AN ALMOST CAUSAL META-MODEL

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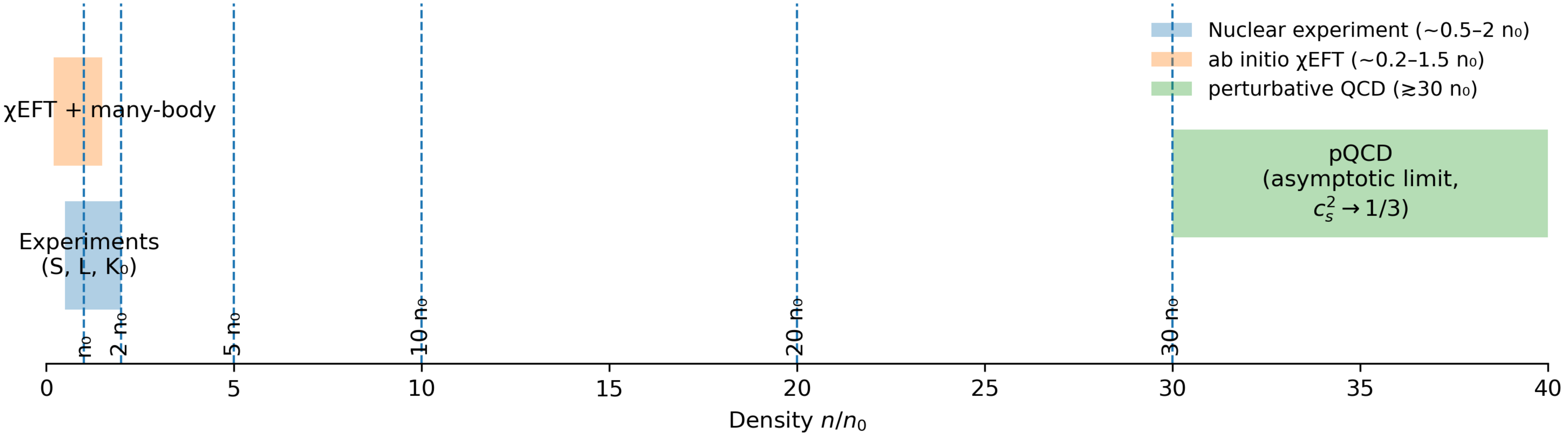
Motivation:
Nuclear EoS uncertainty

Nuclear experiment	χ_{EFT}	pQCD
$0.5 - 2\ n_0$	$0.2 - 1.5\ n_0$	$> 30\ n_0$



Motivation:
Meta model extrapolation

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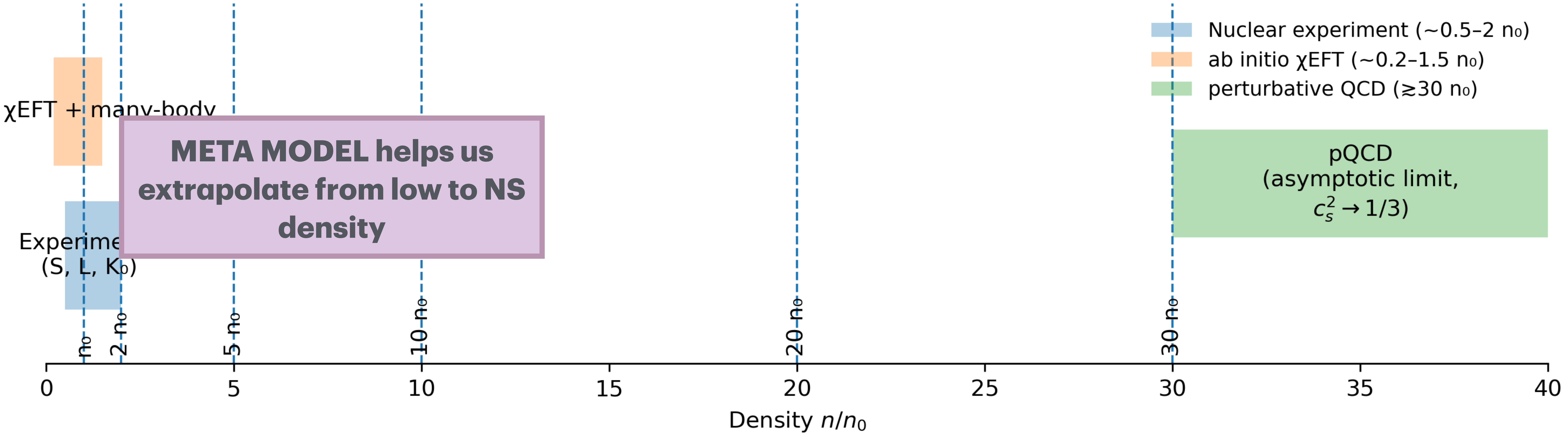

Meta-modelling of the EoS:
Exploring the EoS space

Originally presented in [PRC 97, 025805 (2018)]

Parametric representation of the energy density $\epsilon_X(n_n, n_p, ...)$ as a function of the different species

The variation of the parameters set X makes possible to explore the EoS space compatible with the hypothesis of a matter with the chosen species

Both nuclear and Astro observables are accessible

r_i, BE_i ⟹ $e_X(n_n, n_p, ...)$ ⟸ M, R, Λ

Nuclear observables

Astro observables

Almost causal meta-model:
A possible choice of the energy density

Montefusco et al [in prep]

Causality asymptotically implemented

Starting ansatz:

$$\epsilon(n, x_e, x_\mu) = \epsilon_k(n, x_e, x_\mu) + n\left[\, e_0(n) + \delta^2 e_2(n) + \delta^4\, e_4(n)\right]$$

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$\epsilon_k(n, x_e, x_\mu)$: free fermi gas energy density for $npe\mu$ matter

$e_0(n)$, $e_2(n)$: Nucleonic Potential (per baryon)

$$e_0(x) = V_0(x) + \frac{h_0 + h_1 x + h_2 x^2 + h_3 x^3}{(1 + a_0 x)(1 + b_0 x)(1 + c_0 x)}$$

δ^2, δ^4: Nuclear asymmetry

$$\delta = 1 - 2(x_e + x_\mu)$$

$e_4(n)$: Quartic correction

$$e_4(n) = A\frac{n/n_0}{1 + \left(n/n_0\right)^B}$$

Almost causal meta-model:
EoS reconstruction

Test the flexibility of the model to reproduce β-equilibrated EoS

Constrain the space of the unphysical parameters

We have chosen: Sly4, BSK24, DD2, FSU2 and TM1e

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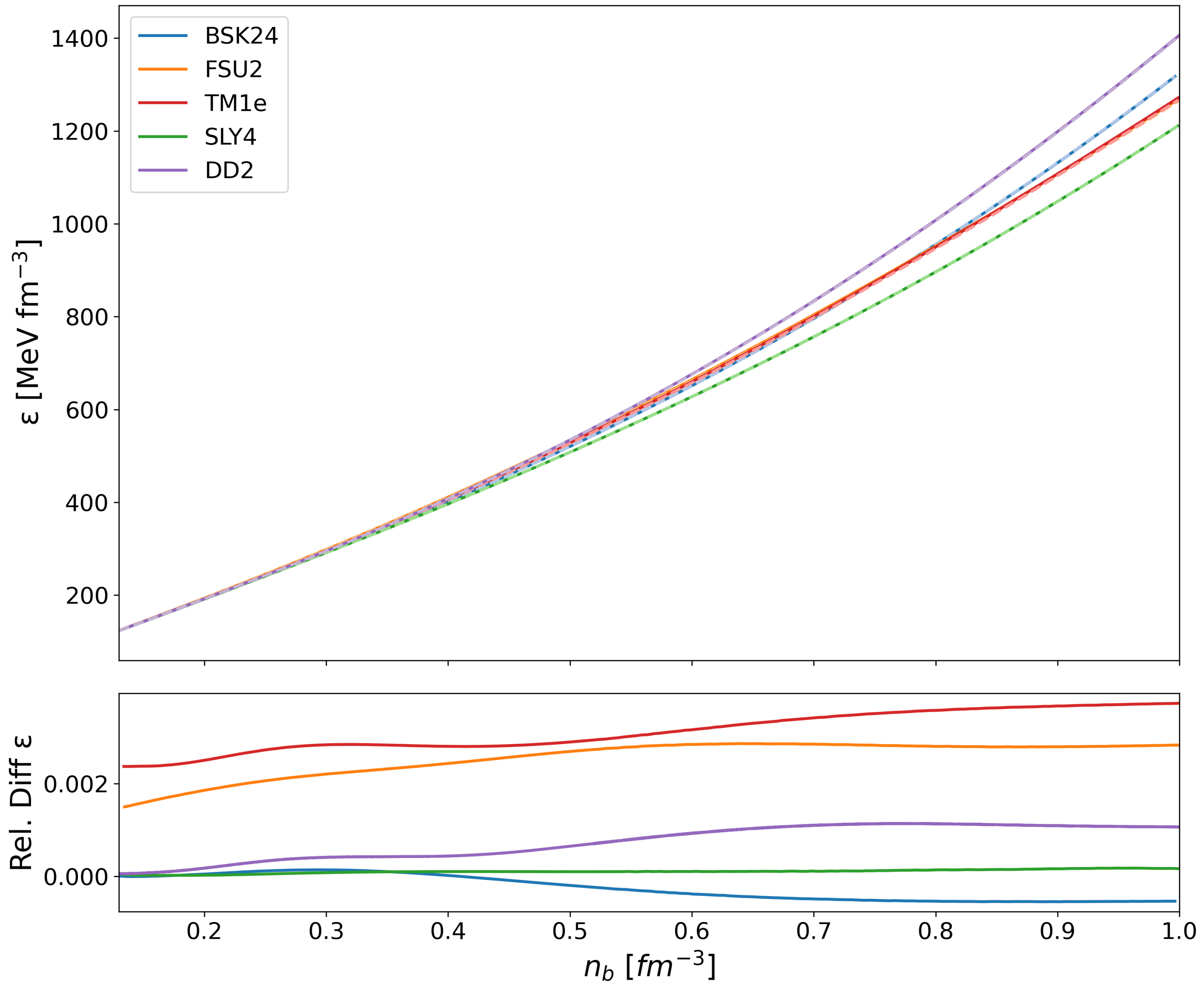
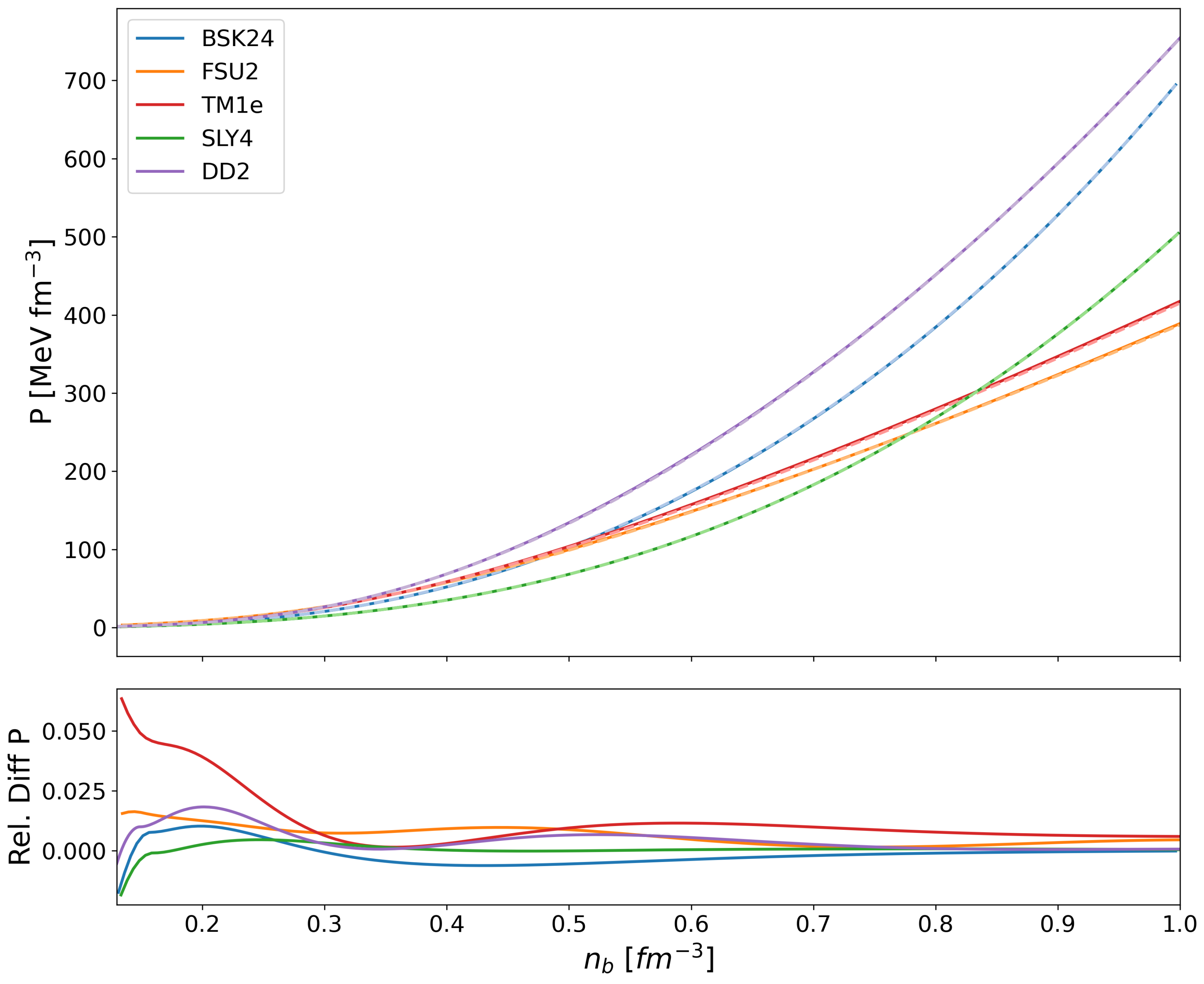
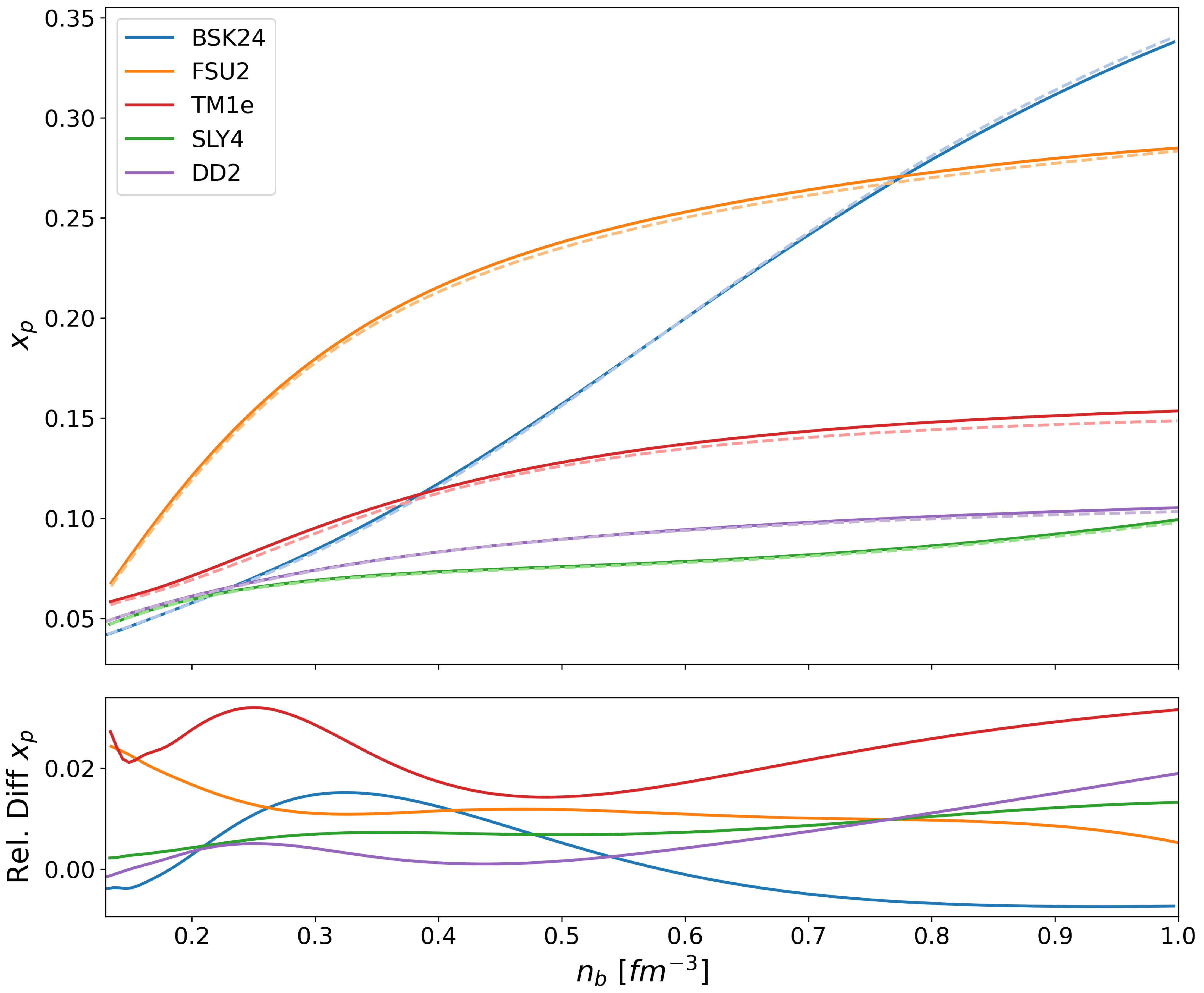


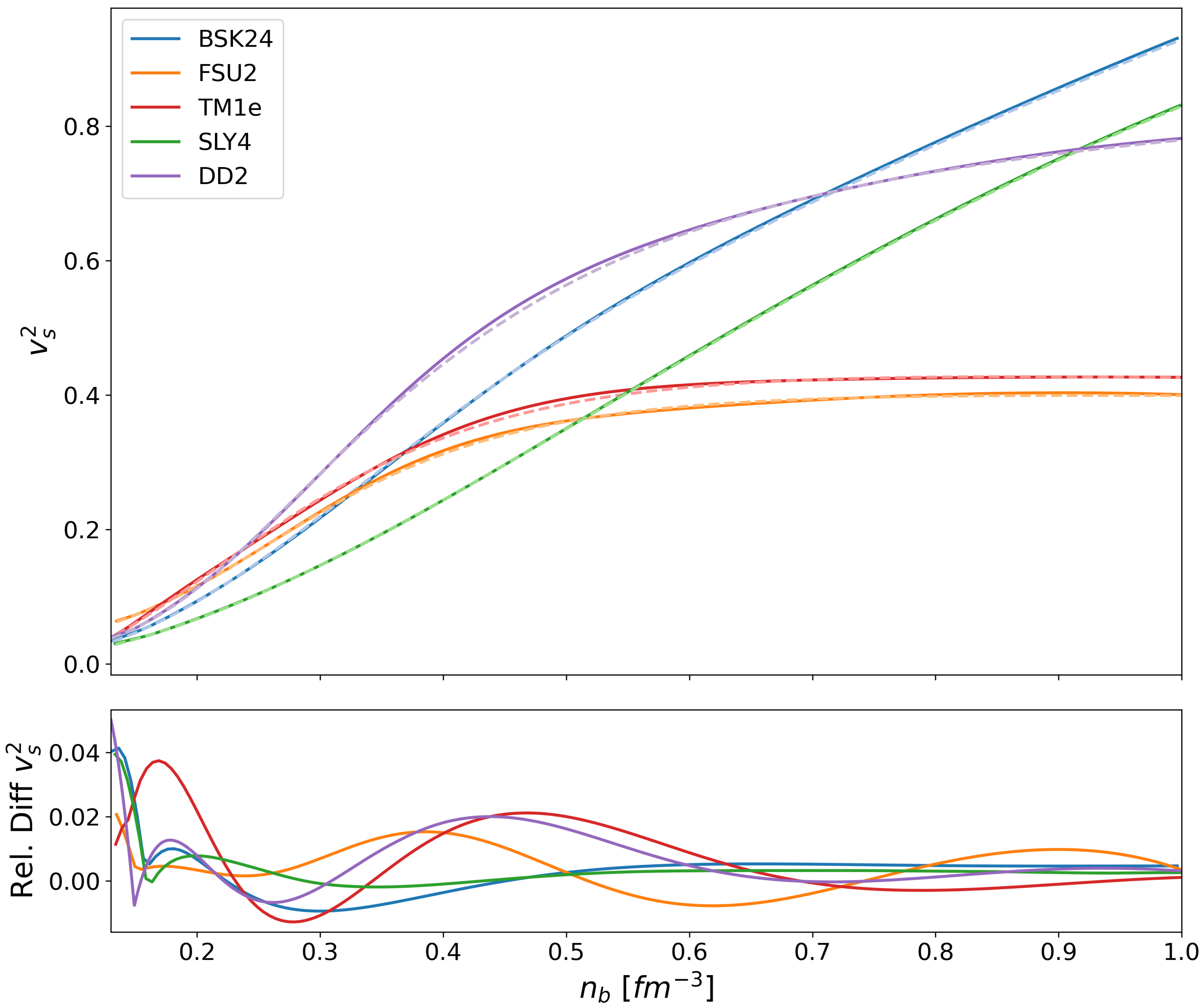


Almost causal meta-model:
EoS reconstruction

Proton fraction



Speed of sound



Bayes inference

$$\mathscr{M} : \mathbf{X} \rightarrow \{\epsilon(n_B), P(n_B), \delta(n_B), v_\beta(n_B), v_{FR}(n_B), \ldots\}$$

$$\mathscr{L}(\mathbf{X}) = \prod_j \mathscr{L}_j(\mathbf{X}) = \prod_j p\left(D_j \,|\, \mathscr{M}(\mathbf{X})\right)$$

Bayes inference

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Informed prior sampling the χ_{EFT} band[1] of PNM energy with a metropolis MCMC

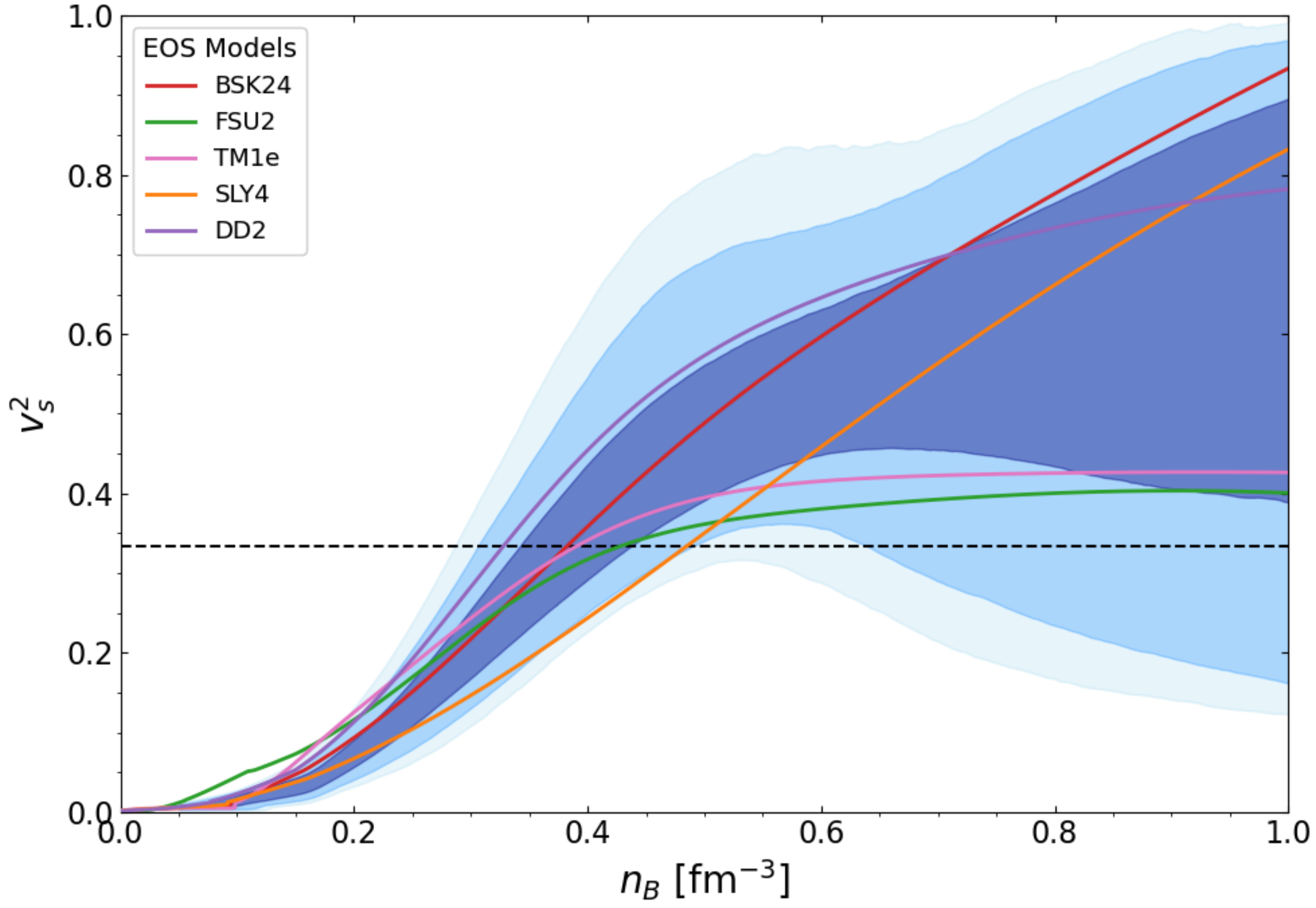
At this stage we have 10^9 models

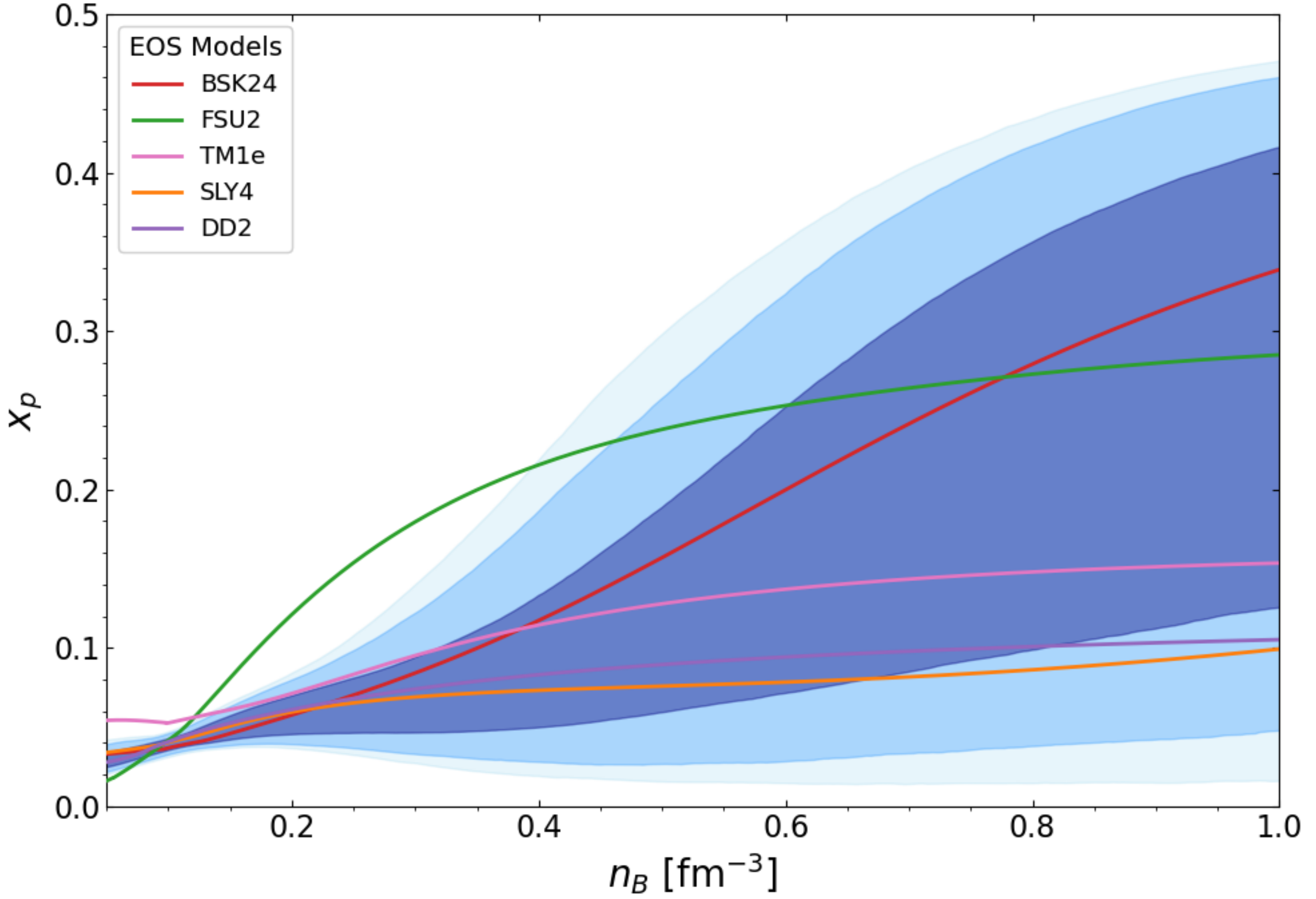
We extract 5×10^5 models that pass through the remaining filter:

- AME2020 nuclear masses table
- Maximum observed NS mass from radio-timing of PSRJ0348 and PSRJ0740
- Tidal deformability from GW170817 event detected by Ligo/Virgo collaboration
- NICER+XMN M-R measurements of PSRJ0030, PSRJ0347, PSRJ0614 and PSRJ0740

[1] Huth et al, 2021, Phys. Rev. C, 103, 025803

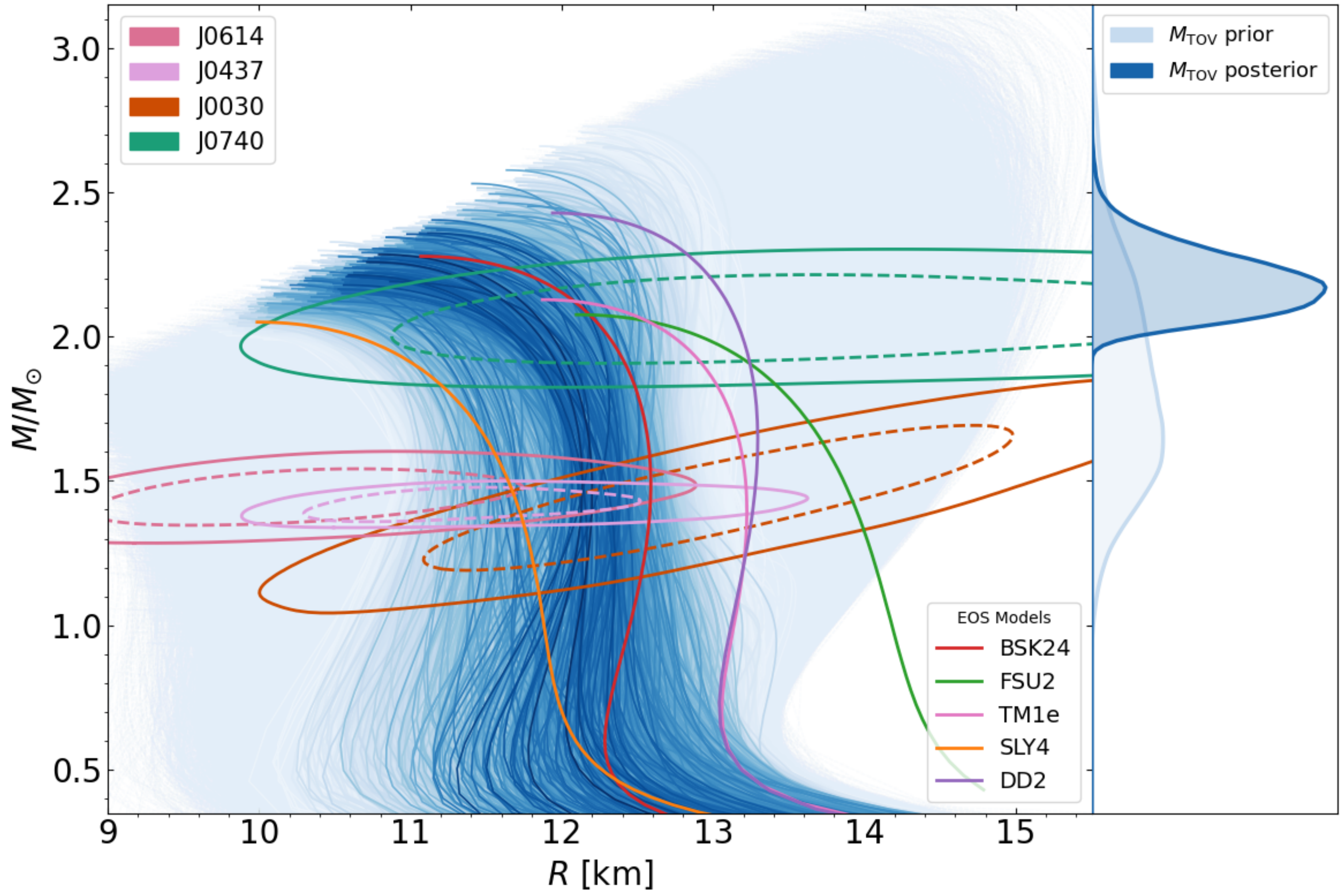
Composition and speed of sound





Mass Radius

We cover a wide range of masses and radii

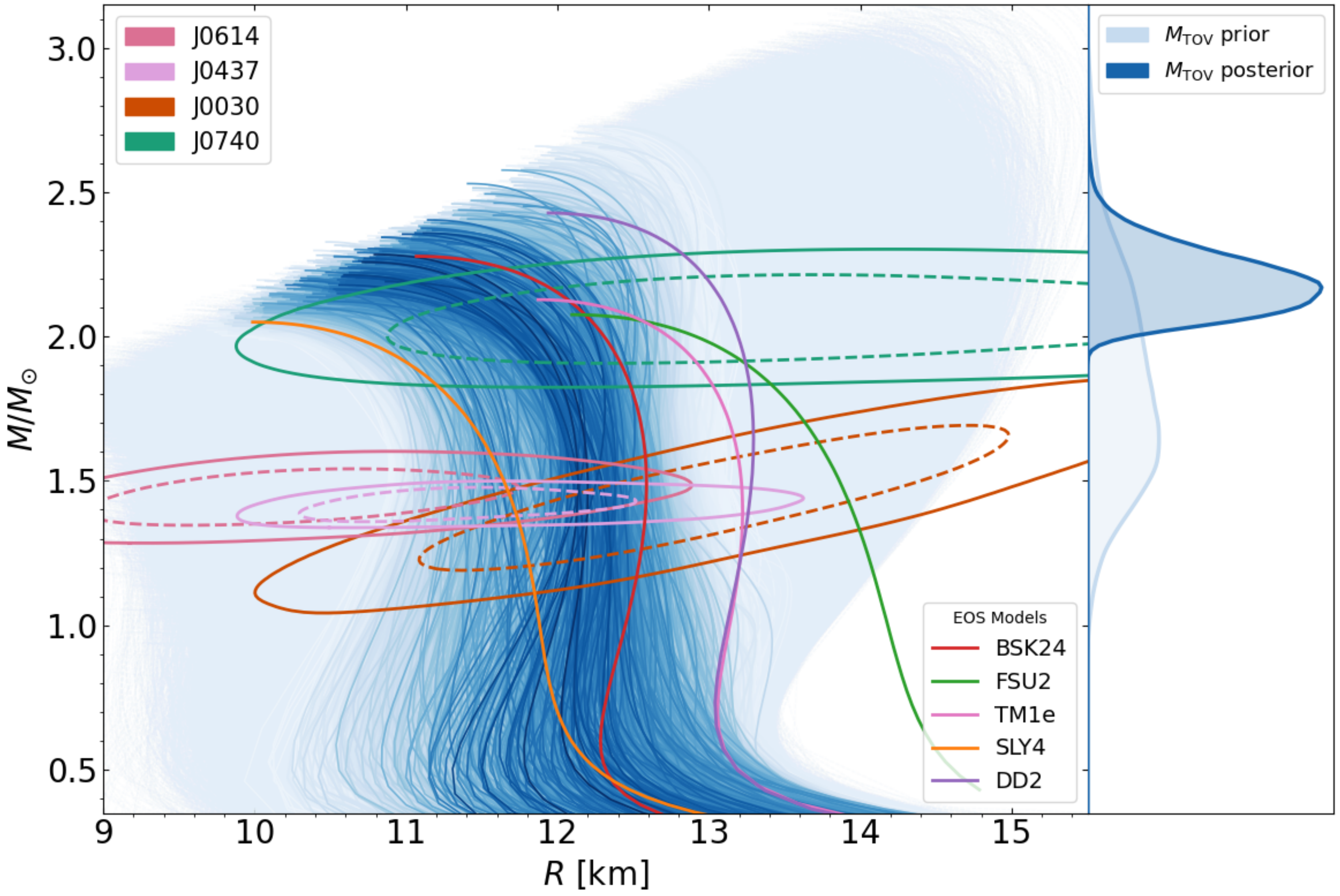


Mass Radius

We cover a wide range of masses and radii

The two newest nicer data suggest a soft EoS

$M_{TOV} > 2.5 M_{\odot}$ is disfavored



NICER softening

Summary

We propose an almost causal meta-model

– Its flexibility was tested by fitting five widely different nucleonic EoSs → It reproduces the energy density, pressure, vs2 and composition within few percent

– We have performed a bayesian inference including the latest nicer measurements → The model offers a wide range on NS features which is pushed towards the softer side from new Nicer results

BACKUP SLIDES

Fitting existing EoS

Test the flexibility of the model to reproduce β-equilibrated EoS

Constrain the space of the unphysical parameters

We have chosen: Sly4, BSK24, DD2, FSU2 and TM1e

Procedures

- Fix the NMP up to second order from COMPOSE
- Fix e_4 parameters from PNM expansion
- Keep e_4 fixed and fit e_0 at least up to n_{tov} on SM
- Repeat the same for e_2 on PNM while keeping both e_0 and e_4 fixed

FIT: results for SNM and PNM

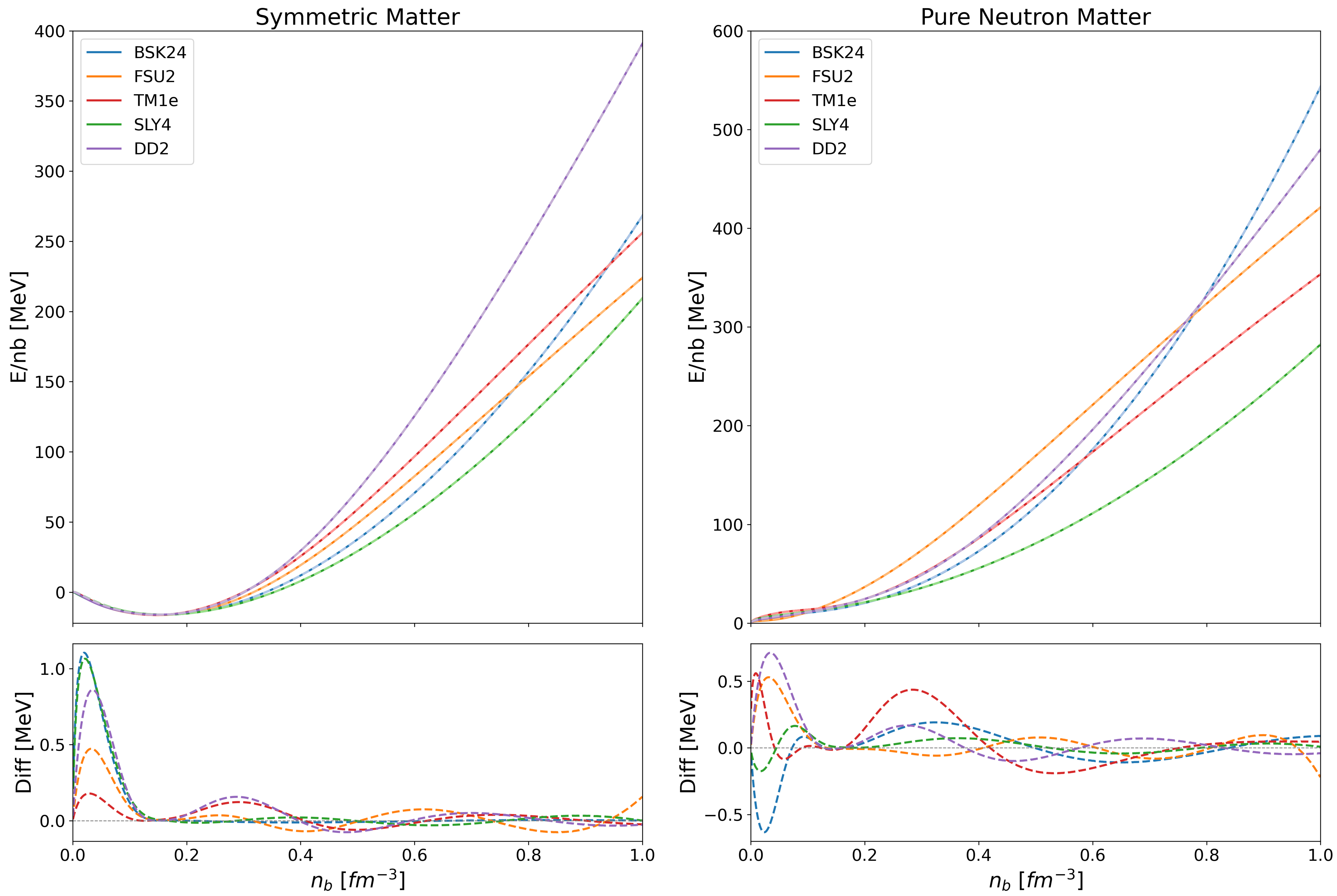
We found satisfying results with

$$g_{0,2} = 0$$

Less Paramaters!

Before saturation the accuracy is limited

Focus on astrophysics



Nucleonic potentials

$$e_0(x) = V_0(x) + \frac{h_0 + h_1 x + h_2 x^2 + h_3 x^3}{(1 + a_0 x)(1 + b_0 x)(1 + c_0 x)}$$

$$V_0(x) = \frac{s_0\, x^3}{1 + w_0 (3x + 1)^{3+g_0}}$$

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$h_{0,1,2}$ are fixed through a simple mapping with the NMP up to second order

h_3 controls the stiffness/softness at high density

a_0, b_0 and c_0 balance the numerator for causality

Nucleonic potentials

$$e_0(x) = V_0(x) + \frac{h_0 + h_1 x + h_2 x^2 + h_3 x^3}{(1 + a_0 x)(1 + b_0 x)(1 + c_0 x)}$$

$$V_0(x) = \frac{s_0\, x^3}{1 + w_0(3x + 1)^{3+g_0}}$$

s_0 is fixed by the request of the energy vanishing at $n = 0$

x is cubic to not modify the mapping with NMP up to second order

g_0 takes care of causality while w_0 tunes the dominant range of the correction

e_2 is built with the same structure

Quartic correction in δ for the PNM

E_{sym} / J, L_{sym} and K_{sym} are usually defined starting from:

$$\left.\frac{\partial^2 e}{\partial \delta^2}\right|_{x=0,\delta=0}$$

By fixing the "sym" parameters of a given EoS with the quadratic expansion in δ, we would not reproduce the PNM ($\delta = 1$) around saturation density

We introduce a term in the energy density to correct this behavior:

$$e_4(n) = A\frac{n/n_0}{1+\left(n/n_0\right)^B}$$

Metamodel representation of the nucleonic EoS

A possible scheme could be

$$\epsilon_X(n_n, n_p) = \sum_{ij} \alpha_{ij} \, n_n^{\xi_i} \, n_p^{\theta_j} \, ,$$

where $X = (\alpha_{ij}, \xi_i, \theta_j)$ can be set to reproduce a given known function $\epsilon(n_n, n_p)$ or to explore novel energy density behaviors beyond those established in the literature

All the other relevant zero-temperature thermodynamic quantities are obtained in the standard way:

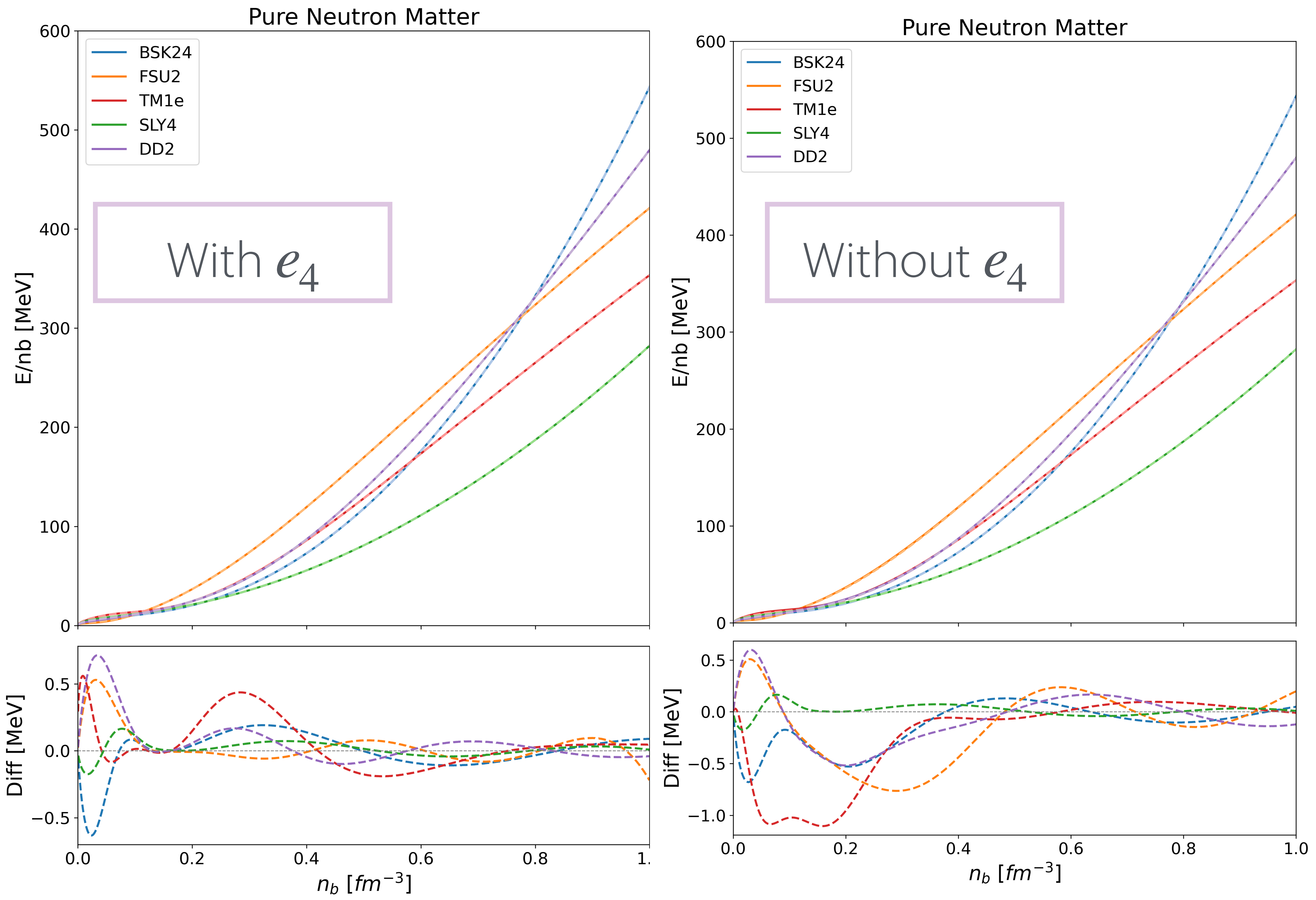
$$\mu_X^q(n_n, n_p) = \partial_q \epsilon_X, \quad P_X(n_n, n_p) = \sum_{q=n,p} n_q \mu_X^q - \epsilon_X,$$

where $q = n, p$

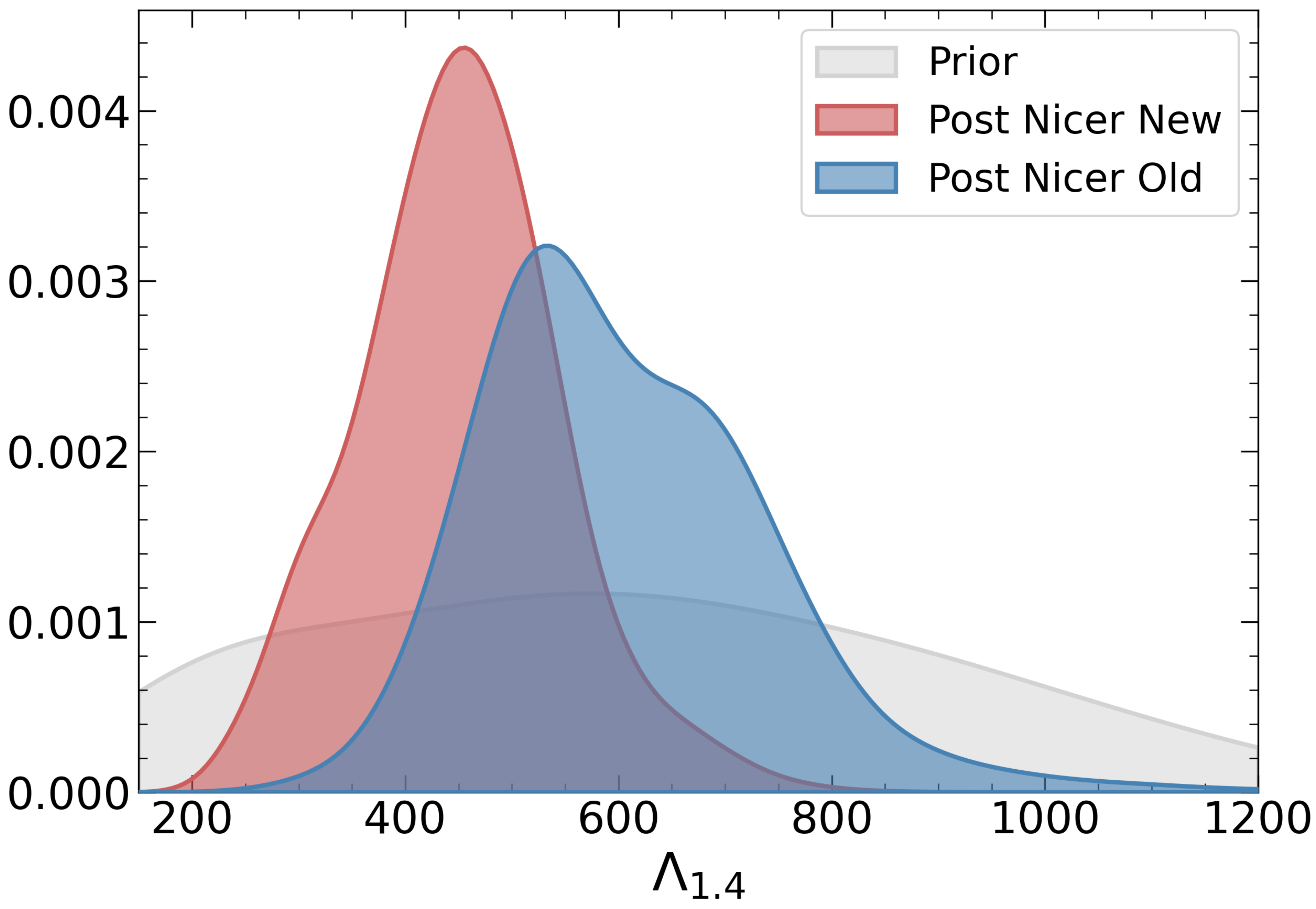
Fit without the linear δ correction

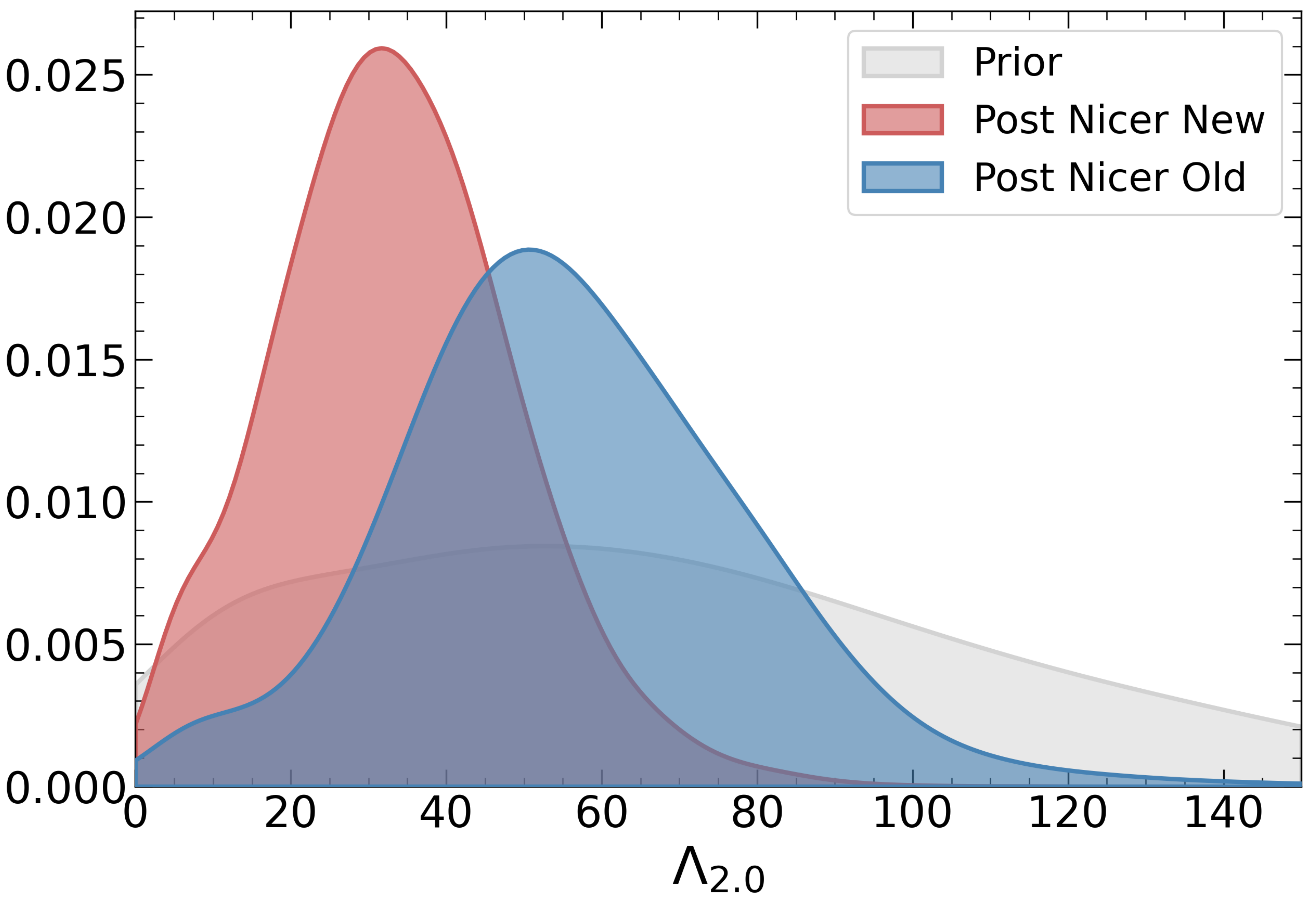
Without the correction around saturation the PNM fit exhibit a $\sim 0.5/1$ MeV of difference

The overall accuracy doesn't change



Nicer old vs nicer new: Tidal deformability





The quest for Nuclear EoS: Complementing with astro-observables

Nuclear experiment

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χ_{EFT}

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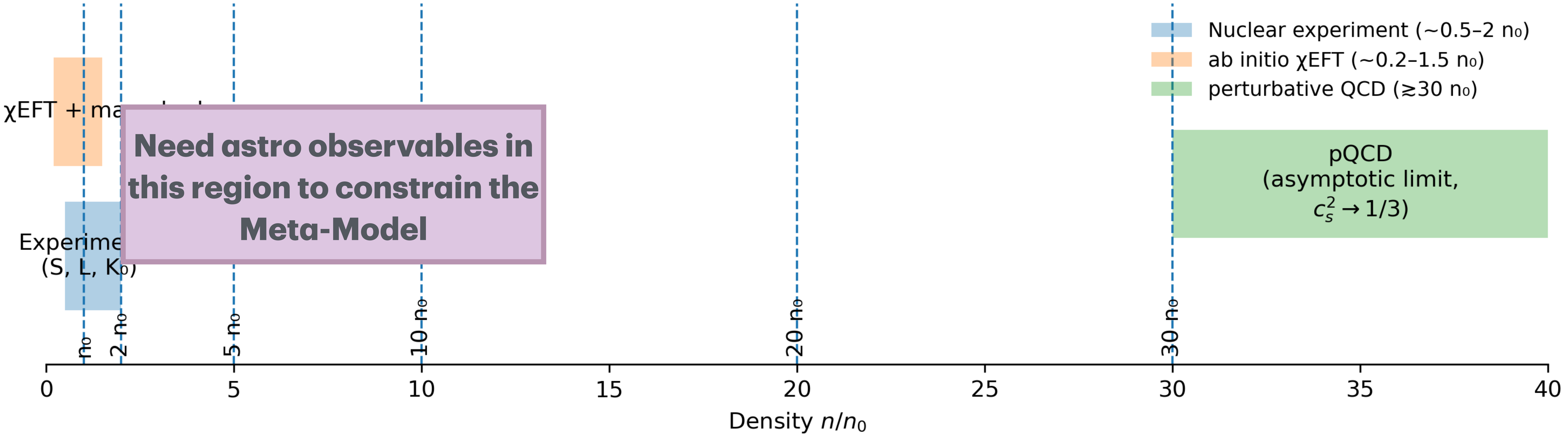
pQCD

$> 30\ n_0$



Need astro observables in this region to constrain the Meta-Model