Soft gluon resummation for four-top production

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With Benjamin Fuks & Hua-Sheng Shao

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Resummation framework: how to improve fixed order

2 Resummation for top physics



$$\mathrm{d}\sigma_{ab\to f} = \mathrm{d}\sigma^{(0)} + \frac{\alpha_s}{2\pi}\mathrm{d}\sigma^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2\mathrm{d}\sigma^{(2)} + \dots$$

- $\mathscr{O}(\alpha_s), \ \mathscr{O}(\alpha) \ll 1$: assuming convergence of perturbative series
- Truncation (finite set of diagrams): the more complex, the more computation ressources needed
- Matrix element computed pertubatively: Leading-Order (LO), Next-to-Leading-Order (NLO), ...
- \bullet Amplitudes not sufficient: d Φ and PDFs

$$\mathbf{LO}$$

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(Collinear) Factorisation theorem \rightarrow Mellin transform

$$d\sigma_{pp \to f}(s, M^2, \mu_F) = \sum_{a,b} \int_0^1 dx_A \, dx_B \, f_{a/p}(x_A, \mu_F) f_{b/p}(x_B, \mu_F)$$
$$d\hat{\sigma}_{ab \to f}(M^2/\hat{s}, M^2, \mu_F) \, \Theta(x_A x_B - M^2/s)$$
$$\Downarrow$$
$$d\sigma_{pp \to f}(N - 1, M^2) = \sum_{a,b} f_{a/p}(N, \mu_F) \, f_{b/p}(N, \mu_F) \, d\hat{\sigma}_{ab}(N, M^2)$$

Partons & Mellin transform

$$F(N) = \int_0^1 dz \ z^{N-1} \ f(z)$$

- $f_{a(b)/p}$: PDFs of a(b) in p
- μ_F : factorisation scale
- $\sqrt{\hat{s}} = \sqrt{sx_A x_B}$: partonic c.o.m. energy
- M: final invariant mass



Backup

About soft gluon threshold resummation



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About soft gluon threshold resummation



- Threshold: $z = \frac{M^2}{\hat{s}} \to 1$
- Large logs arising from soft gluon emissions: log(1 − z) ↔ log(N)
- Soft scale vs. Hard scale
- Factorisation properties: Soft "universal"

Resummation & exponentiation

$$I_{ij} = \int_0^1 \mathrm{d}z \frac{z^{N-1} - 1}{1 - z} \int_{\mu_F^2}^{(1-z)^2 M^2/4} \frac{\mathrm{d}k_t^2}{k_t^2} \frac{\alpha_s(k_t)}{2\pi} \left(A_i^{(1)} + A_j^{(1)}\right)$$

Thanks to factorisation, we can **resum** and exponentiate:

$$\mathscr{G}_{ij} = \sum_{n=0}^{+\infty} \frac{1}{n!} I_{ij}^n = e^{I_{ij}}$$

Generalises to Sudakov factor: depends only on the nature of coloured external massless particles, encodes (collinear-)soft radiations.

$$\mathscr{G}_{ij} = \exp\left(g_{1\,ij}(\lambda)\log N + g_{2\,ij}(\lambda) + \frac{\alpha_s}{2\pi}g_{3\,ij}(\lambda) + \dots\right), \qquad \lambda = \frac{\alpha_s}{2\pi}\beta_0\log N$$

Resummation & exponentiation

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$$\boxed{\mathsf{LL}} \qquad \mathsf{NLL} \qquad \mathsf{NLL} \qquad \mathsf{N}^2\mathsf{LL} \qquad 4/1$$
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General formalism: adding colour structure

$$\mathrm{d}\hat{\sigma}_{ij\to f}^{N,\,res.}(\mu_R) \propto \mathrm{Tr}\Big(\mathbf{H}(\mu_R) \exp\Big[\int_{\mu_S}^{\mu_R} \mathbf{\Gamma}^{\dagger}\Big] \,\mathbf{S}(\mu_S) \exp\Big[\int_{\mu_S}^{\mu_R} \mathbf{\Gamma}\Big]\Big) \mathscr{G}_{ij}$$

- H: hard colour matrix, high energy process part
- S: soft colour matrix, low energy emissions
- $\Gamma:$ soft anomalous dimension colour matrix controlling the evolution over the RG of ${\bf S}$
- \mathcal{G}_{ij} : Sudakov factor encodes (collinear-)soft radiations

$$\mathbf{H} = \sum_{k=0} \left(\frac{\alpha_s}{2\pi}\right)^k \mathbf{H}^{(k)}, \quad \mathbf{S} = \sum_{k=0} \left(\frac{\alpha_s}{2\pi}\right)^k \mathbf{S}^{(k)}, \quad \Gamma = \sum_{k=0} \left(\frac{\alpha_s}{2\pi}\right)^k \Gamma^{(k)}$$

Matching resummation and fixed order

We can expand $d\hat{\sigma}^{res.}$ to NLO in α_s in order to match to the NLO cross section (obtained with MG5AMC) \rightarrow winning on all fronts !



Matching: $d\sigma_{|_{N^kLO}} + d\sigma^{res.} - d\sigma^{res.}_{|_{N^kLO}}$ valid everywhere

 $(\mu_R, \mu_F) \in \mu_0 \times \left\{ (1,1); (1,2); (2,1); (2,2); (1/2,1); (1,1/2); (1/2,1/2) \right\}$

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Top sector: SM & beyond

Up-to-date physics: 4 t

- First mesure by ATLAS and CMS with 2018 data, $\sqrt{s} = 13$ TeV
- Observation $\approx 2 \times SM$: 1.8 σ deviation
- $\mathscr{O}(25\%)$ uncertainty both on exp. & theory
- Higgs laboratory: e.g. Constraint on Γ_H , [ATLAS 2407.10631]
- Connection to NP using SMEFT at LHC and future colliders [Vryonidou *et al.* 2022]



Figure: [ATL-PHYS-PUB-2023-028] Preliminary summary for top physics

• High multiplicity final state \rightarrow NNLO out of reach for now...

Resummation ingredients for $t\bar{t}t\bar{t}$ at NLL

$$\mathrm{d}\hat{\sigma}_{ij\to f}^{N,\,res.}(\mu_R) \propto \mathrm{Tr}\Big(\mathbf{H}(\mu_R)\,\exp\Big[\int_{\mu_S}^{\mu_R} \mathbf{\Gamma}^{\dagger}\Big]\,\mathbf{S}(\mu_S)\,\exp\Big[\int_{\mu_S}^{\mu_R} \mathbf{\Gamma}\Big]\Big)\mathscr{G}_{ij}$$

- Color basis choice $(6 \times 6 \text{ for } q\bar{q} \text{ and } 13 \times 13 \text{ for } gg)$
- **H**: extracted from MadLoop up to $\mathbf{H}^{(1)}$
- Soft matrices from Mathematica code (Eikonal integrals)
- Sudakov well known

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$t\bar{t}t\bar{t}$ production: preliminary results



Figure: Matching of NLO to NLL accuracy, no virtual loops included, $\mu_0 = M_{tt\bar{t}t}/2$

 $\mathbf{H}^{(0)}, \quad \mathbf{S}^{(1)}, \quad \Gamma^{(1)} \quad \in \mathbb{M}_{6/13 \times 6/13}(\mathscr{F});$

Four top production

- Scale uncertainty reduction: +28.6% - 25.2% to +20.6% - 18.2%
 - ightarrow precision
- Central value increase: +7% \rightarrow accuracy
- Multiplicity: challenging phase space integration (Vegas MC integrator)
- Need to be optimized to allow dynamical scale integration

$$g_1, \quad g_2 \quad \in \mathscr{F}(\lambda, \mu_F, \mu_R, M_{t\bar{t}t\bar{t}})$$

$t\bar{t}t\bar{t}$ production: full NLL results (+ EW corrections)



Figure: Matching of NLO to NLL(') accuracy, $\mu_0 = M_{tt\bar{t}t}/2$, [2505.10381]

Four top production

- Scale uncertainty reduction: +30.1% - 22.6% to +15.8% - 14.9% \rightarrow precision
- Central value increase: $+18\% \rightarrow accuracy$
- Inclusion of EW corrections $\mathscr{O}(\alpha_s^3 \alpha^{1,2}), \ \mathscr{O}(\alpha_s^2 \alpha^2), \ \mathscr{O}(\alpha_s^4 \alpha)$
- Different scale choices \rightarrow <u>robustness</u>

 $\mathbf{H}^{(1)}, \quad \mathbf{S}^{(1)}, \quad \Gamma^{(1)} \quad \in \mathbb{M}_{6/13 \times 6/13}(\mathscr{F}); \qquad g_1, \quad g_2 \quad \in \mathscr{F}(\lambda, \mu_F, \mu_R, M_{t\bar{t}t\bar{t}})$

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Summary

Takeaway message

- Higher orders: *accuracy*, **precision** and correct <u>kinematics</u>
- Resummation: cure pathologically large logarimths, better convergence of the series, reduction of the scale uncertainty
- High multiplicity final states computationally demanding but achievable, requires case by case optimization
- Some features remain to be understood (increase or decrease depend on scale factor, proper limit between threshold ranges...)
- Many prospects to continue the study (NNLL, joint resummation, scheme consistency...)

\rightarrow Stay tunned for more soft gluon threshold resummation for high multiplicity channels !

Thank you for your attention!



Backup



Figure: [ATLAS]: SM summary plots

Parton Distribution Functions



Figure: LHAPDF CT18NLO set [PRD.103.014013] displayed for $\mu_F = 10$ GeV

Fixed order calculation: NLO and beyond

Examples of calculations at f. o.

- Well known processes (N³LO): (di)-Higgs production, Drell-Yan
- More complex (N²LO): $t\bar{t}$, vector boson pair (γ , Z, W^{\pm}), $\gamma\gamma\gamma$

• Known only at NLO: $t\bar{t}t\bar{t}$

Multiplicity of the final state increases complexity (\mathcal{M} and $d\Phi$ integration). NLO computations automated in MADGRAPH5_AMC@NLO (MG5AMC), but not beyond.



$$\mathrm{d}\Phi_n(P; \{p_i\}_{i\leq n}) = \prod_{i=1}^m \left(\frac{\mathrm{d}Q_i^2}{2\pi} \mathrm{d}\Phi_{n_i}(Q_i; \{p_k^{(i)}\}_{k\leq n_i})\right) \mathrm{d}\Phi_m(P; \{Q_j\}_{j\leq m})$$

For a single soft gluon emission it gives:

$$\mathrm{d}\Phi_{n+1}(P;\,\{p_i\},k) \simeq \mathrm{d}\Phi_n(M;\,\{p_i\}) \frac{\mathrm{d}^3k}{(2\pi)^3 2E_k} \frac{\mathrm{d}M^2}{\hat{s}} \delta\Big(z-1+\frac{2E_k}{\sqrt{\hat{s}}}\Big)$$

Phase space factorisation: Mellin space

For multiple soft gluon emissions:

$$\mathrm{d}\Phi_{n+n_k}(P; \{p_i\}, \{k_j\}) \propto \mathrm{d}\Phi_n(M; \{p_i\})\delta\Big(z - \prod_j z_j\Big)$$

with $z_j = 1 - \frac{2E_{k_j}}{\sqrt{\hat{s}}}$. The phase space factorises in Mellin space:

$$\mathrm{d}\Phi_{n+n_k}(N) \propto \int_0^1 dz \ z^{N-1} \ \delta\left(z - \prod_j z_j\right) = \prod_j z_j^{N-1}$$

Expected behaviours

• We expect the ratio
$$\frac{1}{d\sigma^0/dM^2} \left(\frac{d\sigma^{res.}}{dM^2} - \frac{d\sigma^{res.}}{dM^2} \Big|_{NLO} \right) \xrightarrow[M^2 \ll S_h]{} 0$$

Away from threshold the logarithmic terms are not important and the behaviour is captured by the first orders of the expansion.

• We expect also
$$\frac{1}{d\sigma^0/dM^2} \left(\frac{d\sigma^{NLO}}{dM^2} - \frac{d\sigma^{res.}}{dM^2} \Big|_{NLO} \right) \xrightarrow[M^2 \to S_h]{} 0$$

In the threshold regime, the resummed expanded reproduces the behaviour of original cross section.

To obtain a sensible cross section in all ranges we may consider the combination: $\sigma_{|_{NLO}} + \sigma^{res.} - \sigma^{res.}_{|_{NLO}}$

Threshold resummation for Drell-Yan-like processes

- Only massless initial quarks emit gluons \rightarrow apply to all similar processes: Drell-Yan-like
- Universal Sudakov soft part known up to N^3LL

$$\begin{split} \Delta_{q\bar{q}}^{\mathrm{res}}(N,M^2,\mu_F^2)\Big|_{\mathrm{N}^{\mathrm{k}}\mathrm{LL}} &= \tilde{g}_{0,q\bar{q}}(M^2,\mu_F^2,\mu_R^2)\Big|_{\mathrm{N}^{\mathrm{k}}\mathrm{LO}} \\ &\times \exp\left(g_{1,q\bar{q}}(\lambda)\ln\overline{N} + \sum_{j=2}^{k+1} a_s^{j-2}g_{j,q\bar{q}}(\lambda)\right) \end{split}$$

• Can be used for BSM studies: exotic lepton pair productions \rightarrow require some fixed order pieces: adapt UFO and use MG5AMC!

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Application to tt production: differential cross section

NLO 0.8 LO NLO⊕NLL 0.6 0.4 0.2 2.0 070 9/07 1.0 360 380 400 420 440 460 480 500 M_{tī} [GeV]

 $t\bar{t}$ production cross section $\sqrt{s} = 7$ TeV

Figure: Effect of resummation for differential cross section

Top pair production

- Central scale: $\mu_0 = M_{t\bar{t}}/2$
- Enhancement between 3% (production thresh.) & 12%at $M_{t\bar{t}} = 500 \text{ GeV}$
- Scale uncertainty reduction: $26.6\% \rightarrow 21.4\%$



Application to $t\bar{t}$ production: differential cross section



Figure: Effect of resummation for differential cross section

Figure: [JHEP09(2010)097] SCET $t\bar{t}$ resummation

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g functions

$$\begin{split} \tilde{g}_{0,q\bar{q}}^{(1)} &= \frac{-64}{3} + \frac{64}{3}\zeta_2 - 8L_{fr} + 8L_{qr} ,\\ \tilde{g}_{0,q\bar{q}}^{(2)} &= \frac{-1291}{9} + \frac{64\zeta_2}{9} + \frac{368\zeta_2^2}{3} + \frac{4528\zeta_3}{27} \\ &\quad + \frac{188L_{fr}^2}{3} + \frac{4L_{qr}^2}{3} \\ &\quad + L_{fr} \left(\frac{1324}{9} - \frac{1888\zeta_2}{9} + \frac{32\zeta_3}{3}\right) \\ &\quad + L_{qr} \left(\frac{148}{9} - 64L_{fr} + \frac{416\zeta_2}{9} - \frac{32\zeta_3}{3}\right) , \end{split}$$
(1)

with $L_{qr} = \ln \frac{M^2}{\mu_R^2}$, $L_{fr} = \ln \frac{\mu_F^2}{\mu_R^2}$ and ζ_n being the Riemann zeta function.