Numerical relativity waveforms in effective field theories of gravity

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Philippe Meyer Junior Research Chair
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October 14, 2025: Le 9ième GdR Ondes Gravitationnelles Challenges & Future Directions
École Normale Supérieure, Paris, France

Motivation: **Gravitational Effective Field Theories**

Part I: **Reassessment of ghostly interactions**

with Cédric Deffayet, Shinji Mukohyama, and Alexander Vikman (2023, 2025)
as well as 2509.18049

Part II: **Well-posed time evolution & black hole binaries**

with Hyun Lim (2021, 2023, 2025)
and with Pau Figueras and Áron Kovács (2024)

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Joint derivative/curvature expansion ...

$$\mathcal{L}^{(1)}_{\mathsf{EFT}} = \mathsf{M}^2_{\mathsf{Pl}}\, \mathsf{R}$$

$$\mathcal{L}^{(2)}_{\mathsf{EFT}} = \left[\alpha_0 \mathsf{R}_{\mathsf{ab}}\mathsf{R}^{\mathsf{ab}} - \beta_0\, \mathsf{R}^2\right]$$

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After reduction via

(i) index symmetries

(ii) geometric identities

(iii) 4D-specific identities (e.g. Gauss-Bonnet)

see Fulling CQG 9 (1992); Martin-Garcia, Yllanes, Portugal, CPC 179 (2008)

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$$g_{ab} \to g_{ab} + c_1\, g_{ab}\, X + c_2\, X_{ab}$$

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Goroff, Sagnotti, Nucl.Phys.B 266 (1986)
Bueno, Cano, PRD 94 (2016) 10
de Rham, Francfort, Zhang, PRD 102 (2020) 2

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Endlich, Gorbenko, Huang, Senatore, JHEP 09, 122 (2017)

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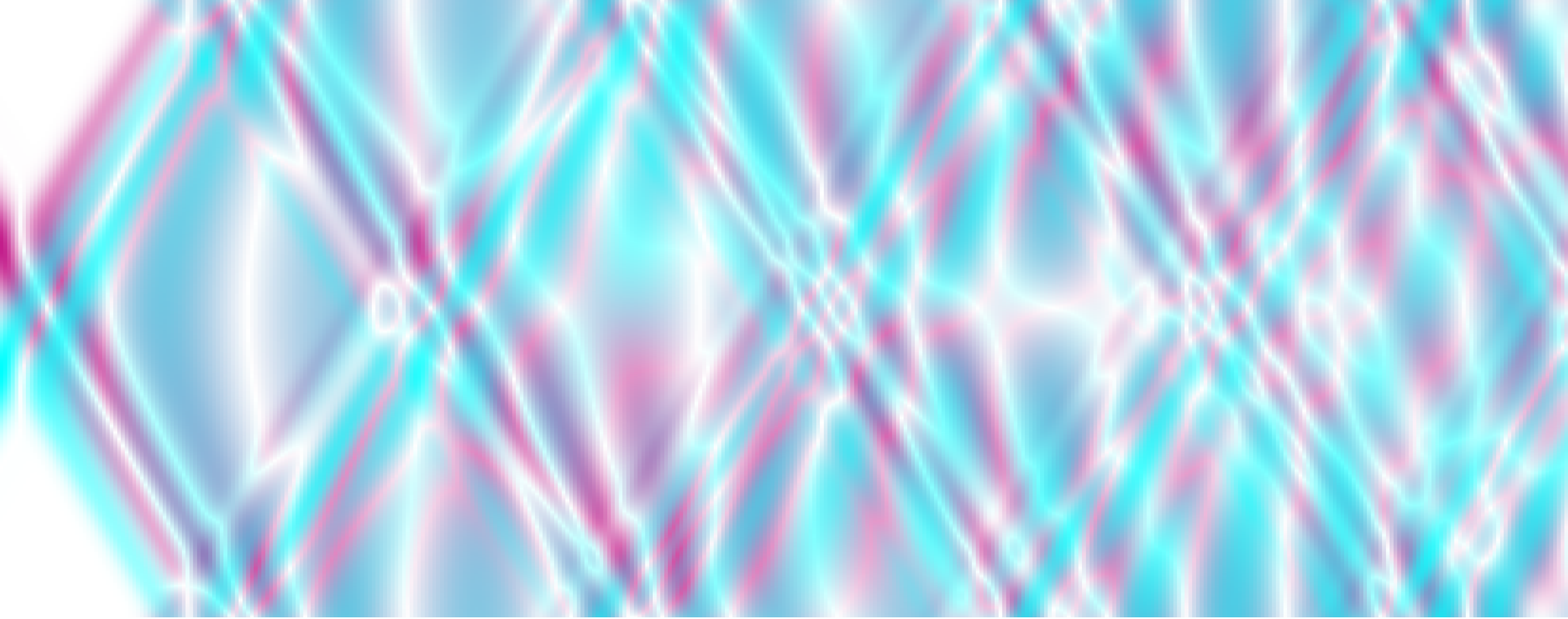

Part I: Controlling ghostly interactions

Deffayet, Held, Mukohyama, Vikman, JCAP 11 (2023) 031
Deffayet, Held, Mukohyama, Vikman, 2504.11437

The **Hamiltonian** of all higher-derivative non-degenerate classical point-particle theories is **unbounded from above and below.**

Ostrogradski 1857

Part I: Ghostly interactions in classical field theory

Deffayet, Held, Mukohyama, Vikman, JCAP 11 (2023) 031
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All non-degenerate **higher-derivative classical point-particle** theories **exhibit runaway** solutions.

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see Deffayet, Mukohyama, Vikman, PRL 128 (2022) 4
for the first proven counter-example

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Ok, but **field theories** will still **decay instantaneously** because of an infinite phase-space volume at high energy.

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Classical field theories do not decay instantaneously and can **exhibit stable EFT vacuum.**

Deffayet, Held, Mukohyama, Vikman, 2504.11437
Held 2509.18049

see also Figueras, Kovacs, Yao, 2505.00082
for an example in the context of EFT

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Opposite-sign kinetic terms …

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The **Helmhotz-Lagrangian** (equivalent 2nd-order Lagrangian) of non-degenerate higher-derivative theories exhibits **opposite-sign kinetic terms.**

Urries, Julve J. Phys. A 31 (1998)

- e.g., Lagrangian

$$\mathcal{L} = -\frac{1}{2}\phi\left[\Box + \mathrm{m}_\phi^2\right]\phi - \frac{\sigma}{2}\chi\left[\Box + \mathrm{m}_\chi^2\right]\chi - \mathrm{V}(\phi,\,\chi)$$

σ = + 1: non-ghostly

σ = – 1: ghostly

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- with field equations

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(non-)ghostly nature (σ = ± 1)
does not affect the principal part

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see Held, 2509.18049 for application

(non-)ghostly nature (σ = ± 1)
does not affect the principal part

see Klainerman 1985, Comm. Pure Appl. Math. 38 for small-data global stability

... do not obstruct from well-posedness.

Higher frequencies ...

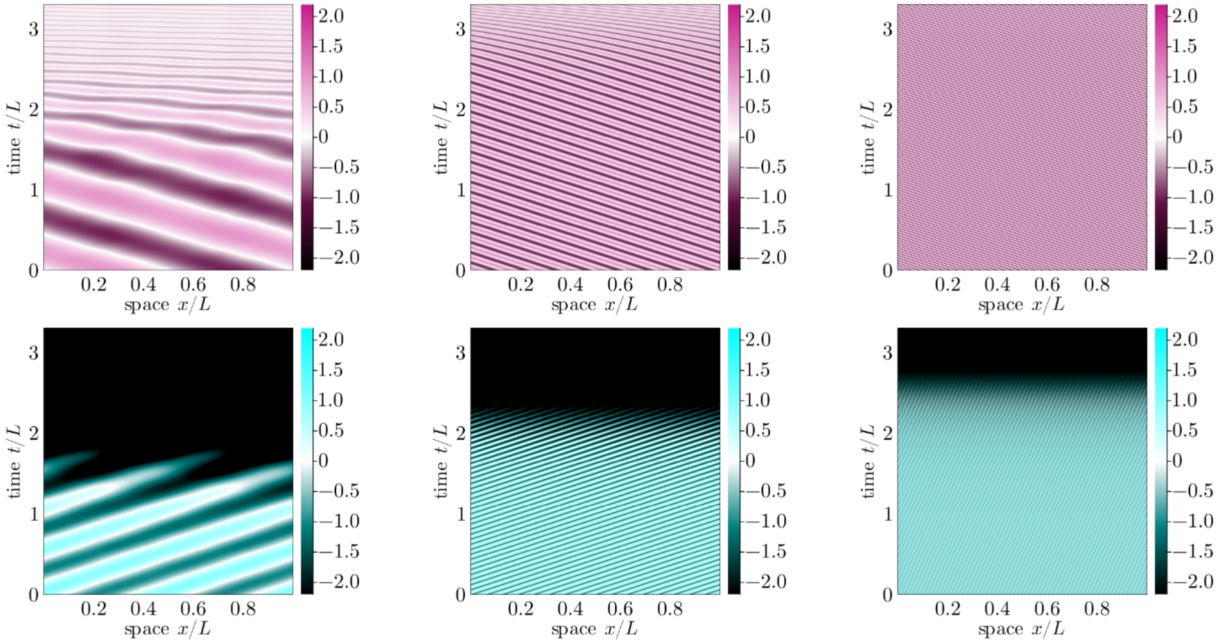
Deffayet, Held, Mukohyama, Vikman, 2504.11437

$$V = \lambda_{nm}\, \phi^n \chi^m$$

$$\partial_t^2 \phi = -\left(k_\phi^2 + m_\phi^2 + \lambda\, \chi^2\right) \phi$$

$$\partial_t^2 \chi = -\left(k_\chi^2 + m_\chi^2 + \sigma\, \lambda\, \phi^2\right) \chi$$

- plane-wave approximation
- **high frequencies dominate potential,** both for the non-ghost and for the ghost case



Deffayet, Held, Mukohyama, Vikman, 2504.11437

... are more stable, not less stable.

Heavy ghost fields …

$V = \lambda \phi^2 \chi^2$

(1+1)D Simulation converges to the solution of the continuum field theory

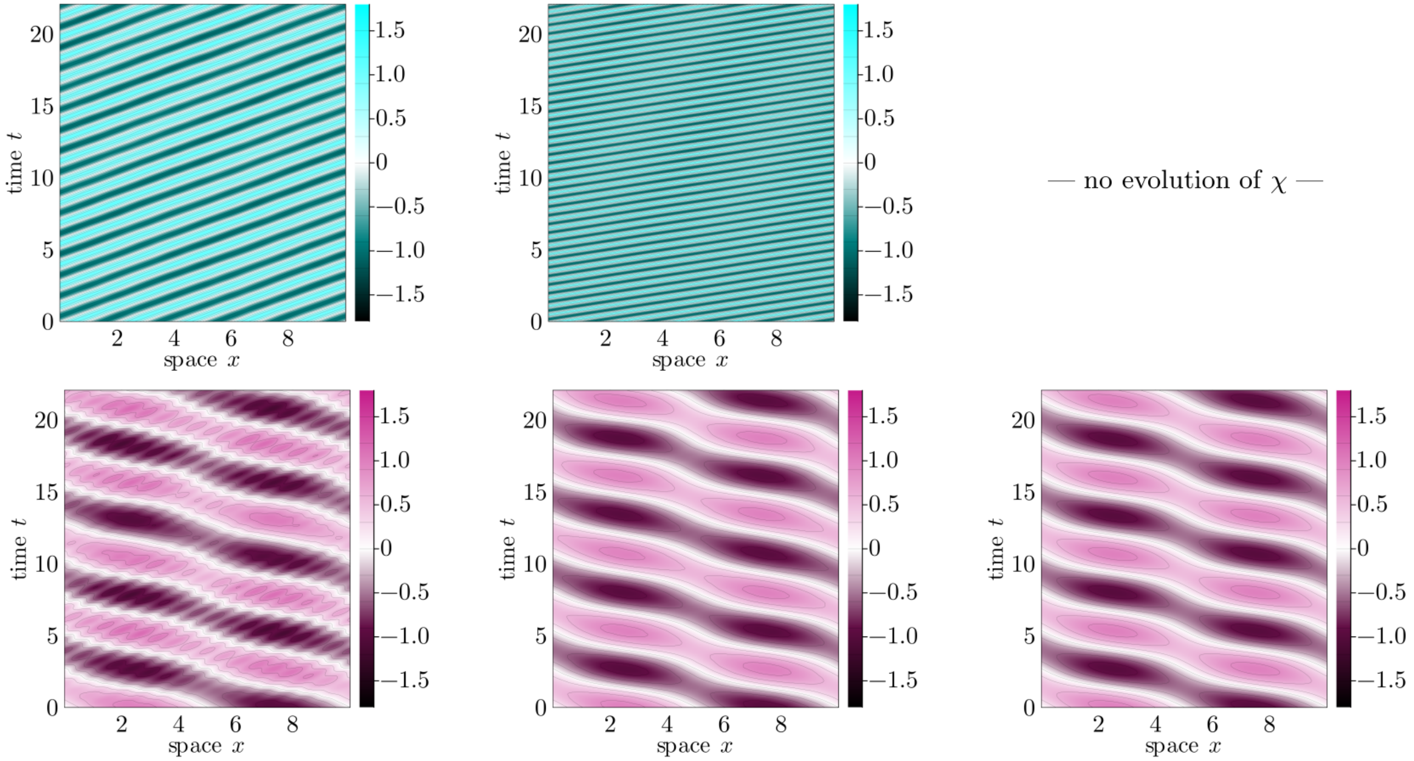
- 4^{th} order FD
- 4^{th} order RK4 timestep
- self-convergence rate verified at all times

Model parameters

- $\lambda = 1$;
- $m_\phi = 0$;
- $m_\chi = 3$; $m_\chi = 10$; $m_\chi = \infty$ (from left to right)

Plane-wave initial data

Periodic boundary



Deffayet, Held, Mukohyama, Vikman, 2504.11437 (see also Figueras, Kovács, Yao, 2505.00082)

… can effectively decouple.

Geometric/dissipative spreading ...

see Held, 2509.18049 for physicists review and scattering solutions in (N+1) dimensions

$V = \lambda_{cross}\, \phi^2 \chi^2$

(3+1)D simulation in spherical symmetry converges to the continuum field theory

- 4^{th} order FD
- 4^{th} order RK4 timestep
- self-convergence rate verified at all times

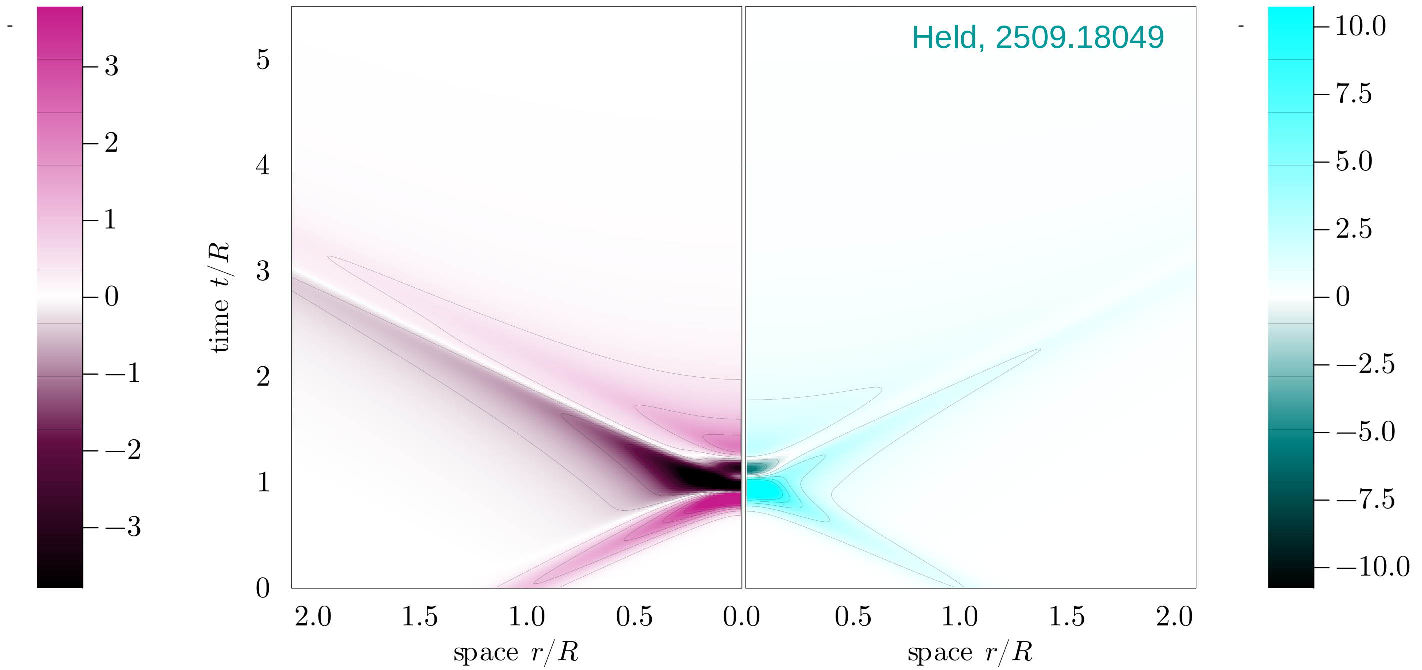
Model parameters

- $\lambda_{self} = \lambda_{cross} = 1$;
- $m_\phi = m_\chi = 0$;

Ingoing scattering initial data with characteristic amplitude

- A = 1,2,4

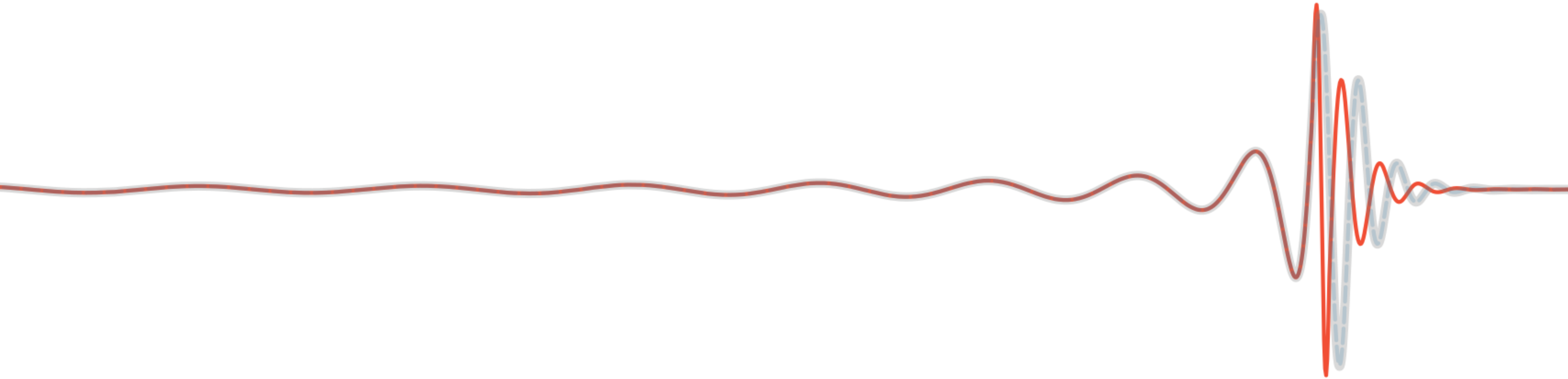
Open boundary conditions



see Klainerman 1985, Comm. Pure Appl. Math. 38 for proof of small-data global stability

and Figueras, Kovacs, Yao, 2505.00082 for a conjecture in effective field theories

... leads to small-data global stability.

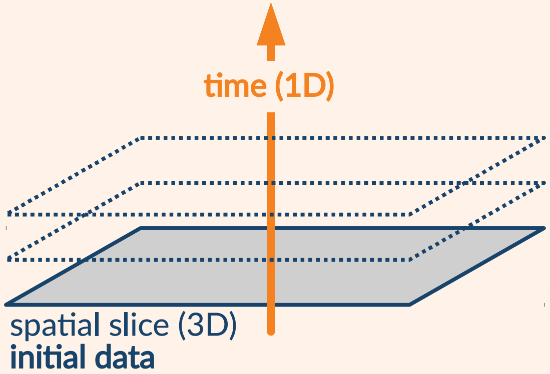


Part II: Nonlinear evolution & black hole binaries

Noakes, JMP 24, 1846 (1983);
Figueras, Held, Kovacs, 2407.08775

Held, Lim, PRD 104 (2021) 8
Held, Lim, PRD 108 (2023) 10
Held, Lim, 2503.13428

A well-posed initial value problem (IVP) ...



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An initial value problem is well-posed if a solution
- **exists**, at least for some future time
- is **unique**
- and depends **continuous**ly on the initial data
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... for General Relativity

Formal proof of existence and uniqueness
Yvonne Choquet-Bruhat '52

(3+1) numerical evolution
Pretorius '05; BSSN '87-'99; Sarbach et.Al '02-'04

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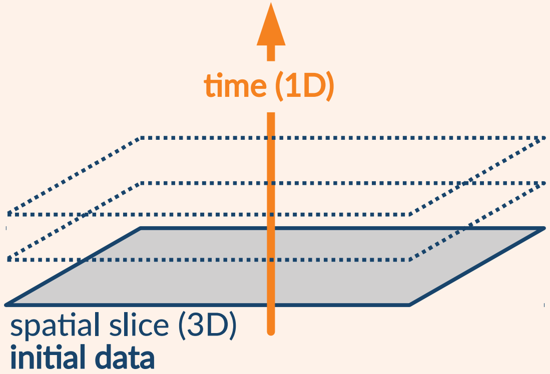

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… and for Quadratic Gravity

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Held, Lim '21, '23, '25; Cayuso '23;
East, Siemonsen '23

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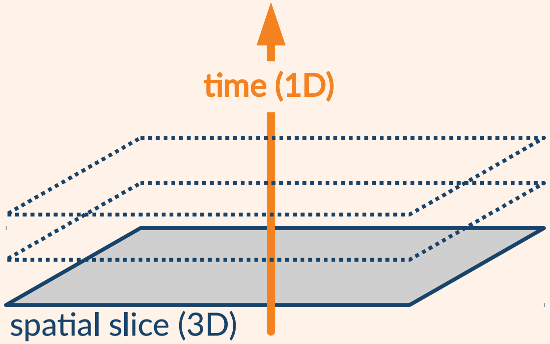

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Held, Lim ‘21, ‘23, ‘25; Cayuso ‘23;
East, Siemonsen ‘23

... and for the EFT (at fixed order)

Figueras, Held, Kovacs, 2407.08775

Leray weights: A simple example ...

- System of PDEs:

$$\Box u = v$$

$$\Box v = \partial_t^2 u$$

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General Relativity (in harmonic gauge) ...

- Gauge potential: $F^a \equiv -g^{cd}\Gamma^a_{cd}$
- Ricci curvature: $R_{ab} \sim \Box g_{ab} + g_{c(a}\nabla_{b)}F^c + \mathcal{O}(g, \partial g) = 0$
- In **harmonic gauge**, i.e., $F^a = 0$
  the vacuum Einstein equations, i.e., $R_{ab} = 0$
  are of **wave-like form**.

For constraint propagation see
Choquet-Bruhat '52

... is already in wave-like form.

Quadratic Gravity ...

Held, Lim, PRD 104 (2021) 8

- recall $\mathcal{L} = M_{Pl}^2 \left[R + \frac{1}{12 m_0^2} R^2 + \frac{1}{4 m_2^2} C_{abcd} C^{abcd} \right]$

Stelle, PRD 16 (1977) 953-969
Noakes, JMP 24, 1846 (1983)
Held, Lim, PRD 104 (2021) 8

2nd-order variables

$$\Box g_{ab} \sim R_{ab} \overset{2|0}{\equiv} S_{ab} + \frac{1}{4} g_{ab} R$$

massless spin-2 (graviton)

$$\Box R \overset{2|0}{=} m_0^2 R$$

massive spin-0 (scalar)

$$\Box S_{ab} \overset{3|1}{=} -\frac{1}{3} \left(\frac{m_2^2}{m_0^2} - 1 \right) (\nabla_a \nabla_b R) - 2 S^{cd} C_{acbd} + \mathcal{O}_{\text{lower order}}$$

massive spin-2 (ghost)

- For equal masses, the 2nd-order field equations of Quadratic Gravity are of wave-like form.
- For unequal masses, one can still find suitable Leray weights.

... admits wave-like 2nd order field equations.

Cubic Gravity (after suitable field redefinitions) ...

Figueras, **Held**, Kovacs, 2407.08775

- recall $\mathcal{L}^{(3)}_{\text{EFT}} = \frac{1}{M_{\text{Pl}}^2}\left[\alpha_1 R^{ab}\Box R_{ab} - \beta_1 R\Box R + \gamma_3 C_{ab}{}^{cd}C_{cd}{}^{ef}C_{ef}{}^{ab}\right]$

order-reduced 2nd-order field equations

$$\Box g_{ab} \sim R_{ab} \overset{2|0}{\equiv} S_{ab} + \frac{1}{4} g_{ab} R$$

$$\Box C_{abde} \overset{3|1}{=} \mathcal{O}^{C}_{abde}(\partial C,\ \partial\partial S,\ \partial\partial R)$$

$$\Box R \overset{2|0}{\equiv} R^{(1)}$$

$$\Box S_{ab} \overset{2|0}{\equiv} S^{(1)}_{ab}$$

$$\Box R^{(1)} \overset{3|1}{\equiv} \mathcal{O}^{R}(\partial C,\ \partial\partial S,\ \partial\partial R)$$

$$\Box S^{(1)}_{ab} \overset{3|1}{\equiv} \left(1 - \frac{2\beta_1}{\alpha_1}\right)\left(\frac{1}{4} g_{ab}\Box - \nabla_a\nabla_b\right) R^{(1)} + \mathcal{O}^{S}_{ab}(\partial C,\ \partial\partial S,\ \partial\partial R)$$

vanishes for equal mass: $\alpha_1 = 2\beta_1$

... admits wave-like 2nd order field equations.

Higher order EFT (after suitable field redefinitions) ...

Figueras, **Held**, Kovacs, 2407.08775

- Inductively, this extends to $\mathcal{L}^{(n)}_{reg} = \sum_{k=0}^{n} \left[\alpha_k R^{ab} \Box^k R_{ab} - \beta_k R \Box^k R\right]$ with $\alpha_n = 2\beta_n$

order-reduced 2nd-order field equations

$$\Box g_{ab} \sim R_{ab} \overset{2|0}{\equiv} S_{ab} + \frac{1}{4} g_{ab} R$$

$$\Box C_{abde} \overset{3|1}{=} \mathcal{O}^{C}_{abde}(\partial C,\, \partial\partial S,\, \partial\partial R)$$

$$\Box R^{(k)} \overset{k+2|k}{\equiv} R^{(k+1)} \qquad \forall 0 \leq k < n$$

$$\Box S^{(k)}_{ab} \overset{k+2|k}{\equiv} S^{(k+1)}_{ab} \qquad \forall 0 \leq k < n$$

$$\Box R^{(n)} \overset{n+2|n}{\equiv} \mathcal{O}^{R}(\partial^{n-1} C,\, \partial^{n-k} S^{(k)},\, \partial^{n-k} R^{(k)})$$

$$\Box S^{(n)}_{ab} \overset{n+2|n}{\equiv} \mathcal{O}^{S}_{ab}(\partial^{n-1} C,\, \partial^{n-k} S^{(k)},\, \partial^{n-k} R^{(k)})$$

... admits wave-like 2nd order field equations.

Higher order EFT (after suitable field redefinitions) ...

Figueras, **Held**, Kovacs, 2407.08775

- Inductively, this extends to $\mathcal{L}^{(n)}_{\text{reg}} = \sum_{k=0}^{n} \left[\alpha_k\, R^{ab}\, \Box^k\, R_{ab} - \beta_k\, R\, \Box^k\, R\right]$ with $\alpha_n = 2\beta_n$

order-reduced 2^{nd}-order field equations

$$\Box g_{ab} \sim R_{ab} \overset{2\,|\,0}{\equiv} S_{ab} + \frac{1}{4} g_{ab}\, R$$

$$\Box C_{abde} \overset{3\,|\,1}{=} \mathcal{O}^{C}_{abde}(\partial C,\ \partial\partial S,\ \partial\partial R)$$

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$$\Box S^{(n)}_{ab} \overset{n+2\,|\,n}{\equiv} \mathcal{O}^{S}_{ab}(\partial^{n-1} C,\ \partial^{n-k} S^{(k)},\ \partial^{n-k} R^{(k)})$$

Not altered if supplemented with an action that only adds to the omitted lower-order terms.

see
Figueras, **Held**, Kovacs, 2407.08775
for a complete proof

... admits wave-like 2^{nd} order field equations.

Well-posed initial value formulation …

$$\mathcal{L}^{(1)}_{\mathrm{EFT}} = M_{Pl}^2\, R$$

✓ Choquet-Bruhat '52

$$\mathcal{L}^{(2)}_{\mathrm{EFT}} = \left[\alpha_0 R_{ab} R^{ab} - \beta_0 R^2\right]$$

order-by-order field redefinitions of the form

$$g_{ab} \to g_{ab} + c_1\, g_{ab}\, X + c_2\, X_{ab}$$

can remove any term containing Ricci variables

$$\mathcal{L}^{(3)}_{\mathrm{EFT}} = \frac{1}{M_{Pl}^2}\Big[\alpha_1 R^{ab}\,\Box\, R_{ab} - \beta_1 R\,\Box\, R + \gamma_3\, C_{ab}{}^{cd} C_{cd}{}^{ef} C_{ef}{}^{ab}$$
$$+ \delta_{3,1}\, C_{abcd} C^{abcd} R + \delta_{3,2}\, C_{abcd} R^{ac} R^{bd} + \delta_{3,3}\, R^b_a R^c_b R^a_c + \delta_{3,4}\, R_{ab} R^{ab} R + \delta_{3,5}\, R^3\Big]$$

$$\mathcal{L}^{(4)}_{\mathrm{EFT}} = \frac{1}{M_{Pl}^4}\Big[\alpha_2 R^{ab}\,\Box^2 R_{ab} - \beta_2 R\,\Box^2 R + \gamma_{4,1}\left(C_{abcd}\, C^{abcd}\right)^2 + \gamma_{4,2}\left(C_{abcd}\, {}^*C^{abcd}\right)^2 + \dots\Big]$$

Well-posed initial value formulation ...

$$\mathcal{L}^{(1)}_{\rm EFT} = M_{\rm Pl}^2\, R$$ ✓ Choquet-Bruhat '52

$$\mathcal{L}^{(2)}_{\rm EFT} = \left[\alpha_0 R_{ab}R^{ab} - \beta_0 R^2\right]$$ ✓

Noakes, JMP 24, 1846 (1983)
Held, Lim '21, '23, '25

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Well-posed initial value formulation …

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✓ Choquet-Bruhat '52

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✓ Noakes, JMP 24, 1846 (1983)
Held, Lim '21, '23, '25

order-by-order field redefinitions of the form

$$g_{ab} \to g_{ab} + c_1 \, g_{ab} \, X + c_2 \, X_{ab}$$

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$$+ \delta_{3,1} \, C_{abcd} C^{abcd} R + \delta_{3,2} \, C_{abcd} R^{ac} R^{bd} + \delta_{3,3} \, R^b_a \, R^c_b \, R^a_c + \delta_{3,4} \, R_{ab} R^{ab} R + \delta_{3,5} \, R^3\Big]$$

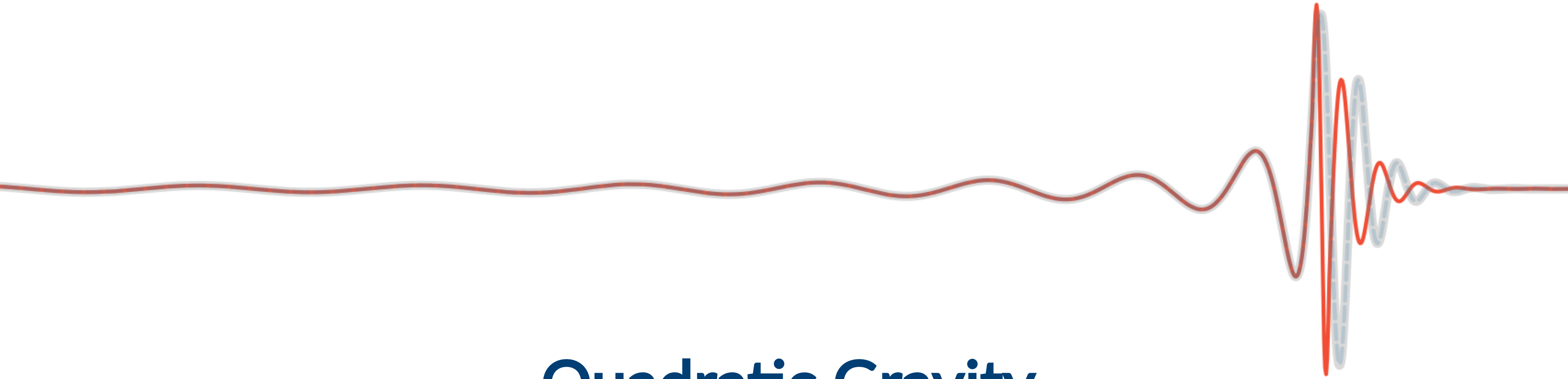
✓ Figueras, Held, Kovacs 2407.08775

$$\mathcal{L}^{(4)}_{\text{EFT}} = \frac{1}{M_{Pl}^4}\Big[\alpha_2 \, R^{ab} \, \Box^2 \, R_{ab} - \beta_2 \, R \, \Box^2 \, R + \gamma_{4,1} \left(C_{abcd} \, C^{abcd}\right)^2 + \gamma_{4,2} \left(C_{abcd} \, {}^*C^{abcd}\right)^2 + \dots\Big]$$

✓ Figueras, Held, Kovacs 2407.08775

Figueras, Held, Kovacs, 2407.08775

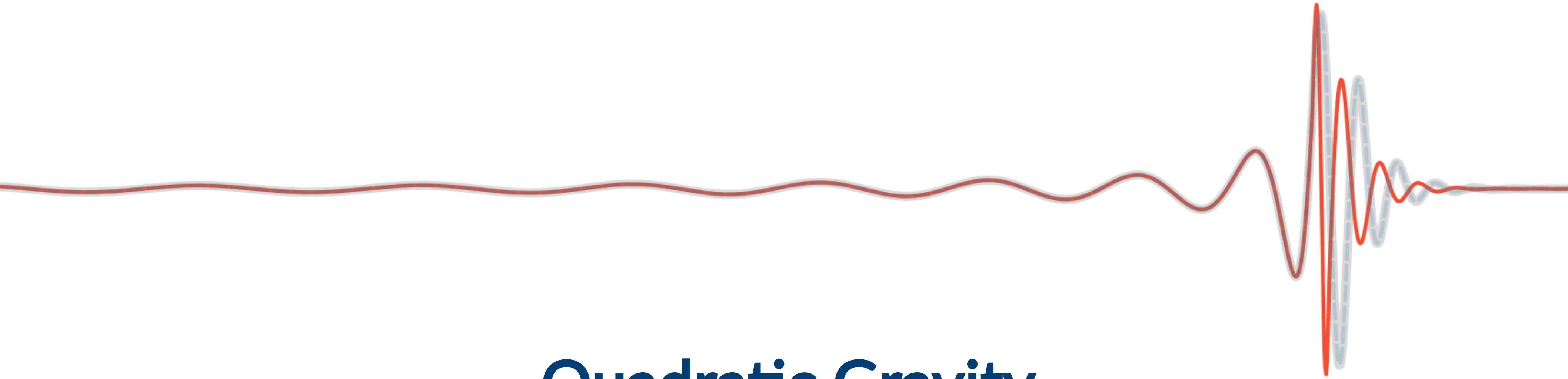
… of general effective field theories of gravity.



Quadratic Gravity

$$S = \int d^4x \sqrt{|g|}\, M_{\text{Planck}}^2 \left[R + \frac{1}{12m_0^2} R^2 + \frac{1}{4m_2^2} C_{abcd} C^{abcd} \right]$$

massless spin-2 massive spin-0 massive spin-2

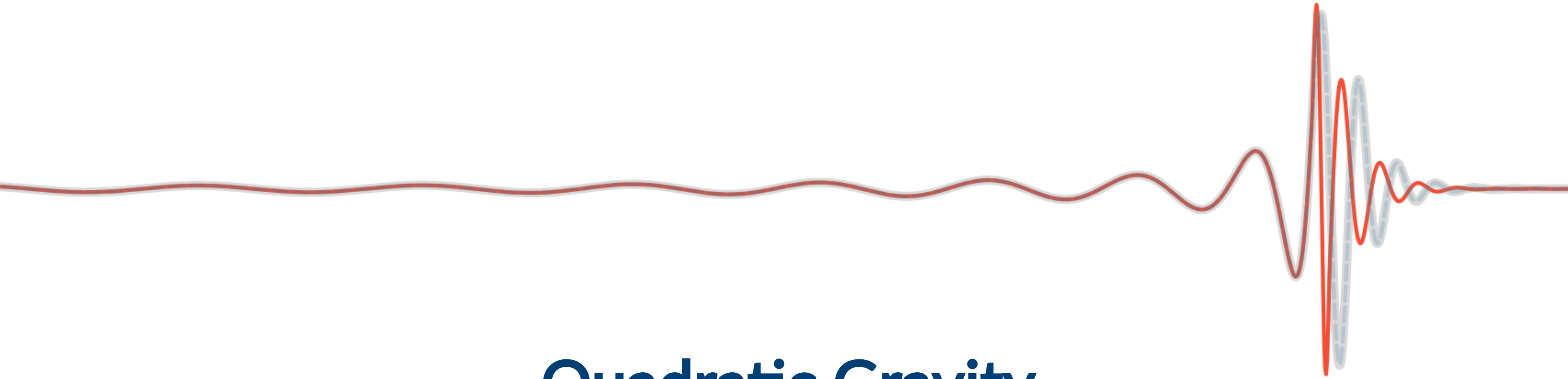


Quadratic Gravity

$$S = \int d^4x \sqrt{|g|}\, M^2_{\text{Planck}} \left[R + \frac{1}{12m_0^2} R^2 + \frac{1}{4m_2^2} C_{abcd} C^{abcd} \right]$$

massless spin-2 massive spin-0 massive spin-2

as a benchmark model to show that heavy ghosts decouple



Quadratic Gravity

$$S = \int d^4x \sqrt{|g|}\, M_{Planck}^2 \left[R + \frac{1}{12 m_0^2} R^2 + \frac{1}{4 m_2^2} C_{abcd} C^{abcd} \right]$$

massless spin-2   massive spin-0   massive spin-2

as a fundamental theory of gravity

Stelle, PRD 16 (1977) 953-969

Avramidi, Barvinsky, PLB 159 (1985) 269-274

Buccio, Donoghue, Menezes, Percacci, PRL 133 (2024) 2, 021604

Waveforms ...

		EFT regime of validity
$GM\, m_2 \gg 1$	no deviations	
$GM\, m_2 \sim 1$	quantitative deviations	
$GM\, m_2 \lesssim 1$	qualitative deviations	

Waveforms ...

		EFT regime of validity
$GM\, m_2 \gg 1$	Held, Lim, PRD 108 (2023) 10 **no deviations**	
$GM\, m_2 \sim 1$	**quantitative deviations**	
$GM\, m_2 \lesssim 1$	**qualitative deviations**	

Waveforms ...

EFT regime of validity

$GM\, m_2 \gg 1$	**no deviations**
$GM\, m_2 \sim 1$	Held, Zhang, PRD 107 (2023) 6 **quantitative deviations** Held, Lim, PRD 108 (2023) 10 East, Siemonsen PRD 108 (2023) 12
$GM\, m_2 \lesssim 1$	**qualitative deviations**

Waveforms ...

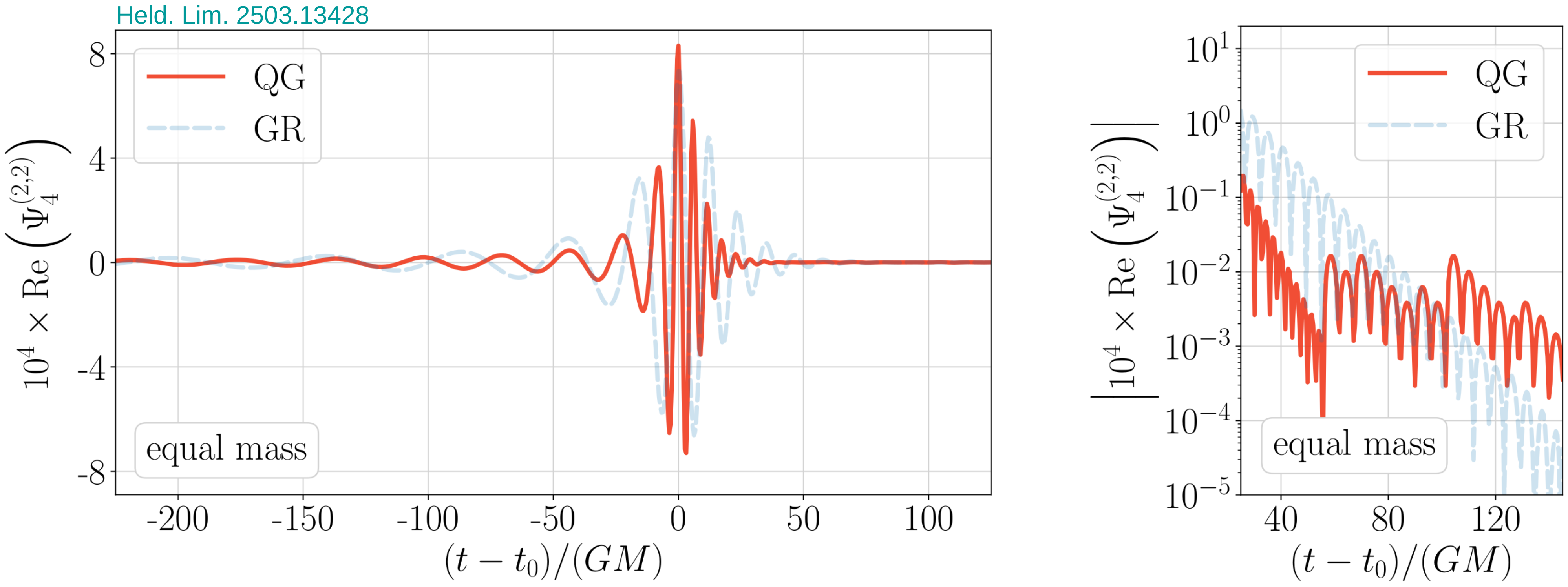
EFT regime of validity

$GM\,m_2 \gg 1$	**no deviations**
$GM\,m_2 \sim 1$	**quantitative deviations**
$GM\,m_2 \lesssim 1$	**qualitative deviations** Held, Lim, 2503.13428

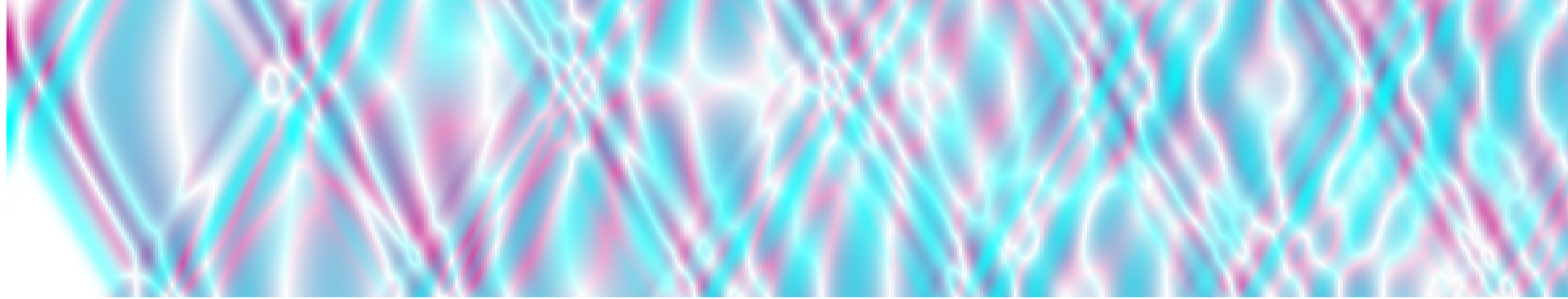
Waveforms for $GM\, m_2 \lesssim 0.43$ ...

Held, Lim, 2503.13428

QG masses		Binary parameters			
$G\, m_0\, M_2$	$G\, m_2\, M_2$	$\sqrt{G}\, M_1$	$q = \frac{M_1}{M_2}$	$a_{z,1}$	$a_{z,2}$
1	0.2	1	1	0	0



... in quadratic higher-derivative gravity.



Heavy ghost fields dynamically decouple.

Ghosts propagation enables well-posed time evolution.

Access to the nonlinear regime of higher-derivative theories.