cosmology with the massive black hole population

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and the LISA cosmology standard sirens group

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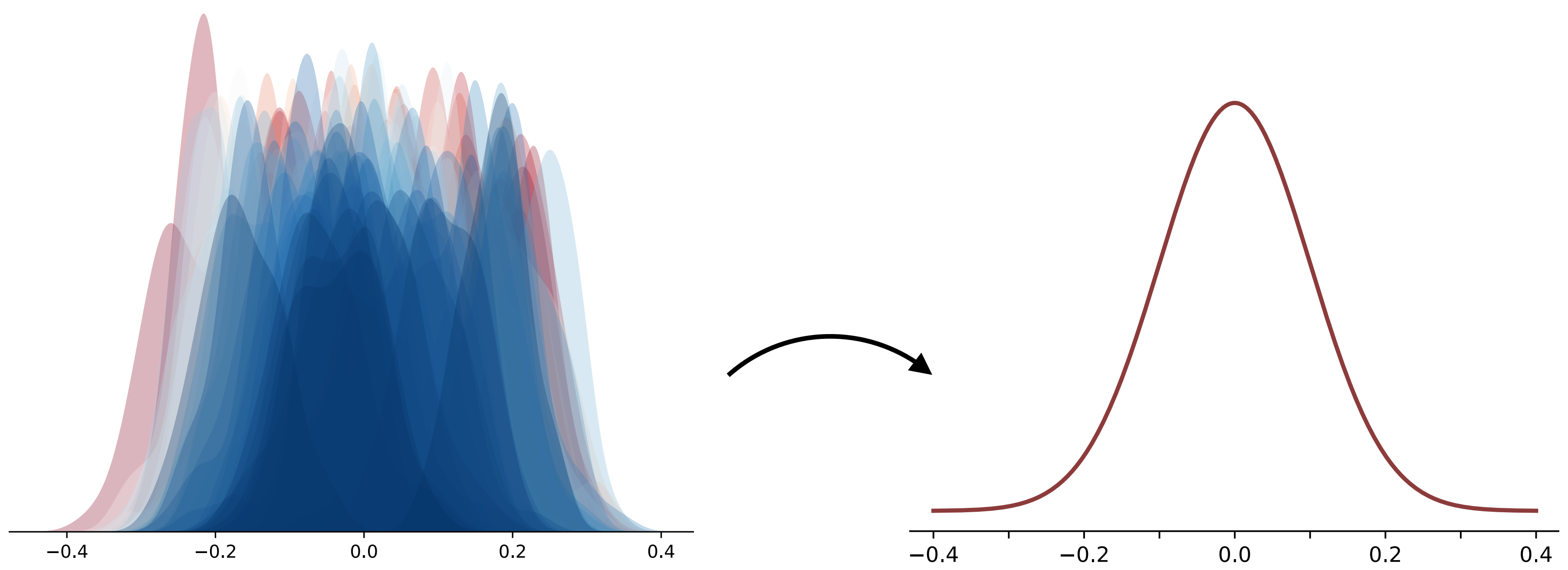
Journée LISA à Toulouse, IRAP

1. the method

HIERARCHICAL INFERENCE

[1809.02063, 2007.05579, 1809.03815]

combine multiple measurements to find the most likely intrinsic probability distribution from which the observed events are drawn



single observations　　　population distribution

- we use Bayesian analysis

$$p(\,\textit{model}\,|\,\textit{data}\,) \longleftrightarrow p(\,\textit{data}\,|\,\textit{model}\,)$$

functional form for the BH distribution

GW events to be combined

$$p(\boldsymbol{\Lambda}|\{\mathbf{d}\}) \propto \mathcal{L}_H(\{\mathbf{d}\}|\boldsymbol{\Lambda})\, p(\boldsymbol{\Lambda})$$

posterior

(hierarchical) likelihood

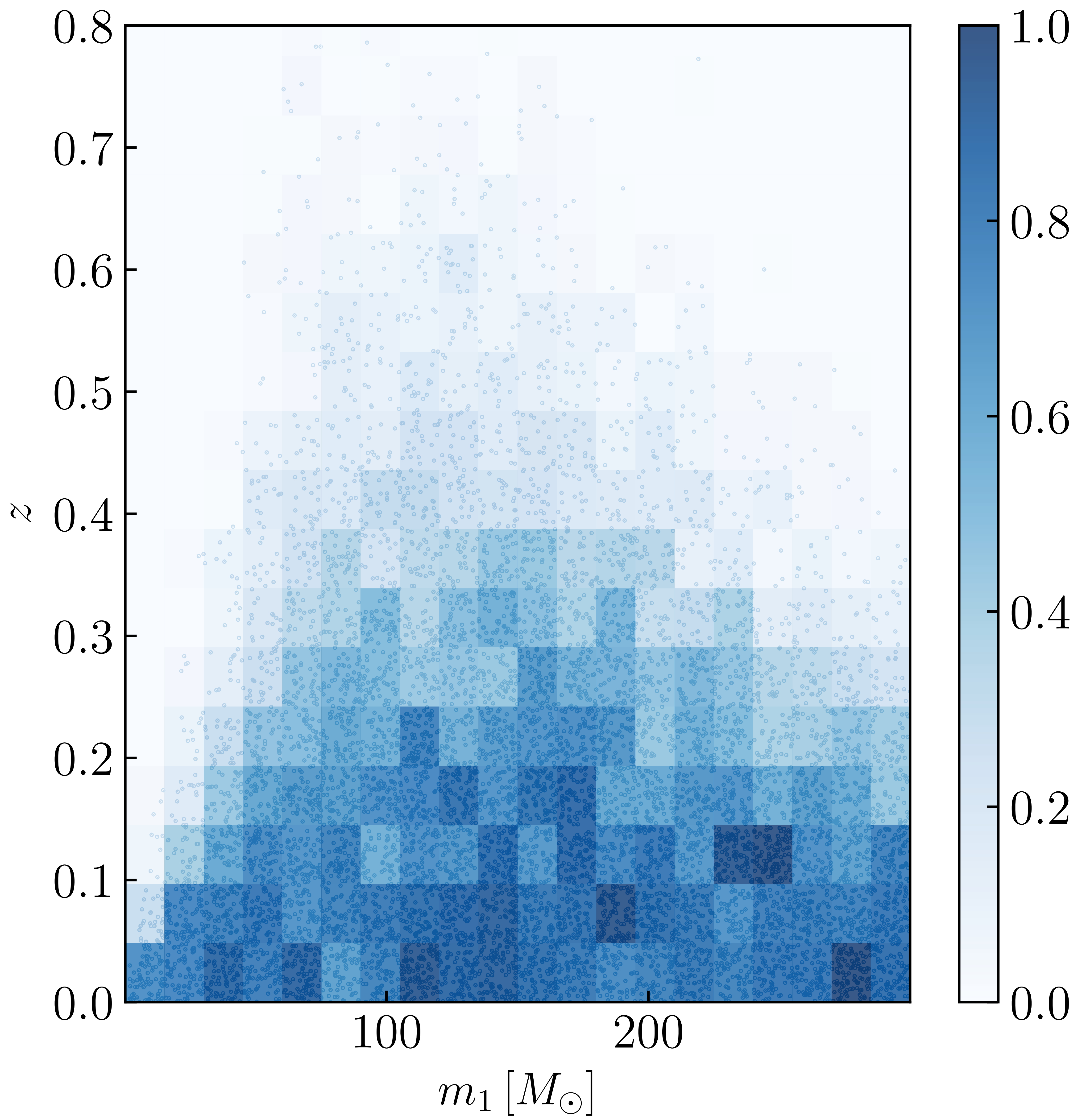
prior

COMPLICATIONS

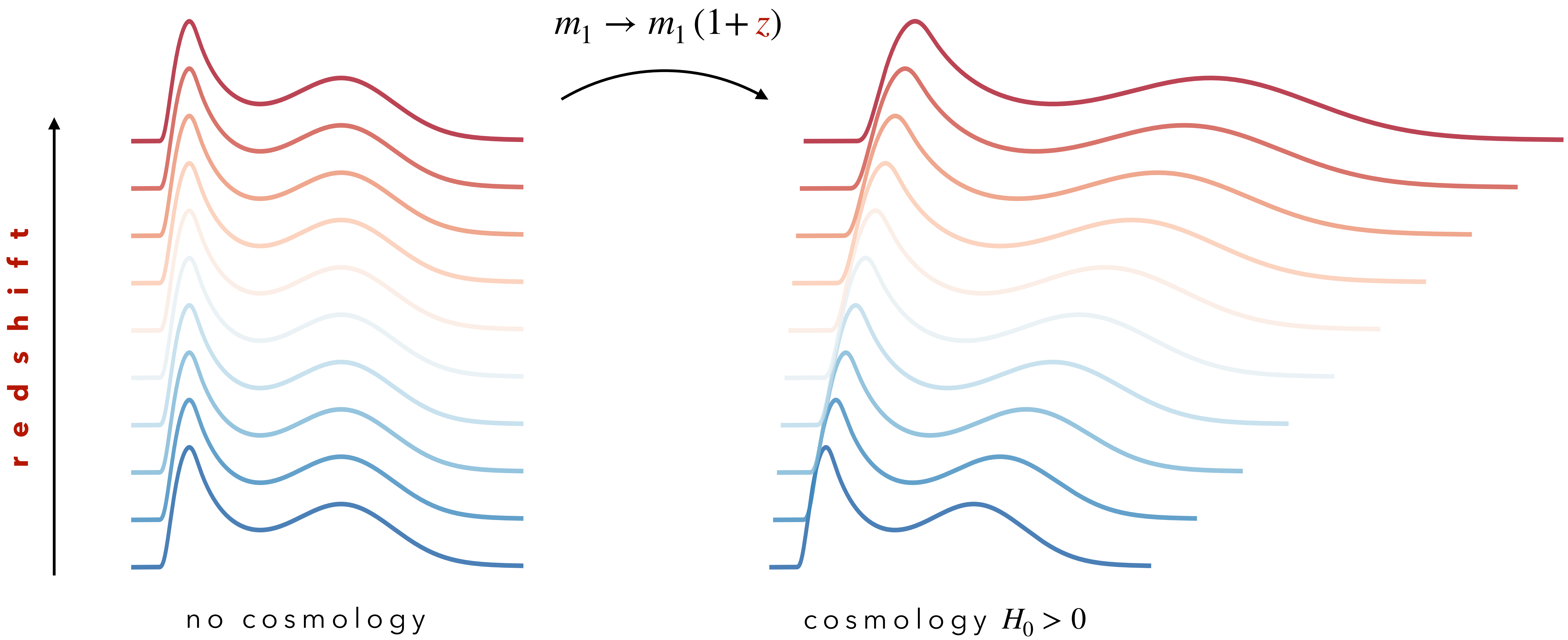
- our detector is not perfect
    - measurement uncertainty
    - selection effects

- we aim at reconstructing the *astrophysical distribution*
    - source frame distribution

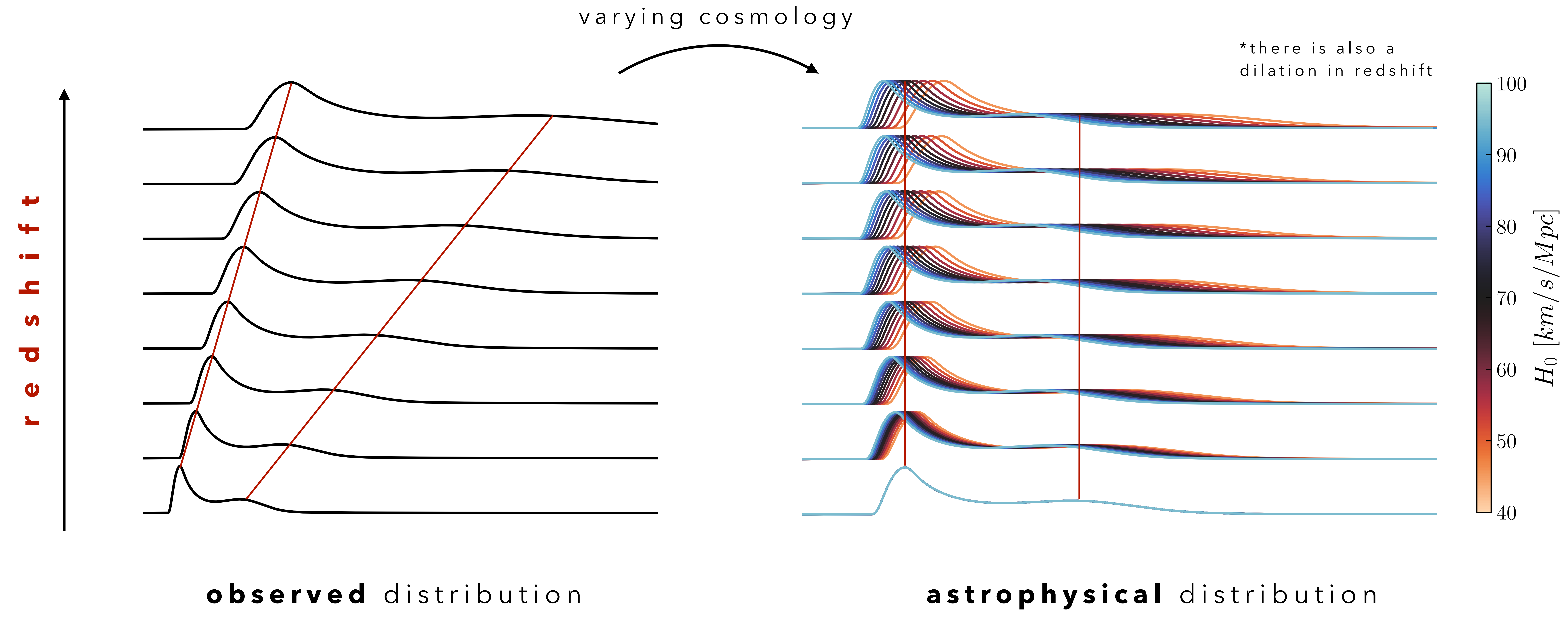
      (not known in advance!)



observed masses are redshifted because of cosmology



we can infer cosmology from the the population features



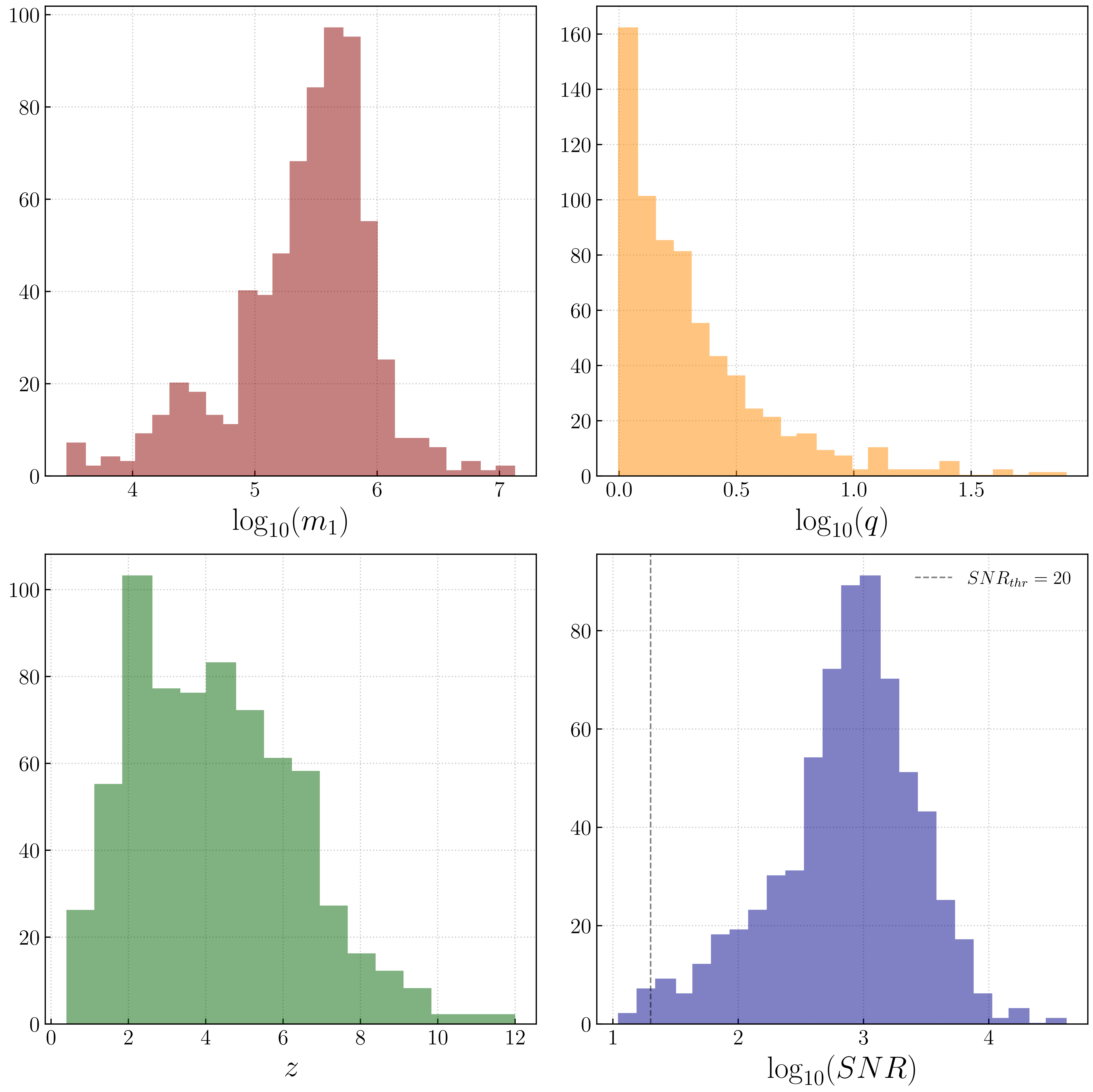
2. the catalog

THE Q3d CATALOG

- ~**700** events for 90yrs of LISA
  - ~**30** events for a 4yrs mission
  - masses spanning large range
  - events up to very high redshift

- hierarchical inference
  - well measured parameters
  - almost **no selection effects**

    (only 12 events with SNR<20)

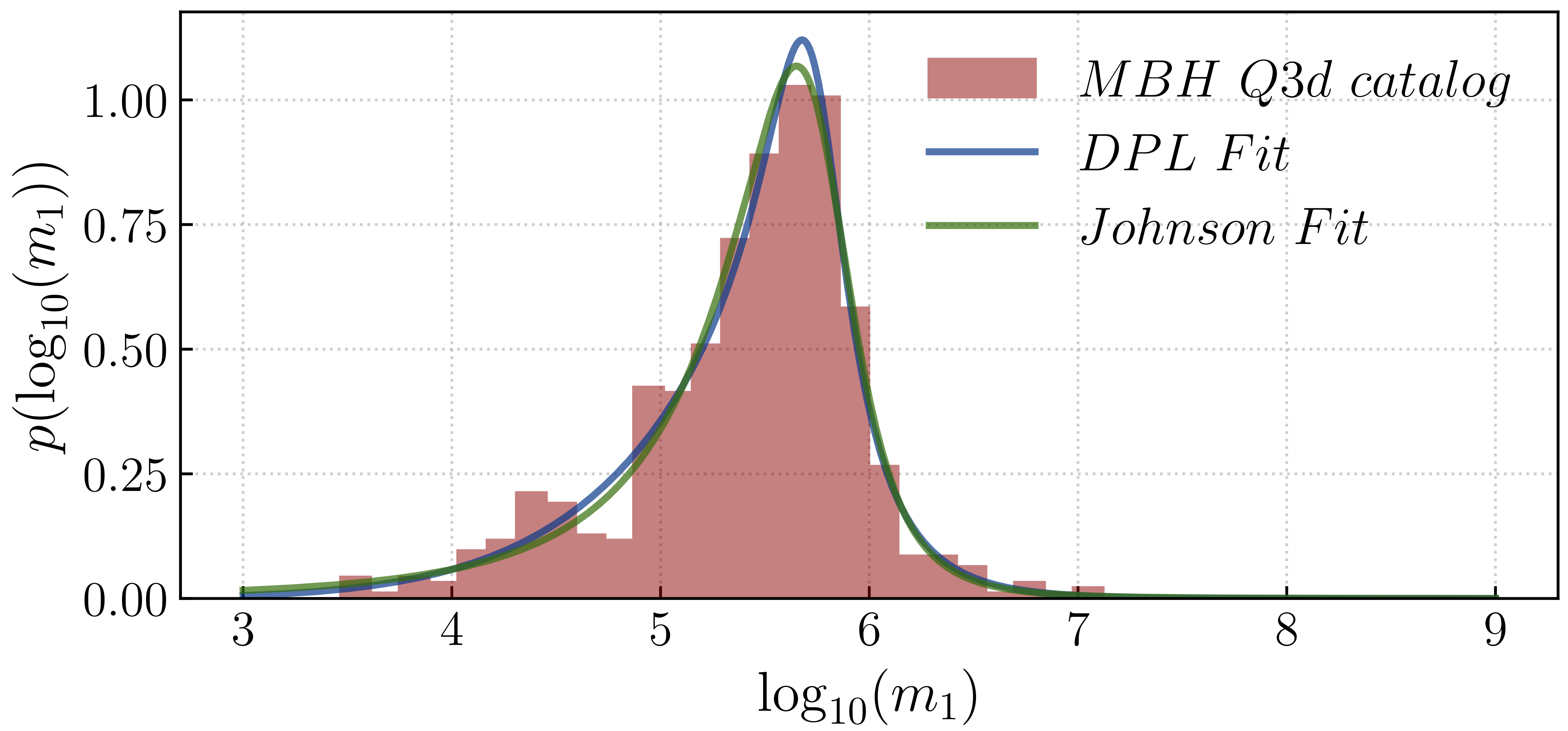


CHOOSING THE MODEL 1D fits of the data

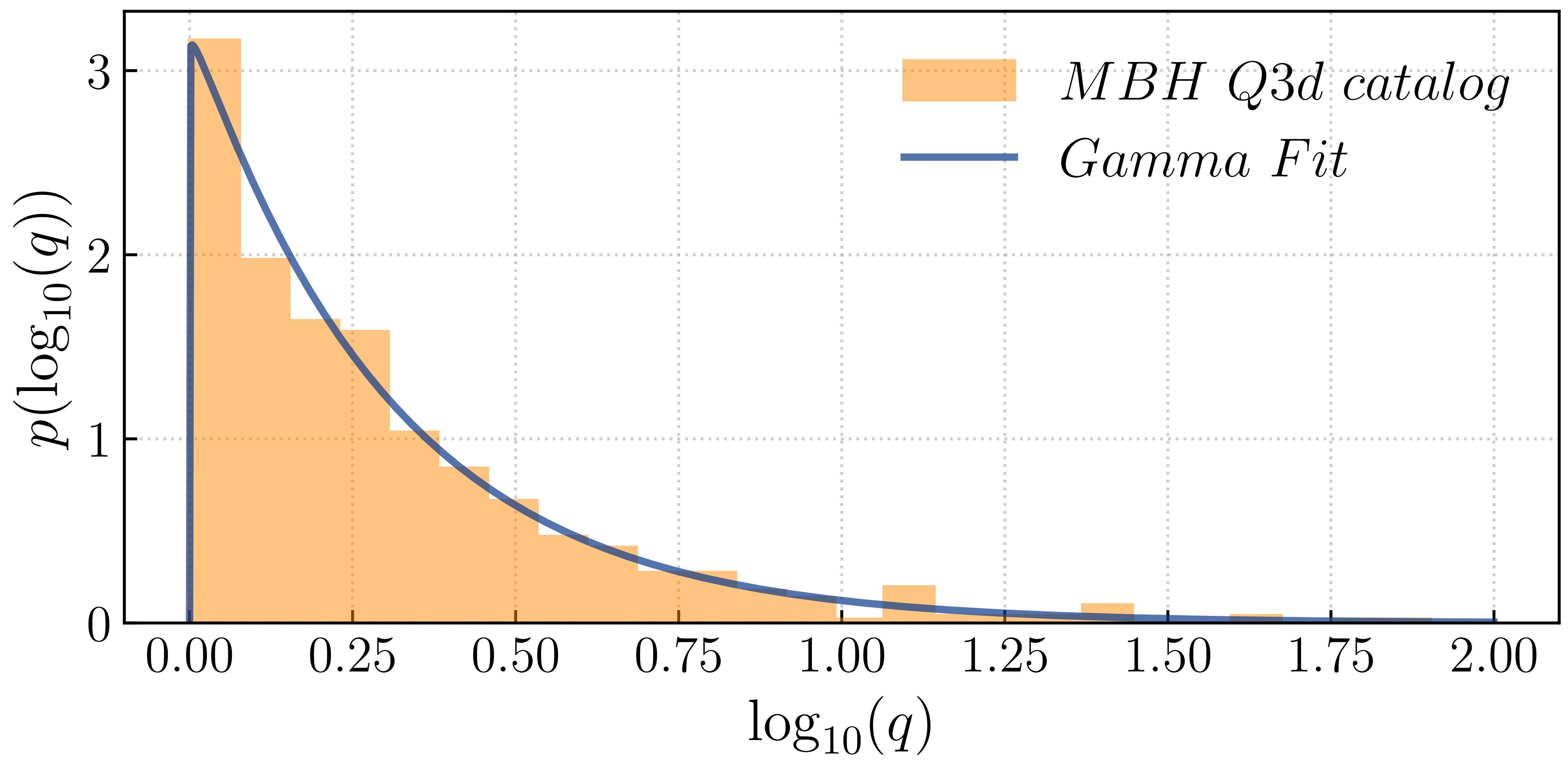
need to find a suitable parametrization for the data

- don't know the model in advance
- it will never be perfect

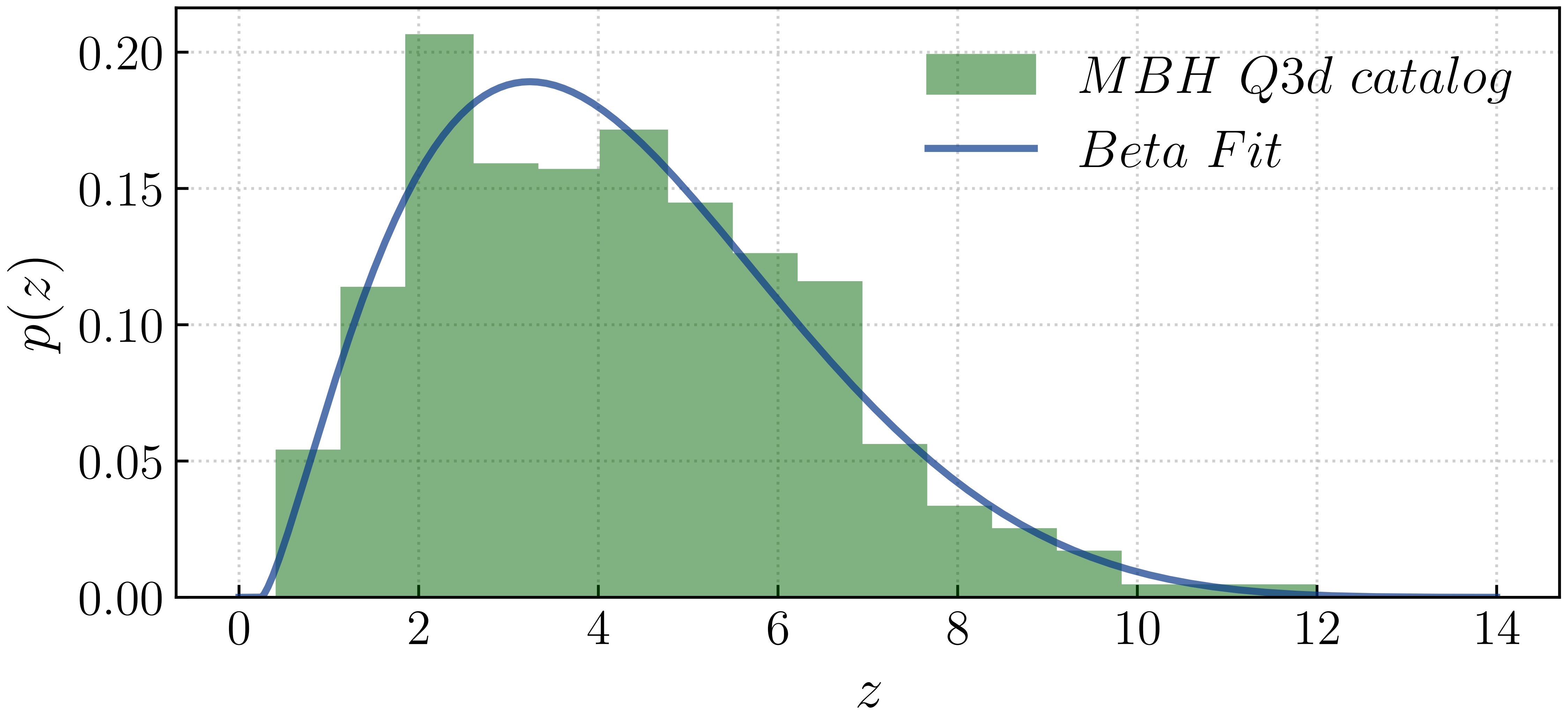
primary BH



mass ratio



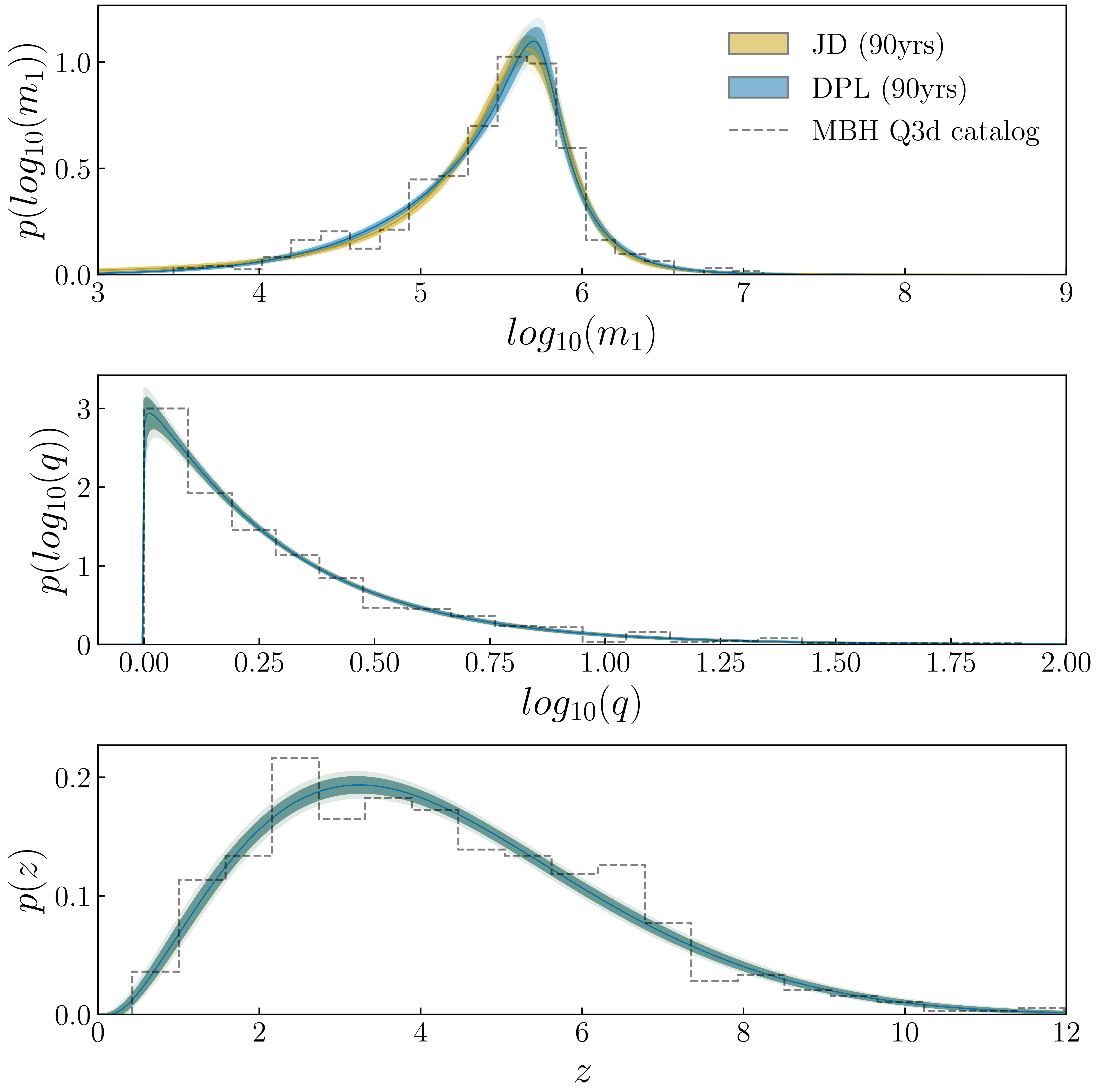
redshift



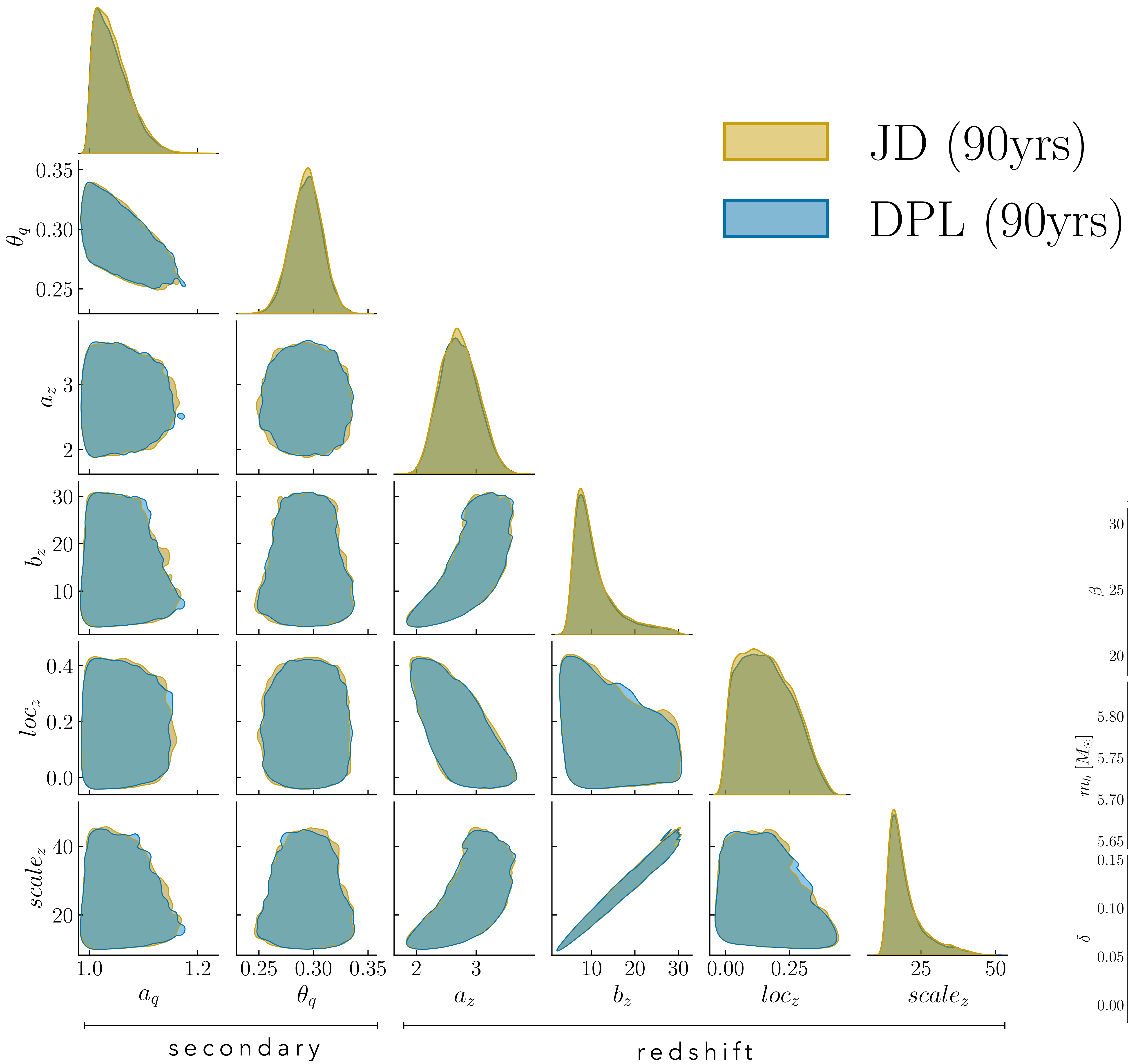
3. the inference

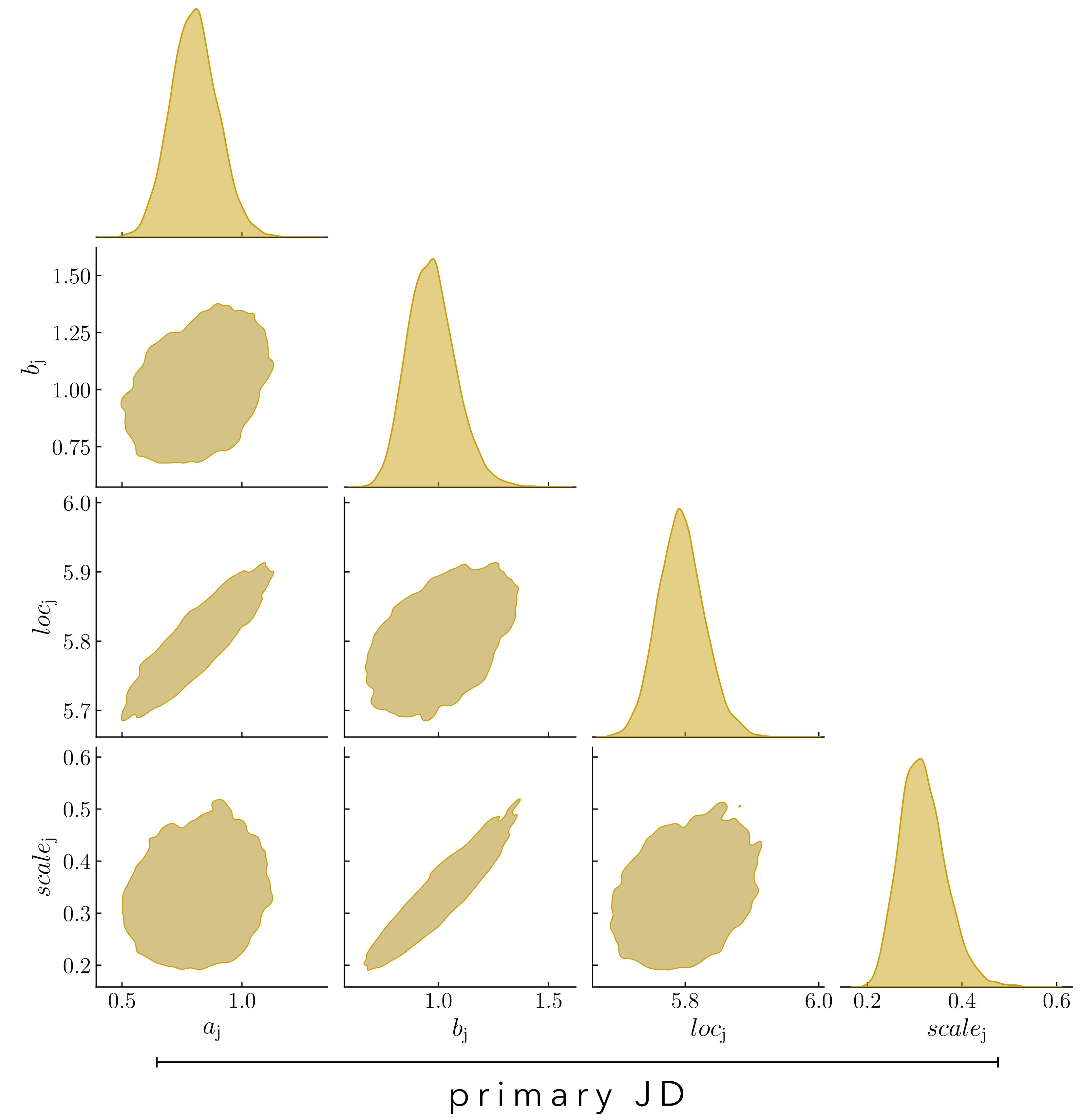
Q3d INFERENCE 90yrs

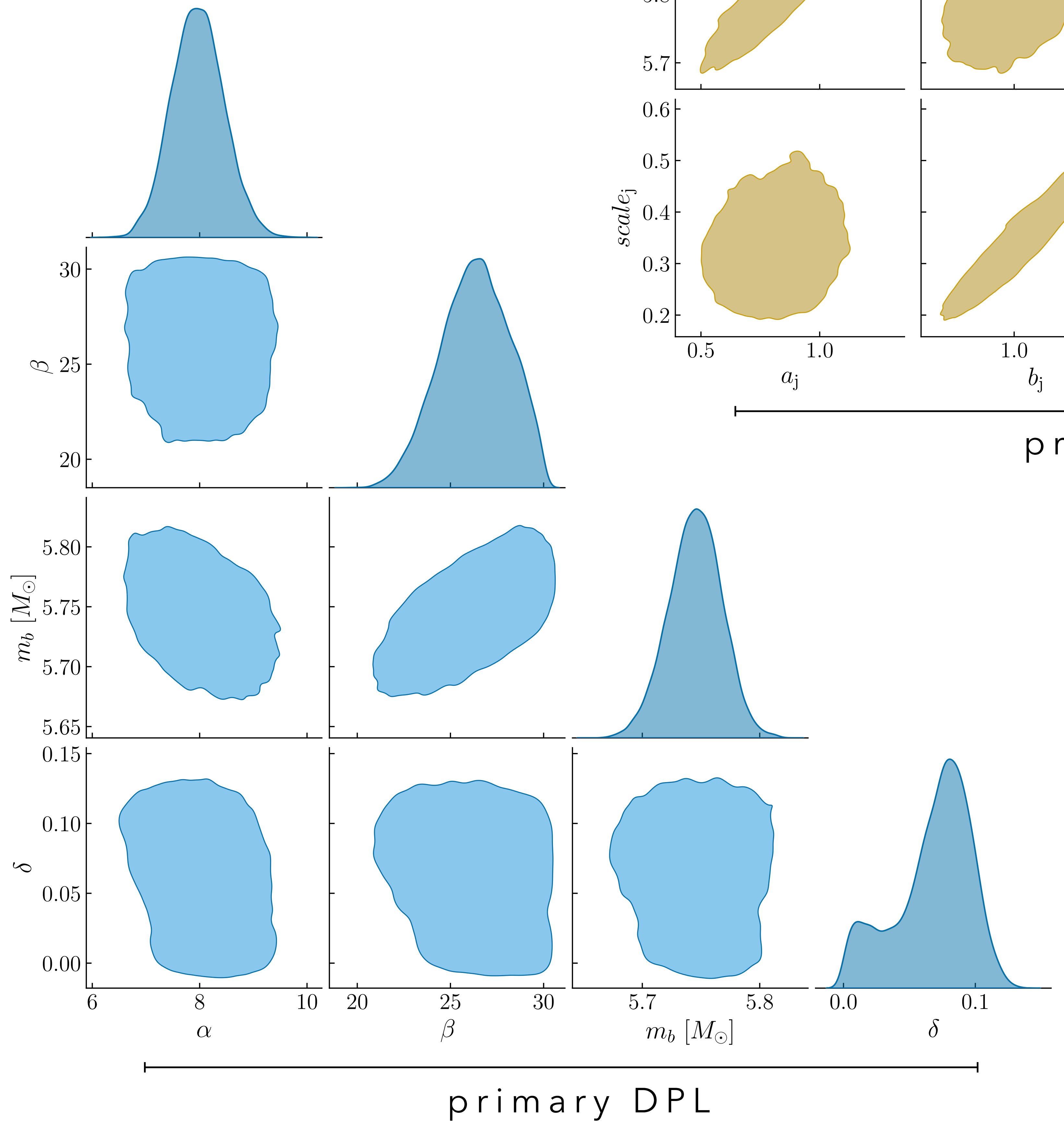
- reconstructed distributions agree with data
- population parameters agree with 1-dimensional fits
- posteriors well measured



Q3d INFERENCE 90yrs

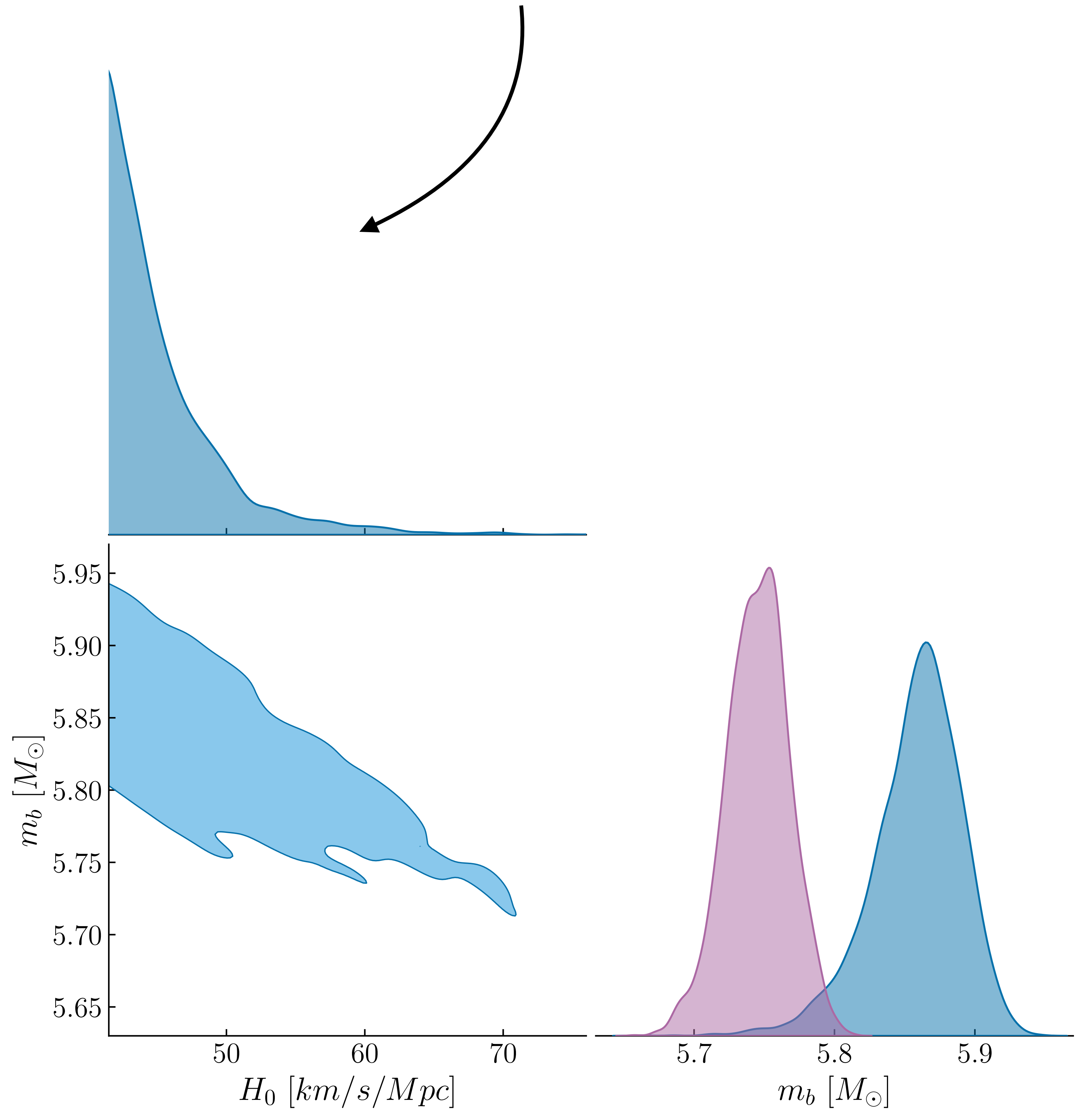


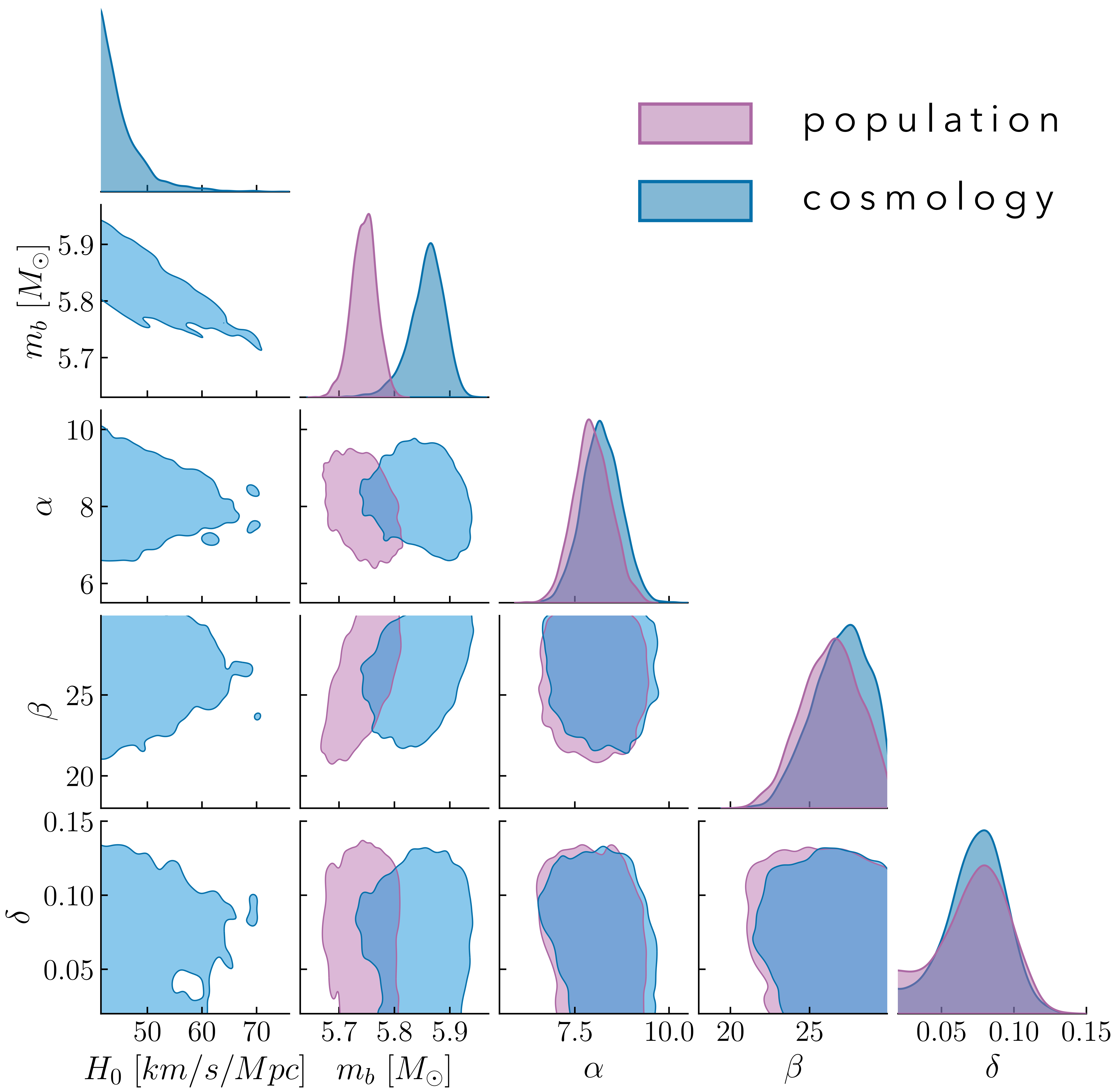




COSMOLOGY?

H0 railing against the prior

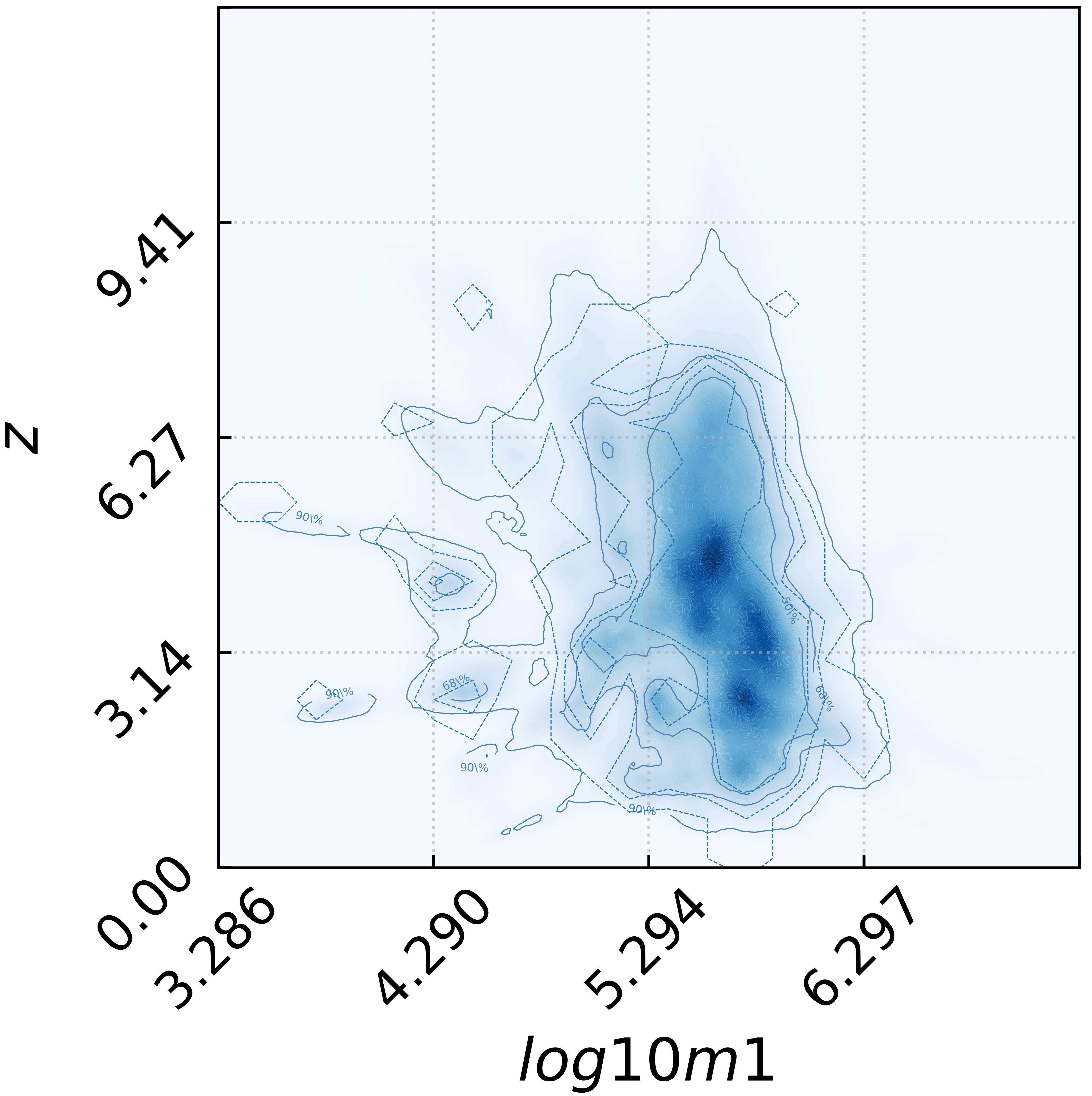




Q3d REDSHIFT EVOLUTION

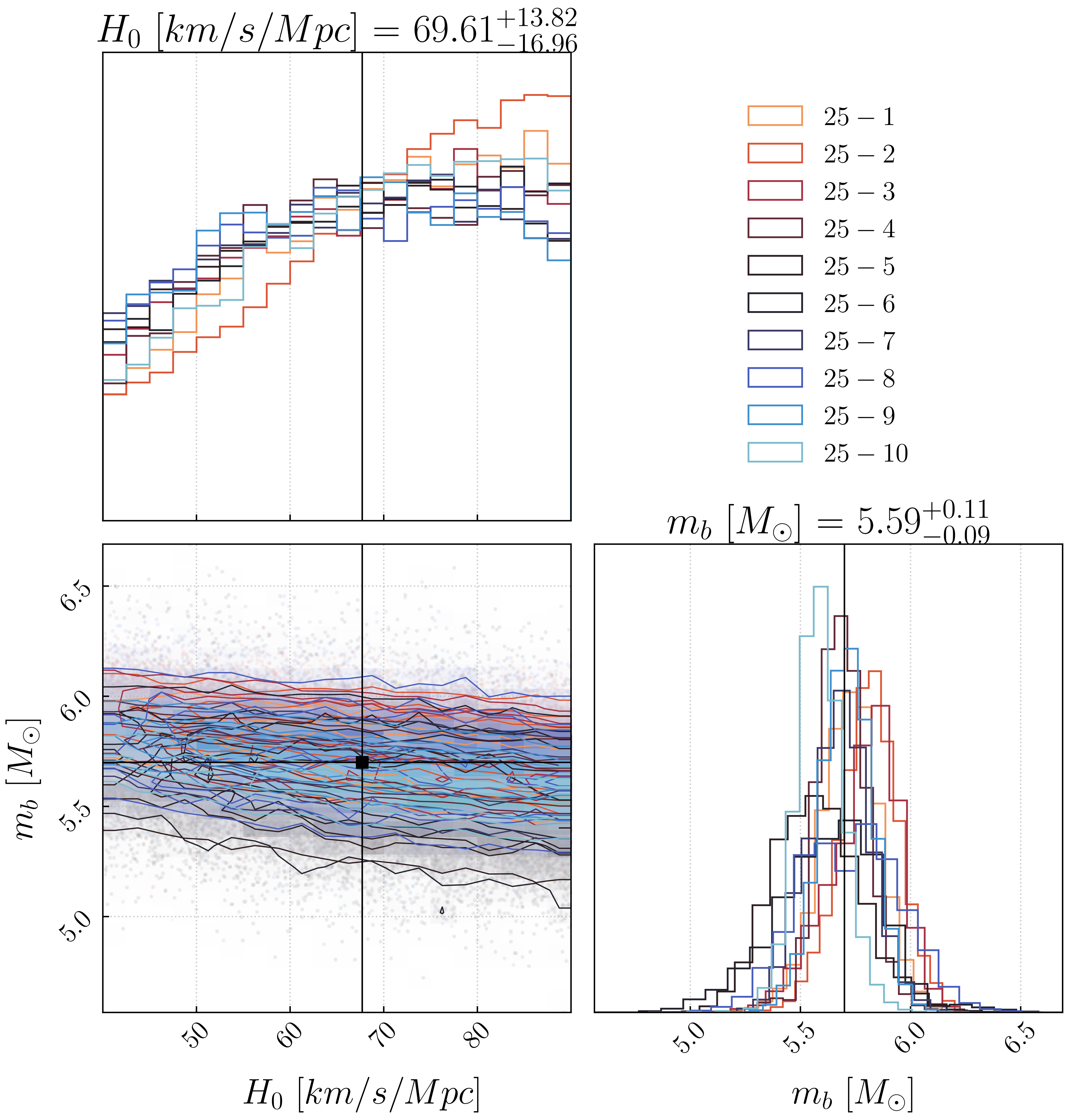
nobody is perfect, especially our parametrized model

- richer unmodelled structure can bias the inference
- probably need more flexible methods



4. simulations

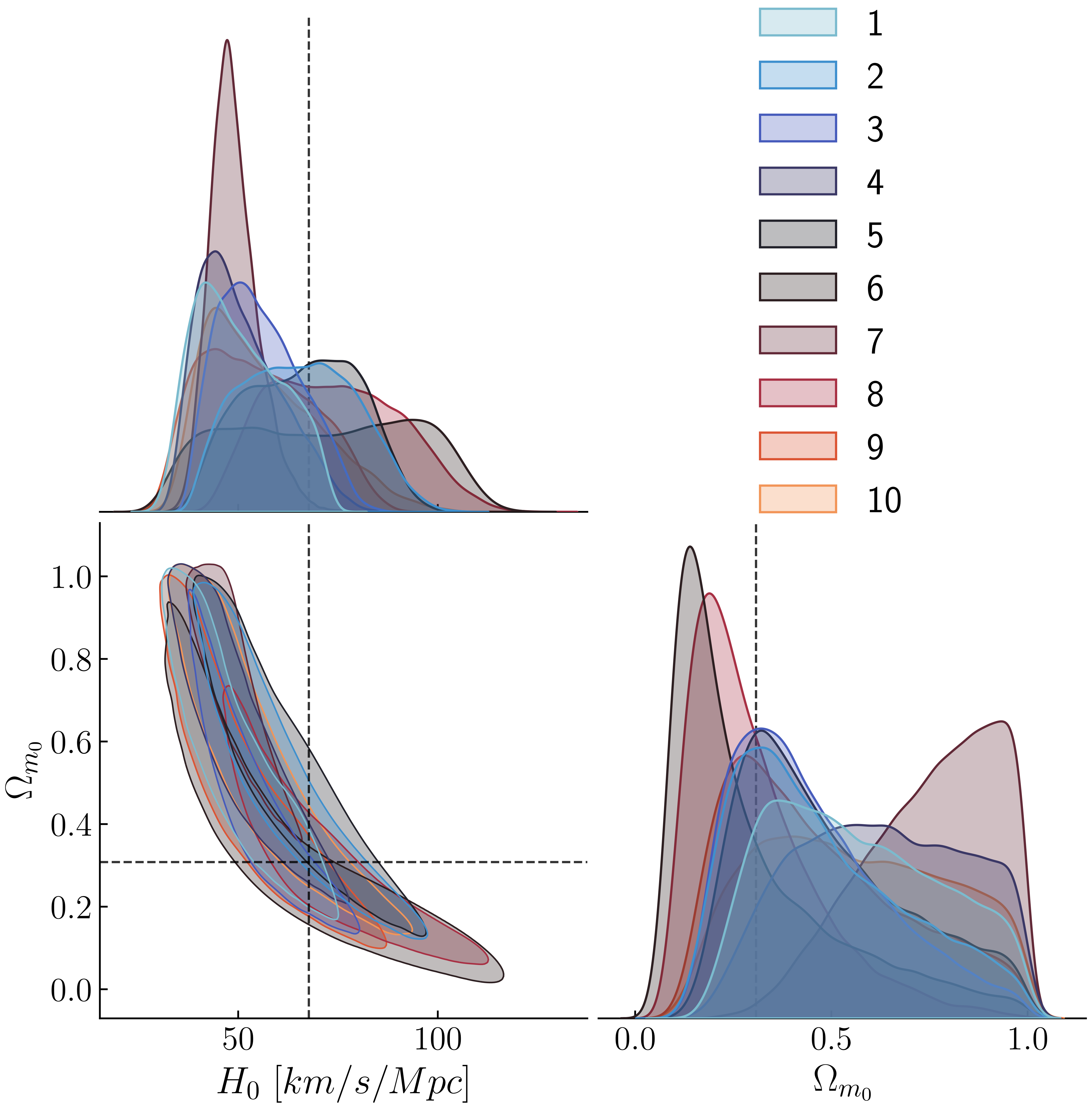
10 realizations of 4 years



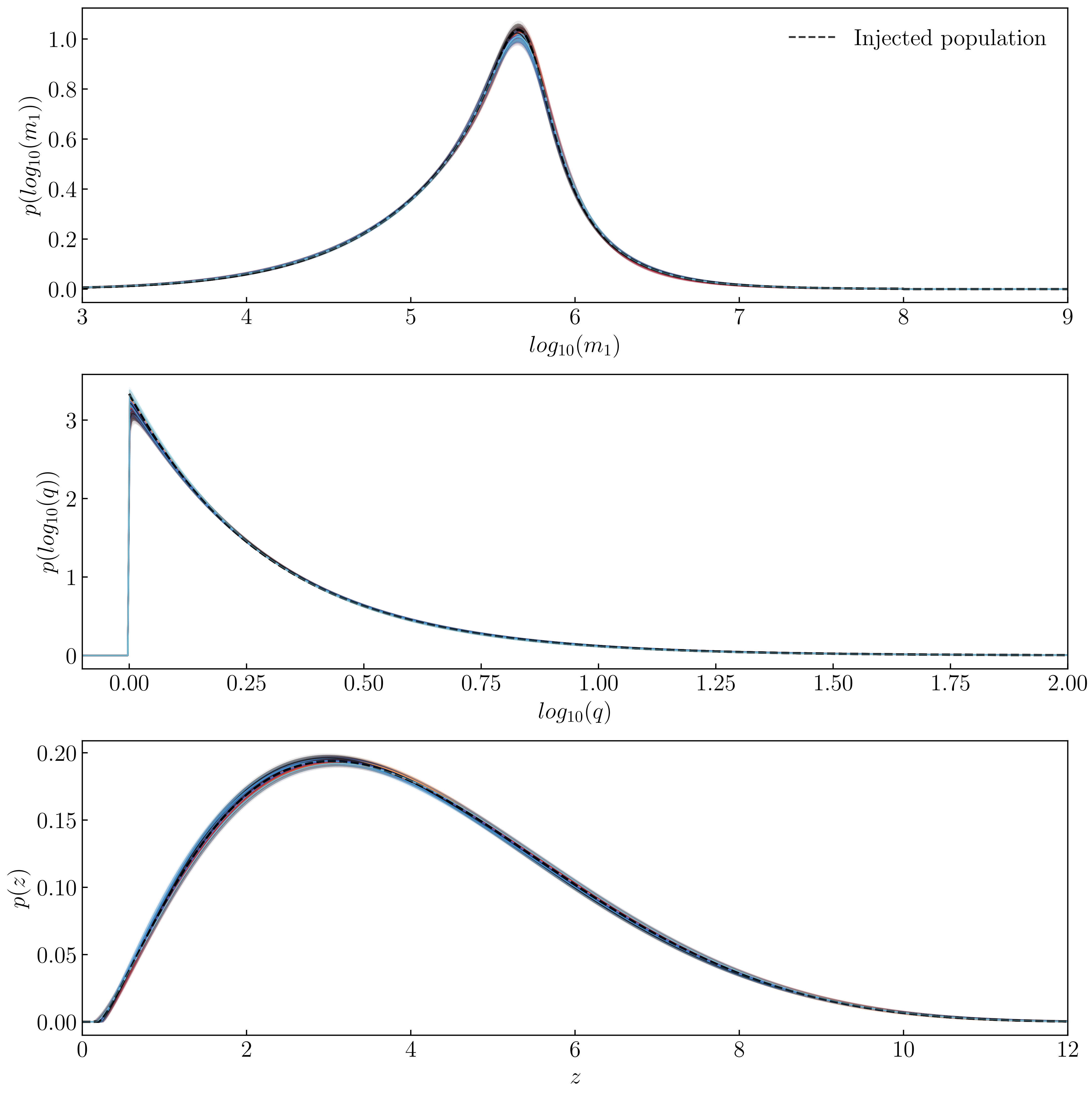
uninformative constraints on H0

- very few events
- only one, very wide feature
- the distribution weakly depends on cosmology

10 realisations of 10000 events Q3d-like

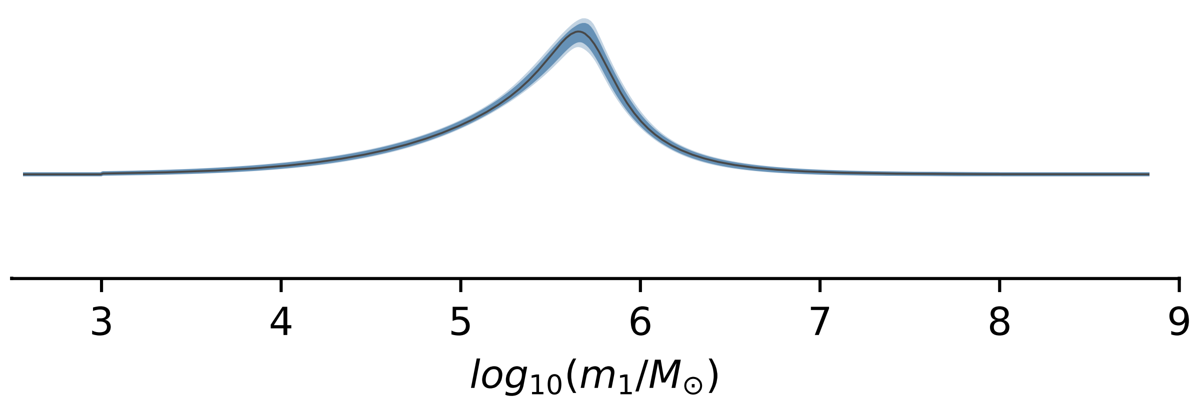


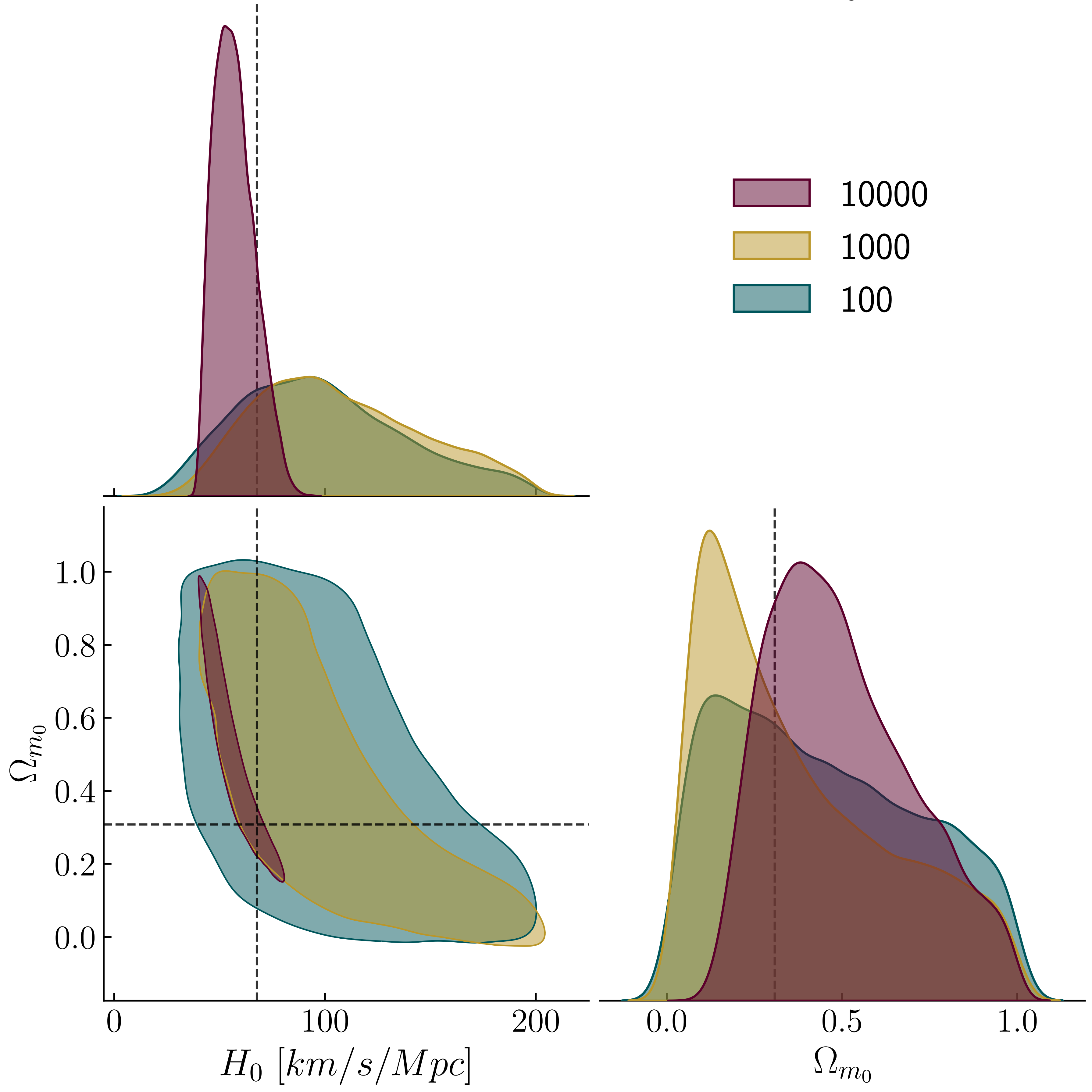
population only



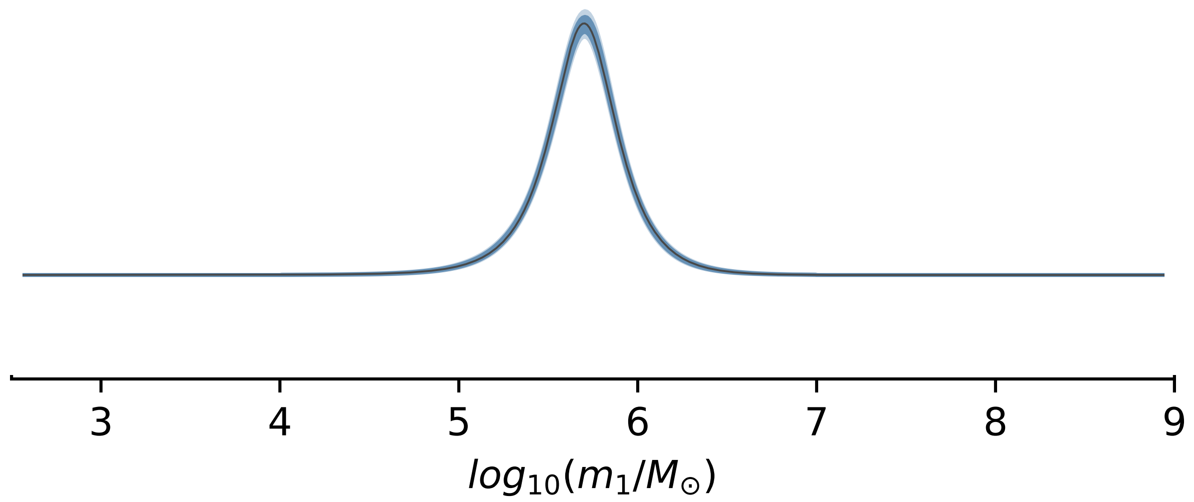
artificially narrowing the feature does improve the H0 measurement

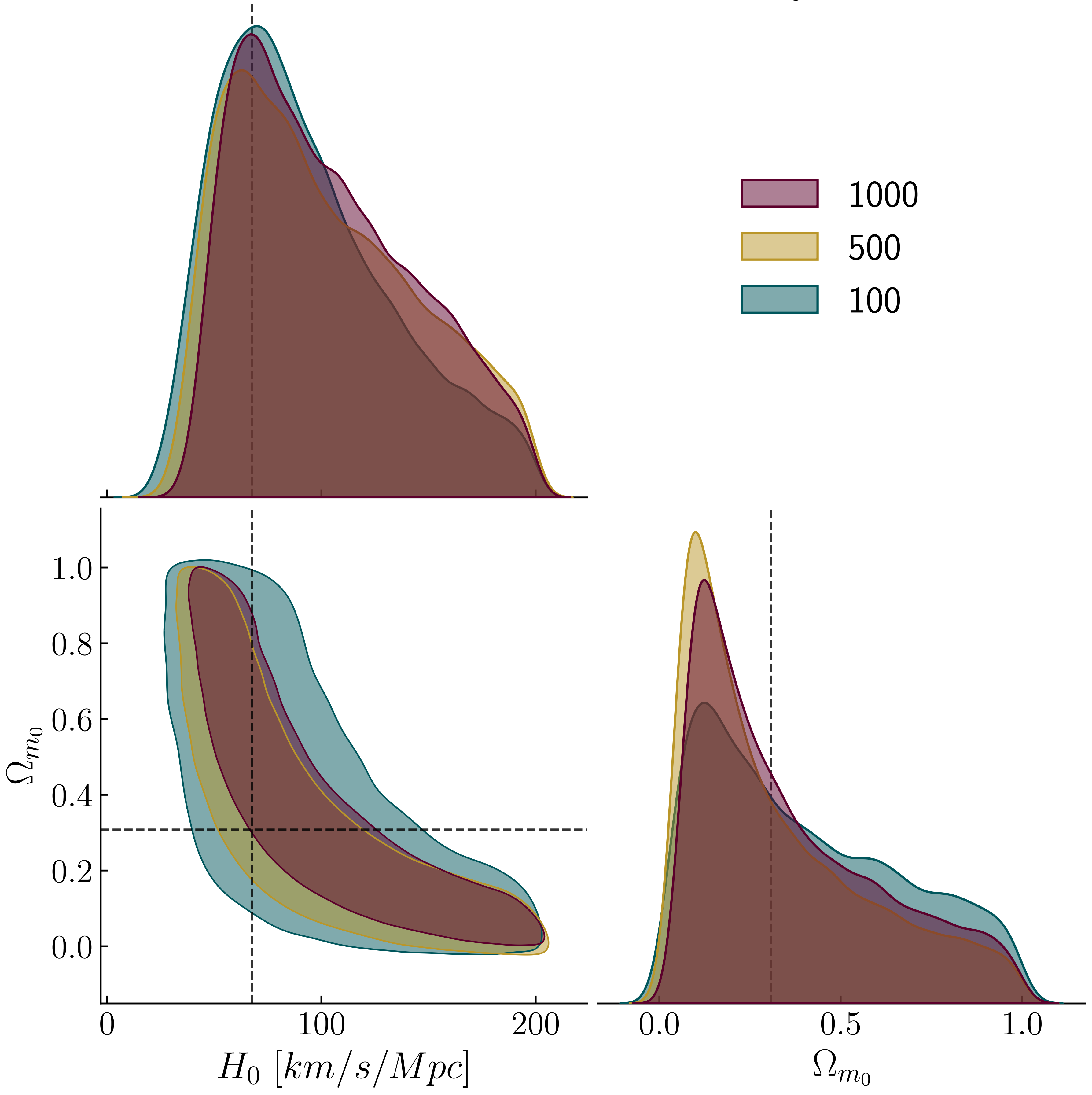
Q3d-like feature





narrow feature





5. conclusions

CONCLUSIONS

- we implemented and tested population models for MBHs
  - first realistic population analysis
  - first realistic cosmological analysis from the population
- unable to measure H0 with the realistic catalog
  - intrinsic structure prevent unbiased analysis
  - need to find develop more flexible strategies
- poor constraints on H0 from simulations
  - intrinsic limitations due to the distribution shape
  - additional feature will make the analysis more informative

backup slides

HIERARCHICAL LIKELIHOOD

the hierarchical likelihood is

$$\mathcal{L}_H(\{\mathbf{d}\}|\boldsymbol{\Lambda}) \propto \prod_i^{N_{\mathrm{gw}}} \frac{\int \mathrm{d}\boldsymbol{\theta}\, \mathcal{L}_{\mathrm{gw}}(\mathbf{d}_i \,|\, \boldsymbol{\theta}, \boldsymbol{\Lambda})\, \boxed{\frac{\mathrm{d}\mathcal{N}}{\mathrm{d}\boldsymbol{\theta}\mathrm{d}t}(\boldsymbol{\Lambda})}}{\int \mathrm{d}\boldsymbol{\theta}\, p_{det}(\boldsymbol{\theta}, \boldsymbol{\Lambda})\, \boxed{\frac{\mathrm{d}\mathcal{N}}{\mathrm{d}\boldsymbol{\theta}\mathrm{d}t}(\boldsymbol{\Lambda})}}$$

with observed rate in the *detector frame*

$$\boxed{\frac{\mathrm{d}\mathcal{N}}{\mathrm{d}\boldsymbol{\theta}\mathrm{d}t}} = \frac{\mathrm{d}\mathcal{N}}{\mathrm{d}m_{1,d}\,\mathrm{d}q\,\mathrm{d}d_L\,\mathrm{d}t_d} = \frac{\mathrm{d}\mathcal{N}}{\mathrm{d}\log m_{1,s}\,\mathrm{d}\log q\,\mathrm{d}z\,\mathrm{d}t_d} \frac{1}{\left|\mathcal{J}_{(m_{1,d},q,d_L)\to(\log m_{1,s},\log q,z)}\right|}$$

$$(1+z)\,\frac{\partial d_L}{\partial z}\,\frac{m_{1,s}\,q}{(\log e)^2}$$

$$\frac{\mathrm{d}\mathcal{N}}{\mathrm{d}\log m_{1,s}\,\mathrm{d}\log q\,\mathrm{d}z\,\mathrm{d}t_d}(\boldsymbol{\Lambda}) \propto \boxed{p(\log m_{1,s}\,|\,\boldsymbol{\Lambda})\,p(\log q\,|\,\boldsymbol{\Lambda})\,p(z\,|\,\boldsymbol{\Lambda})}$$ population model

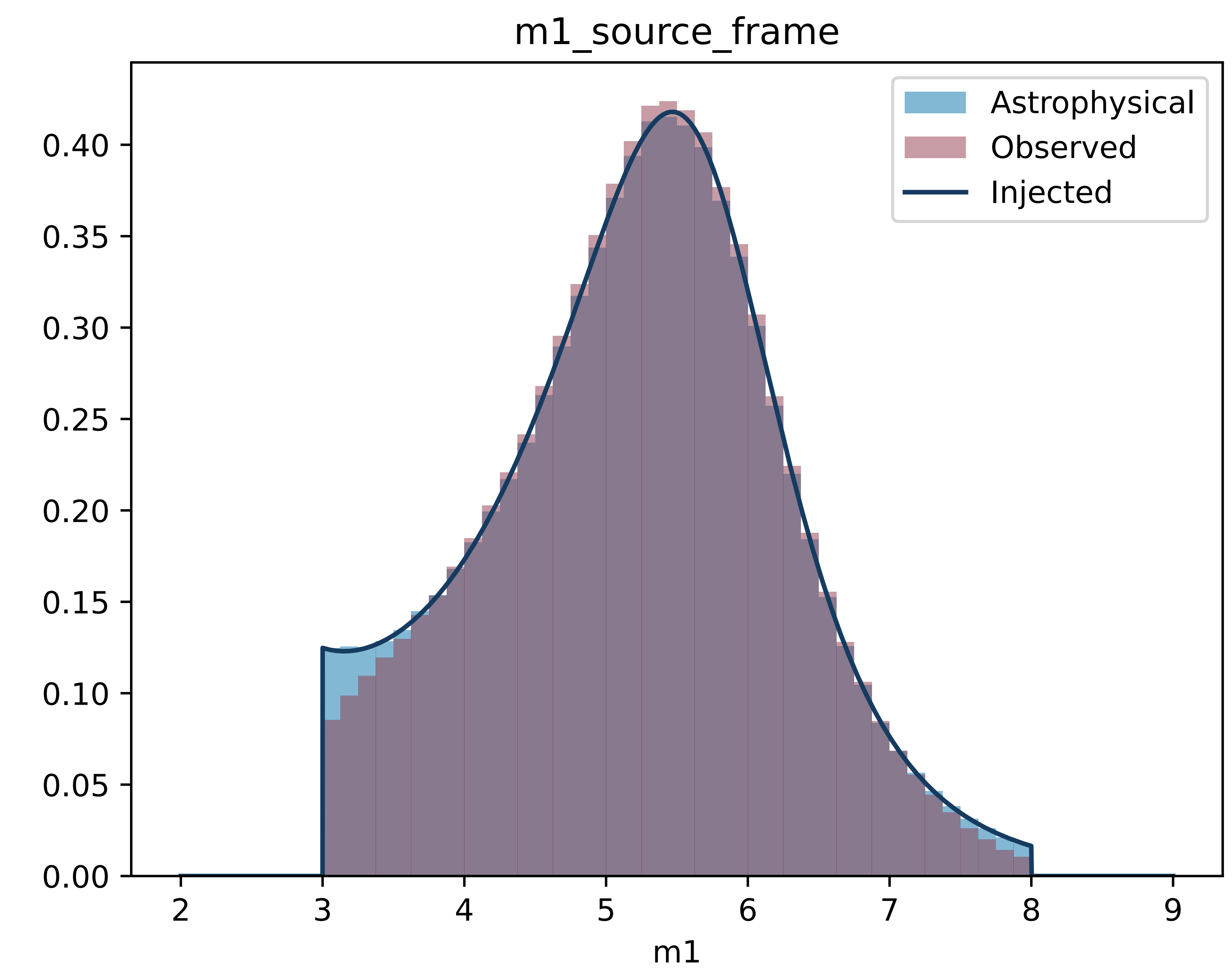
primary **mass ratio** **redshift**

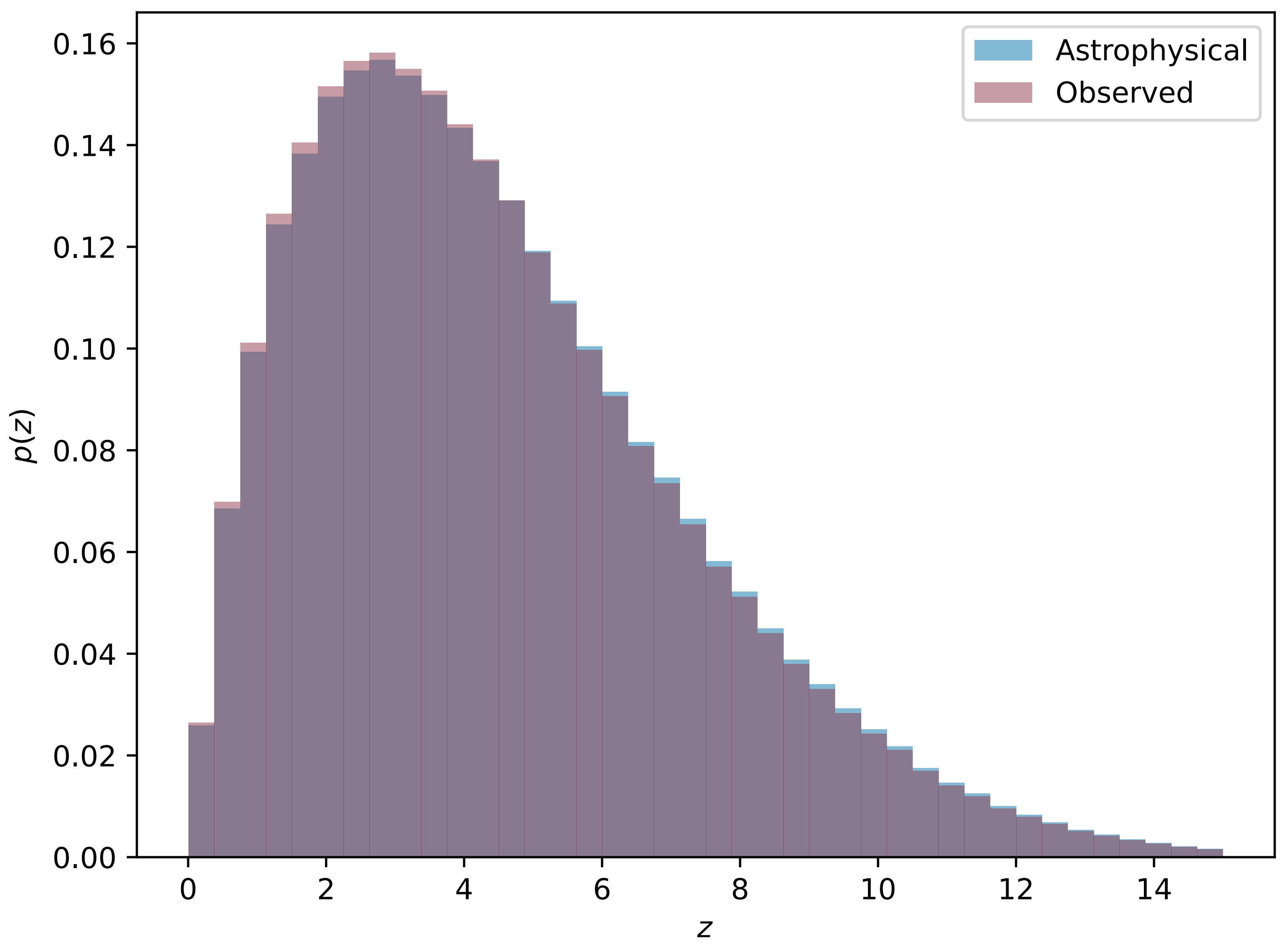
IMPORTANT

we assume (m_1, q, z) to be *uncorrelated* on a population level

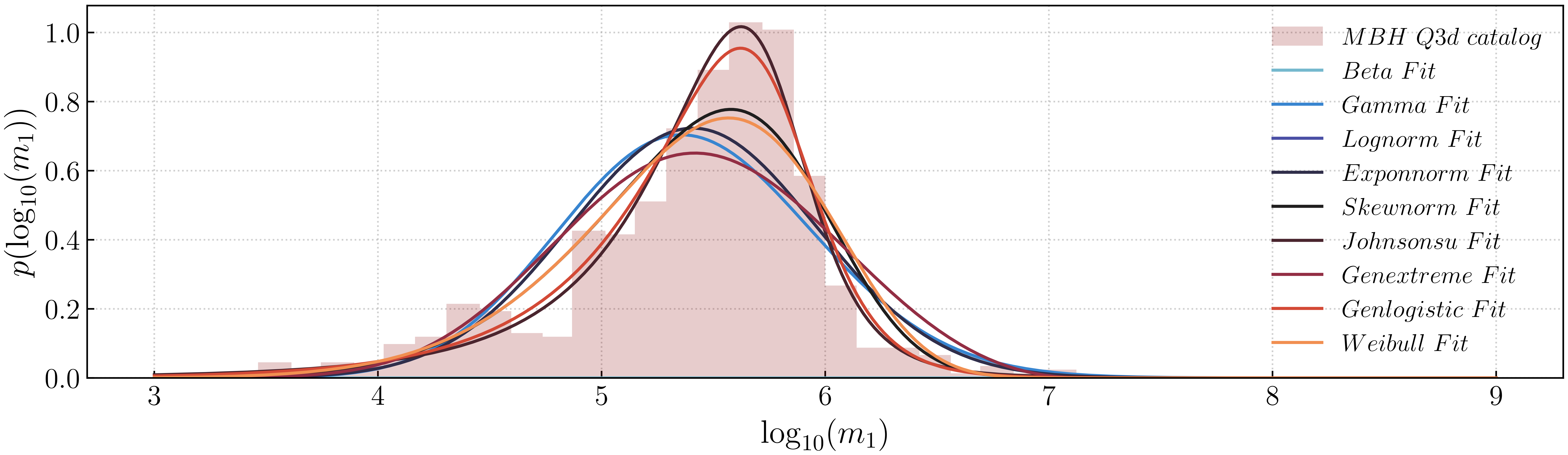
ICAROGW SETTINGS

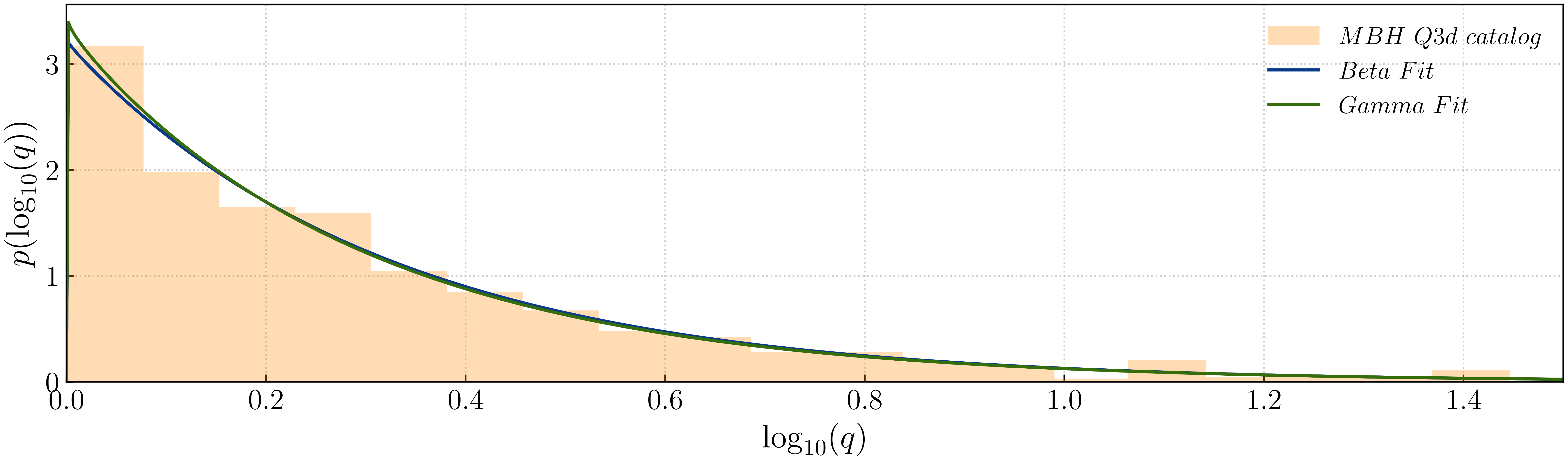
- **injections** (for selection effects)
  - 10^6 injections
  - $SNR > 20$ for detection
- **hierarchical analysis**
  - we use ICAROGW
  - sampler: nessai with 2000 live points
  - $N_{eff,inj}/(4 * N_{events}) > 1$ for numerical stability





Q3d MODEL SELECTION

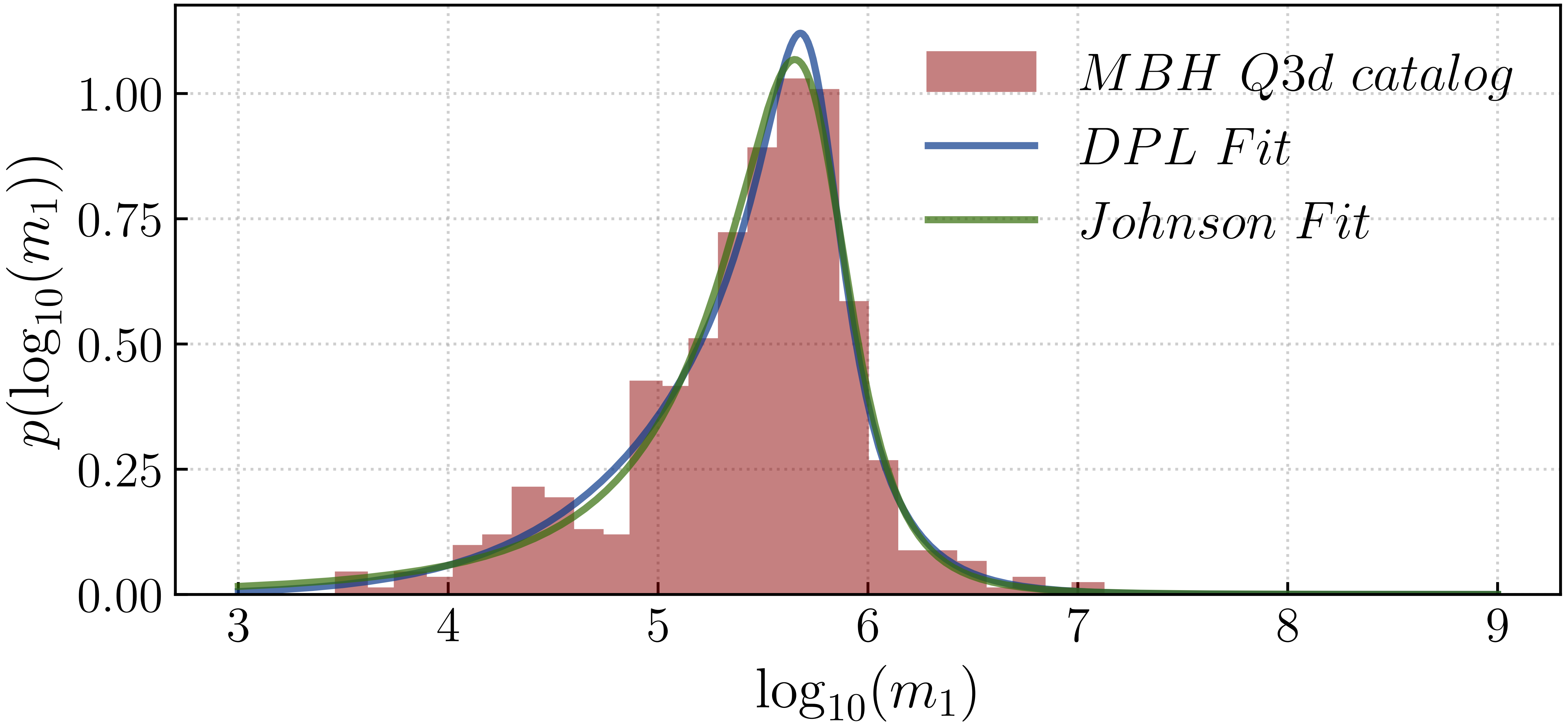




POPULATION MODEL

primary mass

$$p(log_{10} m_{1,s})$$



- double powerlaw (DPL)

$$p(x,\alpha,\beta,m_b,\delta) = \frac{1}{norm}\{\left[1-\sigma(x)\right] x^{\alpha} + \sigma(x)\, x^{-\beta}\, m_b^{\alpha-\beta}\} \qquad \sigma(x) = \frac{1}{1+e^{\frac{m_b - x}{\delta}}}$$

$\alpha = 8.07,\ \beta = 25.05,$
$m_b = 5.74,\ \delta = 0.08$

$a = 0.79,\ b = 1.03,$
$loc = 5.79,\ \lambda = 0.34$

- johnson distribution (JD)

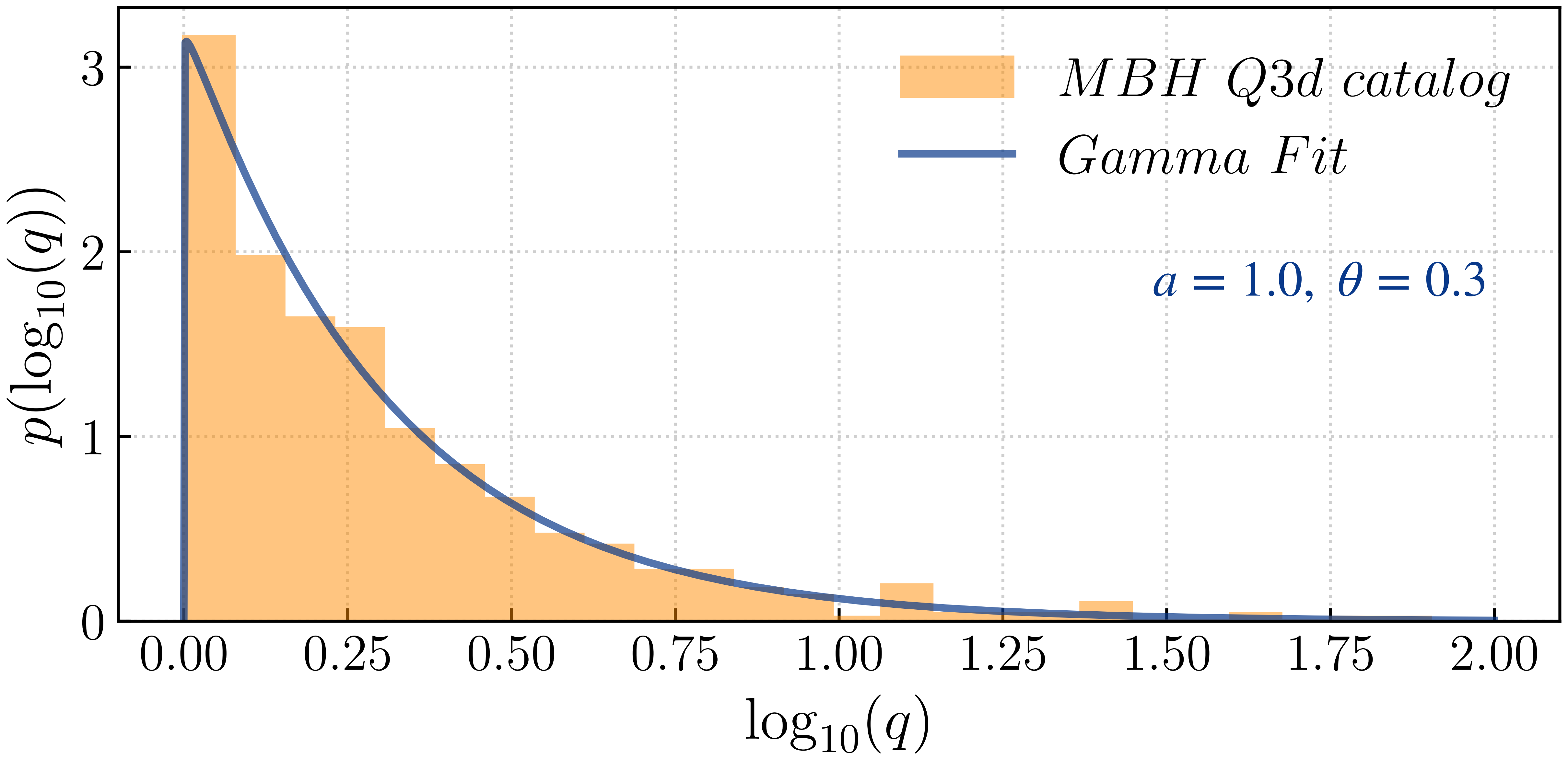
$$p(y,a,b,\lambda) = \frac{b}{\lambda\sqrt{y^2+1}} N(\xi(y),0,1) \qquad N(\xi(y),0,1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{\xi^2(y)}{2}}, \quad \xi(y) = a + b\, ln(y + \sqrt{y^2+1}), \quad y = \frac{x - loc}{\lambda}$$

POPULATION MODEL

mass ratio $p(log_{10} q)$

- gamma distribution

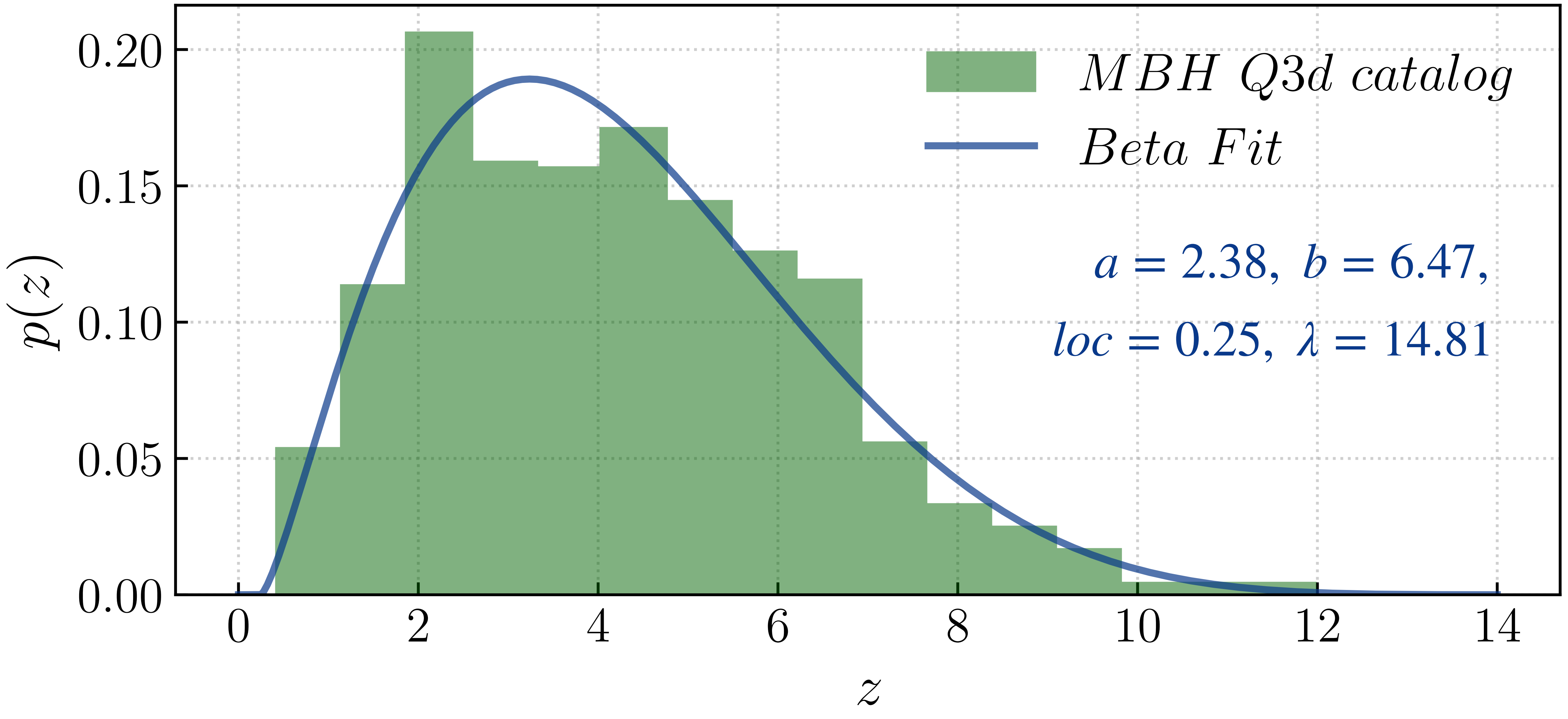
$$p(x, a, \theta) = \frac{\theta^a x^{a-1} e^{-\theta x}}{\Gamma(a)}$$



redshift $p(z)$

- beta distribution

$$p(y, a, b) = \frac{\Gamma(a+b)\, y^{a-1}\, (1-y)^{b-1}}{\Gamma(a)\,\Gamma(b)}$$

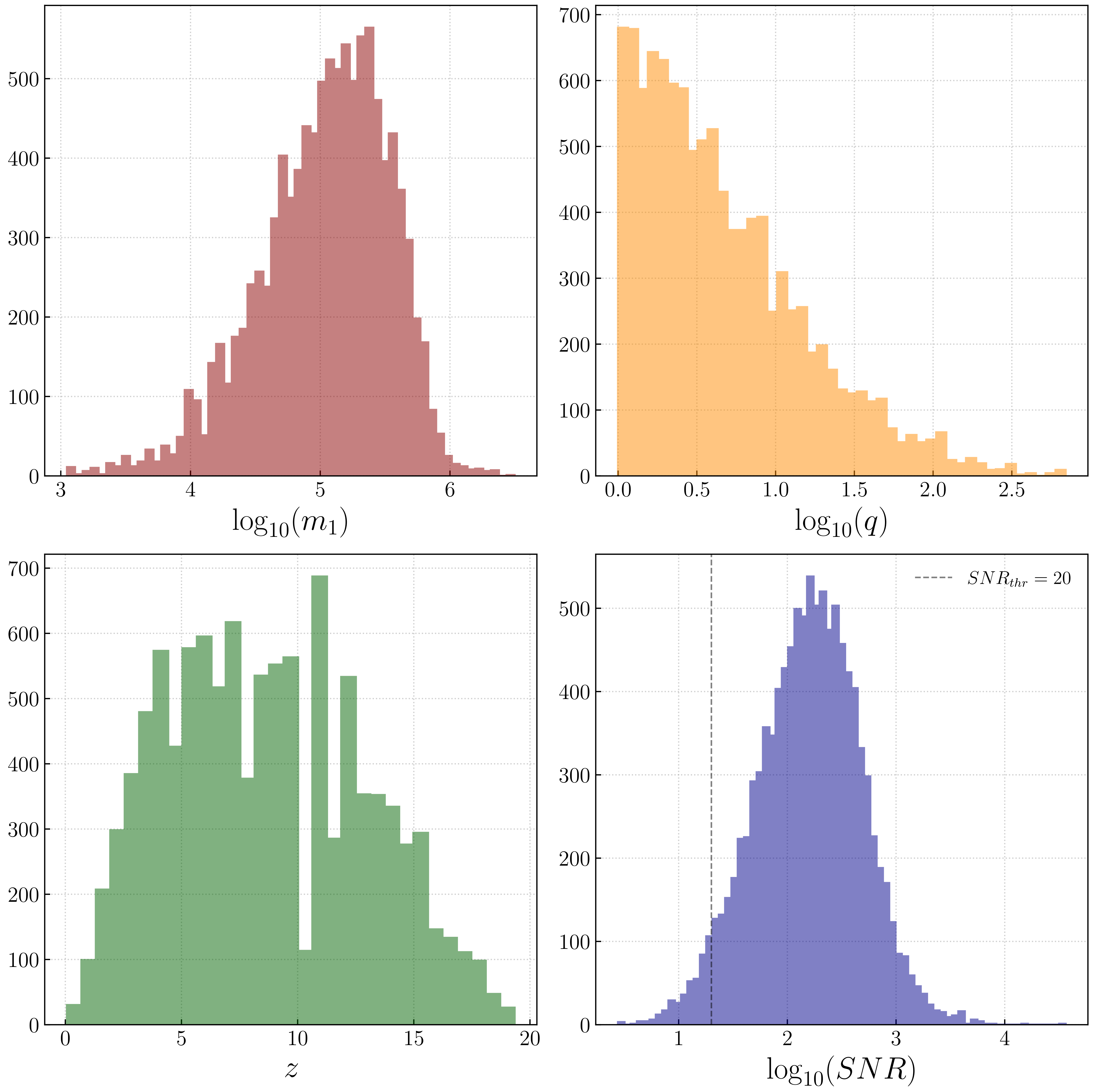


Q3nd CATALOG

- ~**10200** events for 90yrs of LISA
  - ~**110** events for a 4yrs mission
  - masses spanning large range
  - events up to *extremely high* redshift

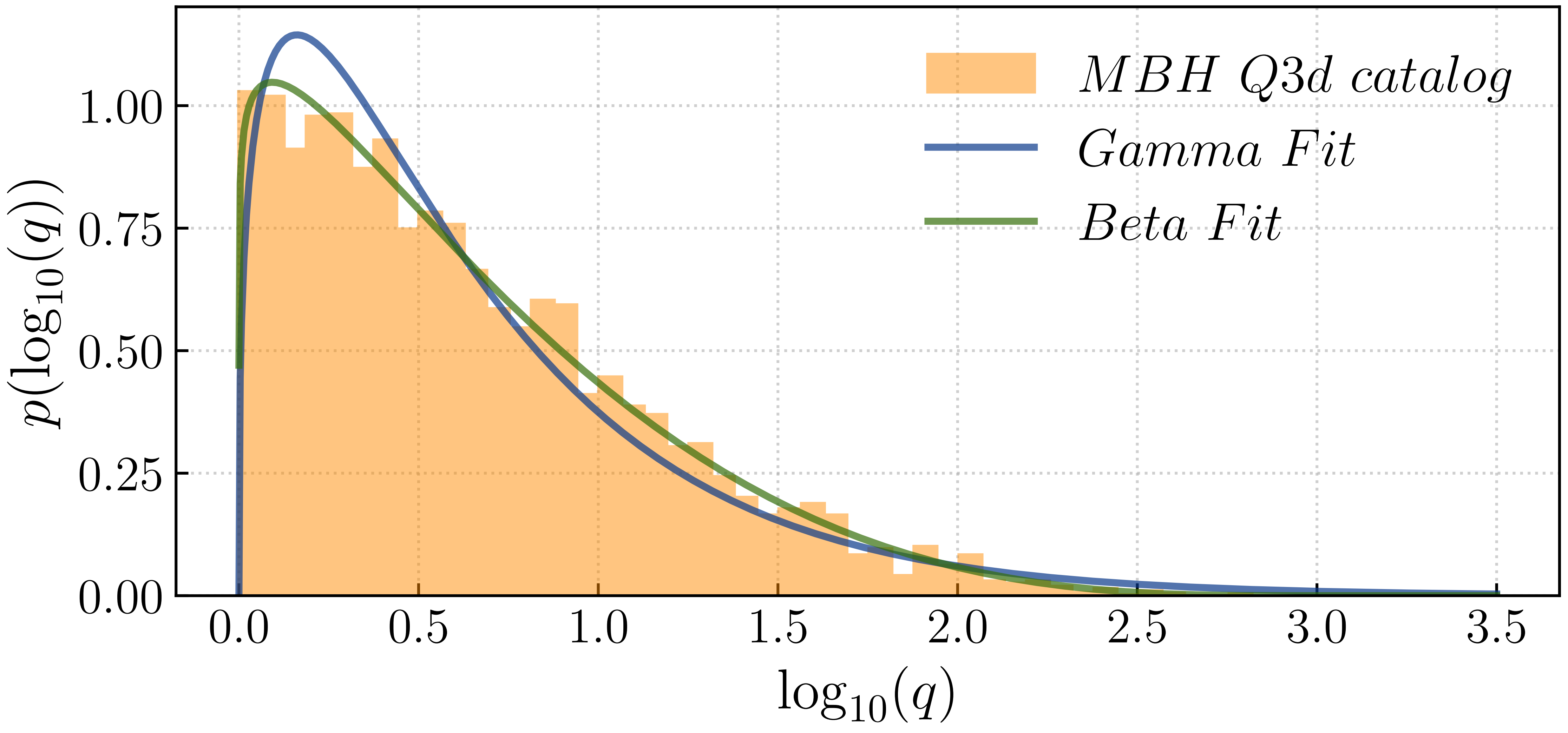
- hierarchical inference
  - well measured parameters
  - mild **selection effects**

    (434 events with SNR<20, ~4%)

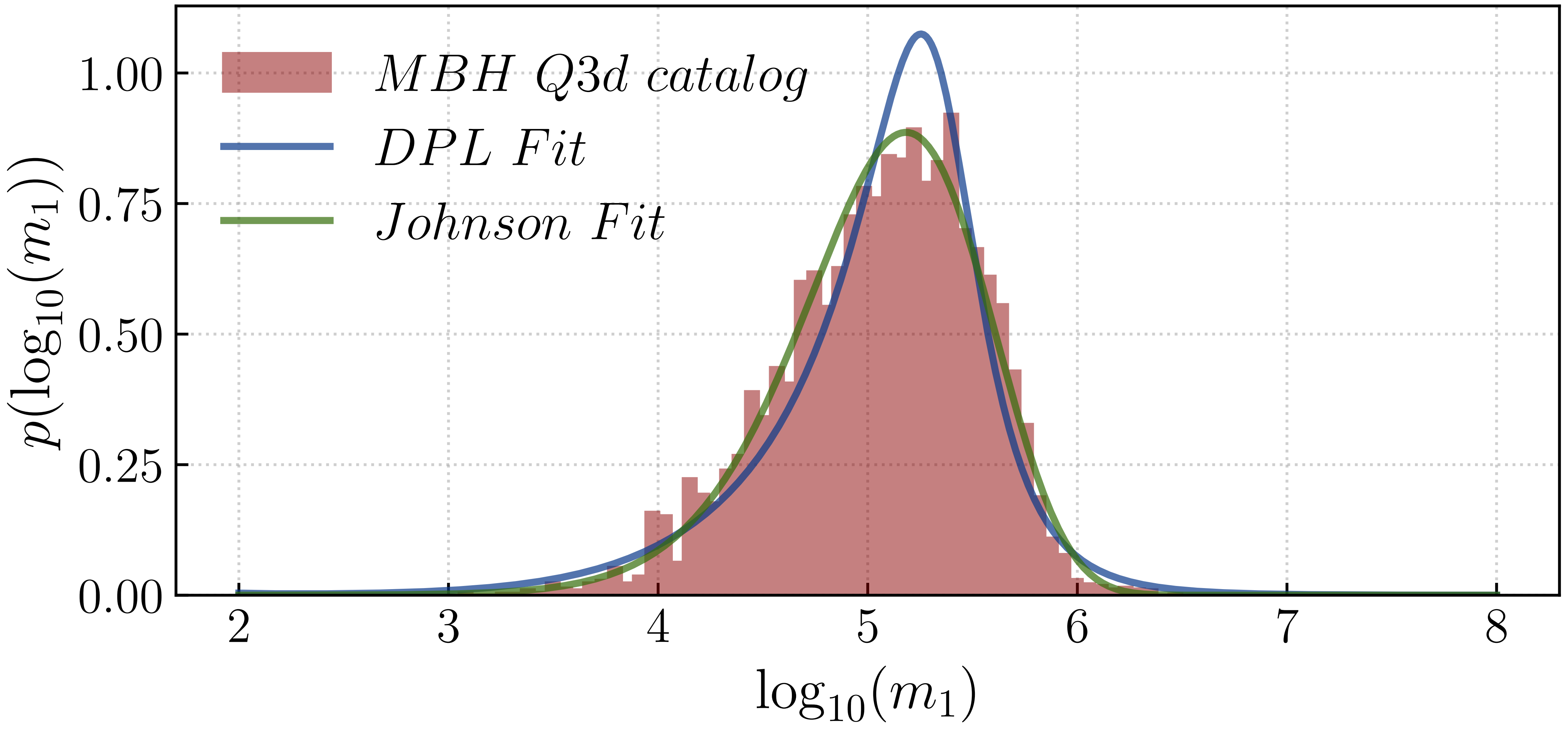


POPULATION MODEL Q3nd

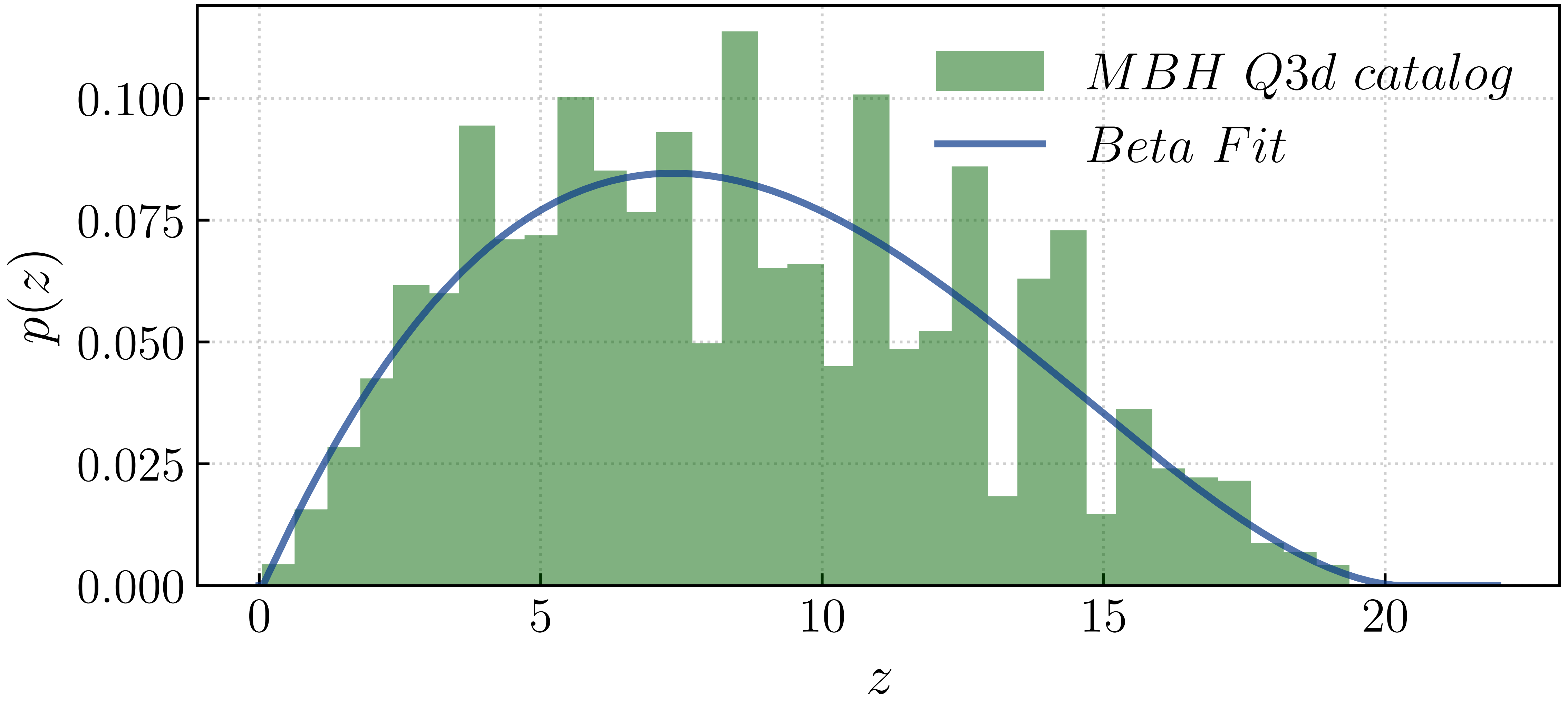
johnson preferred over DPL



same as Q3d with more spikes



beta preferred over gamma

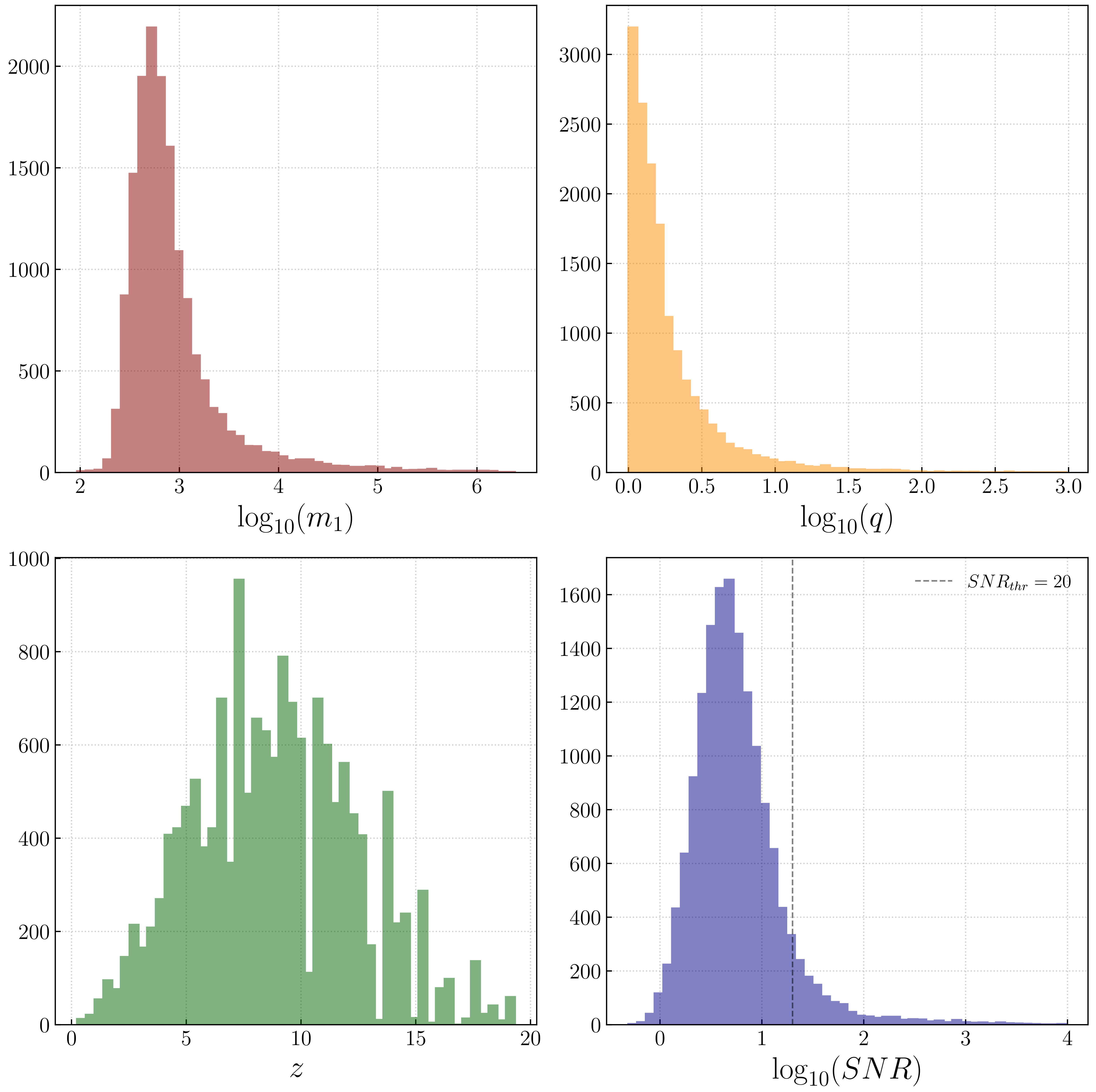


Pop3 CATALOG

- ~**1200** events for 90yrs of LISA
  - ~**50** events for a 4yrs mission
  - low masses compared to Q3d/Q3nd
  - events up to *extremely high* redshift

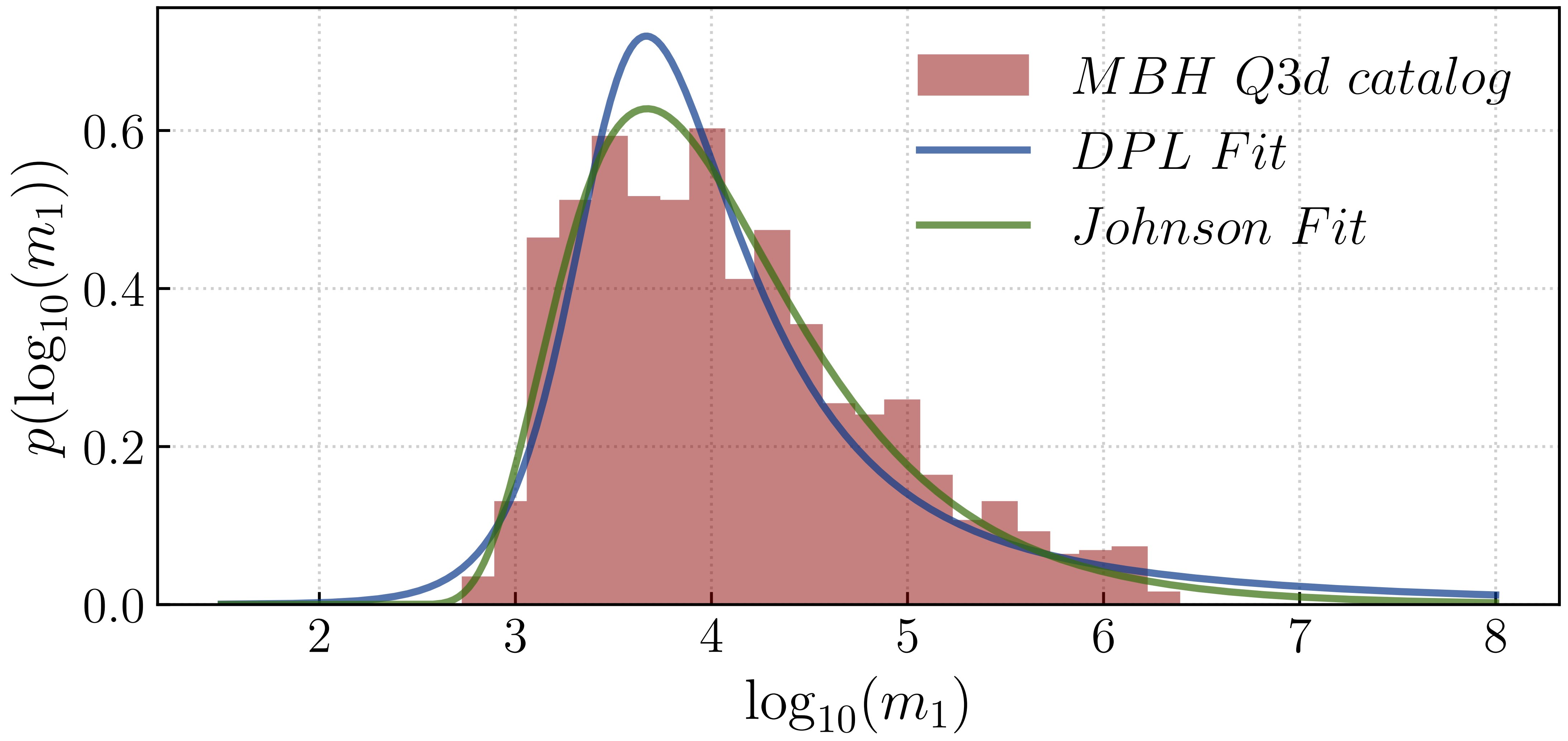
- hierarchical inference
  - problems at low masses
  - *heavy* **selection effects**

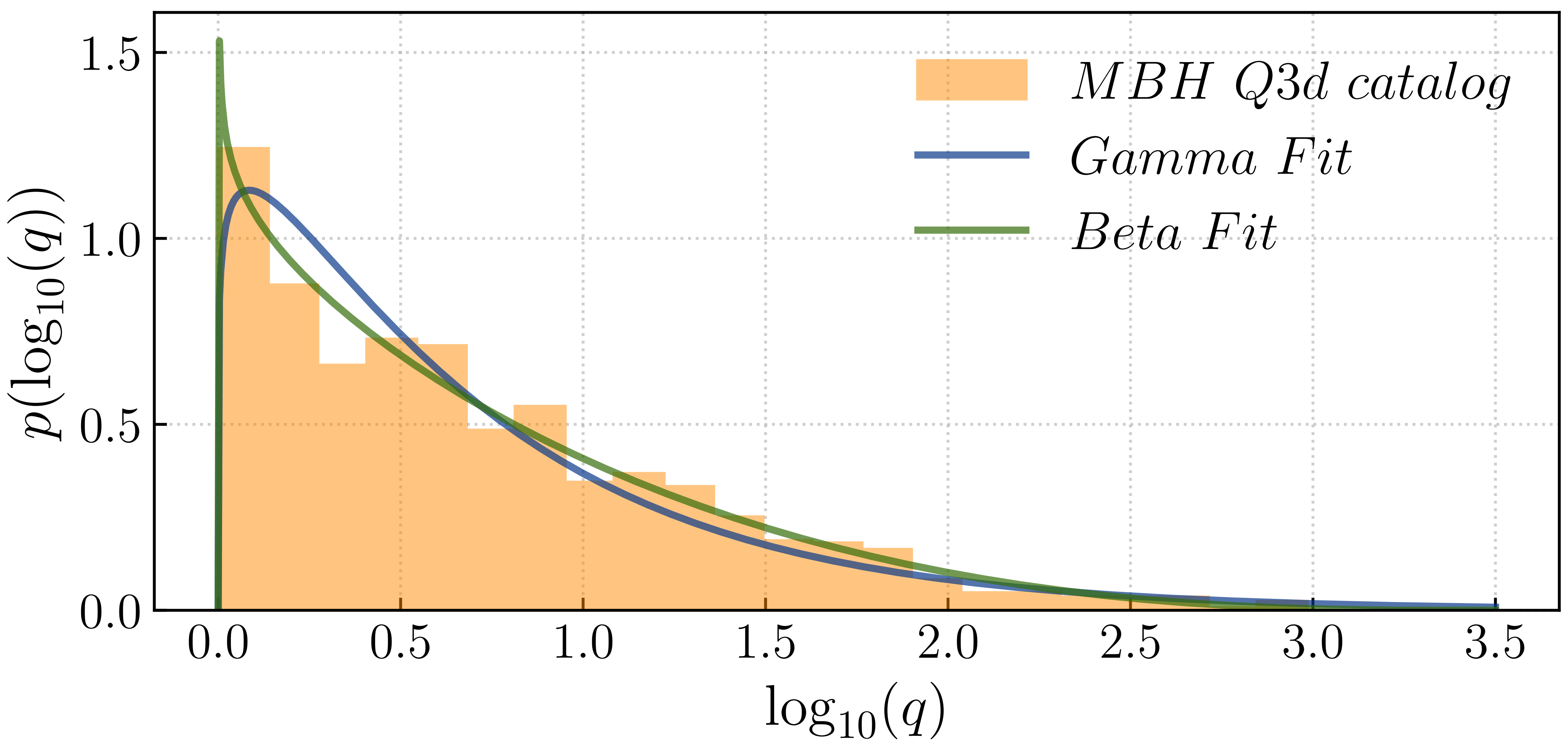
    (14000 events with SNR<20, ~90%)



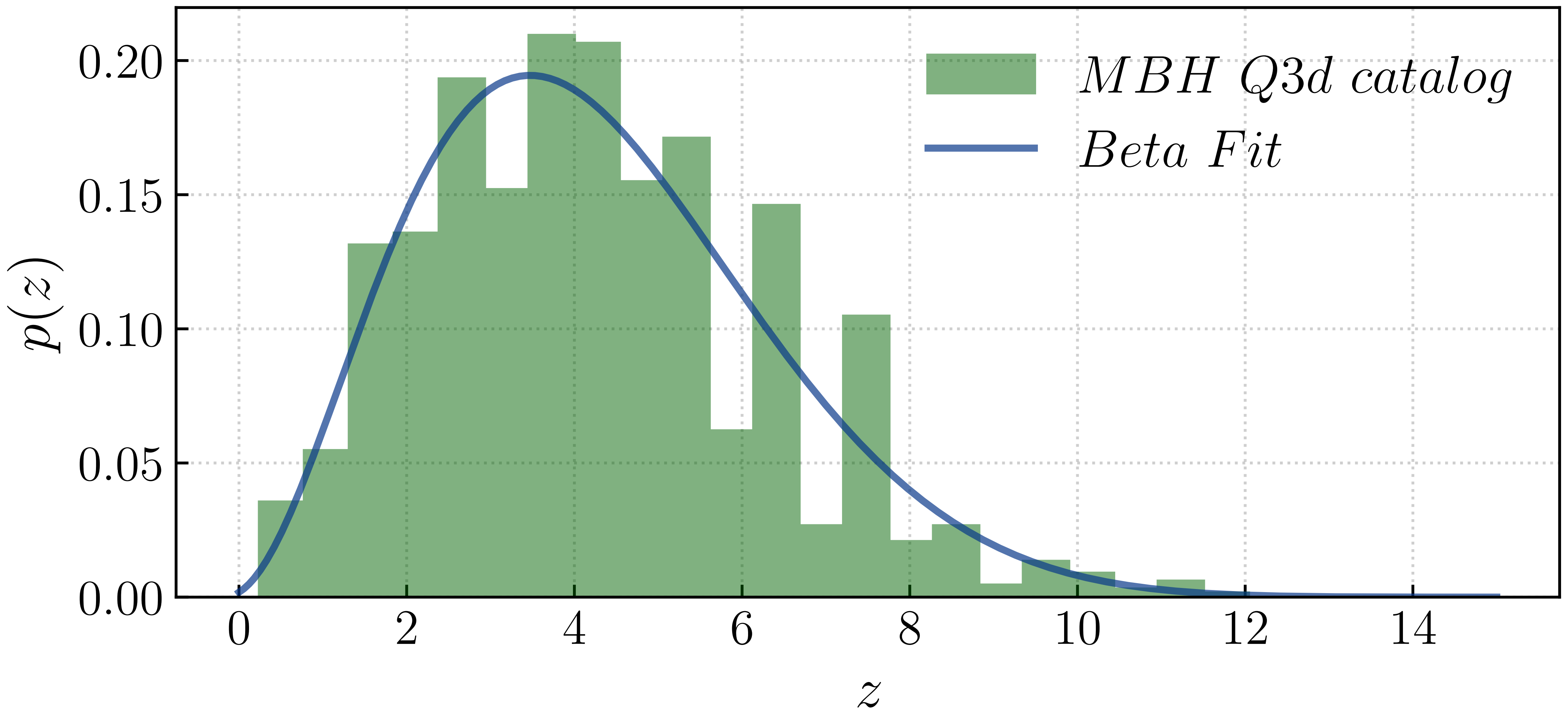
POPULATION MODEL Pop3

johnson preferred over DPL





beta preferred over gamma

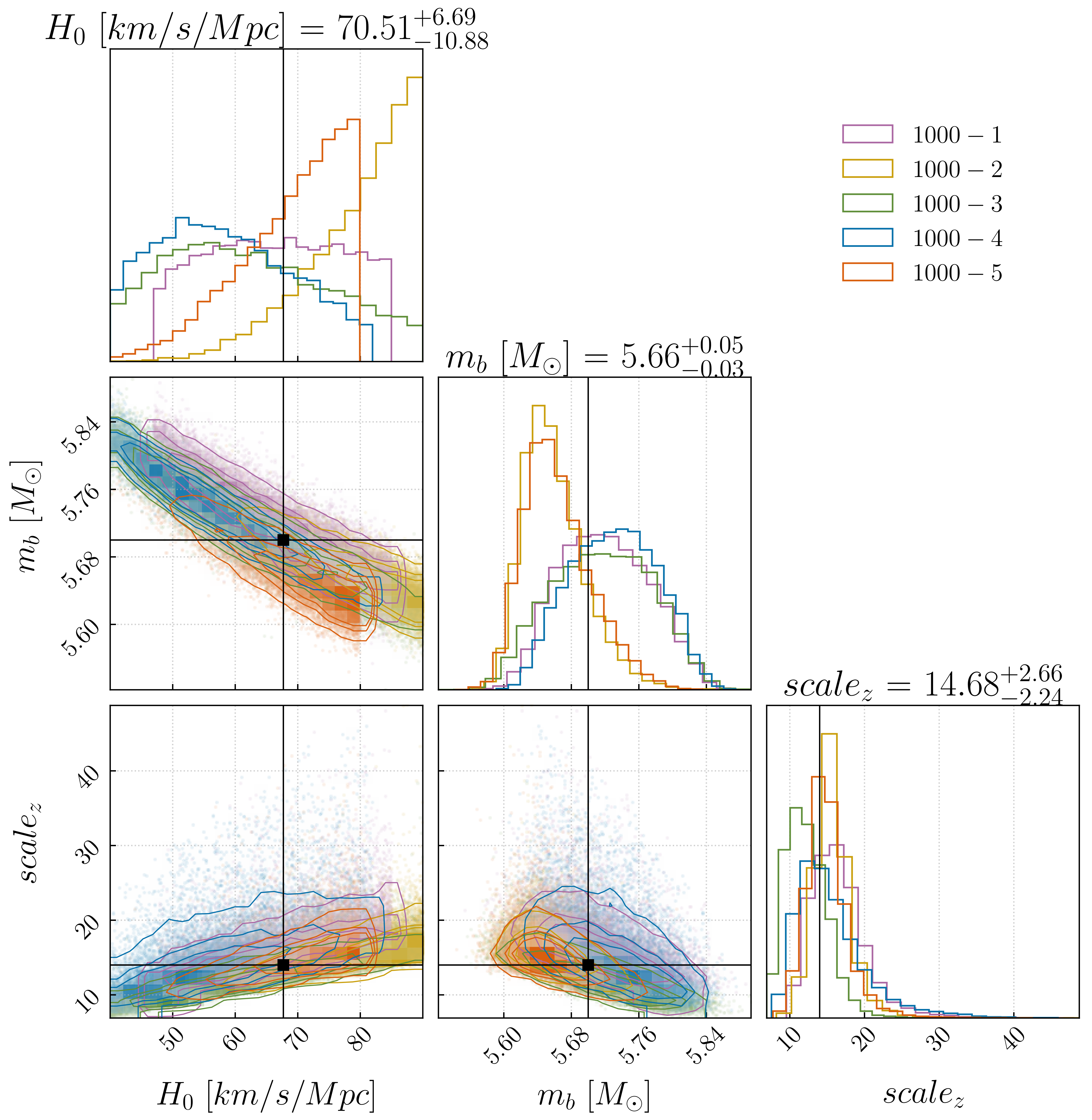


same as Q3d with more spikes

SIMULATIONS Q3d-like 1000 events

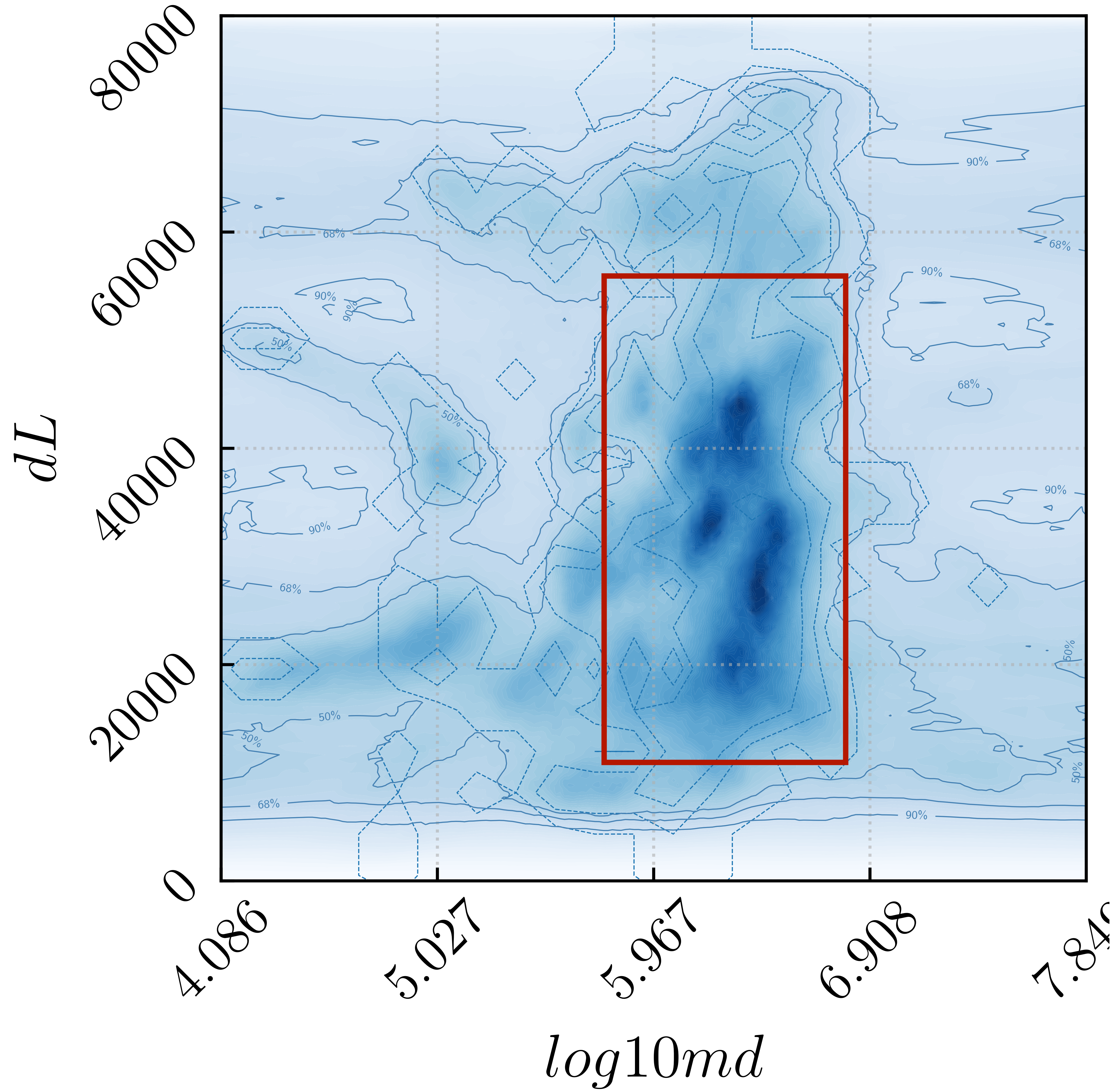
5 realisations of the same population (**1000 events**)

- poor constraints on H0
- increasing the number of simulations

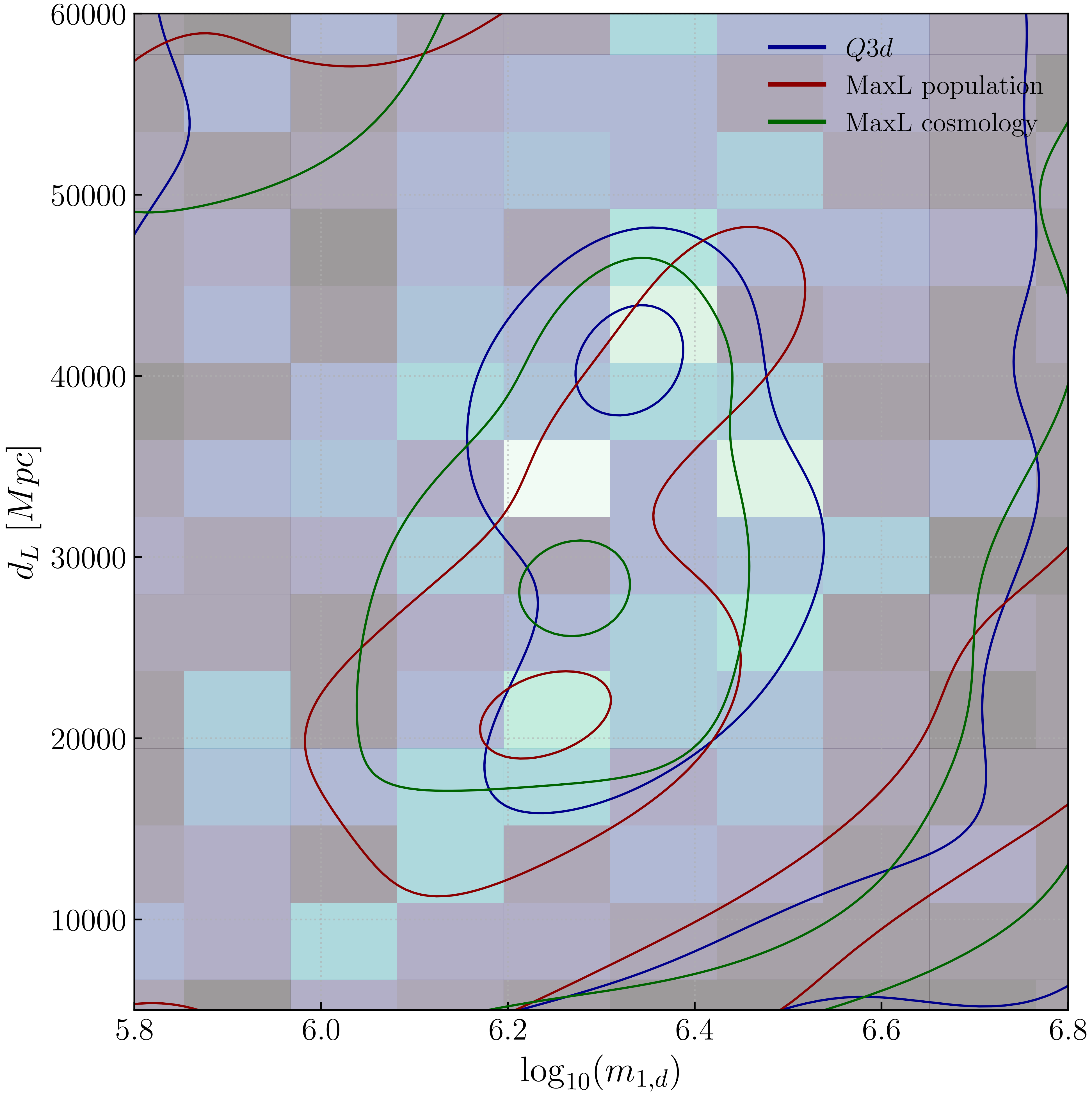


WHY H0 WITH Q3d IS BIASED?

Q3d non-parametric



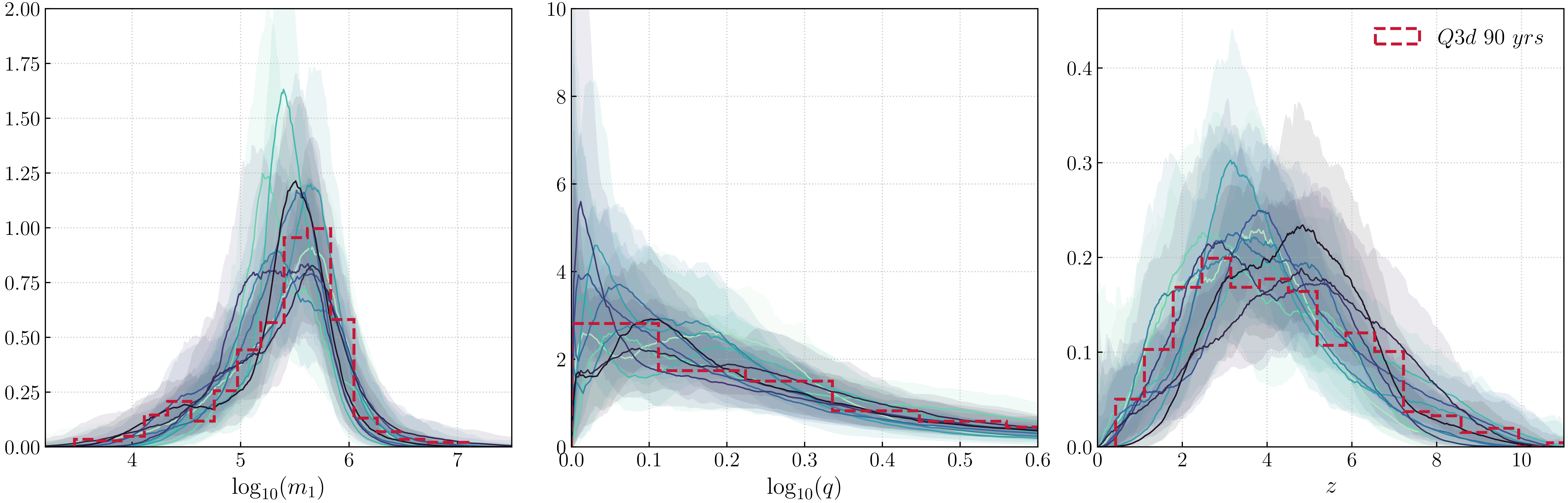
Q3d/inference comparison



possible impact of intrinsic redshift evolution

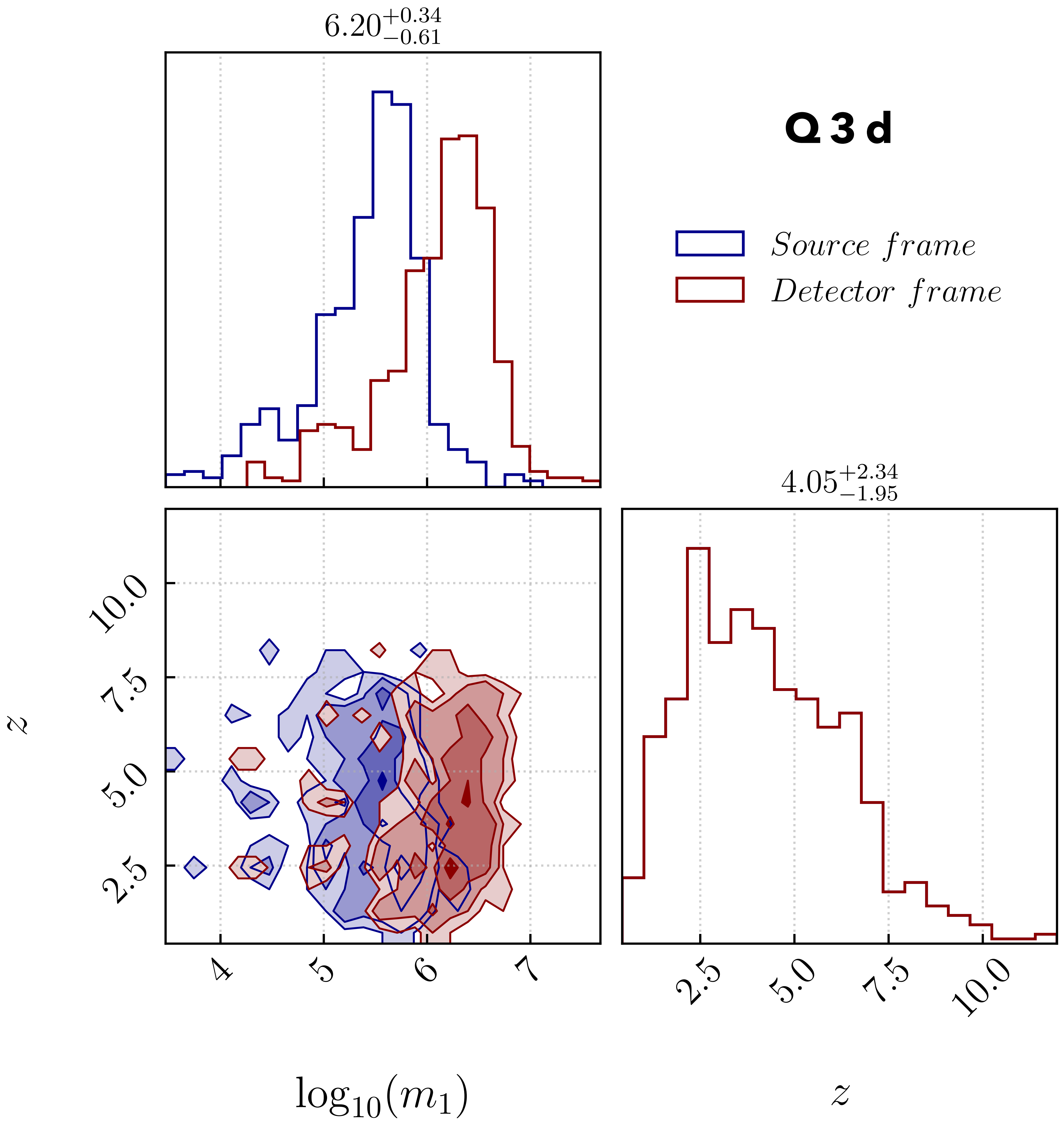
WHAT IS THE IMPACT OF HAVING FEW EVENTS?

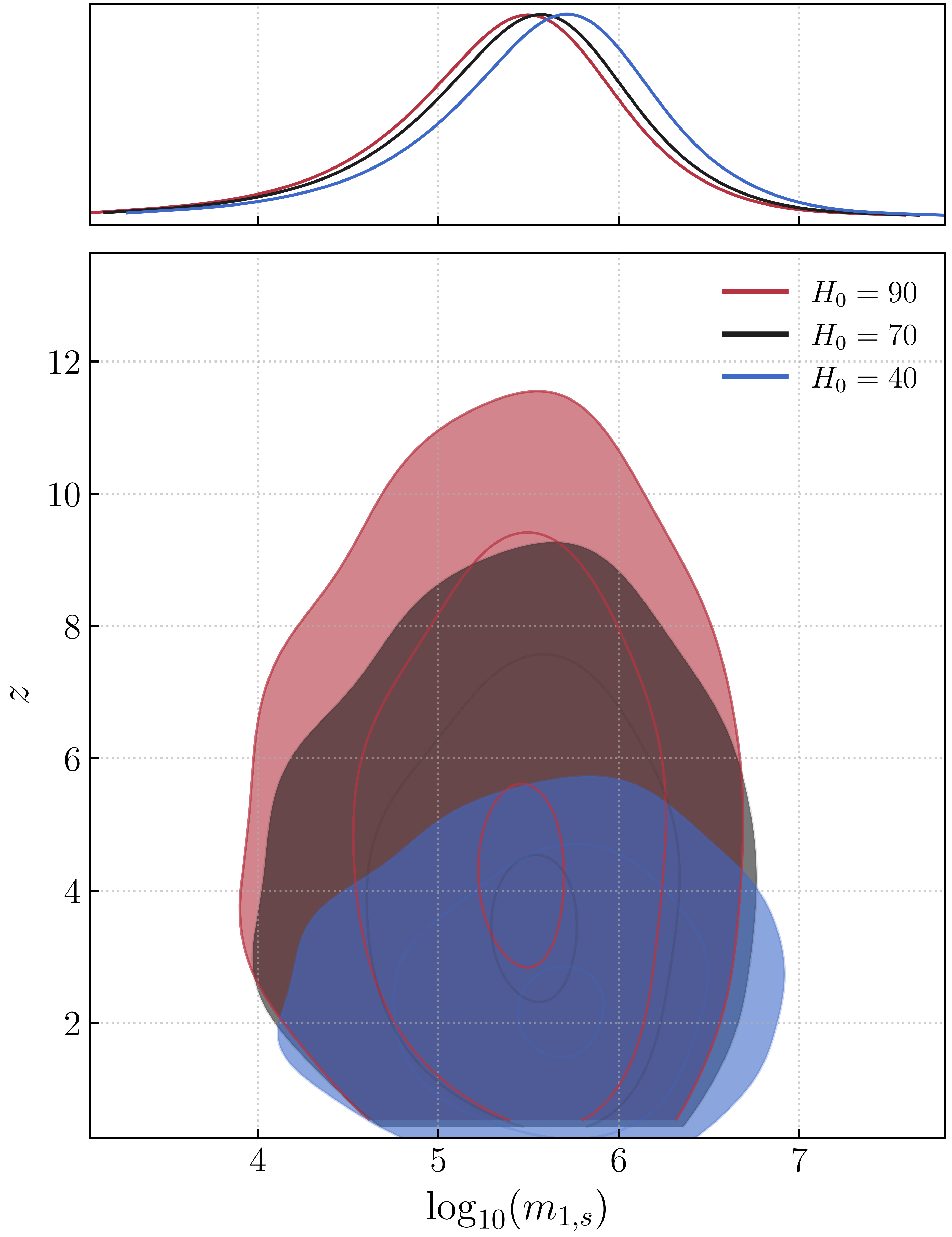
Q3d 4yrs 10 realizations non-parametric



small dataset (~25 events), *large Poisson noise*

COSMOLOGY WITH BROAD FEATURES?





Q3d for different cosmologies