Exercises: Astro probes of BSM at all wavelengths

These problems are designed to be completed in approximately 90 minutes. They emphasize conceptual reasoning and order-of-magnitude estimates over detailed calculation. You may find it helpful to work in natural units ($\hbar = c = k_B = 1$) throughout.

Problem 1: Volume vs. Surface Emission

A key insight is that weakly-coupled BSM particles produced in a stellar interior free-stream out of the star, so their luminosity scales with the *volume* of the emitting region. In contrast, the photon luminosity scales with the *surface area*, since photons have a short mean free path and can only escape from near the stellar surface.

(a) Consider a hypothetical BSM particle ϕ produced uniformly throughout the solar interior with an energy loss rate per unit volume ϵ_ϕ. Write down the total BSM luminosity L_ϕ in terms of ϵ_ϕ and the solar radius $R_\odot$.

(b) Now consider a red giant with a helium core of radius $r_c \sim 0.01\, R_\odot$ and a core temperature $T_c \sim 10\, T_\odot$, where $T_\odot \sim 1$ keV is the solar core temperature. Suppose the production rate scales as $\epsilon_\phi \propto T^n$. For what value of n does the BSM luminosity from the red giant core equal that from the entire Sun, i.e., $L_\phi(\text{RG core}) = L_\phi(\odot)$?

(c) Briefly explain why your answer to (b) means that evolved stars with hot, compact cores can be more powerful probes of BSM physics than the Sun, even though their emitting regions are far smaller.

Problem 2: White Dwarf Cooling and New Physics

The WD luminosity function counts the number of WDs as a function of luminosity. Since we observe the WD population at a single snapshot in time, the number of WDs at a given luminosity is proportional to the time a WD spends cooling through that luminosity.

(a) A WD cools by emitting photons from its surface with luminosity $L_\gamma(T)$, and its total thermal energy is $E(T)$. If a new BSM cooling channel $L_\phi(T)$ is introduced that dominates at high temperatures, at which end of the luminosity function—bright or faint—do you expect a deficit of WDs relative to the SM prediction? Explain your reasoning.

(b) Now consider two different BSM particles: one whose production rate increases steeply with temperature (e.g., $\epsilon_\phi \propto T^8$), and another with a temperature-independent production rate. Which of these would produce a more *distinctive* (and therefore more constrainable) signature in the WD luminosity function? Explain in terms of where and how sharply each affects the luminosity function.

Problem 3: How Stable is "Stable"?

The lecture notes show that DM decay lifetimes are constrained to $\tau \gtrsim 10^{26}$ s across a wide range of masses, despite the age of the universe being only $t_0 \sim 10^{18}$ s.

(a) If DM has a decay lifetime $\tau \gg t_0$, what fraction of DM particles has decayed by time t_0? Express your answer to leading order in t_0/τ.

(b) The notes argue that a fraction $f \sim 10$ eV/5 GeV $\sim 10^{-9}$ of the DM rest mass energy, if deposited into the gas, would be sufficient to fully reionize the universe. Treating full reionization as a conservative (i.e., worst-case) detectability threshold—since a fully reionized universe would be unmistakable—use your answer from (a) to estimate the longest DM lifetime that would produce this level of energy injection. How does it compare to 10^{26} s?

(c) In reality, the CMB is sensitive to much subtler effects than full reionization. The Planck satellite measures the optical depth of the universe to $\sim 1\%$ precision, meaning it can detect a $\sim 1\%$ change in the ionization fraction. Use this to refine your estimate: what is the longest τ detectable by Planck? Comment on why your estimate differs from the actual bound of $\sim 10^{26}$ s (hint: consider the efficiency with which decay energy is converted to ionization).

Problem 4: Choosing Your Target

You are planning a gamma-ray observing campaign to search for DM signals. You have two candidate targets:

- **Target A**: A nearby dwarf galaxy with $J \sim 10^{19}\,\text{GeV}^2/\text{cm}^5$ and $D \sim 10^{18}\,\text{GeV}/\text{cm}^2$. Dwarf galaxies are DM-dominated systems with very little gas, star formation, or other astrophysical activity, so the expected astrophysical gamma-ray background from this target is well below the instrument's sensitivity.
- **Target B**: A galaxy cluster with $J \sim 10^{22}\,\text{GeV}^2/\text{cm}^5$ and $D \sim 10^{18.5}\,\text{GeV}/\text{cm}^2$. Galaxy clusters contain hot intracluster gas, active galaxies, and cosmic ray interactions that produce an astrophysical gamma-ray flux roughly ~ 100 times larger than the expected DM annihilation signal for a thermal relic cross section. This background must be modeled and subtracted.

(a) If you are searching for DM *annihilation*, which target gives a larger expected signal? Despite this, why might you prefer the other target for setting a robust constraint?

(b) If instead you are searching for DM *decay*, how does the signal comparison between the two targets change? Explain why the balance between signal strength and backgrounds shifts relative to the annihilation case.

(c) The notes mention that DM substructure can significantly boost the J-factor but not the D-factor. Explain in a few sentences why $\langle \rho^2 \rangle \neq \langle \rho \rangle^2$ matters for annihilation but not for decay, connecting this to the mathematical form of the two integrals.

Problem 5: The p-Wave Penalty

Some DM models feature p-wave annihilation, where $\langle \sigma v \rangle \propto v^2$.

(a) At thermal freeze-out, the DM temperature is $T_f \sim m_\chi/20$. Using the equipartition relation for a non-relativistic gas, estimate the typical DM velocity at freeze-out (in units of c) and the corresponding v^2 factor.

(b) In a Milky Way–like galaxy today, DM has typical velocities $v \sim 10^{-3}\,c$. What is the v^2 factor in the present-day halo?

(c) Compute the ratio of the present-day annihilation rate to the freeze-out rate for a p-wave model. What does this imply for the prospects of indirectly detecting p-wave DM using gamma-ray observations of galaxies?