(Tidal effects on) primordial black hole capture in neutron stars

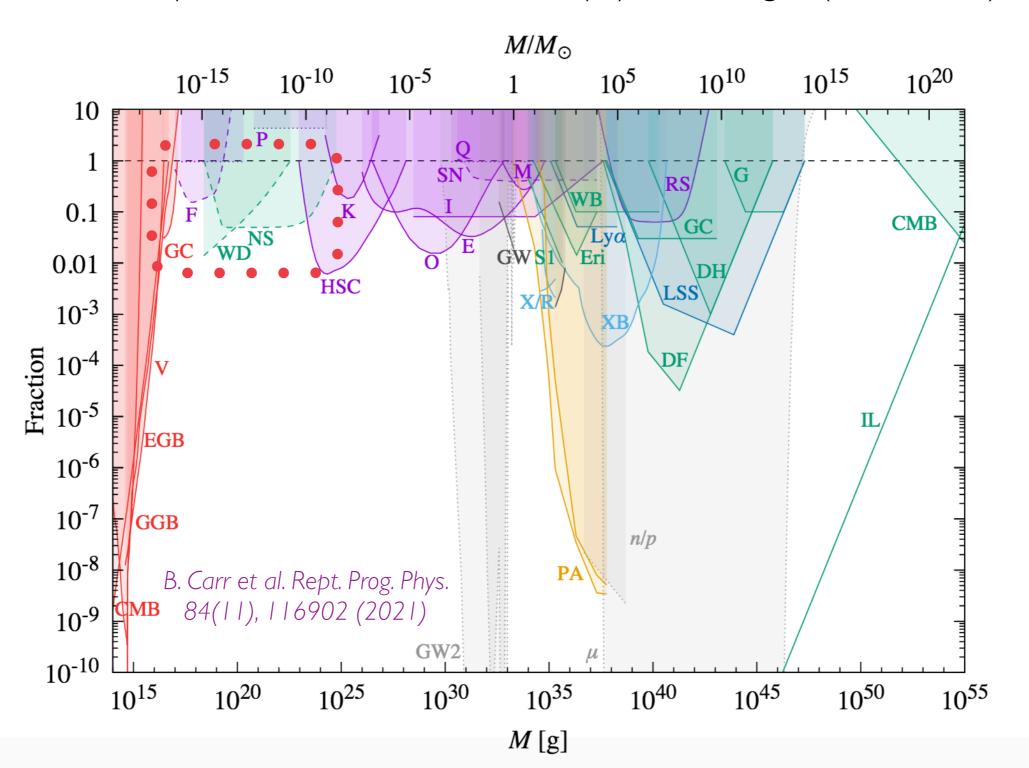


Assemblée Générale Enigmass+ 07/11/2025, Annecy

Pasquale Dario Serpico しんうてん

Motivation for 'asteroid-mass' PBH

- ▶ PBH in this mass range could constitute the totality of the dark matter (DM)
- Can be obtained (at price of fine-tuning!) even with single-field polynomial inflation
- ▶ Can be probed with stochastic GW searches (2nd order perturbation, LISA band)
- A number of potential effects on stellar astrophysics envisaged (dashed lines)



Plan of the talk

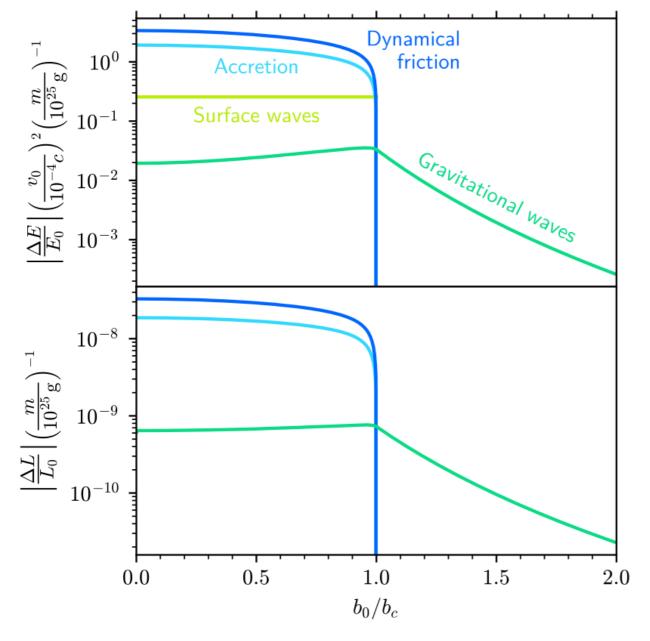
- Energy loss mechanisms & capture
- Phenomenological consequences
- ▶ Beyond the 2-body problem: formalism for tidal effects of far perturber
- Consequences
- Considerations for a near perturber (notably, planet!?)
- ▶ Conclusions

Recap of PBH E-loss processes

As a PBH passes through (or near) a star, it loses energy (& angular momentum wrt the NS)

$$\Delta E = \int_{\mathcal{C}} (\mathbf{v} \cdot \mathbf{F}) \, dt,$$

$$\Delta \mathbf{L} = \int_{\mathcal{C}} (\mathbf{r} \times \mathbf{F}) \, dt,$$



via a number of processes:

dynamical friction (dominant) "gravitational pull from the wake of the medium"

$$\mathbf{F}_{\rm dyn} = -4\pi G^2 m^2 \rho \ln \Lambda_{\rm dyn}(v) \frac{\boldsymbol{v}}{v^3}$$

Accretion (Bondi-like) from the NS medium

$$\mathbf{F}_{\mathrm{acc}} = -\dot{m} \, \boldsymbol{v}$$

Surface waves (small)

$$|\Delta E|_{\rm surf} \sim \frac{G \, m^2}{R_{\star}}$$

Gravitational waves (small, finite also without crossing)

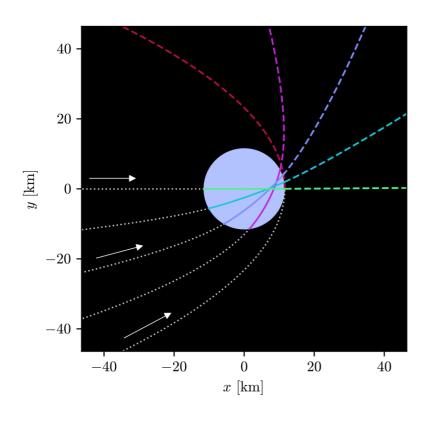
$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{G}{5\,c^5} \langle \ddot{Q}_{ij} \ddot{Q}_{ij} \rangle \ \, \to \ \, \Delta E_{\mathrm{gw}} = \frac{8}{15} \frac{m^2 M_*^2}{M^3} v_i^7 \frac{p(e)}{(e-1)^{7/2}}$$

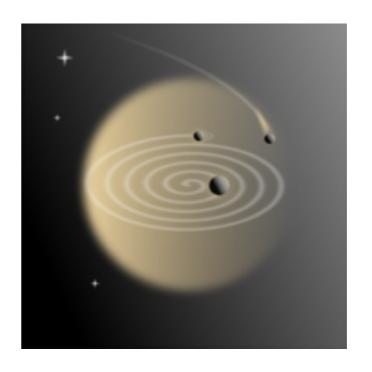
p: periastron distance

Y. Génolini, P. D.S. and P.Tinyakov, Phys. Rev. D 102 (2020) no.8, 083004 [2006.16975]

PBH crossing, capture & NS transmutation

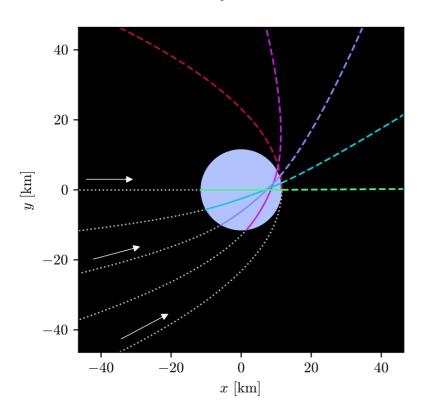
If, as a result of losses, the PBH is captured, it will sink towards the NS centre, accrete matter and eventually swallow & destroy the star, leaving a BH behind (transmutation)

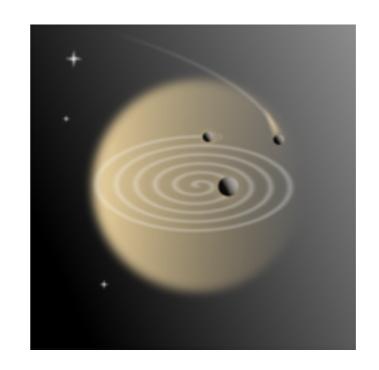




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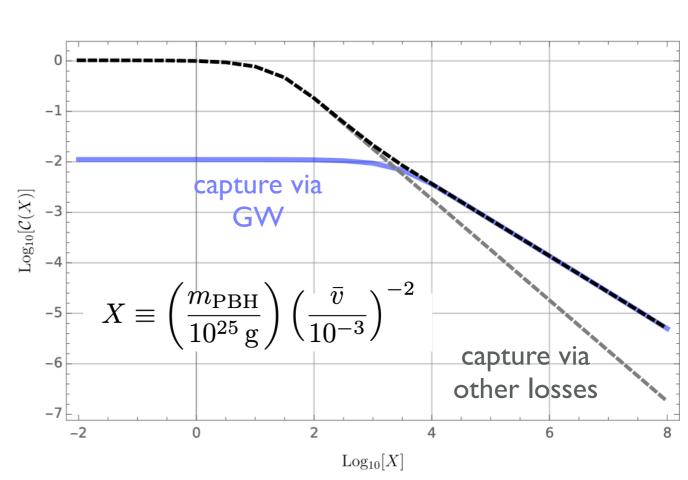
Note that the NS crossing rate

$$\Gamma_{\star} = \int \frac{\mathrm{d}^{3} n}{\mathrm{d}v^{3}} \pi b_{c}^{2}(v) v \, \mathrm{d}^{3}v =$$

$$\simeq 3.8 \times 10^{-16} \left(\frac{\rho_{\mathrm{BH}}}{1 \mathrm{GeV \, cm^{-3}}}\right) \left(\frac{10^{25} \mathrm{g}}{m}\right) \left(\frac{10^{-3}}{\bar{v}}\right) \mathrm{yr^{-1}} \stackrel{\text{So}}{\underset{\text{colo}}{\overset{\text{So}}{\sim}}} {\overset{\text{So}}{\sim}} {}^{-3}$$

is much larger, at small mpbH, than the capture rate

$$\mathcal{G}_{\star} \simeq 2.1 \times 10^{-17} \left(\frac{\rho_{\mathrm{PBH}}}{\mathrm{GeV \, cm^{-3}}} \right) \left(\frac{10^{-3}}{\bar{v}} \right)^{3} \mathcal{C}\left[X\right] \mathrm{yr^{-1}}$$



Phenomenological consequences

Stellar survival constraints (e.g. observing NS in globular clusters)

F. Capela, M. Pshirkov and P.Tinyakov, Phys. Rev. D 87 (2013) 123524 [1301.4984]

not robust against relaxing hypotheses on DM density there.

The transit of a PBH through a carbon/oxygen white dwarf will lead to localized heating by dynamical friction, which could ignite the carbon and potentially cause a runaway explosion

P.W. Graham, S. Rajendran and J. Varela, Phys. Rev. D 92 (2015) 063007 [1505.04444]

Triggering explosion harder than thought for 'low' masses not excluded otherwise, see

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Perhaps more promising: a 'positive' evidence

The explosion delivers observable energy at least associated to the B-field. In NS:

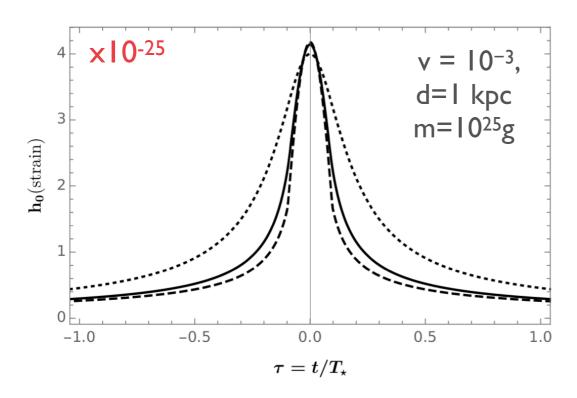
$$E_B = \frac{B^2}{8\pi} \frac{4\pi}{3} R_\star^3 \simeq 2 \times 10^{41} \left(\frac{B}{10^{12} \mathrm{G}}\right)^2 \left(\frac{R_\star}{10 \, \mathrm{km}}\right)^3 \mathrm{erg}$$
 (This benchmark \approx energy emitted by the Sun in 1 yr...in a few ms!)

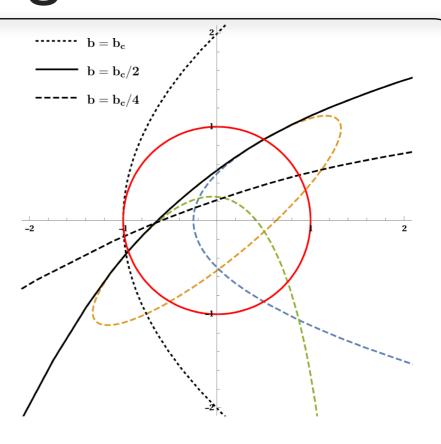
Signature(?)

Poynting flux + small amount of ejecta, since virtually no kilo-nova is found in simulations of W. E. East and L. Lehner, Phys. Rev. D 100 (2019) 124026 [1909.07968]

Gravitational wave signals?

Teardrop signal associated to first transit



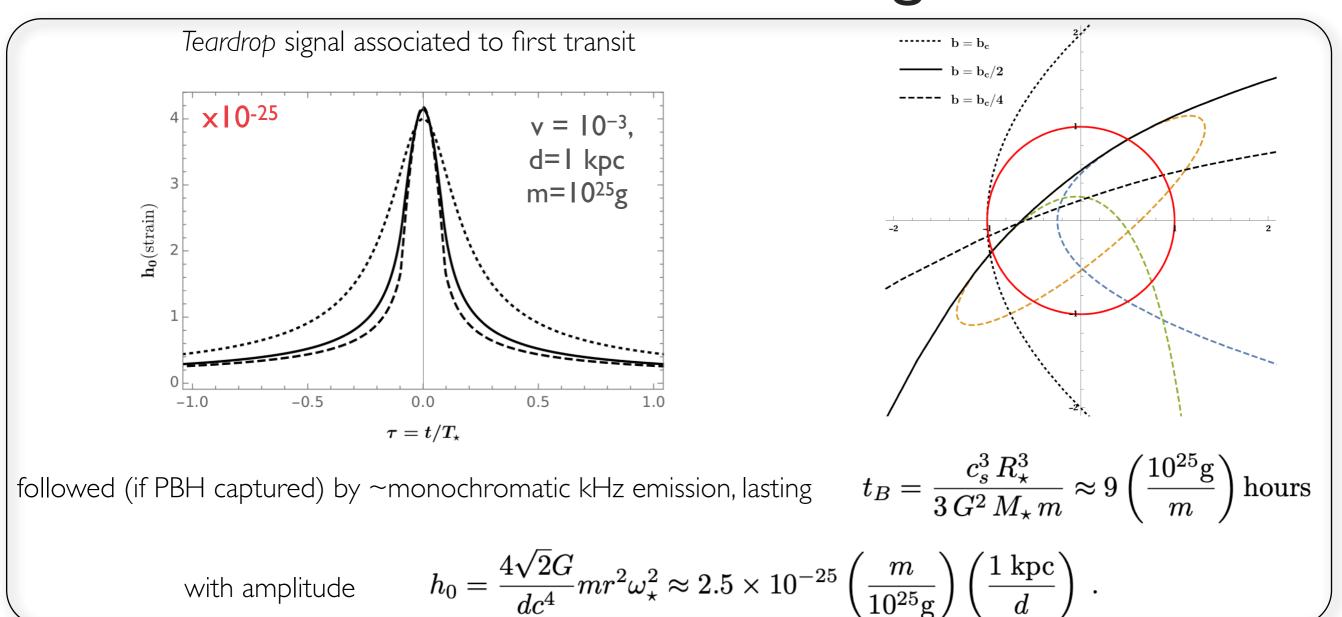


followed (if PBH captured) by ~monochromatic kHz emission, lasting

$$t_B = \frac{c_s^3 \, R_{\star}^3}{3 \, G^2 \, M_{\star} \, m} \approx 9 \left(\frac{10^{25} \text{g}}{m}\right) \text{hours}$$

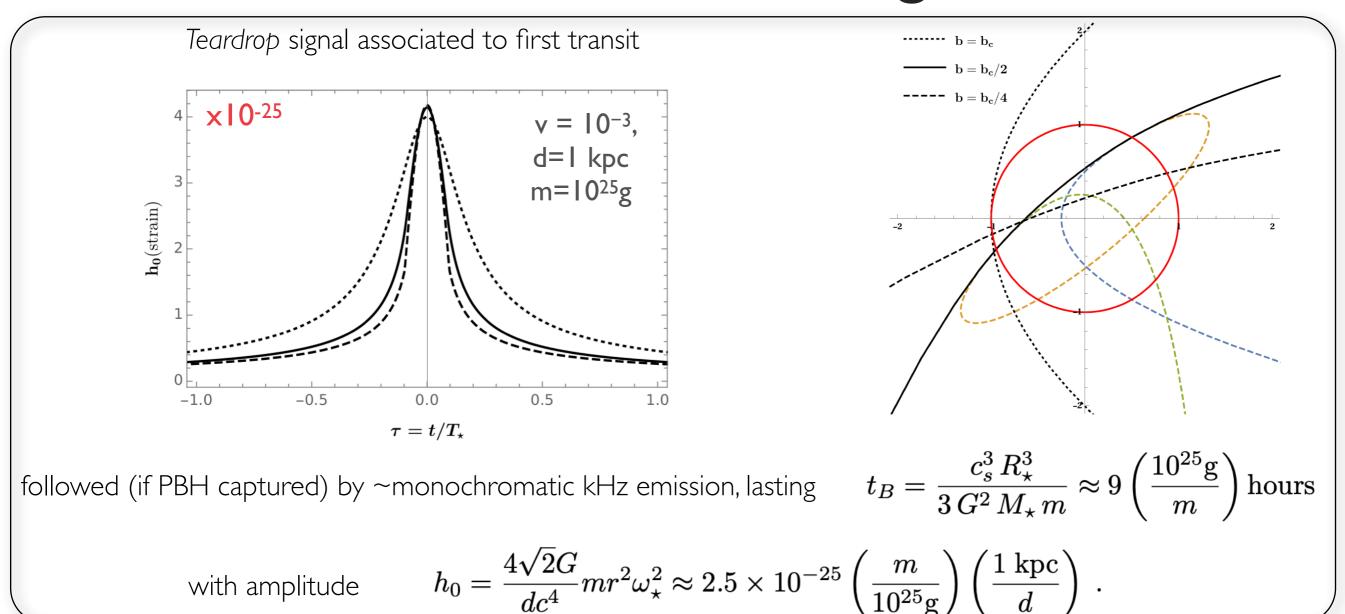
$$h_0 = \frac{4\sqrt{2}G}{dc^4} mr^2 \omega_{\star}^2 \approx 2.5 \times 10^{-25} \left(\frac{m}{10^{25} \text{g}}\right) \left(\frac{1 \text{ kpc}}{d}\right) .$$

Gravitational wave signals?



Eventually, additional GW can be associated to the transmutation event, requiring ad hoc simulations [Perhaps current dim perspectives are too pessimistic, assuming PBH exactly at the center]

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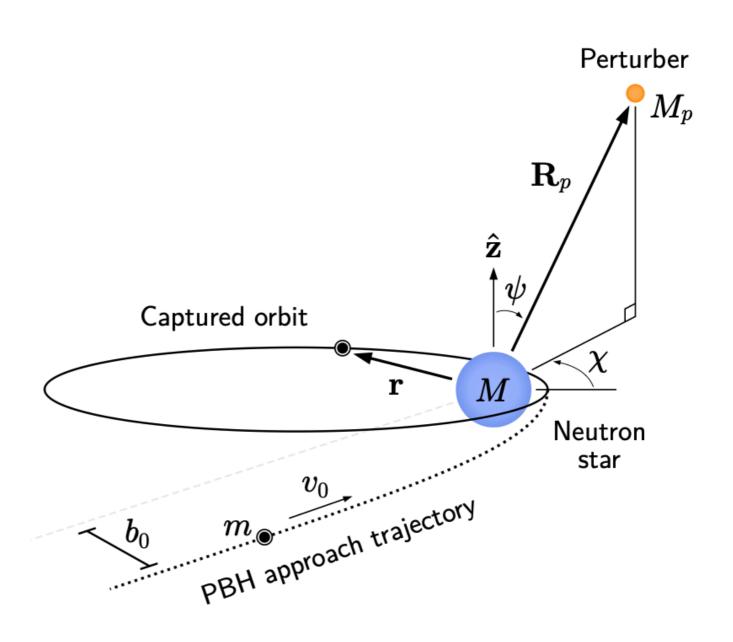
Alternative: any chance to see (e.g. via GW from binaries) BHs with mass significantly below 3 M_{\odot} ?

No bounds from all that, but possible signals in GW/E.M. if one is lucky*...interesting to dig further (*In general, sufficiently frequent events are too dim/quiet, bright/loud events are rare)

Beyond 2-body problem: setting the stage

Previous treatment limited to "two body problem"

Quid of tidal effects due to neighbour stellar (or planetary, see later) bodies?



Average over pdf of PBH velocities, perturber distance, stellar mass...

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Tidal Force

$$\begin{aligned} \mathbf{F}_t &= m(\mathbf{a}_{\mathrm{BH}} - \mathbf{a}_{\mathrm{NS}}) \\ &= GM_p m \left(\frac{\mathbf{R}_p - \mathbf{r}}{|\mathbf{R}_p - \mathbf{r}|^3} - \frac{\mathbf{R}_p}{|\mathbf{R}_p|^3} \right) \\ &\approx \frac{GM_p m}{R_p^3} \left(\frac{3(\mathbf{r} \cdot \mathbf{R}_p)}{R_p^2} \mathbf{R}_p - \mathbf{r} \right). \end{aligned}$$

Last step

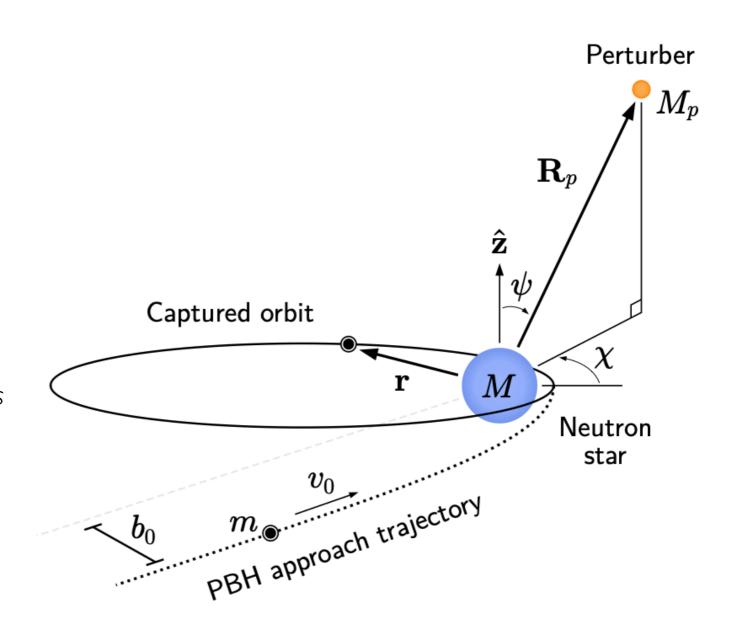
$$R_p \gg r$$

Stationary perturber during a PBH orbit; requires

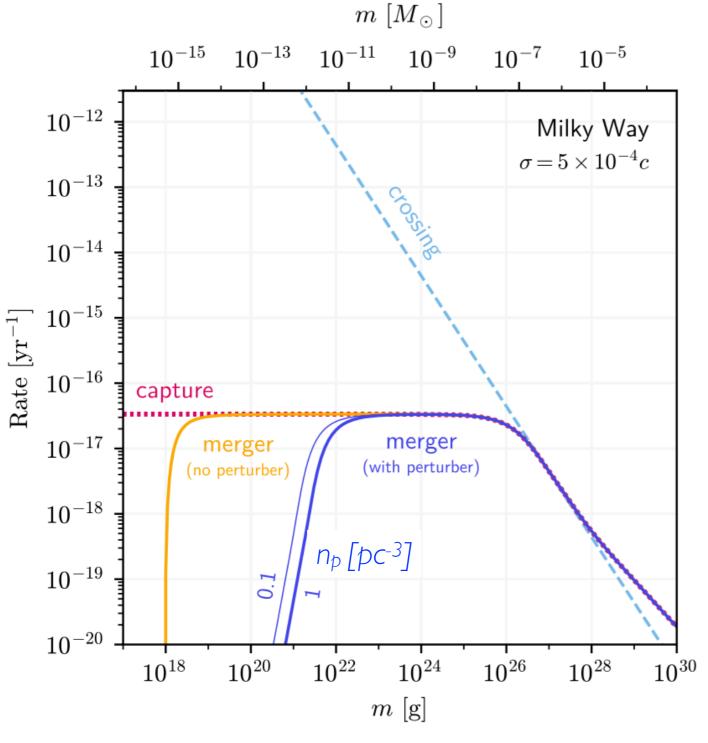
$$m \gg 2 \times 10^{19} \,\mathrm{g} \times (\mathrm{pc}/R_p)$$

Torque due to the perturber

$$oldsymbol{ au} = \mathbf{r} imes \mathbf{F}_t = rac{3GM_pm}{R_p^5} (\mathbf{r} \cdot \mathbf{R}_p) (\mathbf{r} imes \mathbf{R}_p)$$



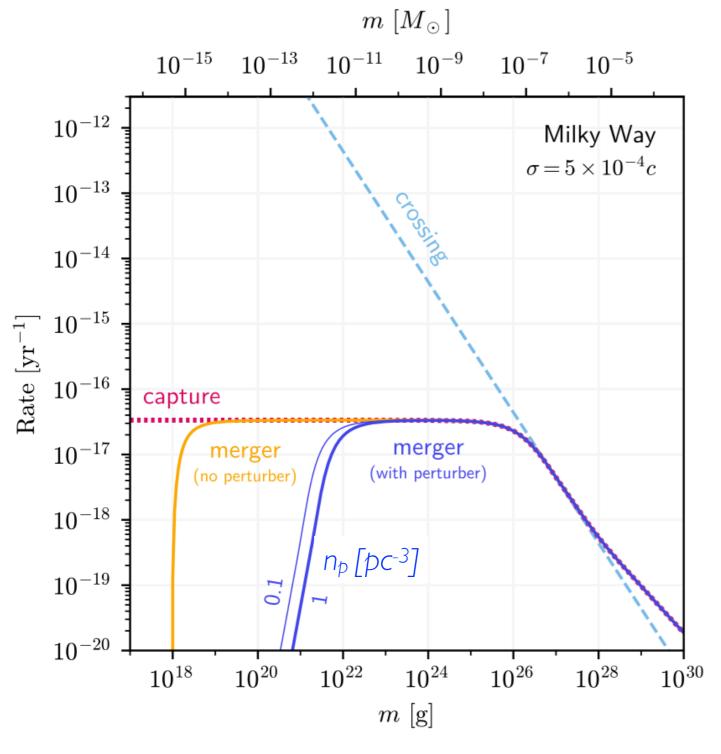
Average over pdf of PBH velocities, perturber distance, stellar mass...



Analytical approximations

$$\Gamma_{\rm cross} \approx 5.0 \, \frac{GMR \, \rho}{\sigma m}$$

$$\Gamma_{ ext{cross}} = rac{
ho}{m} \int\limits_0^\infty \mathrm{d} v_0 \, p(v_0) v_0 \int\limits_0^{b_{
m c}} \mathrm{d} b_0 \, 2\pi b_0$$



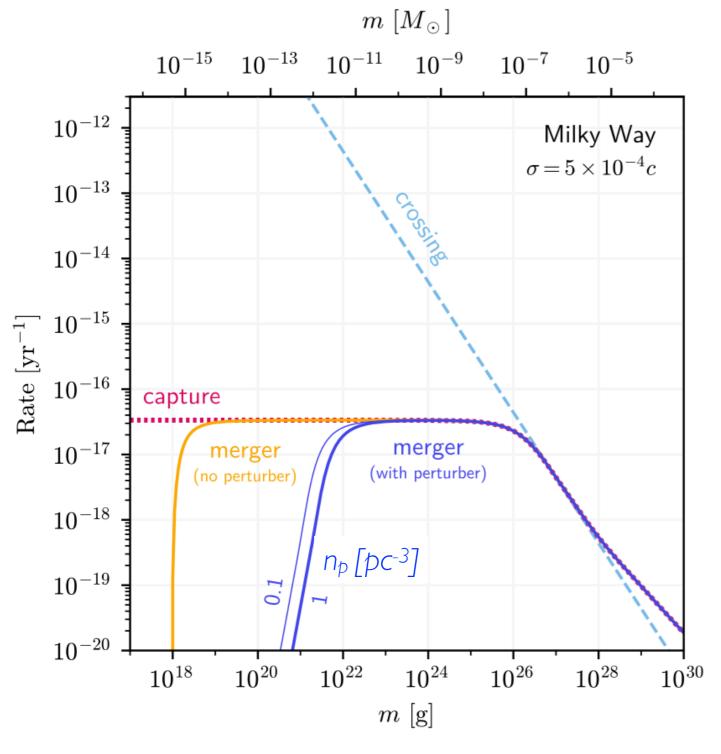
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$$\Gamma_{\rm cross} \approx 5.0 \, \frac{GMR \, \rho}{\sigma m}$$

$$\Gamma_{
m cap} pprox 151 \, rac{G^2 M
ho}{\sigma^3} \qquad > \Gamma_{
m cross} \, {
m at \ large \ m} \ {
m due \ to \ GW \ losses}$$

$$\Gamma_{\text{cap}} = \frac{\rho}{m} \left(\int_{0}^{\infty} \mathrm{d}v_0 \, p(v_0) v_0 \, \int_{0}^{b_{\text{max}}} \mathrm{d}b_0 \, 2\pi b_0 \right)$$

$$\Theta(|\Delta E| - |E_0|)$$



Analytical approximations

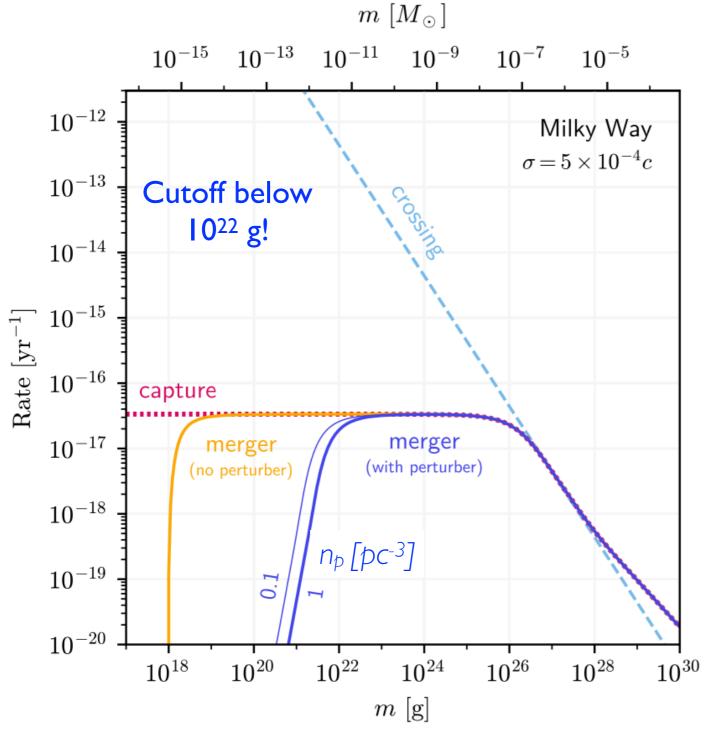
$$\Gamma_{\rm cross} \approx 5.0 \, \frac{GMR \, \rho}{\sigma m}$$

$$\Gamma_{\rm cap} \approx 151 \, \frac{G^2 M \rho}{\sigma^3}$$

$$m_{\rm cut} \approx 1.4 \times 10^{18} \,\mathrm{g} \, \left(\frac{t_{\rm age}}{10^{10} \,\mathrm{yr}}\right)^{-2/3}$$

$$\Gamma_{\text{mrg}} = \frac{\rho}{m} \left(\int_{0}^{\infty} dv_0 \, p(v_0) v_0 \, \int_{0}^{b_{\text{max}}} db_0 \, 2\pi b_0 \right)$$

$$\Theta(t_{\rm age}-T_{\rm ins})$$



Analytical approximations

$$\Gamma_{\rm cross} \approx 5.0 \, \frac{GMR \, \rho}{\sigma m}$$

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 $m_{\rm cut} \approx 1.4 \times 10^{18} \,\mathrm{g} \,\left(\frac{t_{\rm age}}{10^{10} \,\mathrm{yr}}\right)^{-2/3}$

$$m_{\text{pert}} \approx 0.080 \, M^{5/7} R^{6/7} M_{\text{min}}^{2/7} n_p^{2/7}$$

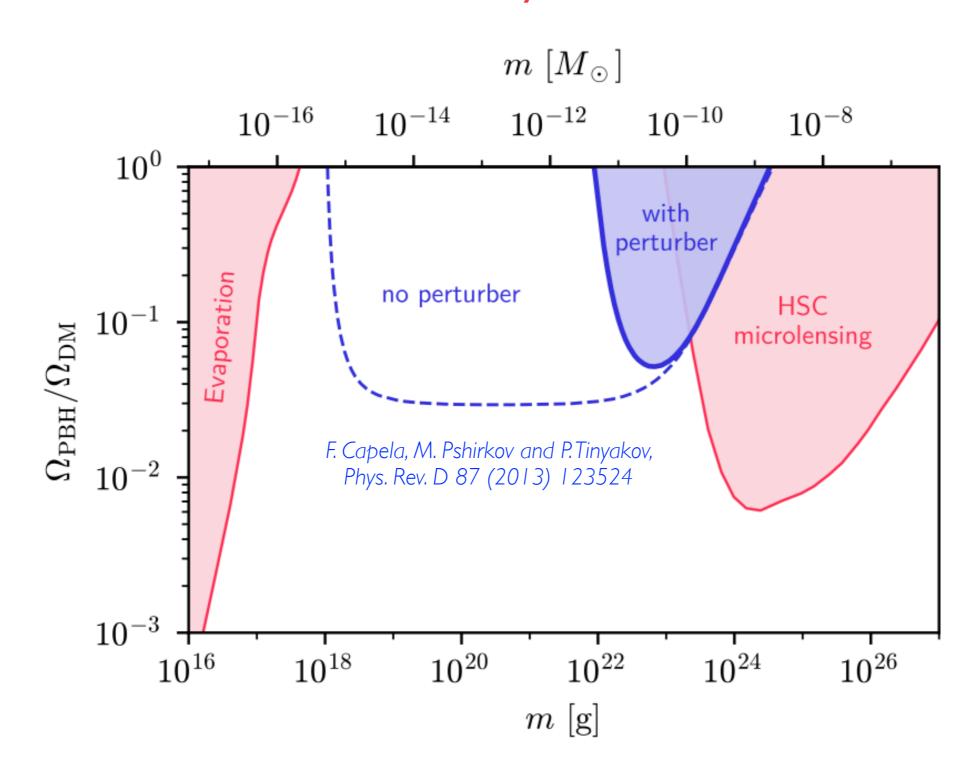
Stationary perturber approximation verified

$$m \gg 2 \times 10^{19} \,\mathrm{g} \times (\mathrm{pc}/R_p)$$

$$\Gamma_{\text{mrg}} = \frac{\rho}{m} \left(\int_{0}^{\infty} dv_0 \, p(v_0) v_0 \int_{0}^{b_{\text{max}}} db_0 \, 2\pi b_0 \int_{0}^{\infty} dY \, p(Y) \frac{1}{4\pi} \int_{0}^{2\pi} d\chi \int_{0}^{\pi} d\psi \sin \psi \, \, \Theta(t_{\text{age}} - T_{\text{ins}}) \right)$$

Impact on potential/putative constraints

Extremely relevant!



What if perturber is "near" the NS?

 $(R_p \gg r \text{ not true})$ Notable case: Planetary bodies (around NS?! Yes, a few...)

Can rely on analogous (well-studied) problem: Influence of planets on comets in the solar system!

R.A. Lyttleton, J.M. Hammersley, MNRAS 127-3 (1964), 257; S. Yabushita, A&A, 16, (1972); 471 P. Wiegert & S. Tremaine, Icarus 137-1, (1999), 84

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"Energy kick" expected in a single passage in the inner system ($\leq R_p$)

Merger possible only if

singling out

$$\delta E \sim \frac{GM_p m}{R_p} \qquad \rightarrow \qquad a_1 < \frac{M}{M_p} R_p \qquad \rightarrow \qquad m > 0.017 \frac{R}{R_p} M_p$$

$$\frac{10^{-12}}{10^{-14}} \frac{\text{Milky Way}}{\sigma = 5 \times 10^{-4}c} \frac{10^{-12}}{10^{-18}} \frac{\text{Dwarf Galaxy}}{10^{-18}} \frac{\text{Dwarf Galaxy}}{10^{-18}} \frac{\sigma = 3 \times 10^{-5}c}{10^{-18}} \frac{\sigma = 3 \times 10$$

Potentially important, but only small fraction of NS believed to be in such a situation

Conclusions

Unavoidable tidal effects of nearby stars crucial in altering proposed capture and transmutation statistics of NS by PBHs (opens parameter space at low masses)

We provide analytical approximations of the relevant rates and critical masses, to be used for rescaling to the system of your interest

Comparable or larger effect due to planets/close companions, but expected to be rare based on current knowledge (updates expected?)

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On a future to-do-list...

If the latter more important than currently thought, might deserve dedicated analyses for assessing *cumulative* effects of perturbations, via simulations

Study "true" three-body encounters, with no/little hierarchy (expected to be a small fraction)

Effects of perturbations due to other PBHs (not larger, at least not in the MW)

Backup

Perturbative calculation

If, after losses, PBH is bound on elliptical orbit

(If not, lost for the purpose of capture...neglect tidal effect!)

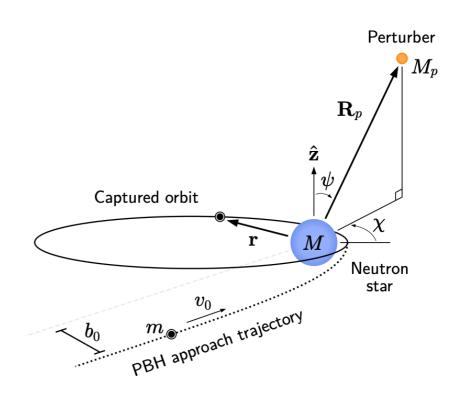
$$E = -GMm/a$$

$$L = m\sqrt{GMa(1 - e^2)},$$

Effect of torque on the unperturbed orbit

(by construction, then, $\Delta E=0$)

$$\begin{split} \delta \mathbf{L} &= \int_0^{T_1} \mathrm{d}t \, \boldsymbol{\tau} = \int_0^{2\pi} \mathrm{d}\phi \, \left(\frac{mr^2}{L}\right) \boldsymbol{\tau} \\ &= \frac{3\pi G M_p m}{\sqrt{GM/a}} \left(\frac{a}{R_p}\right)^3 \begin{pmatrix} (1-e^2)\sin\psi\cos\psi\sin\chi \\ -(1+4e^2)\sin\psi\cos\psi\cos\chi \\ 5e^2\sin^2\psi\sin\chi\cos\chi \end{pmatrix} \end{split}$$



New orbital parameters

(before the PBH re-enters/re-approaches the NS)

$$a'_1 = a_1$$
 since $E'_1 = E_1$, $b'_1 = \frac{|\mathbf{L}'_1|}{\sqrt{-2mE'_1}}$ with $\mathbf{L}'_1 = \mathbf{L}_1 + \delta \mathbf{L}$

New orbital parameters

(after the PBH re-enters/re-approaches the NS)

$$a_2 = -rac{GMm}{2E_2}, \quad ext{with} \quad E_2 = E_1' + \Delta E_2,$$
 $b_2 = rac{|\mathbf{L}_2|}{\sqrt{-2mE_2}} \quad ext{with} \quad \mathbf{L}_2 = \mathbf{L}_1' + \Delta \mathbf{L}_2.$

Truncating the effect

In principle, we could iterate $(a_i, b_i) \rightarrow (a'_i, b'_i) \rightarrow (a_{i+1}, b_{i+1}) \rightarrow (a'_{i+1}, b'_{i+1}) \rightarrow \dots$

but beyond the second pass through the NS, we ignore further effects of the perturber, and only consider the effect of subsequent energy loss

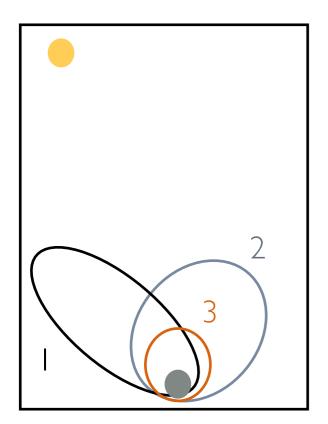
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Rationale

- ▶ Perturber effects do not add up coherently: "precession" when passing inside the NS
 → orientation of the perturber e.g. wrt apoapsis changes chaotically
- Due to repeated E-losses, the orbit circularises → more moderate tidal perturbations after 1st passage: largest contribution to the torque ~ quadratic with apoapsis



$$oldsymbol{ au} = \mathbf{r} imes \mathbf{F}_t = rac{3GM_pm}{R_p^5} (\mathbf{r} \cdot \mathbf{R}_p) (\mathbf{r} imes \mathbf{R}_p)$$

Disruption Criterion: estimate

PBH settles into the NS when $a \sim R_*$, final energy

$$E_f = -\frac{GMm}{2R_*}$$

Number of orbits before attaining that: (assuming constant loss per orbit)

$$N_{
m orb} pprox rac{E_f - E_1}{\Lambda E'}$$

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Inspiral time:

(Dominated by first few orbits)

 $\zeta_{\rm H}$ = Hurwitz zeta function

$$T_{\text{ins}} = \sum_{j=1}^{N_{\text{orb}}} T_j = \sum_{j=1}^{N_{\text{orb}}} 2\pi \sqrt{\frac{a_j^3}{GM}}$$

$$= \sum_{j=1}^{N_{\text{orb}}} 2\pi GM \left[-\frac{2}{m} \left(E_1 + (j-1)\Delta E' \right) \right]^{-3/2}$$

$$= \frac{2\pi GM m^{3/2}}{(-2\Delta E')^{3/2}} \left[\zeta_H \left(\frac{3}{2}, \frac{E_1}{\Delta E'} \right) - \zeta_H \left(\frac{3}{2}, \frac{E_f}{\Delta E'} \right) \right]$$

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Disrupted if either $T_{
m ins}>t_{
m age}pprox 10^{10}\,{
m yr}$ or $|\deltaec L|>|ec L_1|$ "Breakdown of perturbativity"

$$|\delta \vec{L}| > |\vec{L}_1|$$

Assessing the impact (probabilistic distribution of parameters)

MB distribution of PBH velocities

$$p(v_0) = \sqrt{\frac{2}{\pi}} \frac{v_0^2}{\sigma^3} \exp\left(-\frac{v_0^2}{2\sigma^2}\right)$$

Salpeter distribution of stellar masses

$$\gamma$$
~2.3, $M_{
m min}=0.5\,M_{\odot},~M_{
m max}=10\,M_{\odot}$

$$p(M_p) = \frac{(\gamma - 1)}{M_{\min}^{1 - \gamma} - M_{\max}^{1 - \gamma}} M_p^{-\gamma}$$

Poissonian distribution of perturber distance

$$p(R_p) = 4\pi n_p R_p^2 \exp\left(-\frac{4}{3}\pi n_p R_p^3\right)$$

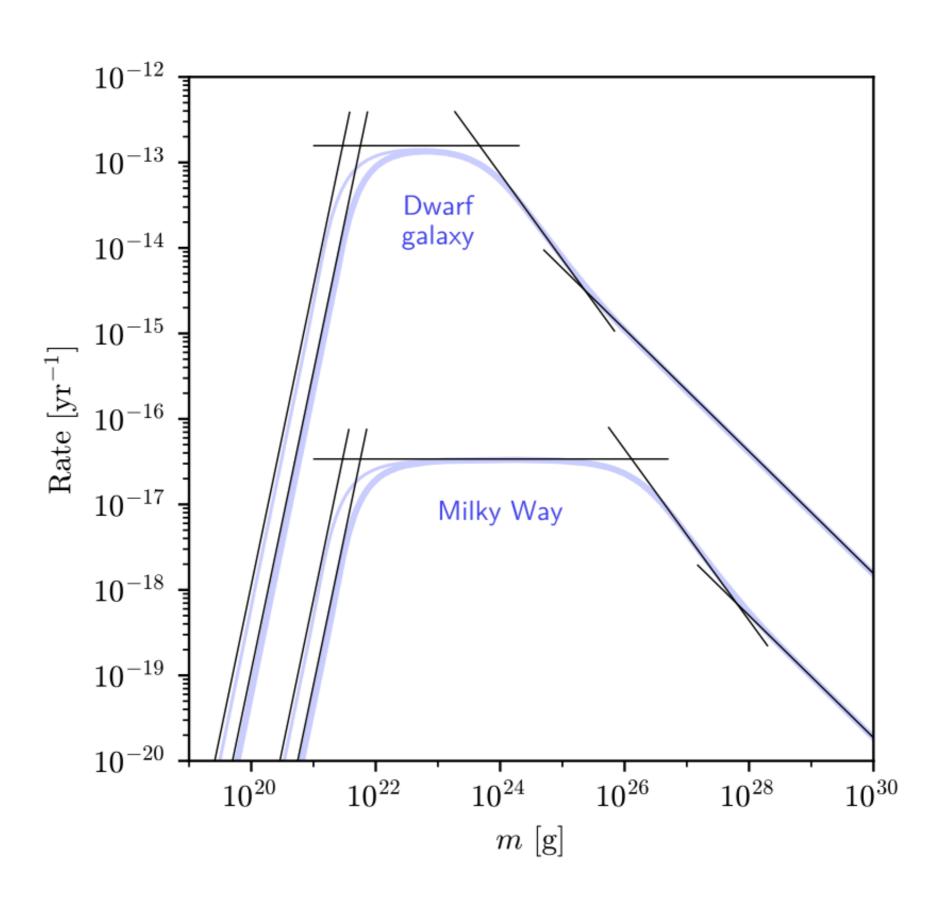
Mass & distances only enter via

$$Y \equiv \frac{M_p}{R_p^3}$$

$$p(Y) = \frac{(\gamma - 1)}{Y} \frac{\Gamma(2 - \gamma, \frac{Y_{\min}}{Y}) - \Gamma(2 - \gamma, \frac{Y_{\max}}{Y})}{\left(\frac{Y_{\min}}{Y}\right)^{1 - \gamma} - \left(\frac{Y_{\max}}{Y}\right)^{1 - \gamma}}$$

 $\Gamma(n,x)$ = upper incomplete gamma function

Gauging analytical approximations



Effect of v-dispersion

