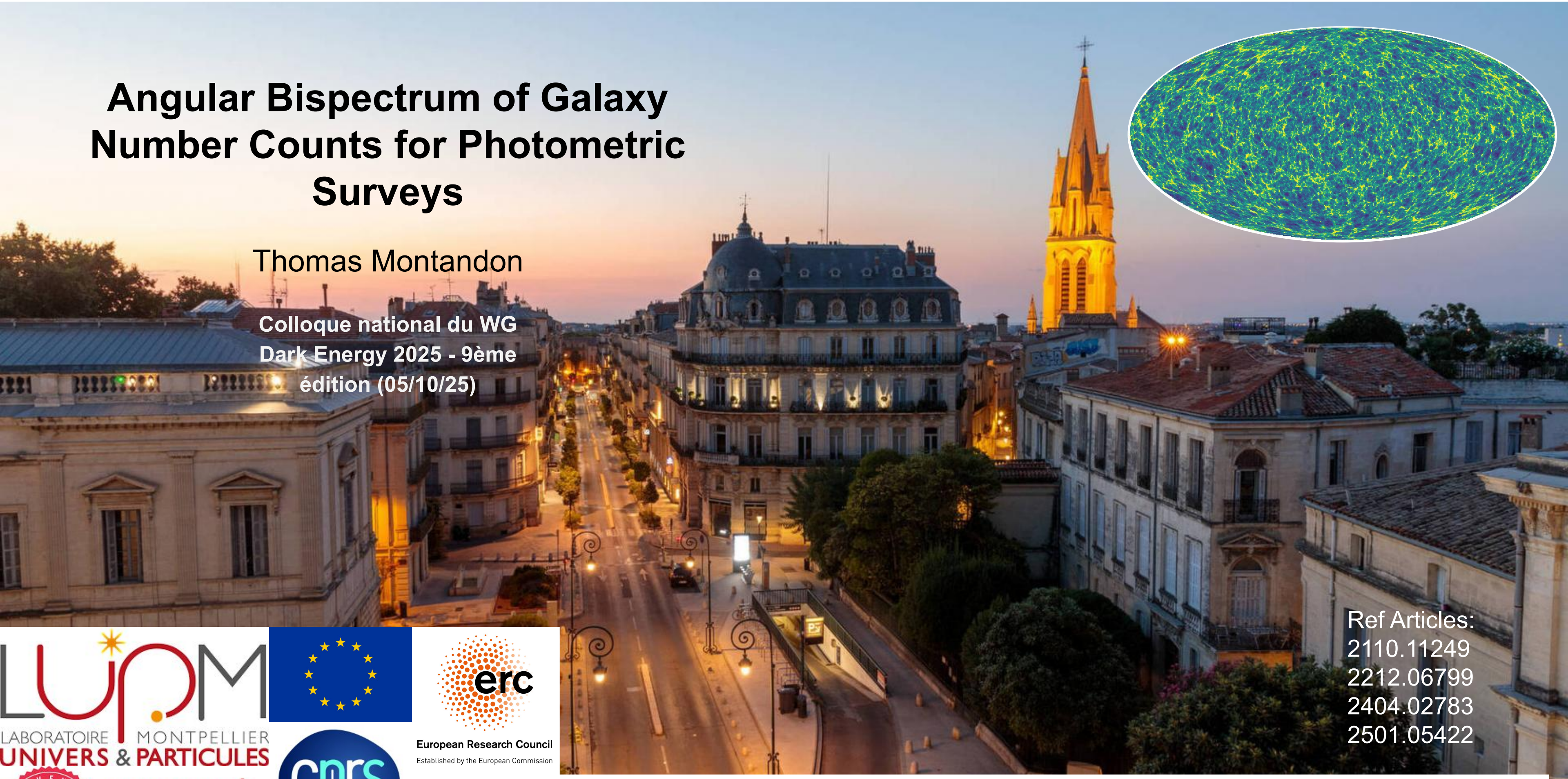

Angular Bispectrum of Galaxy Number Counts for Photometric Surveys

Thomas Montandon

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Ref Articles:
2110.11249
2212.06799
2404.02783
2501.05422

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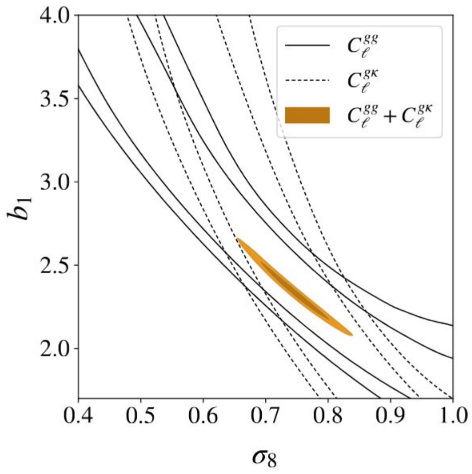
Angular Bispectrum of Galaxy Number Counts for Photometric Surveys

- Why?

Angular Bispectrum of Galaxy Number Counts for Photometric Surveys

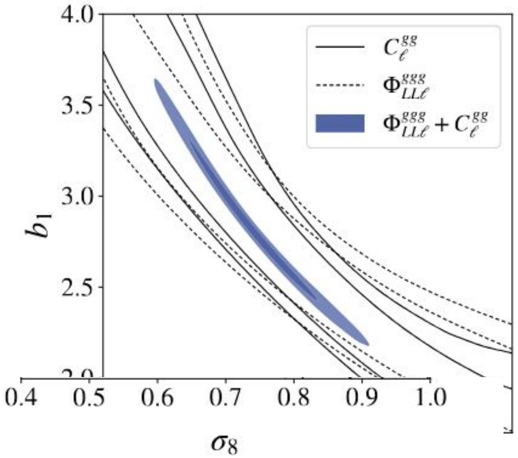
- Why?
  - LCDM predicts an “Intrinsic Bispectrum due to nonlinearities ”

$$B(t, k_1, k_2, k_3) = 2T_1(k_1)T_1(k_2)T_2(k_1, k_2, k_3)P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_2) + 2 \times \text{perm.}$$



Harscouet et al. 2507.07968

$$C_\ell^{gg} \propto \sigma_8^2 b_1^2$$
$$B^{ggg}_{\ell_1\ell_2\ell_3} = \sigma_8^4 b_1^3$$



Angular Bispectrum of Galaxy Number Counts for Photometric Surveys

- Why?
  - LCDM predicts an "Intrinsic Bispectrum due to nonlinearities "
  - it is annoying for inflation

$$B(t, k_1, k_2, k_3) = 2T_1(k_1)T_1(k_2)T_2(k_1, k_2, k_3)P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_2) + 2 \times \text{perm.}$$
$$+ 2T_1(k_1)T_1(k_2)T_1(k_3)B(k_1, k_2, k_3)$$

Primordial non-Gaussianities / inflation

GR and Radiation effects are degenerate in time and in momentum space with PNG!

Angular Bispectrum of Galaxy Number Counts for Photometric Surveys

Spectroscopic survey

- fsky = ⅓
- ~30 millions of galaxies
- accurate redshifts

- Why?
  - LCDM predicts an "Intrinsic Bispectrum due to nonlinearities "
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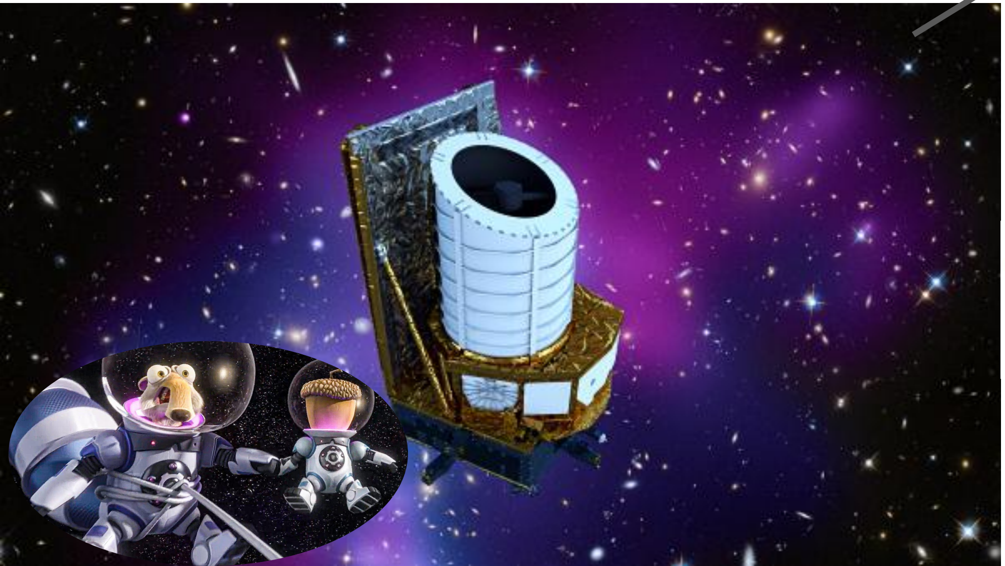
  - Photometry



FFT based estimator:

Scoccimarro estimator (Scoccimaro 1506.02729)

$$\hat{B}_\ell(k_1,k_2,k_3) = \frac{2\ell+1}{V_B V}\prod_{i=1}^{3}\left[\int_{k_i} d^3q_i \int_V d^3x_i\, e^{-i\mathbf{q}_i\cdot\mathbf{x}_i}\right]\delta_D(\mathbf{q}_{123})\delta_g(\mathbf{x}_1)\delta_g(\mathbf{x}_2)\delta_g(\mathbf{x}_3)\mathcal{L}_\ell(\hat{\mathbf{q}}_1\cdot\hat{\mathbf{x}}_3)$$

Tri-polar estimator (Sugiyama et al 1803.02132)

$$B_{\ell_1\ell_2L}(k_1,k_2) = N_{\ell_1\ell_2L}H_{\ell_1\ell_2L}\sum_{m_1m_2M}\begin{pmatrix}\ell_1 & \ell_2 & L\\ m_1 & m_2 & M\end{pmatrix} \times\int\frac{\mathrm{d}^2\hat{\mathbf{k}}_1}{4\pi}\frac{\mathrm{d}^2\hat{\mathbf{k}}_2}{4\pi}\frac{\mathrm{d}^2\hat{\mathbf{n}}}{4\pi}\,y_{\ell_1}^{m_1*}(\hat{\mathbf{k}}_1)\,y_{\ell_2}^{m_2*}(\hat{\mathbf{k}}_2)\,y_L^{M*}(\hat{\mathbf{n}})\,B(\mathbf{k}_1,\mathbf{k}_2,\hat{\mathbf{n}})$$

Wide-angle effect is hard to include!

Bardede, Di Dio and Castorina (2302.12789)

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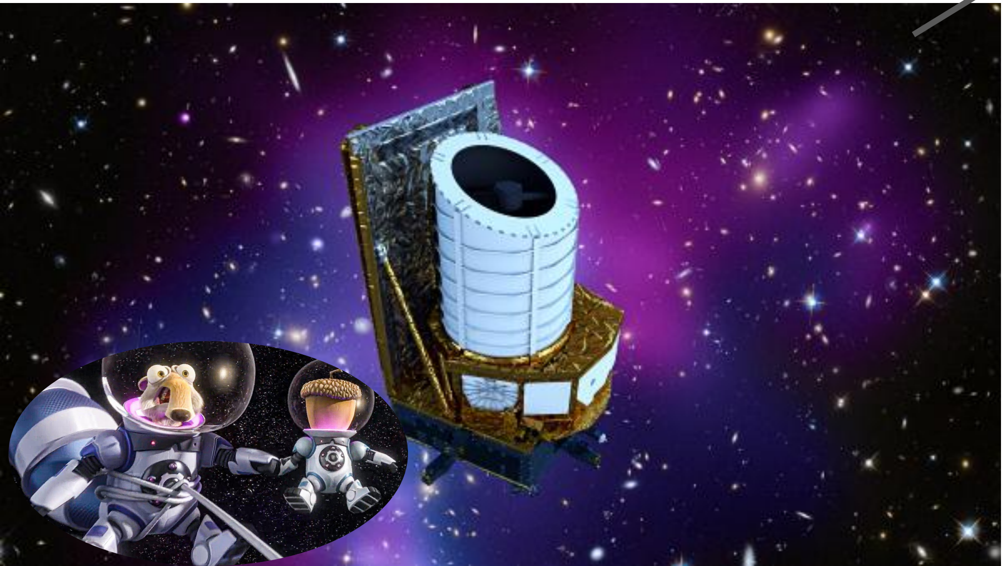

Spherical harmonics based estimator:

Tomographic Spherical Harmonic” (Assassi et al 1705.05022 and Bucher et al 1509.08107 (For CMB))

$$\Delta_i^z(\hat{\boldsymbol{n}}) = \sum_{\ell=\ell_i^{\min}}^{\ell_i^{\max}} \sum_{m=-\ell}^{\ell} a^z_{\ell m} Y_{\ell m}(\hat{\boldsymbol{n}})$$

Too many bins
bad data compression!

$$B^{z_1 z_2 z_3}_{i_1 i_2 i_3} = \frac{1}{N_{i_1 i_2 i_3}} \int d\hat{\Omega} \Delta^{z_1}_{i_1}(\hat{\Omega}) \Delta^{z_2}_{i_2}(\hat{\Omega}) \Delta^{z_3}_{i_3}(\hat{\Omega})$$

Spherical Fourier-Bessel estimator (Peebles 1973, Benabou et al 2312.15992)

$$\delta_{\ell m}(k) \equiv \int \mathrm{d}^3\boldsymbol{r} \left[\sqrt{\frac{2}{\pi}}\, k\, j_\ell(kr)\, Y^*_{\ell m}(\hat{\boldsymbol{r}})\right] \delta(\boldsymbol{r})\,.$$

$$B^{\mathrm{SFB}}_{l_1 l_2 l_3}(k_1,k_2,k_3) \equiv \sum_{m_1,m_2,m_3} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \left\langle \delta^{g,\mathrm{obs}}_{l_1 m_1}(k_1) \delta^{g,\mathrm{obs}}_{l_2 m_2}(k_2) \delta^{g,\mathrm{obs}}_{l_3 m_3}(k_3) \right\rangle .$$

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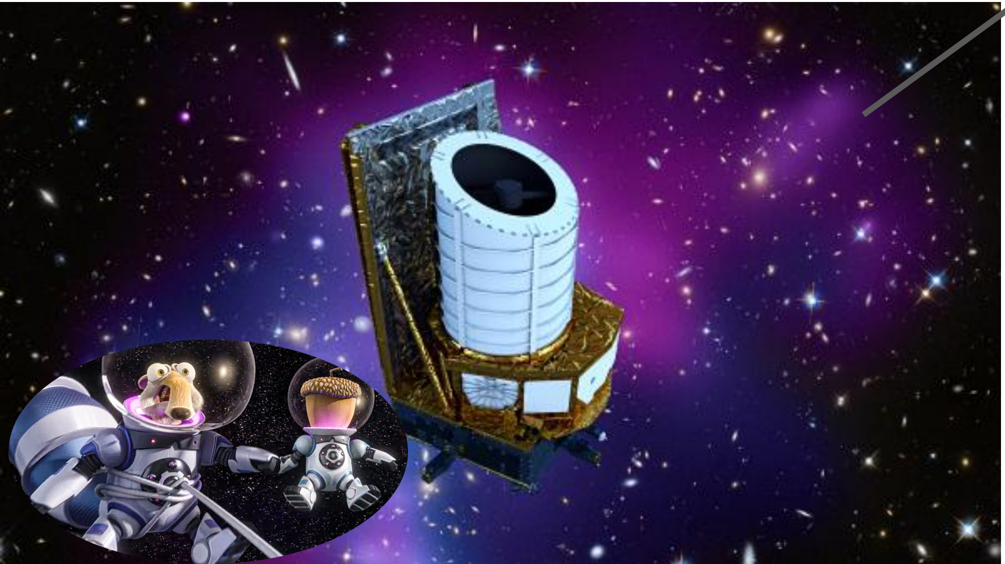

Photometric survey

- fsky = ⅓
- ~1.5 billion of galaxies
- ~6-10 redshift bins
- Weak lensing

Matter 3D tomography

Galaxy number count

Spherical harmonics based estimator:

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$$\Delta_i^z(\hat{\boldsymbol{n}}) = \sum_{\ell=\ell_i^{\min}}^{\ell_i^{\max}} \sum_{m=-\ell}^{\ell} a_{\ell m}^z Y_{\ell m}(\hat{\boldsymbol{n}})$$

$$B_{i_1 i_2 i_3}^{z_1 z_2 z_3} = \frac{1}{N_{i_1 i_2 i_3}} \int d\hat{\Omega} \Delta_{i_1}^{z_1}(\hat{\Omega}) \Delta_{i_2}^{z_2}(\hat{\Omega}) \Delta_{i_3}^{z_3}(\hat{\Omega})$$

Spherical Fourier-Bessel estimator (Peebles 1973, Benabou et al 2312.15992)

$$\delta_{\ell m}(k) \equiv \int \mathrm{d}^3\boldsymbol{r} \left[\sqrt{\frac{2}{\pi}}\, k\, j_\ell(kr)\, Y_{\ell m}^*(\hat{\boldsymbol{r}})\right] \delta(\boldsymbol{r})\,.$$

Too few bins not efficient!

$$B_{l_1 l_2 l_3}^{\mathrm{SFB}}(k_1, k_2, k_3) \equiv \sum_{m_1, m_2, m_3} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \left\langle \delta_{l_1 m_1}^{g,\mathrm{obs}}(k_1) \delta_{l_2 m_2}^{g,\mathrm{obs}}(k_2) \delta_{l_3 m_3}^{g,\mathrm{obs}}(k_3) \right\rangle .$$

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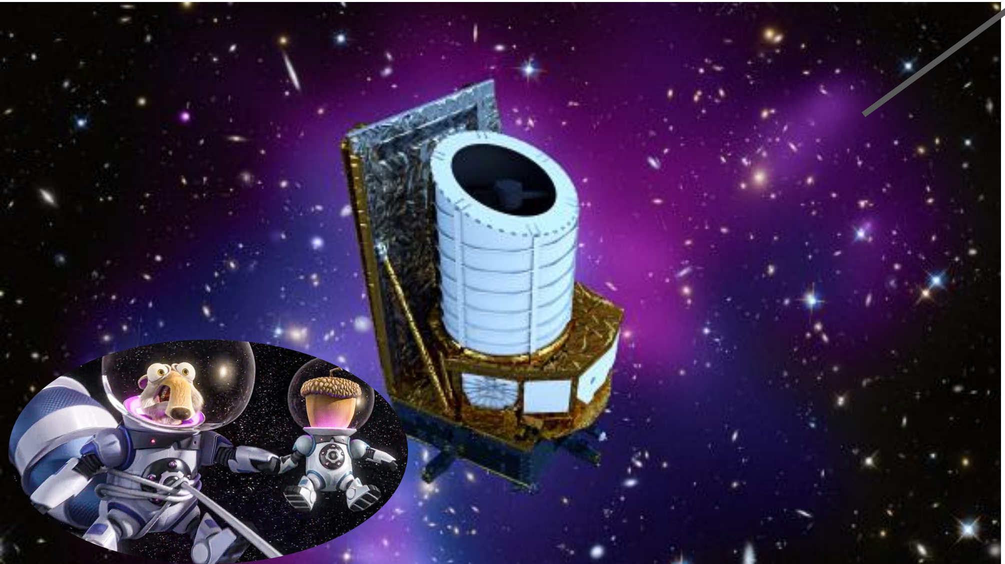


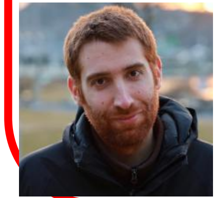
This is the main subject of this talk

Tomographic Spherical Harmonic” (Assassi et al 1705.05022 and Bucher et al 1509.08107 (For CMB))

$$\Delta_i^z(\hat{\boldsymbol{n}}) = \sum_{\ell=\ell_i^{\min}}^{\ell_i^{\max}} \sum_{m=-\ell}^{\ell} a_{\ell m}^z Y_{\ell m}(\hat{\boldsymbol{n}})$$

$$B_{i_1 i_2 i_3}^{z_1 z_2 z_3} = \frac{1}{N_{i_1 i_2 i_3}} \int d\hat{\Omega} \Delta_{i_1}^{z_1}(\hat{\Omega}) \Delta_{i_2}^{z_2}(\hat{\Omega}) \Delta_{i_3}^{z_3}(\hat{\Omega})$$

Enea di Dio



Stefano Camera

Apply this estimator for the photometric survey of Euclid

Angular Bispectrum of Galaxy Number Counts for Photometric Surveys

- How?
  - Theory
    - First order
      Yoo 1009.3021, Challinor and Lewis 1105.5292, Bonvin and Durrer 1105.5280, Jeong et al 1107.5427

$$\Delta_1 = \Delta_1^{\mathrm{N}} + P_1^{\mathrm{R}} + D_1^{\mathrm{R}}$$

- Newtonian terms $\Delta_1^{\mathrm{N}} = \delta_1^{\mathrm{N}} - \mathcal{H}^{-1}\partial_r^2 v_1 \, ,$
- Relativistic projection $P_1^{\mathrm{R}} = -\mathcal{R}\partial_r v_1 - 2\phi_1 + (\mathcal{R}+1)\psi_1 + \mathcal{H}^{-1}\dot{\phi}_1$
- GR dynamics $D_1^{\mathrm{R}} = \delta_1^{\mathrm{GR}} \, ,$

Angular Bispectrum of Galaxy Number Counts for Photometric Surveys

- How?
  - Theory
    - First order Yoo 1009.3021, Challinor and Lewis 1105.5292, Bonvin and Durrer 1105.5280, Jeong et al 1107.5427
    - Second order Di Dio 1407.0376, Yoo 1406.4140, , Bertacca 1405.4403， Magi 2204.01751

$$\Sigma_{IS} = \left(-\frac{2}{\mathcal{H}_s r_s} - \frac{\mathcal{H}'_s}{\mathcal{H}_s^2}\right)\left\{-v^{(2)}_{||s} - \frac{1}{2}\phi^{(2)}_s - \frac{1}{2}\int_{\eta_s}^{\eta_0} d\eta' \partial_{\eta'}\left[\phi^{(2)}(\eta') + \psi^{(2)}(\eta')\right] + \frac{1}{2}(v_{||s})^2\right.$$

$$\Delta = \ )^2 + (-v_{||s} - \psi^I_s)\left(-\psi^I_s - 2\int_{\eta_s}^{\eta_o} d\eta' \partial_{\eta'}\psi^I(\eta')\right) + \frac{1}{2} v^a_{\perp s} v_{\perp a\, s}$$

$$-2a\, v^a_{\perp s}\partial_a \int_{\eta_s}^{\eta_o} d\eta' \psi^I(\eta') + 4\int_{\eta_s}^{\eta_0} d\eta' \left[\psi^I(\eta')\partial_{\eta'}\psi^I(\eta') + \partial_{\eta'}\psi^I(\eta')\int_{\eta'}^{\eta_o} d\eta'' \partial_{\eta''}\psi^I(\eta'')\right.$$

$$\left. +\psi^I(\eta')\int_{\eta'}^{\eta_o} d\eta'' \partial^2_{\eta''}\psi^I(\eta'') - \gamma_0^{ab}\partial_a\left(\int_{\eta'}^{\eta_o} d\eta''\psi^I(\eta'')\right)\partial_b\left(\int_{\eta'}^{\eta_o} d\eta''\partial_{\eta''}\psi^I(\eta'')\right)\right]$$

$$+2\partial_a(v_{||s} + \psi^I_s)\int_{\eta_s}^{\eta_o} d\eta'\gamma_0^{ab}\partial_b\int_{\eta'}^{\eta_o} d\eta''\psi^I(\eta'')$$

$$\left. +4\int_{\eta_s}^{\eta_o} d\eta'\partial_a\left(\partial_{\eta'}\psi^I(\eta')\right)\int_{\eta_s}^{\eta_o} d\eta''\gamma_0^{ab}\partial_b\int_{\eta''}^{\eta_o} d\eta'''\psi^I(\eta''')\right\}$$

$$+\left[\frac{1}{2}\frac{\mathcal{H}'_s}{\mathcal{H}_s^2} + \frac{3}{2}\left(\frac{\mathcal{H}'_s}{\mathcal{H}_s^2}\right)^2 - \frac{1}{2}\frac{\mathcal{H}''_s}{\mathcal{H}_s^3} + \frac{1}{\mathcal{H}_s r_s}\left(1 + 3\frac{\mathcal{H}'_s}{\mathcal{H}_s^2} + \frac{1}{\mathcal{H}_s r_s}\right)\right]\left[(v_{||s})^2 + (\psi^I_s)^2 + 2\psi^I_s v_{||s}\right.$$

$$\left. +4(v_{||s} + \psi^I_s)\int_{\eta_s}^{\eta_o} d\eta'\partial_{\eta'}\psi^I(\eta') + 4\left(\int_{\eta_s}^{\eta_o} d\eta'\partial_{\eta'}\psi^I(\eta')\right)^2\right] - \psi^{(2)}_s + \frac{1}{2}\phi^{(2)}_s + \frac{1}{2\mathcal{H}_s}\partial_\eta\psi^{(2)}_s$$

$$+\frac{1}{\mathcal{H}_s}\partial_r v^{(2)}_{||s} - \frac{1}{2}\frac{1}{r_s}\int_{\eta_s}^{\eta_o} d\eta'\frac{\eta' - \eta_s}{\eta_o - \eta'}\Delta_2\left[\psi^{(2)} + \phi^{(2)}\right](\eta') + \frac{1}{r_s}\int_{\eta_s}^{\eta_o} d\eta'\left[\psi^{(2)} + \phi^{(2)}\right](\eta')$$

$$+2\left(1 - \frac{1}{\mathcal{H}_s r_s}\right)\left\{-\frac{2}{\mathcal{H}_s}v_{||s}\partial_r v_{||s} - (v_{||s})^2 - v_{\perp a\, s}v^a_{\perp s} + \left[-\frac{1}{\mathcal{H}_s}\partial_\eta\psi^I_s - 2\int_{\eta_s}^{\eta_o} d\eta'\partial_{\eta'}\psi^I(\eta')\right.\right.$$

$$\left. -\frac{2}{r_s}\int_{\eta_s}^{\eta_o} d\eta'\psi^I(\eta') + \frac{2}{r_s}\int_{\eta_s}^{\eta_o} d\eta'\frac{\eta' - \eta_s}{\eta_o - \eta'}\Delta_2\psi^I(\eta')\right] v_{||s} + \left[-2\psi^I_s - 4\int_{\eta_s}^{\eta_o} d\eta'\partial_{\eta'}\psi^I(\eta')\right.$$

$$\left. -2\mathcal{H}_s\int_{\eta_s}^{\eta_o} d\eta'\psi^I(\eta')\right]\frac{1}{\mathcal{H}_s}\partial_r v_{||s} + a v^a_{\perp s}\partial_a\int_{\eta_s}^{\eta_o} d\eta'\psi^I(\eta') + \left[\partial_r\psi^I_s + 2\partial_\eta\psi^I_s\right.$$

$$\left. +2\int_{\eta_s}^{\eta_o} d\eta'\partial^2_{\eta'}\psi^I(\eta')\right]\left(-2\int_{\eta_s}^{\eta_o} d\eta'\psi^I(\eta')\right)$$

$$-\left(-\psi^I_s - 2\int_{\eta_s}^{\eta_o} d\eta'\partial_{\eta'}\psi^I(\eta')\right)\left[\frac{2}{r_s}\int_{\eta_s}^{\eta_o} d\eta'\frac{\eta' - \eta_s}{\eta_o - \eta'}\Delta_2\psi^I(\eta') - \frac{2}{r_s}\int_{\eta_s}^{\eta_o} d\eta'\psi^I(\eta')\right.$$

$$\left.\left. -\frac{1}{\mathcal{H}_s}\partial_\eta\psi^I_s - \psi^I_s\right]\right\} + \frac{3}{2}v_{\perp a\, s}v^a_{\perp s} + \frac{2}{\mathcal{H}_s}a v^a_{\perp s}\partial_a v_{||s} + \left(\frac{5}{2} + \frac{\mathcal{H}'_s}{\mathcal{H}_s^2}\right)(v_{||s})^2$$

$$+\left(5 + 3\frac{\mathcal{H}'_s}{\mathcal{H}_s^2}\right)\frac{1}{\mathcal{H}_s}v_{||s}\partial_r v_{||s} + \frac{1}{\mathcal{H}_s^2}\left[v_{||s}\partial_r^2 v_{||s} + (\partial_r v_{||s})^2\right] + \left[-\frac{1}{\mathcal{H}_s^2}\left(\partial_r^2\psi^I_s + \partial_\eta^2\psi^I_s - \partial_\eta\partial_r\right.\right.$$

$$-\frac{1}{\mathcal{H}_s}\partial_r\psi^I_s - \frac{3}{\mathcal{H}_s}\left(-1 - \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} - \frac{2}{3}\frac{1}{\mathcal{H}_s r_s}\right)\partial_\eta\psi^I_s - \frac{4}{r_s}\left(-1 - \frac{\mathcal{H}'_s}{\mathcal{H}_s^2}\right)\int_{\eta_s}^{\eta_o} d\eta'\psi^I(\eta')$$

$$+\left(-2 - \frac{\mathcal{H}'_s}{\mathcal{H}_s^2}\right)\frac{2}{r_s}\int_{\eta_s}^{\eta_o} d\eta'\frac{\eta' - \eta_s}{\eta_o - \eta'}\Delta_2\psi^I(\eta') - 2\left(-2 - \frac{\mathcal{H}'_s}{\mathcal{H}_s^2}\right)\int_{\eta_s}^{\eta_o} d\eta'\partial_{\eta'}\psi^I(\eta')$$

$$\left. +\frac{4}{\mathcal{H}_s r_s}\psi^I_s - \frac{2}{\mathcal{H}_s r_s^2}\int_{\eta_s}^{\eta_o} d\eta'\Delta_2\psi^I(\eta')\right] v_{||s} + \left[2\left(2 + \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} + \frac{2}{\mathcal{H}_s r_s}\right)\partial_r v_{||s}\right.$$

$$\left. +\frac{2}{\mathcal{H}_s}\partial_r^2 v_{||s}\right]\int_{\eta_s}^{\eta_o} d\eta'\psi^I(\eta') + \left[\frac{2}{\mathcal{H}_s}\left(5 + 3\frac{\mathcal{H}'_s}{\mathcal{H}_s^2}\right)\partial_r v_{||s} + \frac{2}{\mathcal{H}_s^2}\partial_r^2 v_{||s}\right]\int_{\eta_s}^{\eta_o} d\eta'\partial_{\eta'}\psi^I(\eta')$$

$$-\frac{2}{\mathcal{H}_s}\partial_r v_{||s}\frac{1}{r_s}\int_{\eta_s}^{\eta_o} d\eta'\frac{\eta' - \eta_s}{\eta_o - \eta'}\Delta_2\psi^I(\eta') + \frac{2}{\mathcal{H}_s^2}\partial_\eta\psi^I_s\partial_r v_{||s} + \frac{1}{\mathcal{H}_s}\left[\frac{1}{\mathcal{H}_s}\partial_r^2 v_{||s}\right.$$

$$\left. +\left(6 + 3\frac{\mathcal{H}'_s}{\mathcal{H}_s^2}\right)\partial_r v_{||s}\right]\psi^I_s - \frac{2}{\mathcal{H}_s r_s}a v^a_{\perp s}\partial_a\int_{\eta_s}^{\eta_o} d\eta'\psi^I(\eta') + \frac{1}{\mathcal{H}_s}a v^a_{\perp s}\partial_a\psi^I_s$$

$$-\frac{6}{\mathcal{H}_s}\gamma_{0s}^{ab}\partial_a v_{||s}\partial_b\int_{\eta_s}^{\eta_o} d\eta'\psi^I(\eta') + \frac{4}{r_s^2}\left(\int_{\eta_s}^{\eta_o} d\eta'\psi^I(\eta')\right)^2 + \left\{\left[2\left(-2 - \frac{\mathcal{H}'_s}{\mathcal{H}_s^2}\right)\psi^I_s\right.\right.$$

$$\left. +4\left(-2 - \frac{\mathcal{H}'_s}{\mathcal{H}_s^2}\right)\int_{\eta_s}^{\eta_o} d\eta'\partial_{\eta'}\psi^I(\eta') - \frac{2}{\mathcal{H}_s}\partial_\eta\psi^I_s\right]\frac{1}{r_s} + 2\left(-2 - \frac{\mathcal{H}'_s}{\mathcal{H}_s^2}\right)\int_{\eta_s}^{\eta_o} d\eta'\partial^2_{\eta'}\psi^I(\eta')$$

$$\left. +2\left(-2 - \frac{\mathcal{H}'_s}{\mathcal{H}_s^2}\right)\partial_\eta\psi^I_s + \left(-1 - \frac{\mathcal{H}'_s}{\mathcal{H}_s^2}\right)\partial_r\psi^I_s - \frac{1}{\mathcal{H}_s}\partial_\eta\partial_r\psi^I_s\right\}\left(-2\int_{\eta_s}^{\eta_o} d\eta'\psi^I(\eta')\right)$$

$$+\left[\frac{3}{\mathcal{H}_s}\left(-1 - \frac{\mathcal{H}'_s}{\mathcal{H}_s^2} - \frac{2}{3}\frac{1}{\mathcal{H}_s r_s}\right)\partial_\eta\psi^I_s + \frac{1}{\mathcal{H}_s}\partial_r\psi^I_s - \left(2 - \frac{\mathcal{H}'_s}{\mathcal{H}_s^2}\right)\psi^I_s\right.$$

$$\left. +\frac{1}{\mathcal{H}_s^2}\left(\partial_r^2\psi^I_s + \partial_\eta^2\psi^I_s - \partial_\eta\partial_r\psi^I_s\right)\right]\left(-2\int_{\eta_s}^{\eta_o} d\eta'\partial_{\eta'}\psi^I(\eta')\right) + \left[\left(-2 - 2\frac{\mathcal{H}'_s}{\mathcal{H}_s^2}\right)\psi^I_s\right.$$

$$+4\left(-2 - \frac{\mathcal{H}'_s}{\mathcal{H}_s^2}\right)\int_{\eta_s}^{\eta_o} d\eta'\partial_{\eta'}\psi^I(\eta') - \frac{8}{r_s}\int_{\eta_s}^{\eta_o} d\eta'\psi^I(\eta')$$

$$\left. -\frac{2}{\mathcal{H}_s}\partial_\eta\psi^I_s\right]\frac{1}{r_s}\int_{\eta_s}^{\eta_o} d\eta'\frac{\eta' - \eta_s}{\eta_o - \eta'}\Delta_2\psi^I(\eta') + 2\left(\frac{1}{r_s}\int_{\eta_s}^{\eta_o} d\eta'\frac{\eta' - \eta_s}{\eta_o - \eta'}\Delta_2\psi^I(\eta')\right)^2$$

$$+\left[\frac{1}{\mathcal{H}_s\Delta\eta}\left(-\psi^I_s - 2\int_{\eta_s}^{\eta_o} d\eta'\partial_{\eta'}\psi^I(\eta')\right) - \frac{1}{\Delta\eta}\int_{\eta_s}^{\eta_o} d\eta'\psi^I(\eta')\right]\frac{2}{r_s}\int_{\eta_s}^{\eta_o} d\eta'\Delta_2\psi^I(\eta')$$

$$-2\left[\int_{\eta_s}^{\eta_o} d\eta'\frac{1}{(\eta_o - \eta')^2}\psi^I(\eta') + 2\int_{\eta_s}^{\eta_o} d\eta'\frac{1}{(\eta_o - \eta')^2}\int_{\eta'}^{\eta_o} d\eta''\partial_{\eta''}\psi^I(\eta'')\right]\int_{\eta_s}^{\eta_o} d\eta'\Delta_2\psi^I(\eta')$$

$$+\left(-\frac{1}{2} - \frac{\mathcal{H}'_s}{\mathcal{H}_s^2}\right)(\psi^I_s)^2 + \frac{1}{\mathcal{H}_s^2}(\partial_\eta\psi^I_s)^2 + \left[-\frac{1}{\mathcal{H}_s^2}\left(\partial_r^2\psi^I_s + \partial_\eta^2\psi^I_s - \partial_\eta\partial_r\psi^I_s\right)\right.$$

$$\left. +\frac{1}{\mathcal{H}_s}\left(4 + 3\frac{\mathcal{H}'_s}{\mathcal{H}_s^2} + \frac{2}{\mathcal{H}_s r_s}\right)\partial_\eta\psi^I_s - \frac{1}{\mathcal{H}_s}\partial_r\psi^I_s\right]\psi^I_s$$

$$+2\partial_a\left[-\psi^I_s - \frac{1}{\mathcal{H}_s}\left(\partial_\eta\psi^I_s + \partial_r v_{||s}\right)\right]\int_{\eta_s}^{\eta_o} d\eta'\gamma_0^{ab}\partial_b\int_{\eta'}^{\eta_o} d\eta''\psi^I(\eta'')$$

$$-2a v^a_{\perp s}\partial_a\int_{\eta_s}^{\eta_o} d\eta'\psi^I(\eta') + \frac{4}{r_s}\int_{\eta_s}^{\eta_o} d\eta'\left[\psi^I(\eta')\left(-\psi^I(\eta') - 2\int_{\eta'}^{\eta_o} d\eta''\partial_{\eta''}\psi^I(\eta'')\right)\right.$$

$$\left. +\gamma_0^{ab}\partial_a\left(\int_{\eta'}^{\eta_o} d\eta''\psi^I(\eta'')\right)\partial_b\left(\int_{\eta'}^{\eta_o} d\eta''\psi^I(\eta'')\right)\right] + 4\partial_a\psi^I_s\int_{\eta_s}^{\eta_o} d\eta'\gamma_0^{ab}\partial_b\int_{\eta'}^{\eta_o} d\eta''\psi^I(\eta'')$$

$$-\frac{8}{r_s}\left[\int_{\eta_s}^{\eta_o} d\eta'\partial_a\psi^I(\eta')\int_{\eta_s}^{\eta_o} d\eta''\gamma_0^{ab}\partial_b\int_{\eta''}^{\eta_o} d\eta'''\psi^I(\eta''')\right]$$

$$+2\partial_a\left(\int_{\eta_s}^{\eta_o} d\eta'\psi^I(\eta')\right)\left[4\int_{\eta_s}^{\eta_o} d\eta'\frac{1}{(\eta_o - \eta')}\gamma_0^{ab}\int_{\eta'}^{\eta_o} d\eta''\partial_b\psi^I(\eta'') - 3\int_{\eta_s}^{\eta_o} d\eta'\gamma_0^{ab}\partial_b\psi^I(\eta')\right.$$

$$\left. -6\int_{\eta_s}^{\eta_o} d\eta'\gamma_0^{ab}\int_{\eta'}^{\eta_o} d\eta''\partial_b\partial_{\eta''}\psi^I(\eta'')\right]$$

$$+2\partial_a\left(\int_{\eta_s}^{\eta_o} d\eta'\gamma_0^{bc}\partial_c\int_{\eta'}^{\eta_o} d\eta''\psi^I(\eta'')\right)\partial_b\left(\int_{\eta_s}^{\eta_o} d\eta'\gamma_0^{ad}\partial_d\int_{\eta'}^{\eta_o} d\eta''\psi^I(\eta'')\right)$$

$$-4\left(\int_{\eta_s}^{\eta_o} d\eta'\psi^I(\eta')\right)\int_{\eta_s}^{\eta_o} d\eta'\left[-\frac{1}{(\eta_o - \eta')^3}\int_{\eta'}^{\eta_o} d\eta''\Delta_2\psi^I(\eta'') + \frac{1}{(\eta_o - \eta')^2}\left(\frac{1}{2}\Delta_2\psi^I(\eta')\right.\right.$$

$$\left.\left. +\int_{\eta'}^{\eta_o} d\eta''\partial_{\eta''}\left(\Delta_2\psi^I(\eta'')\right)\right)\right] + \frac{2}{(\sin\theta_o)^2}\left[\frac{1}{r_s}\int_{\eta_s}^{\eta_o} d\eta'\frac{\eta' - \eta_s}{\eta_o - \eta'}\partial_{\theta_o}\psi^I(\eta')\right]^2$$

$$+2\partial_a\left\{\frac{1}{\mathcal{H}_s}\left[-\psi^I_s - 2\int_{\eta_s}^{\eta_o} d\eta'\partial_{\eta'}\psi^I(\eta')\right] - \int_{\eta_s}^{\eta_o} d\eta'\psi^I(\eta')\right\}\gamma_{0s}^{ab}\int_{\eta_s}^{\eta_o} d\eta'\partial_b\psi^I(\eta')$$

$$+\frac{4}{r_s}\int_{\eta_s}^{\eta_o} d\eta'\frac{\eta' - \eta_s}{\eta_o - \eta'}\partial_b\left[\Delta_2\left(\psi^I(\eta')\right)\right]\int_{\eta_s}^{\eta_o} d\eta''\gamma_0^{ab}\partial_a\int_{\eta''}^{\eta_o} d\eta'''\psi^I(\eta''')$$

$$-\frac{2}{r_s}\int_{\eta_s}^{\eta_o} d\eta'\frac{\eta' - \eta_s}{\eta_o - \eta'}\Delta_2\left[\psi^I(\eta')\left(-\psi^I(\eta') - 2\int_{\eta'}^{\eta_o} d\eta''\partial_{\eta''}\psi^I(\eta'')\right)\right.$$

$$\left. +\gamma_0^{ab}\partial_a\left(\int_{\eta'}^{\eta_o} d\eta''\psi^I(\eta'')\right)\partial_b\left(\int_{\eta'}^{\eta_o} d\eta''\psi^I(\eta'')\right)\right]$$

$$-2\int_{\eta_s}^{\eta_o} d\eta'\left\{-2\psi^I(\eta')\frac{1}{\eta_o - \eta'}\int_{\eta'}^{\eta_o} d\eta''\frac{\eta'' - \eta'}{\eta_o - \eta''}\Delta_2\partial_{\eta''}\psi^I(\eta'')\right.$$

$$+2\gamma_0^{ab}\partial_b\left(\int_{\eta'}^{\eta_o} d\eta''\psi^I(\eta'')\right)\frac{1}{\eta_o - \eta'}\int_{\eta'}^{\eta_o} d\eta''\frac{\eta'' - \eta'}{\eta_o - \eta''}\partial_a\Delta_2\psi^I(\eta'')$$

$$-\left(-2\psi^I(\eta') - 2\int_{\eta'}^{\eta_o} d\eta''\partial_{\eta''}\psi^I(\eta'')\right)\frac{1}{(\eta_o - \eta')^2}\int_{\eta'}^{\eta_o} d\eta''\Delta_2\psi^I(\eta'')$$

$$-2\partial_a\psi^I(\eta')\int_{\eta'}^{\eta_o} d\eta''\gamma_0^{ab}\partial_b\int_{\eta''}^{\eta_o} d\eta'''\partial_{\eta'''}\psi^I(\eta''')$$

$$+2\partial_a\left[\gamma_0^{db}\partial_b\int_{\eta'}^{\eta_o} d\eta''\psi^I(\eta'')\right]\int_{\eta'}^{\eta_o} d\eta''\partial_d\left[\gamma_0^{ac}\partial_c\int_{\eta''}^{\eta_o} d\eta'''\psi^I(\eta''')\right]$$

$$\left. +2\gamma_0^{ab}\partial_a\left(\psi^I(\eta') + \int_{\eta'}^{\eta_o} d\eta''\partial_{\eta''}\psi^I(\eta'')\right)\partial_b\int_{\eta'}^{\eta_o} d\eta''\psi^I(\eta'')\right\}$$

$$+\left[\left(\frac{2}{\mathcal{H}_s r_s} + \frac{\mathcal{H}'_s}{\mathcal{H}_s^2}\right)\left(v_{||s} + \psi^I_s + 2\int_{\eta_s}^{\eta_o} d\eta'\partial_{\eta'}\psi^I(\eta')\right) - 3v_{||s} + \frac{1}{\mathcal{H}_s}\partial_r v_{||s} - 4\psi^I_s\right.$$

$$\left. -6\int_{\eta_s}^{\eta_o} d\eta'\partial_{\eta'}\psi^I(\eta') + \frac{4}{r_s}\int_{\eta_s}^{\eta_o} d\eta'\psi^I(\eta') - \frac{2}{r_s}\int_{\eta_s}^{\eta_o} d\eta'\frac{\eta' - \eta_s}{\eta_o - \eta'}\Delta_2\psi^I(\eta') + \frac{1}{\mathcal{H}_s}\partial_\eta\psi^I_s\right]\delta^{(1)}_\rho$$

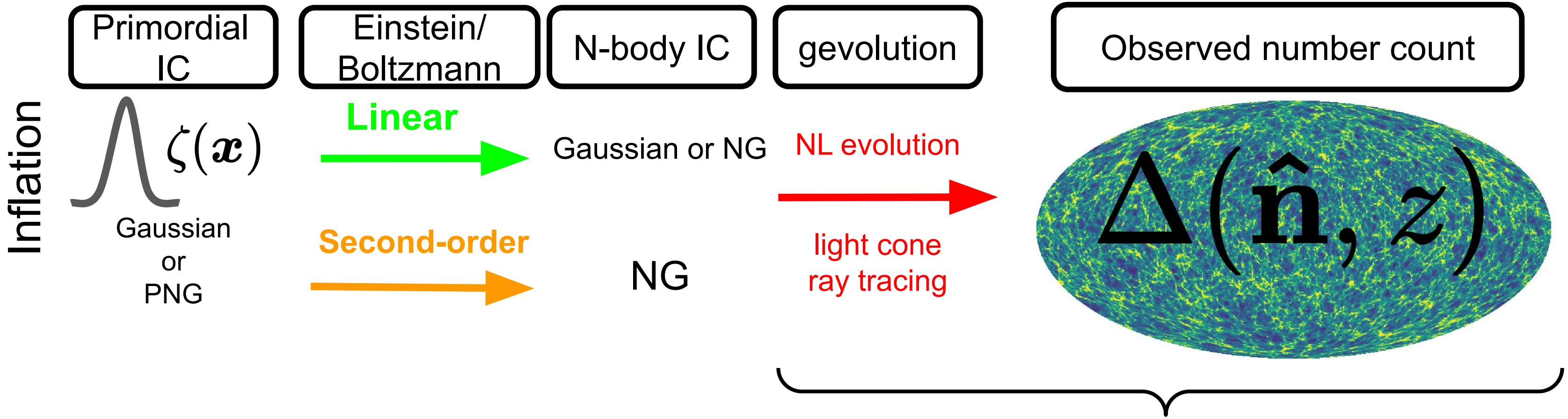
$$+\left[\frac{1}{\bar{\rho}}\partial_\eta\left(\bar{\rho}\,\delta^{(1)}_\rho\right) - \partial_r\delta^{(1)}_\rho\right]\frac{1}{\mathcal{H}_s}\left(-v_{||s} - \psi^I_s - 2\int_{\eta_s}^{\eta_o} d\eta'\partial_{\eta'}\psi^I(\eta')\right)$$

$$+2\partial_r\delta^{(1)}_\rho\int_{\eta_s}^{\eta_o} d\eta'\psi^I(\eta') - 2\partial_a\delta^{(1)}_\rho\int_{\eta_s}^{\eta_o} d\eta'\gamma_0^{ab}\partial_b\int_{\eta'}^{\eta_o} d\eta''\psi^I(\eta'') + \delta^{(2)}_\rho. \qquad (4.4$$

Angular Bispectrum of Galaxy Number Counts for Photometric Surveys

- How?
  - Theory
    - First order
      Yoo 1009.3021, Challinor and Lewis 1105.5292, Bonvin and Durrer 1105.5280, Jeong et al 1107.5427
    - Second order Di Dio 1407.0376, Yoo 1406.4140, Bertacca 1405.4403, Magi 2204.01751
  - Simulation!
    - gevolution GR or Newtonian Adamek 1604.06065
    - relativistic ray tracing

Simulation pipeline



- Second-order weak field GR
- Linear radiation effects
- Nonlinear relativistic ray tracing

Montandon 2212.06799

Simulation pipeline

Primordial IC → Einstein/ Boltzmann → N-body IC → gevolution → Observed number count

Inflation

$\zeta(\boldsymbol{x})$

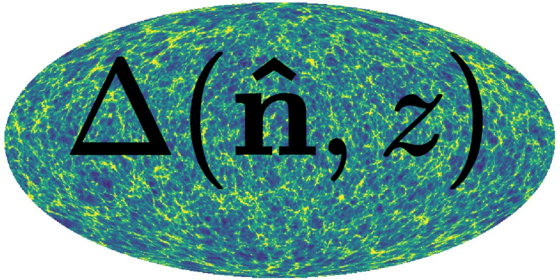
Gaussian or PNG

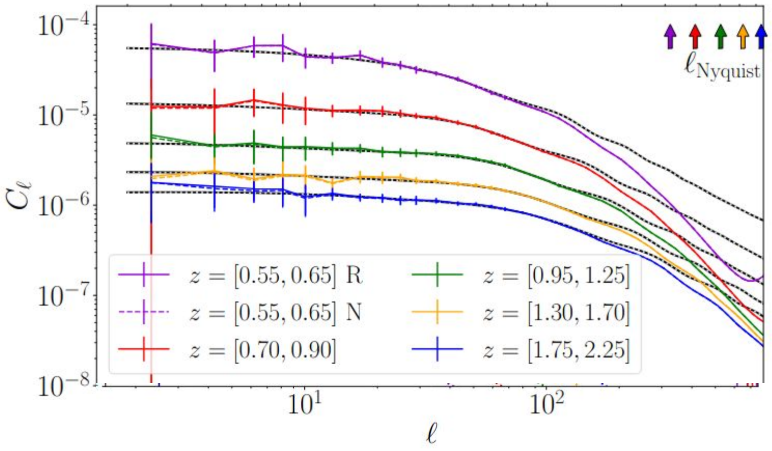
Linear → Gaussian or NG

Second-order → NG

NL evolution

light cone ray tracing

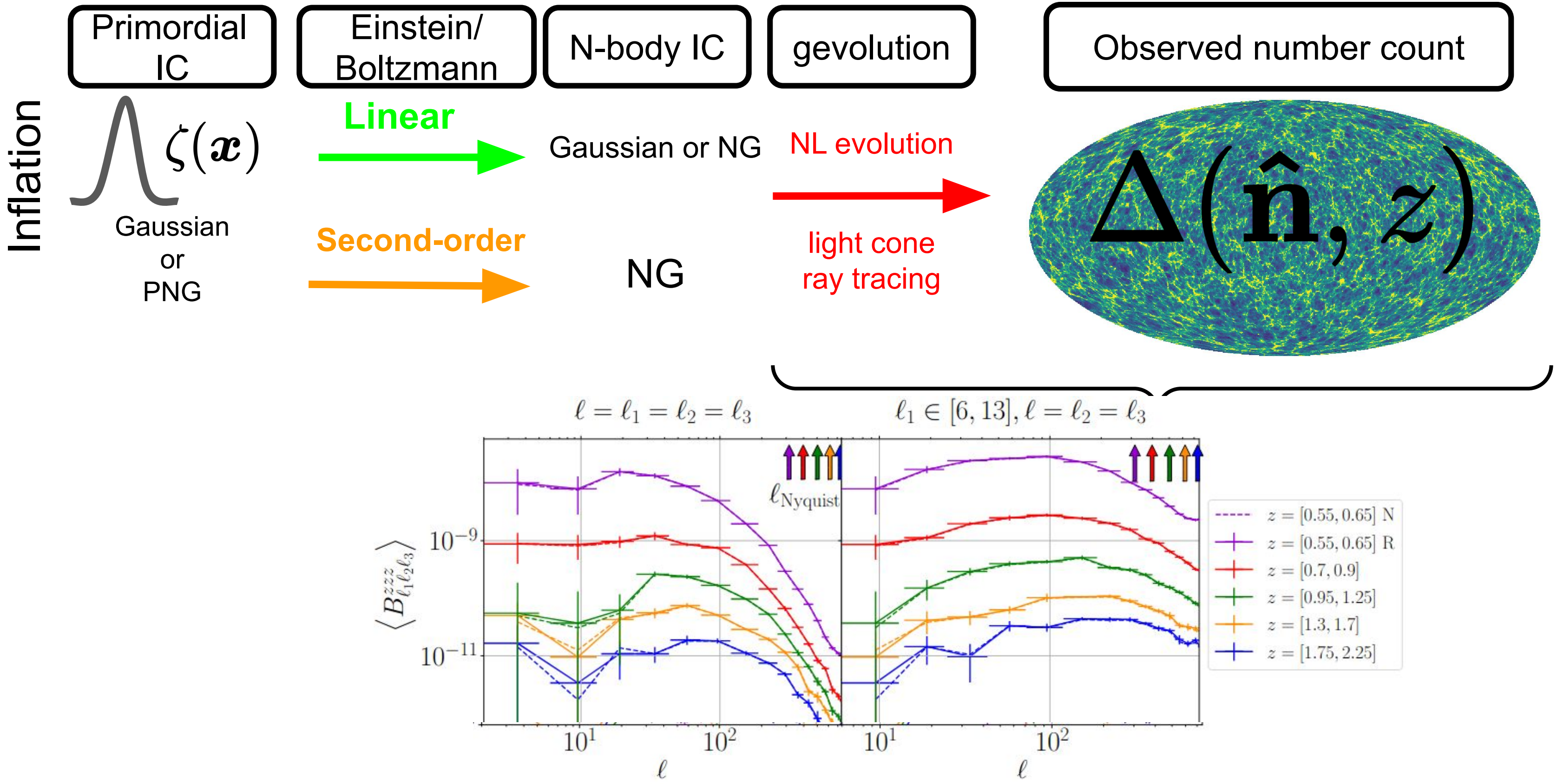




- Second-order weak field GR
- Linear radiation effects
- Nonlinear relativistic ray tracing

Montandon 2212.06799

Simulation pipeline



Angular bispectrum calculation

- How?
  - Theory
    - First order
    - Second order
      - no integrated term
      - projection up to H/k
  - Simulation! Done

Angular bispectrum calculation

- How?
  - Theory
    - First order
    - Second order
      - no integrated term
      - projection up to H/k

$$
\begin{aligned}
\Delta =& \left(-\frac{2}{\mathcal{H}_s r_s}-\frac{\mathcal{H}'_s}{\mathcal{H}_s^2}\right)\Bigg\{-v_{\|s}^{(2)}-\frac{1}{2}\phi_s^{(2)}-\frac{1}{2}\int_{\eta_s}^{\eta_o} d\eta'\partial_{\eta'}\left[\phi^{(2)}(\eta')+\psi^{(2)}(\eta')\right]+\frac{1}{2}(v_{\|s})^2 \\
&+\frac{1}{2}(\psi_s^I)^2+\left(-v_{\|s}-\psi_s^I\right)\left(-\psi_s^I-2\int d\eta'\partial_{\eta'}\psi^I(\eta')\right)+\frac{1}{2}v^a_{\perp s}v_{\perp a\, s} \\
&-2a v^a_{\perp s}\partial_a\int_{\eta_s}^{\eta_o} d\eta'\psi^I(\eta')+4\int_{\eta_s}^{\eta_o} d\eta'\left[\psi^I(\eta')\partial_{\eta'}\psi^I(\eta')+\partial_{\eta'}\psi^I(\eta')\int_{\eta'}^{\eta_o} d\eta''\partial_{\eta''}\psi^I(\eta'')\right. \\
&\left.+\psi^I(\eta')\int_{\eta'}^{\eta_o} d\eta''\partial^2_{\eta''}\psi^I(\eta'')-\gamma_0^{ab}\partial_a\left(\int_{\eta'}^{\eta_o} d\eta''\psi^I(\eta'')\right)\partial_b\left(\int_{\eta'}^{\eta_o} d\eta''\partial_{\eta''}\psi^I(\eta'')\right)\right] \\
&+2\partial_a\left(v_{\|s}+\psi_s^I\right)\int_{\eta_s}^{\eta_o} d\eta'\gamma_0^{ab}\partial_b\int_{\eta'}^{\eta_o} d\eta''\psi^I(\eta'') \\
&+4\int_{\eta_s}^{\eta_o} d\eta'\partial_a\left(\partial_{\eta'}\psi^I(\eta')\right)\int_{\eta_s}^{\eta'} d\eta''\gamma_0^{ab}\partial_b\int_{\eta''}^{\eta_o} d\eta'''\psi^I(\eta''')\Bigg\} \\
&+\left[\frac{1}{2}\frac{\mathcal{H}'_s}{\mathcal{H}_s^2}+\frac{3}{2}\left(\frac{\mathcal{H}'_s}{\mathcal{H}_s^2}\right)^2-\frac{1}{2}\frac{\mathcal{H}''_s}{\mathcal{H}_s^3}+\frac{1}{\mathcal{H}_s r_s}\left(1+3\frac{\mathcal{H}'_s}{\mathcal{H}_s^2}+\frac{1}{\mathcal{H}_s r_s}\right)\right]\Big[(v_{\|s})^2+(\psi_s^I)^2+2\psi_s^I v_{\|s} \\
&+4\left(v_{\|s}+\psi_s^I\right)\int_{\eta_s}^{\eta_o} d\eta'\partial_{\eta'}\psi^I(\eta')+4\left(\int_{\eta_s}^{\eta_o} d\eta'\partial_{\eta'}\psi^I(\eta')\right)^2\Big]-\psi_s^{(2)}-\frac{1}{2}\phi_s^{(2)}+\frac{1}{2\mathcal{H}_s}\psi_s^{(2)\prime} \\
&+\frac{1}{\mathcal{H}_s}\partial_r v_{\|s}^{(2)}-\frac{1}{2}\frac{1}{r_s}\int_{\eta_s}^{\eta_o} d\eta'\frac{\eta'-\eta_s}{\eta_o-\eta'}\Delta_2\left[\psi^{(2)}+\phi^{(2)}\right](\eta')+\frac{1}{r_s}\int_{\eta_s}^{\eta_o} d\eta'\left[\psi^{(2)}+\phi^{(2)}\right](\eta') \\
&+2\left(1-\frac{1}{\mathcal{H}_s r_s}\right)\Bigg\{-\frac{2}{\mathcal{H}_s}v_{\|s}\partial_r v_{\|s}-(v_{\|s})^2-v_{\perp a\, s}v^a_{\perp s}+\Big[-\frac{1}{\mathcal{H}_s}\partial_\eta\psi_s^I-2\int_{\eta_s}^{\eta_o} d\eta'\partial_{\eta'}\psi^I(\eta') \\
&-\frac{2}{r_s}\int_{\eta_s}^{\eta_o} d\eta'\psi^I(\eta')+\frac{2}{r_s}\int_{\eta_s}^{\eta_o} d\eta'\frac{\eta'-\eta_s}{\eta_o-\eta'}\Delta_2\psi^I(\eta')\Big]v_{\|s}+\Big[-2\psi_s^I-4\int_{\eta_s}^{\eta_o} d\eta'\partial_{\eta'}\psi^I(\eta') \\
&-\frac{2}{\mathcal{H}_s}\int_{\eta_s}^{\eta_o} d\eta'\psi^I(\eta')\Big]\frac{1}{\mathcal{H}_s}\partial_r v_{\|s}+a v^a_{\perp s}\partial_a\int_{\eta_s}^{\eta_o} d\eta'\psi^I(\eta')+\left[\partial_r\psi_s^I+2\partial_\eta\psi_s^I\right.
\end{aligned}
$$

Angular bispectrum calculation

- How?
  - Theory
    - First order
    - Second order
      - no integrated term
      - projection up to H/k

Newtonian

GR/Radiation dynamics up to (H/k)^4

Projection effect up to H/k

$$\begin{aligned}\Delta_2 = {}& \delta_2 - \mathcal{H}^{-1}\partial_r^2 v_2 \\ &+ \mathcal{H}^{-2}\left[\left(\partial_r^2 v_1\right)^2 + \partial_r v_1 \partial_r^3 v_1\right] - \mathcal{H}^{-1}\left[\partial_r v_1 \partial_r \delta_1 + \partial_r^2 v_1 \delta_1\right] \\ &- \mathcal{R}\partial_r v_2 + \mathcal{H}^{-1}\left(1 + 3\frac{\mathcal{H}}{\mathcal{H}^2} + \frac{4}{\mathcal{H}r}\right)\partial_r v_1 \partial_r^2 v_1 - \mathcal{R}\partial_r v_1 \delta_1 + \partial_r v_1 \dot{\delta}_1 + 2\mathcal{H}^{-1} v_1^\alpha \partial_\alpha \partial_r v_1 \\ &- \mathcal{H}^{-2}\psi_1 \partial_r^3 v_1 + \mathcal{H}^{-1}\psi_1 \partial_r \delta_1 + \mathcal{H}^{-2}\partial_r v_1 \partial_r^2 \psi_1 \,, \qquad (2.12)\end{aligned}$$

Angular bispectrum calculation

$$b_{\ell_1\ell_2\ell_3}^{z_1z_2z_3} = \frac{8}{\pi^3}\int dz_1' dz_2' dz_3' W(z_1,z_1')W(z_2,z_2')W(z_3,z_3')\int dk_1 dk_2 dk_3 d\chi (k_1k_2k_3\chi)^2 B^{z_1'z_2'z_3'}(k_1,k_2,k_3)$$
$$j_{\ell_1}^{(p)}(k_1 r(z_1'))j_{\ell_2}^{(q)}(k_2 r(z_2'))j_{\ell_3}^{(m)}(k_3 r(z_3'))j_{\ell_1}(k_1\chi)j_{\ell_2}(k_2\chi)j_{\ell_3}(k_3\chi)$$

Angular bispectrum calculation

$$b_{\ell_1\ell_2\ell_3}^{z_1z_2z_3} = \frac{8}{\pi^3}\int dz_1' dz_2' dz_3' W(z_1,z_1')W(z_2,z_2')W(z_3,z_3')\int dk_1 dk_2 dk_3 d\chi (k_1k_2k_3\chi)^2 B^{z_1'z_2'z_3'}(k_1,k_2,k_3)$$
$$j_{\ell_1}^{(p)}(k_1 r(z_1'))j_{\ell_2}^{(q)}(k_2 r(z_2'))j_{\ell_3}^{(m)}(k_3 r(z_3'))j_{\ell_1}(k_1\chi)j_{\ell_2}(k_2\chi)j_{\ell_3}(k_3\chi)$$
$$= \frac{8}{\pi^2}\mathcal{N}^4\sum_{mn}\int d\chi\chi^2$$
$$\left[\int dr_2' dk_2 k_2^{4+n}\left(W(r_2,r_2')D_{r_2'}\left(k_2^2+3f_{r_2'}\mathcal{H}_{r_2'}^2\right)+\frac{d^2}{dr_2'^2}\left(W(r_2,r_2')D_{r_2'}f_{r_2'}\right)\right)P_{\phi_0}(k_2)j_{\ell_2}(k_2r_2')j_{\ell_2}(k_2\chi)\right] \longrightarrow C_{\ell_2}^{(n)}(\chi)$$
$$\left[\int dr_3' dk_3 k_3^{4+m}\left(W(r_3,r_3')D_{r_3'}\left(k_3^2+3f_{r_3'}\mathcal{H}_{r_3'}^2\right)+\frac{d^2}{dr_3'^2}\left(W(r_3,r_3')D_{r_3'}f_{r_3'}\right)\right)P_{\phi_0}(k_3)j_{\ell_3}(k_3r_3')j_{\ell_3}(k_3\chi)\right] \longrightarrow C_{\ell_3}^{(m)}(\chi)$$
$$\int \frac{dr_1'}{r_1'^2}D^2(r_1')W(r_1,r_1')\left[\delta_D(\chi-r_1')+\mathcal{D}_{\ell_1}[\delta_D(r_1'-\chi)]+\mathcal{D}_{\ell_1}^2[\delta_D(r_1'-\chi)]\right]$$

- Generalised Power spectra $C_\ell^{(n)}(\chi) = -\frac{2}{\pi}D(r)\mathcal{N}^2\int dr'\,dk\,F(r')k^{4+n}P_{\phi_0}(k)j_\ell(kr)j_\ell(k\chi)$

Angular bispectrum calculation

$$b_{\ell_1\ell_2\ell_3}^{z_1z_2z_3} = \frac{8}{\pi^3}\int dz_1'dz_2'dz_3' W(z_1,z_1')W(z_2,z_2')W(z_3,z_3')\int dk_1dk_2dk_3d\chi(k_1k_2k_3\chi)^2 B^{z_1'z_2'z_3'}(k_1,k_2,k_3)$$
$$j_{\ell_1}^{(p)}(k_1r(z_1'))j_{\ell_2}^{(q)}(k_2r(z_2'))j_{\ell_3}^{(m)}(k_3r(z_3'))j_{\ell_1}(k_1\chi)j_{\ell_2}(k_2\chi)j_{\ell_3}(k_3\chi)$$
$$= \frac{8}{\pi^2}\mathcal{N}^4\sum_{mn}\int d\chi\chi^2$$
$$\left[\int dr_2'dk_2k_2^{4+n}\left(W(r_2,r_2')D_{r_2'}\left(k_2^2+3f_{r_2'}\mathcal{H}_{r_2'}^2\right)+\frac{d^2}{dr_2'^2}\left(W(r_2,r_2')D_{r_2'}f_{r_2'}\right)\right)P_{\phi_0}(k_2)j_{\ell_2}(k_2r_2')j_{\ell_2}(k_2\chi)\right] \longrightarrow C_{\ell_2}^{(n)}(\chi)$$
$$\left[\int dr_3'dk_3k_3^{4+m}\left(W(r_3,r_3')D_{r_3'}\left(k_3^2+3f_{r_3'}\mathcal{H}_{r_3'}^2\right)+\frac{d^2}{dr_3'^2}\left(W(r_3,r_3')D_{r_3'}f_{r_3'}\right)\right)P_{\phi_0}(k_3)j_{\ell_3}(k_3r_3')j_{\ell_3}(k_3\chi)\right] \longrightarrow C_{\ell_3}^{(m)}(\chi)$$
$$\int\frac{dr_1'}{r_1'^2}D^2(r_1')W(r_1,r_1')\left[\delta_D(\chi-r_1')+\mathcal{D}_{\ell_1}[\delta_D(r_1'-\chi)]+\mathcal{D}_{\ell_1}^2[\delta_D(r_1'-\chi)]\right]$$

- Generalised Power spectra $C_\ell^{(n)}(\chi) = -\frac{2}{\pi}D(r)\mathcal{N}^2\int dr'dk\, F(r')k^{4+n}P_{\phi_0}(k)j_\ell(kr)j_\ell(k\chi)$
- FFTLog: $P_{\phi_0}(k) = \sum c_p k^{b+i\eta_p}$

Assassi 1705.05022, Simonovic 1708.08130

$$\Rightarrow \int dkP_{\phi_0}(k)j_\ell(kr)j_\ell(k\chi) = \frac{\pi}{2r^2}\sum c_p^{\phi_0}I_\ell(\nu_p,r,\chi)$$

Angular bispectrum calculation

$$b_{\ell_1\ell_2\ell_3}^{z_1z_2z_3} = \frac{8}{\pi^3}\int dz_1' dz_2' dz_3' W(z_1,z_1')W(z_2,z_2')W(z_3,z_3')\int dk_1 dk_2 dk_3 d\chi (k_1k_2k_3\chi)^2 B^{z_1'z_2'z_3'}(k_1,k_2,k_3)$$
$$j_{\ell_1}^{(p)}(k_1 r(z_1'))j_{\ell_2}^{(q)}(k_2 r(z_2'))j_{\ell_3}^{(m)}(k_3 r(z_3'))j_{\ell_1}(k_1\chi)j_{\ell_2}(k_2\chi)j_{\ell_3}(k_3\chi)$$
$$= \frac{8}{\pi^2}\mathcal{N}^4\sum_{mn}\int d\chi\chi^2$$
$$\left[\int dr_2' dk_2 k_2^{4+n}\left(W(r_2,r_2')D_{r_2'}\left(k_2^2+3f_{r_2'}\mathcal{H}_{r_2'}^2\right)+\frac{d^2}{dr_2'^2}\left(W(r_2,r_2')D_{r_2'}f_{r_2'}\right)\right)P_{\phi_0}(k_2)j_{\ell_2}(k_2r_2')j_{\ell_2}(k_2\chi)\right] \longrightarrow C_{\ell_2}^{(n)}(\chi)$$
$$\left[\int dr_3' dk_3 k_3^{4+m}\left(W(r_3,r_3')D_{r_3'}\left(k_3^2+3f_{r_3'}\mathcal{H}_{r_3'}^2\right)+\frac{d^2}{dr_3'^2}\left(W(r_3,r_3')D_{r_3'}f_{r_3'}\right)\right)P_{\phi_0}(k_3)j_{\ell_3}(k_3r_3')j_{\ell_3}(k_3\chi)\right] \longrightarrow C_{\ell_3}^{(m)}(\chi)$$
$$\int \frac{dr_1'}{r_1'^2}D^2(r_1')W(r_1,r_1')\left[\delta_D(\chi-r_1')+\mathcal{D}_{\ell_1}[\delta_D(r_1'-\chi)]+\mathcal{D}_{\ell_1}^2[\delta_D(r_1'-\chi)]\right]$$

- Generalised Power spectra $C_\ell^{(n)}(\chi) = -\frac{2}{\pi}D(r)\mathcal{N}^2\int dr'\, dk\; F(r')k^{4+n}P_{\phi_0}(k)j_\ell(kr)j_\ell(k\chi)$
- FFTLog:
  Assassi 1705.05022, Simonovic 1708.08130
  $$P_{\phi_0}(k) = \sum c_p k^{b+i\eta_p}$$
  $$\Rightarrow \int dk P_{\phi_0}(k)j_\ell(kr)j_\ell(k\chi) = \frac{\pi}{2r^2}\sum c_p^{\phi_0} I_\ell(\nu_p,r,\chi)$$
- Integration by part:
  Assassi 1705.05022, Simonovic 1708.08130
  $$\int dr\; F(r)\mathcal{D}_\ell^n[\delta_D(r-\chi)] = \mathcal{D}_\ell^n[F(\chi)]$$

Angular bispectrum calculation

- Generalised Power spectra $C_\ell^{(n)}(\chi) = -\frac{2}{\pi} D(r) \mathcal{N}^2 \int dr'\, dk\, F(r') k^{4+n} P_{\phi_0}(k) j_\ell(kr) j_\ell(k\chi)$
- FFTLog:

Assassi 1705.05022, Simonovic 1708.08130

$$P_{\phi_0}(k) = \sum c_p k^{b+i\eta_p}$$

$$\Rightarrow \int dk P_{\phi_0}(k) j_\ell(kr) j_\ell(k\chi) = \frac{\pi}{2r^2} \sum c_p^{\phi_0} I_\ell(\nu_p, r, \chi)$$

- Integration by part:

Assassi 1705.05022, Simonovic 1708.08130

$$\int dr\, F(r) \mathcal{D}_\ell^n [\delta_D(r-\chi)] = \mathcal{D}_\ell^n [F(\chi)]$$

$$b_{\ell_1\ell_2\ell_3}^{\delta_2} = 2 \sum_{mn} \int d\chi C_{\ell_2}^{(n,0)}(\chi) C_{\ell_3}^{(m,0)}(\chi) \left(f_{nm}^{(0)}(\chi) D_\chi^2 \tilde{W}_\chi + \mathcal{D}_{\ell_1} \left[f_{nm}^{(2)}(\chi) D_\chi^2 \tilde{W}_\chi \right] + \mathcal{D}_{\ell_1}^2 \left[f_{nm}^{(4)}(\chi) D_\chi^2 \tilde{W}_\chi \right] \right) + 2\times \circlearrowleft,$$

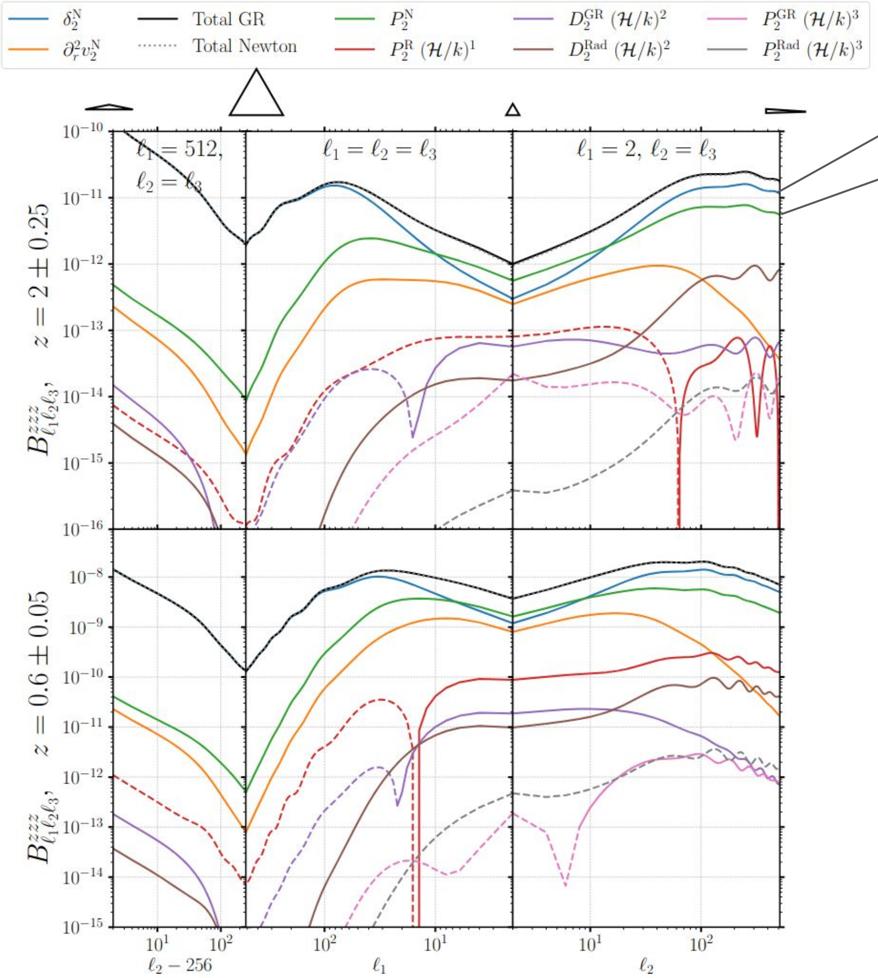
$$b_{\ell_1\ell_2\ell_3}^{XY} = \int dr_1 \tilde{W}_{r_1} C_{\ell_2}^X(r_1) C_{\ell_3}^Y(r_1) + 5\times \circlearrowleft,$$

$$b_{\ell_1\ell_2\ell_3}^{\partial_r^2 v_2} = -2 \sum_{mn} \int d\chi\, C_{\ell_2}^{(n,0)}(\chi) C_{\ell_3}^{(m,0)}(\chi) \left(\chi^2 \int \frac{dr_1}{r_1^2} \frac{d^2}{dr_1^2} \left[D_{r_1}^2 \tilde{W}_{r_1} f_{nm}^{(0)}(r_1) \right] A_{\ell_1}(r_1, \chi) + \frac{d^2}{d\chi^2} \left[f_{nm}^{(2)}(\chi) D_\chi^2 \tilde{W}_\chi \right] + \mathcal{D}_{\ell_1} \left[\frac{d^2}{d\chi^2} \left[f_{nm}^{(4)}(\chi) D_\chi^2 \tilde{W}_\chi \right] \right] \right) + 2\times \circlearrowleft$$

ang_bispec v0.1: Python code for galaxy number count angular bispectrum https://github.com/TomaMTD/ang_bispec

- **Complete theoretical model**
  Based on first- and second-order number count expressions from the literature, covering:
  1. Newtonian terms (density, RSD, quadratic terms)
  2. Non-integrated projection effects
  3. Radiative and dynamical GR effects
- **Efficient hypergeometric evaluation**
  Translated the Mathematica expressions from Assassi et al. (2017) into numba-accelerated Python for fast computation of the ${}_2F_1$ hypergeometric function.
- **Pipeline overview**
  1. **Linear cosmology module**
     - Solves growth functions **D(z), f(z), v(z), w(z)**
     - Interfaces with **CLASS** to extract potential transfer functions and power spectra
  2. **FFTLog module**
     - Computes Hankel transforms of the potential power spectrum and transfer function
  3. **Generalized power spectra (C_ℓ(χ))**
     - Precomputes 7 spectra across ℓ and χ values—this is the main bottleneck
  4. **Precomputation of radial integrals**
     - Integrals over (r_1) for all (f_{n,m}) coefficients
  5. **Main bispectrum loop**
     - Computes contributions for all (ℓ_1, ℓ_2, ℓ_3) triplets, including:
       - Density, RSD, projection, quadratic, and cross terms

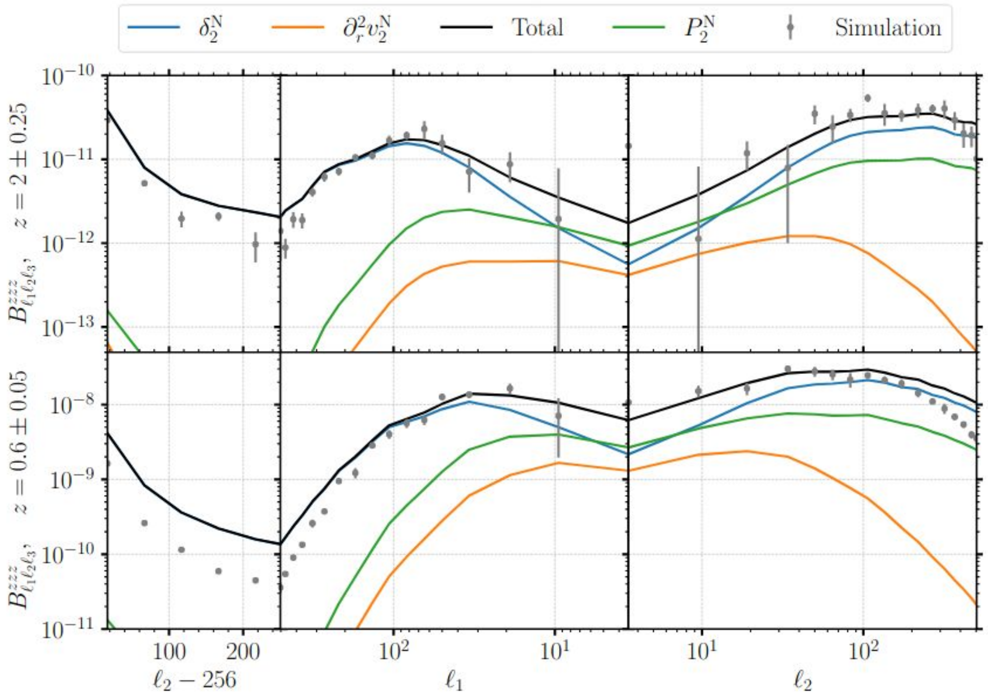
Theoretical results



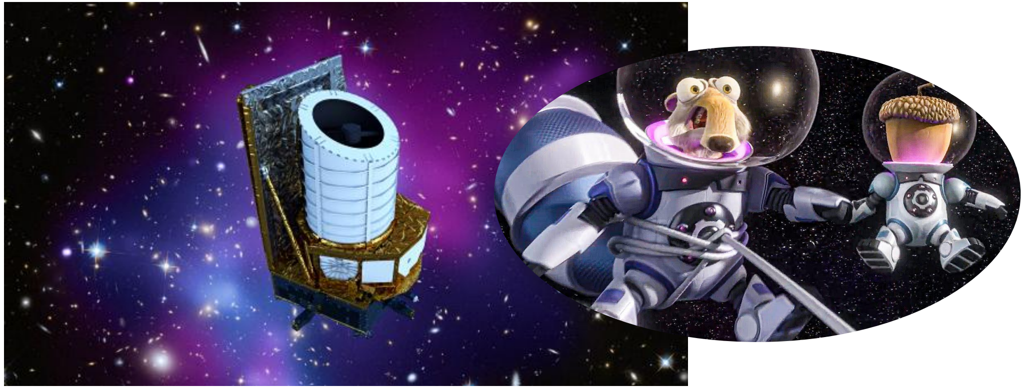
Partial computation in Assassi 1705.05022

Inefficient computation in di Dio et al 1510.04202

Theory vs. simulation



Apply the formalism to Euclid (on going work)



Missing ingredients:

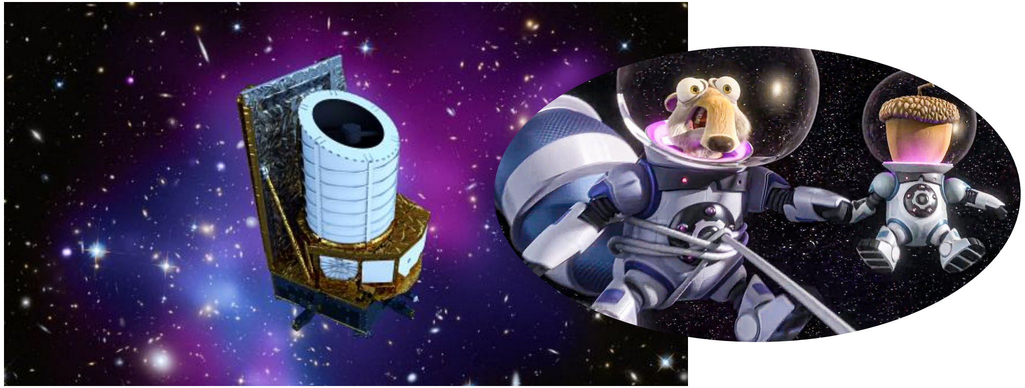
- Bias
- Lensing
- Photometric Window Function
- Forget about relativistic effects

- Linear order

$$\Delta_1 = b_1\delta_1 - \mathcal{H}^{-1}\partial_r^2 v_1 - (2-5s)\kappa_1 \,.$$

Bias

Apply the formalism to Euclid (on going work)



Missing ingredients:

- Bias
- Lensing
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- Forget about relativistic effects

- Linear order

Lensing

$$\kappa_1 = \frac{1}{2}\int dz W(z)\Delta_\Omega \int_0^{r(z)} dr \frac{r(z)-r}{r(z)r}(\phi_1+\psi_1)$$

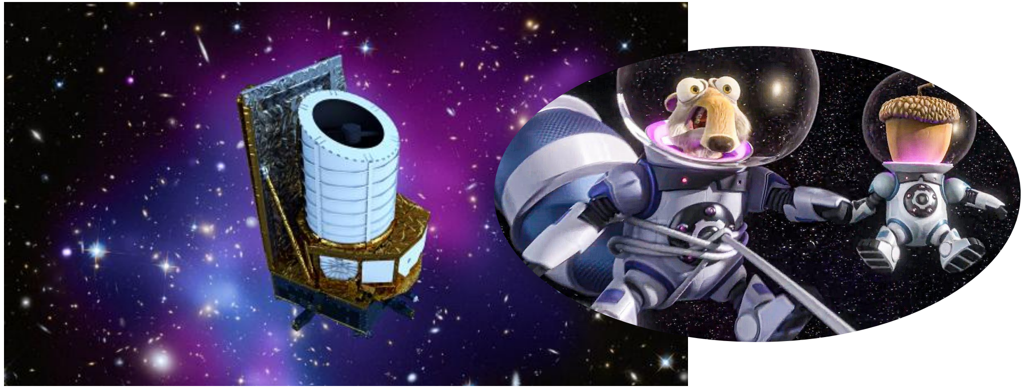
$$= \int dr \frac{D(r)}{a(r)} \hat{W}_\phi(r)\Delta_\Omega \phi_0\,,$$

$$\Delta_1 = b_1\delta_1 - \mathcal{H}^{-1}\partial_r^2 v_1 - (2-5s)\kappa_1\,.$$

Bias

$$\hat{W}_\phi = \int_r^\infty dr\; W(r)\frac{r(z)-r}{r(z)r}$$

Apply the formalism to Euclid (on going work)



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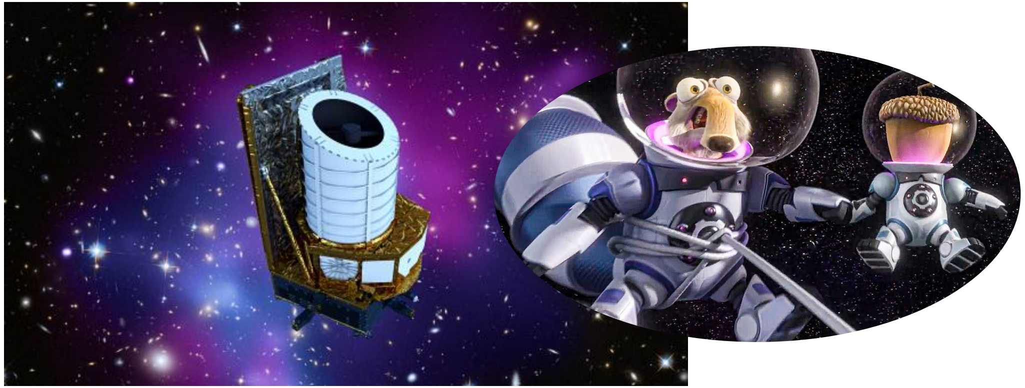
$$\hat{W}_\phi = \int_r^\infty dr\; W(r)\frac{r(z)-r}{r(z)r}$$

$$\Delta_1 = b_1\delta_1 - \mathcal{H}^{-1}\partial_r^2 v_1 - (2-5s)\kappa_1\,.$$

Bias

$$C_\ell(\chi) = \int dr dk k^4 \left[b_1(r)\hat{W}_r D_r k^2 + \frac{d^2}{dr^2}\left(\hat{W}_r D_r f_r\right) + \frac{D_r \hat{W}_\phi(r)(2-5s_r)\ell(\ell+1)}{a(r)}\right] P_{\phi_0}(k)\, j_\ell(kr) j_\ell(k\chi)$$

Apply the formalism to Euclid (on going work)



Missing ingredients:

- Bias
- Lensing
- Photometric Window Function
- Forget about relativistic effects

- Second order

Bias

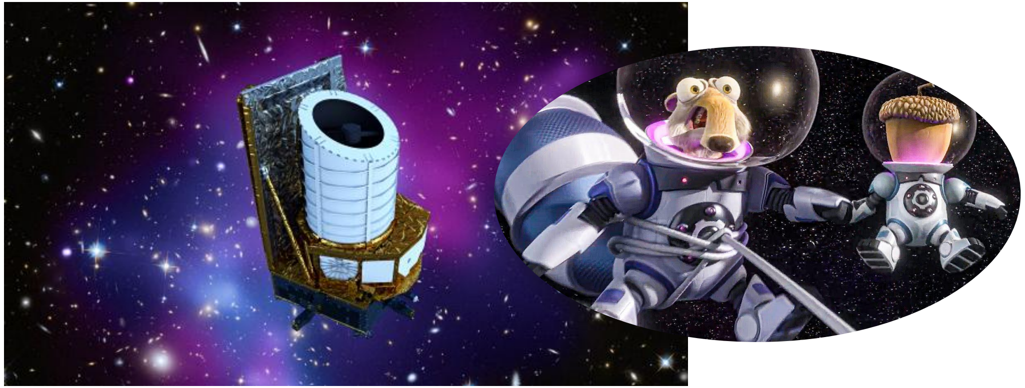
$$\Delta_2^{\rm N} = b_1\delta_2 + \frac{b_2}{2}\delta_1^2 + b_s\left(\mathcal{N}^2 D^2(\partial_i\partial_j\phi_0)^2 + \frac{1}{9}\delta_1^2 - \frac{2}{3}\mathcal{N} D\delta_1\Delta\phi_0\right) - \mathcal{H}^{-1}\partial_r^2 v_2$$

$$+ \mathcal{H}^{-2}\left[\left(\partial_r^2 v_1\right)^2 + \partial_r v_1\partial_r^3 v_1\right] - \mathcal{H}^{-1}\left[\partial_r v_1\partial_r\delta_1 + \partial_r^2 v_1\delta_1\right]$$

Apply the formalism to Euclid (on going work)

Missing ingredients:

- Bias
- Lensing
- Photometric Window Function
- Forget about relativistic effects



- Second order

Bias

$$\Delta_2^{\rm N} = b_1\delta_2 + \tfrac{b_2}{2}\delta_1^2 + b_s\left(\mathcal{N}^2 D^2(\partial_i\partial_j\phi_0)^2 + \tfrac{1}{9}\delta_1^2 - \tfrac{2}{3}\mathcal{N}D\delta_1\Delta\phi_0\right) - \mathcal{H}^{-1}\partial_r^2 v_2$$
$$+\mathcal{H}^{-2}\left[\left(\partial_r^2 v_1\right)^2 + \partial_r v_1\partial_r^3 v_1\right] - \mathcal{H}^{-1}\left[\partial_r v_1\partial_r\delta_1 + \partial_r^2 v_1\delta_1\right]$$

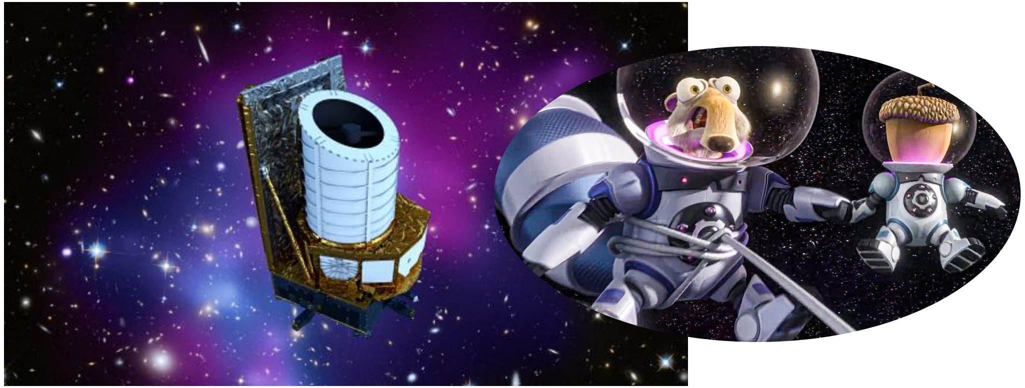
Lensing

$$\Delta_2^{\rm L} = -2\left(1-\frac{5}{2}s\right)\left(\kappa_2 + \nabla_b\kappa_1\nabla^b\phi_1\right) - 2\delta_1\kappa_1 + 5(\delta s)^{(1)}\kappa_1 + \nabla_a\delta_1\nabla^a\phi_1$$
$$+\mathcal{H}^{-1}\left[-2\left(1-\frac{5}{2}s\right)\partial_r^2 v_1\kappa_1 + \nabla_a\partial_r^2 v_1\nabla^a\phi_1\right] + 2\left(1-5s+\frac{25}{4}s^2-\frac{5}{2}t\right)\kappa_1^2$$

Apply the formalism to Euclid (on going work)

Missing ingredients:

- Bias
- Lensing
- Photometric Window Function
- Forget about relativistic effects



- Second order

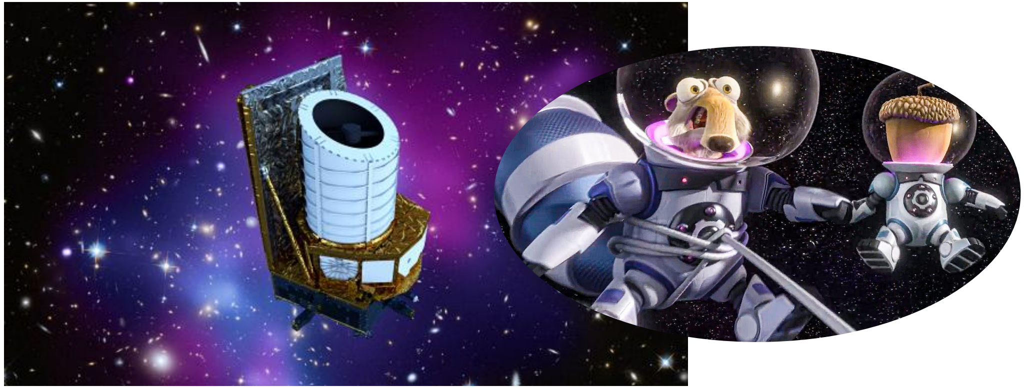
Bias

$$\Delta_2^{\rm N} = b_1\delta_2 + \frac{b_2}{2}\delta_1^2 + b_s\left(\mathcal{N}^2 D^2(\partial_i\partial_j\phi_0)^2 + \frac{1}{9}\delta_1^2 - \frac{2}{3}\mathcal{N}D\delta_1\Delta\phi_0\right) - \mathcal{H}^{-1}\partial_r^2 v_2$$
$$+\mathcal{H}^{-2}\left[\left(\partial_r^2 v_1\right)^2 + \partial_r v_1\partial_r^3 v_1\right] - \mathcal{H}^{-1}\left[\partial_r v_1\partial_r\delta_1 + \partial_r^2 v_1\delta_1\right]$$

Lensing

$$\Delta_2^{\rm L} = -2\left(1-\frac{5}{2}s\right)\left(\kappa_2 + \nabla_b\kappa_1\nabla^b\phi_1\right) - 2\delta_1\kappa_1 + 5(\delta s)^{(1)}\kappa_1 + \nabla_a\delta_1\nabla^a\phi_1$$
$$+\mathcal{H}^{-1}\left[-2\left(1-\frac{5}{2}s\right)\partial_r^2 v_1\kappa_1 + \nabla_a\partial_r^2 v_1\nabla^a\phi_1\right] + 2\left(1-5s+\frac{25}{4}s^2-\frac{5}{2}t\right)\kappa_1^2$$

Apply the formalism to Euclid (on going work)



Missing ingredients:

- Bias
- Lensing
- Photometric Window Function
- Forget about relativistic effects

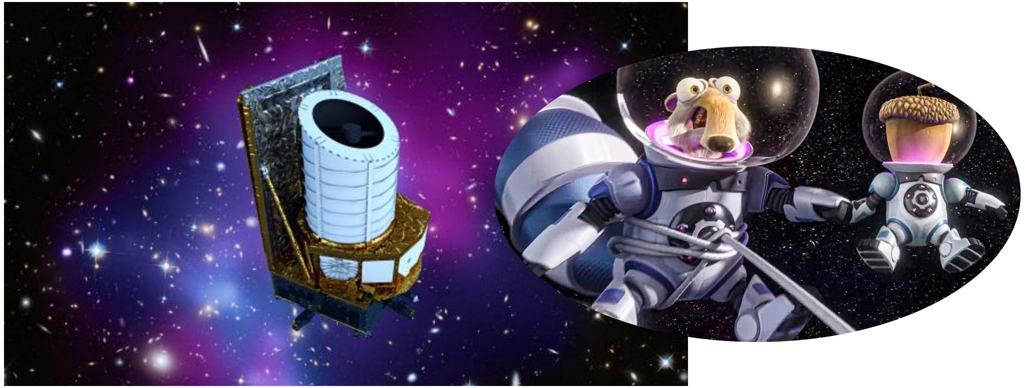
Pure second-order lensing term

$$b^{\kappa_2}_{\ell_1\ell_3\ell_3} = -2\ell_1(\ell_1+1)\sum_{mn}\int d\chi C^{(n)}_{\ell_2}(\chi)C^{(m)}_{\ell_3}(\chi)\Big[\chi^2\int\frac{dr_1}{r_1^2}\hat{W}_\phi(r_1)D_1^2(r_1)f^{(0)}_{nm}(r_1)A_{\ell_1}(r_1,\chi)$$
$$+\hat{W}_\phi(\chi)D_1^2(\chi)f^{(2)}_{nm}(\chi)+\mathcal{D}_{\ell_1}[\hat{W}_\phi(\chi)D_1^2(\chi)f^{(2)}_{nm}(\chi)]\Big]$$

other lensing terms are quadratic:

$$b^{XY}_{\ell_1\ell_2\ell_3} = \int dr_1\tilde{W}_{r_1}C^X_{\ell_2}(r_1)C^Y_{\ell_3}(r_1) + 5\times \circlearrowleft,$$

Apply the formalism to Euclid (on going work)



Missing ingredients:

- Bias
- Lensing
- Photometric Window Function
- Forget about relativistic effects

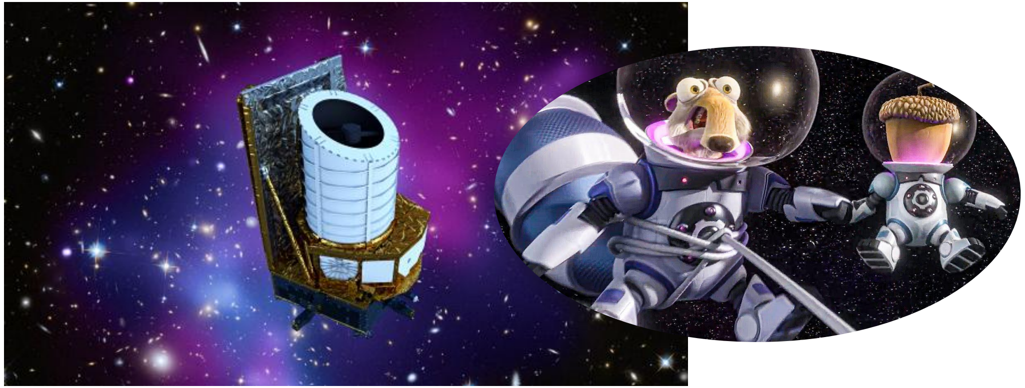
Photometric Window Function (Euclid Collaboration: Blanchard et al. (2020))

$$n(z) \propto \left(\frac{z}{z_0}\right)^2 \exp\left[-\left(\frac{z}{z_0}\right)^{3/2}\right], \qquad z_i = \{0.0010, 0.42, 0.56, 0.68, 0.79, 0.90, 1.02, 1.15, 1.32, 1.58, 2.50\},$$

Apply the formalism to Euclid (on going work)

Missing ingredients:

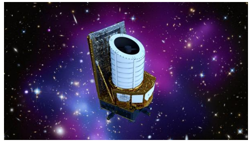
- Bias
- Lensing
- Photometric Window Function
- Forget about relativistic effects

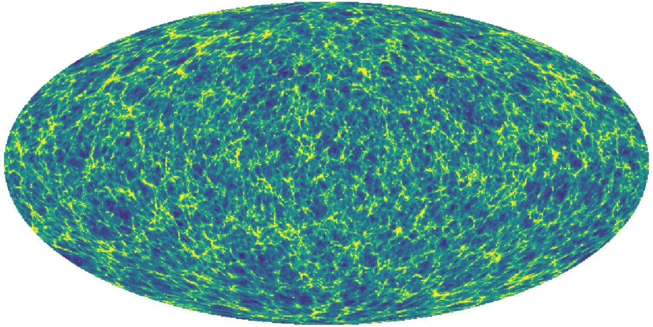


Soon: ang_bispec v1.0

Design for efficient and parallelize computation of the angular bispectrum for Euclid-like photometric surveys including bias and lensing terms.

Conclusion





- Why?
  - LCDM predicts an
    “Intrinsic Bispectrum due to nonlinearities ”
  - it is annoying for inflation
  - Photometry

- How?
  - Theory
    - First order
    - Second order
  - Simulation
    - N-body/ray tracing
    - Binned bispectrum estimator
- Results
  - Newtonian part agrees well with simulation!
  - Relativistic effects are hard
- Euclid (ongoing)
  - Including bias
  - Lensing
  - Photometric window

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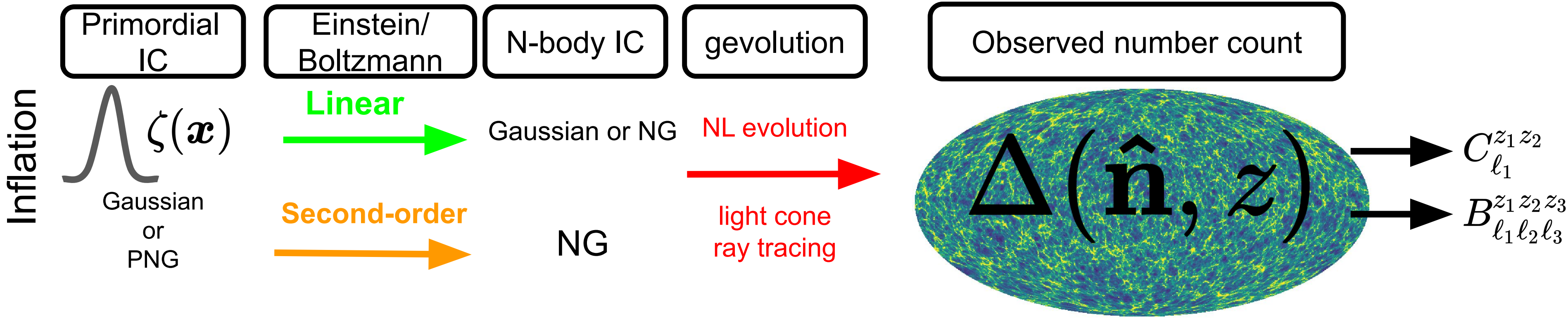
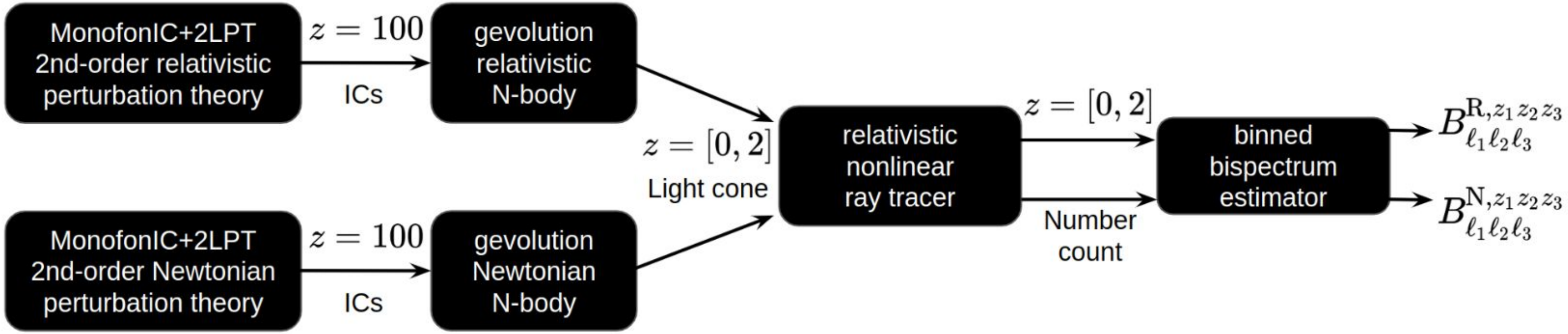
Voila!

Simulation pipeline

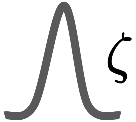




Simulation pipeline

Primordial IC → Einstein/ Boltzmann → N-body IC → gevolution → Observed number count

Inflation



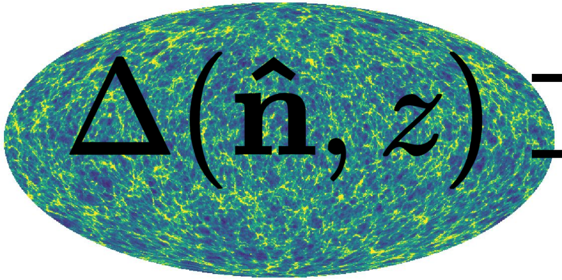
$\zeta(\boldsymbol{x})$

Gaussian or PNG

Linear → Gaussian or NG

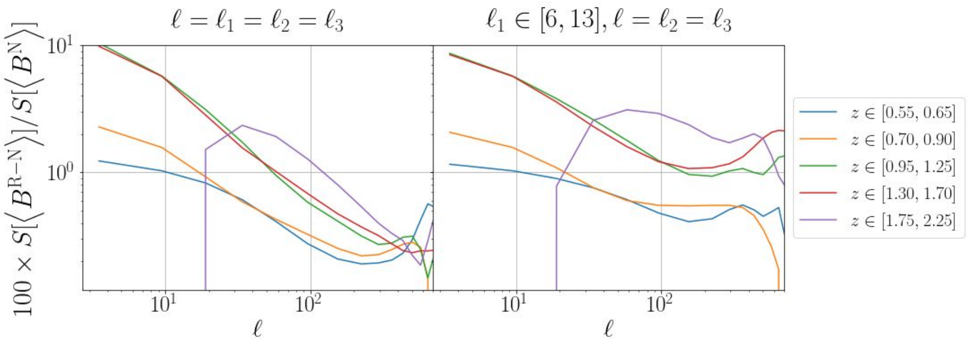
Second-order → NG

NL evolution → light cone ray tracing



→ $C_{\ell_1}^{z_1 z_2}$

→ $B_{\ell_1 \ell_2 \ell_3}^{z_1 z_2 z_3}$



IMPORTANT: Gauge invariance!

Can we now compute what we measure?

Angular bispectrum calculation

- How?
  - Theory
    - First order
    - Second order
      - no integrated term
      - projection up to H/k

Newtonian

GR/Radiation dynamics up to (H/k)^4

Projection effect up to H/k

$$
\begin{aligned}
\Delta_2 = {} & \delta_2 - \mathcal{H}^{-1}\partial_r^2 v_2 \\
& + \mathcal{H}^{-2}\left[\left(\partial_r^2 v_1\right)^2 + \partial_r v_1 \partial_r^3 v_1\right] - \mathcal{H}^{-1}\left[\partial_r v_1 \partial_r \delta_1 + \partial_r^2 v_1 \delta_1\right] \\
& - \mathcal{R}\partial_r v_2 + \mathcal{H}^{-1}\left(1 + 3\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{4}{\mathcal{H}r}\right)\partial_r v_1 \partial_r^2 v_1 - \mathcal{R}\partial_r v_1 \delta_1 + \partial_r v_1 \dot{\delta}_1 + 2\mathcal{H}^{-1} v_1^\alpha \partial_\alpha \partial_r v_1 \\
& - \mathcal{H}^{-2}\psi_1 \partial_r^3 v_1 + \mathcal{H}^{-1}\psi_1 \partial_r \delta_1 + \mathcal{H}^{-2}\partial_r v_1 \partial_r^2 \psi_1 \,, \qquad (2.12)
\end{aligned}
$$

Angular bispectrum calculation

- How?
  - Theory
    - First order
    - Second order
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Newtonian

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& - \mathcal{R}\partial_r v_2 + \mathcal{H}^{-1}\left(1 + 3\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{4}{\mathcal{H}r}\right)\partial_r v_1 \partial_r^2 v_1 - \mathcal{R}\partial_r v_1 \delta_1 + \partial_r v_1 \dot{\delta}_1 + 2\mathcal{H}^{-1} v_1^\alpha \partial_\alpha \partial_r v_1 \\
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\end{aligned}
$$