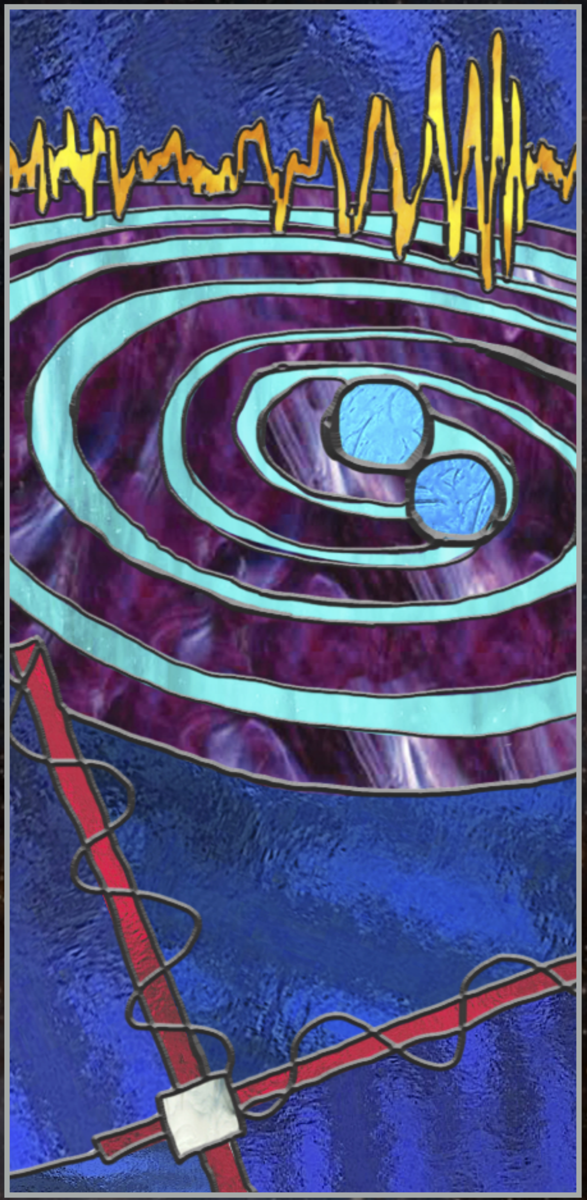
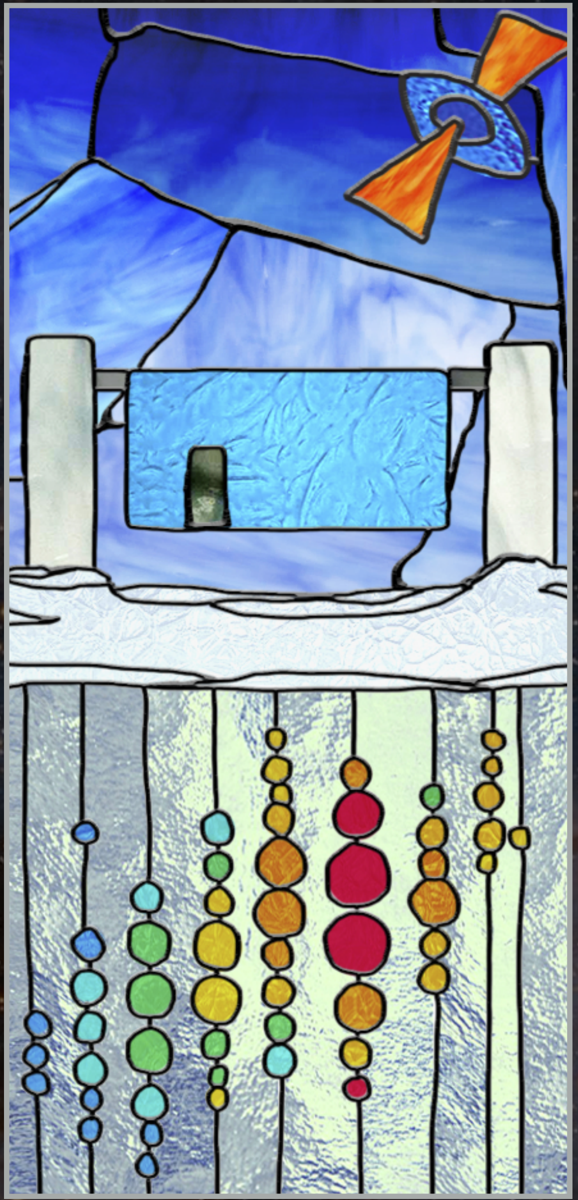
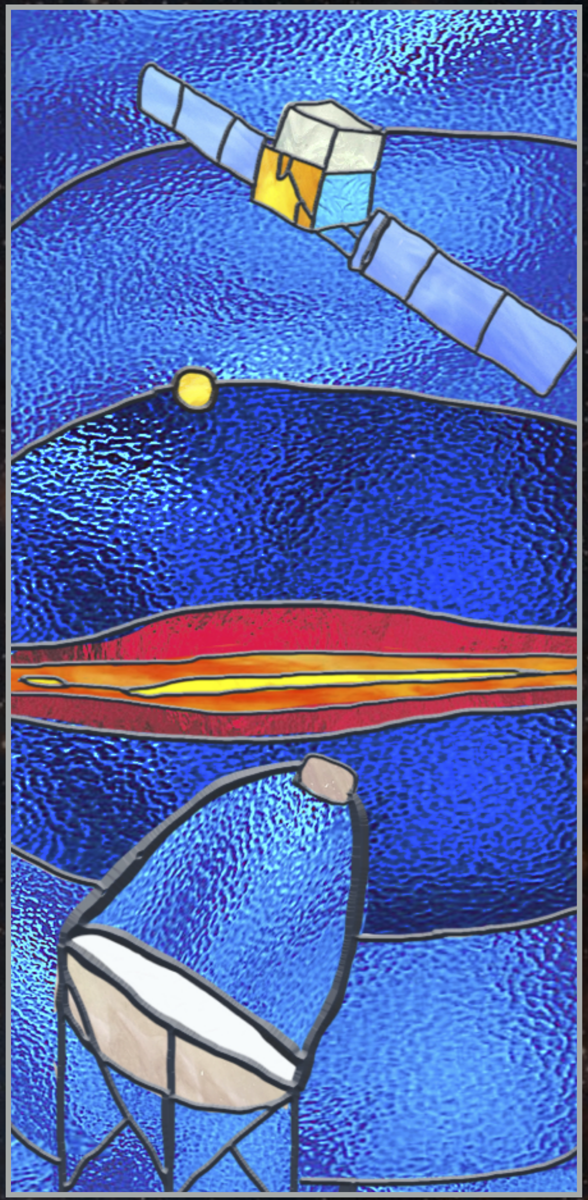




Lecture 2: *Hadronic Processes*

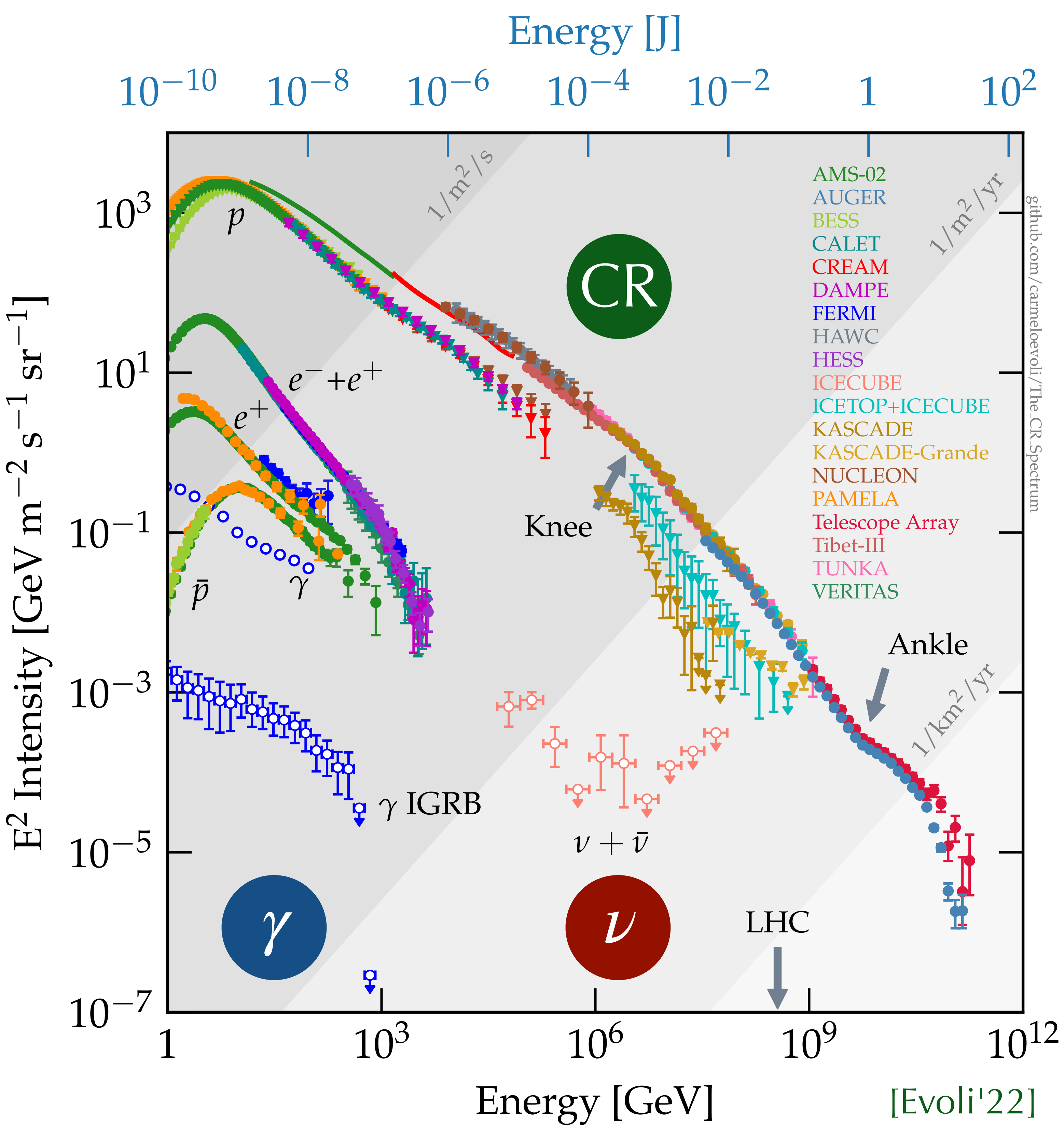
Markus Ahlers
Niels Bohr Institute
2nd UNDARK School 2026

CARLSBERG FOUNDATION

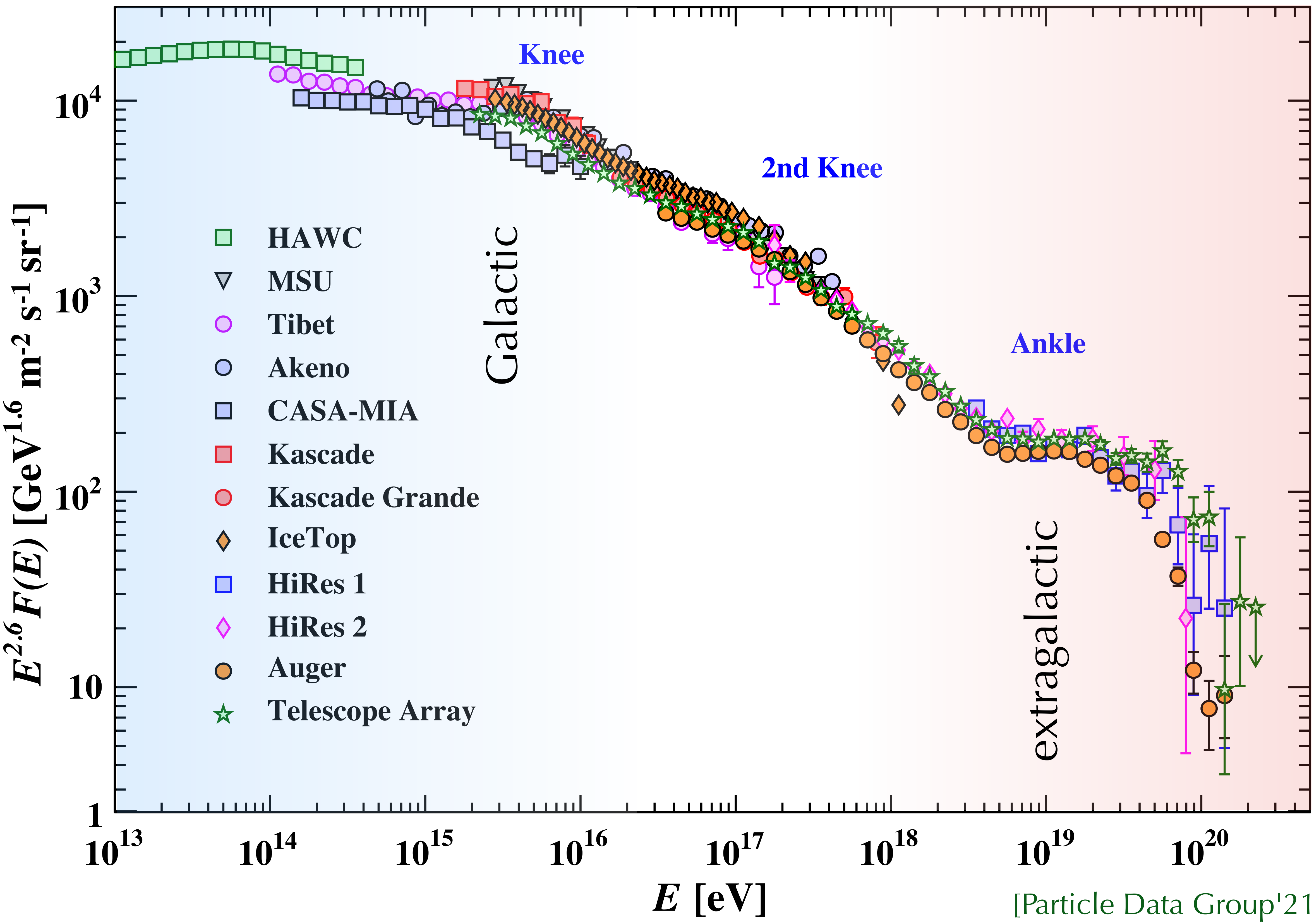
KØBENHAVNS UNIVERSITET

Cosmic Rays

- Cosmic rays (CRs) are **energetic nuclei** and (at a lower level) leptons.
- Spectrum follows a **power-law** over many orders of magnitude, indicating a **non-thermal origin**.
- **Direct observation** with satellite and balloon-borne experiments up to TeV
- **Indirect observation** as air showers above 10 TeV



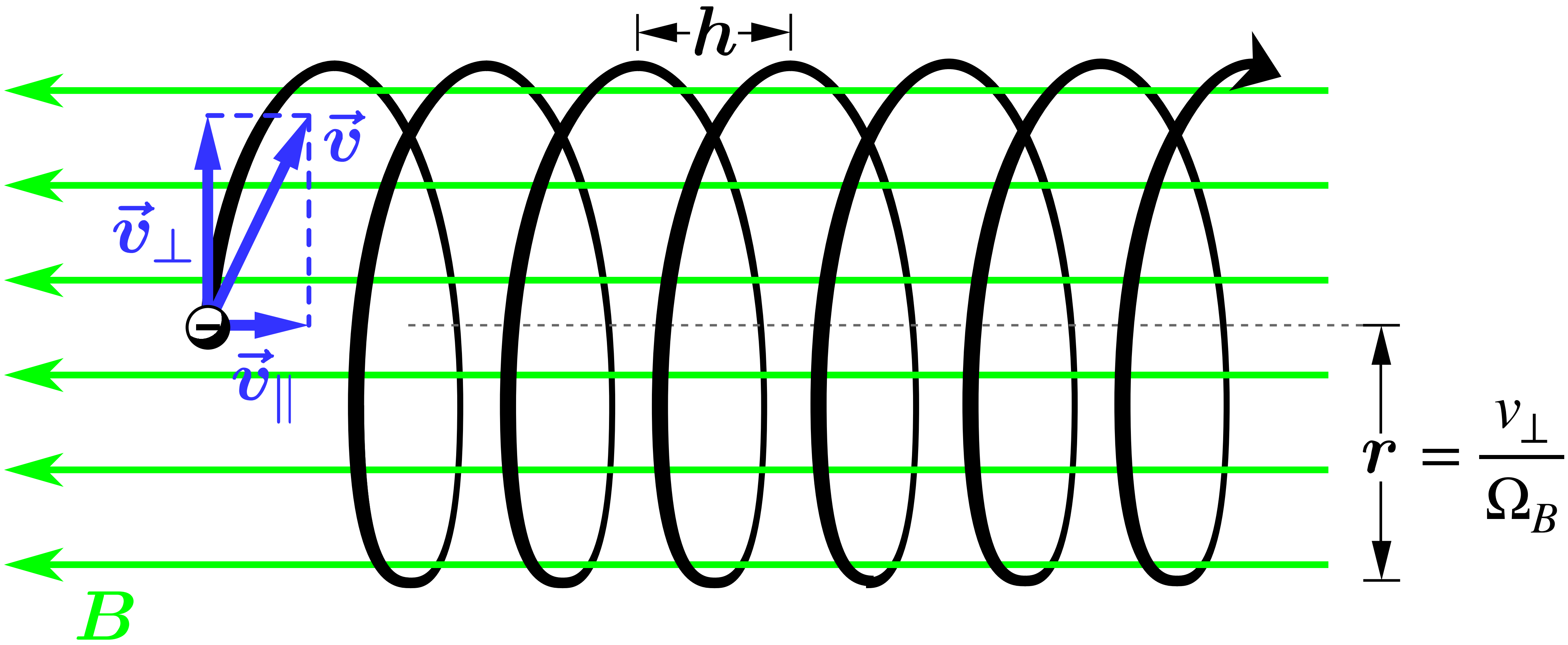
What is the Origin of Cosmic Rays?



[Particle Data Group'21]

Cosmic Ray Transport

Cosmic Ray Gyration



Pitch angle α between $\mathbf{v}(t)$ and $\mathbf{B}_0$ remains constant in time.

Path is a superposition of circular motion in the plane perpendicular to $\mathbf{B}_0$ and linear motion along $\mathbf{B}_0$ with velocity:

$$v_\parallel = \cos\alpha v \equiv \mu v.$$

Magnetic Turbulence

- Consider cosmic magnetic fields with a coherent background field $B_0\mathbf{e}_z$ and a **homogenous and isotropic** turbulent component $\delta\mathbf{B}(\mathbf{r})$:

$$\mathbf{B}(\mathbf{r}) = \underbrace{B_0\mathbf{e}_z}_{\text{ordered}} + \underbrace{\delta\mathbf{B}(\mathbf{r})}_{\text{turbulent}}$$

- Turbulence can be characterized by its **two-point correlation function**:

$$\langle \delta B_i(\mathbf{r})\delta B_j(\mathbf{r}')\rangle = C_{ij}(\mathbf{r}-\mathbf{r}')$$

- To characterize the turbulence we look into the Fourier modes:

$$\delta B_i(\mathbf{r}) = \int \mathrm{d}^3k\, \delta\tilde{B}_i(\mathbf{k})e^{i\mathbf{k}\mathbf{r}}$$

Magnetic Turbulence

- The two-point correlation function can now be expressed in Fourier space:

$$\langle \delta\tilde{B}_i(\mathbf{k})\delta\tilde{B}_j^*(\mathbf{k}')\rangle = \delta(\mathbf{k}-\mathbf{k}')\left(\delta_{ij} - \frac{k_ik_j}{k^2}\right)\frac{\mathcal{P}(k)}{4\pi k^2}$$

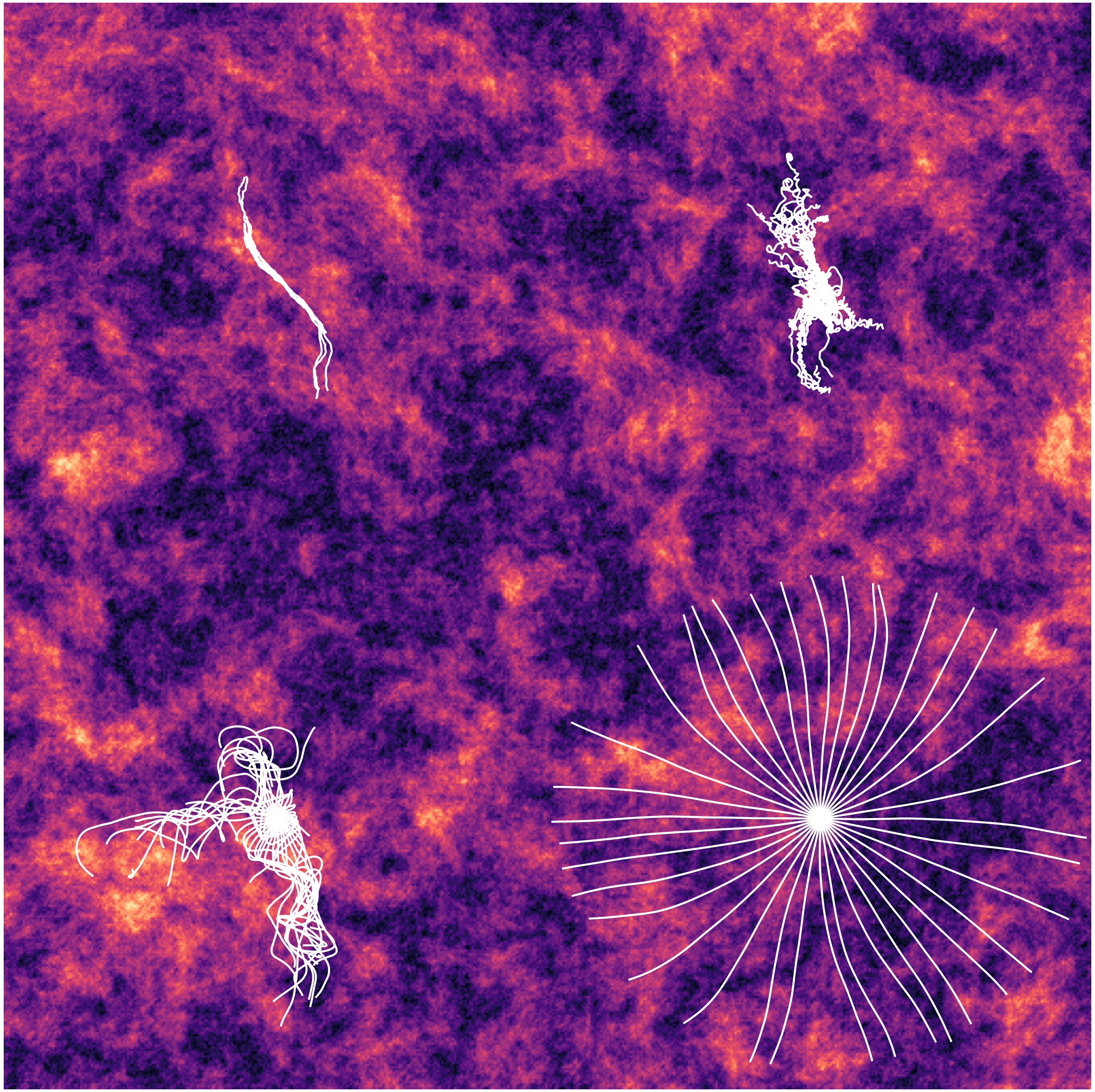
- The **power spectrum** $\mathcal{P}(k)$ is normalized to the energy density of the turbulence:

$$U_{\delta B} = \frac{1}{8\pi}\langle\delta\mathbf{B}^2\rangle = \int \mathrm{d}k\mathcal{P}(k)$$

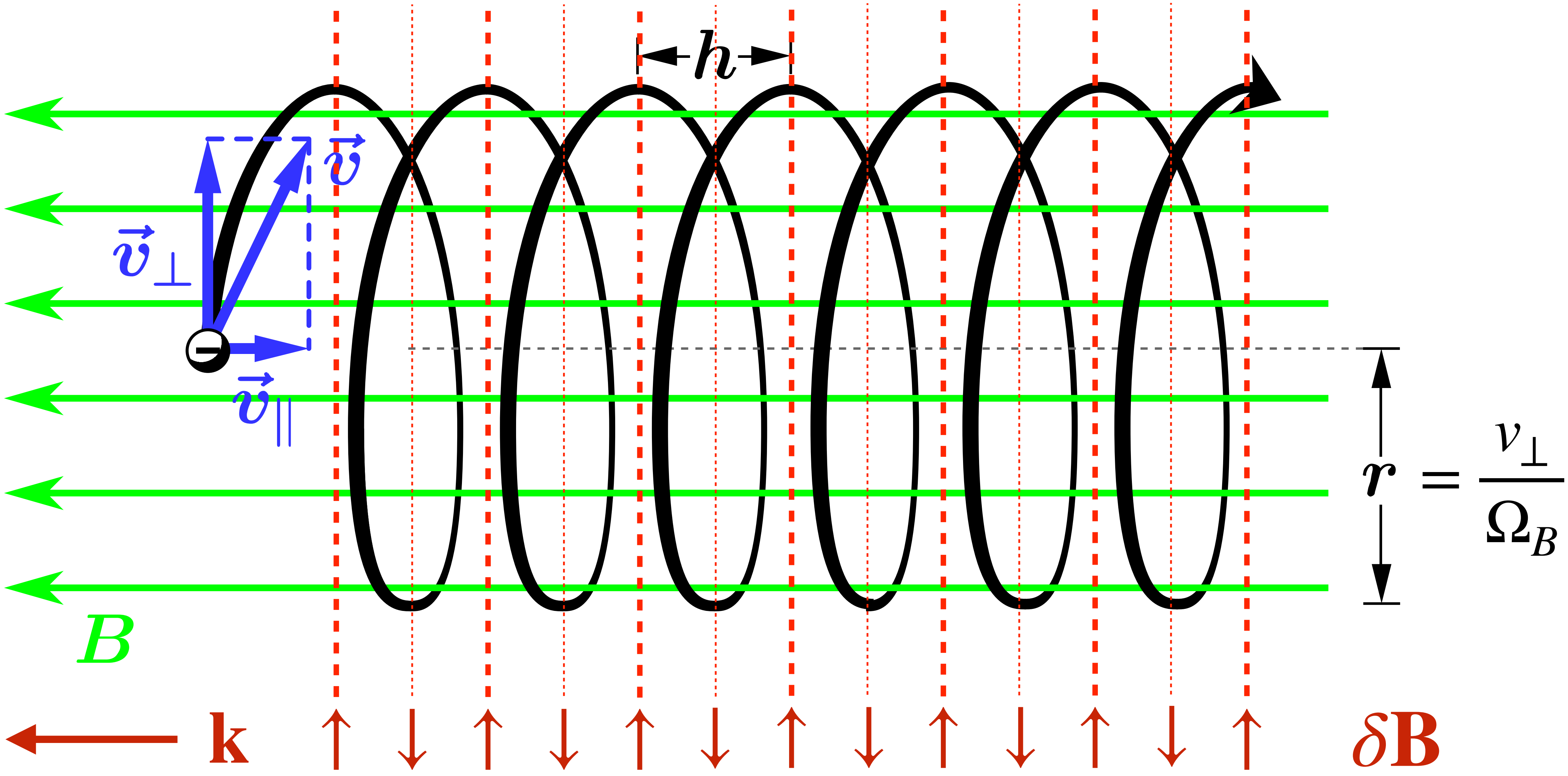
- For instance, in **Kolmogorov turbulence**:

$$\mathcal{P}(k) \propto k^{-5/3} \quad (k_{\rm min} < k < k_{\rm max})$$

Magnetic Turbulence



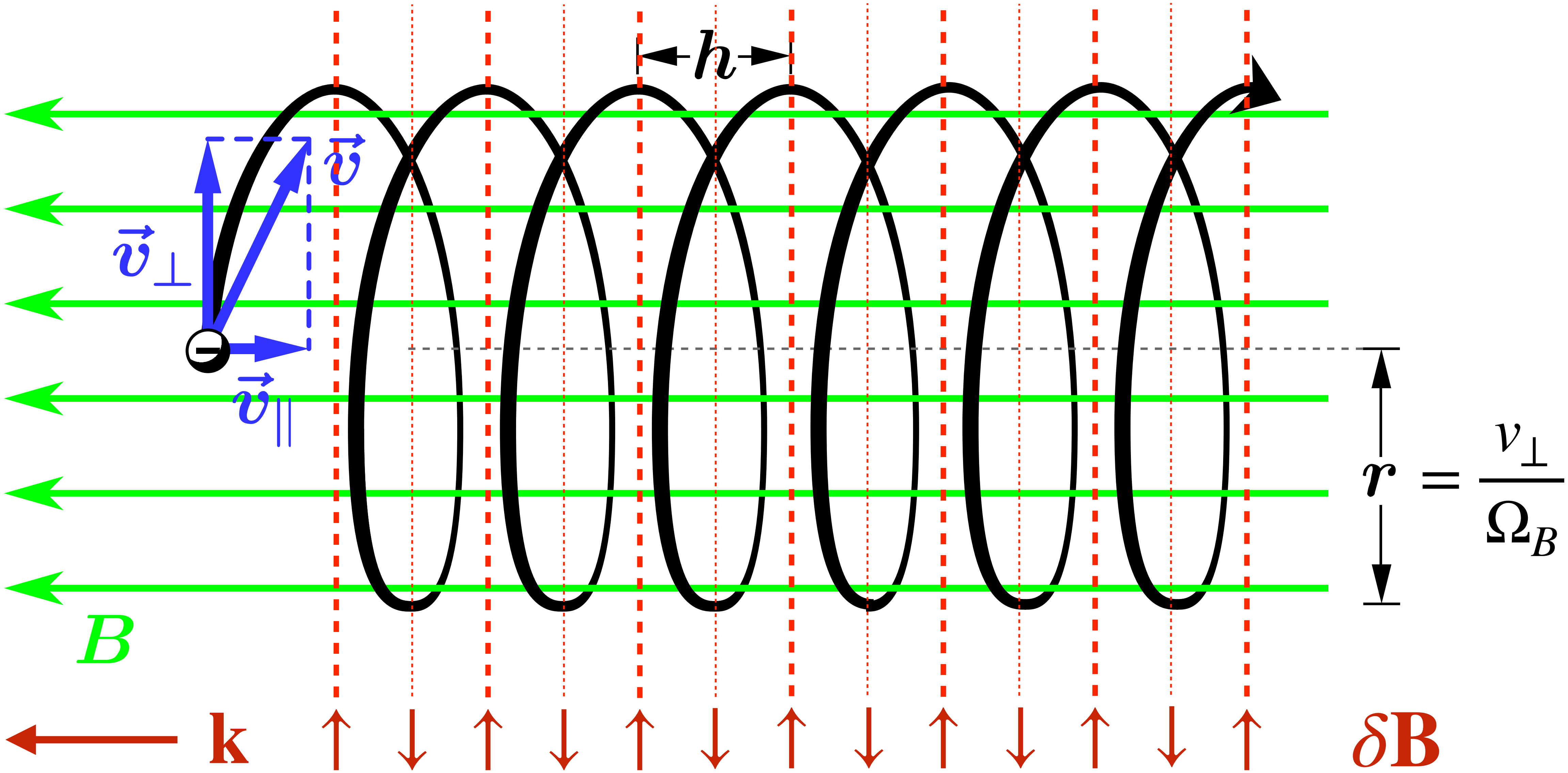
Resonant Scattering



Consider now a **magnetic perturbation** in form of a plane wave:

$$\delta\mathbf{B} = \delta B\mathbf{e}_x \cos(kz + \alpha)$$

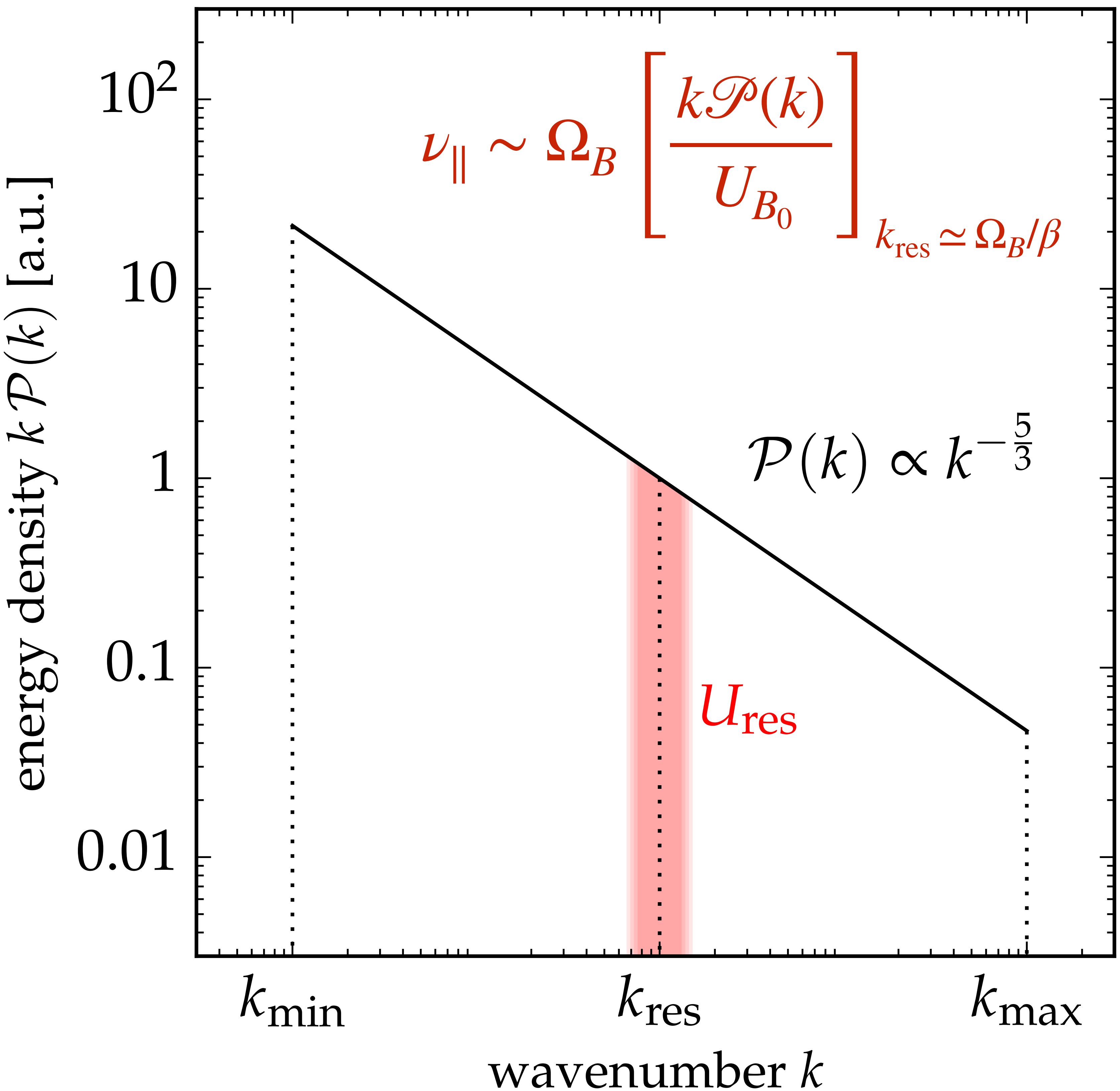
Resonant Scattering



The time-averaged Lorentz force $\delta\mathbf{F}_L = q\boldsymbol{\beta}\times\delta\mathbf{B}$ along the path has the strongest contribution at the **resonance**:

$$kv_\parallel = \pm\,\Omega_B$$

Resonant Scattering



Diffusion Approximation

- Treating the effect of magnetic turbulence as a collision term with rate $\nu_\parallel$ that isotropizes the PSD, we arrive at the **diffusion equation**:

$$\partial_t n - \partial_i \left(K_{ij} \partial_j n \right) = 0$$

- For background field $B_0\mathbf{e}_z$ the **diffusion tensor** can be approximated:

$$\mathbf{K} \simeq \frac{\beta^2}{3} \begin{pmatrix} \nu_\perp^{-1} & \nu_A^{-1} & 0 \\ -\nu_A^{-1} & \nu_\perp^{-1} & 0 \\ 0 & 0 & \nu_\parallel^{-1} \end{pmatrix}$$

$$\nu_\perp \simeq \nu_\parallel + \Omega_B^2/\nu_\parallel$$

$$\nu_A \simeq \Omega_B + \nu_\parallel^2/\Omega_B$$

- For instance, parallel diffusion in **Kolmogorov turbulence**:

$$\nu_\parallel \sim \Omega_B \left[\frac{k\mathscr{P}(k)}{U_{B_0}} \right]_{k_{\rm res} \simeq \Omega/\beta} \sim \Omega_B^{2-\frac{5}{3}} \sim R_L^{-\frac{1}{3}}$$

Example: Transient Point-Source

- Consider now a CR source term: $$\partial_t n - \partial_i\left(K_{ij}\partial_j n\right) = q(t, \mathbf{r}, p)$$

- **Green's function** for $q(t, \mathbf{r}, p) = \delta(\mathbf{r} - \mathbf{r_s})\delta(t - t_s)$:

$$G(t, \mathbf{r}; t_s, \mathbf{r}_s) = (4\pi\Delta t)^{-3/2}(\det \mathbf{K}_s)^{-1/2}\exp\left(-\frac{\Delta\mathbf{r}^T\mathbf{K}_s^{-1}\Delta\mathbf{r}}{4\Delta t}\right)$$

- General solution:

$$n(t, \mathbf{r}, p) = \int \mathrm{d}^3 r_s \int \mathrm{d}t_s G(t, \mathbf{r}; t_s, \mathbf{r}_s)Q(t_s, \mathbf{r}_s, p)$$

- **Impulsive source**, $q = Q_\star(p)\delta(t - t_s)\delta(\mathbf{r} - \mathbf{r}_s)$, in isotropic diffusion:

$$n(t, \mathbf{r}, p) = \frac{Q_\star(p)}{(4\pi\Delta t K_{\rm iso})^{3/2}}\exp\left(-\frac{\Delta\mathbf{r}^2}{4\Delta t K_{\rm iso}}\right) \qquad \langle\Delta\mathbf{r}^2\rangle = 6K_{\rm iso}\Delta t$$

Cosmic Ray Transport

$$\frac{\partial n_i}{\partial t} = \frac{\partial}{\partial r_a}\left[K_{ab}\frac{\partial}{\partial r_b}n_i\right] \qquad \text{(spatial diffusion)}$$

$$+\frac{\partial}{\partial p}\left[p^2\tilde{K}\frac{\partial}{\partial p}\left(\frac{n_i}{p^2}\right)\right] \qquad \text{(momentum diffusion)}$$

$$-\frac{\partial}{\partial r_a}\left(u_a n_i\right) \qquad \text{(convection)}$$

$$-\frac{\partial}{\partial p}\left[\dot{p}n_i - \frac{p}{3}\left(\frac{\partial u_a}{\partial r_a}\right)n_i\right] \qquad \text{(continuous energy loss)}$$

$$-\Gamma_i^{\rm dec}n_i \qquad \text{(CR decay)}$$

$$-c\rho_{\rm bgr}\sigma_i n_i \qquad \text{(loss from CR collisions)}$$

$$+c\rho_{\rm bgr}\sum_j\int {\rm d}E_j\frac{{\rm d}\sigma_{j\to i}}{{\rm d}E_i}n_j \qquad \text{(gain from CR collisions)}$$

$$+Q_i \qquad \text{(source term)}$$

Galactic Cosmic Rays

Galactic Cosmic Rays

- *Standard paradigm*: Galactic CRs accelerated in supernova remnants
  [Baade & Zwicky'34]
  [Ginzburg & Sirovatskii'64]

- diffusive shock acceleration:

$$n_{\rm CR} \propto E^{-\Gamma}$$

- rigidity-dependent escape from Galaxy:

$$n_{\rm CR} \propto E^{-\Gamma-\delta}$$

★ Cosmic-ray arrival directions are randomized.

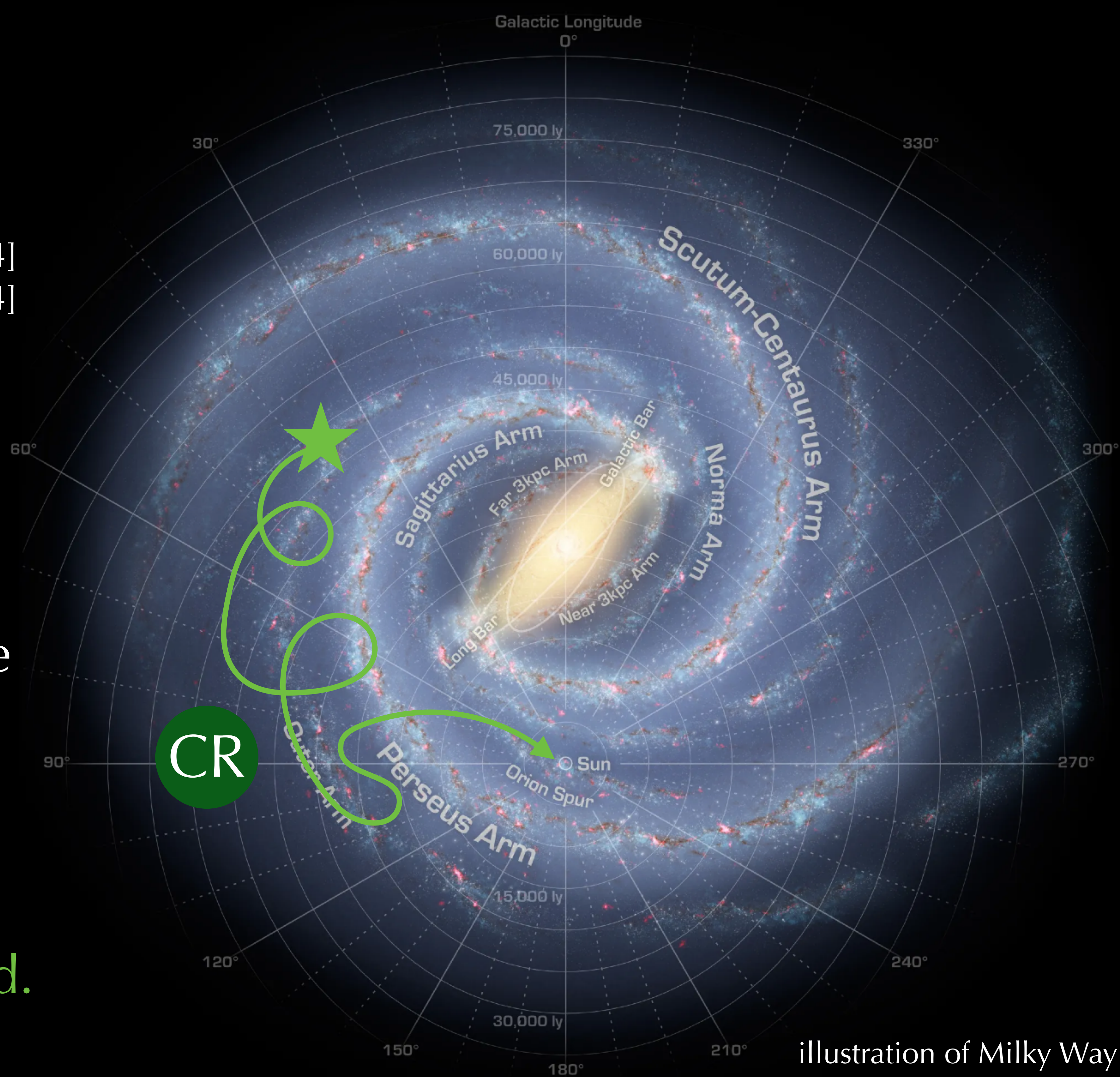
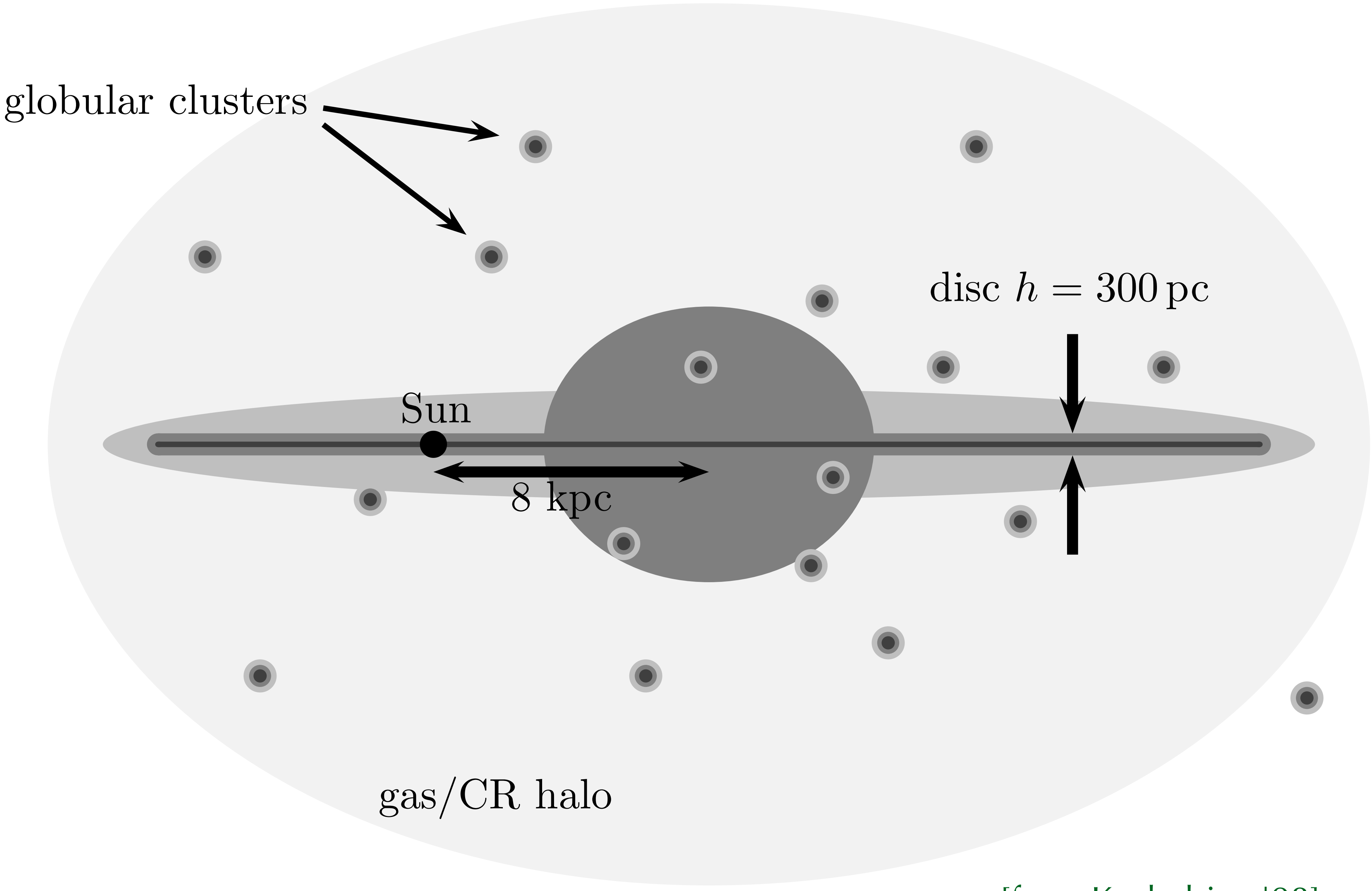


illustration of Milky Way
[Credit: NASA]

Leaky Box Model



[from Kachelriess'08]

Secondary-To-Primary Ratio

- Abundance of Galactic CRs in the Li-Be-B group is larger than observed from solar system measurement.

- Phenomenon is related to production of secondary CRs (n_s) in primary CR (n_p) collisions in background molecular gas:

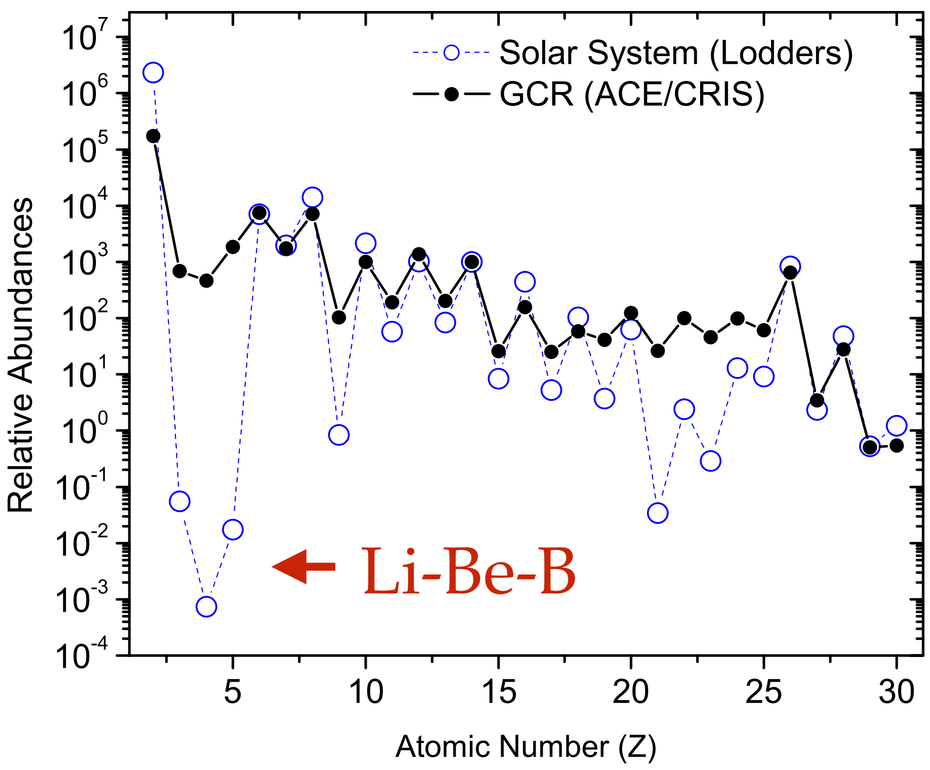
$$\partial_t N_s = -\frac{N_s}{\tau_{\rm loss}} + c\rho\sigma_{p\to s}N_p$$

- Steady-state solution ($\partial_t N_s = \partial_t N_p = 0$):

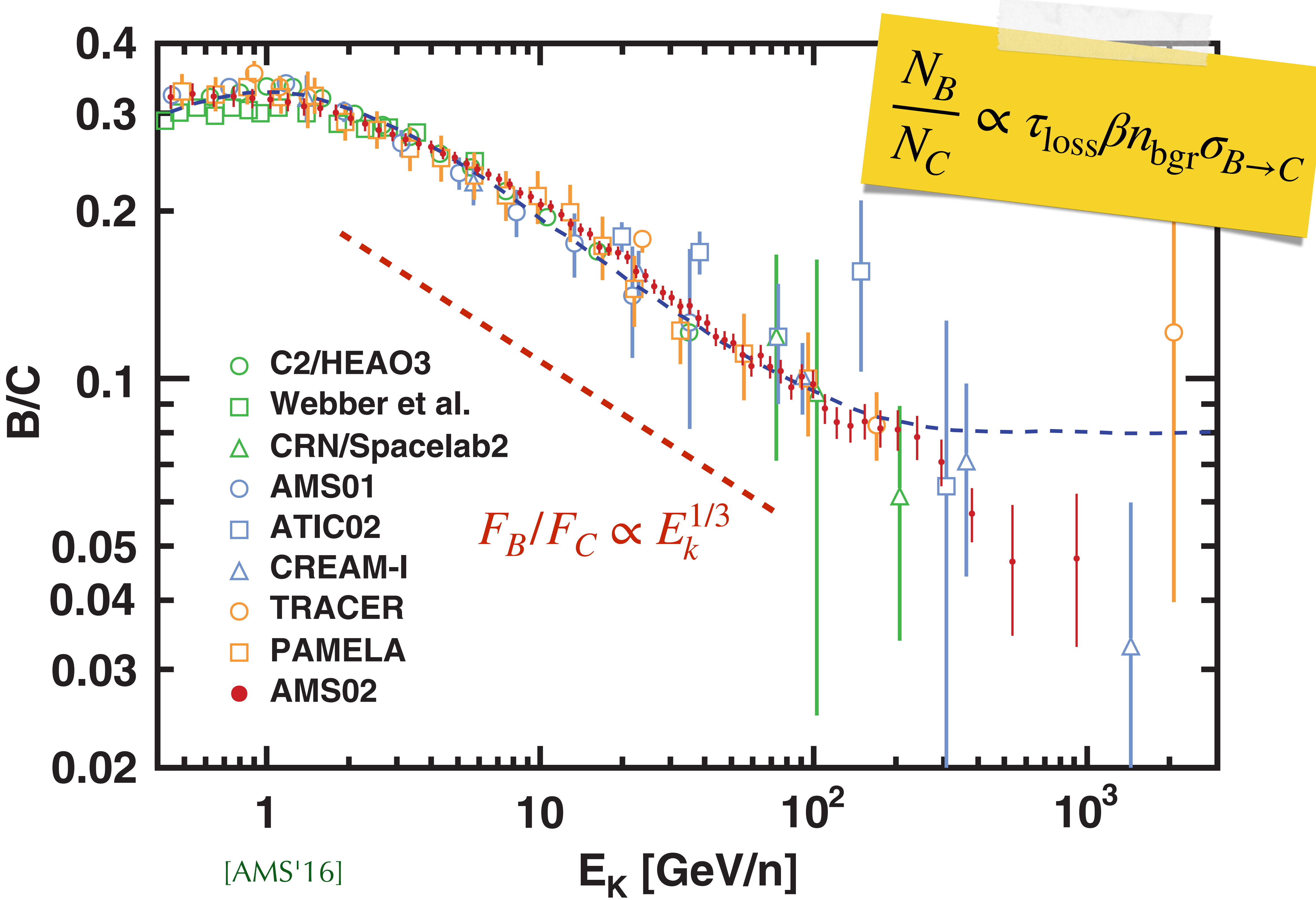
$$N_s = \tau_{\rm loss}c\rho\sigma_{p\to s}N_p$$

- **Secondary-to-primary ratio**:

$$\frac{N_s}{N_p} = \tau_{\rm loss}(E)c\rho\sigma_{p\to s} \propto E^{-\delta_{\rm diff}}$$



Borton-To-Carbon Ratio



[AMS'16]

Radioactive Isotopes

- Secondary isotopes produces in CR interactions can be unstable, *e.g.* ^{10}Be with a lifetime $\tau_{\rm decay} \simeq 1.5 \times 10^6$ years:

$$\partial_t N_s = -\frac{N_s}{\tau_{\rm loss}} - \frac{N_s}{\tau_{\rm decay}} + c\rho\sigma_{p\to s}N_p$$

- Steady-state solution is now:

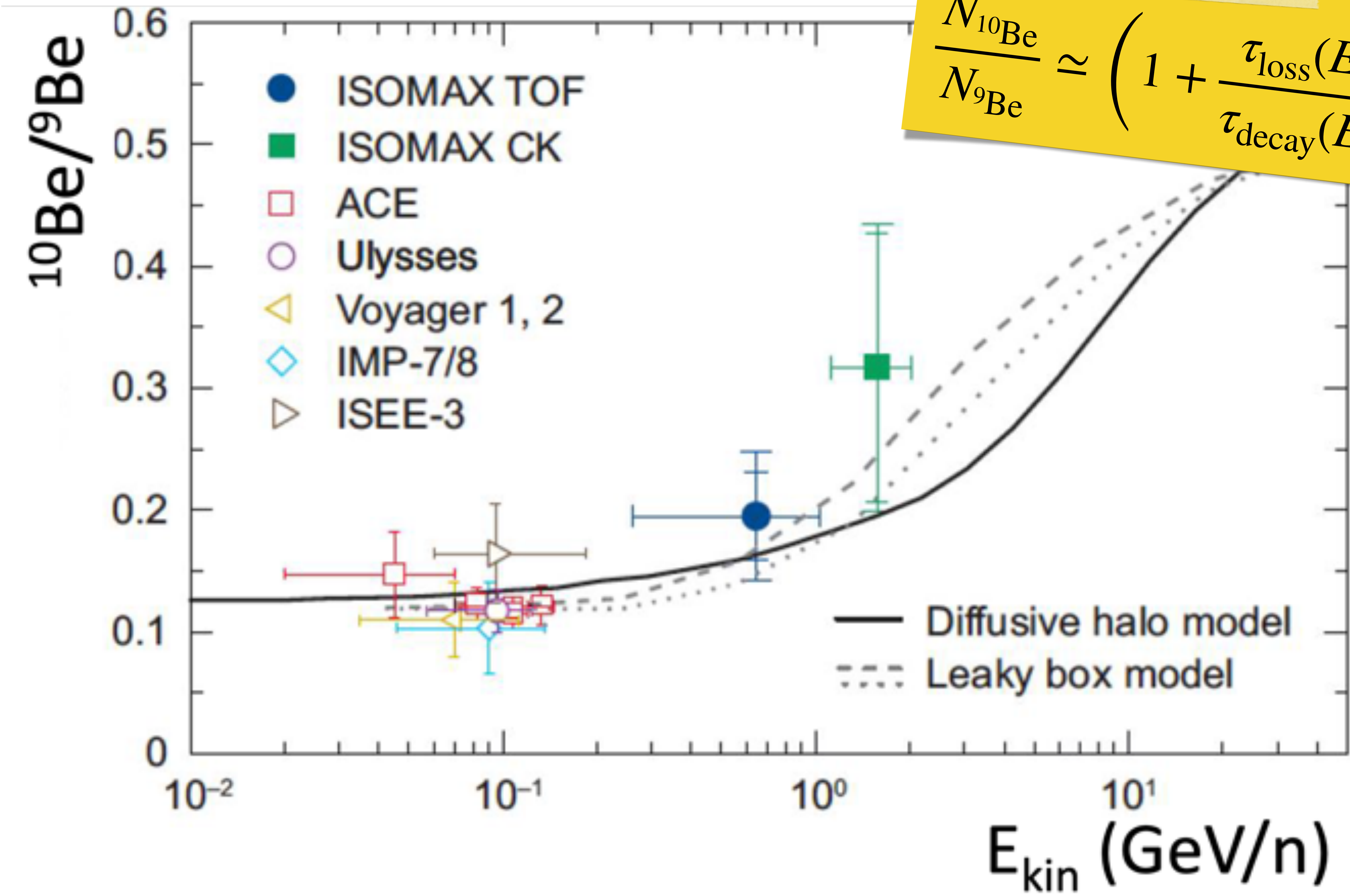
$$N_s = \frac{c\rho\sigma_{p\to s}}{\tau_{\rm loss}^{-1} + \tau_{\rm decay}^{-1}}N_p$$

- Beryllium is dominated by secondary CRs, so ratio of isotopes is:

$$\frac{N_{^{10}\rm Be}}{N_{^{9}\rm Be}} \simeq \left(1 + \frac{\tau_{\rm loss}}{\tau_{\rm decay}}\right)^{-1}\frac{\sigma_{p\to {}^{10}\rm Be}}{\sigma_{p\to {}^{9}\rm Be}}$$

- For $E \simeq 100$ MeV we get $\tau_{\rm loss} \simeq 15 \times 10^6$ years.

Be^{10}-to-Be^{9} Ratio



$$\frac{N_{^{10}\mathrm{Be}}}{N_{^{9}\mathrm{Be}}} \simeq \left(1 + \frac{\tau_{\mathrm{loss}}(E)}{\tau_{\mathrm{decay}}(E)}\right)^{-1}$$

Energy Budget of Galactic CRs

- **Energy density** of observed CRs:

$$w_{\rm CR} = \int {\rm d}E E n_{\rm CR}(E) \simeq 1 \frac{\rm eV}{\rm cm^3}$$

- Diffusion volume in leaky-box: $V \simeq 10{\rm kpc} \times \pi(15{\rm kpc})^2 \simeq 7000{\rm kpc}^3$

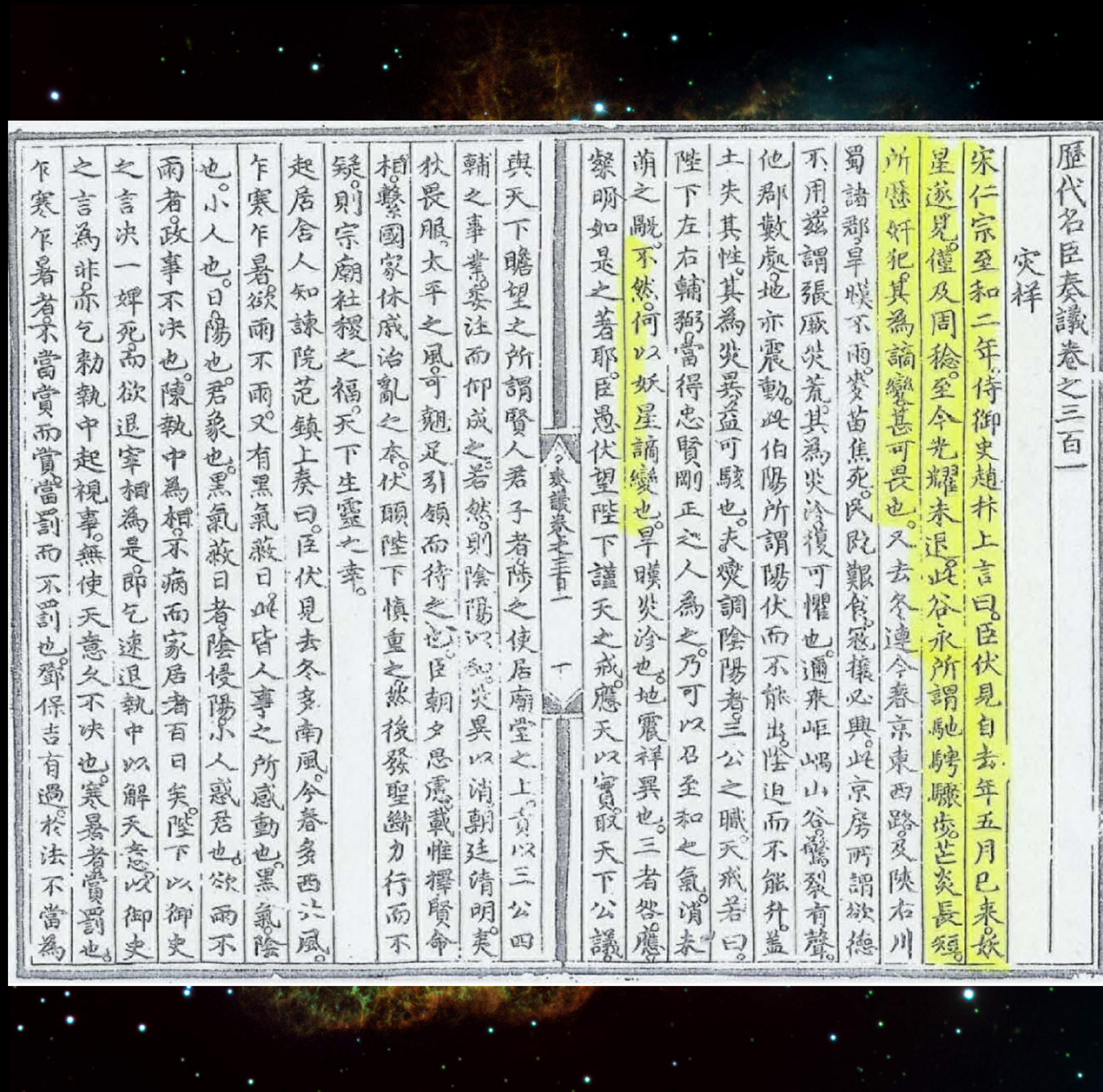
- With $\tau_{\rm loss} \simeq 15 \times 10^6$ years we can estimate the **required luminosity of Galactic CR sources**:

$$\mathscr{L}_{\rm CR} = \frac{V\omega_{\rm CR}}{\tau_{\rm loss}} \simeq 5 \times 10^{53} \frac{\rm eV}{\rm s} \simeq 7 \times 10^{41} \frac{\rm erg}{\rm s}$$

- **Galactic core-collapse supernovae** release gravitational binding energy at the level of $W_{\rm kin} \simeq 3 \times 10^{51}\,{\rm erg}$.

- **Can account for $\mathscr{L}_{\rm CR}$ if $\mathscr{O}(10)\%$ of $W_{\rm kin}$ is converted to CRs.**

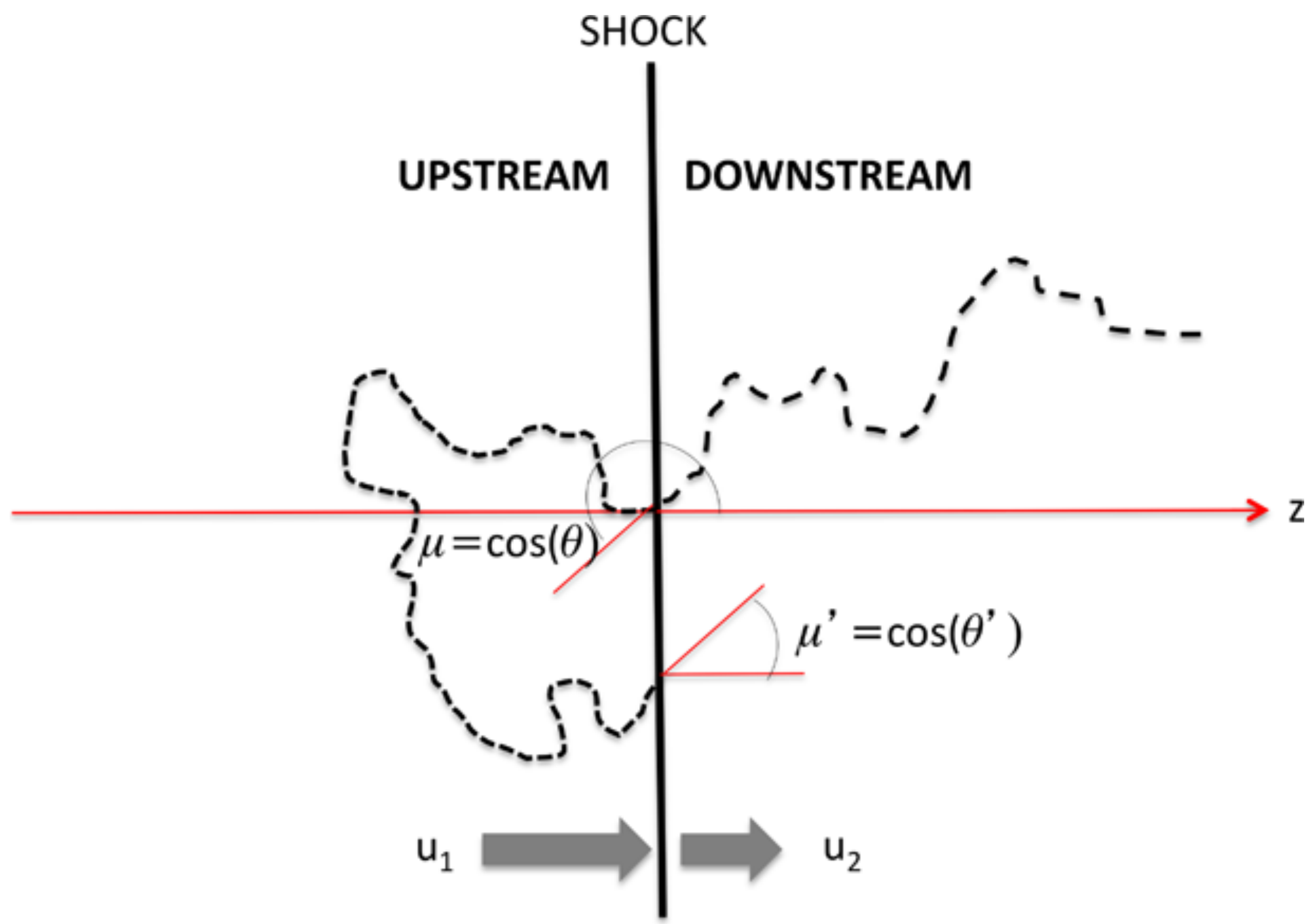
Supernovae



歷代名臣奏議卷之三百一
災祥
宋仁宗至和二年侍御史趙抃上言曰臣伏見自去年五月已來妖
星遂見僅及周稔至今光耀未退此谷永所謂馳騁驟步芒炎長短
所歷奸犯其為譴變甚可畏也又去冬連今春京東西路及陝右川
蜀諸郡旱暵不雨麥苗焦死民既艱食寇攘必興此京房所謂欲德
不用茲謂張厥災荒其為災沴複可懼也邇來岷峨山谷驚裂有聲
他郡數處地亦震動此伯陽所謂陽伏而不能出陰迫而不能升蓋
土失其性其為災異益可駭也夫變調陰陽者三公之職天戒若曰
陛下左右輔弼當得忠賢剛正之人為之乃可以召至和之氣消未
萌之亂不然何以妖星譴變也旱暵災沴也地震祥異也三者咎應
繁明如是之著耶臣愚伏望陛下謹天之戒應天以實取天下公議
奏議卷之三百一　十
與天下瞻望之所謂賢人君子者除之使居廟堂之上責以三公四
輔之事業委注而仰成之若然則陰陽以和災異以消朝廷清明夷
狄畏服太平之風可翹足引領而待之也臣朝夕思慮載惟擇賢命
相繫國家休戚治亂之本伏願陛下慎重之然後發聖斷力行而不
疑則宗廟社稷之福天下生靈之幸
起居舍人知諫院范鎮上奏曰臣伏見去冬多南風今春多西北風
乍寒乍暑欲雨不雨又有黑氣蔽日此皆人事之所感動也黑氣陰
也小人也日陽也君象也黑氣蔽日者陰侵陽小人惑君也欲雨不
雨者政事不決也陳執中為相不病而家居者百日矣陛下以御史
之言決一婢死而欲退宰相為是耶乞速退執中以解天意以御史
之言為非耶亦乞勅執中起視事無使天意久不決也寒暑者賞罰也
乍寒乍暑者不當賞而賞當罰而不罰也鄧保吉有過於法不當為

Crab Nebula (SN 1054)

Diffusive Shock Acceleration



$$\underbrace{u\partial_z f}_{\text{convection}} - \underbrace{\partial_z\left(K_{\parallel}\partial_z f\right)}_{\text{diffusion}} - \underbrace{\frac{1}{3}\frac{du}{dz}p\partial_p f}_{\text{compression}} = Q$$

Diffusive Shock Acceleration

- **CR injection at shock** ($z = 0$): $Q(p,z) = \mathcal{N} u_1 \delta(p - p_{\rm inj})\delta(z)$
- Integration of diffusion equation (DE) in the **shock vicinity**:

$$\left[K\partial_z f\right]_2 - \left[K\partial_z f\right]_1 + \frac{1}{3}(u_2 - u_1)p\partial_p f_0 + \mathcal{N} u_1 \delta(p - p_{\rm inj}) = 0$$

- In **upstream region** $f \to 0$ **for** $z \to -\infty$ **,** so DE yields:

$$\partial_z \left[uf - K\partial_z f\right] = 0 \quad \to \quad \left[K\partial_z f\right]_1 = u_1 f_0$$

- In **downstream region** $\partial_z f = 0$, so DE in **shock vicinity** becomes:

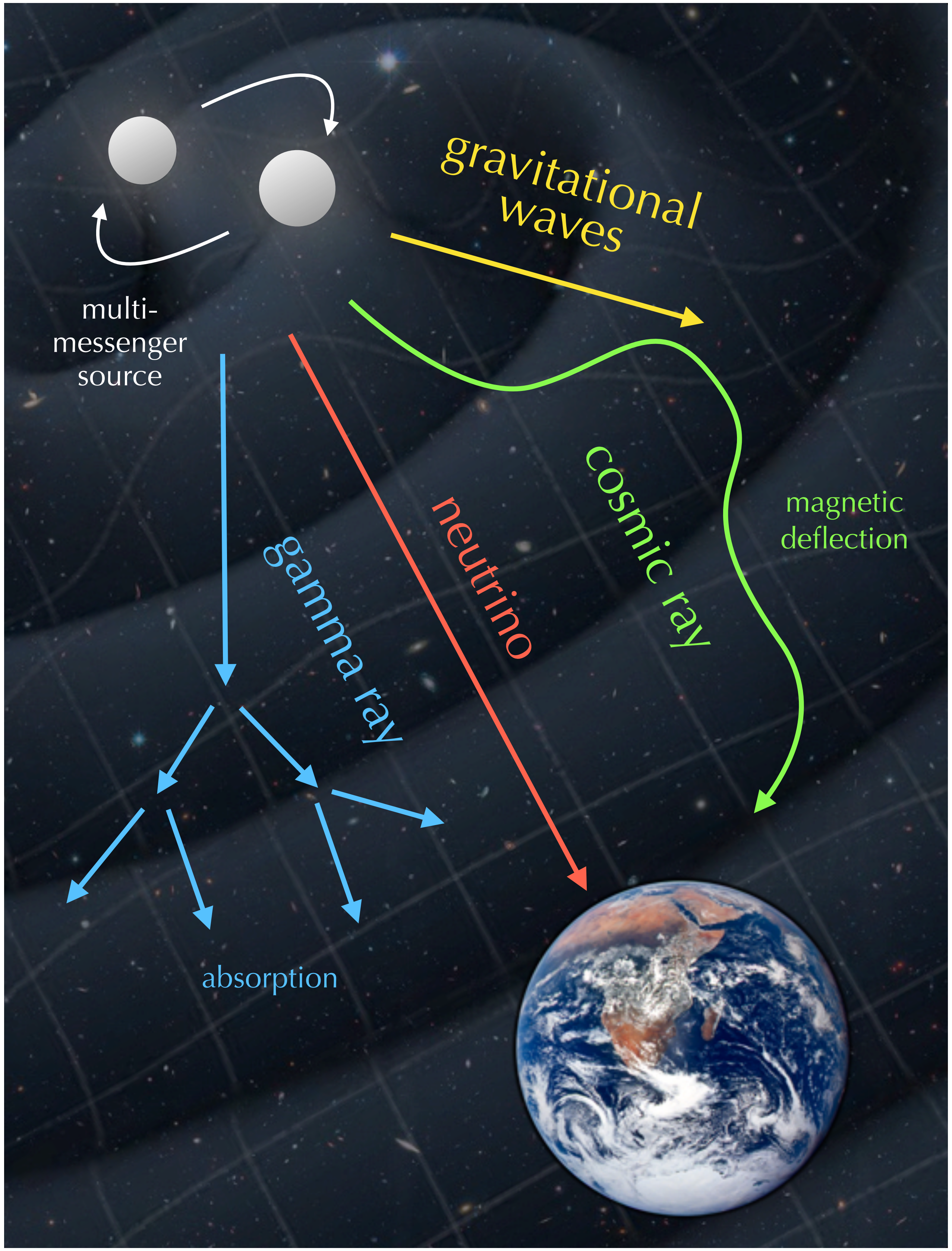
$$f_0 = \frac{u_2 - u_1}{3u_1} p\partial_p f_0 + \mathcal{N}\delta(p - p_{\rm inj})$$

- Solution is **power-law in momentum**:

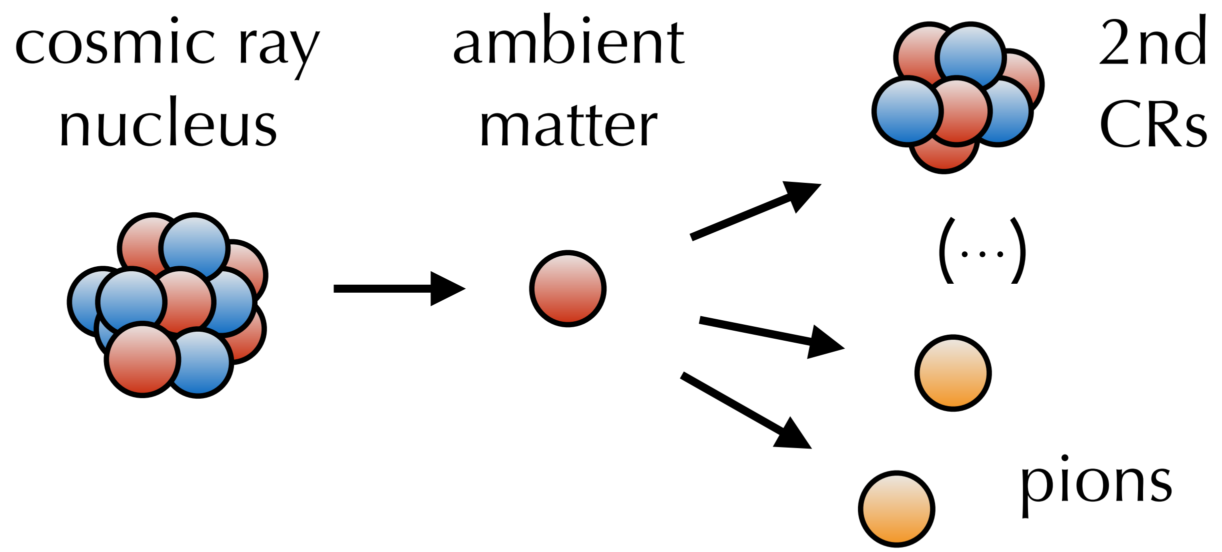
$$f_0(p) = \theta(p - p_{\rm inj})\frac{3u_1}{u_1 - u_2}\frac{\mathcal{N}}{p_{\rm inj}}\left(\frac{p}{p_{\rm inj}}\right)^{-\frac{3u_1}{u_1 - u_2}}$$

Hadronic Emission

Multi-Messenger Interfaces



Acceleration of **cosmic rays** (CRs) - especially in the aftermath of cataclysmic events, sometimes visible in **gravitational waves** (GW).

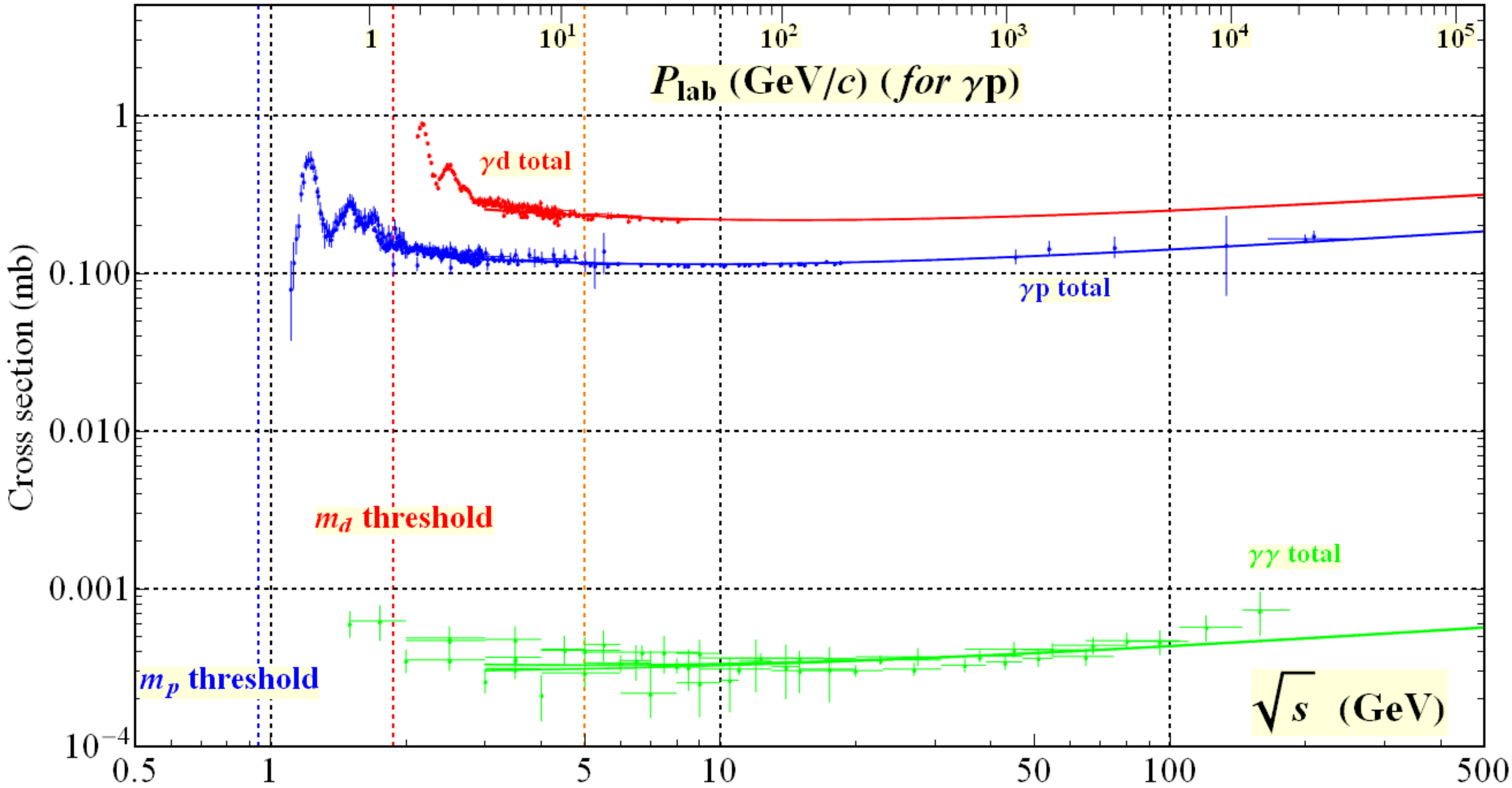


Secondary **neutrinos** and **gamma-rays** from pion decays:

$$\pi^+ \rightarrow \mu^+ + \nu_\mu \qquad \pi^0 \rightarrow \gamma + \gamma$$

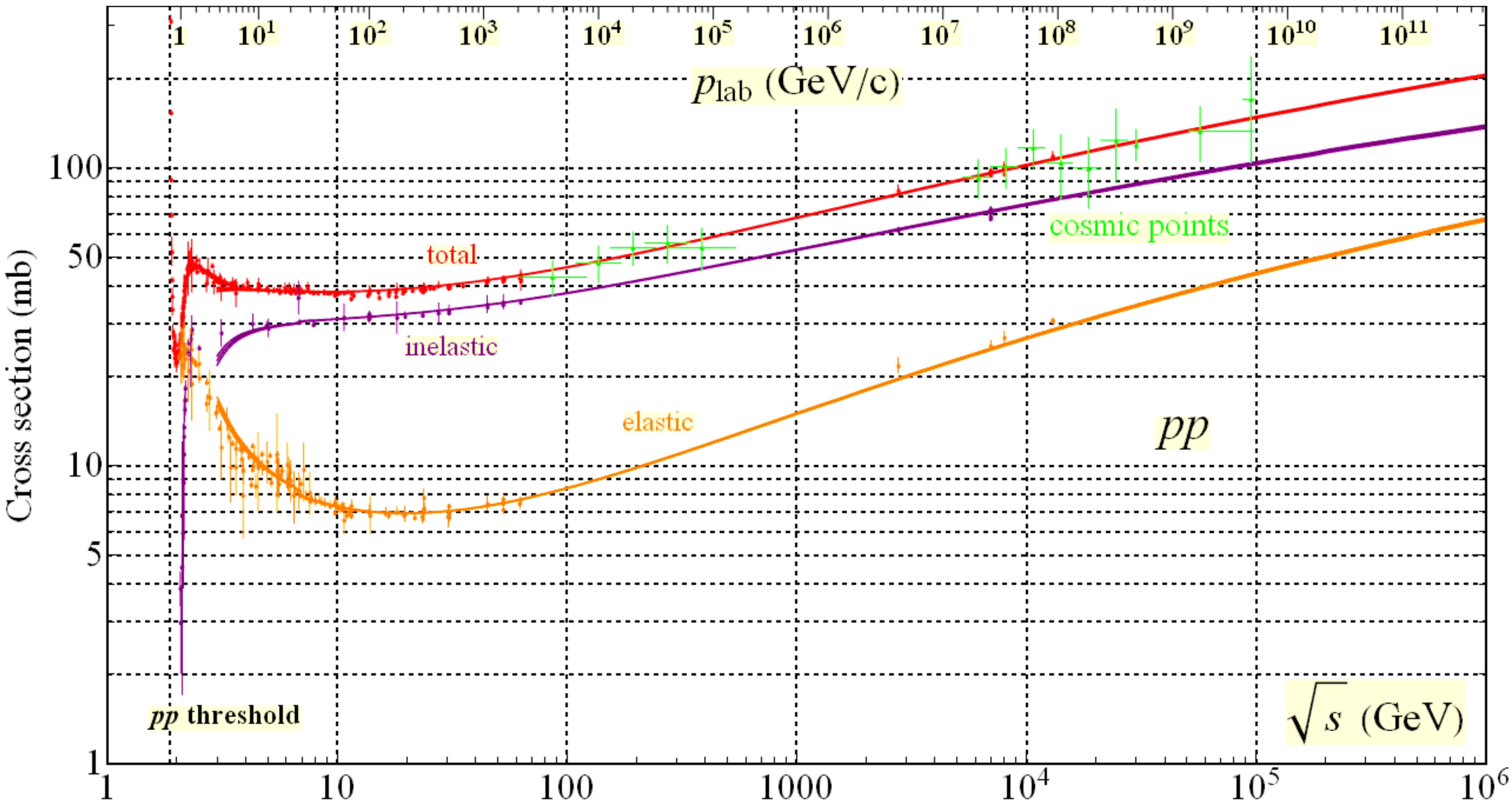
$$\hookrightarrow e^+ + \nu_e + \bar{\nu}_\mu$$

Proton-Photon ($p\gamma$) Interactions



[Particle Data Group; pdg.lbl.gov]

Proton-Proton (pp) Interactions



[Particle Data Group; pdg.lbl.gov]

Pion Production Efficiency

- Pion production depend on **target opacity** $\tau = \ell\sigma n$
- Bolometric **pion production efficiency** (with inelasticity κ):

$$f_\pi = 1 - e^{-\kappa\tau}$$

- Inelasticity per pion: $\kappa_\pi = \kappa/\langle N_\pi\rangle \simeq 0.17 - 0.2$
- Bolometric relation of the production rates Q:

$$E_\pi^2 Q_{\pi^\pm} \simeq \frac{\langle N_{\pi^\pm}\rangle}{\langle N_{\pi^0}\rangle + \langle N_{\pi^\pm}\rangle}\left[f_\pi E_N^2 Q_N(E_N)\right]_{E_N = E_\pi/\kappa_\pi}$$

- **Charged-to-neutral pion ratio** K_π:

$$E_\pi^2 Q_{\pi^\pm} \simeq \frac{K_\pi}{1 + K_\pi}\left[f_\pi E_N^2 Q_N(E_N)\right]_{E_N = E_\pi/\kappa_\pi} \qquad K_\pi = \frac{\langle N_{\pi^\pm}\rangle}{\langle N_{\pi^0}\rangle} = \begin{cases} 2 & \text{pp} \\ 1 & \text{p}\gamma \end{cases}$$

Gamma-Ray vs. Neutrinos

- **Neutrino emission** from charged pion decay:

$$\frac{1}{3}\sum_\alpha E_\nu Q_{\nu_\alpha}(E_\nu) \simeq \left[E_\pi Q_{\pi^\pm}(E_\pi)\right]_{E_\pi \simeq 4E_\nu}$$

$$\pi^+ \to \mu^+ + \nu_\mu$$
$$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$$

- **Gamma-ray emission** from neutral pion decay:

$$\frac{1}{2}E_\gamma Q_\gamma(E_\nu) \simeq \left[E_\pi Q_{\pi^0}(E_\pi)\right]_{E_\pi \simeq 2E_\gamma}$$

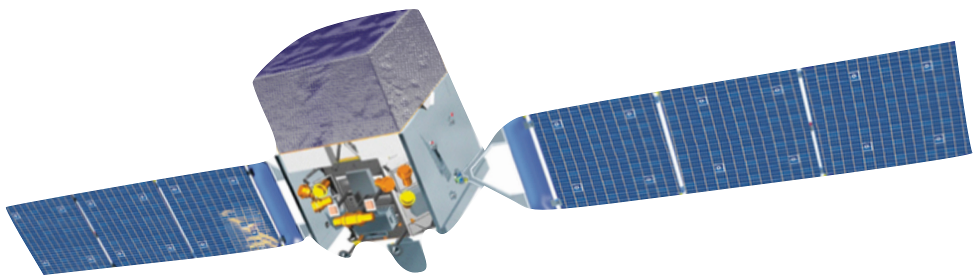
$$\pi^0 \to \gamma + \gamma$$

- **Multi-messenger relation** between neutrino and γ-ray emission:

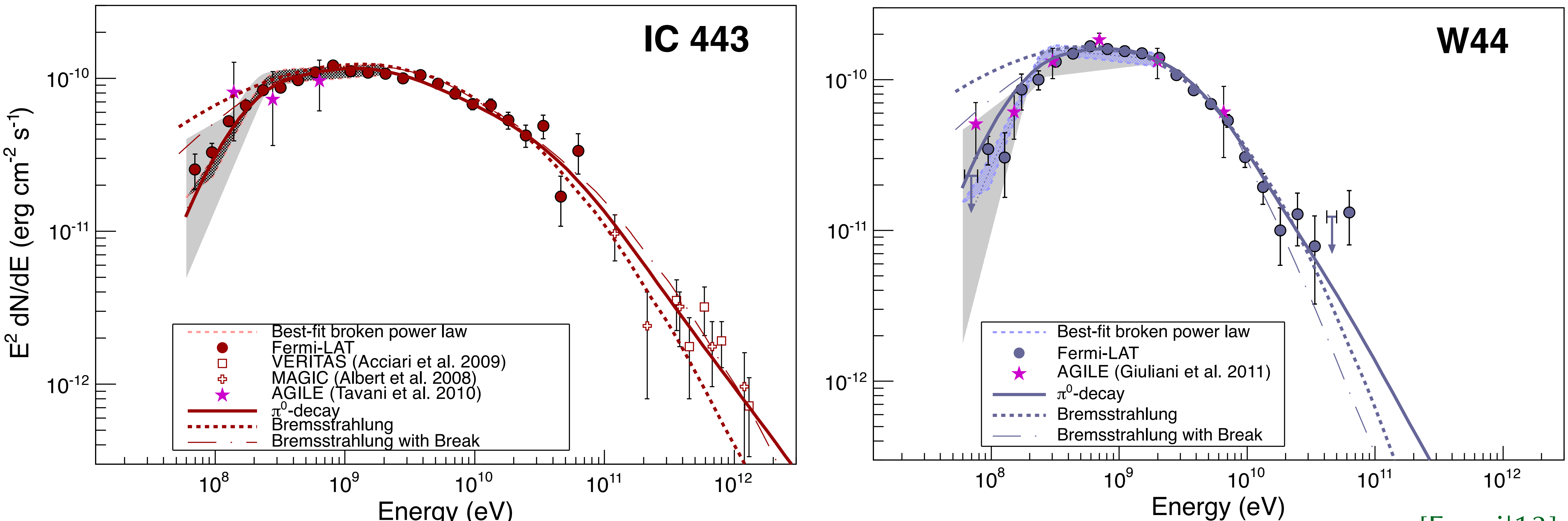
$$\frac{1}{3}\sum_\alpha E_\nu^2 Q_{\nu_\alpha}(E_\nu) \simeq \frac{1}{4}K_\pi \left[E_\gamma^2 Q_\gamma(E_\gamma)\right]_{E_\gamma \simeq 2E_\nu}$$

- ***Note:*** Observable γ-ray emission is attenuated in sources and, in particular, in extragalactic background radiation.

Indirect Evidence



gamma-ray spectra observed from two Galactic supernova remnants



[Fermi'13]

- π^0 production in CR collisions with gas: $p + p \to \pi^0 + X$.
- γ-ray in the rest-frame of the pion takes $E^*_\gamma = m_\pi/2 \simeq 67.5\,\mathrm{MeV}$.
- Kinematics of the interaction produces a break at $E_\gamma \simeq 200\,\mathrm{MeV}$.

Average Neutrino Energies

- Average energy fraction of pions from CR nucleons:

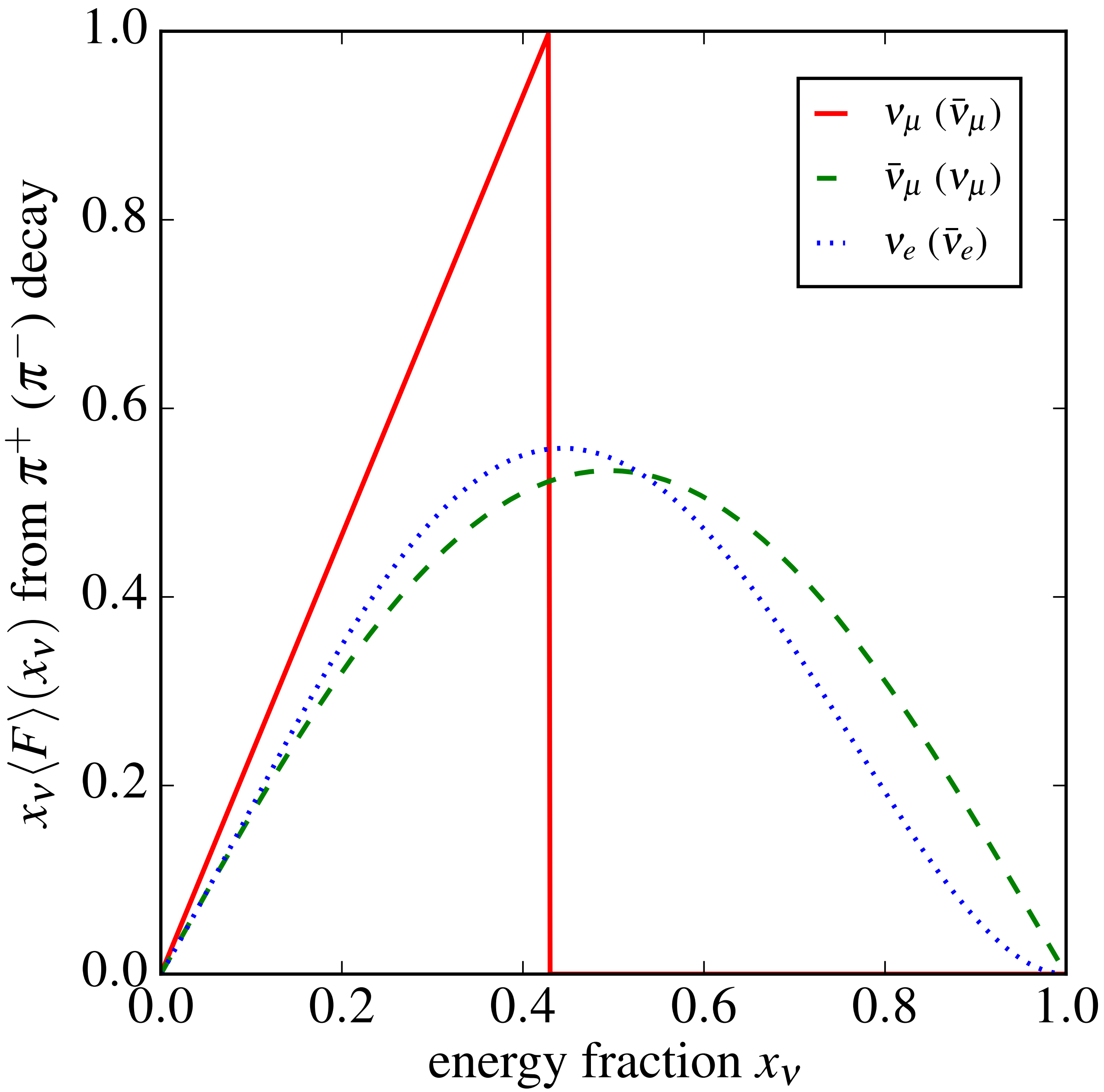
$$\langle x_\pi \rangle = \kappa_\pi \simeq 20\,\%$$

- Average energy fraction from relativistic pions ($r_\pi = (m_\mu/m_\pi)^2$)

$$\langle x_{\nu_\mu} \rangle = \frac{1 - r_\pi}{2} \simeq 21\,\%$$

$$\langle x_{\overline{\nu}_\mu} \rangle = \frac{3 + 4r_\pi}{20} \simeq 26\,\%$$

$$\langle x_{\nu_e} \rangle = \frac{2 + r_\pi}{10} \simeq 26\,\%$$



- **Approximately:**

$$\langle E_\nu \rangle \simeq \frac{1}{2} \langle E_\gamma \rangle \simeq \frac{1}{20} E_{\rm N}$$

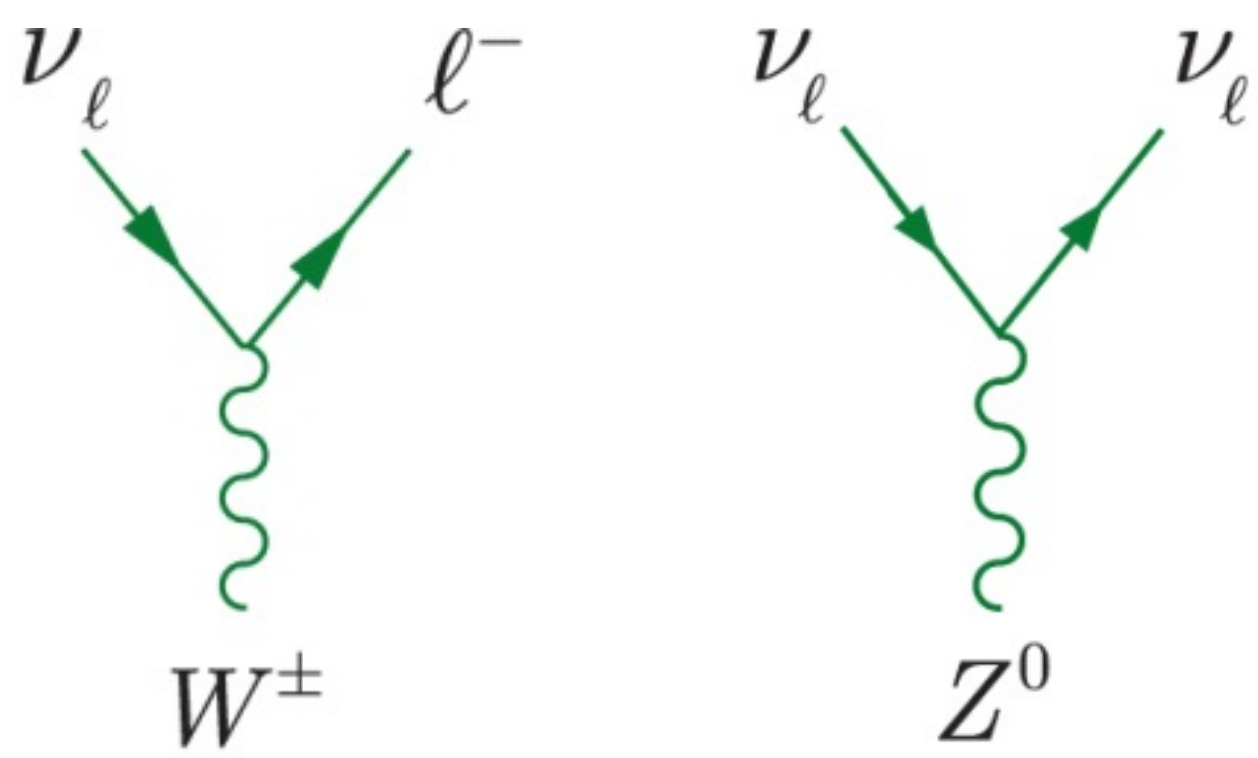
[e.g. Lipari, Lusignoli & Meloni '07]

Neutrinos in the Standard Model

Neutrinos are part of weak isospin doublets and anti-doublets:

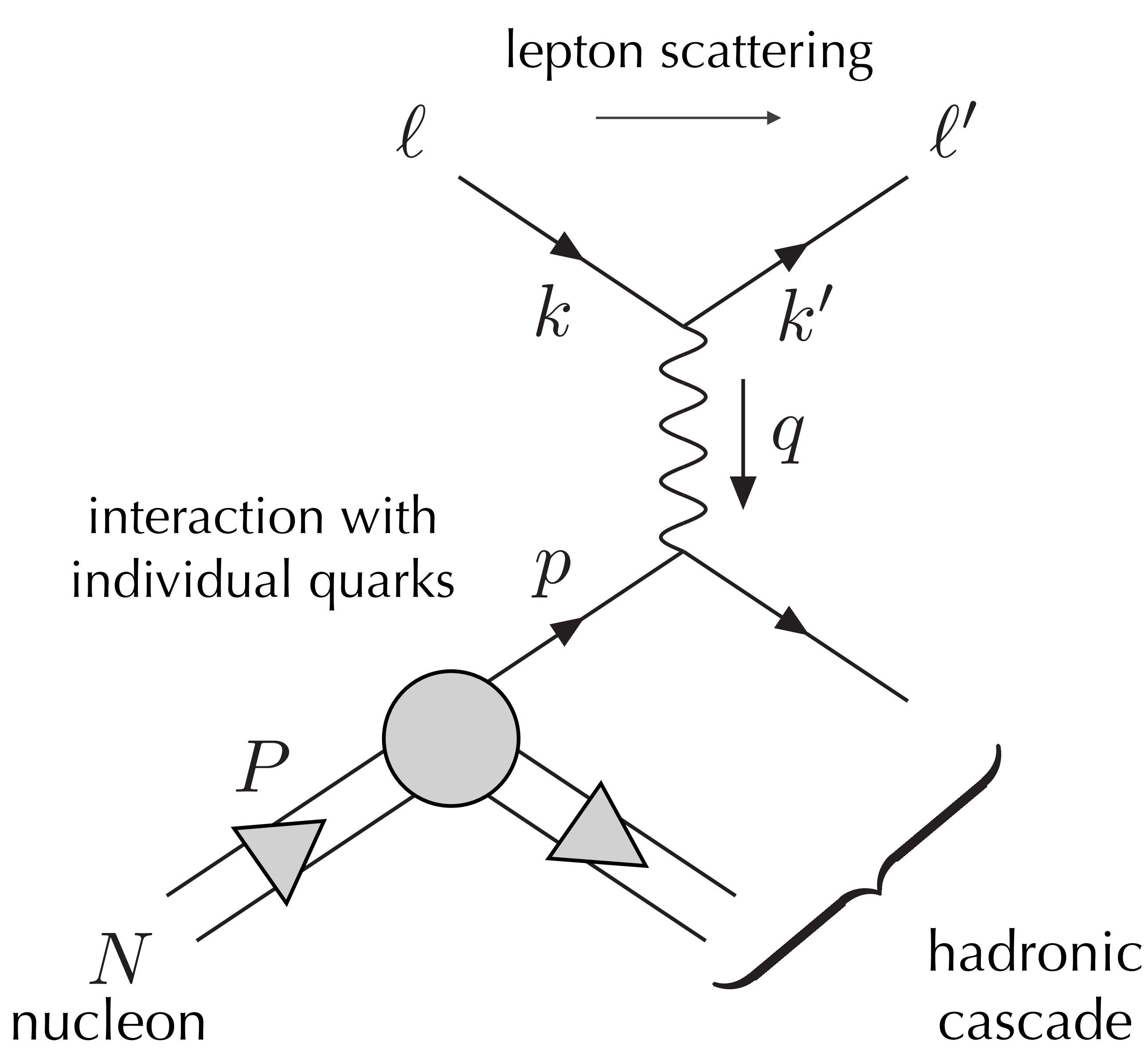
$$\begin{pmatrix}\nu_e\\ e^-\end{pmatrix}_L \quad \begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}_L \quad \begin{pmatrix}\nu_\tau\\ \tau^-\end{pmatrix}_L \quad \begin{pmatrix}e^+\\ \overline{\nu}_e\end{pmatrix}_R \quad \begin{pmatrix}\mu^+\\ \overline{\nu}_\mu\end{pmatrix}_R \quad \begin{pmatrix}\tau^+\\ \overline{\nu}_\tau\end{pmatrix}_R$$

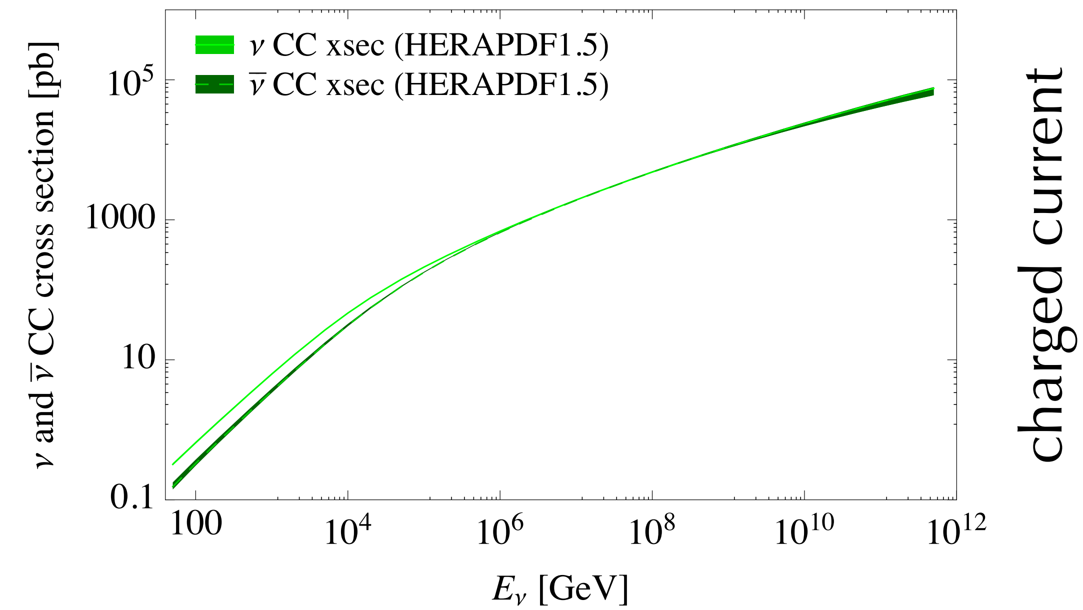
Participate in **charged** (W) **and neutral** (Z) **current** interactions:

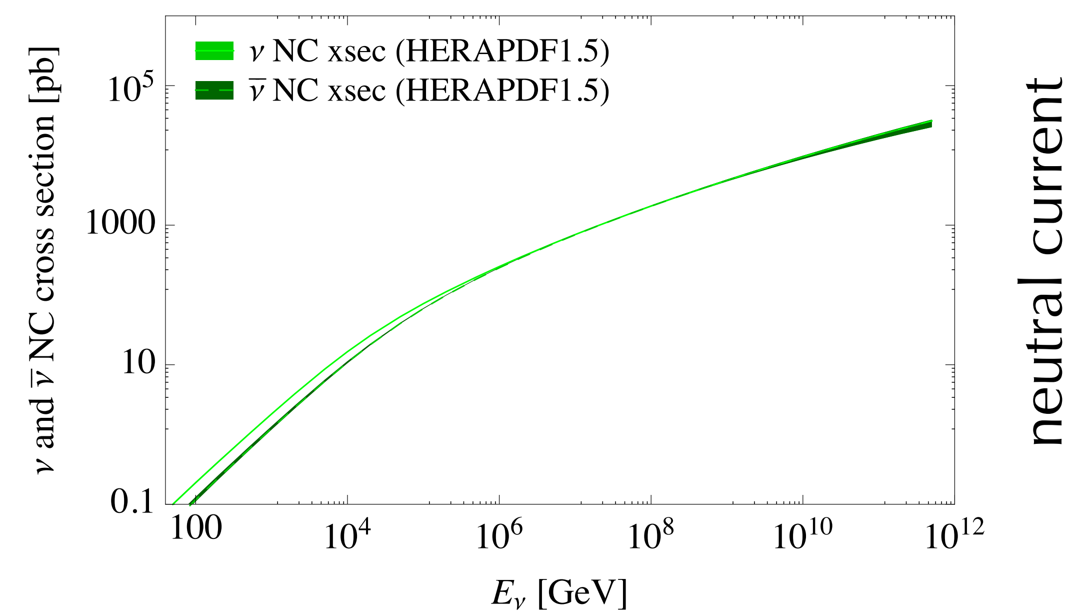


Neutrino Interactions

- Low-energy (<10GeV) neutrino interaction with matter in coherent, quasi-elastic or resonant interactions.
- High-energy neutrinos interact with nuclei via **deep inelastic scattering.**

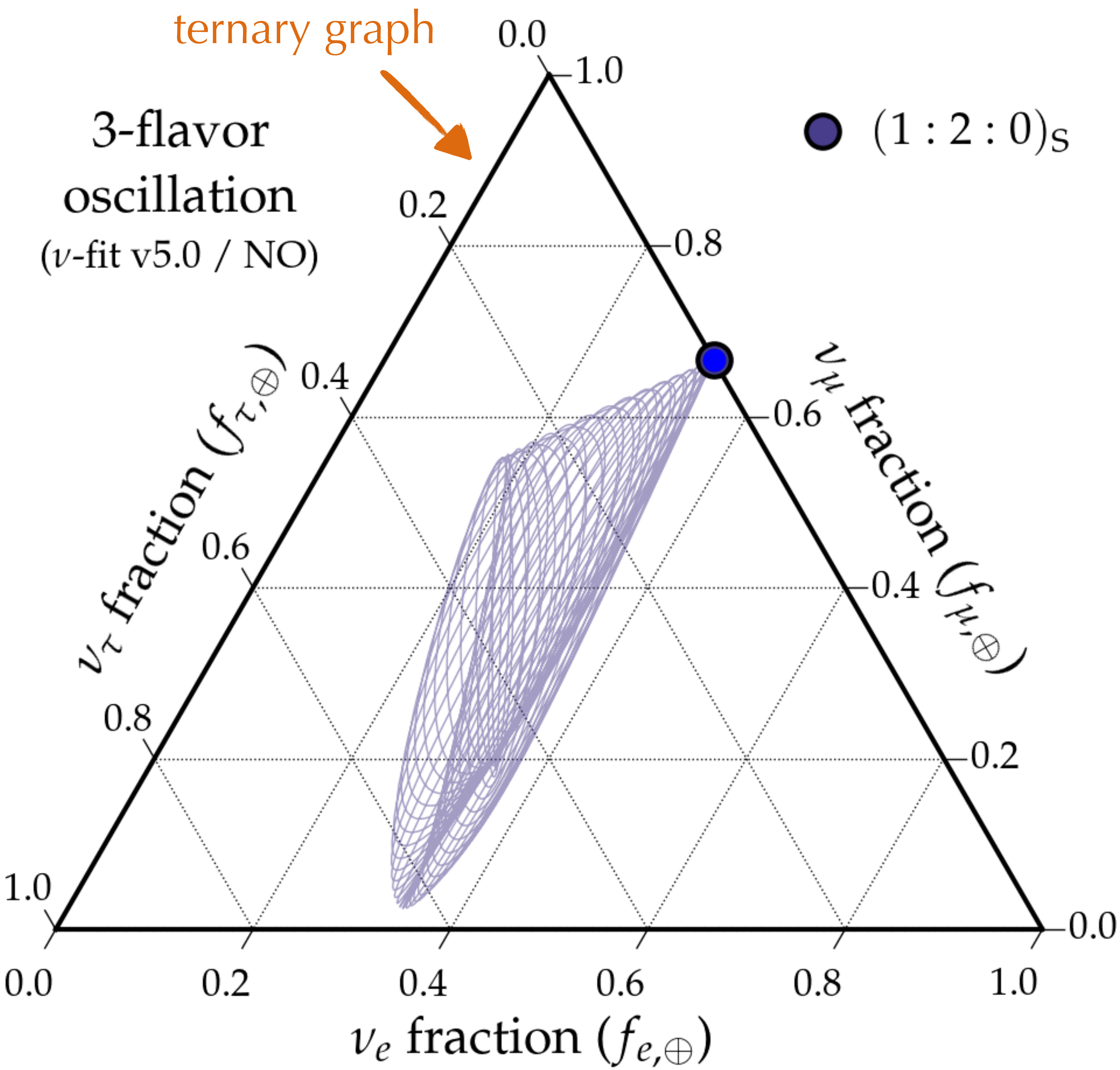






[Cooper-Sarkar, Mertsch & Sarkar'11]

Astrophysical Flavours



flavor ratios
on production

Superposition of flavor and mass states induce oscillations.