Theoretical predictions for tau-pair production in ultraperipheral heavy-ion collisions

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In collaboration with Stefan Dittmaier, Mathieu Pellen, and Tim Engel

[arxiv: 2504.11391, to appear in JHEP]

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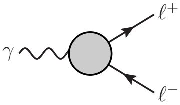

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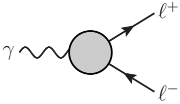

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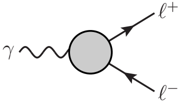

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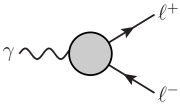

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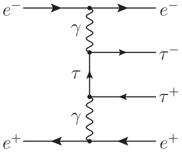
- $a_\tau \rightarrow$ No precise determination ($\tau_\tau = 2.9 \cdot 10^{-13}$ s)
  [A. Keshavarzi, D. Nomura, and T.Teubner, 2020]

$$-0.0042 < a_\tau < 0.0062$$ [CMS, 2024]

- Challenging experimental determination due to the small lifetime of the τ-lepton, $\tau_\tau = 2.9 \cdot 10^{-13}\,\mathrm{s}$

LEP

[DELPHI, 2004]



$-0.052 < a_\tau < 0.013$

LHC

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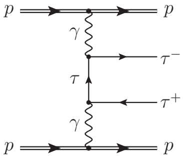

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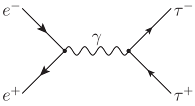
SuperKEKB

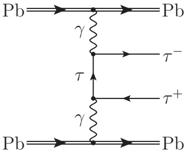
[Belle II, 2024]



Not competitive

LHC

[ATLAS, 2022; CMS, 2022]



Competitive with LEP

Experimental determination of a_τ

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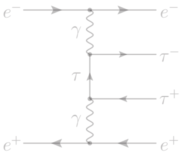

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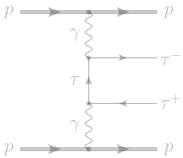

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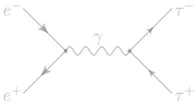

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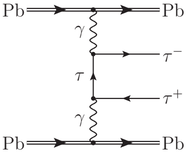
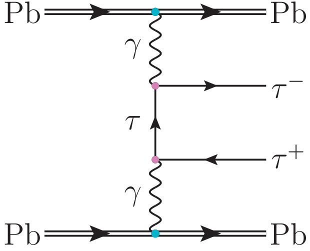


Competitive with LEP

Ultraperipheral collisions (UPC)



- Two $\gamma\tau\tau$ vertices
  ↪ Larger sensitivity to a_τ
- Elastic collision (nuclei do not break up)
  ↪ Clean final state
- Long-distance interaction
  ↪ Photons with low virtuality
- Photon flux $\propto Z^2$
  ↪ Cross section $\propto Z^4$

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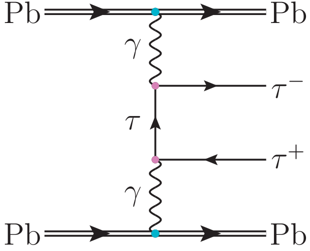

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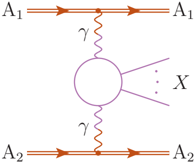
- Previous studies
  ↪ Sensitivity to a_τ (effective $\gamma\tau\tau$ coupling)
  [F. del Aguila, *et al.*, 1991] [S. Atag and A. A. Billur, 2010] [L. Beresford and J. Liu, 2019] [M. Dyndal, *et al.*, 2020] [M. Verducci, *et al.*, 2024]
  ↪ Higher-order corrections ($\mathrm{PbPb} \to \mathrm{PbPb}\tau^+\tau^-$)
  [H.-S. Shao and D. d'Enterria, 2024] [J. Jiang, *et al.*, 2024] [H.-S. Shao and L. Simon, 2025]

Equivalent-photon approximation

- Equivalent-photon approximation (EPA)
  - ↪ Provides a theoretical framework for the treatment of UPCs
  - ↪ Describes the electromagnetic field of the accelerated charged particle as a flux of quasireal photons [C. F. von Weizsacker, 1934] [E. J. Williams, 1934]



$$\sigma\left(\sqrt{s_{\mathrm{A}_1\mathrm{A}_2}}\right) = \int \frac{\mathrm{d}E_{\gamma_1}}{E_{\gamma_1}} \frac{\mathrm{d}E_{\gamma_2}}{E_{\gamma_2}} \frac{\mathrm{d}^2 N^{(\mathrm{A}_1\mathrm{A}_2)}_{\gamma_1/Z_1,\gamma_2/Z_2}}{\mathrm{d}E_{\gamma_1}\mathrm{d}E_{\gamma_2}} \hat{\sigma}\left(\sqrt{s_{\gamma\gamma}}\right)$$

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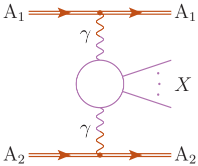

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- Photon flux
  - ↪ Probability of γ_1 being emitted by $\mathrm{A_1}$ with an energy E_{γ_1} and γ_2 being emitted by $\mathrm{A_2}$ with an energy E_{γ_2} without breaking up the ions
  - ↪ Computed with `gamma-UPC` using the charge form factor of the ions [H.-S. Shao and D. d'Enterria, 2022]

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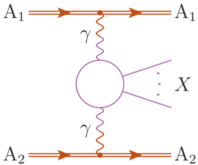

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- Hard process
  - ↪ Describes the production of a final state X via photon–photon scattering, *e.g.* $\gamma\gamma \to \tau^+\tau^-$

Summary of perturbation theory

- Perturbation theory
  - ↪ Compute observables as a perturbative expansion in small couplings
  - ↪ Perturbation parameters: $\alpha \sim 0.01$ and $\alpha_\mathrm{s} \sim 0.1$
  - ↪ Cross section $\hat{\sigma}$

$$\hat{\sigma} = \hat{\sigma}^{\mathrm{LO}} \left(1 + \delta_\mathrm{s}^{(1)} + \delta_\mathrm{s}^{(2)} + \delta_\mathrm{EW}^{(1)} + \ldots\right)$$

- LO contribution $\hat{\sigma}^{\mathrm{LO}}$

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- LO contribution $\hat{\sigma}^{\mathrm{LO}}$
  ↪ Provides the bulk of the prediction

$$\hat{\sigma}^{\mathrm{LO}} = \frac{1}{F} \int \mathrm{d}\Phi_n \overline{\left|\mathcal{M}^{(0)}(\Phi_n)\right|^2}$$

  · F: Flux factor
  · $\mathrm{d}\Phi_n$: Differential phase-space volume
  · $\mathcal{M}^{(0)}$: LO matrix element

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↪ Small corrections to the LO contribution

$$\delta = \frac{\hat{\sigma}^{\mathrm{NLO}} - \hat{\sigma}^{\mathrm{LO}}}{\hat{\sigma}^{\mathrm{LO}}}$$

↪ Include extra powers in the perturbative parameter

- NLO QCD correction $\delta_\mathrm{s}^{(1)} \propto \alpha_\mathrm{s} \sim 10\,\%$
- NNLO QCD correction $\delta_\mathrm{s}^{(2)} \propto \alpha_\mathrm{s}^2 \sim 1\,\%$
- NLO EW correction $\delta_\mathrm{EW}^{(1)} \propto \alpha \sim 1\,\%$

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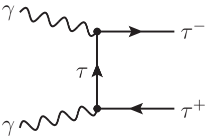
Hard process

- Hard process: $\gamma\gamma \to \tau^+\tau^-$



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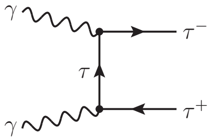

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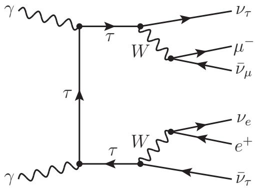

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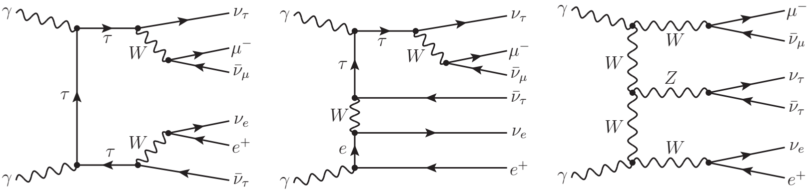

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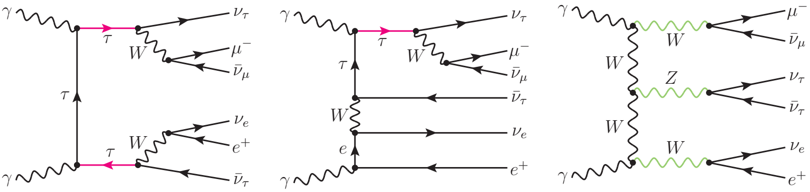

- Classification attending to the resonant pattern

$$\frac{1}{|p^2-m^2+im\Gamma|^2} \underset{\Gamma \ll m}{\widetilde{\ \ }} \frac{\pi}{m\Gamma}\delta(p^2-m^2) + \mathcal{O}\left(\frac{\Gamma}{m}\right)$$

- τ-resonance
  - $\hookrightarrow$ Missing $\Rightarrow$ suppression of $\Gamma_\tau/m_\tau \sim 10^{-12}$
- Weak-gauge-boson resonance
  - $\hookrightarrow$ Enhancement of $M_V/\Gamma_V \sim 40$ ($V=$ W,Z)
  - $\hookrightarrow$ Photon-flux suppression ($s_{\gamma\gamma} \gtrsim M_{\mathrm{W}}^2$)

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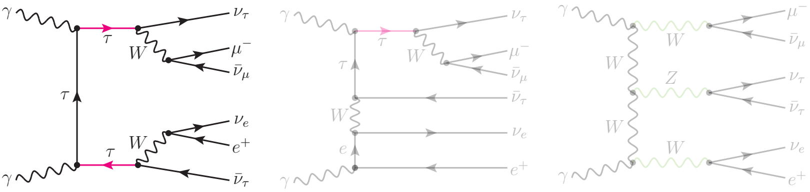

- Classification according to the resonant pattern

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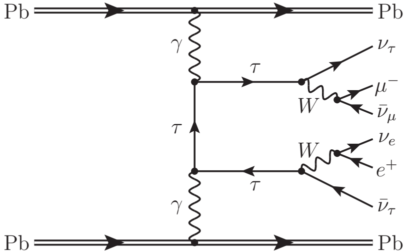
- Main contribution: $\gamma\gamma \to \tau^+\tau^- \to e^+\mu^-\bar{\nu}_\tau\nu_\tau\bar{\nu}_\mu\nu_e$

Process

- Process: $\gamma\gamma \to \tau^+\tau^- \to e^+\mu^-\bar{\nu}_\tau\nu_\tau\bar{\nu}_\mu\nu_e$ induced by UPCs of lead ions
  $\hookrightarrow \sqrt{s_{\mathrm{PbPb}}} = 5.02\,\mathrm{TeV}$



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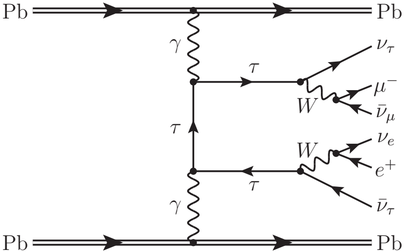
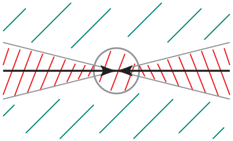


- Fiducial phase-space region ($\ell = \mu, e$) [ATLAS,2022]

  - $|\eta_\ell| < 2.5$
  - $p_{\mathrm{T},\ell} > 4\,\mathrm{GeV}$



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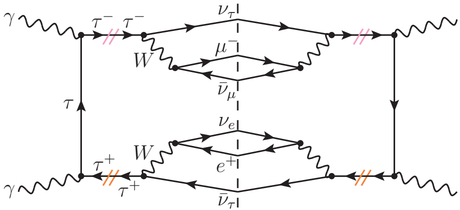
Narrow-width approximation

- Narrow-width approximation (NWA)
  ↪ Keeps only the main contributions $\gamma\gamma \to \tau^+\tau^- \to e^+\mu^-\bar{\nu}_\tau\nu_\tau\bar{\nu}_\mu\nu_e$
  ↪ Takes the narrow-width limit $\Gamma_\tau/m_\tau \to 0$ $(\Gamma_\tau/m_\tau \sim 10^{-12})$

$$\frac{1}{|p_\tau^2 - m_\tau^2 + \mathrm{i}m_\tau\Gamma_\tau|^2} \sim \frac{\pi}{m_\tau\Gamma_\tau}\delta(p_\tau^2 - m_\tau^2)$$



$$\overline{|\mathcal{M}|^2} \xrightarrow[\text{NWA}]{} \left(\frac{\pi}{m_\tau\Gamma_\tau}\right)\left(\frac{\pi}{m_\tau\Gamma_\tau}\right)\overline{|\widetilde{\mathcal{M}}|^2}\delta(p_\tau^2 - m_\tau^2)\delta(\bar{p}_\tau^2 - m_\tau^2)$$

Naive vs. improved NWA

- Narrow-width approximation (NWA)

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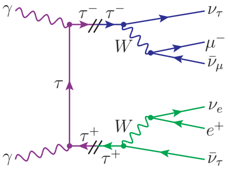

- Naive NWA

  ↪ Does not transfer the spin information of the τ-leptons to the decays

$$\overline{|\widetilde{\mathcal{M}}_{\text{NWA}}|^2} = \overline{|\mathcal{M}_{\text{P}}|^2}\;\overline{|\mathcal{M}_{\text{D}}|^2}\;\overline{|\mathcal{M}_{\overline{\text{D}}}|^2}$$

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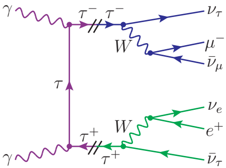

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- Improved NWA
  - $\hookrightarrow$ Includes spin correlations between decaying τ-leptons

$$\overline{|\widetilde{\mathcal{M}}_{\mathrm{iNWA}}|^2} = \overline{\left|\mathcal{A}_{\mathrm{D}}\left(\not{p}_\tau + m_\tau\right)\mathcal{A}_{\mathrm{P}}\left(\not{\bar{p}}_\tau - m_\tau\right)\mathcal{A}_{\overline{\mathrm{D}}}\right|^2}$$

$$= \overline{\left|\mathcal{A}_{\mathrm{D}}\left(\textstyle\sum_\sigma u_\sigma \bar{u}_\sigma\right)\mathcal{A}_{\mathrm{P}}\left(\textstyle\sum_{\bar{\sigma}} v_{\bar{\sigma}}\bar{v}_{\bar{\sigma}}\right)\mathcal{A}_{\overline{\mathrm{D}}}\right|^2}$$

$$\widetilde{\mathcal{M}}_{\mathrm{iNWA}} = \textstyle\sum_{\sigma,\bar{\sigma}} \mathcal{M}_{\mathrm{P},\sigma\bar{\sigma}}\mathcal{M}_{\mathrm{D},\sigma}\mathcal{M}_{\overline{\mathrm{D}},\bar{\sigma}}$$

  - $\hookrightarrow$ Relevant if the final-state kinematics are not fully integrated over

Numerical results

- Spin-correlations effects: $\Delta_{\text{spin}} = \frac{\sigma^{\text{LO}}_{\text{NWA}} - \sigma^{\text{LO}}_{\text{iNWA}}}{\sigma^{\text{LO}}_{\text{iNWA}}}$

  - Fiducial cross section

	σ^{LO} [nb]	Δ_{spin} [%]
Spin corr.	45.869(4)	-
No spin corr.	43.282(4)	$-5.64(1)$

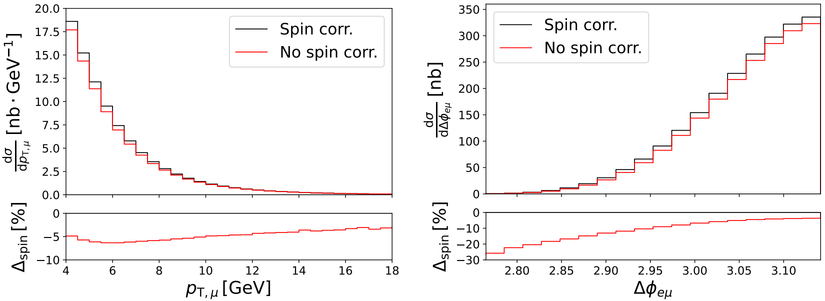
Numerical results

- Spin-correlations effects: $\Delta_{\text{spin}} = \frac{\sigma^{\text{LO}}_{\text{NWA}} - \sigma^{\text{LO}}_{\text{iNWA}}}{\sigma^{\text{LO}}_{\text{iNWA}}}$

  - Fiducial cross section

	σ^{LO} [nb]	Δ_{spin} [%]
Spin corr.	45.869(4)	-
No spin corr.	43.282(4)	$-5.64(1)$

  - Differential distributions



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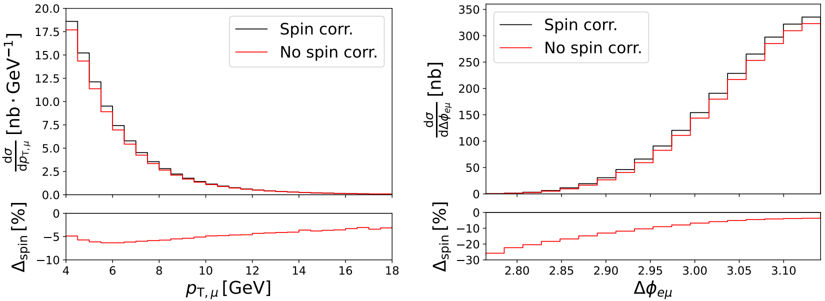

- NLO EW corrections $\mathcal{O}(1\,\%)$

  $\hookrightarrow$ Necessary in a precise study of $\gamma\gamma \to \tau^+\tau^- \to e^+\mu^-\bar{\nu}_\tau\nu_\tau\bar{\nu}_\mu\nu_e$

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Next-to-leading-order corrections

- Perturbation expansion for $\hat{\sigma}$

$$\hat{\sigma} = \hat{\sigma}^{\mathrm{LO}}\left(1 + \delta_{\mathrm{s}}^{(1)} + \delta_{\mathrm{s}}^{(2)} + \delta_{\mathrm{EW}}^{(1)} + \dots\right)$$

- LO contribution $\hat{\sigma}^{\mathrm{LO}}$

- Higher-order corrections δ

$\hookrightarrow$ No quark nor gluon lines in LO diagrams $\Rightarrow \delta_{\mathrm{s}}^{(1)} = \delta_{\mathrm{s}}^{(2)} = 0$

$\hookrightarrow$ NLO corrections $\rightarrow$ EW corrections $\delta_{\mathrm{EW}}^{(1)}$

Next-to-leading-order corrections

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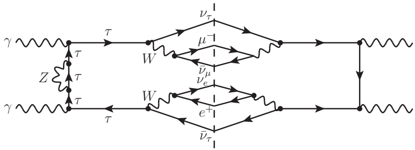
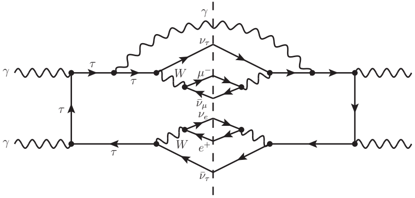
↪ NLO corrections → EW corrections $\delta_{\mathrm{EW}}^{(1)}$

One-loop corrections



Real-emission corrections



One-loop corrections

- Narrow-width approximation

  ↪ Keeps only the corrections to $\gamma\gamma \to \tau^+\tau^- \to e^+\mu^-\bar{\nu}_\tau\nu_\tau\bar{\nu}_\mu\nu_e$ that can be factorized into corrections to τ-pair production and corrections to τ-decays
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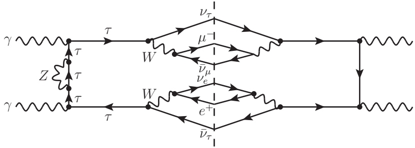

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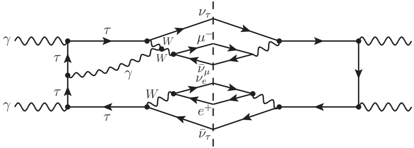

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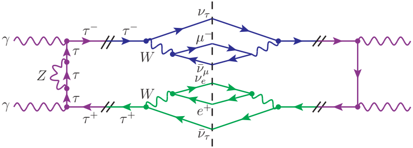

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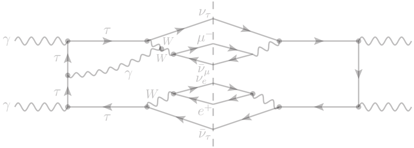

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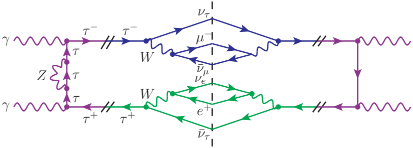

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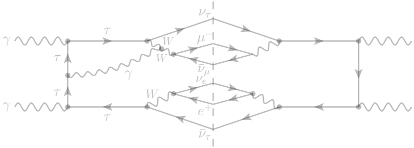
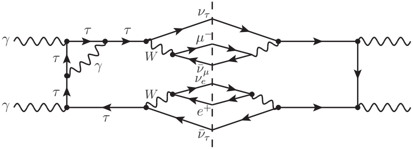
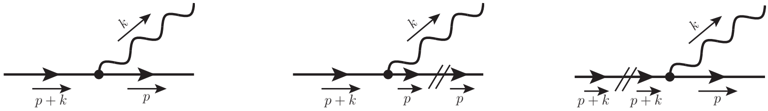




Partial fractioning

- Used to split diagrams

$$\frac{1}{[(p+k)^2-m^2][p^2-m^2]} = \frac{1}{(2pk)[p^2-m^2]} - \frac{1}{[(p+k)^2-m^2](2pk)}$$



One-loop corrections

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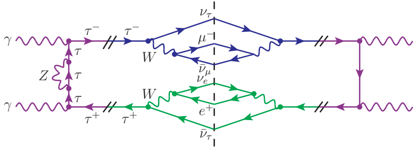

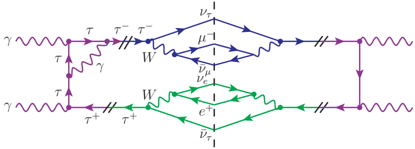

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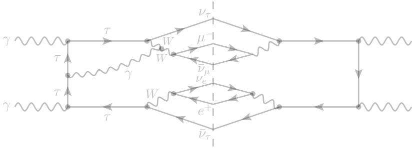

. . .

Real-emission corrections

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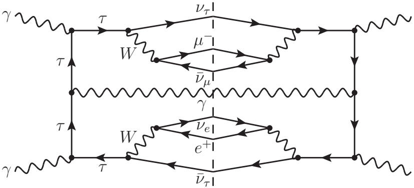

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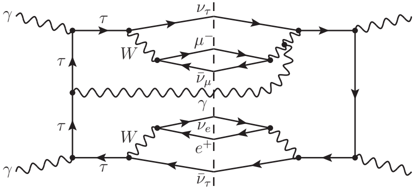

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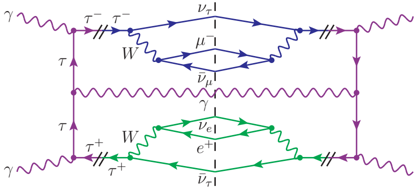

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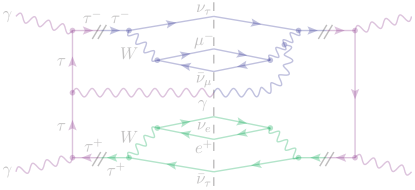

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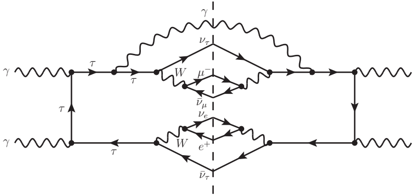
Factorizable — Non-factorizable



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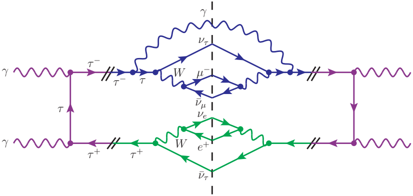	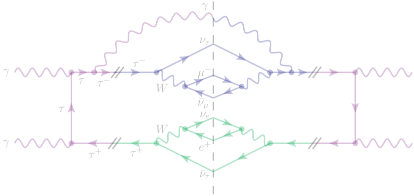
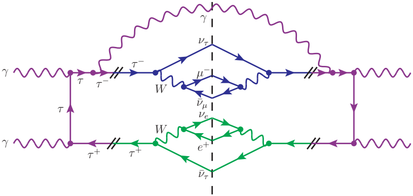	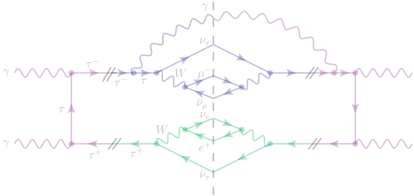

Numerical results

- Relative next-to-leading-order correction: $\delta = \frac{\sigma^{\mathrm{NLO}} - \sigma^{\mathrm{LO}}}{\sigma^{\mathrm{LO}}}$

  - Fiducial cross section

	σ [nb]	δ [%]
LO	45.869(4)	-
NLO EW	45.327(4)	$-1.182(1)$

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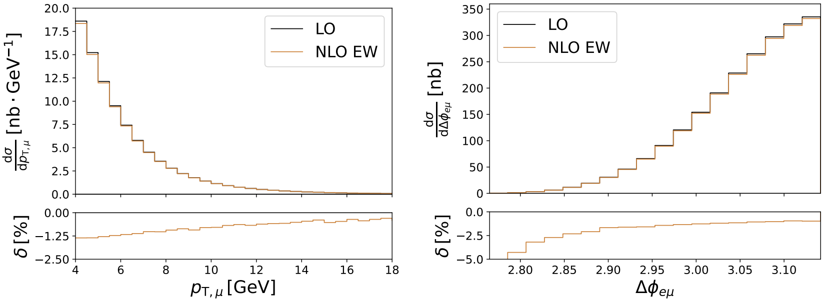

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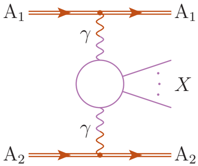
- Equivalent-photon approximation (EPA)
  - ↪ Provides a theoretical framework for the treatment of UPCs
  - ↪ Describes the electromagnetic field of the accelerated charged particle as a flux of quasireal photons [C. F. von Weizsacker, 1934] [R. N. Cahn and J. D. Jackson, 1990]



$$\sigma\left(\sqrt{s_{\mathrm{A_1A_2}}}\right) = \int \frac{\mathrm{d}E_{\gamma_1}}{E_{\gamma_1}} \frac{\mathrm{d}E_{\gamma_2}}{E_{\gamma_2}} \frac{\mathrm{d}^2 N^{(\mathrm{A_1A_2})}_{\gamma_1/Z_1,\gamma_2/Z_2}}{\mathrm{d}E_{\gamma_1}\mathrm{d}E_{\gamma_2}} \hat{\sigma}\left(\sqrt{s_{\gamma\gamma}}\right)$$

- Photon flux
  - ↪ Probability of γ_1 being emitted by A$_1$ with an energy E_{γ_1} and γ_2 being emitted by A$_2$ with an energy E_{γ_2} without breaking up the ions
  - ↪ Computed with `gamma-UPC` using the charge form factor of the ions [H.-S. Shao and D. d'Enterria, 2022]
- Hard process
  - ↪ Describes the production of a final state X via photon–photon scattering, *e.g.* $\gamma\gamma \to \tau^+\tau^-$

Photon flux

- Photon flux

  ↪ Probability of γ_1 being emitted by A_1 with an energy E_{γ_1} and γ_2 being emitted by A_2 with an energy E_{γ_2} without breaking up the ions

$$\frac{\mathrm{d}^2 N^{(\mathrm{A}_1\mathrm{A}_2)}_{\gamma_1/Z_1,\gamma_2/Z_2}}{\mathrm{d}E_1\mathrm{d}E_2} = \int \mathrm{d}^2\mathbf{b}_1\mathrm{d}^2\mathbf{b}_2\, P_{\mathrm{elas}}(\mathbf{b}_1,\mathbf{b}_2)\, N_{\gamma_1/Z_1}(E_{\gamma_1},\mathbf{b}_1)\, N_{\gamma_2/Z_2}(E_{\gamma_2},\mathbf{b}_2)$$

- Probability of elastic scattering P_{elas}

  ↪ Probability of hadrons A_1 and A_2 to remain intact after the interaction at given impact parameters $\mathbf{b}_1$ and $\mathbf{b}_2$

- Photon number density N_{γ_i/Z_i}

  ↪ Probability of γ_i being emitted with an energy E_{γ_i} by A_i at impact parameter $\mathbf{b}_i$

  ↪ Can be parameterized using the **Charge Form Factor (ChFF)** or the Electric Dipole Form Factor (EDFF) of the ion A_i

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Numerical results

- Different parametrization of the photon flux: $\Delta_{\mathrm{PF}} = \frac{\sigma^{\mathrm{LO}}_{\mathrm{EDFF}} - \sigma^{\mathrm{LO}}_{\mathrm{ChFF}}}{\sigma^{\mathrm{LO}}_{\mathrm{ChFF}}}$

  - Fiducial cross section

	σ^{LO} [nb]	Δ_{PF} [%]
ChFF	45.87(1)	-
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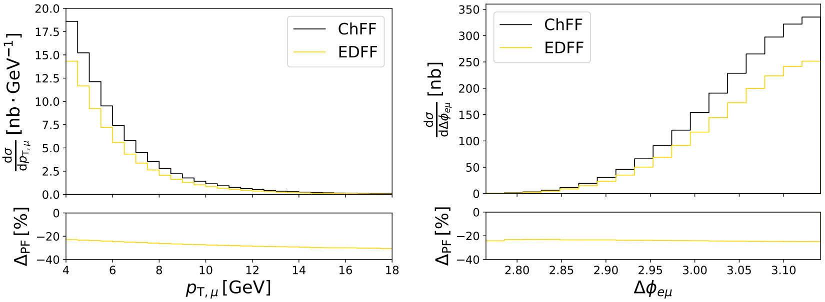

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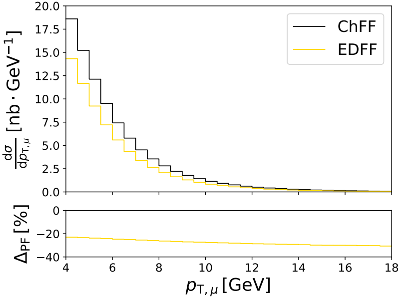
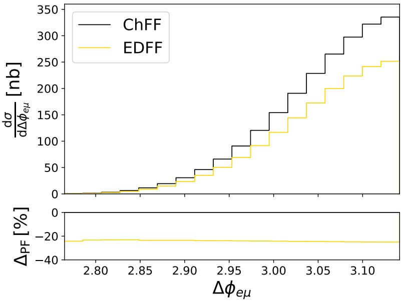




- Largest source of uncertainty $\sim 25\,\%$
  ↪ Reduce it by using ratios of cross sections

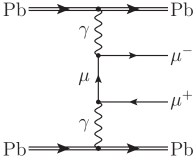
Auxiliary observable

- The parametrization of the photon flux as largest source of uncertainty

  ↪ Define observables based on ratios of cross sections

$$\mathcal{O} = \frac{\sigma^{\mathrm{LO}}}{\sigma^{\mathrm{LO}}_{\mu\mu}} \qquad\qquad \frac{\mathrm{d}\mathcal{O}}{\mathrm{d}X} = \frac{1}{\sigma^{\mathrm{LO}}_{\mu\mu}} \frac{\mathrm{d}\sigma^{\mathrm{LO}}}{\mathrm{d}X}$$

- $\sigma^{\mathrm{LO}}_{\mu\mu}$: LO cross section for $\gamma\gamma \to \mu^+\mu^-$ induced by UPCs of lead ions



· $p_{\mathrm{T},\mu} > 4\,\mathrm{GeV}$

· $|\eta_\mu| < 2.5$

Auxiliary observable

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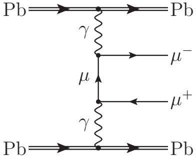

· $p_{\mathrm{T},\mu} > 4\,\mathrm{GeV}$

· $|\eta_\mu| < 2.5$

- Numerical results $\sigma^{\mathrm{LO}}_{\mu\mu}$

	$\sigma^{\mathrm{LO}}_{\mu\mu}$ [μb]	Δ_{PF} [%]
ChFF	57.24(2)	-
EDFF	45.64(1)	$-20.28(1)$

Numerical results

- Different parametrization of the photon flux: $\Delta_{\rm PF} = \frac{\sigma^{\rm LO}_{\rm EDFF} - \sigma^{\rm LO}_{\rm ChFF}}{\sigma^{\rm LO}_{\rm ChFF}}$

  - Auxiliary observable

	$\mathcal{O}^{\rm LO} \cdot 10^4$	$\Delta_{\rm PF}$ [%]
ChFF	8.013(3)	-
EDFF	7.584(3)	$-5.36(5)$

Numerical results

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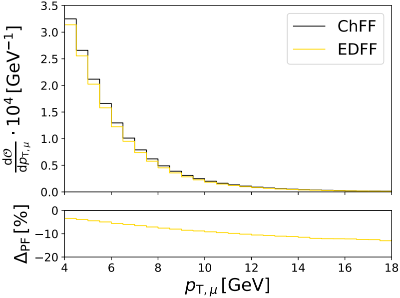

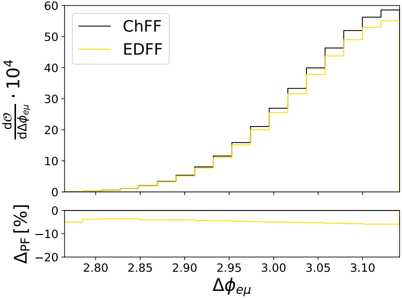

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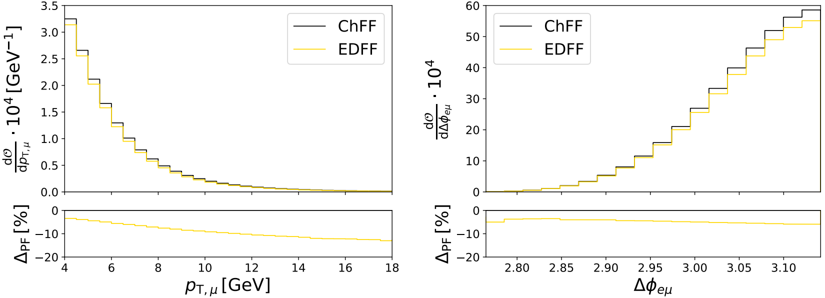

- Uncertainty reduce to $\sim 5\,\%$

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Conclusions

- In this talk

  $\hookrightarrow$ τ-pair production in UPCs $\rightarrow$ determination of a_τ

  $\hookrightarrow$ We provide state-of-the-art SM prediction assuming leptonic τ-decays

  $\hookrightarrow$ Predictions used in the on-going ATLAS analyses

  $\hookrightarrow$ Spin-correlation effects $\sim 5\,\%$

  $\hookrightarrow$ NLO EW corrections $\sim 1\,\%$

  $\hookrightarrow$ Parametrization of the photon flux $\sim 25\,\%$ $\rightarrow$ $\sim 5\,\%$

Conclusions

- In this talk
  - ↪ τ-pair production in UPCs → determination of a_τ
  - ↪ We provide state-of-the-art SM prediction assuming leptonic τ-decays
  - ↪ Predictions used in the on-going ATLAS analyses
  - ↪ Spin-correlation effects $\sim 5\,\%$
  - ↪ NLO EW corrections $\sim 1\,\%$
  - ↪ Parametrization of the photon flux $\sim 25\,\% \rightarrow \sim 5\,\%$

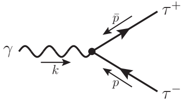
- Further studies in our paper [2504.11391]
  - ↪ Finite-mass effects
  - ↪ Importance of a proper input-parameter scheme
  - ↪ Gauge-invariant subsets of the NLO EW corrections
  - ↪ Non-inclusive treatment of collinear radiation

Outlook

- Include a model-independent parametrization of the $\gamma\tau\tau$ vertex

  $\hookrightarrow$ Study the sensitivity to a_τ

  - In the literature:

$\Gamma^\mu(k^2) =$ 

$$= F_{\mathrm{E}}(k^2)\gamma^\mu + F_{\mathrm{A}}(k^2)\left(\gamma^\mu - \frac{2m}{k^2}k^\mu\right)\gamma_5 + F_{\mathrm{M}}(k^2)\frac{\mathrm{i}\sigma^{\mu\nu}k_\nu}{2m} + F_{\mathrm{D}}(k^2)\frac{\sigma^{\mu\nu}k_\nu}{2m}\gamma_5$$

· $F_{\mathrm{E}}(k^2)$: Electric charge form factor $\rightarrow F_{\mathrm{E}}(0) = 1$

· $F_{\mathrm{A}}(k^2)$: Anapole moment (P violating) $\rightarrow F_{\mathrm{A}}(0) = 0$

· $F_{\mathrm{M}}(k^2)$: Magnetic form factor $\rightarrow F_{\mathrm{M}}(0) = a_\tau$

· $F_{\mathrm{D}}(k^2)$: Dipole form factor (CP violating) $\rightarrow F_{\mathrm{D}}(0) = -2md_\tau$

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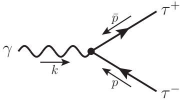
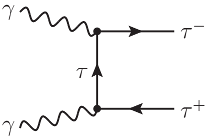


$$= F_{\mathrm{E}}(k^2)\gamma^\mu + F_{\mathrm{A}}(k^2)\left(\gamma^\mu - \frac{2m}{k^2}k^\mu\right)\gamma_5 + F_{\mathrm{M}}(k^2)\frac{\mathrm{i}\sigma^{\mu\nu}k_\nu}{2m} + F_{\mathrm{D}}(k^2)\frac{\sigma^{\mu\nu}k_\nu}{2m}\gamma_5$$

  - Problem: Assumes both τ-leptons to be on-shell



Outlook

- Include a model-independent parametrization of the $\gamma\tau\tau$ vertex

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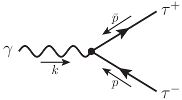
$\Gamma^\mu(k^2) =$ 

$$= F_\mathrm{E}(k^2)\gamma^\mu + F_\mathrm{A}(k^2)\left(\gamma^\mu - \tfrac{2m}{k^2}k^\mu\right)\gamma_5 + F_\mathrm{M}(k^2)\tfrac{\mathrm{i}\sigma^{\mu\nu}k_\nu}{2m} + F_\mathrm{D}(k^2)\tfrac{\sigma^{\mu\nu}k_\nu}{2m}\gamma_5$$

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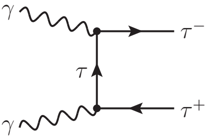


  - Solution:

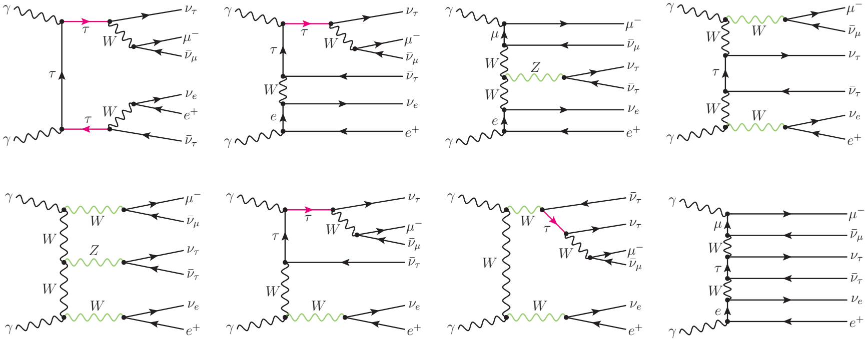
    $\hookrightarrow$ More general decomposition of the $\gamma\tau\tau$ vertex function

    $\hookrightarrow$ Classify form factors according to CP properties

    $\hookrightarrow$ Look for kinematical configurations in which $F_\mathrm{M}(0)$ is dominant

Leading-order contributions

- Process: $\gamma\gamma \to e^+\mu^-\bar{\nu}_\tau\nu_\tau\bar{\nu}_\mu\nu_e$



Importance of a proper input-parameter scheme

- Input-parameter schemes for α:

  - $\alpha(0)$-scheme: $\alpha = \alpha(0)$, $\alpha^{-1}(0) \approx 137$
    - · Processes without internal gauge-boson lines and external photons
      $\hookrightarrow$ No universal corrections from coupling renormalization $\propto \alpha \ln m_f$
    - · Processes with internal gauge-boson lines and no external photons
      $\hookrightarrow$ Large logarithmic corrections $\propto \alpha \ln m_f$ from the running of α
      $\hookrightarrow$ Corrections to the ρ-parameter from the renormalization of θ_{w}

  - G_μ-scheme: $\alpha = \alpha_{G_\mu} = \frac{\sqrt{2}}{\pi} G_\mu M_{\mathrm{W}}^2 \left(1 - \frac{M_{\mathrm{W}}^2}{M_{\mathrm{Z}}^2}\right)$, $G_\mu \approx 1.17 \times 10^{-5}\,\mathrm{GeV}^{-2}$
    - · Processes without internal gauge-boson lines and external photons
      $\hookrightarrow$ Corrections to the ρ-parameter erroneously absorbed into α
    - · Processes with internal gauge-boson lines and no external photons
      $\hookrightarrow$ Absorbs universal corrections to $\alpha(M_{\mathrm{Z}}^2)/\sin\theta_{\mathrm{w}}$ into the value of α

Importance of a proper input-parameter scheme

$\gamma\gamma \to \tau^+\tau^-$	$\alpha(0)$-scheme		G_μ-scheme	
	σ or $\Delta\sigma$ [mb]	δ [%]	σ or $\Delta\sigma$ [mb]	δ [%]
σ^{LO}	1.063(2)	-	1.136(3)	-
$\Delta\sigma^{\mathrm{NLO}}_{\mathrm{QED}}$	0.010(3)	0.94(3)	0.012(1)	1.08(6)
$\Delta\sigma^{\mathrm{NLO}}_{\mathrm{weak}}$	$9.1(7) \times 10^{-8}$	$8.5(6) \times 10^{-6}$	$-0.009(3)$	$-0.84(1)$
$\Delta\sigma^{\mathrm{NLO}}_{\mathrm{ferm}}$	$6.6(1) \times 10^{-7}$	$6.2(6) \times 10^{-5}$	$-0.058(1)$	$-5.10(2)$
σ^{NLO}	1.073(2)	0.94(3)	1.081(3)	$-4.86(6)$

$\tau^- \to e^-\nu_\tau\bar{\nu}_e$	$\alpha(0)$-scheme		G_μ-scheme	
	Γ or $\Delta\Gamma$ [ns^{-1}]	δ [%]	Γ or $\Delta\Gamma$ [ns^{-1}]	δ [%]
Γ^{LO}	573.35(8)	-	615.28(9)	-
$\Delta\Gamma^{\mathrm{NLO}}_{\mathrm{bos}}$	2.18(1)	0.38(1)	$-2.69(1)$	$-0.44(1)$
$\Delta\Gamma^{\mathrm{NLO}}_{\mathrm{ferm}}$	28.18(3)	4.92(1)	$1.1(1) \times 10^{-2}$	$1.9(1) \times 10^{-3}$
Γ^{NLO}	603.71(9)	5.30(1)	612.60(9)	$-0.44(1)$

Effects

- Effects: $\Delta_{\text{LO}} = \frac{\sigma^{\text{LO}} - \sigma^{\text{LO}}_{\text{Best}}}{\sigma^{\text{LO}}_{\text{Best}}}$

	σ^{LO}[nb]	Δ_{LO} [%]
Best	45.869(4)	-
No spin corr.	43.282(4)	$-5.64(1)$
$m_e = 0$	45.873(4)	0.01(1)
$m_\mu = m_e = 0$	46.446(4)	1.26(1)

Effects

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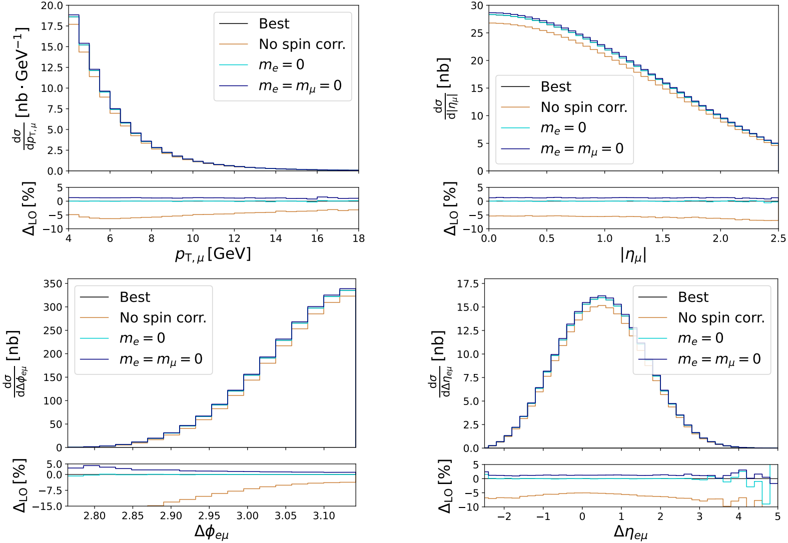


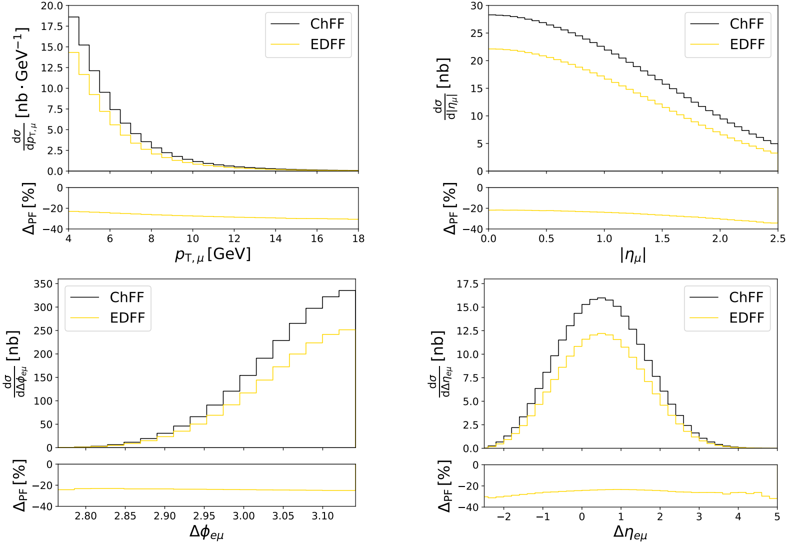
Parametrization of the photon flux

- Effects: $\Delta_{\rm PF} = \frac{\sigma^{\rm LO}_{\rm EDFF} - \sigma^{\rm LO}_{\rm ChFF}}{\sigma^{\rm LO}_{\rm ChFF}}$, $\mathcal{O}_i \equiv \frac{\sigma^{\rm LO}_i}{\sigma^{\rm LO}_{\mu\mu,i}}$

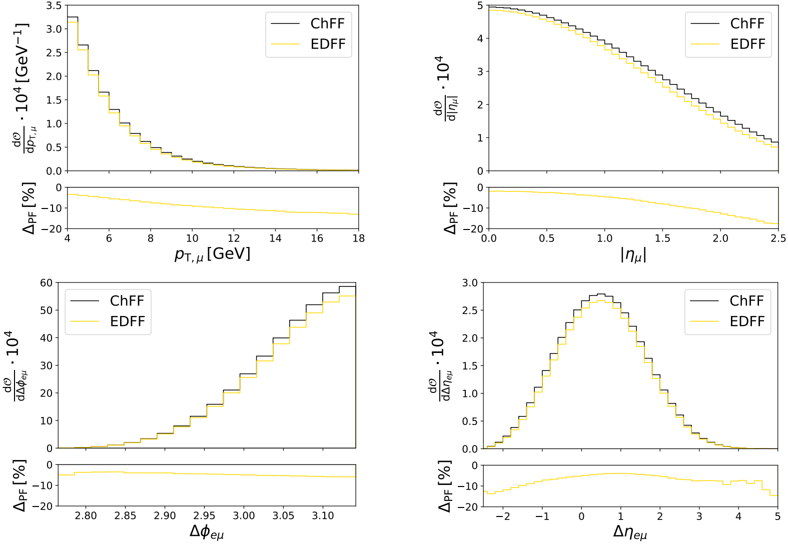
	ChFF	EDFF	$\Delta_{\rm PF}[\%]$
$\sigma^{\rm LO}$[nb]	45.87(1)	34.61(1)	$-24.55(1)$
$\sigma^{\rm LO}_{\mu\mu}$[μb]	57.24(2)	45.64(1)	$-20.28(4)$
$\mathcal{O}^{\rm LO} \cdot 10^4$	8.013(3)	7.584(3)	$-5.36(5)$

Parametrization of the photon flux

- Effect: $\Delta_{\mathrm{PF}} = \frac{\sigma^{\mathrm{LO}}_{\mathrm{EDFF}} - \sigma^{\mathrm{LO}}_{\mathrm{ChFF}}}{\sigma^{\mathrm{LO}}_{\mathrm{ChFF}}}$



- Effect: $\Delta_{\mathrm{PF}} = \frac{\sigma^{\mathrm{LO}}_{\mathrm{EDFF}} - \sigma^{\mathrm{LO}}_{\mathrm{ChFF}}}{\sigma^{\mathrm{LO}}_{\mathrm{ChFF}}}$, $\frac{\mathrm{d}\mathcal{O}_i}{\mathrm{dX}} \equiv \frac{1}{\sigma^{\mathrm{LO}}_{\mu\mu,i}} \frac{\mathrm{d}\sigma^{\mathrm{LO}}_i}{\mathrm{dX}}$



NLO EW correction

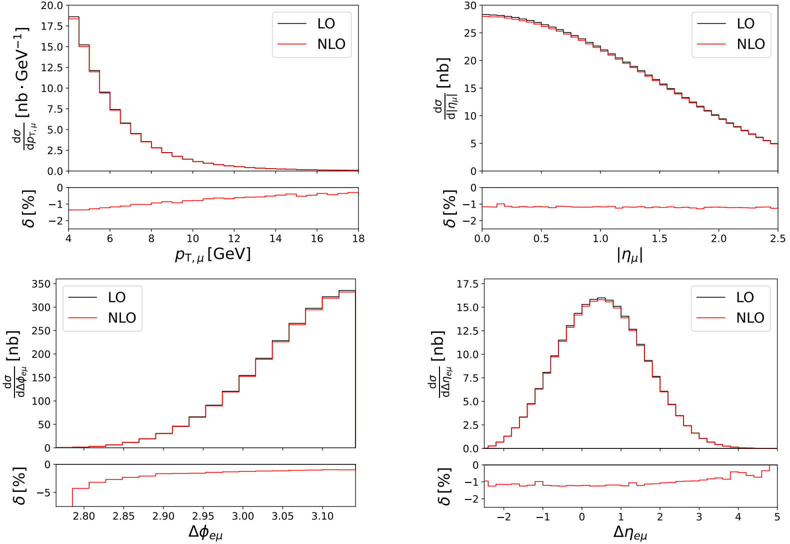
- Relative correction: $\delta = \frac{\sigma^{\mathrm{NLO}} - \sigma^{\mathrm{LO}}}{\sigma^{\mathrm{LO}}}$

	σ [nb]	δ [%]
LO	45.869(4)	-
NLO EW	45.327(4)	$-1.182(1)$

subprocess	correction	$\Delta\sigma^{\mathrm{NLO}}$ [nb]	δ [%]
$\gamma\gamma \to \tau^+\tau^-$	QED	0.1733(3)	0.3778(7)
	weak	0.00082(1)	0.0018(1)
	fermionic	0.00005(1)	0.0001(1)
	sum	0.1741(3)	0.3797(7)
$\tau^- \to \mu^-\nu_\tau\bar{\nu}_\mu$	bosonic	$-0.3342(4)$	$-0.7286(8)$
	fermionic	0.0010(1)	0.0023(1)
	sum	$-0.3332(3)$	$-0.7263(8)$
$\tau^+ \to e^+\bar{\nu}_\tau\nu_e$	bosonic	$-0.3840(4)$	$-0.8372(9)$
	fermionic	0.0010(1)	0.0023(1)
	sum	$-0.3830(4)$	$-0.8349(9)$
sum		$-0.5421(6)$	$-1.182(1)$

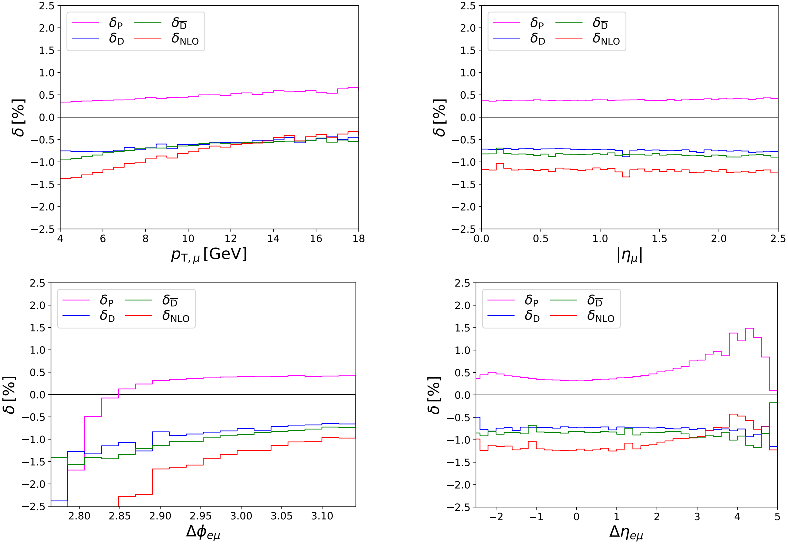
NLO EW correction

- Relative correction: $\delta = \frac{\sigma^{\mathrm{NLO}} - \sigma^{\mathrm{LO}}}{\sigma^{\mathrm{LO}}}$



NLO EW correction

- Relative correction: $\delta = \frac{\sigma^{\text{NLO}} - \sigma^{\text{LO}}}{\sigma^{\text{LO}}}$



Non-inclusive treatment of collinear radiation

- Effects: $\Delta_{\text{drs/bare}} = \frac{\sigma^{\text{NLO}}_{\text{drs}} - \sigma^{\text{LO}}_{\text{bare}}}{\sigma^{\text{LO}}} = \delta_{\text{drs}} - \delta_{\text{bare}}$

- Massive muons

subprocess	$\Delta\sigma^{\text{NLO}}_{\text{bare}}$ [nb]	δ_{bare} [%]	$\Delta\sigma^{\text{NLO}}_{\text{drs}}$ [nb]	δ_{drs} [%]	$\Delta_{\text{drs/bare}}$ [%]
$\gamma\gamma \to \tau^+\tau^-$	0.1666(3)	0.363(1)	0.1745(3)	0.380(1)	0.017(1)
$\tau^- \to \mu^-\nu_\tau\bar{\nu}_\mu$	−0.4799(2)	−1.046(1)	−0.3332(2)	−0.726(1)	0.320(1)
$\tau^+ \to e^+\bar{\nu}_\tau\nu_e$	−0.3821(3)	−0.833(1)	−0.3828(3)	−0.835(1)	−0.002(1)
sum	−0.6954(5)	−1.516(1)	−0.5417(5)	−1.181(1)	0.335(2)

- Massless limit, $m_\mu \to 0$

subprocess	$\Delta\sigma^{\text{NLO}}_{\text{bare}}$ [nb]	δ_{bare} [%]	$\Delta\sigma^{\text{NLO}}_{\text{drs}}$ [nb]	δ_{drs} [%]	$\Delta_{\text{drs/bare}}$ [%]
$\gamma\gamma \to \tau^+\tau^-$	0.1685(2)	0.363(1)	0.1762(2)	0.379(1)	0.017(1)
$\tau^- \to \mu^-\nu_\tau\bar{\nu}_\mu$	−0.4531(3)	−0.976(1)	−0.3595(3)	−0.774(1)	0.202(1)
$\tau^+ \to e^+\bar{\nu}_\tau\nu_e$	−0.3594(3)	−0.774(1)	−0.3597(3)	−0.774(1)	0.001(1)
sum	−0.6440(5)	−1.387(1)	−0.5429(5)	−1.169(1)	0.218(2)

Non-inclusive treatment of collinear radiation

- Relative difference: $\Delta_{\text{drs/bare}} = \frac{\sigma^{\text{NLO}}_{\text{drs}} - \sigma^{\text{LO}}_{\text{bare}}}{\sigma^{\text{LO}}} = \delta_{\text{drs}} - \delta_{\text{bare}}$