Inference of cosmological parameters from the SGBW

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COSMOLOGY WITH GWs

GWs: independent and **direct measurement of redshift**

$$d_L \propto \frac{f(z)}{H_0}$$

Strain amplitude depends on masses and distance:

$$h(t) \propto M_c^{\frac{5}{6}}\, d_L^{-1}\, F(f)$$

$$\Downarrow$$

z measurements: **GWs sirens!**

SIRENS

- Bright sirens $\Rightarrow$ *z* from EM
- Dark sirens $\Rightarrow$ *z* from galaxy
- **Spectral sirens** $\Rightarrow$ RESOLVED

  $\Rightarrow$ **UNRESOLVED : SGWB**

Strain h_{ij} at t, x:

$$h_{ij}(t,x) = \int_{-\infty}^{+\infty} df \int_{S^2} d\hat{n} \sum_{P=+,\times} h_P(f,\hat{n})\, \epsilon_{ij}^{P}(\hat{n})\, e^{i2\pi f(\hat{n}\cdot x - t)}$$

SGWB

Stochastic = non-deterministic $\Rightarrow$ amplitude h_p = complex number from Gaussian

- **cosmological component:** cosmological processes generating GWs

- **astrophysical component:** superposition astrophysical unresolved signals.

$\Downarrow$

BBH

SGWB AS SIREN

$$\propto \frac{1}{H_0}$$

$$\boxed{\Omega_{GW} = \frac{1}{\rho_C} \frac{d\rho_{GW}}{d\, ln(f)} \quad \propto \quad \frac{1}{H_0^3}}$$

$$\propto \frac{1}{H_0^2}$$

Ok…

It is 4.20 p.m.

…

Why should we care about it?

3G detectors will measure it!

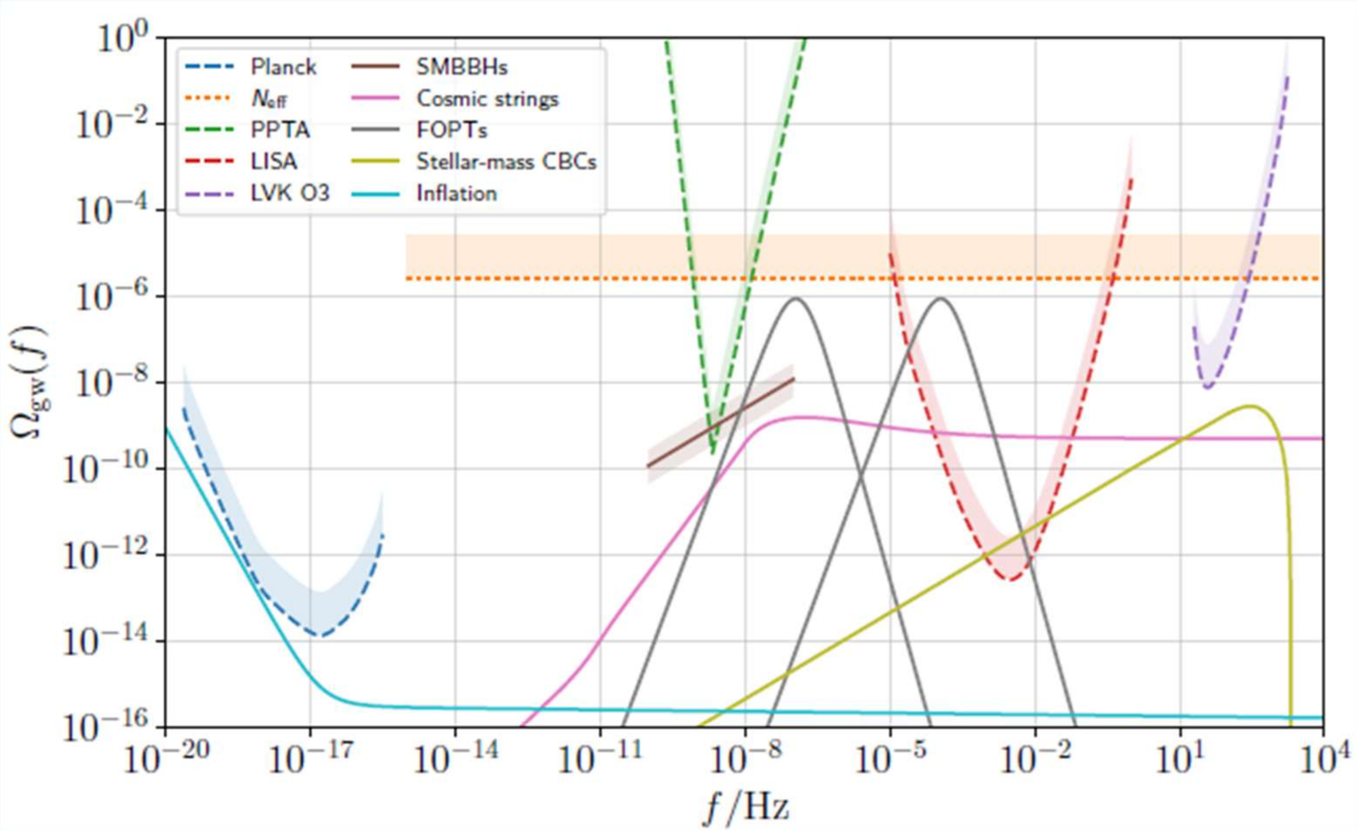


Fig. 1: Predicted SGWB contributions, compared with detectors' sensitivity. *Renzini et al., 2022*

- **Yellow curve** = SGWB by **stellar-mass compact binaries**
- **Red curve:** LISA sensitivity

O3 LVK results:

$$\boldsymbol{\Omega_{GW}\,|_{25Hz} < 0.7 \pm 2.7 \cdot 10^{-8}}$$

ESTIMATION

$$S_h \propto < \widetilde{h_i} \widetilde{h_j} >$$

$$S_h(f) = \frac{3H_0^3}{2\pi^2 f^3} \Omega_{GW}(f)$$

Correlation between channels $\Rightarrow$ measurement of the energy density

$\Downarrow$

Recent interest in the topic:
--- Capurri G., 2023, BNS in ET, analitical
--- Ferraiuolo S., 2025, BBH in LVK

catalogue approach
to simulate the SGWB in LVK, O5

BBH CATALOGUE APPROACH

BBH population models = 15 astrophysical parameters, $\{\theta\}$

Intrinsic parameters = fundamental properties: $m_1, m_2,\ \overrightarrow{a_1},\ \overrightarrow{a_2}$

Extrinsic parameters = depend on measure of event: binary system position: RA, decl, z (or d_L), ψ, ϕ_C, i ...

Prior for $\{\theta\}$ parametrized by set of hyperparameters Λ: $\pi(\theta|\Lambda)$

$\Downarrow$

Catalogue 10^8 sources extracted

Mass prior: $p(m_1, m_2|\Lambda) = \pi_1(m_1|\Lambda)\, \pi_2(q|m_1, \Lambda)$

$$\pi(m_1|\lambda_{\rm peak}, \alpha, m_{\rm min}, \delta_m, m_{\rm max}, \mu_m, \sigma_m) = \Big[(1-\lambda_{\rm peak})\mathfrak{P}(m_1|-\alpha, m_{\rm max}) + \lambda_{\rm peak} G(m_1|\mu_m, \sigma_m)\Big] S(m_1|m_{\rm min}, \delta_m)$$

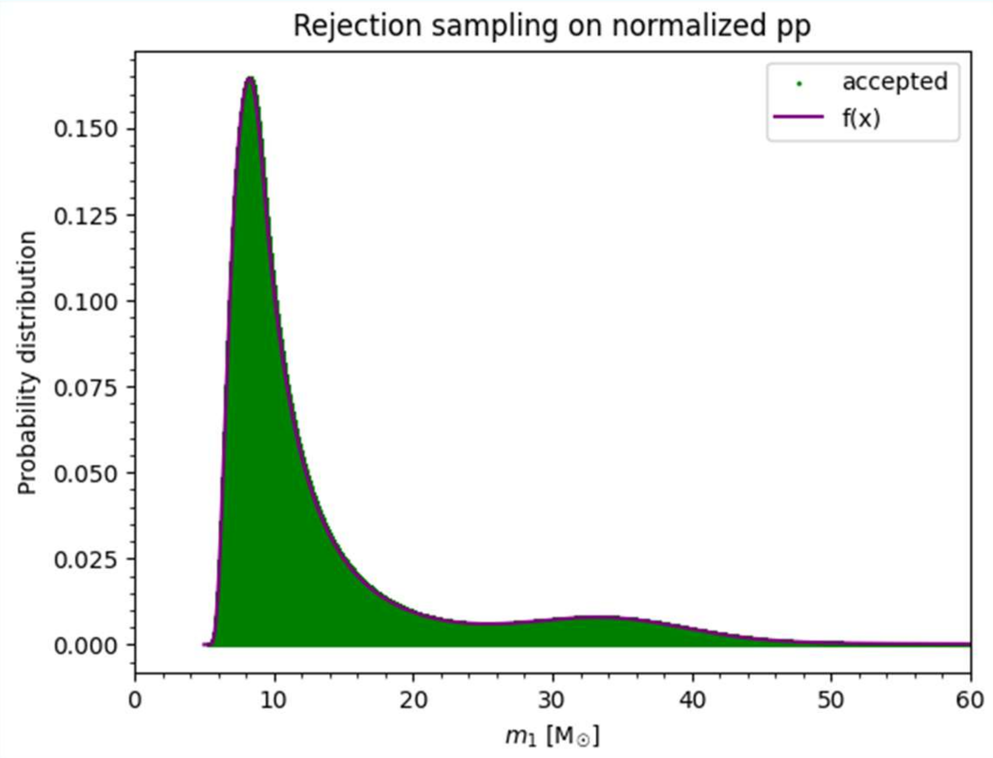


Fig. 2: Rejection sampling on pp model for m_1

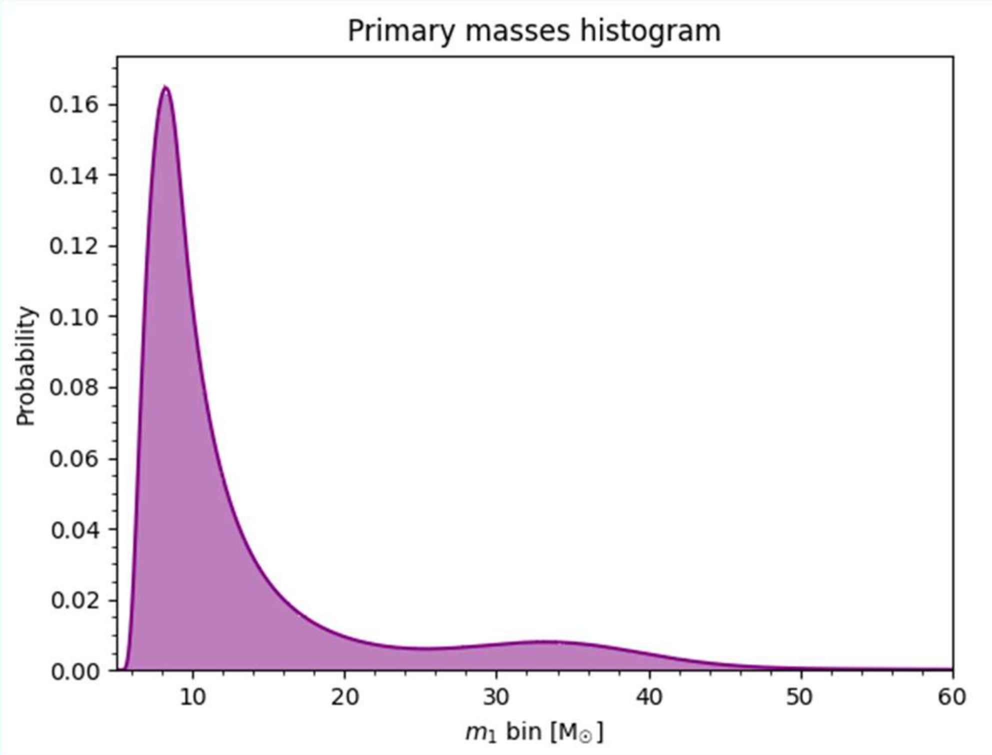


Fig. 3: m_1 histogram – following the curve as expected

Secondary mass model

Conditioned probability: $\pi(q \mid \beta, m_1, m_{\min}, \delta_m) \propto q^{\beta_q} S(q m_1 \mid m_{\min}, \delta_m)$

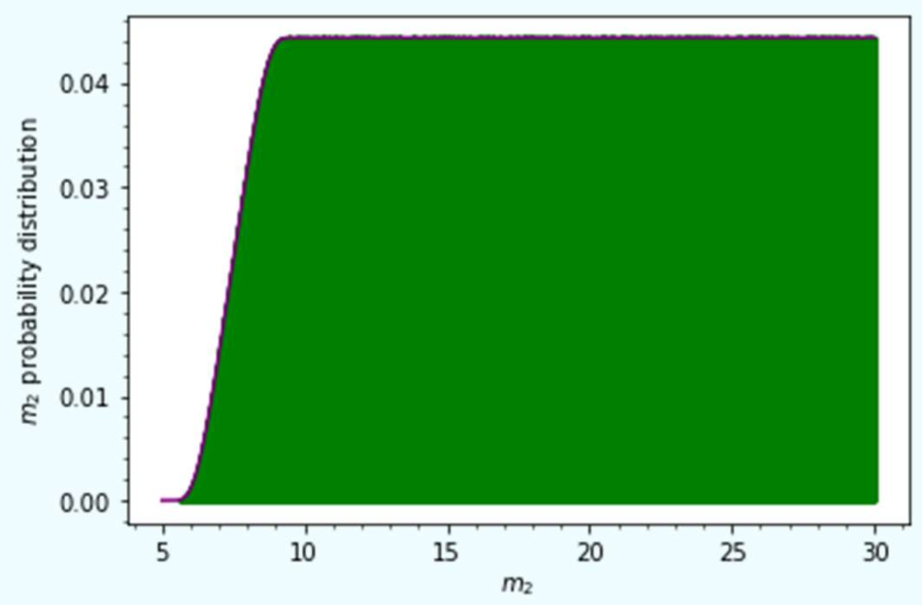


Fig. 4: Rejection sampling on smoothing part of the model for m_2

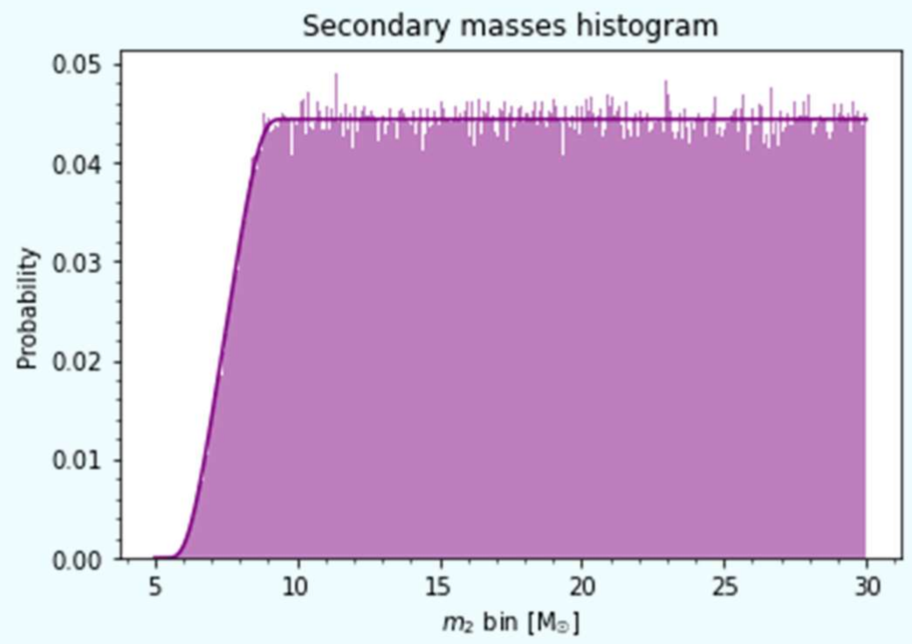


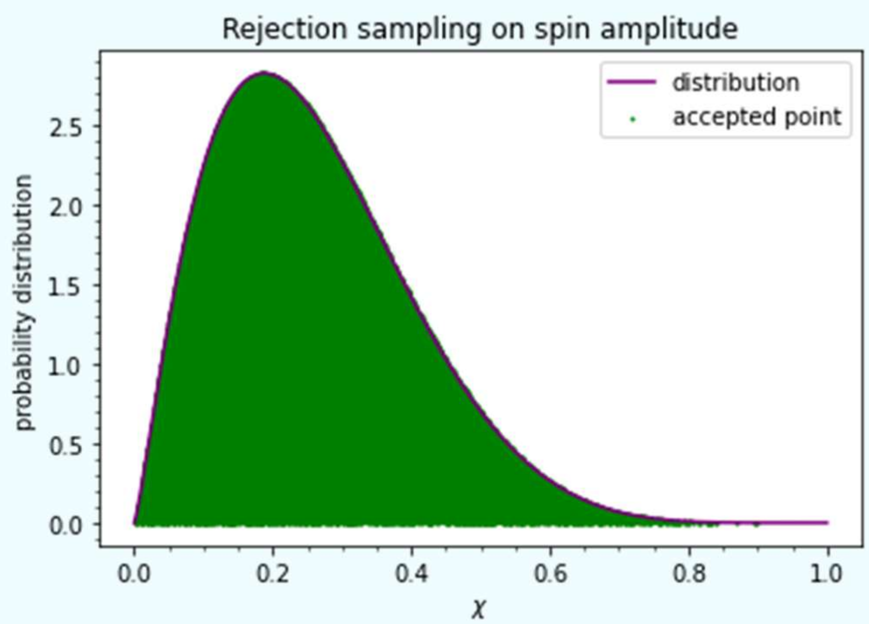
Fig. 5: m_2 distribution histogram

Spins prior:

Product two PDFs: one for spin amplitudes and one for spin tilts

$$p\left(\chi_i|\alpha_\chi,\beta_\chi\right)=\frac{\chi_i^{\alpha_\chi-1}\left(1-\chi_i\right)^{\beta_\chi-1}}{B\left(\alpha_\chi,\beta_\chi\right)}$$

$$p(\cos(t_1),\cos(t_2)|\sigma_1,\sigma_2,\zeta)=\frac{1-\zeta}{4}+\frac{2\zeta}{\pi}\prod_{i\in 1,2}\frac{\exp\left\{-\left[1-\cos(t_i)\right]^2/(2\sigma_i^2)\right\}}{\sigma_i\,\mathrm{erf}(\sqrt{2}/\sigma_i)}$$



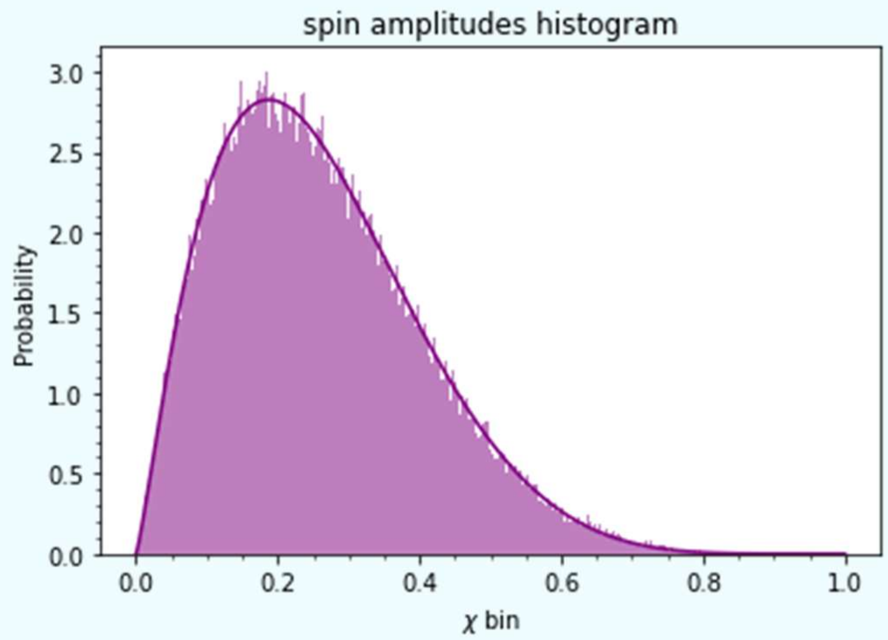
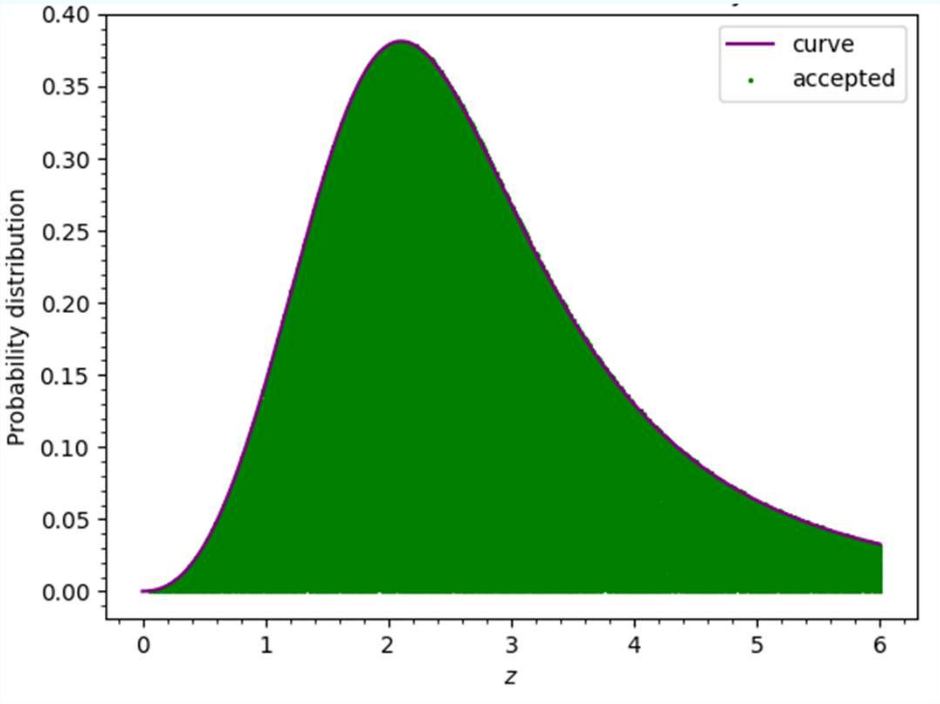


Fig. 6, 7: Rejection sampling and histogram for spin amplitude χ

Extrinsic parameters: position distribution. Uniform for α, δ, angles

Merger rate and redshift

$$p\left(z|\alpha,\beta,z_p\right) \propto \frac{1}{1+z}R(z)\frac{dV}{dz}$$



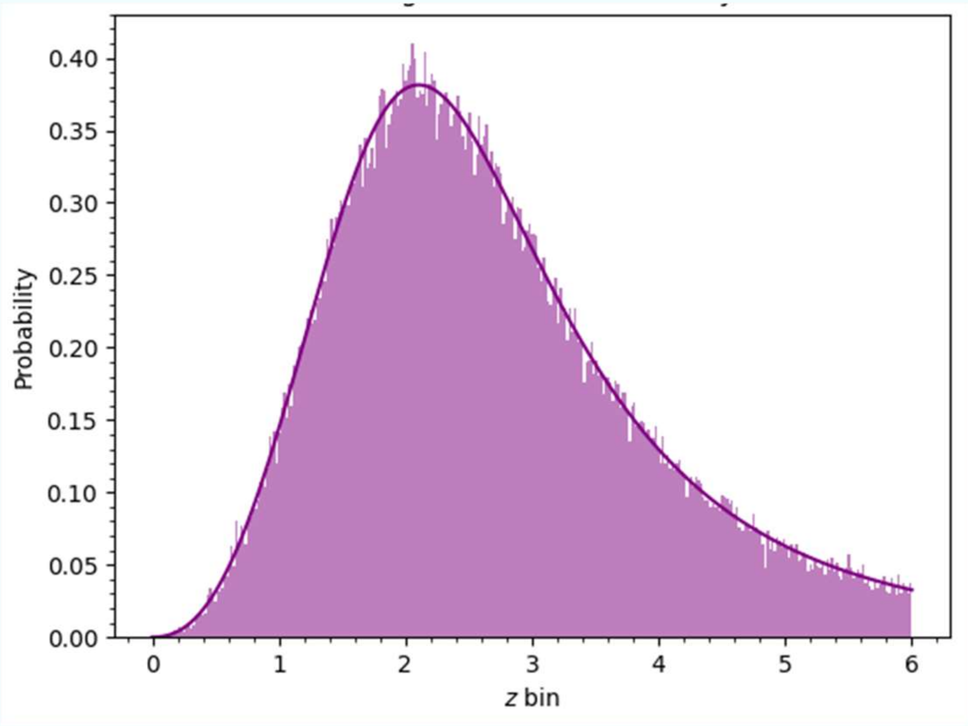


Fig. 8, 9: Redshift distribution and rejection sampling

Projection of the waveform for each source in LVK at O5 sensitivity

Distinction between resolved and unresolved sources

SNR = 12 threshold for distinction resolved signals – background

From projected waveforms: **cross-correlation measurements**

$$\Omega_{GW} = \sum_i \frac{1}{T} \sum_i \frac{f^3}{8G\rho_0} \left(|h_+(f)|^2 + |h_X(f)|^2\right)$$

$$\hat{C}_{IJ}(f) = \frac{2}{T_{\text{obs}}} \frac{10\pi^2}{3H_0^2} \frac{f^3}{\gamma_{IJ}(f)} \tilde{s}_I(f) \tilde{s}_J^*(f)$$

LIKELIHOOD

$$\mathcal{L}(\vec{d}|\theta) = \mathcal{L}(res|\theta)\mathcal{L}(SGWB|\theta)$$

$$\mathcal{L} = \prod_{I,J} \frac{1}{\sqrt{2\pi\Sigma_{IJ}}} \exp\left[-\frac{1}{2}\frac{(\hat{C}_{IJ} - \bar{C}_{IJ})^2}{\Sigma_{IJ}}\right]$$

$$\mathcal{L}_{\mathrm{tot}}(\vec{d}|\vec{\theta}) = \prod_{i}^{N} \mathcal{L}(d_i|\theta_i)$$

Inference of H0 from the optimal estimator for the SWGB

$$\hat{C}_{IJ}(f) = \frac{2}{T_{\mathrm{obs}}} \frac{10\pi^2}{3H_0^2} \frac{f^3}{\gamma_{IJ}(f)} \tilde{s}_I(f) \tilde{s}_J^*(f).$$

FUTURE

LISA, ET: will the assumption for the product of the likelihoods still hold?

Independence resolved-unresolved signals: stops holding

Short signals: stops holding

And so on

Thank you!