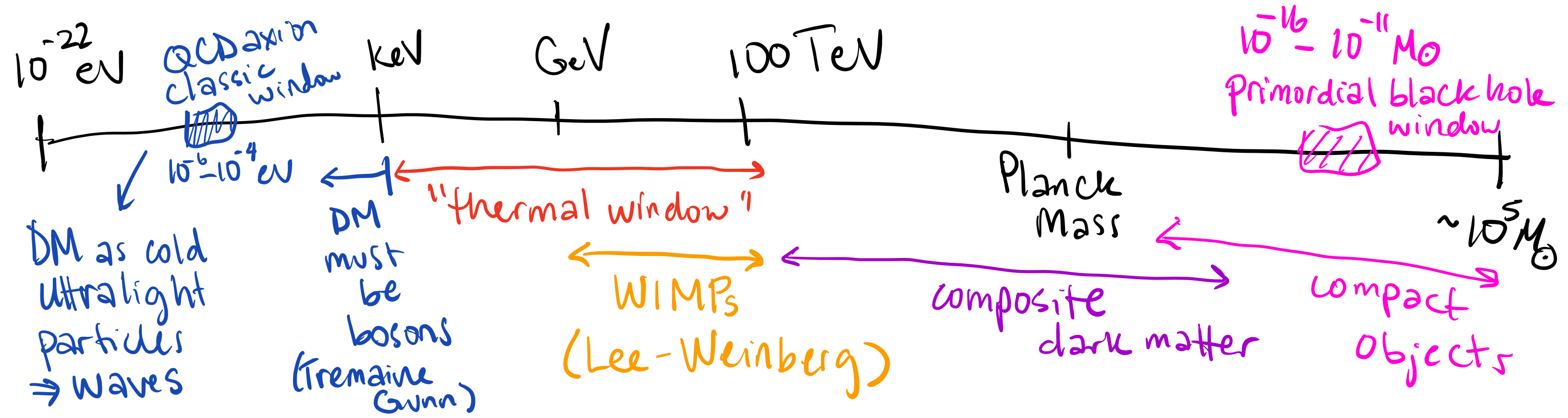
Summary of things we know about dark matter:

- ↳ how much there is (CMB, see later lectures) relative to Ω_b and Ω_m (most precise thing we know!)
  also BBN + K_{eq}! Or BBN + expansion history!
- ↳ locally, $\rho_{DM} \sim 0.4\ \mathrm{GeV/cm^3}$ w/ errors of $\sim 0.1\ \mathrm{GeV/cm^3}$
- ↳ lower bound on lifetime ($\tau \gg$ age of universe)
  DM was present well before recombination K_{eq}.
- ↳ upper bound on interaction rates (or we would have seen interactions)
- ↳ upper bound on DM velocity / free streaming distance or small halos get erased
- ↳ upper bound on mass, $m_{DM} \ll$ observed DM halo masses
- ↳ lower bound on mass
  - ↳ bosons – deBroglie wavelength must fit inside smallest DM halos, $m \gtrsim 10^{-22}$ eV (constraints these days more like 10^{-19} eV)
  - ↳ fermions – Pauli exclusion can't prevent existence of smallest DM halos (which have limited size and $v < v_{esc}$), $m \gtrsim$ keV (Tremaine-Gunn)

Huge range of possible masses, ~90 OOM



How to approach huge space of possibilities?

- ↳ total anarchy/apathy (no thanks)
- ↳ very phenominological (good when we have a lot of data)
- ↳ try to explain anomaly ("ambulance chasing")
- ↳ try to make DM part of solution to other known issues w/ standard model (hierarchy problem, strong-CP problem, neutrino mass, etc.)
- ↳ focus on explaining things we know, especially abundance Ω_{DM}

Focus of today: explaining DM abundance from thermal physics in expanding universe @ early times

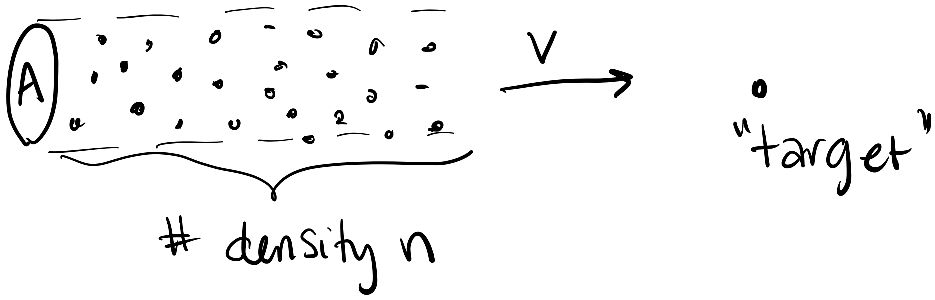
Note: I work in units where $\hbar = c = k_B = 1$!

Suppose DM has a number-changing process. For instance, DM DM → X where X is in SM

- if interaction cross section is zero, number of DM particles is constant, $n_{DM} a^3 = \text{const.} \Rightarrow \frac{d}{dt}(n_{DM} a^3) = 0$

$$\Rightarrow a^3 \frac{dn}{dt} + 3a^2 n \frac{da}{dt} = 0 \Rightarrow \dot{n} + 3Hn = 0$$

- when we turn on annihilation, we have to account for 2 more terms for depletion & production of DM



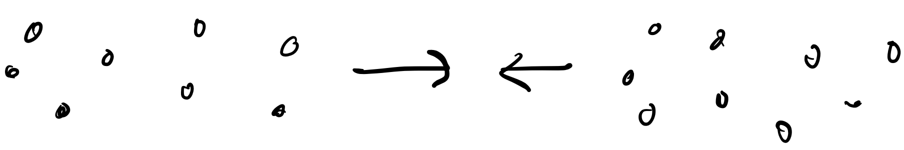
how many particles hit target?

nv is a flux $\frac{\#}{\text{area} \times \text{time}}$

to get a rate of "hits", multiply by an area σ, "cross section," tells you probability for particles to hit target $P \sim \frac{\sigma}{A}$ (like throwing darts to do Monte Carlo integration)

So interaction rate is $n\sigma v$. In general $\sigma(v) \neq$ const., depending on particle physics model, so we do "thermal average" over distribution of particles' speeds so the rate is $\underbrace{n\langle\sigma v\rangle}_{\text{thermal interaction rate}}$ ← thermal average

w/ dark matter, you don't have 1 target, you have a density of targets



scales as n^2

so $\dot{n}+3Hn = \underset{\text{depletion of DM}}{-}\underbrace{n^2\langle\sigma v\rangle}_{\text{has right units}} + \underset{\text{unknown}}{x}\underbrace{\langle\sigma v\rangle}_{\text{independent of } n\text{, accounts for DM production}}$

we can figure out what x is using detailed balance: in equilibrium, depletion & production should cancel so there is no net change to N_{DM}

so in equilibrium, $\langle\sigma v\rangle(x-n^2)=0 \Rightarrow x=n_{eq}^2$

thus, $\boxed{\dot{n}+3Hn = -\langle\sigma v\rangle(n^2-n_{eq}^2)}$ Ricatti differential equation

if DM is produced symmetrically in particle-antiparticle pairs, it has zero chemical potential

so $n_{eq} \sim \begin{cases} (mT)^{3/2}e^{-m/T}, & m_{DM} \gg T \text{ (non-relativistic)} \\ T^3, & m_{DM} \ll T \text{ (relativistic)} \end{cases}$

(there are <u>many</u> variations on DM production that violate this)

To solve for DM # density in closed form is not possible but we can understand qualitative behaviour in two regimes

1. $n\langle\sigma v\rangle \ll H \Rightarrow$ expansion dominates
"DM particles have a hard time finding each other to interact as universe expands"

$$\Rightarrow n \sim 1/a^3$$

2. $n\langle\sigma v\rangle \gg H \Rightarrow n$ follows equilibrium distribution, any deviation drives $n \to n_{eq}$ on RHS of equation
"DM interacting a lot & maintaining equilibrium"

Crossover occurs when $n\langle\sigma v\rangle = H$ and if DM was in equilibrium, we say this is when it "freezes out". (If DM was never in equilibrium and n is far from n_{eq}, we say dark matter "freezes in")

at freeze-out, $T = T_f$, for $T \ll T_f$

$$n \sim \frac{n_f a_f^3}{a^3} \sim \frac{n_{f,eq.} T^3}{T_f^3}$$ (ignoring time-dependent g_* to get rough estimates)

two cases to consider, 1. $T_f \gg m_{DM}$

2. $T_f \ll m_{DM}$

① $T_f \gg m_{DM}$ (freezeout while relativistic)

$n_{eq,f} \sim T_f^3 \Rightarrow n \sim n_{eq,f}\left(\frac{T}{T_f}\right)^3 \sim T^3$ after freezeout

(this is just like neutrinos! See Vicky's lectures!)

if $T_{DM} \sim T_{CMB}$ then based on $\eta = \frac{n_b}{n_\gamma} \sim \frac{n_b}{n_{DM}} \sim 10^{-9}$

then $n_{DM} m_{DM} \sim 5\, n_b m_b \Rightarrow m_{DM} \sim 10^{-8} m_b \sim 10\,eV$
(since $\Omega_{DM} \sim 5\Omega_b$)

if $m_{DM} \gtrsim 10\,eV$ & decoupled while relativistic w/ $T_{DM} \sim T_{CMB}$ then we make too much DM

but w/ $m_{DM} \lesssim 10\,eV$ & $V_{RMS} \sim \sqrt{\frac{T}{m}} \Rightarrow$ DM goes fast

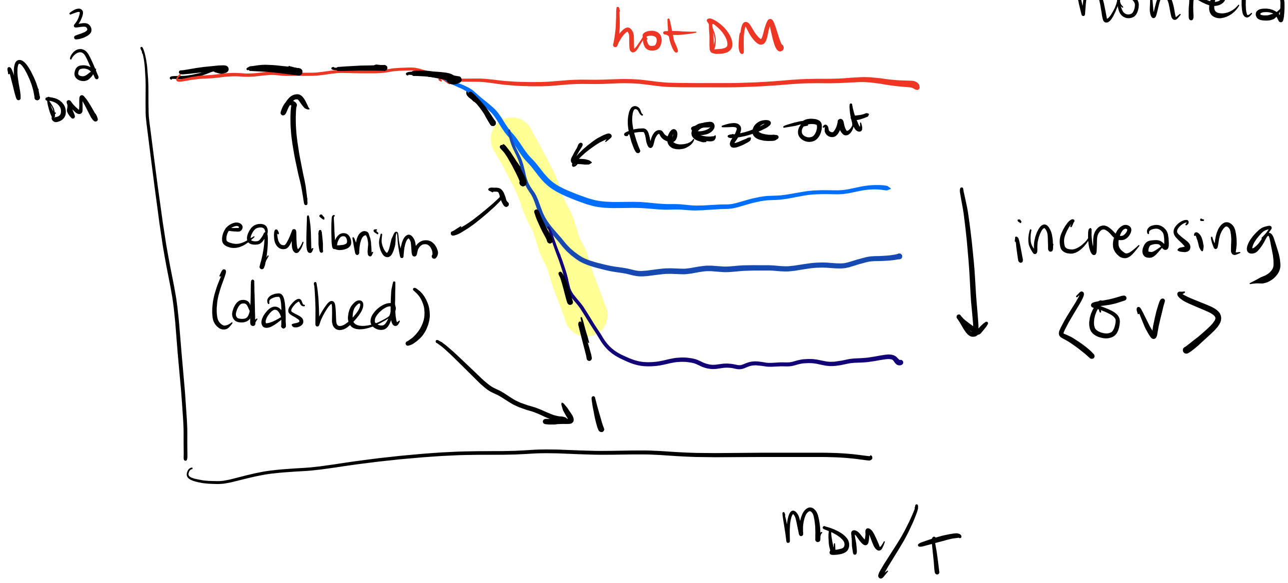
fast moving DM ("hot DM") is ruled out by structure formation.

Loophole, what if $T_{DM} \neq T_{CMB}$ (similar to neutrino decoupling at higher g_* than g_*^0)

On homework, show $\Omega_{DM} = \frac{m_{DM}}{94\,eV} \times \frac{11}{4} \times \left(\frac{T_{DM}}{T_\gamma}\right)^3$

"Warm dark matter" current structure formation constraints are $m \gtrsim 5\,keV \Rightarrow g_* \sim 10^3 - 10^4$
(Standard Model only has $g_* \sim 10^2$)

② What about $T_f \ll m_{DM}$? (freeze-out while nonrelativistic)



freezeout temp not very sensitive to $\langle \sigma v \rangle$

$T_f \sim m_f$ up to log factors

What value of $\langle \sigma v \rangle$ gives observed DM abundance?

to estimate note that @ matter radiation equality $\rho_m = \rho_r \Rightarrow m_{DM}\, n_{DM}(T_{MRE}) \sim T_{MRE}^4$

$$\Rightarrow m_{DM}\, n_{f,eq} \left(\frac{T_{MRE}}{T_f}\right)^3 \sim T_{MRE}^4 \Rightarrow \frac{n_{f,eq}}{T_f^3} \sim \frac{T_{MRE}}{m_{DM}}$$

but also @ T_f, $n_{f,eq} \langle \sigma v \rangle \sim H \sim \frac{T_f^2}{M_{Pl}}$ (Hubble's law during radiation domination)

so $$\frac{\cancel{T_f}^{2}}{M_{Pl} \langle \sigma v \rangle \cancel{T_f^3}} \sim \frac{T_{MRE}}{m_{DM}} \Rightarrow \langle \sigma v \rangle \sim \underbrace{\cancel{\left(\frac{m_{DM}}{T_f}\right)}}_{\sim 1} \frac{1}{M_{Pl} \times T_{MRE}}$$

plug in $M_{Pl} \sim 10^{19}$ GeV $T_{MRE} \sim 1$ eV $\Rightarrow$ $\langle \sigma v \rangle \sim 10^{-27}\, cm^3/s$

in units where $\hbar = c = 1$, mass has units of energy and length has units of mass^{-1} so

$[\langle \sigma v \rangle] = \text{mass}^{-2}$ if $\langle \sigma v \rangle$ doesn't depend on velocity ("s-wave") then by dimensional analysis, $\langle \sigma v \rangle \sim \frac{\alpha^2}{m_{DM}^2}$ ← coupling of theory (i.e. fine structure)

in electroweak theory, $\alpha \sim 10^{-2}$ so to get $\langle \sigma v \rangle \sim 10^{-27} \text{cm}^3/\text{s} \Rightarrow m_{DM} \sim$ few × 100 GeV

↳ scale of weak interaction

"WIMP miracle"! (Huge coincidence!)

if $\alpha = 1$ (maximum for perturbativity)

then $m_{DM} \sim 100$ TeV "unitarity bound"

WIMPs have $m \lesssim 100$ TeV

if $m_{DM} \sim 1$ MeV then $T_f \sim T_{BBN}$ & DM affects entropy during BBN so since we've measured BBN & it agrees w/ SM then

$m_{DM} \gtrsim 1$ MeV for thermal freeze out