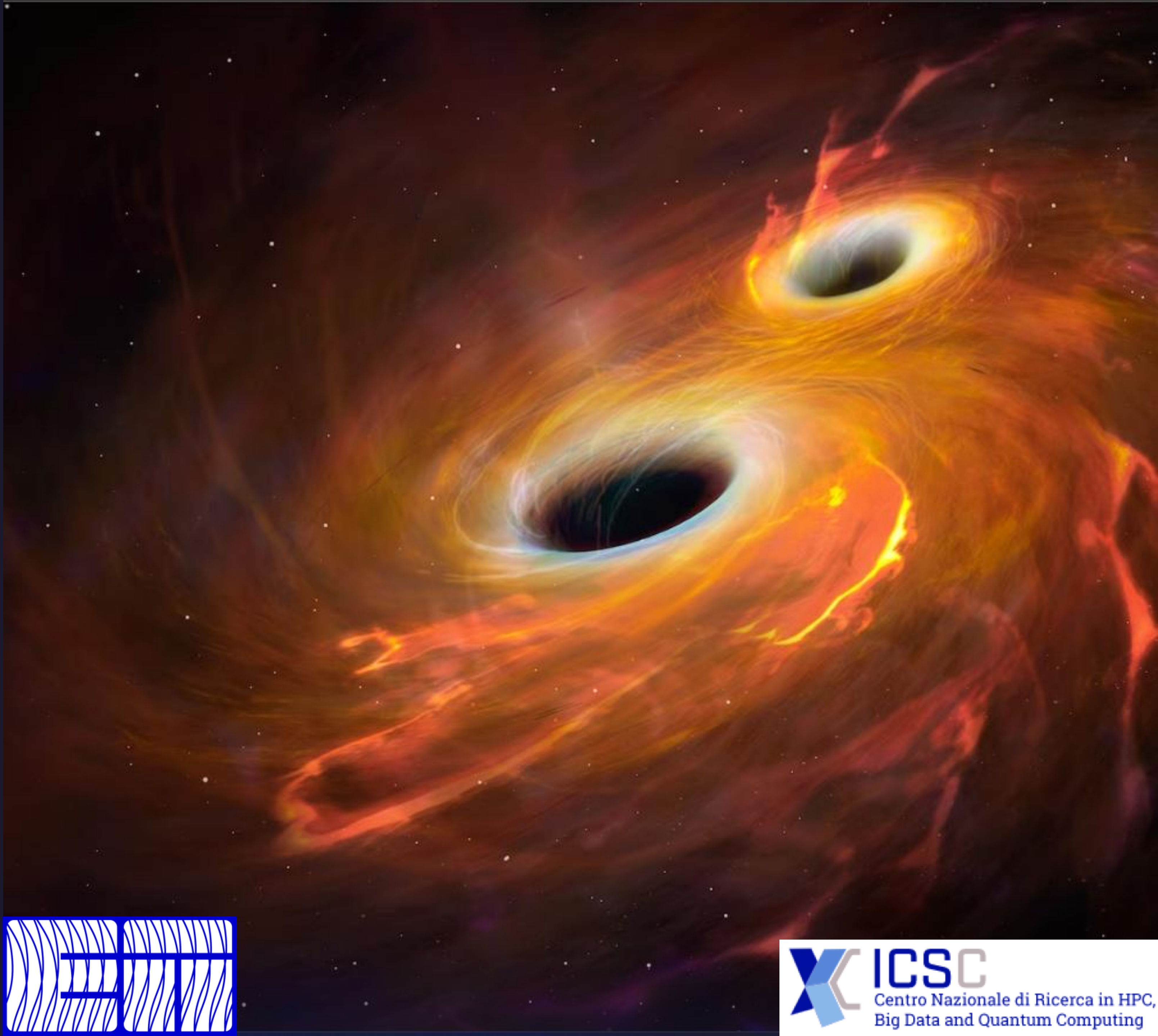
Beyond GR Tests with the Einstein Telescope



GdR - IAP

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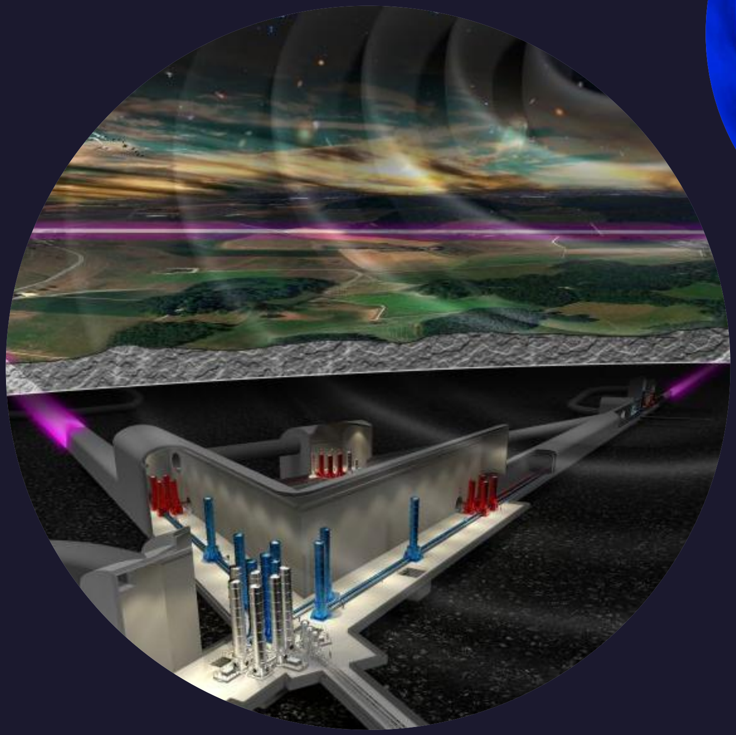
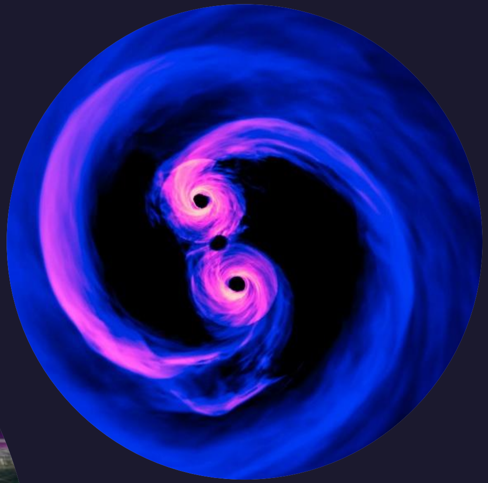
17/10/25

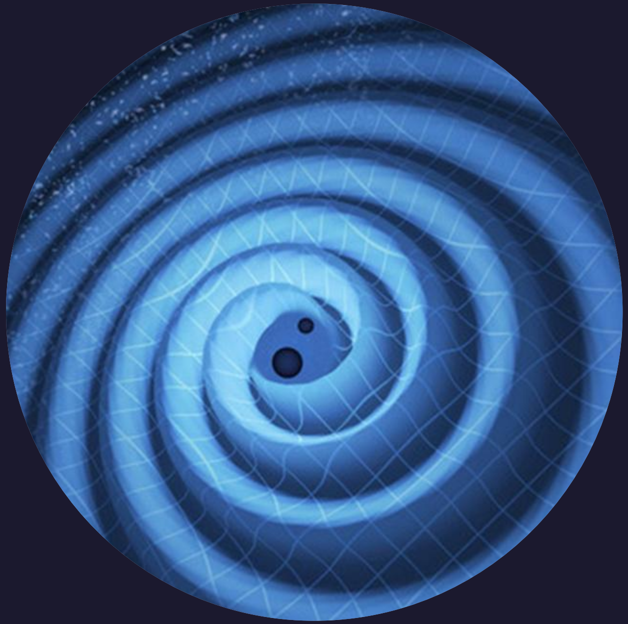
Collaborators

All the work presented was done in collaboration with:

- Stefano Anselmi, Unipd
- Walter Del Pozzo, Unipi
- Matteo Pegorin, Unipd
- Mauro Pieroni, IEM
- Joachim Pomper, Unipi
- Alessandro Renzi, Unipd
- Angelo Ricciardone, Unipi

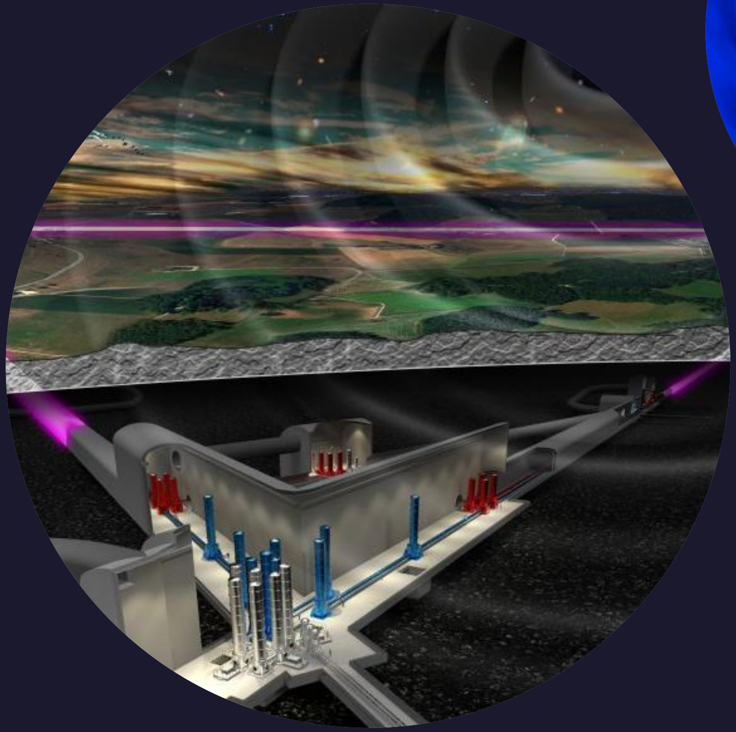


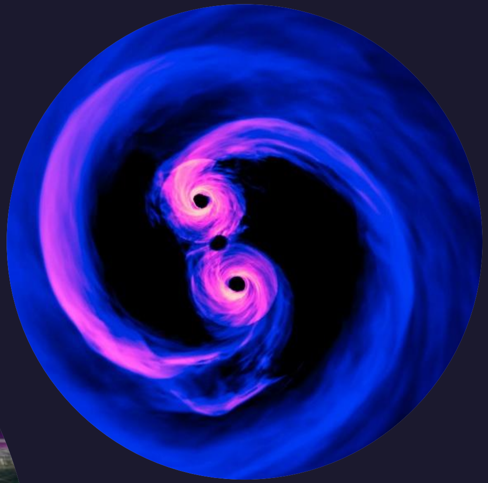


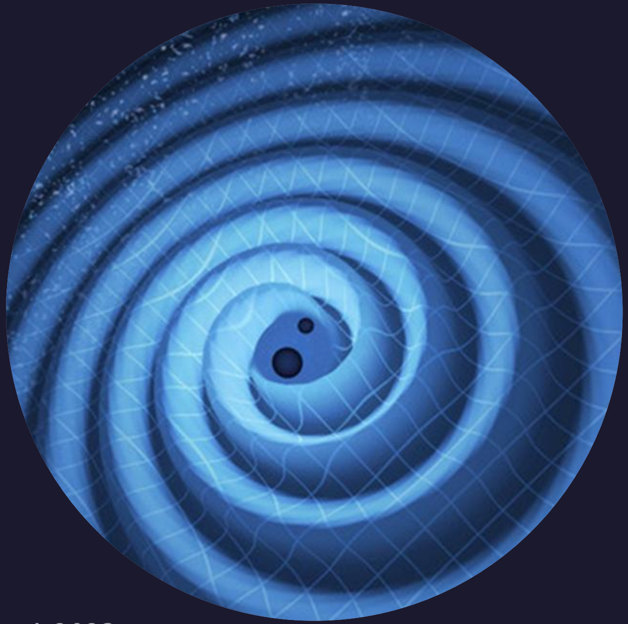


Einstein Telescope

- ET will be part of the third generation of ground-based interferometers
- Will be operative in the late 2030s
- The design is still under scrutiny (triangular vs 2L) [1-2]



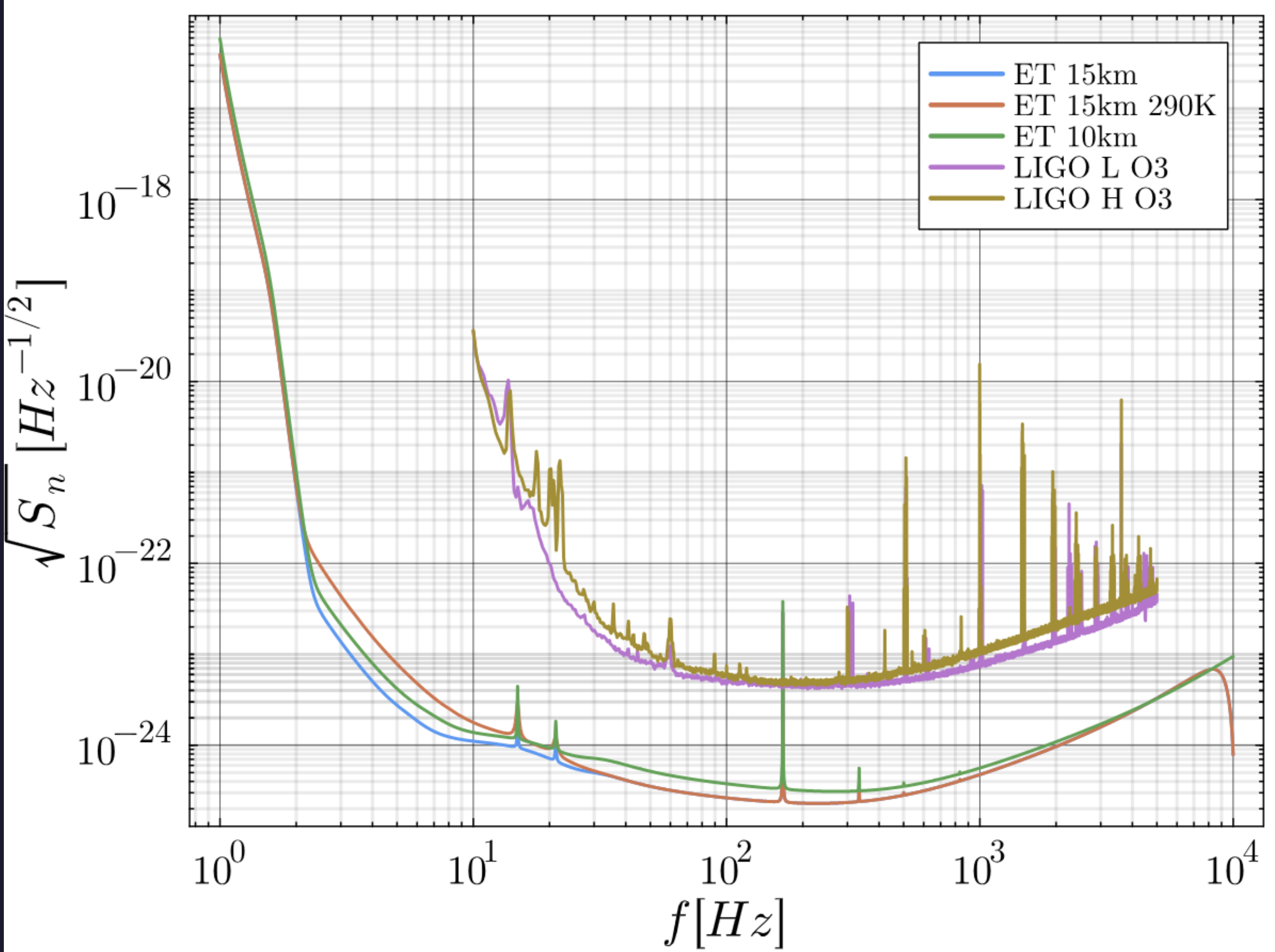




[1] Branchesi et al. 2023
[2] Abac et al, 2025

What ET brings to the table

- Larger frequency space
- Detections up to 50k BBH per year
- Detections up to $z \sim 10$
- SGWB: $\Omega_{\mathrm{GW}} \sim 10^{-12}$

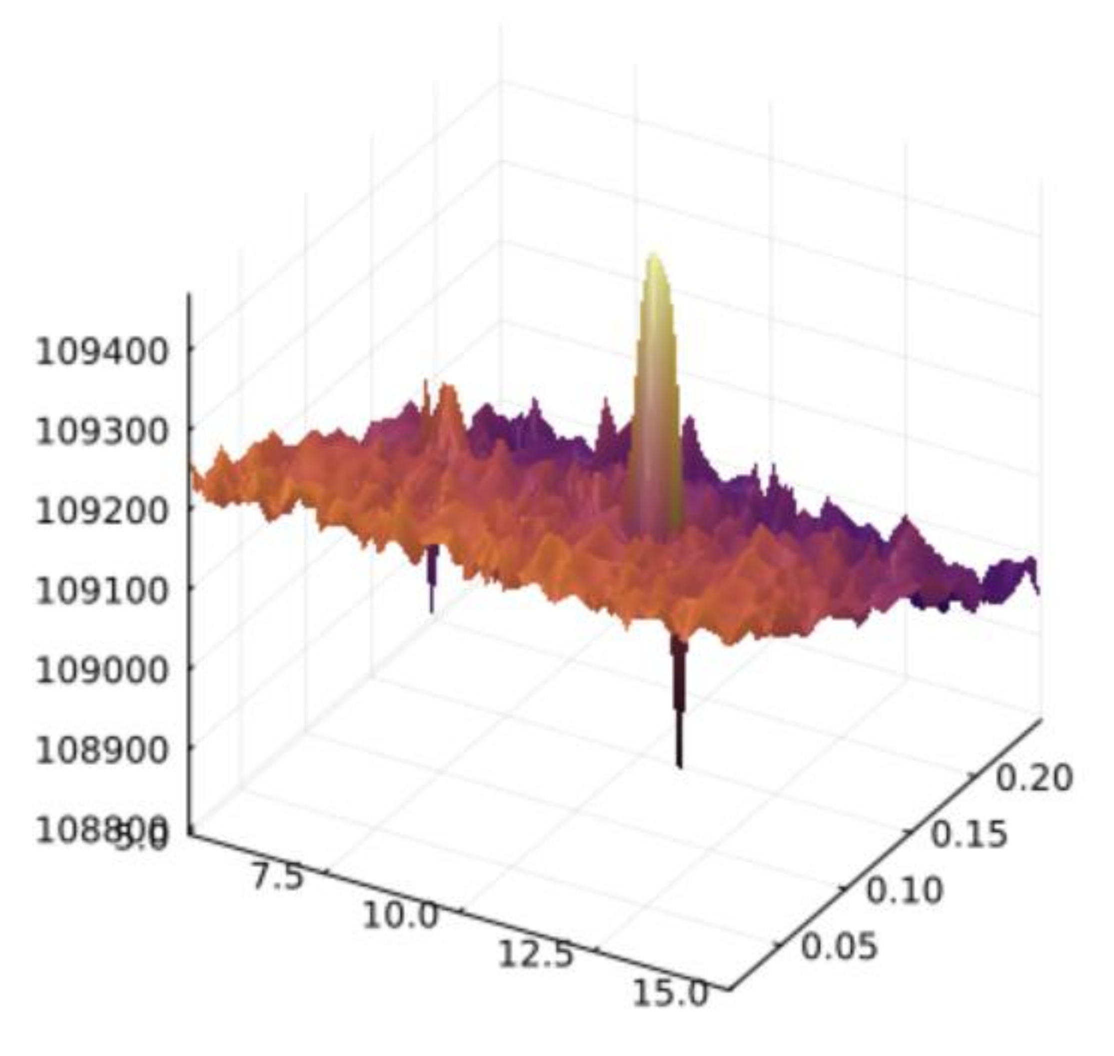


Bayesian analysis

- The gaussian likelihood is expressed as

$$\mathcal{L}(d \mid \boldsymbol{\theta}) \propto \exp\{-(d - s(\boldsymbol{\theta}) \mid d - s(\boldsymbol{\theta}))/2\}$$

- It is very hard to sample (extremely multimodal and long evaluation)
- In LVK nested sampling is used



Fisher Matrix

- To compute the Fisher Matrix which is defined as

$$\Gamma_{ab} = -\langle \partial_a \partial_b \log \mathcal{L}(d|\theta) \rangle \Big|_{\theta=\theta_0} = (h_a \mid h_b)$$

  where we need to derive the likelihood of the data realization
- Each derivative requires calling the expensive waveform function at least a few times (for numerical diff methods)

Automatic Differentiations

- Accurate (at machine level)
- If a derivative exists, it will find it
- Very fast (2x the evaluation of the target function in our case)
- Adopted from ML

Automatic Differentiations

Variable = [value, ∂_xvariable, ∂_yvariable]

$$\langle u, u' \rangle + \langle v, v' \rangle = \langle u + v, u' + v' \rangle$$
$$\langle u, u' \rangle - \langle v, v' \rangle = \langle u - v, u' - v' \rangle$$
$$\langle u, u' \rangle * \langle v, v' \rangle = \langle uv, u'v + uv' \rangle$$
$$\langle u, u' \rangle / \langle v, v' \rangle = \left\langle \frac{u}{v}, \frac{u'v - uv'}{v^2} \right\rangle \quad (v \neq 0)$$

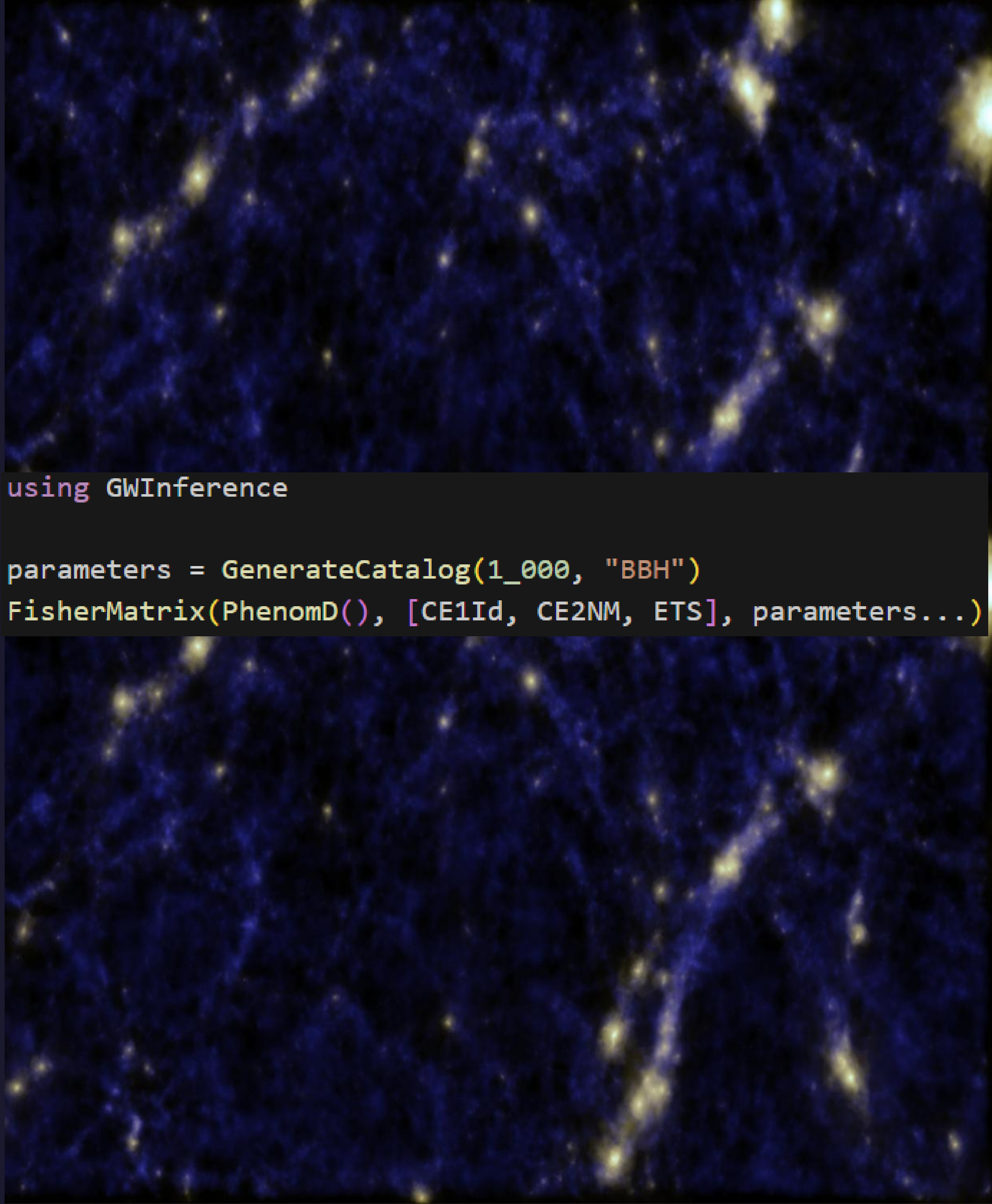
Automatic Differentiations

```
x = Dual(1.0, 1.0, 0.)
y = Dual(2.0, 0.0, 1.)
z = x*y + 3y^2
```

✓ 1.6s

```
Dual{Nothing}(14.0,2.0,13.0)
```

- Easy to learn
- Using automatic differentation (superior to numerical differentation)
- Features advanced waveforms (e.g. PhenomXHM, PhenomD), all written in Julia
- Very fast, e.g., using XHM 3 detectors ~ 0.5 sec per core)
- Does not rely on external packages (all is written in Julia)
- Based on arXiv:2506.21530

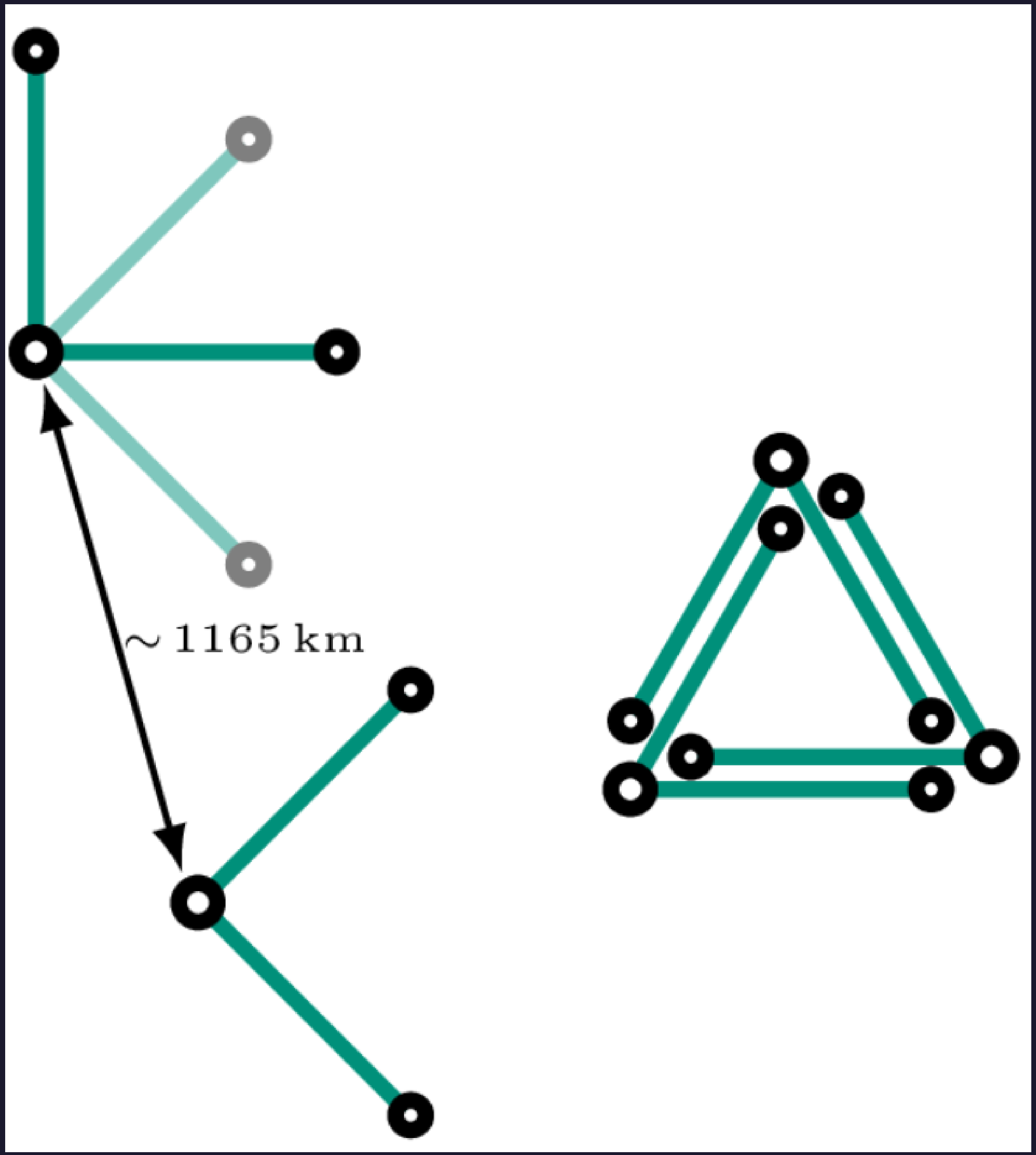


```
using GWInference

parameters = GenerateCatalog(1_000, "BBH")
FisherMatrix(PhenomD(), [CE1Id, CE2NM, ETS], parameters...)
```

Case study – Detector design comparison

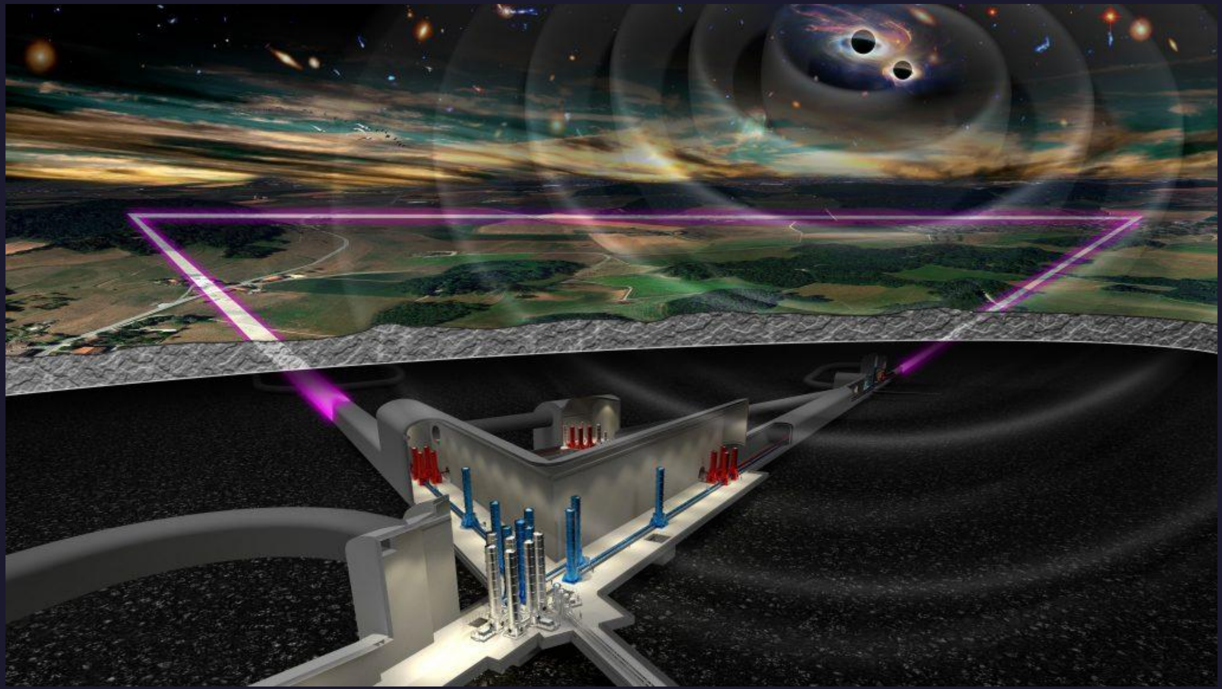
- T: triangular ET with 10 km arms featuring cryogenic technology.
- 2L_0: two aligned 15 km L-shape interferometers one in Sardinia, one in the Meuse–Rhine (MR) Euroregion, both with the cryogenic technology.
- 2L_45: same as 2L_0 with the exception that the orientations lead to $\beta = 45^\circ$.
- 2L_290K_0: same as 2L_0, only one detector features the cryogenic technology.
- 2L_290K_45: same as 2L_45, only one detector features the cryogenic technology.



Credits: [1]

[1] Branchesi et al. 2023
[2] Abac et al, 2025

Detector design comparison - What to expect



What are we interested in?

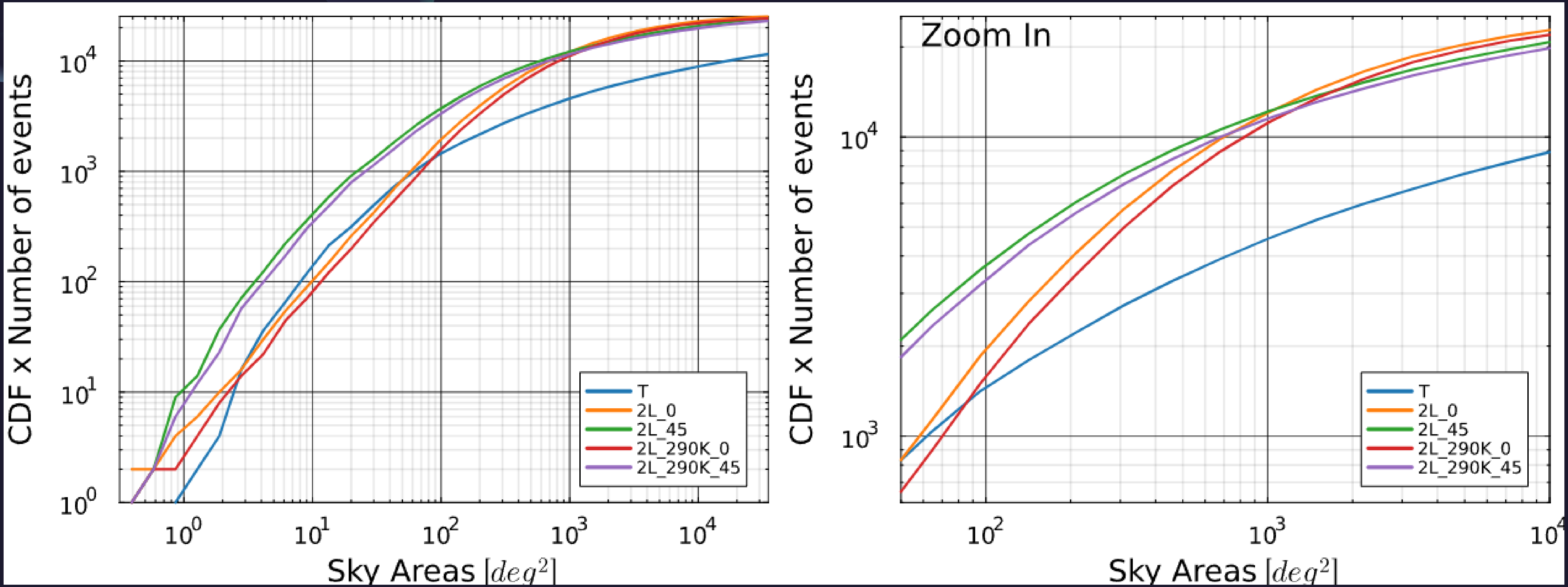
- Cosmology with CBC
- Multimessanger with CBC
- Source properties of CBC
- SGWB

Detector preferences: [1,2]

- CBC : 2L_45 > 2L_0 > T
- SGWB : T ≈ 2L_0 > 2L_45

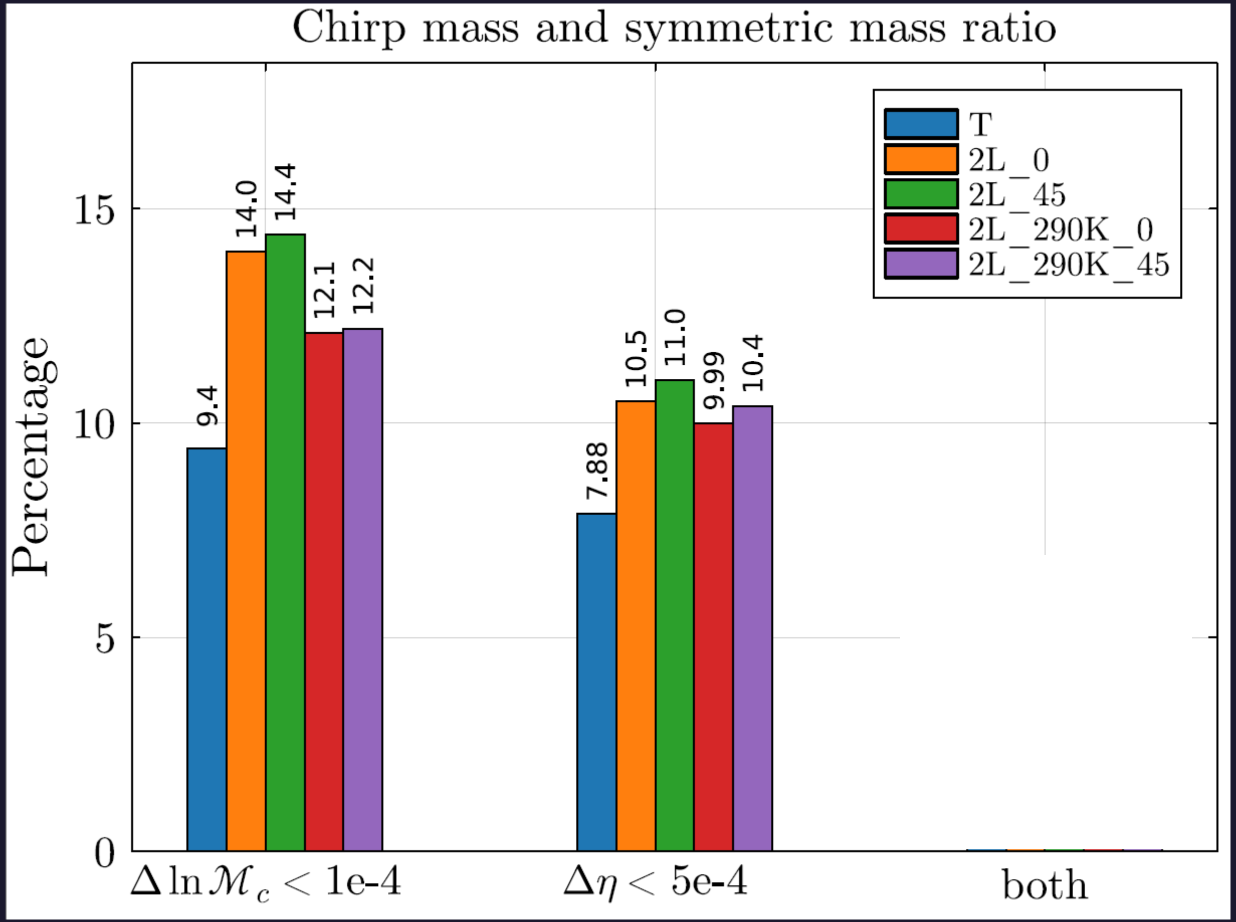
[1] Branchesi et al. 2023
[2] Abac et al. 2025

Case study – Angular Precision



Cumulative density function times the number of events as a function of the 90% sky area. *Right*: zoom in of the left plot

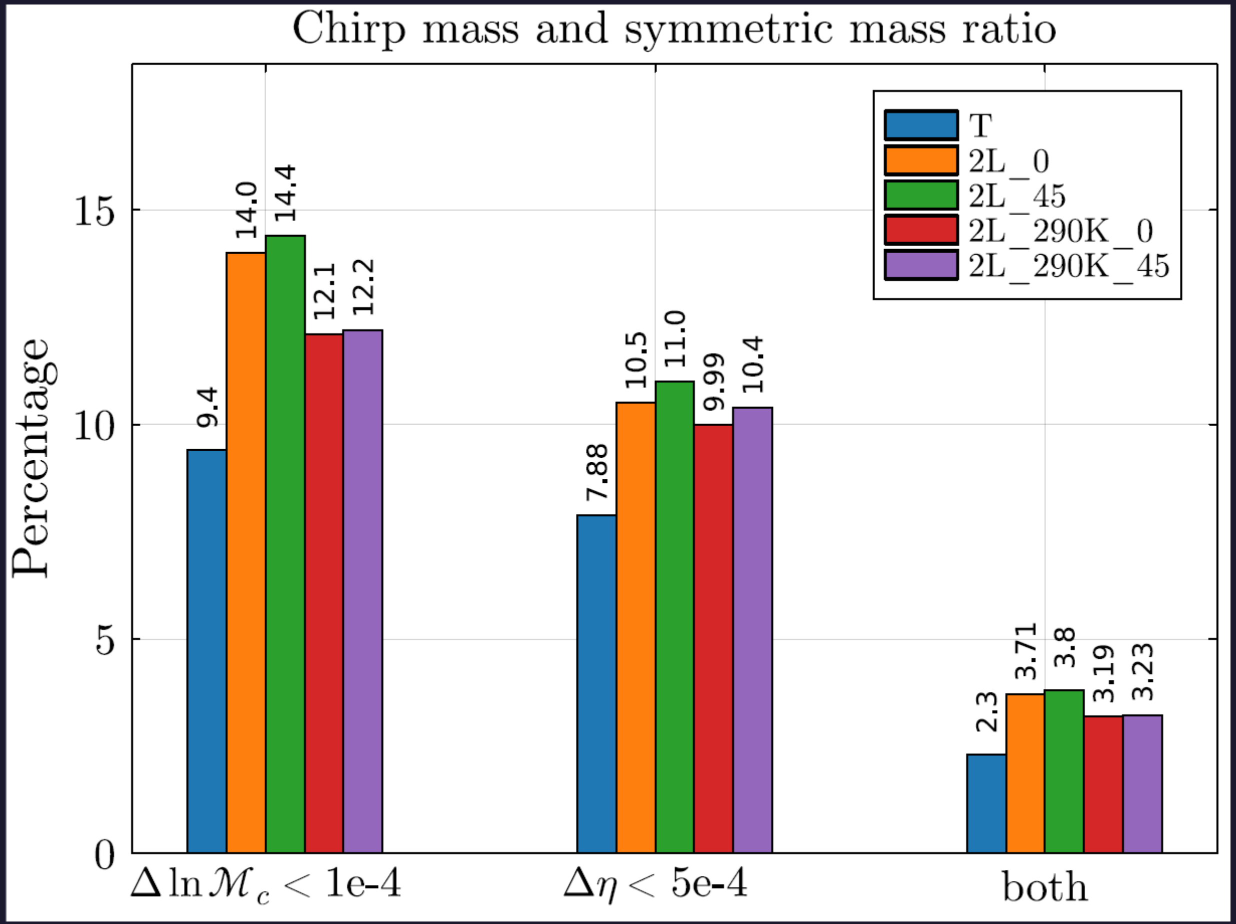
Source properties



The plot represents the percentage of events of which the 1-sigma error is below 1e-4 for the detector chirp mass and 5e-4 for the symmetric mass ratio. Only $\sim 1/3$ of sources that satisfy the mass ratio requirement satisfy both requirements.

$$\mathcal{M}_{c,source} = \frac{m_1^{3/5} m_2^{3/5}}{(m_1 + m_2)^{1/5}} \qquad \eta = \frac{m_1 m_2}{(m_1 + m_2)^2}$$

Source properties



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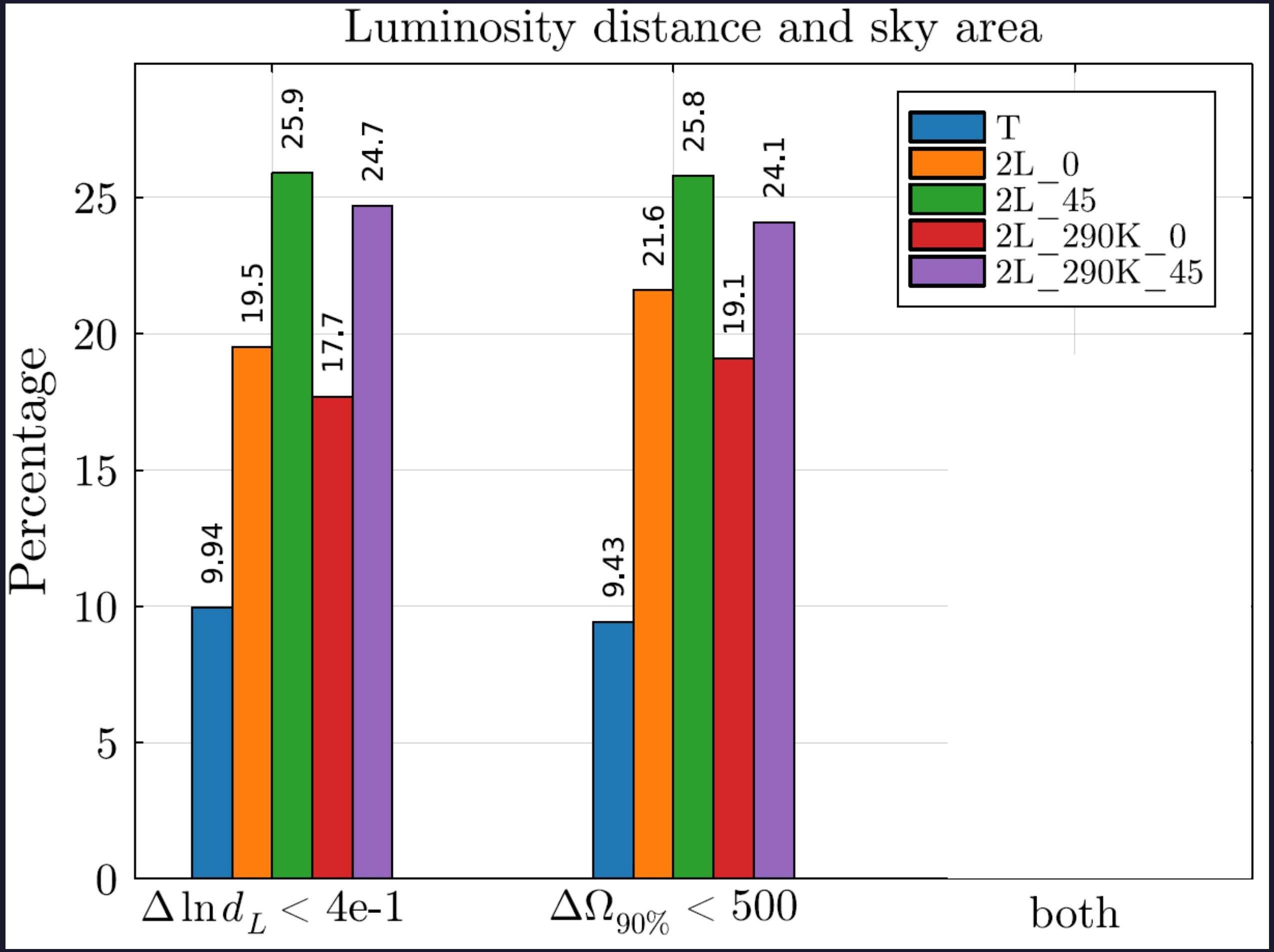
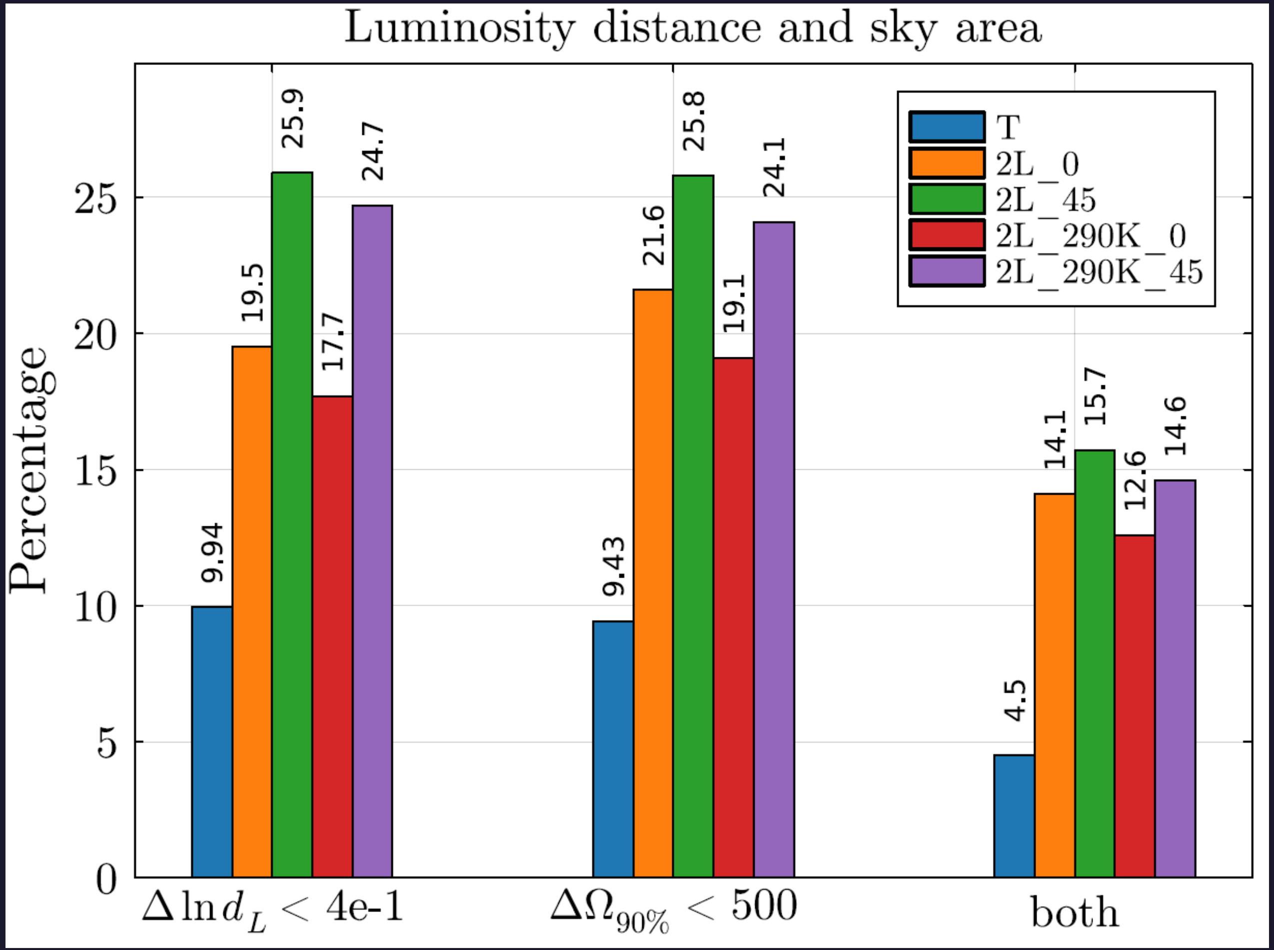
Cosmology



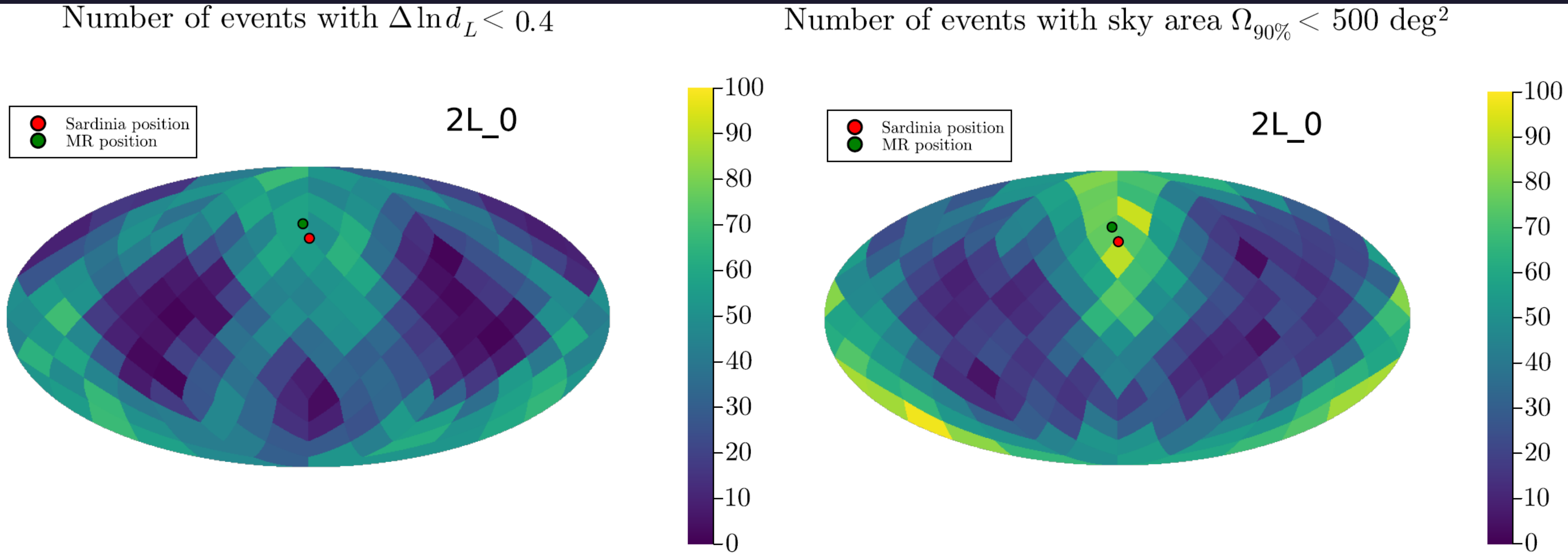
The advantage of the 2L_45 over 2L_0 in the luminosity distance and in the sky area is erased when we require both requirements to be satisfied at the same time.

Cosmology



The advantage of the 2L_45 over 2L_0 in the luminosity distance and in the sky area is erased when we require both requirements to be satisfied at the same time.

Sky area and luminosity distance – 2L_0

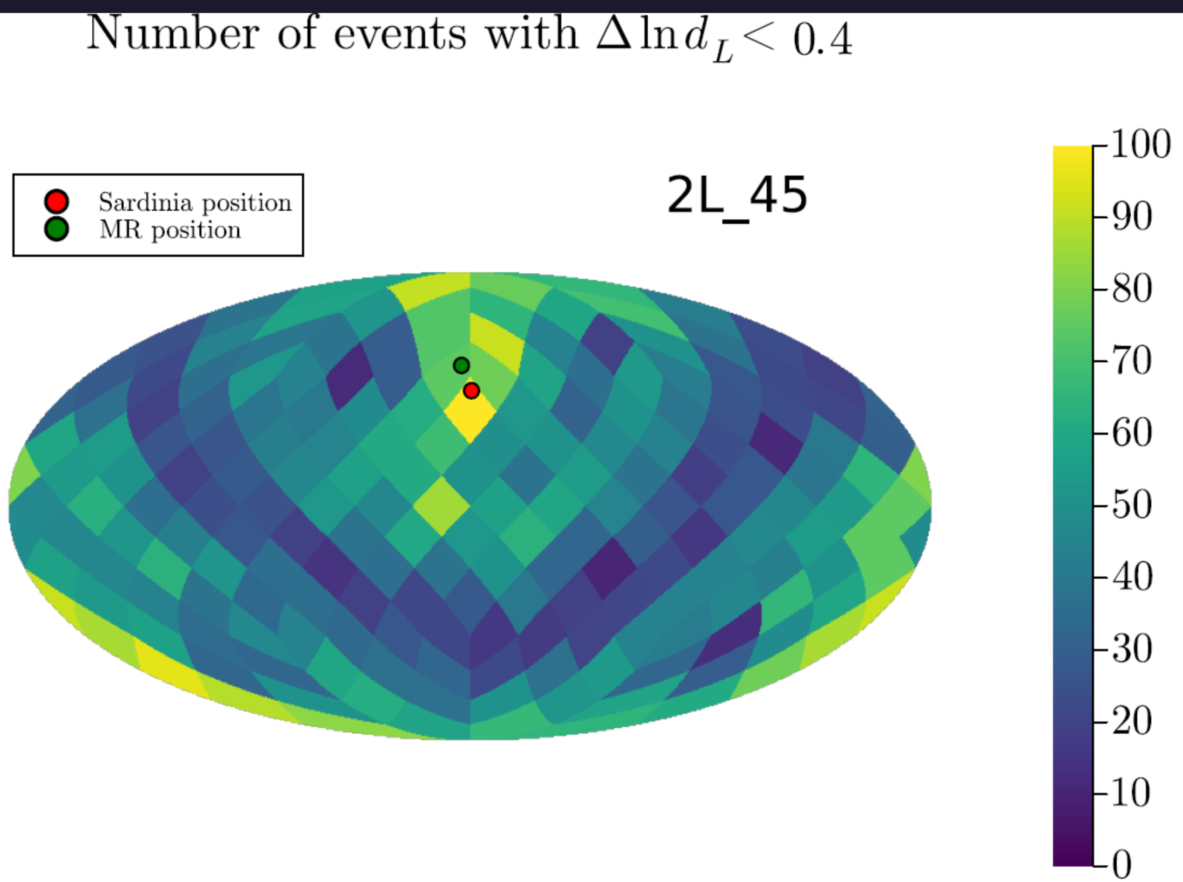


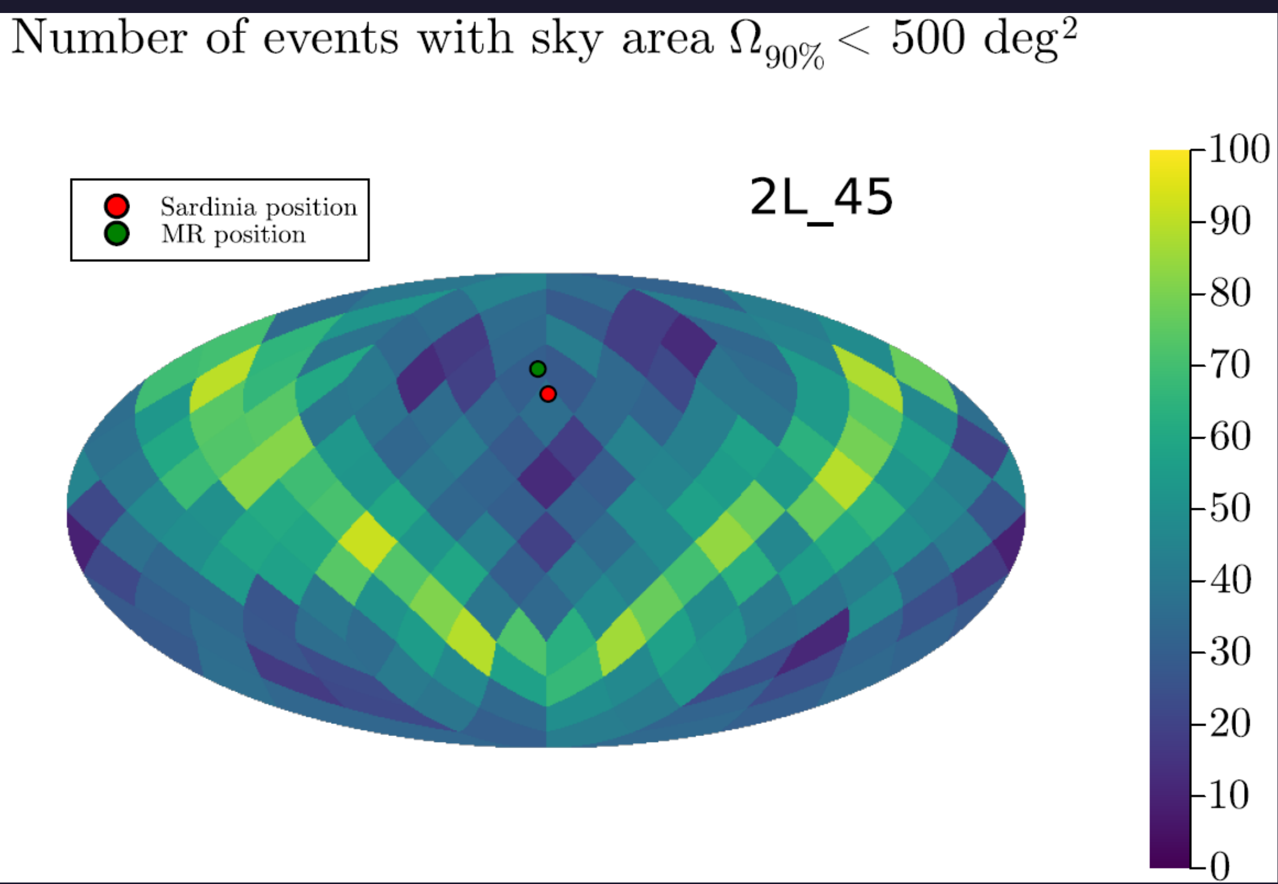
Sky maps in the detector frame for 2L_0.

Left: number of events for which the relative luminosity distance1-sigma error is under 40%

Right: number of events for which the 90% sky area is under 500 sqdeg.

Sky area and luminosity distance – 2L_45





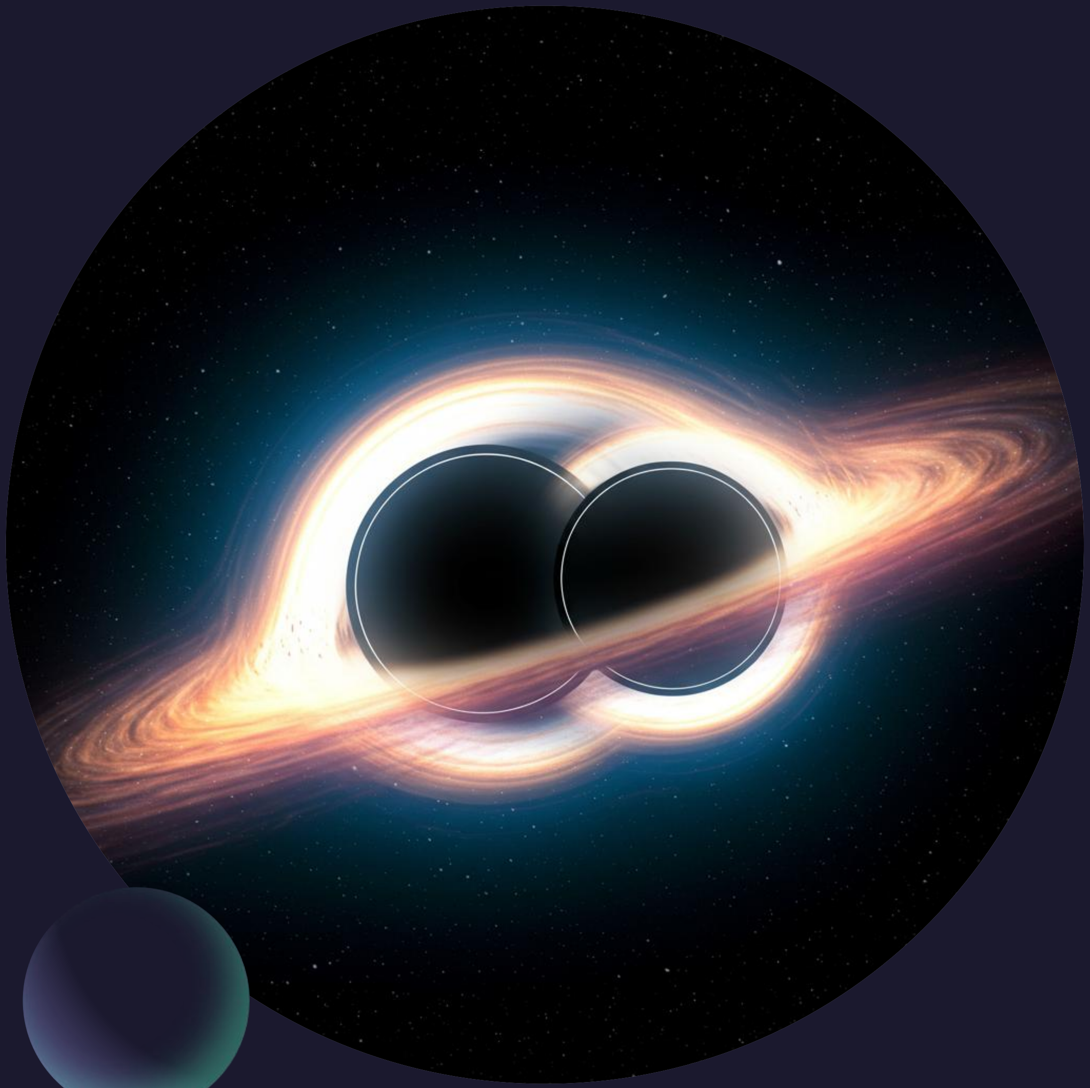
Sky maps in the detector frame for 2L_45.

Left: number of events for which the relative luminosity distance1-sigma error is under 40%

Right: number of events for which the 90% sky area is under 500 sqdeg.

Beyond GR

- GW from CBC can be important test of GR
- Many possible aspects of GR can be tested, e.g., tidal deviations, spin-induced quadrupole moment
- In this work, we focus on the Post-Newtonian terms



Post-Netwonian expansion

- Approximation valid for weak gravitational fields and low velocities
- Used for the inspiral phase of comparable mass CBC
- Expansion in terms of (v/c)
- Analytical GR predictions for

$$\{\varphi_0, \varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5, \varphi_{5l}, \varphi_6, \varphi_{6l}, \varphi_7\}$$

where the index represents two times the PN order

TIGER framework

In the TIGER framework[9], *one PN term at a time* is modified as

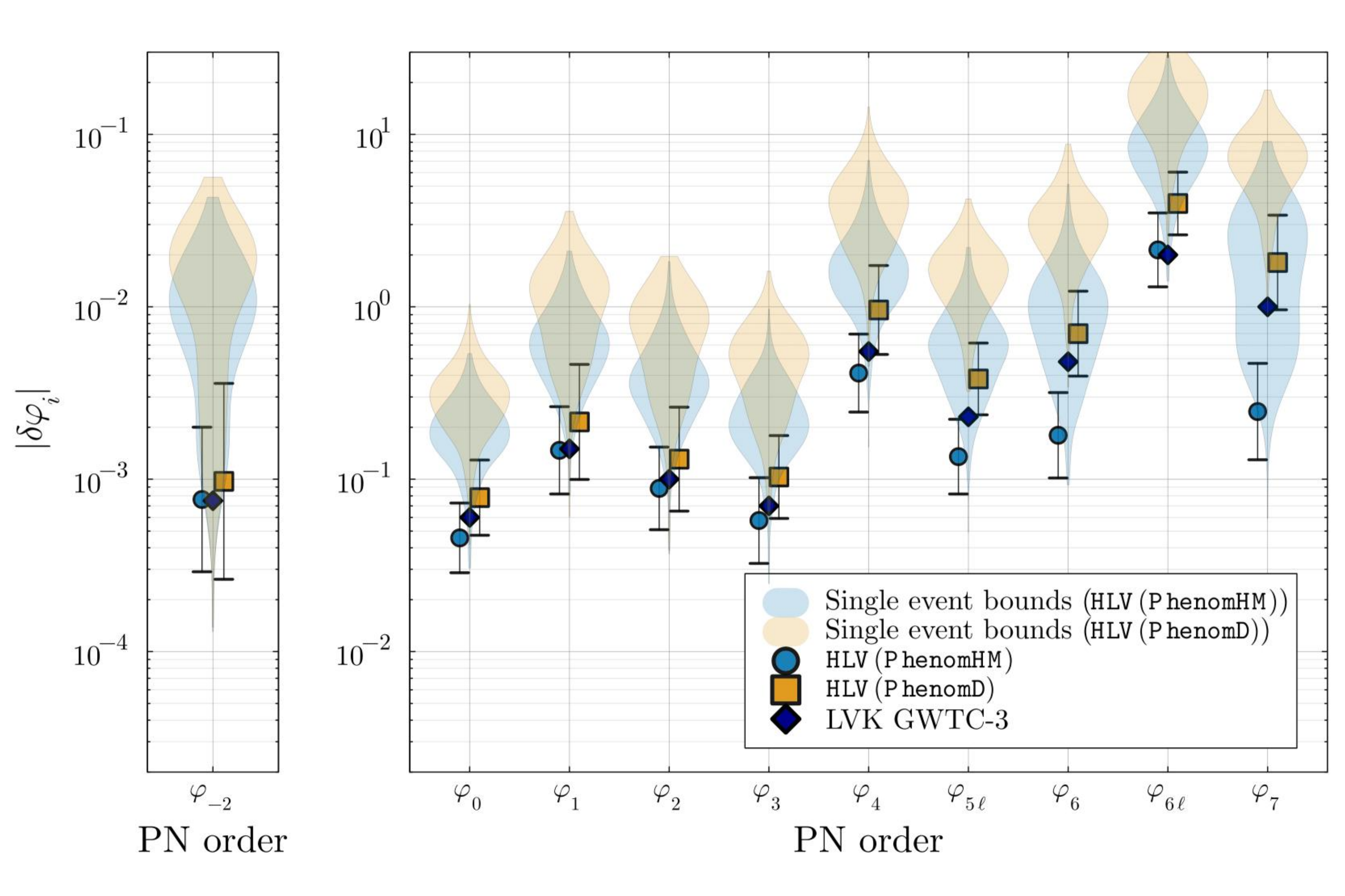
$$\varphi_k^{\mathrm{GR}} \rightarrow (1 + \delta\varphi_k)\, \varphi_k^{\mathrm{GR}}$$

We modified accordingly the waveforms PhenomD and PhenomHM, ensuring that they remain C^1.

[9] Agathos et al. 2014

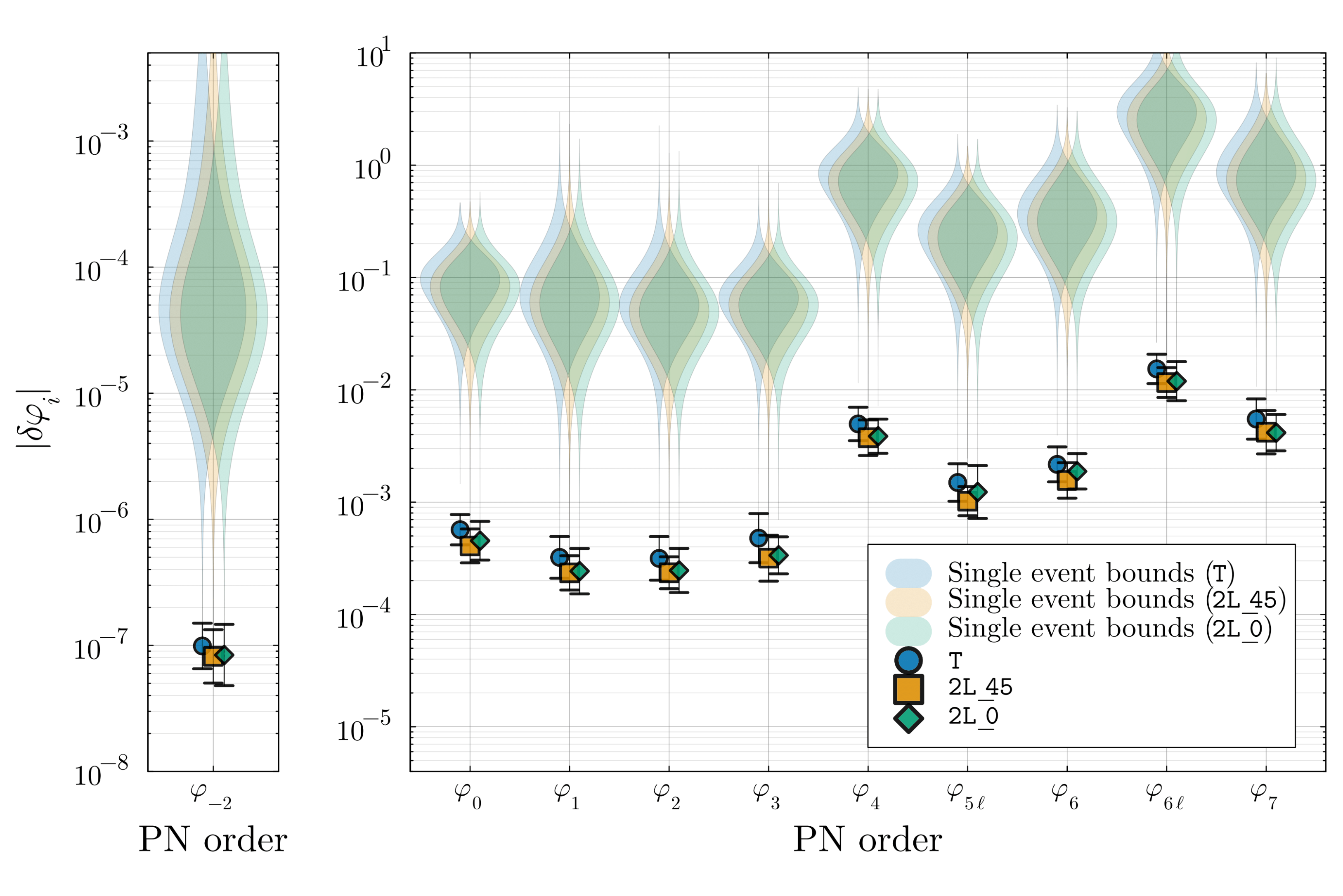
Comparison with GWTC-3

- We test the 90% upper bound from the Fisher analysis with the actual LVK results. The events analyzed are 9 and the different catalog realizations give the error bars. The violin plots are obtained by the single events bounds.
- HLV indicates that the detectors used are Livingston, Hanford and Virgo.
- There is great agreement between the Fisher and the GWTC-3 collaboration results



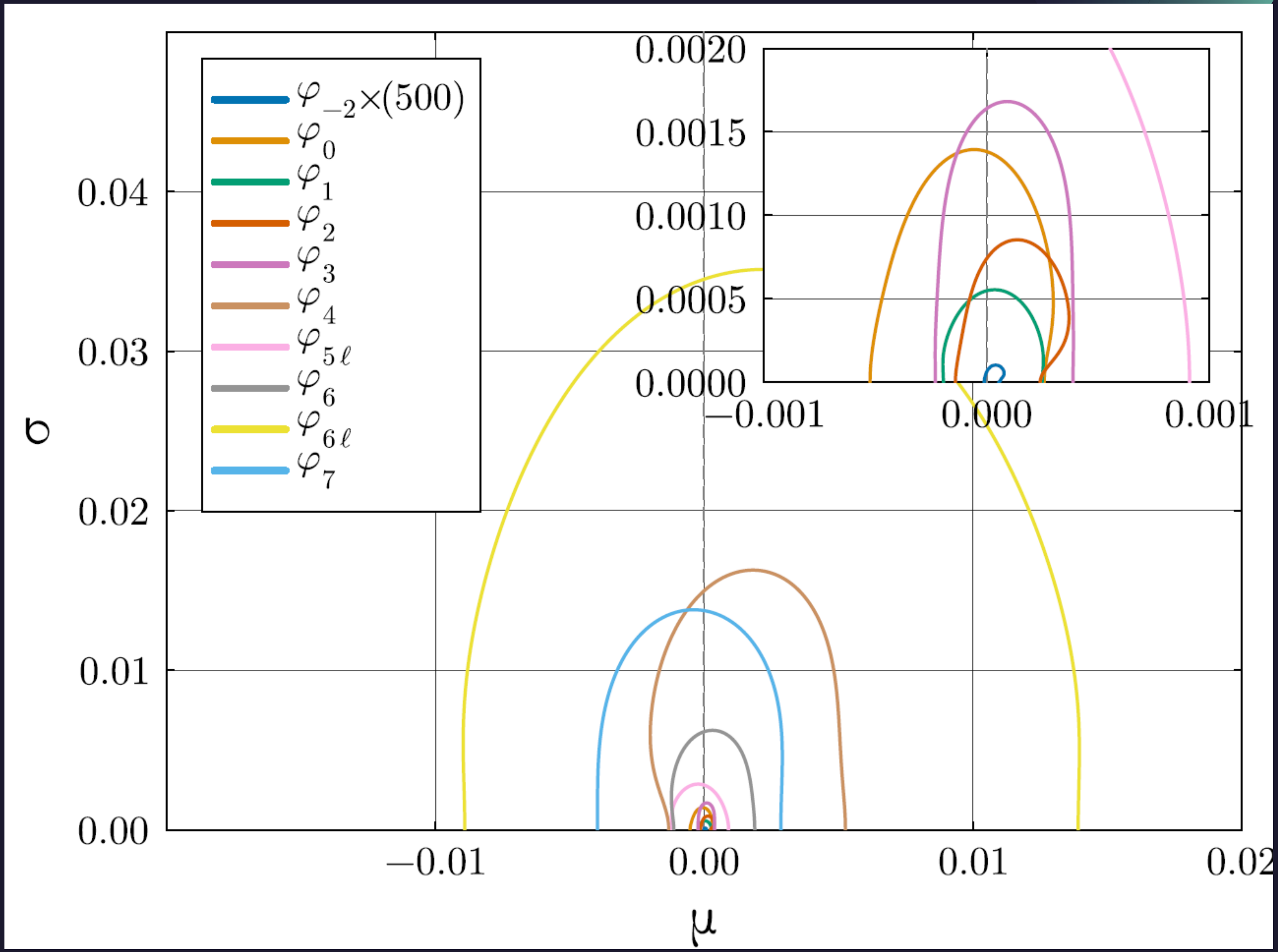
Forecast for ET

- We test 3 detector designs for ET
- All detectors improve significantly the LVK constraint (2 orders of magnitude) while -1 PN improves by 4 orders due to the lower frequency minimum of ET
- These results correspond to approx. 4 months of observations



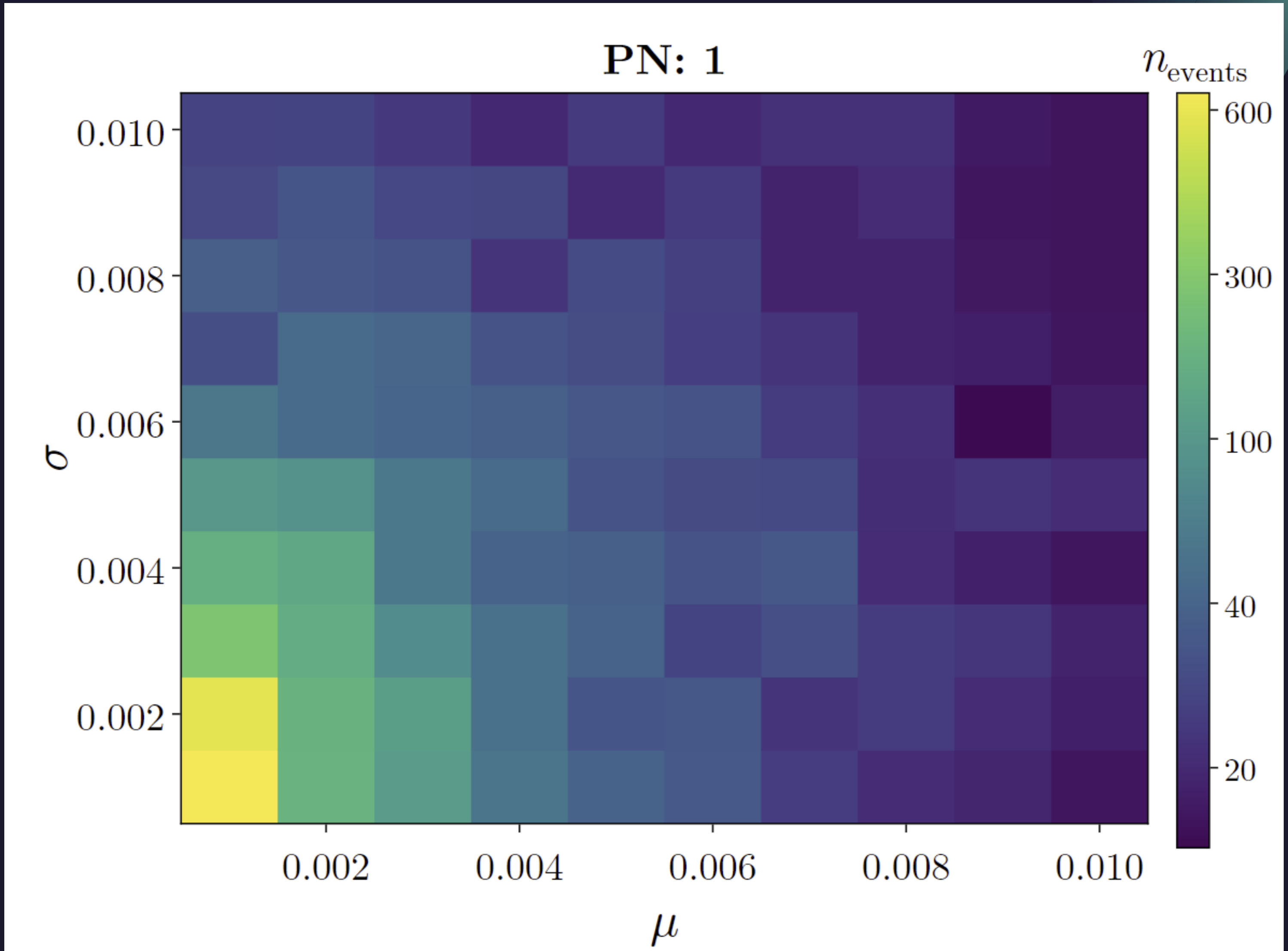
Forecast for ET

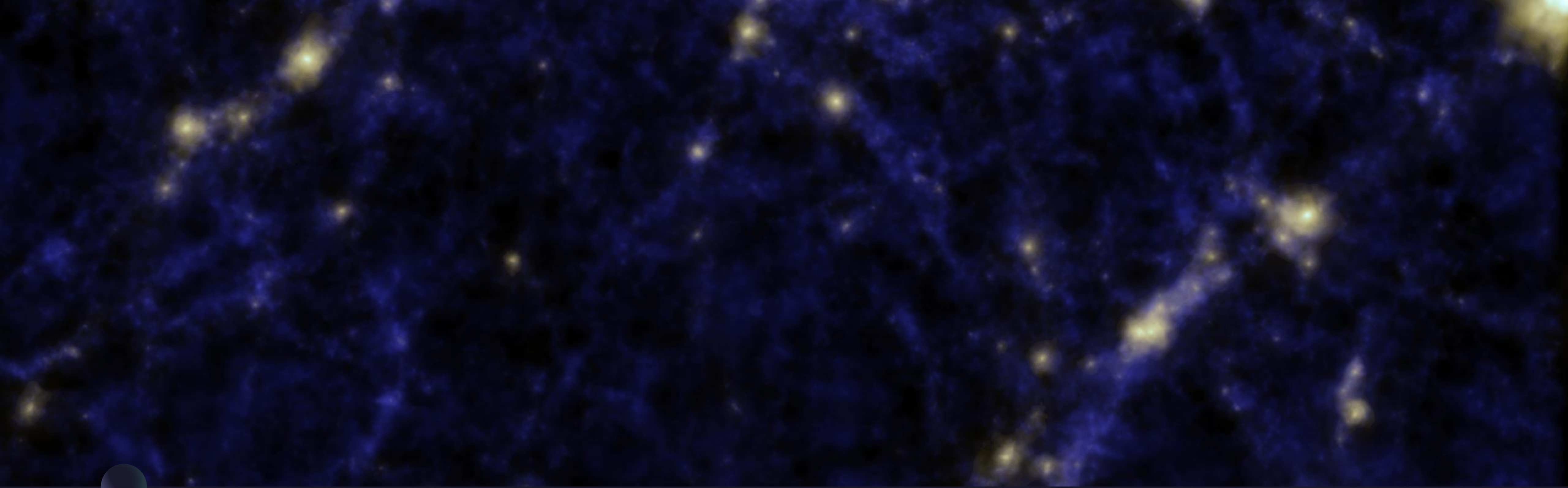
- We model the deviation as a gaussian with mean μ and std σ.
- We inject events with no GR deviations, and we recover the hyperparameters (μ, σ).
- We combine the observations in a hyperparameter framework
- These results corresponds to 4/5 months of observations



Forecast for ET

- How many detections are needed to detect a GR deviation at 90% confidence level?
- We inject events with GR deviations drawn from a gaussian with given μ and σ
- These constrains can be achieved in weeks or days of observations
- We used 2L_45 and the PN 1 term

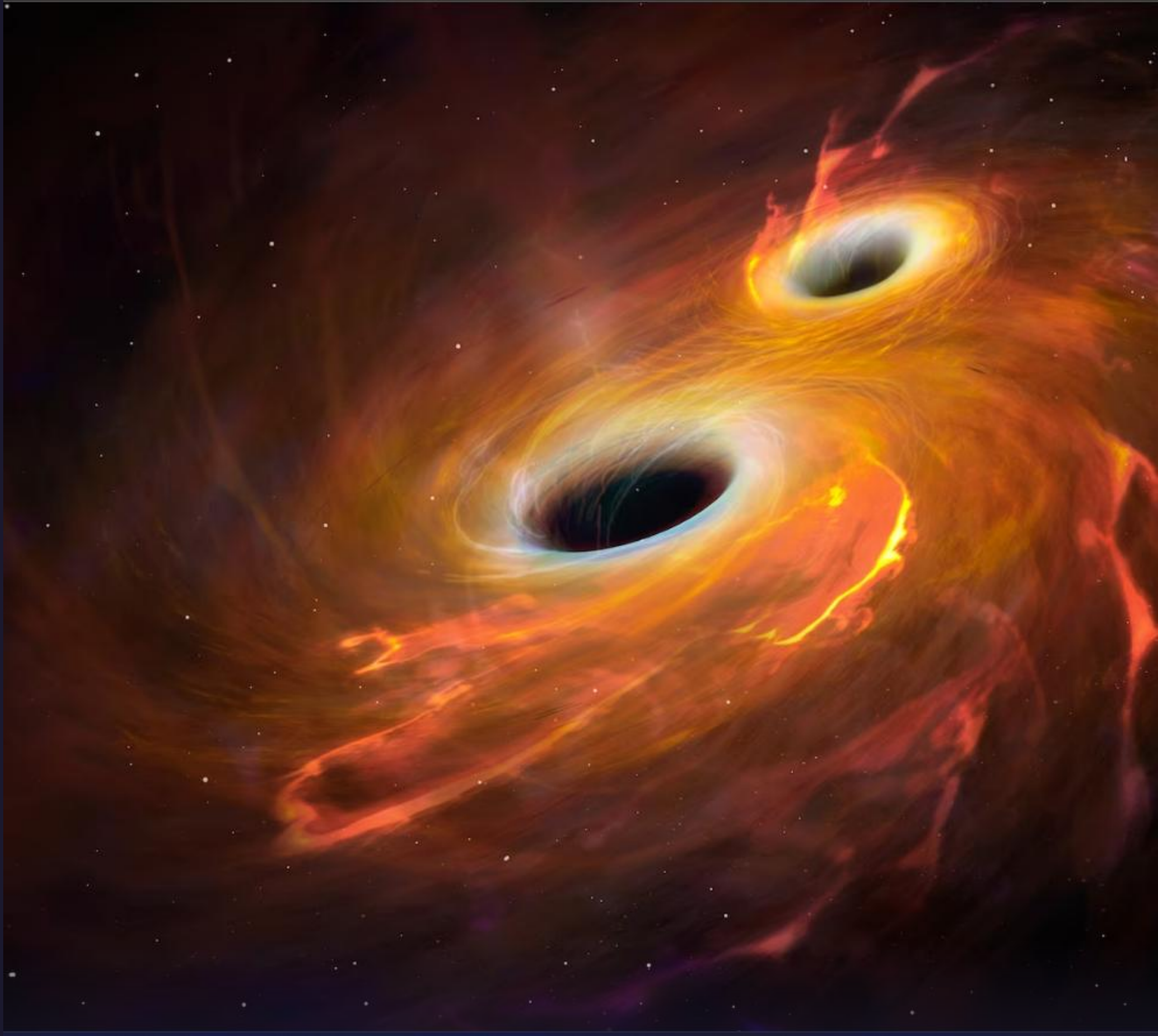




Take home messages

- GWJulia is a fast open-source tool to evaluate Fisher matrices of CBC
- 2L_45 is the best network for CBC with a slight margin over 2L_0, at the cost of losing SGWB
- With ET, BGR test of the PN terms will reach an unprecedented level of precision with just a handful of sources

Thank you!



Case study – SNR

Network	SNR > 8	SNR > 12	SNR > 20	SNR > 50	SNR > 100
T	87.6 %	71.1 %	43.3 %	9.1 %	1.7 %
2L_0	89.3 %	78.6 %	56.5 %	15.7 %	3.6 %
2L_45	94.1 %	82.9 %	58.1 %	15.6 %	3.5 %
2L_290K_0	87.4 %	75.5 %	52.3 %	13.4 %	2.9 %
2L_290K_45	92.3 %	79.6 %	53.5 %	13.3 %	2.8 %

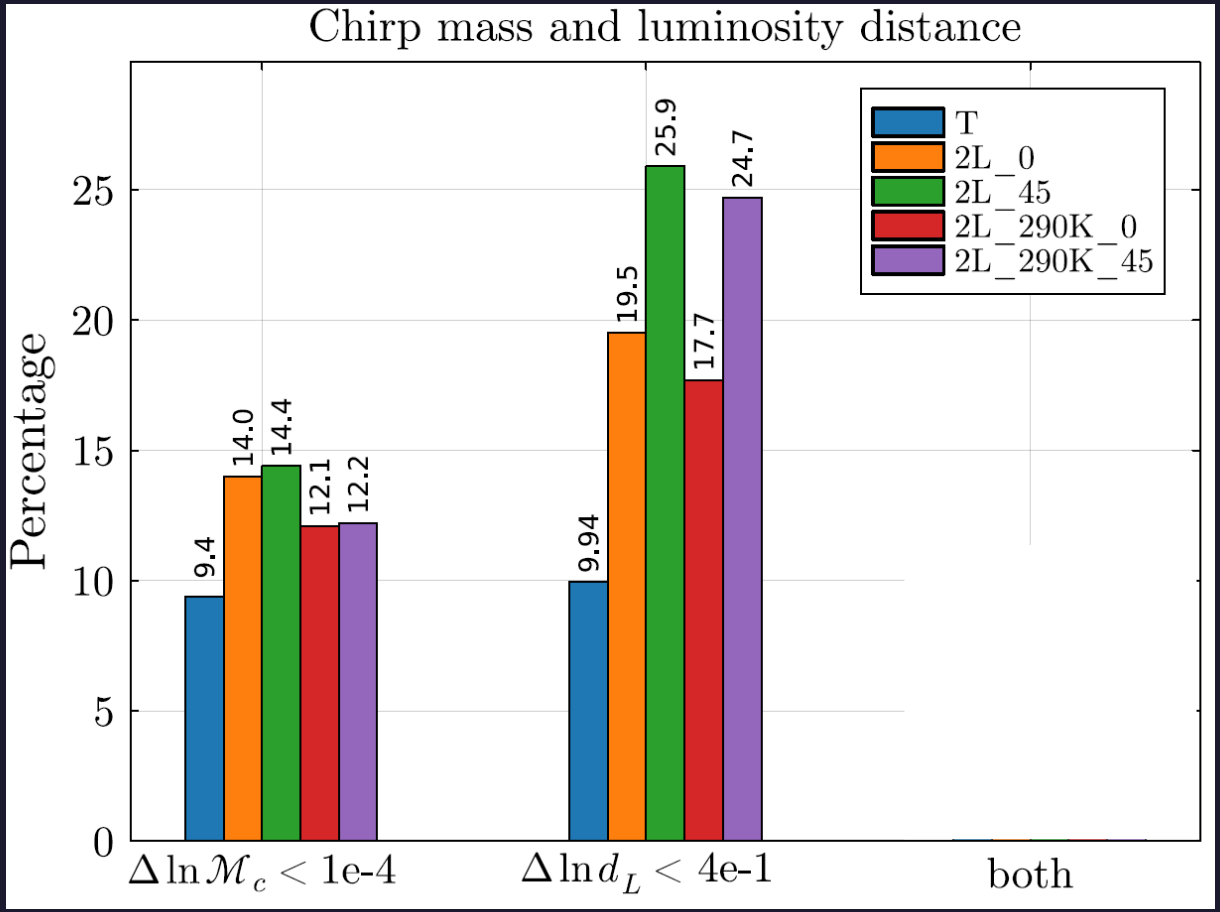
The table represents the percentage of events above the threshold written on top of the column

Case study – Angular Precision

Network	$\Omega < 10$ deg^2	$\Omega < 100$ deg^2	$\Omega < 1000$ deg^2	$\Omega <$ Whole sky
T	0.31 %	3.86 %	12.65 %	34.24 %
2L_0	0.23 %	4.68 %	32.91 %	75.43 %
2L_45	1.01 %	9.63 %	34.04 %	72.22 %
2L_290K_0	0.18 %	3.79 %	30.14 %	72.49 %
2L_290K_45	0.81 %	8.67 %	32.18 %	68.72 %

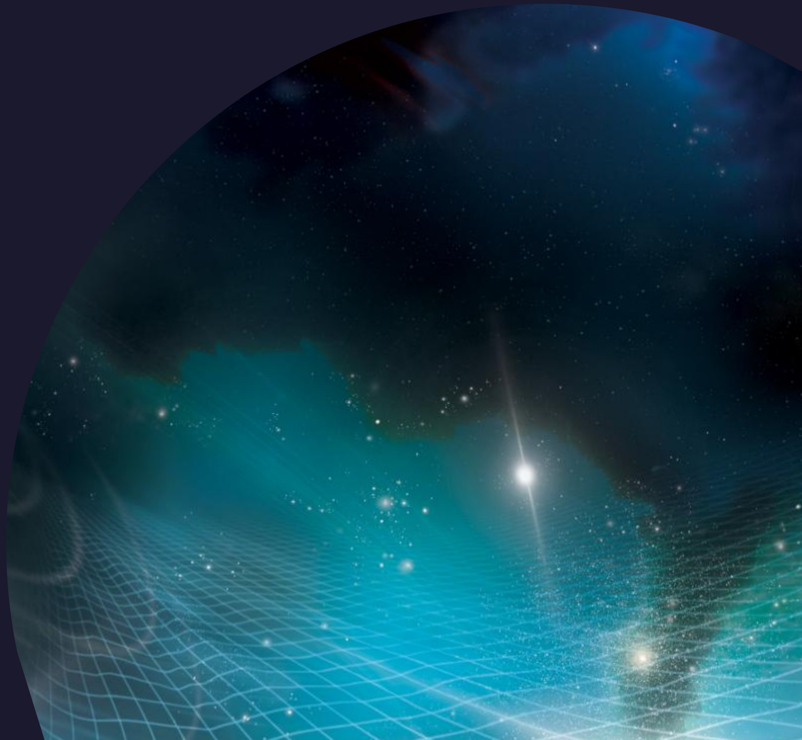
The table represents the percentage of events of which the 90% sky areas are less than the threshold indicated in each column. The 45° networks outperform the other networks and the T configuration. In particular, the T is comparable when considering the few high-precision sources, however, the performance degrades significantly for sources with precision worse than 1000 deg^2.

Source properties



The advantage of the 2L_45 over 2L_0 in the luminosity distance is erased when we require also the chirp mass to satisfy the requirement.

$$\mathcal{M}_{c,det} = (1+z)\mathcal{M}_{c,source}$$



Post-Netwonian terms

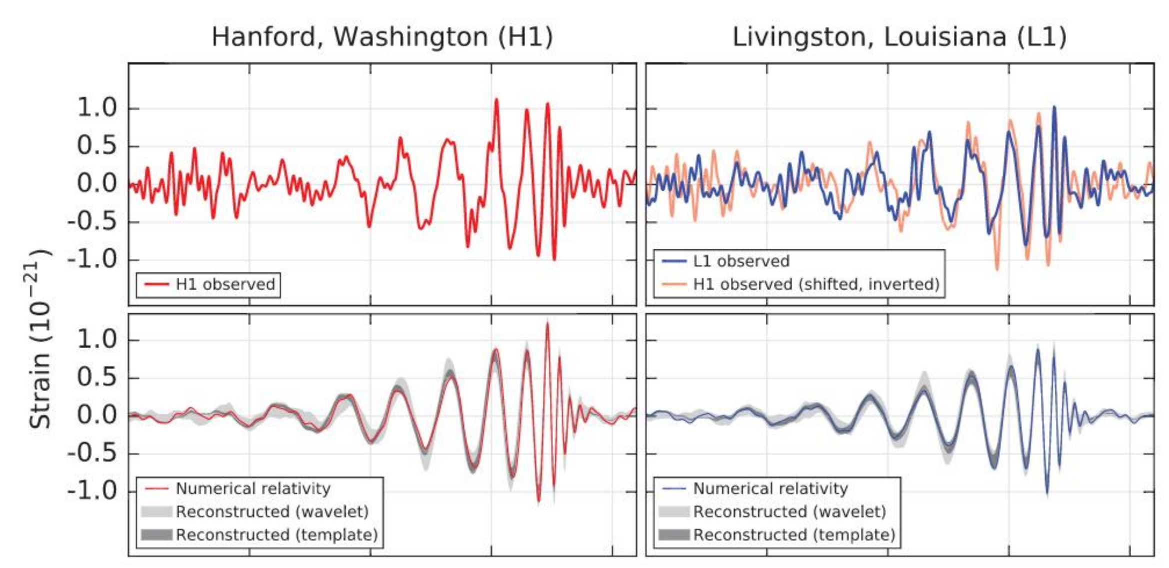
- In frequency domain the phase of a GW can be expressed as

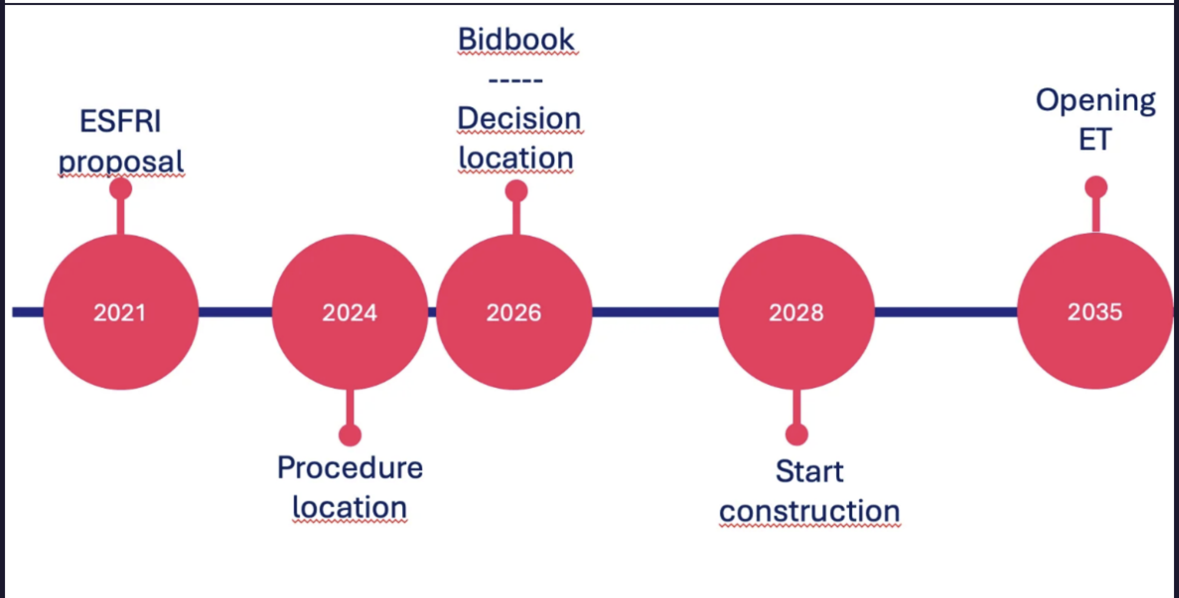
$$\Phi(f) \propto \sum_{j=0}^{7} \left[\varphi_j + \varphi_j^{(l)} \ln f\right] f^{(j-5)/3}$$

The dawn of GW astronomy

- Are we getting closer to noon? -

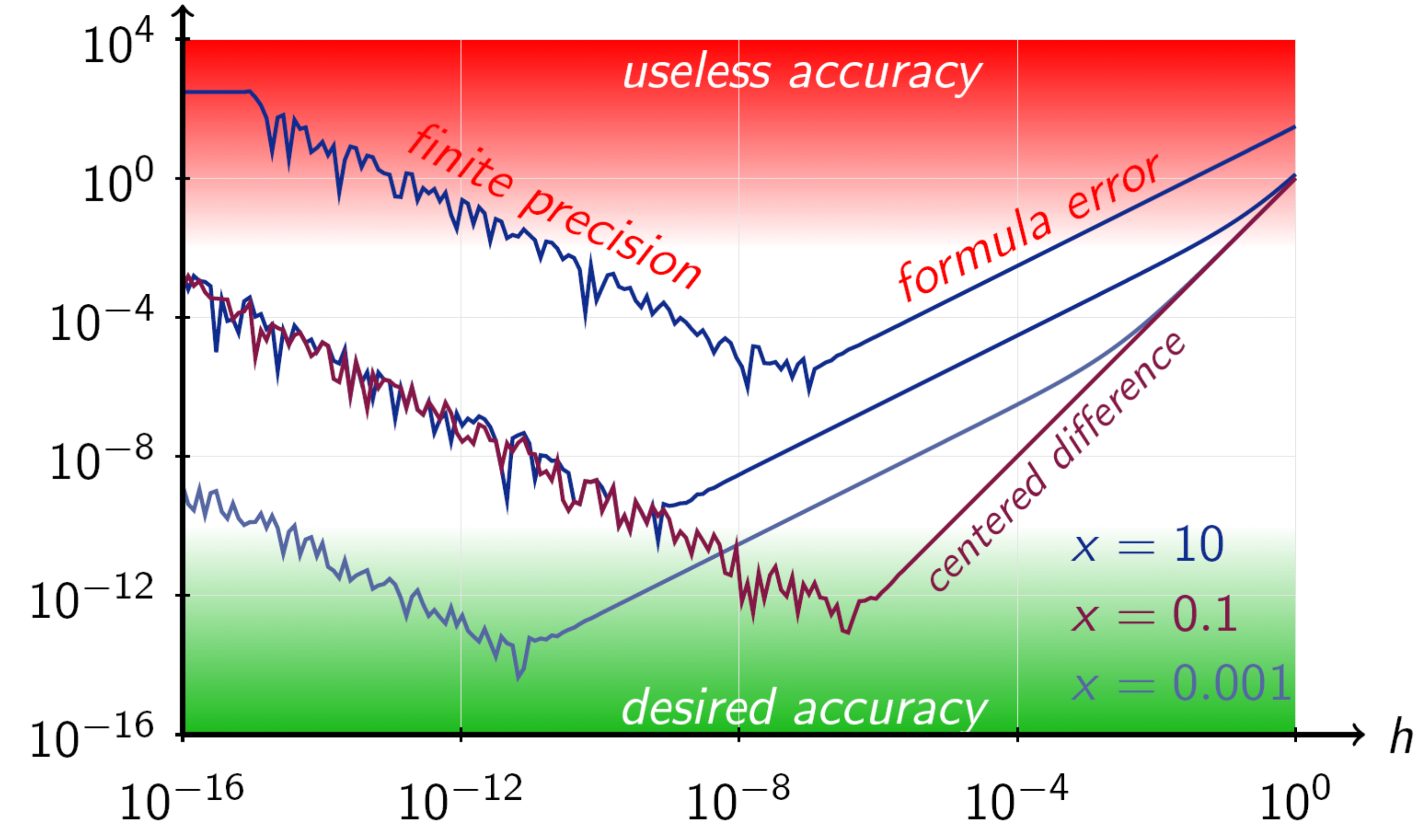
- The first GW detection was almost 10 years ago
- During the O4 run from Ligo Virgo Kagra (LVK) detected more than 200 sources in 2 years
- PTA discovery of a Stochastic GW background (SGWB)
- ET procedure location is ongoing
- LISA entered the hardware implementation phase (B2)





Numerical derivatives

- There is an optimal step size to maximize the accuracy
- The step size is usually dependent on both the function and the point where the derivative is calculated
- There are better alternatives to finite difference methods



Credits: Wikipedia

Waveforms

- The measured data consists of the GW signal plus a noise

$$d(t) = s(t) + n(t)$$

- Where the signal is the projection of the strain onto the detector

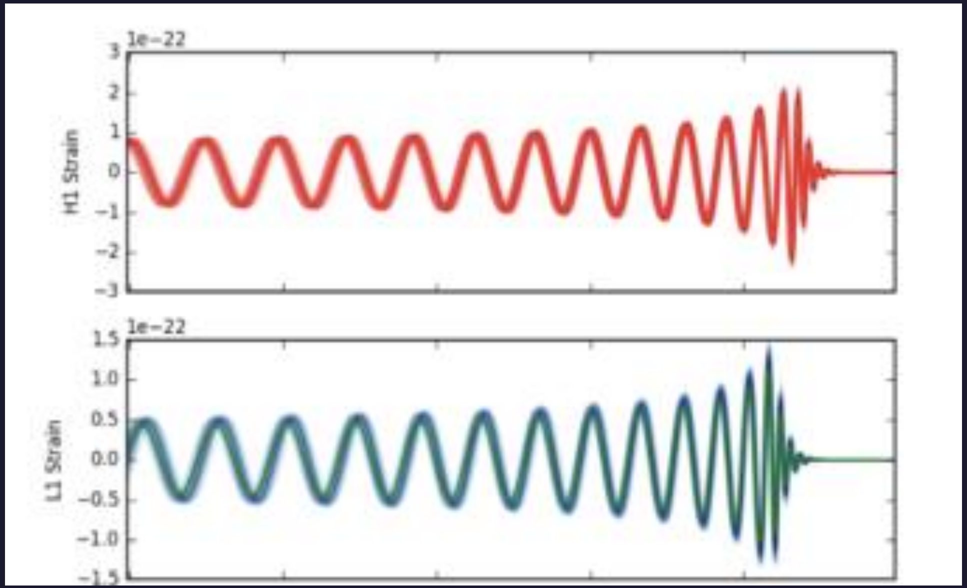
$$s = h_+ e_{+ij} D^{ij} + h_\times e_{\times ij} D^{ij} \equiv h_+ F_+ + h_\times F_\times$$

- A waveform is a function that associates to the source parameters the GW, in time or frequency domain

- Here we use the IMRPhenom waveforms [3-8] (up to XHM) that work in the frequency domain.

$$h = \sum_k A_k(f) e^{i\Phi(f)}$$

[3] Khan et al. 2015
[4] London et al. 2018
[5] García-Quirós et al. 2020
[6] Pratten et al. 2020a
[7] Pratten et al. 2020b
[8] Dietrich et al. 2019



Julia

- Compiled (Python is interpreted)
- Designed for scientific computing
- As easy to write as Python

```python
def factorial(n):
    if n == 0 or n == 1:
        return 1
    else:
        return n * factorial(n - 1)

# Example usage
num = 5
print(f"The factorial of {num} is {factorial(num)}")
```

```julia
function factorial(n)
    if n == 0 || n == 1
        return 1
    else
        return n * factorial(n - 1)
    end
end

# Example usage
num = 5
println("The factorial of $num is $(factorial(num))")
```

Future plans – Fisher informed HMC

- Bayesian data analysis of CBC is very computationally demanding (e.g., using nested sampling)

- If we are interested in future observations, we can simplify the inference problem, i.e., we know an approximate location of the maximum posterior

- With automatic differentiation we can perform fast and accurate gradients

- We can perform Hamiltonian Monte Carlo sampling, injecting information from the Fisher Matrix