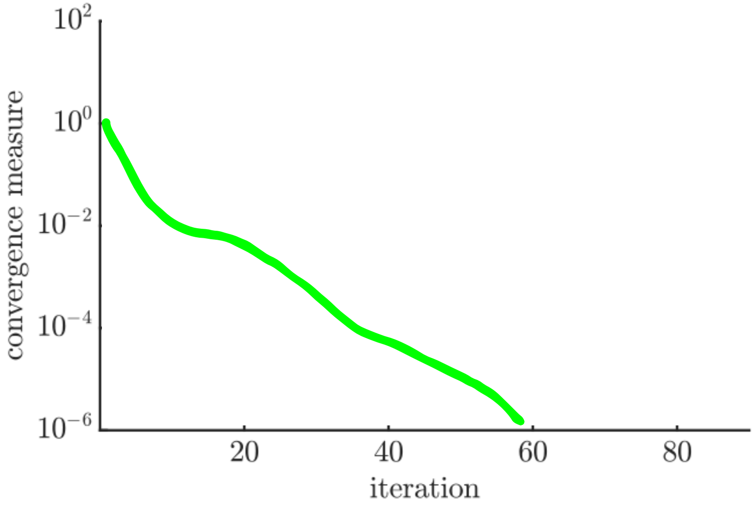

Vanhecke, **JC** et al (2021)

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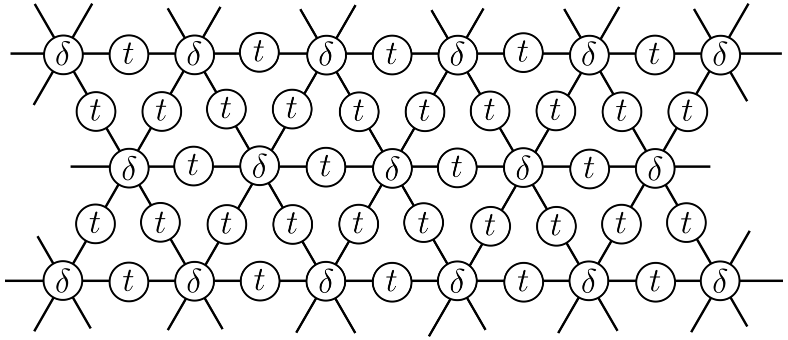
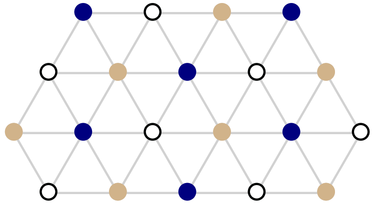
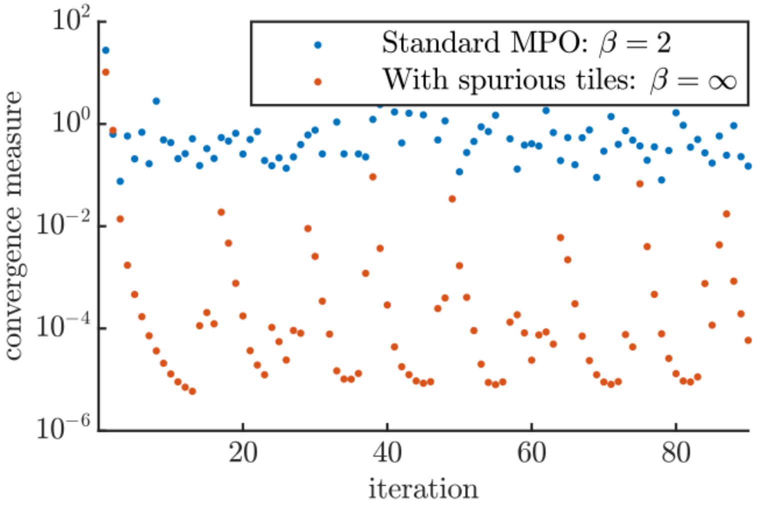




Fails in the presence of frustration and macroscopic g.s. degeneracy



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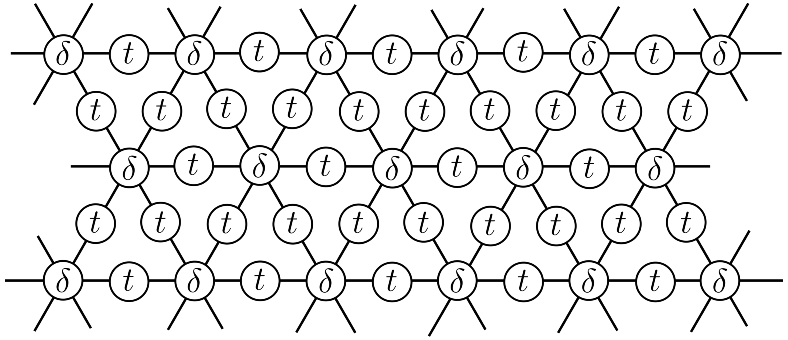

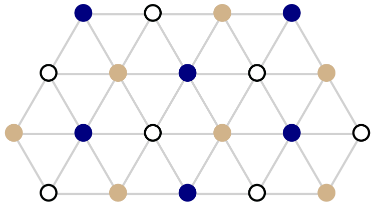

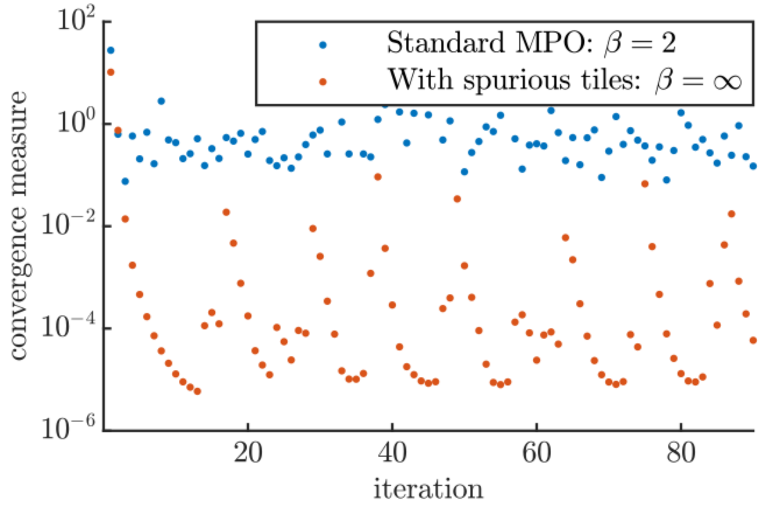

Vanhecke, **JC** et al (2021)

Fails in the presence of frustration and macroscopic g.s. degeneracy

→ in spin glasses

C. Wang, S.-M. Qin, H.-J. Zhou, PRB **90,** (2014)
Z. Zhu, H. G. Katzgraber, arXiv:1903.07721 (2019)
J. G. Liu, L. Wang, P. Zhang, PRL **126**, (2021)

→ in translation-invariant frustrated Ising models

B. Vanhecke, **JC**, et al. PRR **3**, (2021)

→ in lattice gas models

S. A. Akimenko, PRE **107**, (2023)

→ in frustrated XY models

F.F. Song, T.-Y. Lin, G. M. Zhang, PRB (2023)

THE PROBLEM

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Numerical problem

Cancellation of small and large factors

$$\tilde{t} = e^{\beta \frac{E_{\text{G.S.}}}{N_{\text{bonds}}}} t = \begin{pmatrix} e^{-\frac{4}{3}\beta J} & e^{\frac{2}{3}\beta J} \\ e^{\frac{2}{3}\beta J} & e^{-\frac{4}{3}\beta J} \end{pmatrix} \qquad \mathcal{Z}_{\triangle} = \tilde{t}^3$$

$\rightarrow$ precision?

C. Wang, S.-M. Qin, H.-J. Zhou, PRB **90,** (2014)
Z. Zhu, H. G. Katzgraber, arXiv:1903.07721 (2019)

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Bad gauge

The transfer matrix is badly conditioned (e.g. not hermitian, ...)

W. Tang et al, (2024, 2025)

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W. Tang et al, (2024, 2025)

Ground-state rule
Failure to minimize simultaneously **all** local Hamiltonians.

B. Vanhecke, **JC**, et al. PRR **3**, (2021)
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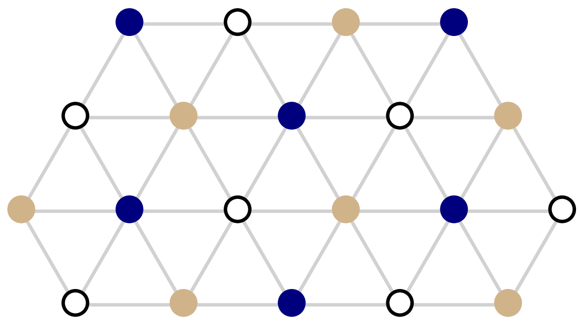
SIMPLE CASE : TRIANGULAR LATTICE ISING ANTIFERROMAGNET

Ground state:



Kasteleyn, (1961), Fisher (1966)

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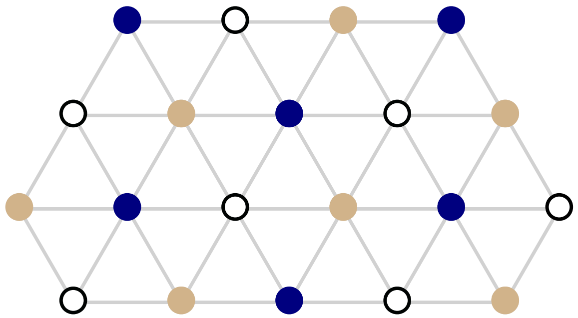

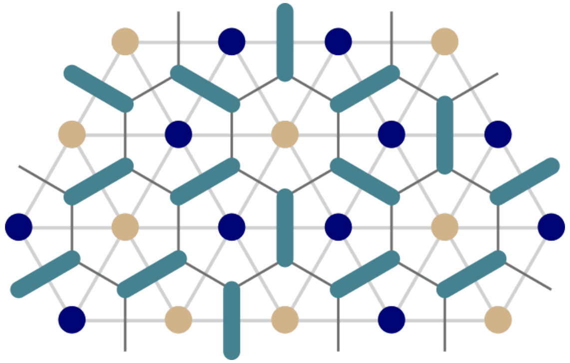

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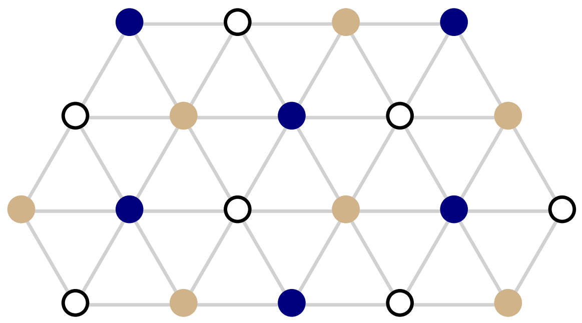
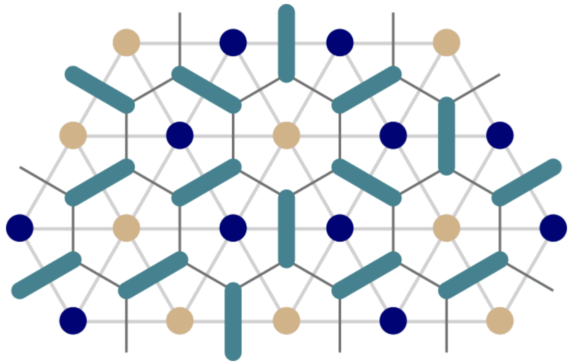


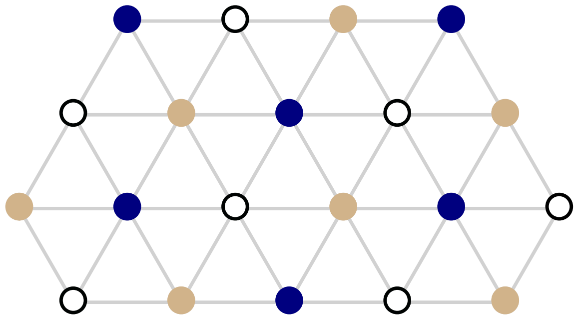
Kasteleyn, (1961), Fisher (1966)

Dimer coverings!



SIMPLE CASE : TRIANGULAR LATTICE ISING ANTIFERROMAGNET

Ground state:



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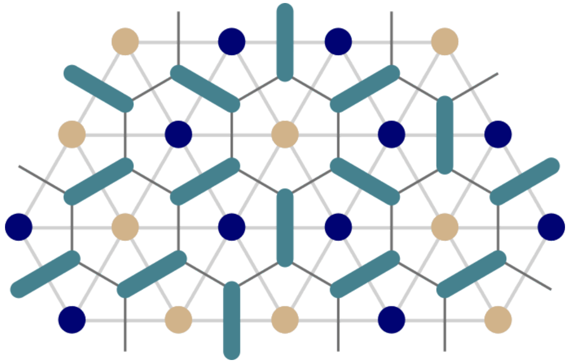

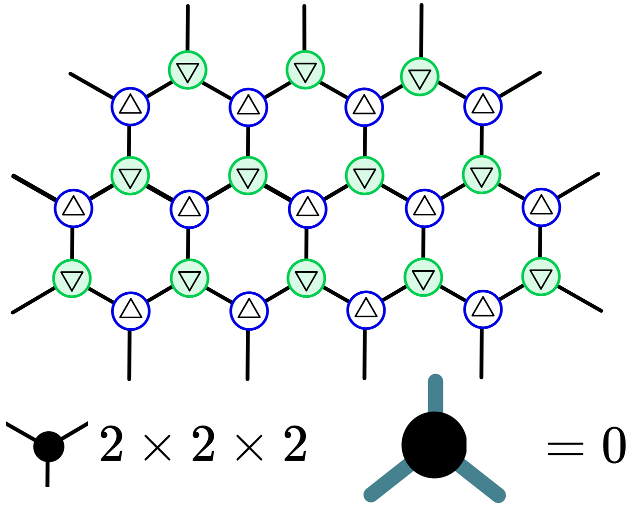

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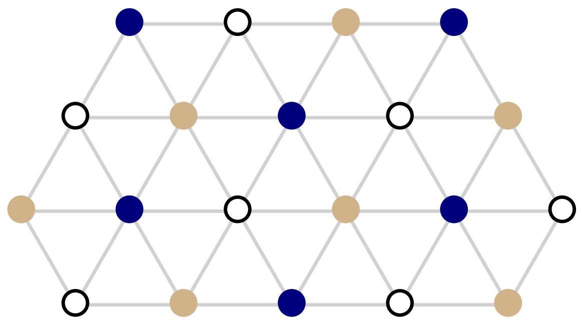

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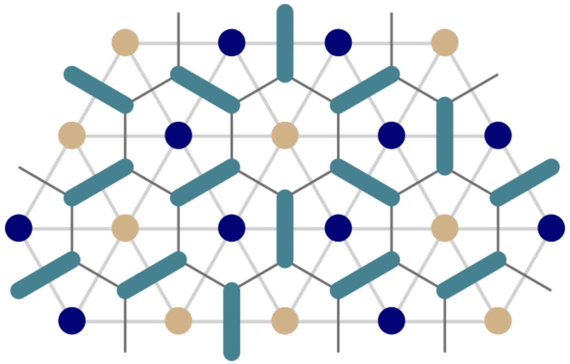

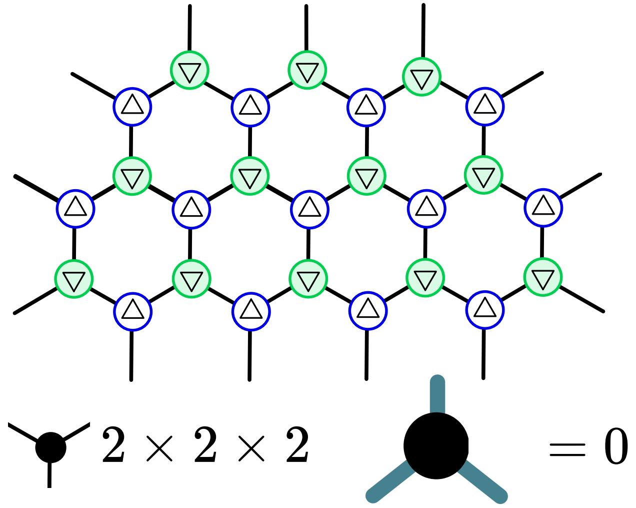

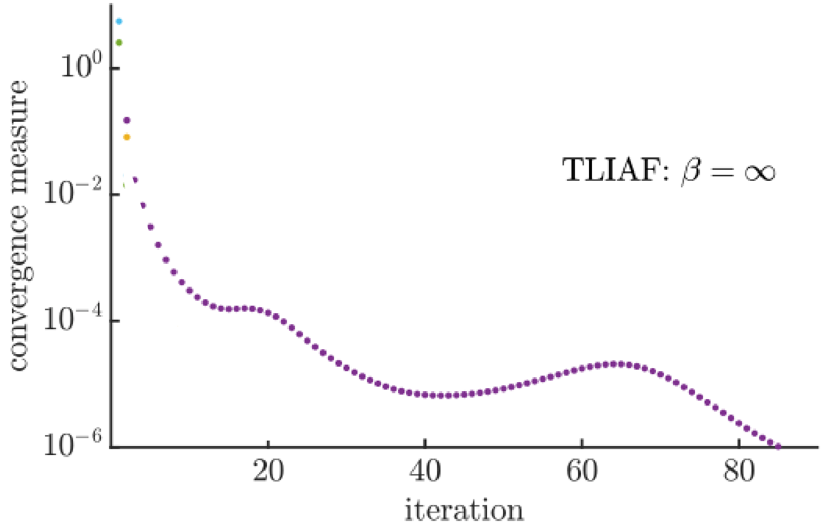

Vanhecke, **JC** et al (2021)

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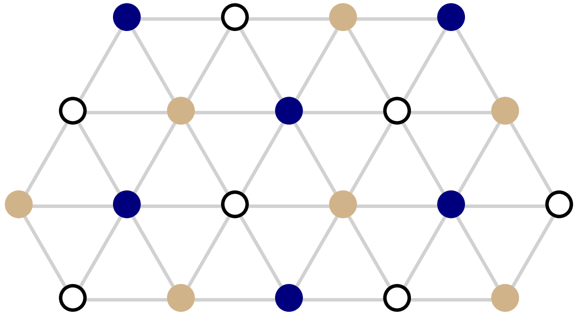

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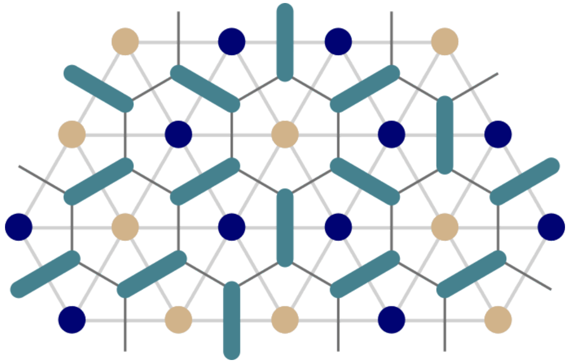
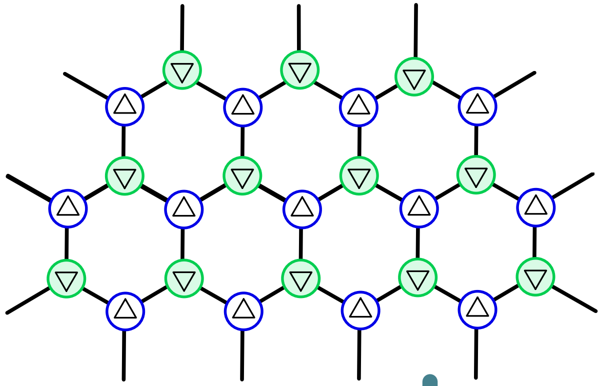
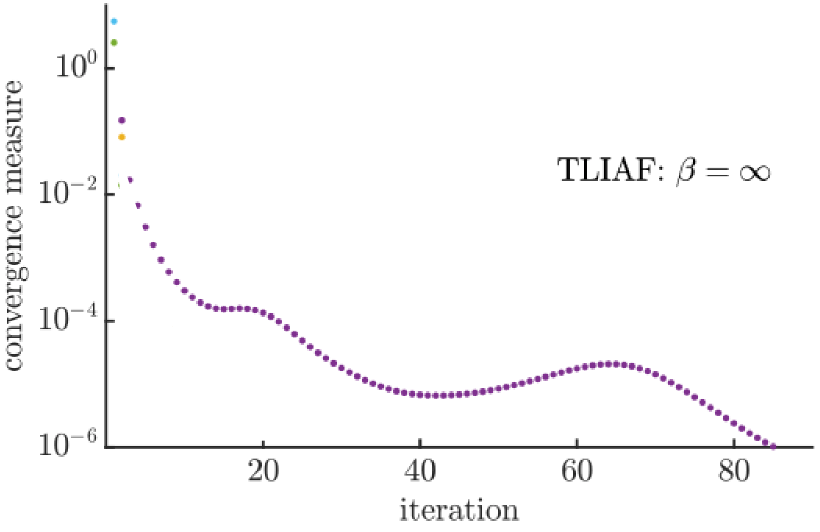
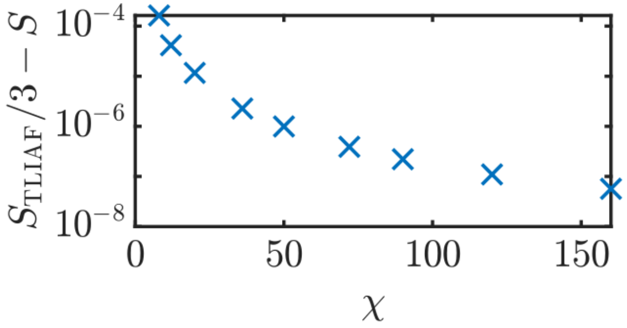


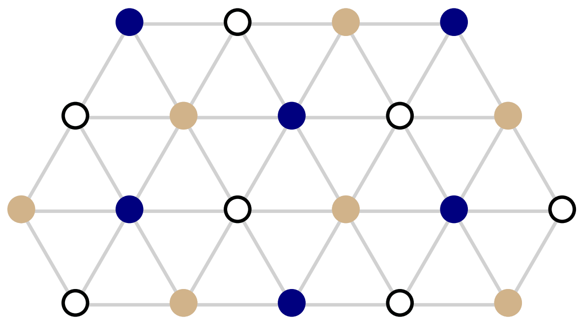


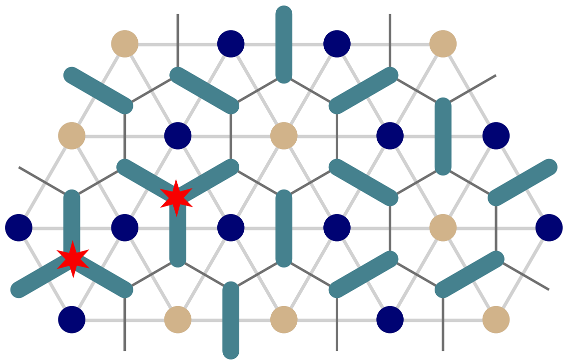


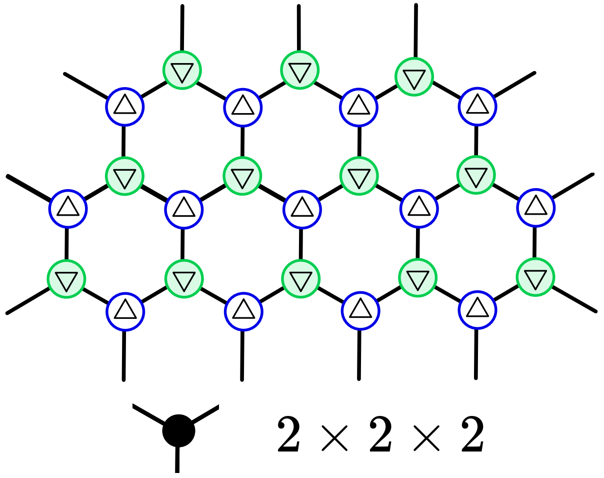


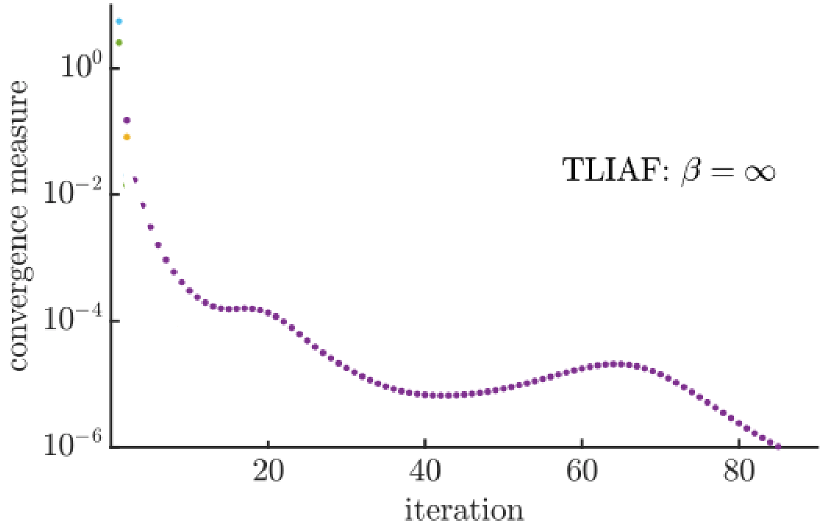
Vanhecke, **JC** et al (2021)

FINITE TEMPERATURE









Vanhecke, **JC** et al (2021)

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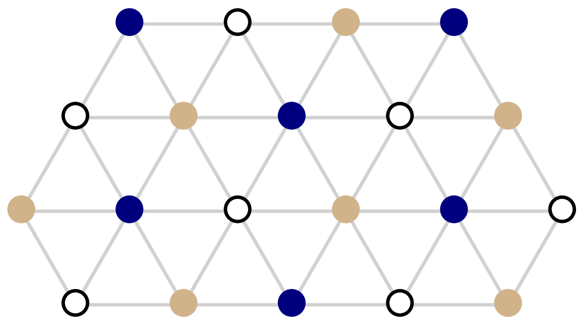
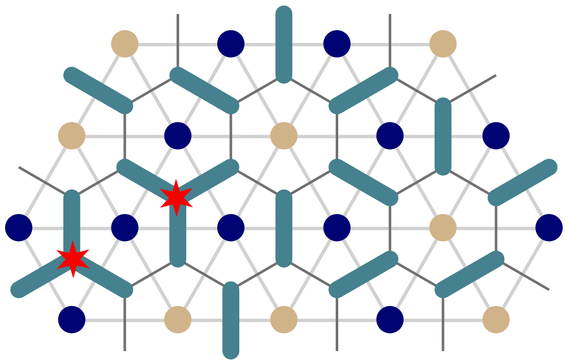
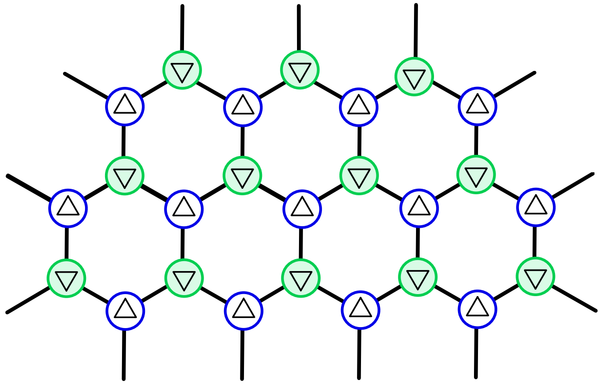
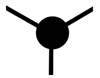


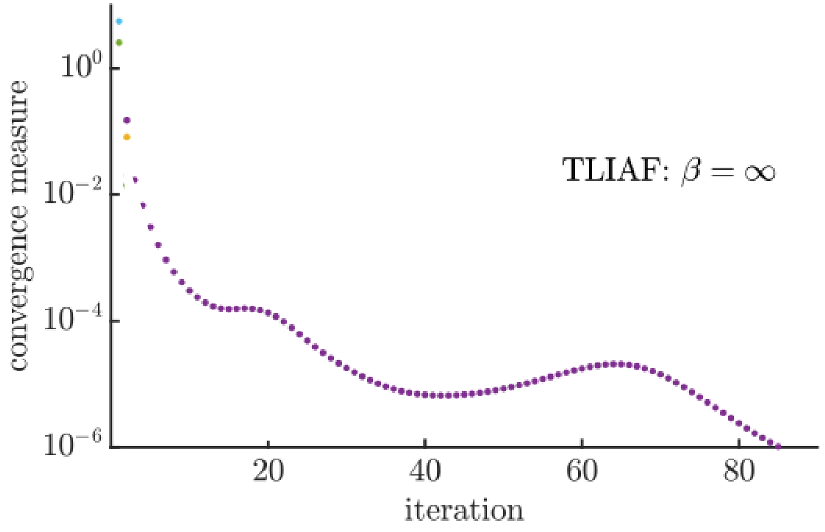


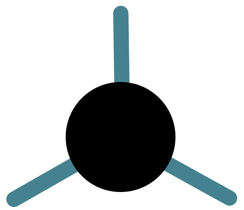




$2 \times 2 \times 2$

Same structure and size



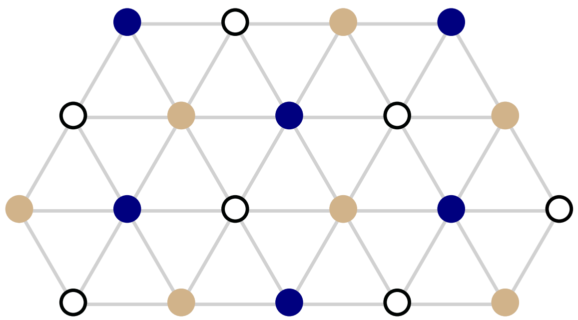


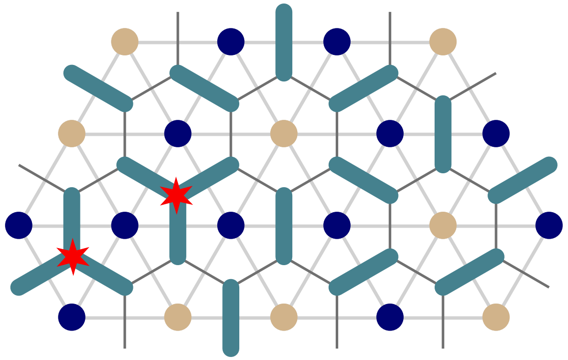
$= e^{-\beta J}$

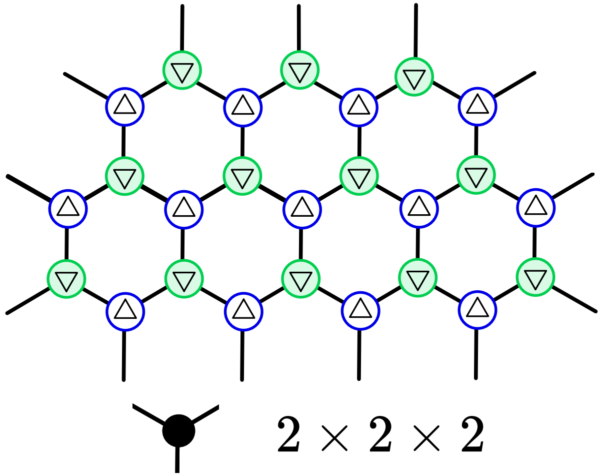
Different entries

Vanhecke, **JC** et al (2021)

FINITE TEMPERATURE

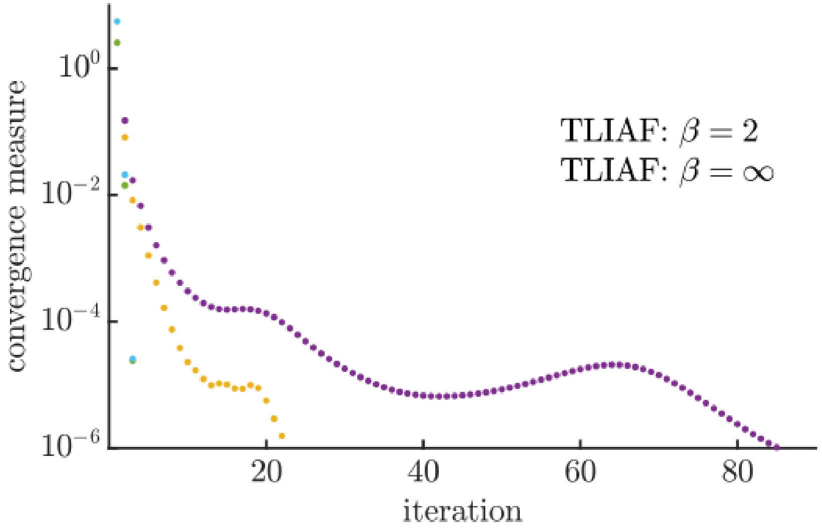


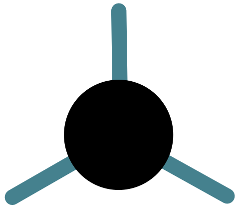




$2 \times 2 \times 2$

Same structure and size



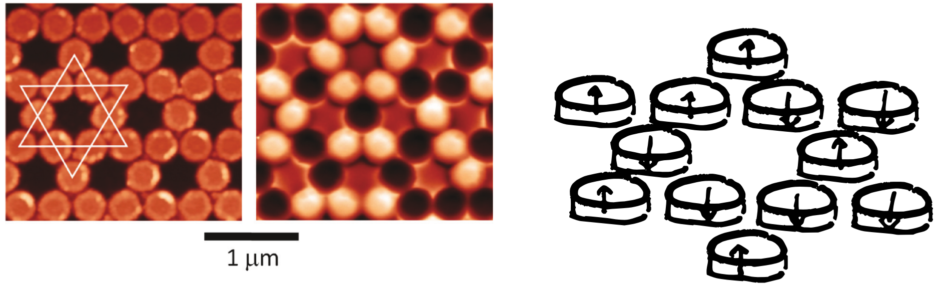


$= e^{-\beta J}$

Different entries

Vanhecke, **JC** et al (2021)

MORE COMPLEX ISING MODELS?



I. A. Chioar, N. Rougemaille, B. Canals, PRB **93,** (2016)
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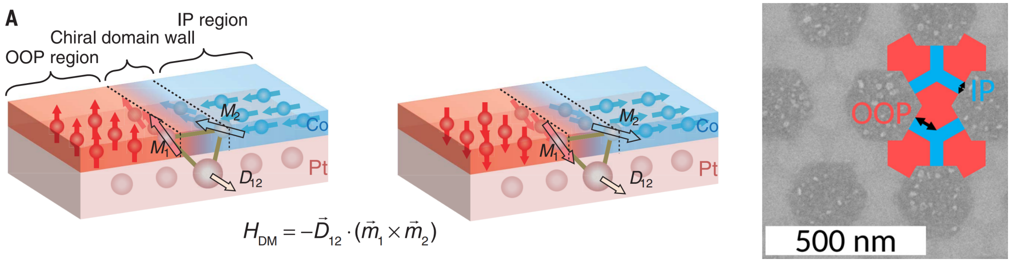
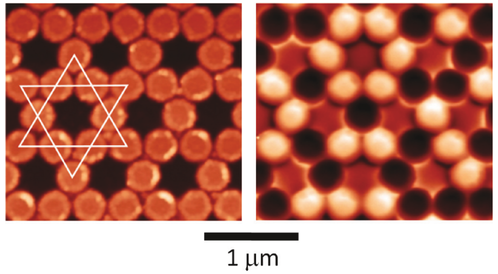
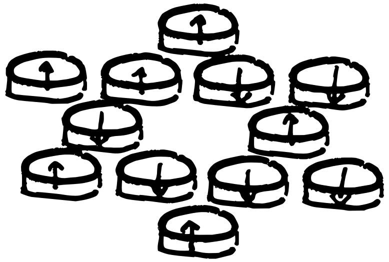
L. Cugliandolo, L. Foini, M. Tarzia, PRB **101** (2020)

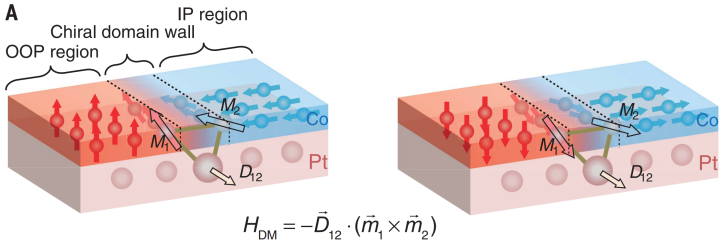


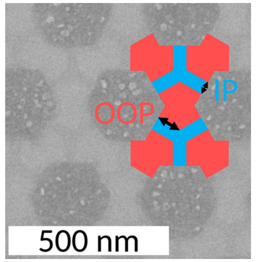
Z. Luo et al. Science **363**, (2019)
JC et al., PRB **104** (2021)

MORE COMPLEX ISING MODELS?









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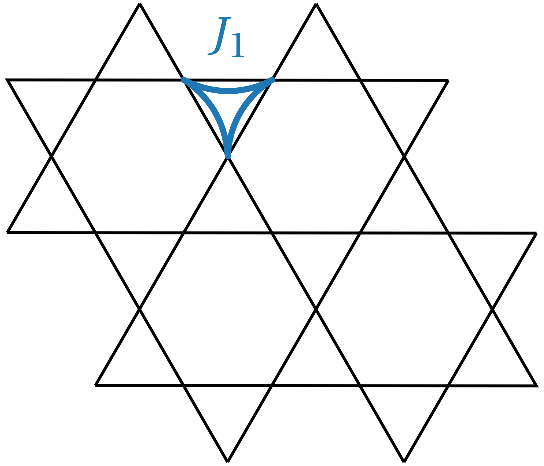
Z. Luo et al. Science **363**, (2019)
JC et al., PRB **104** (2021)

$$H = J_1 \sum_{\langle i,j \rangle_1} \sigma_i \sigma_j$$



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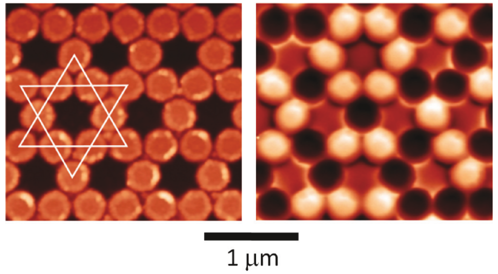

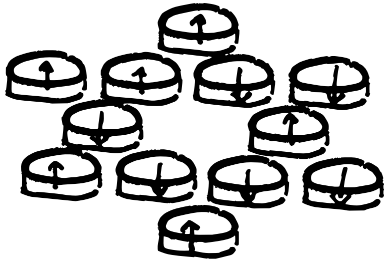

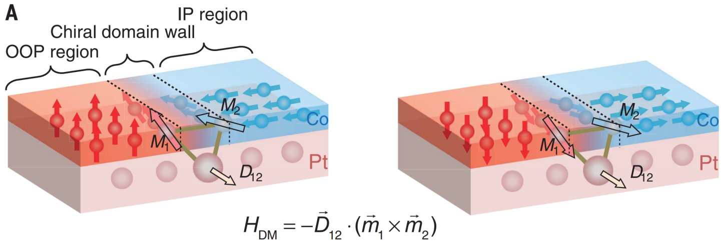

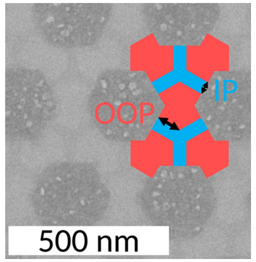

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Z. Luo et al. Science **363**, (2019)
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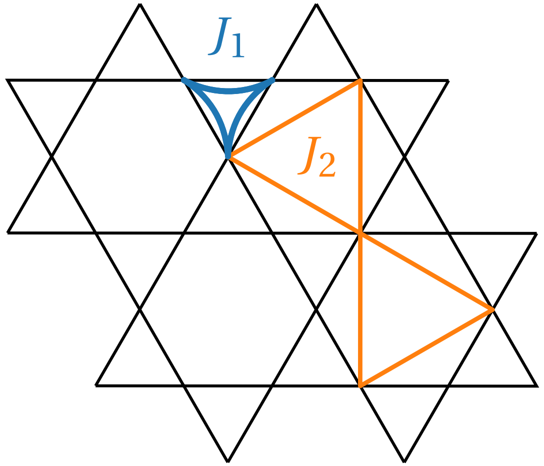
$$H = J_1 \sum_{\langle i,j \rangle_1} \sigma_i \sigma_j + J_2 \sum_{\langle i,j \rangle_2} \sigma_i \sigma_j$$



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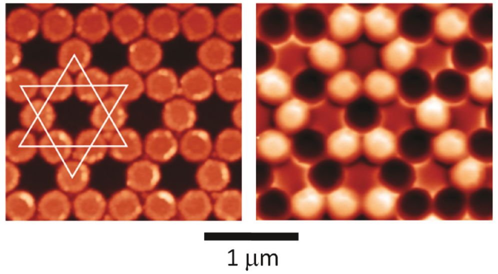

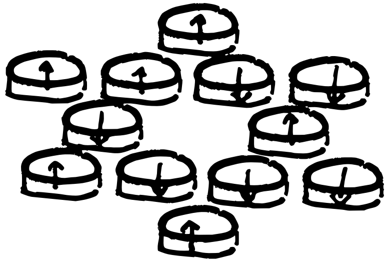

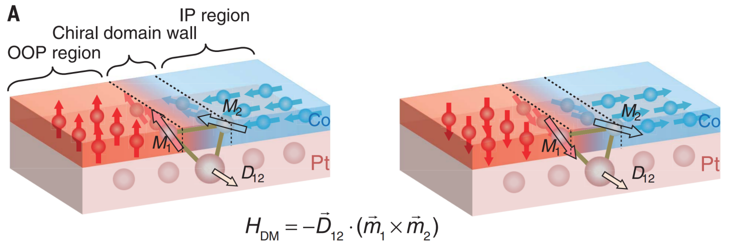

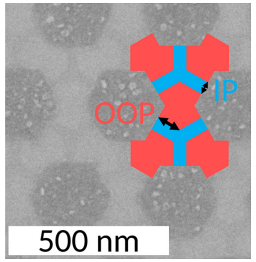

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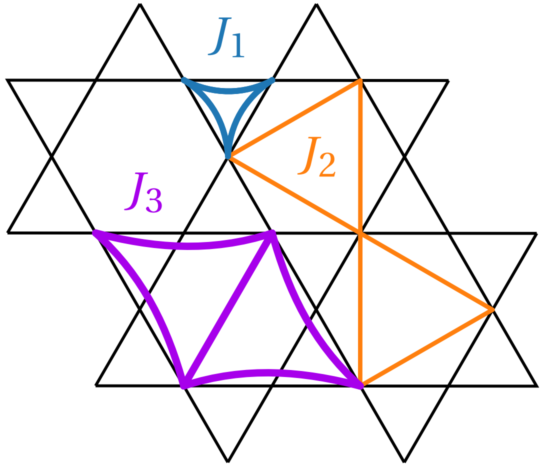
Z. Luo et al. Science **363**, (2019)
JC et al., PRB **104** (2021)

$$H = J_1 \sum_{\langle i,j \rangle_1} \sigma_i \sigma_j + J_2 \sum_{\langle i,j \rangle_2} \sigma_i \sigma_j + J_3 \sum_{\langle i,j \rangle_3} \sigma_i \sigma_j$$



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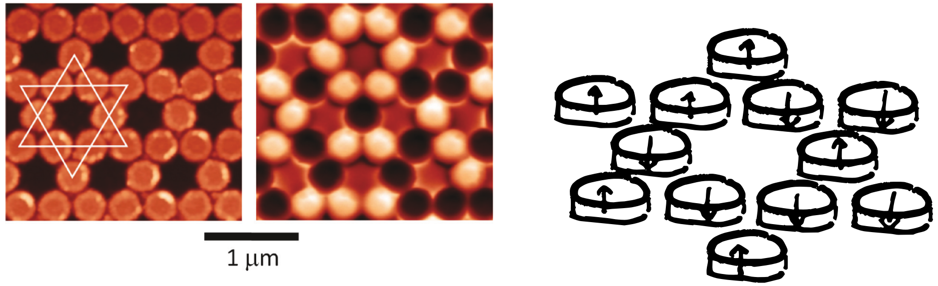

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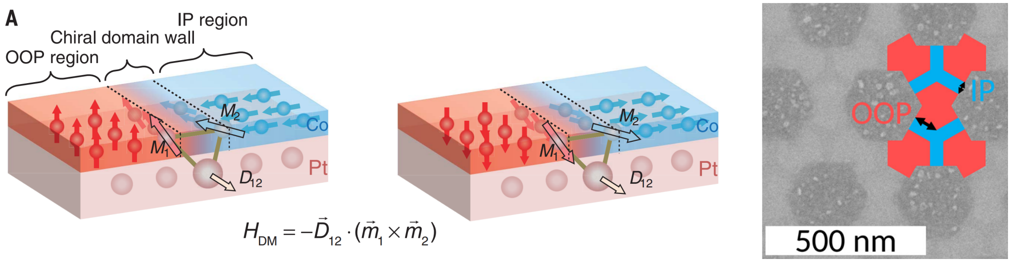

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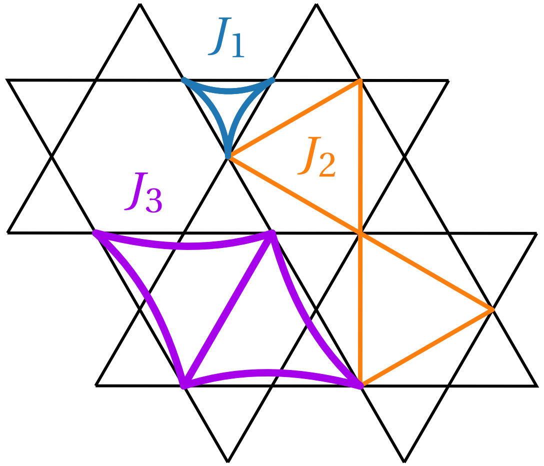

CAN WE STILL FIND THE CONSTRAINT?

ISING MODELS AS WEIGTHED COUNTING PROBLEMS

Essential idea : Anderson bounds

$$H = \sum_{\langle i,j \rangle} \sigma_i \sigma_j = \frac{J}{2} \sum_{\triangle i,j,k \triangledown i,j,k} (\sigma_i \sigma_j + \sigma_j \sigma_k + \sigma_k \sigma_i)$$

C. K. Majumdar and D. K. Ghosh, J. Math. Phys. **10**, (1969); M. Kaburagi, J. Kanamori, Prog. Theor. Phys. **54** , (1975);
B. Sriram Shastry and B. Sutherland, Physica **108** B+C, (1981); W. Huang, D. A. Kitchaev, *et. al.* , Phys. Rev. B **94**, (2016);
B. Vanhecke, **JC**, L. Vanderstraeten, F. Verstraete, F. Mila, PRR **3**, (2021)
Nagy *et al*; PRE **109** (2024)

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LINEAR PROGRAM:

C. K. Majumdar and D. K. Ghosh, J. Math. Phys. **10**, (1969); M. Kaburagi, J. Kanamori, Prog. Theor. Phys. **54** , (1975);
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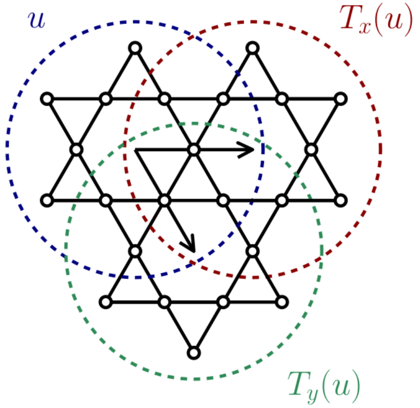
LINEAR PROGRAM:



1. Split **with clusters that overlap** $H = \sum_c H_c^{\alpha}(\vec{\sigma}|_c)$

C. K. Majumdar and D. K. Ghosh, J. Math. Phys. **10**, (1969); M. Kaburagi, J. Kanamori, Prog. Theor. Phys. **54** , (1975); B. Sriram Shastry and B. Sutherland, Physica **108** B+C, (1981); W. Huang, D. A. Kitchaev, *et. al.* , Phys. Rev. B **94**, (2016); B. Vanhecke, **JC**, L. Vanderstraeten, F. Verstraete, F. Mila, PRR **3**, (2021) Nagy *et al*; PRE **109** (2024)

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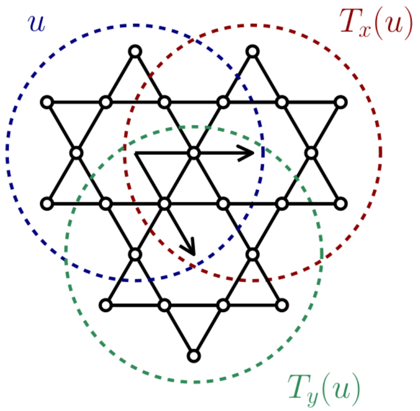
LINEAR PROGRAM:



1. Split **with clusters that overlap** $H = \sum_c H_c^\alpha(\vec{\sigma}|_c)$

2. Minimize : G.S. lower-bound $\min_{\vec{\sigma}|_c} H_c^\alpha(\vec{\sigma}|_c)$

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ISING MODELS AS WEIGTHED COUNTING PROBLEMS

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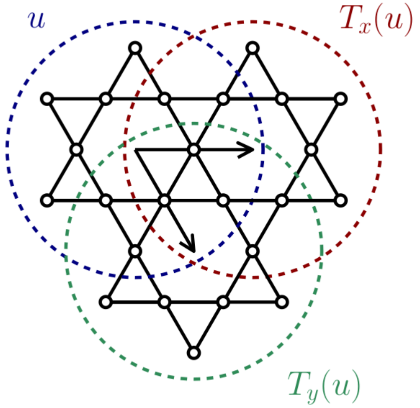

1. Split **with clusters that overlap** $H = \sum_c H_c^\alpha(\vec{\sigma}|_c)$
2. Minimize : G.S. lower-bound $\min_{\vec{\sigma}|_c} H_c^\alpha(\vec{\sigma}|_c)$
3. Maximize w.r.t the weights: $\max_\alpha \; \min_{\vec{\sigma}|_c} H_c^\alpha(\vec{\sigma}|_c)$

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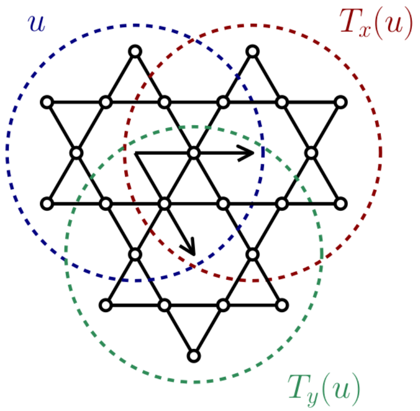
ISING MODELS AS WEIGTHED COUNTING PROBLEMS

Essential idea : Anderson bounds

$$H = \sum_{\langle i,j \rangle} \sigma_i \sigma_j = \frac{J}{2} \sum_{\triangle i,j,k \nabla i,j,k} (\sigma_i \sigma_j + \sigma_j \sigma_k + \sigma_k \sigma_i)$$

LINEAR PROGRAM:



1. Split **with clusters that overlap** $\quad H = \sum_c H_c^\alpha(\vec{\sigma}|_c)$

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Obtain the ground states by tiling!

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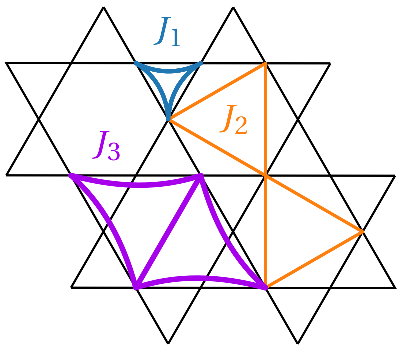
Nagy *et al*; PRE **109** (2024)

APPLICATIONS AND PERSPECTIVES

THREE UNEXPECTED SPIN LIQUID PHASES

$$H = J_1 \sum_{\langle i,j\rangle_1} \sigma_i\sigma_j + J_2 \sum_{\langle i,j\rangle_2} \sigma_i\sigma_j + J_3 \sum_{\langle i,j\rangle_3} \sigma_i\sigma_j$$



THREE UNEXPECTED SPIN LIQUID PHASES

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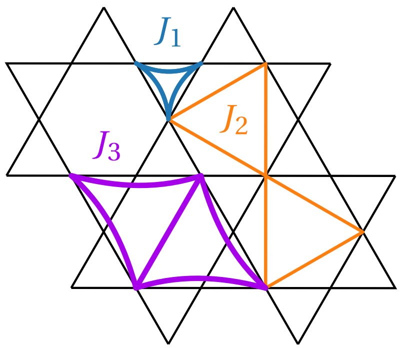

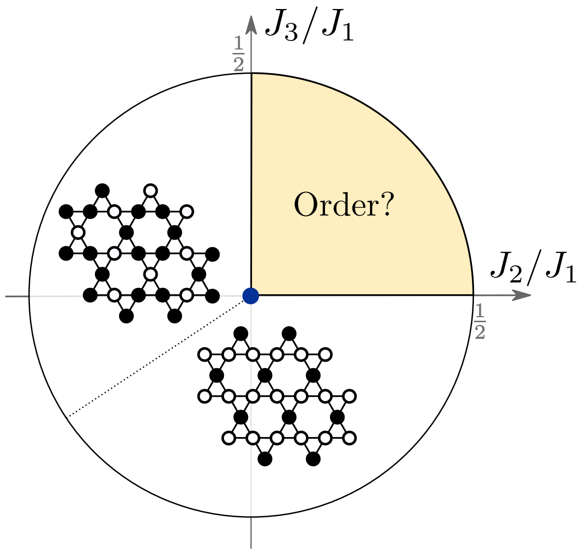

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THREE UNEXPECTED SPIN LIQUID PHASES

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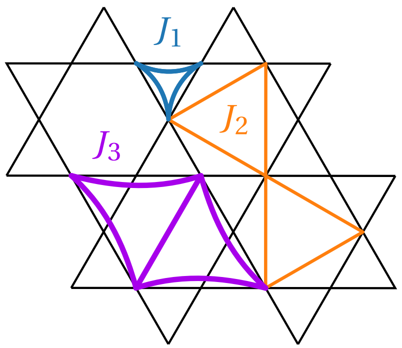

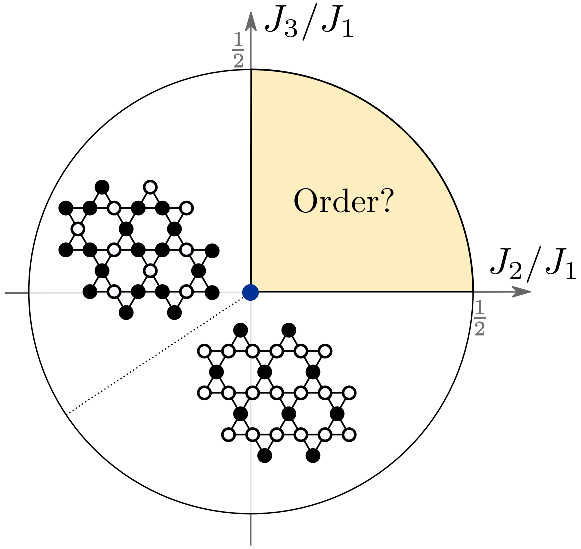

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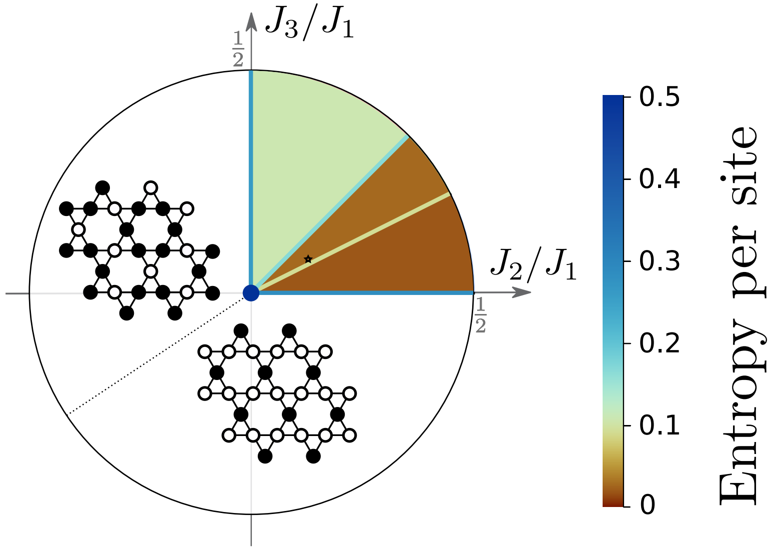
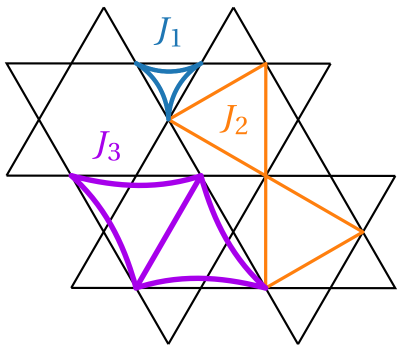
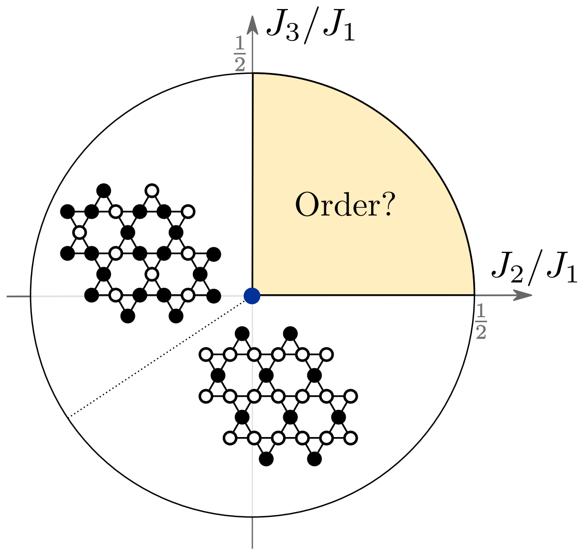


JC, B. Vanhecke et. al., PRB **106** (2022)

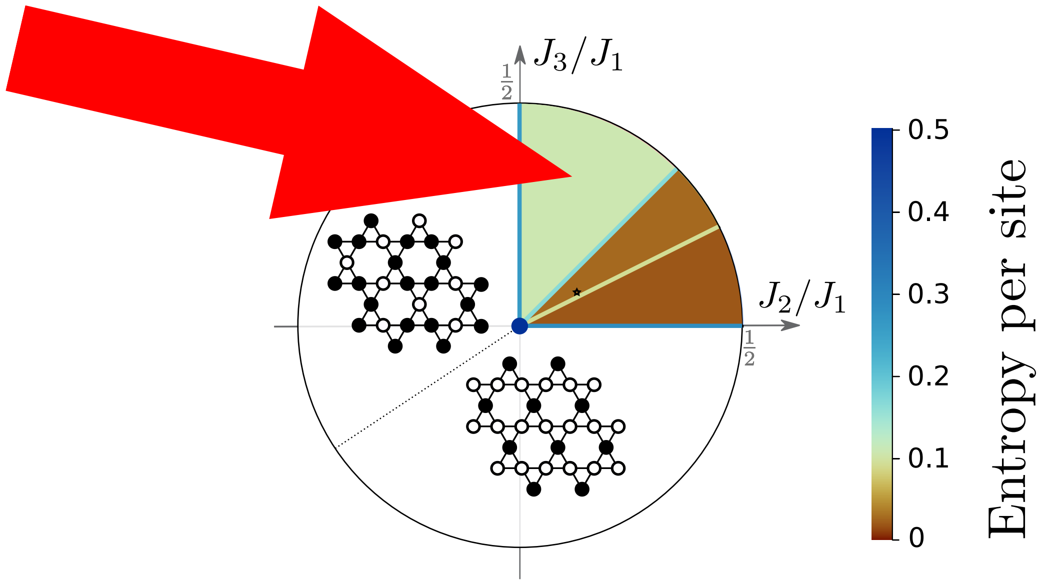
THREE UNEXPECTED SPIN LIQUID PHASES

$$H = J_1 \sum_{\langle i,j\rangle_1} \sigma_i\sigma_j + J_2 \sum_{\langle i,j\rangle_2} \sigma_i\sigma_j + J_3 \sum_{\langle i,j\rangle_3} \sigma_i\sigma_j$$





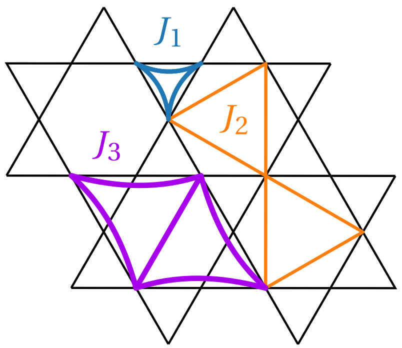
I. A. Chioar, N. Rougemaille, B. Canals, PRB **93,** (2016)
J. Hamp, C. Castelnovo, R. Moessner, PRB **98**, (2018)



JC, B. Vanhecke et. al., PRB **106** (2022)

A CASCADE OF TOPOLOGICAL PHASE TRANSITIONS

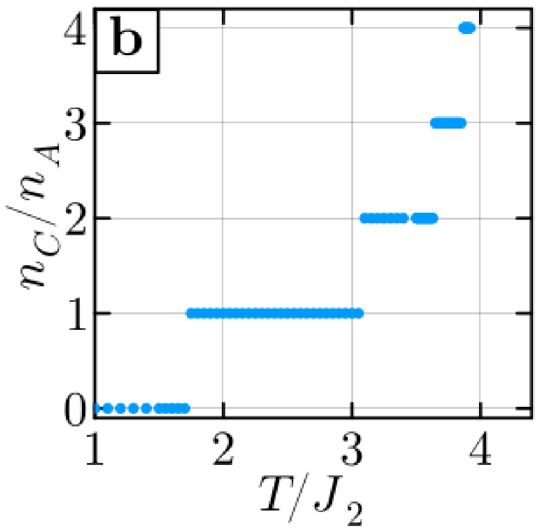
$$H = J_1 \sum_{\langle i,j\rangle_1} \sigma_i\sigma_j + J_2 \sum_{\langle i,j\rangle_2} \sigma_i\sigma_j + J_3 \sum_{\langle i,j\rangle_3} \sigma_i\sigma_j$$



An **infinite** series of plateaus

in the ratios of **densities of 2 types of system-spanning strings**

A. Rufino, S. Nyckees, **JC**, F. Mila, arXiv:2505.05889 (2025)



PERSPECTIVES

Tensor network methods

"Classical" and quantum
frustrated magnetism

PERSPECTIVES

Tensor network methods

Consequences for studying 2D quantum many-body problems ?
Wei Tang et al (2024, 2025)

Dealing with non-local constraints ?
Châtelain & Gendiar (2020)

Combining with Monte Carlo methods?
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"Classical" and quantum frustrated magnetism

Range of frustration: hard versus "weak" frustration ?
Ronceray & Le Floch (2020)

Interpretation of "classical" entanglement ?
Carignano et al (2024)

A route to quantum-classical correspondences?
Allegra et al (2016)

TAKE-HOME MESSAGE

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Tensor networks:

a way to capture complex behavior in statistical mechanics

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***Constrained* models in statistical mechanics shed light**

on tensor network methods (and challenges)

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THANK YOU FOR YOUR ATTENTION!

BONUS SLIDES

IMAGE COMPRESSION

Wikipedia, CC BY license

Julia Yeomans

IMAGE COMPRESSION

$$M = U \quad S \quad V$$

IMAGE COMPRESSION

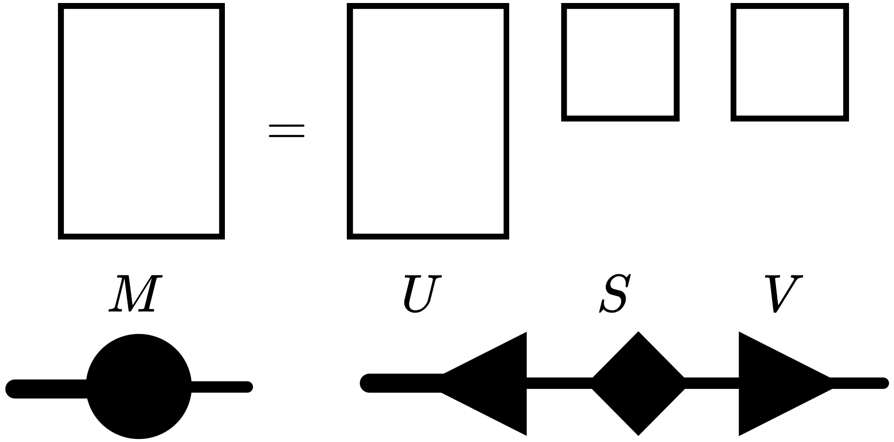


IMAGE COMPRESSION

Wikipedia, CC BY license

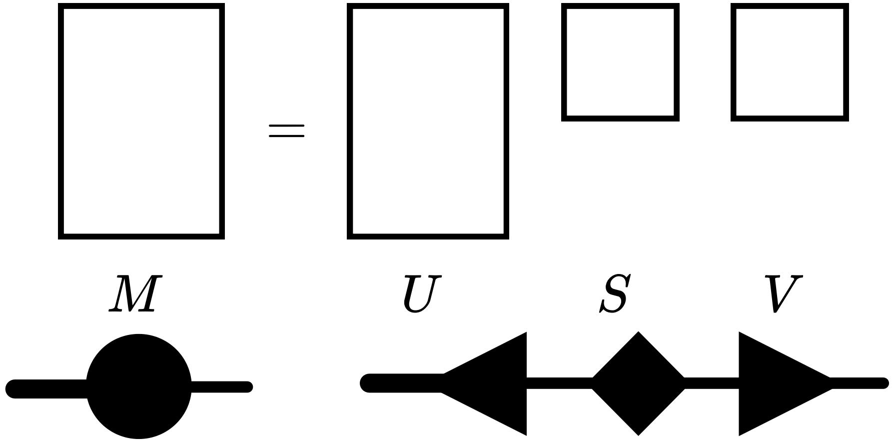
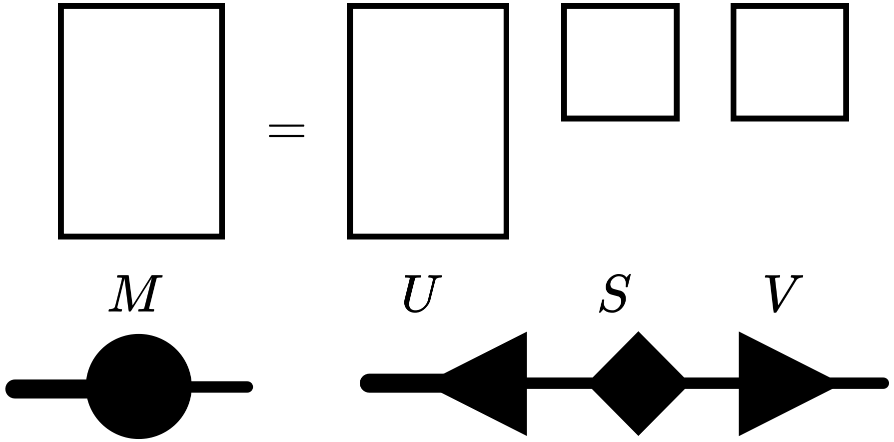


IMAGE COMPRESSION



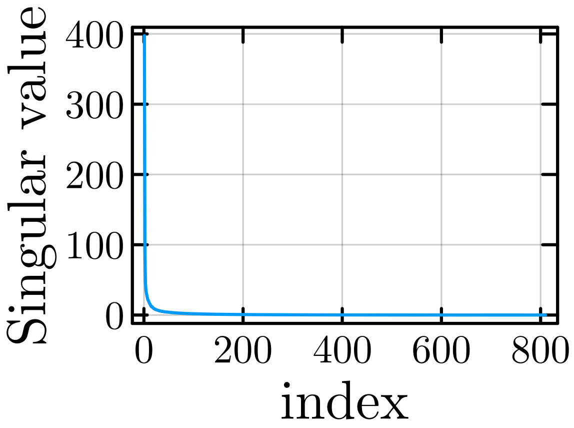
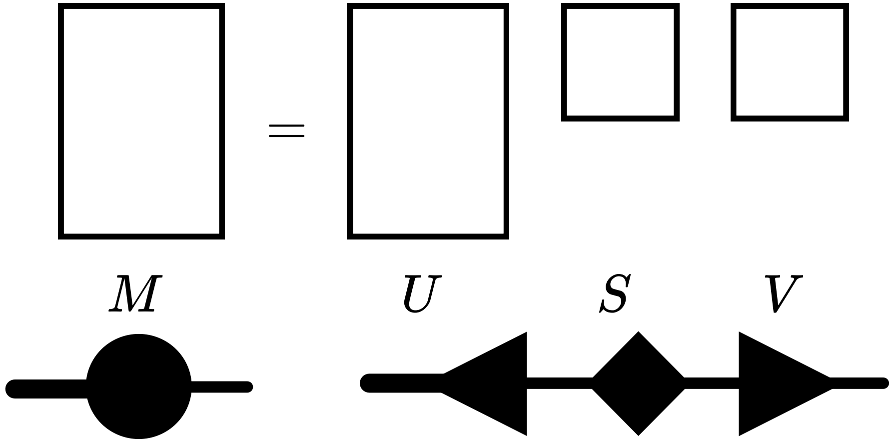
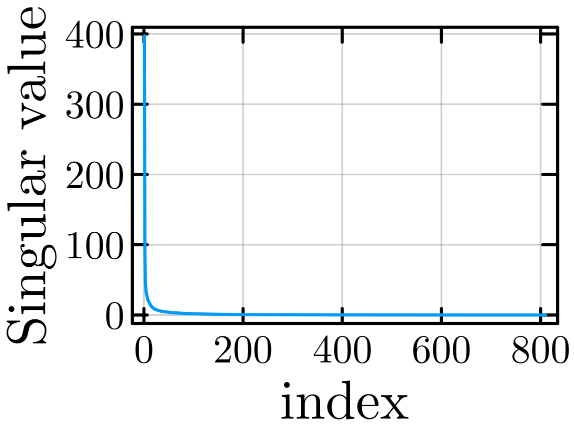


IMAGE COMPRESSION





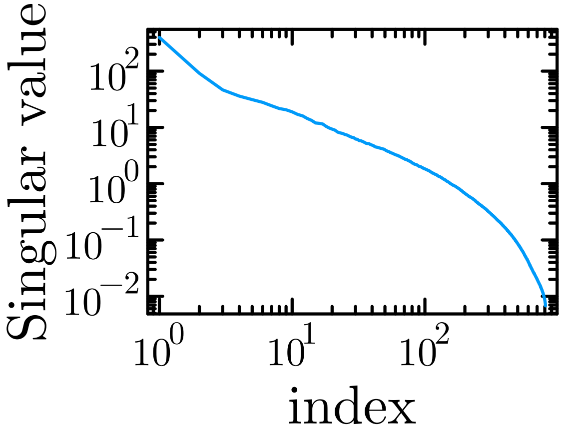
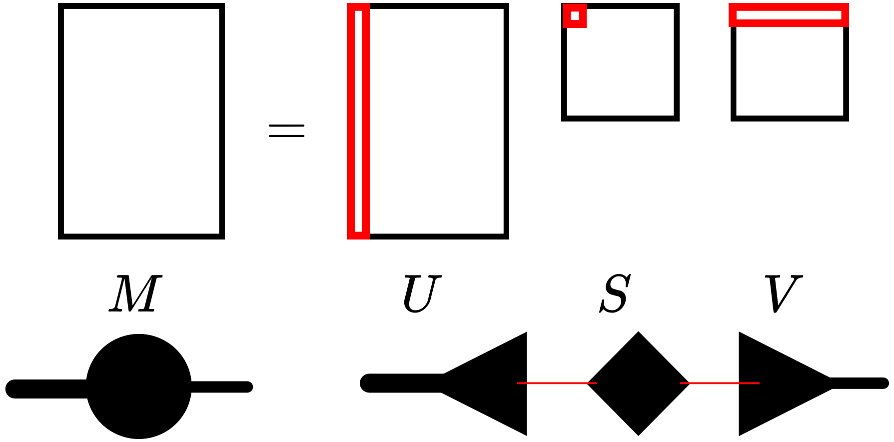
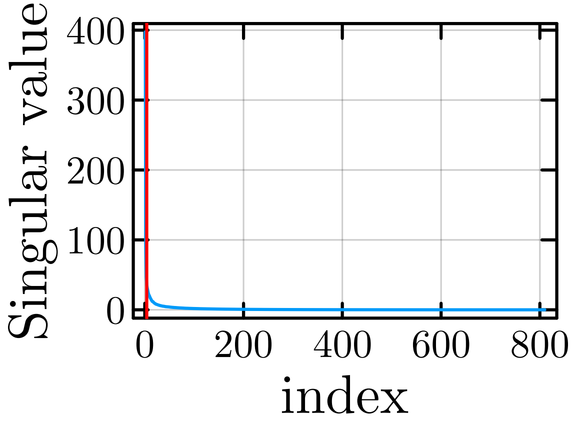


IMAGE COMPRESSION





$\chi = 4$

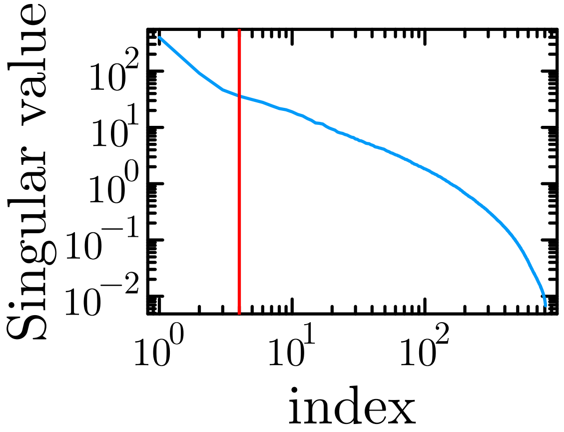
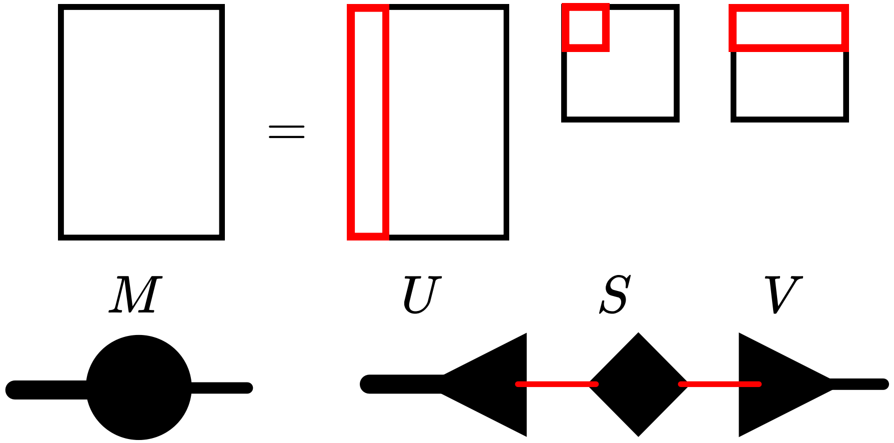
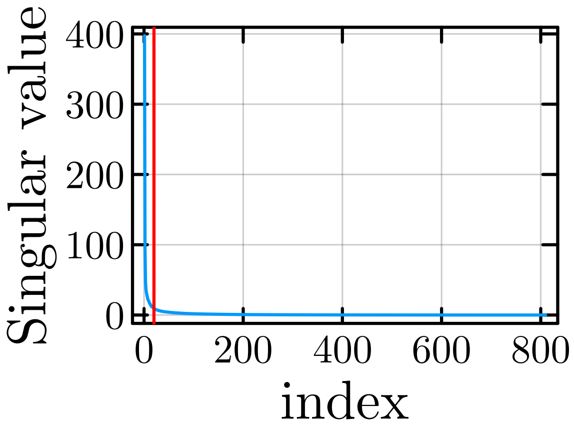


IMAGE COMPRESSION



$\chi = 4$

$\chi = 20$



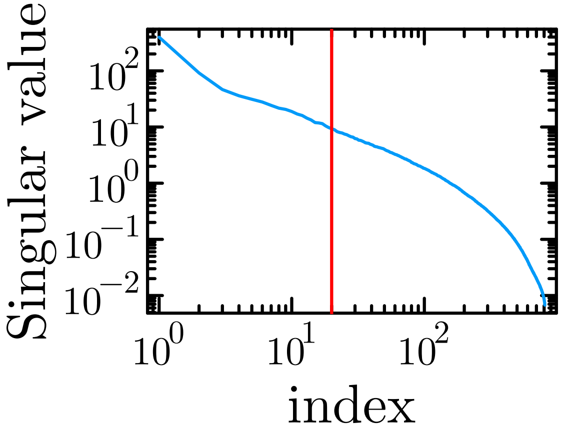
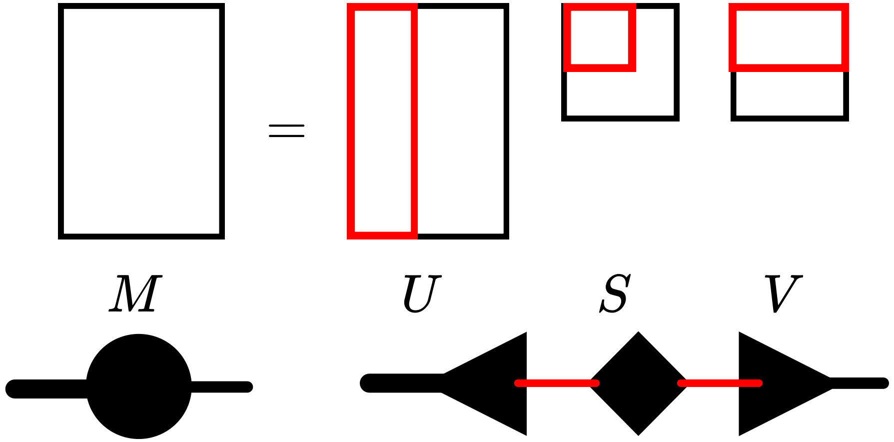
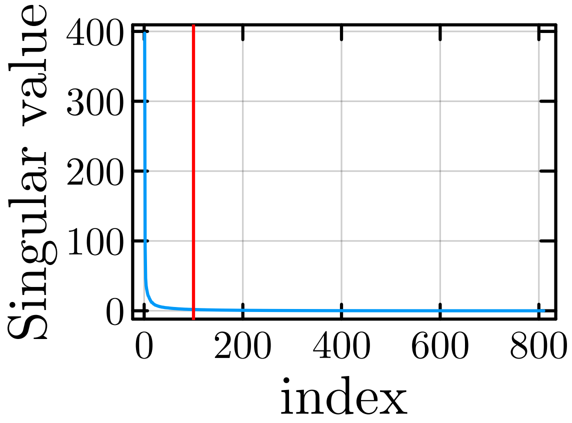
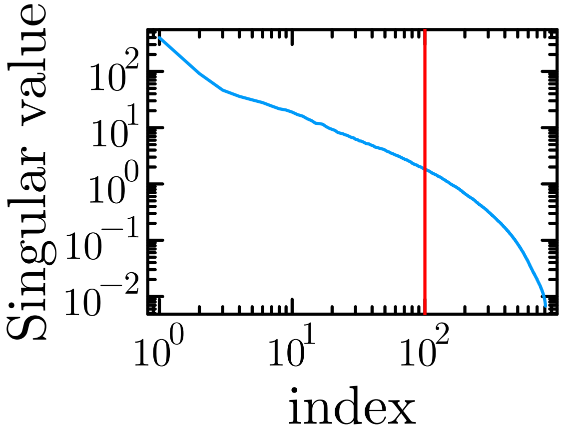


IMAGE COMPRESSION





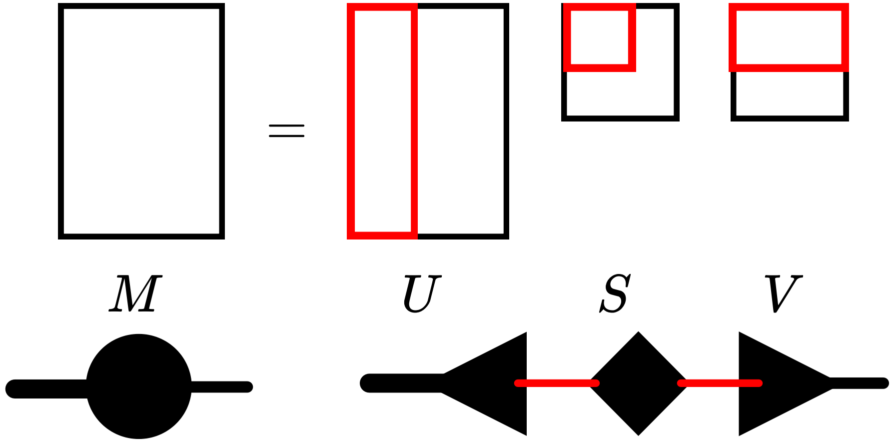


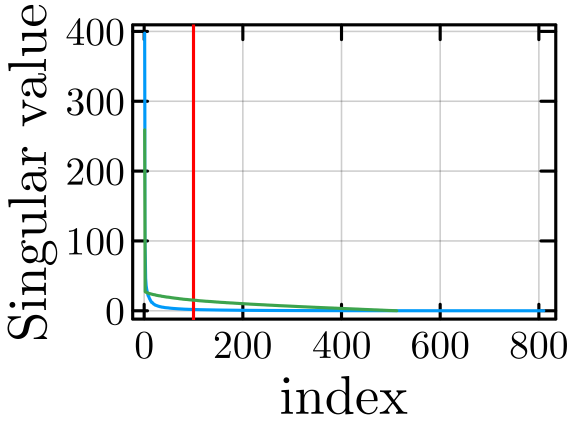
$\chi = 4$

$\chi = 20$

$\chi = 100$

IMAGE COMPRESSION





$\chi = 4$

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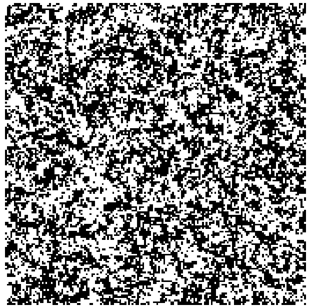


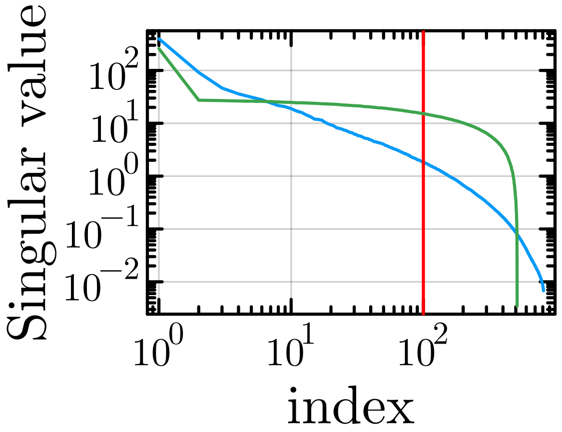
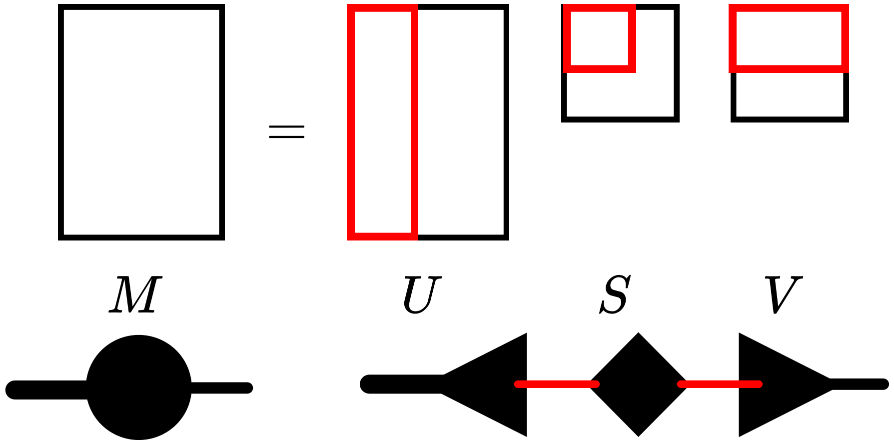
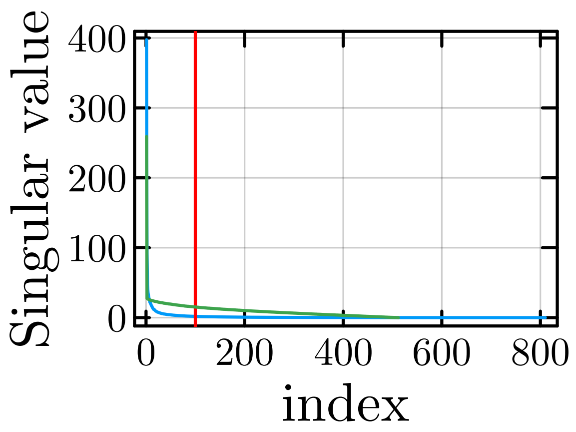
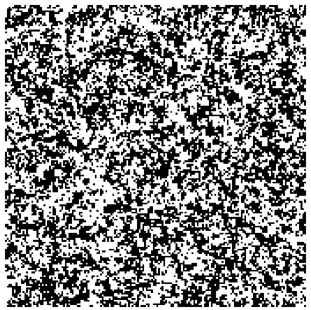


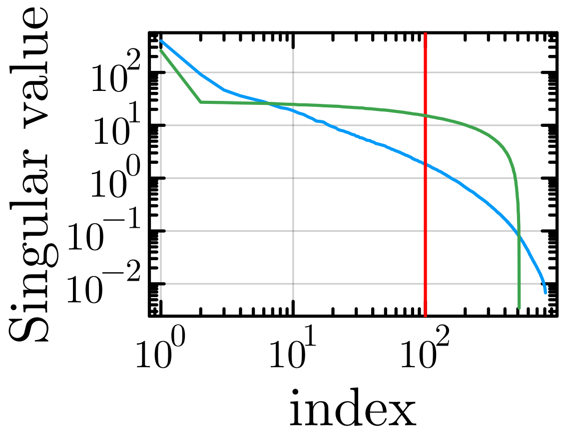
IMAGE COMPRESSION





$\chi = 4$ $\chi = 20$ $\chi = 100$





ALWAYS ASK : *WHY* DO I EXPECT THE BOND DIMENSION TO BE LIMITED?