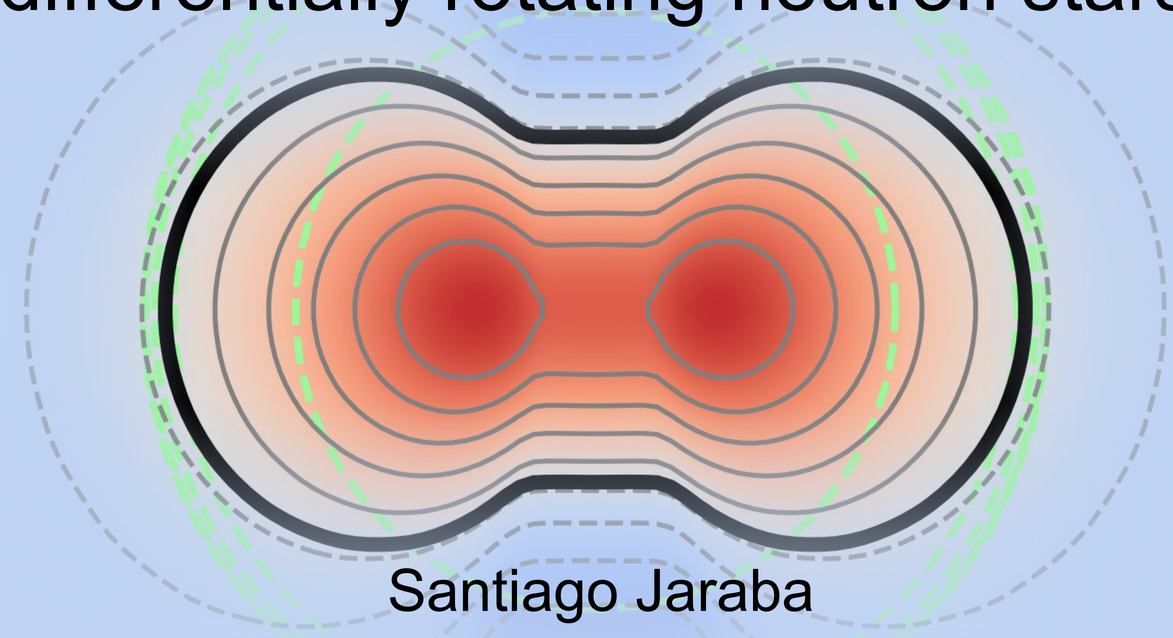
Numerical simulations of oscillations in differentially rotating neutron stars



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Work with Jérôme Novak and Micaela Oertel

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Motivation and history

- Simulating perturbed, isolated neutron stars → BNS post-merger dynamics.
- Studied since the 1960s.
  - Chandrasekhar 1964 → radial oscillations.
  - Thorne, Campolattaro 1967 → non-radial modes.
- Progress on the numerical side has been drastic in the last few decades.
  - Valencia formulation (Banyuls et al., 1997): conservative formulation of GR hydrodynamical eqs.
  - First 3D binary neutron star merger simulation (Shibata, Uryu 2000).
  - Proliferation of different codes for Numerical Relativity:
    - CoCoNut
    - Einstein Toolkit
    - BAM
    - SpEC
    - SACRA
    - …

ROXAS

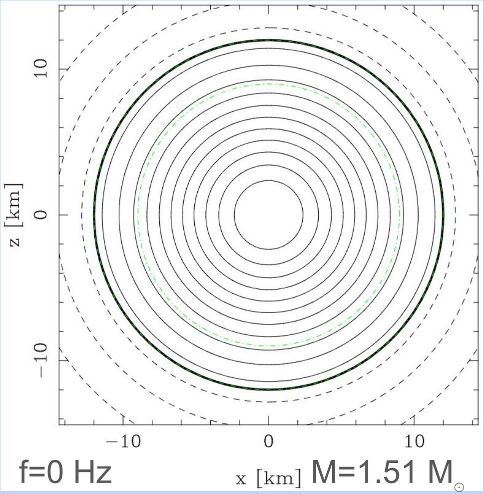
- **R**elativistic **O**scillations of non-a**X**isymmetric neutron st**A**r**S**.
- Developed mainly by Gaël Servignat (Servignat, Novak 2025).
- Aims to be as lightweight as possible. Runs in laptops, O(hours, few days).
- Based on formalism developed in Servignat et al. 2023 using primitive variables.
  - Energy density e, pressure p, baryon density n_B, log-enthalpy H, $H = \ln\left(\frac{e+p}{m_B n_B}\right)$
  - Coordinate velocity v_i, Eulerian velocity U_i.
  - Opposed to conserved variables, $D = m_B n_B \Gamma^2, \quad S_j = (e+p)\Gamma^2 U_j, \quad \tau = (e+p)\Gamma^2 - p$

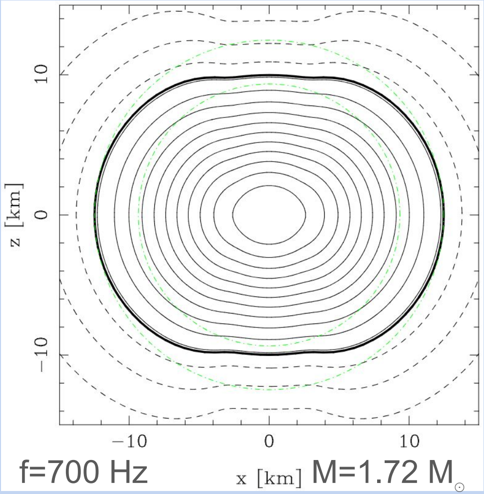
$$\Gamma = (1 - U_j U^j)^{-1/2}$$

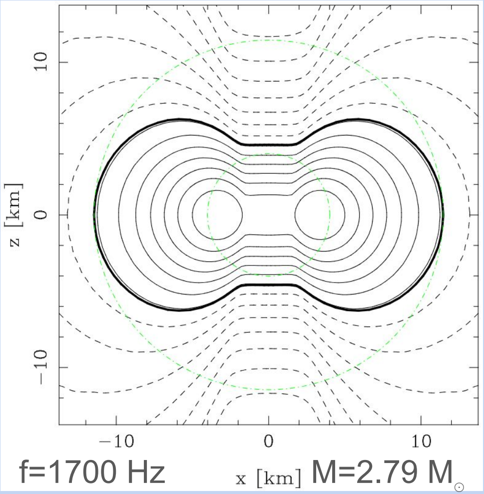
- Uses spectral methods (Chebyshev).
- Publicly available at https://zenodo.org/records/14849547

LORENE (Langage Objet pour la RElativité NumériquE)

- ROXAS is implemented on top of LORENE.
- LORENE: set of C++ classes for numerical relativity.
- Continuous development from 1997, mainly in Paris Observatory.
- LORENE can be used to find the configurations of rotating NSs in equilibrium.







First-step assumptions

- Cold NS (T=0) in β-equilibrium → one-parameter equation of state (EoS), p=p(ρ).
  - ROXAS currently supports 2-parameter (ρ, Y_e) EoS, accounting for out of β-equilibrium NS.
  - 3-parameter EoS (ρ, Y_e, T) are planned to be implemented.
  - Assumed polytropic ($p=\kappa\rho^\gamma$) for the results in this presentation.
- No electromagnetic fields.
- Conformal flatness (CFC): in the 3+1 formalism,

$g_{\mu\nu}dx^\mu dx^\nu = -N^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$, $\gamma_{ij} = \Psi^4 \tilde{\gamma}_{ij}$ with $\tilde{\gamma}_{ij}$ flat.

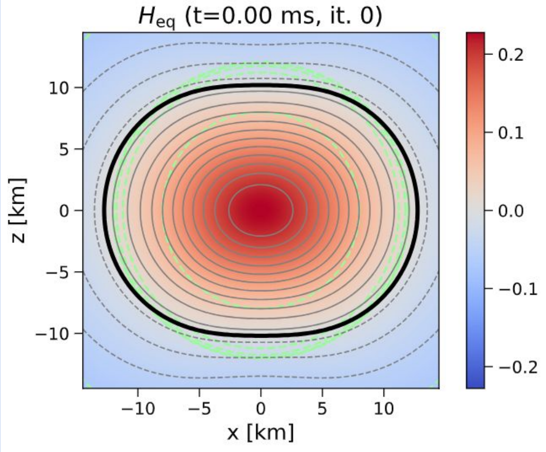
  - Can be done as long as:
    - The deviation from axisymmetry is small enough.
    - The impact of GWs on the dynamics is negligible (no backreaction).
- Equilibrium stars are rigidly rotating, $\vec{v}(r,\theta) = \Omega_0\, r \sin\theta\, \hat{\varphi}$

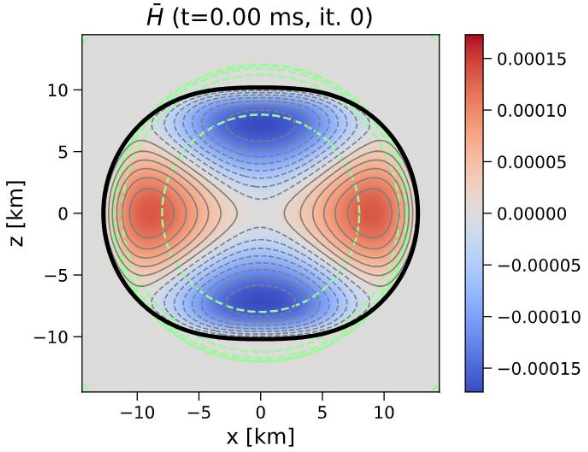
Dynamical evolution with ROXAS

- The equilibrium star is generated with LORENE.
- A perturbation combining l=m=0 and l=m=2 modes is added.

$$H = \ln\left(\frac{e+p}{m_B n_B}\right)$$

$$\delta H = \varepsilon \left(x^2 - y^2 - \frac{2}{3}r^2\right)\left(1 - \frac{r^2}{R_S(\theta)^2}\right)$$





- ROXAS dynamically evolves the perturbed star.

Well-balanced formulation

- The hydrodynamical equations are $$\partial_t H = -v^i D_i H - c_s^2 \frac{\Gamma^2 N}{\Gamma^2 - c_s^2(\Gamma^2-1)} \left[K_{ij} U^i U^j - K + D_i U^i - \frac{U^i}{\Gamma^2} D_i H \right]$$

$$\begin{aligned}\partial_t U_i = &- v^j D_j U_i - U_j D_i v^j + N U_j D_i U^j - \frac{N}{\Gamma^2} D_i (H + \ln N) + U_i U^j D_j N \\ &+ \frac{c_s^2 N U_i}{\Gamma^2 - c_s^2(\Gamma^2-1)} (D_j U^j - K) + U_i \frac{\Gamma^2 (c_s^2 - 1)}{\Gamma^2 - c_s^2(\Gamma^2-1)} N U^l U^j K_{lj} + \frac{N(1-c_s^2)}{\Gamma^2 - c_s^2(\Gamma^2-1)} U_i U^j \frac{D_j p}{e+p}\end{aligned}$$

- We divide quantities in equilibrium term + “perturbation”

$$U_i = U_{i,\text{eq}} + \bar{U}_i, \quad H = H_{\text{eq}} + \bar{H}, \quad \beta^i = \beta^i_{\text{eq}} + \bar{\beta}^i, \quad N = N_{\text{eq}} + \bar{N}$$

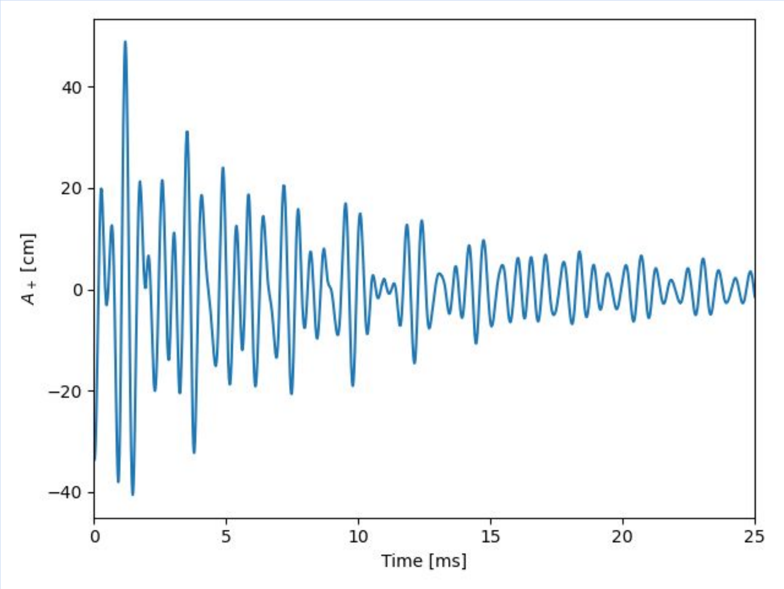
- Equilibrium identities can be used to eliminate some terms (order 0).

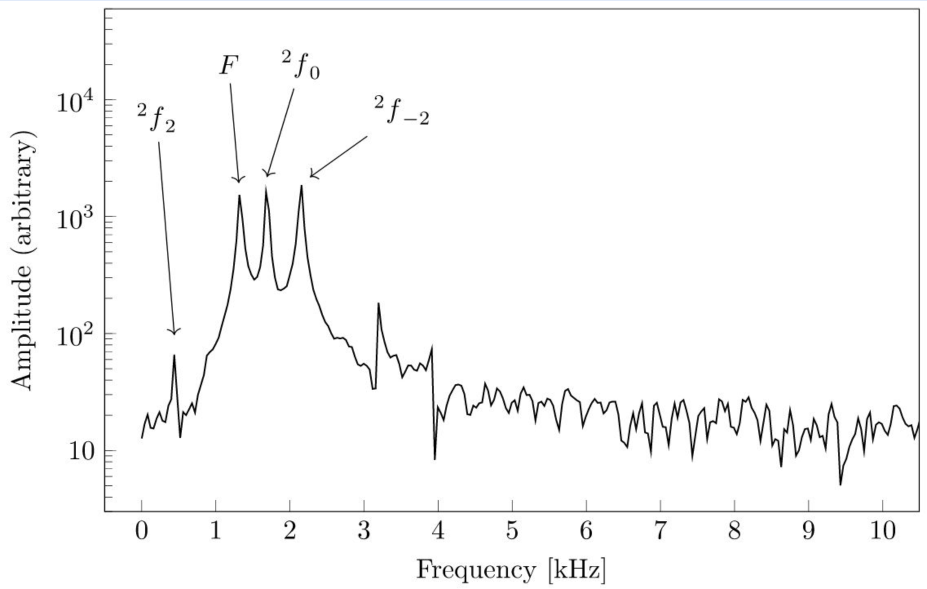
$$H_{\text{eq}} + \ln N_{\text{eq}} - \ln \Gamma_{\text{eq}} = \text{const.} \qquad v^j_{\text{eq}} D_j U_{i,\text{eq}} + U_{j,\text{eq}} D_i v^j_{\text{eq}} = 0 \qquad v^i_{\text{eq}} D_i H_{\text{eq}} = 0$$

- We only need to evolve the perturbation, which gains numerical accuracy.

Waveforms

- CFC removes the radiative degrees of freedom → no GW from the metric.
- GW extraction is still possible through the quadrupole formula.





Oscillation frequencies in rigid rotation (Servignat, Novak 2025)

- Comparison with axisymmetric modes from CFC simulations (Dimmelmeier et al. 2006) and non-axisymmetric modes from GR (Krüger, Kokkotas 2020) simulations.
- Most relative errors of order 1% or below.

Model	f (Hz)	F	H_1	${}^2f_0$	${}^2f_2$	${}^2f_{-2}$	δF	δH_1	$\delta\, {}^2f_0$	$\delta\, {}^2f_2$	$\delta\, {}^2f_{-2}$
BU0	0	1.446	3.956	1.596	1.565	1.565	0.83	0.38	0.63	0.83	0.83
BU1	347	1.423	3.883	1.609	1.080	1.953	0.70	0.82	0.12	0.09	0.56
BU2	487	1.396	3.884	1.642	0.840	2.052	1.15	0.59	0.43	0.48	0.10
BU3	590	1.360	3.915	1.680	0.640	2.123	1.25	0.15	0.65	0.63	0.24
BU4	673	1.322	3.934	1.715	0.451	2.153	1.36	0.40	0.99	1.11	0.09
BU5	740	1.282	3.958	1.723	0.280	2.158	0.08	0.15	0.52	4.64	1.44
BU6	793	1.258	3.998	1.744	0.130	2.143	3.10	0.30	0.86	12.3	1.82
BU7	831	1.202	4.001	1.751	×	2.105	0.42	0.57	1.77	×	3.94

Mode frequencies in kHz, relative errors in %.

Differential rotation

$$\vec{v}(r,\theta) = \Omega(r,\theta)\, r \sin\theta\, \hat{\varphi}$$

- Additional terms need to be taken into account in equilibrium wrt rigid rotation.

$$H_{\mathrm{eq}} + \ln N_{\mathrm{eq}} - \ln \Gamma_{\mathrm{eq}} + \int_{\Omega_p}^{\Omega} F(\Omega) d\Omega = \mathrm{const.}$$

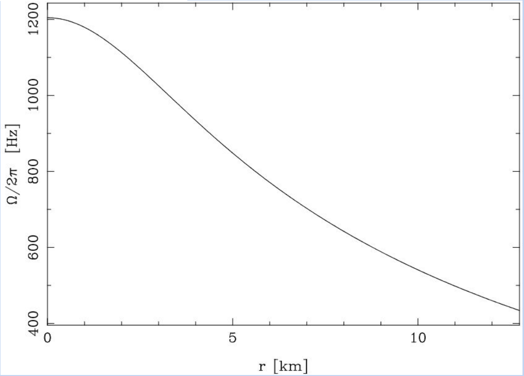
$$v^j_{\mathrm{eq}} D_j U_{i,\mathrm{eq}} + U_{j,\mathrm{eq}} D_i v^j_{\mathrm{eq}} = \frac{U_{\varphi,\mathrm{eq}} v^{\varphi}_{\mathrm{eq}}}{\Omega} D_i \Omega$$

$$F = u^t u_{\varphi}$$

- Must be accounted for both
  - LORENE equilibrium solvers → implemented and tested with literature.
  - Hydrodynamical equations in ROXAS: extra term $\frac{\bar{N}}{N^2_{\mathrm{eq}}} \Psi^4_{\mathrm{eq}} r \sin\theta (\Omega r \sin\theta + \beta^{\varphi}_{\mathrm{eq}}) D_i \Omega$ in equation for U_i.
- We need a rotation profile F(Ω), which we choose to be the KEH profile (Komatsu, Eriguchi, Hachisu 1989)

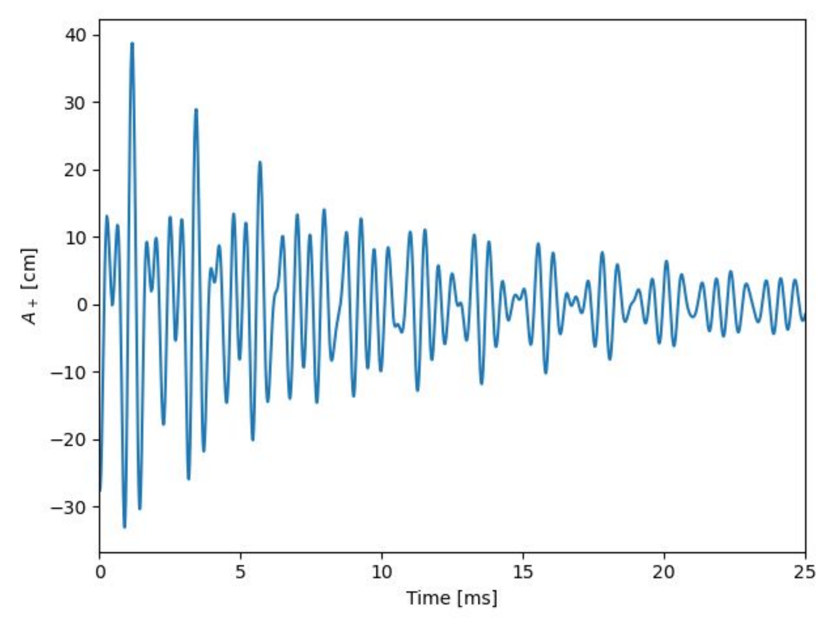
$$F(\Omega) = A^2(\Omega_c - \Omega)$$

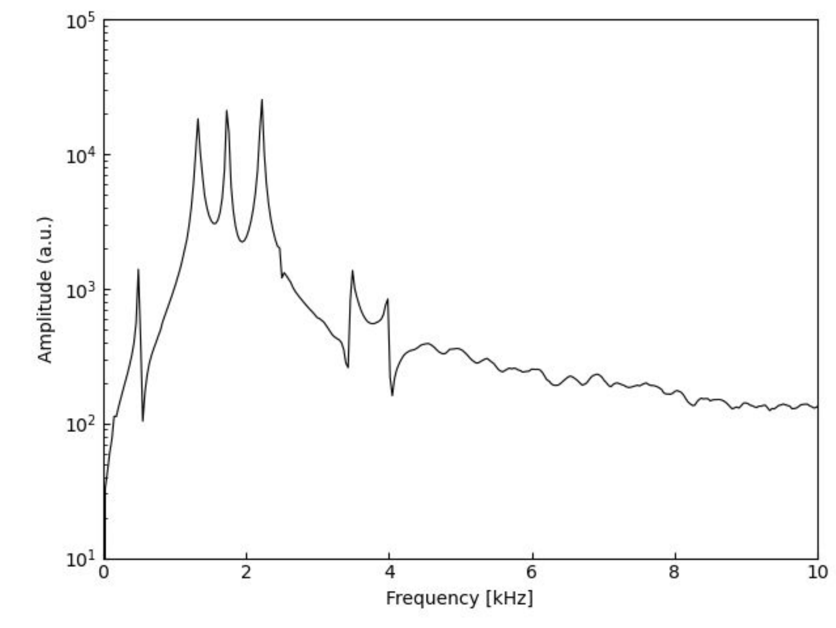
  - Traditional choice due to simplicity, but more complex models available.



Differential rotation: results

- Differential rotation has been successfully implemented for KEH profiles.





Differential rotation: comparison with previous works

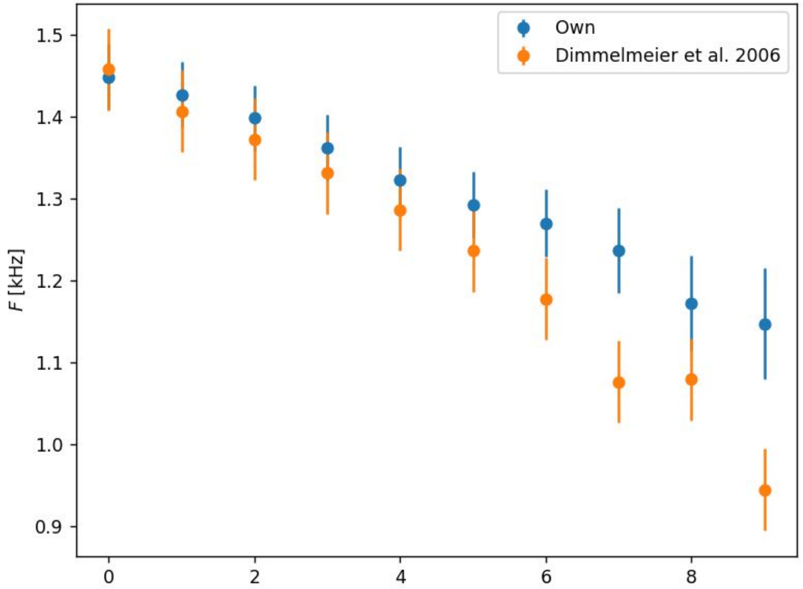
- Consistency of equilibrium values with Dimmelmeier et al. 2006 is excellent.

Model	f_c (Hz)	f_e (Hz)	M $(M_\odot)$	r_p/r_e	δf_e (%)	δM (%)	$\delta(r_p/r_e)$ (%)
B0	0	0	1.400	1.000	0.00	0.01	0.00
B1	582	215	1.437	0.950	0.00	0.03	0.00
B2	832	305	1.478	0.900	0.06	0.02	0.00
B3	1030	375	1.525	0.850	0.02	0.03	0.01
B4	1205	434	1.578	0.800	0.07	0.02	0.01
B5	1366	486	1.640	0.750	0.09	0.02	0.01
B6	1521	534	1.713	0.700	0.16	0.01	0.03
B7	1675	579	1.798	0.650	0.17	0.00	0.03
B8	1836	622	1.900	0.600	0.23	0.03	0.07
B9	2014	665	2.022	0.549	0.30	0.08	0.09

Differential rotation: comparison with previous works

- Consistency of axisymmetric freqs. with [Dimmelmeier et al. 2006] also excellent for H_1 and $^2f_0$, worse for the fundamental mode F in the most demanding cases.
    - Reasons still unclear: different formalisms, resolutions, etc.

Model	F (kHz)	H_1 (kHz)	$^2f_0$ (kHz)	$^2f_2$ (kHz)	$^2f_{-2}$ (kHz)	δF (%)	δH_1 (%)	$\delta\,^2f_0$ (%)
B0	1.449	3.938	1.545	1.564	1.564	0.63	0.83	2.56
B1	1.427	3.904	1.602	1.115	1.939	1.43	0.57	1.60
B2	1.399	3.906	1.642	0.881	2.072	1.86	0.54	1.65
B3	1.363	3.930	1.685	0.681	2.159	2.29	0.86	1.43
B4	1.323	3.993	1.743	0.514	2.216	2.82	0.52	0.25
B5	1.293	4.040	1.775	0.320	2.276	4.51	0.78	0.78
B6	1.270	4.096	1.815	0.138	2.312	7.85	0.53	0.21
B7	1.237	4.147	1.865	-	2.352	14.83	0.77	0.57
B8	1.172	4.196	1.876	-	2.392	8.56	0.37	1.28
B9	1.148	4.164	1.936	-	2.450	21.47	6.83	0.98



Summary

- ROXAS: lightweight code to simulate isolated neutron star oscillations.
- Based on primitive variables, uses spectral methods, CFC, well-balanced formulation.
- Extracts waveforms through quadrupole formula.
- Consistent with the literature with rigid rotation.
- Mostly consistent with the literature with differential rotation.
    - Moderate discrepancies on only one mode, needs further investigation.
- Ongoing work:
    - Simulations with more realistic differential rotation profiles.
    - Improving accuracy and stability.
- Next steps:
    - Using realistic equations of state.
    - Generalizing beyond CFC? Magnetic fields?