Mitigation of the Flexing Filtering Effect in TDI for LISA

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10.06.2025

GDR “développement des détecteurs”

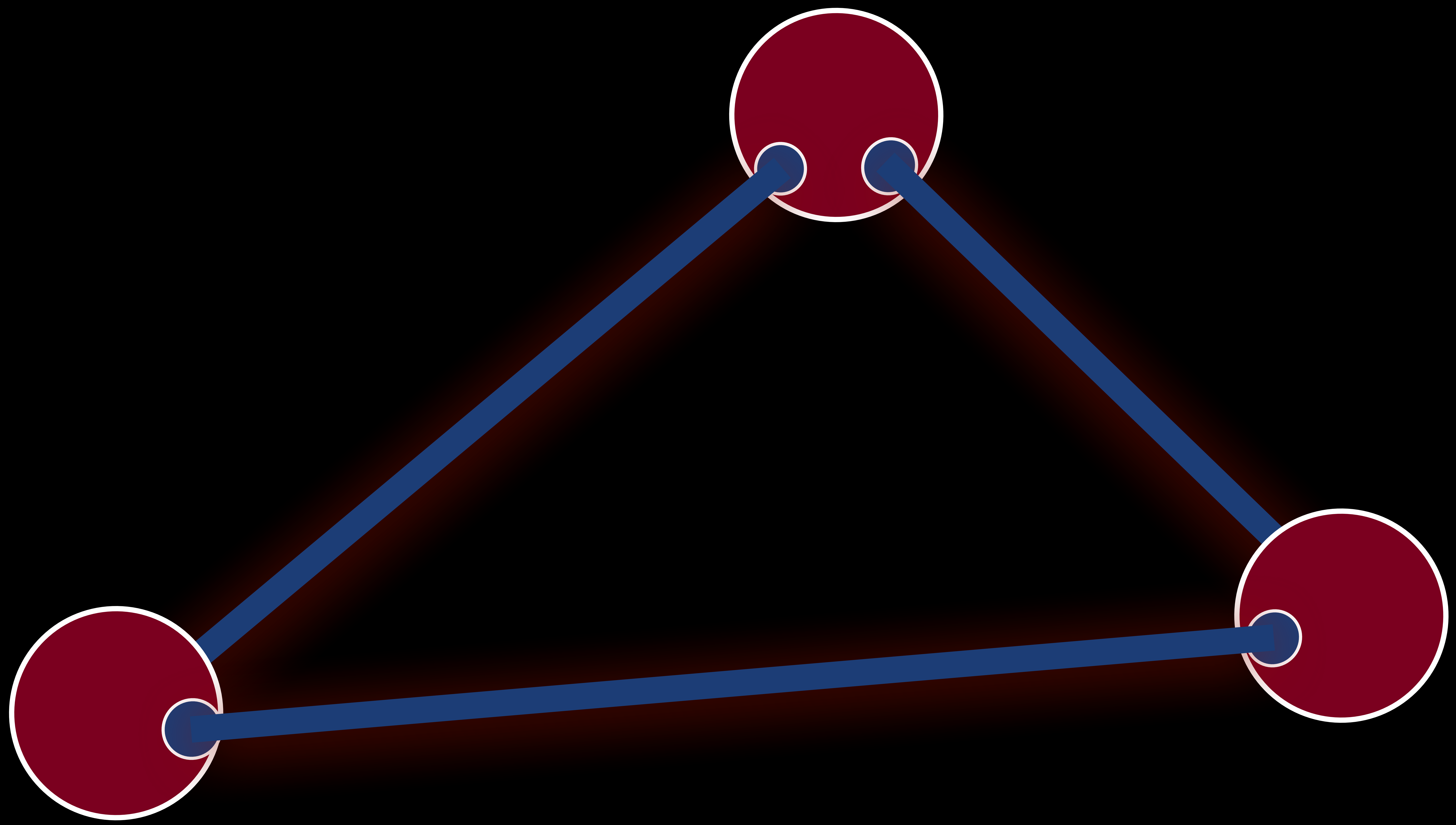
[1]APC Paris [2] LTE Paris [3] GRASP Utrecht

The What

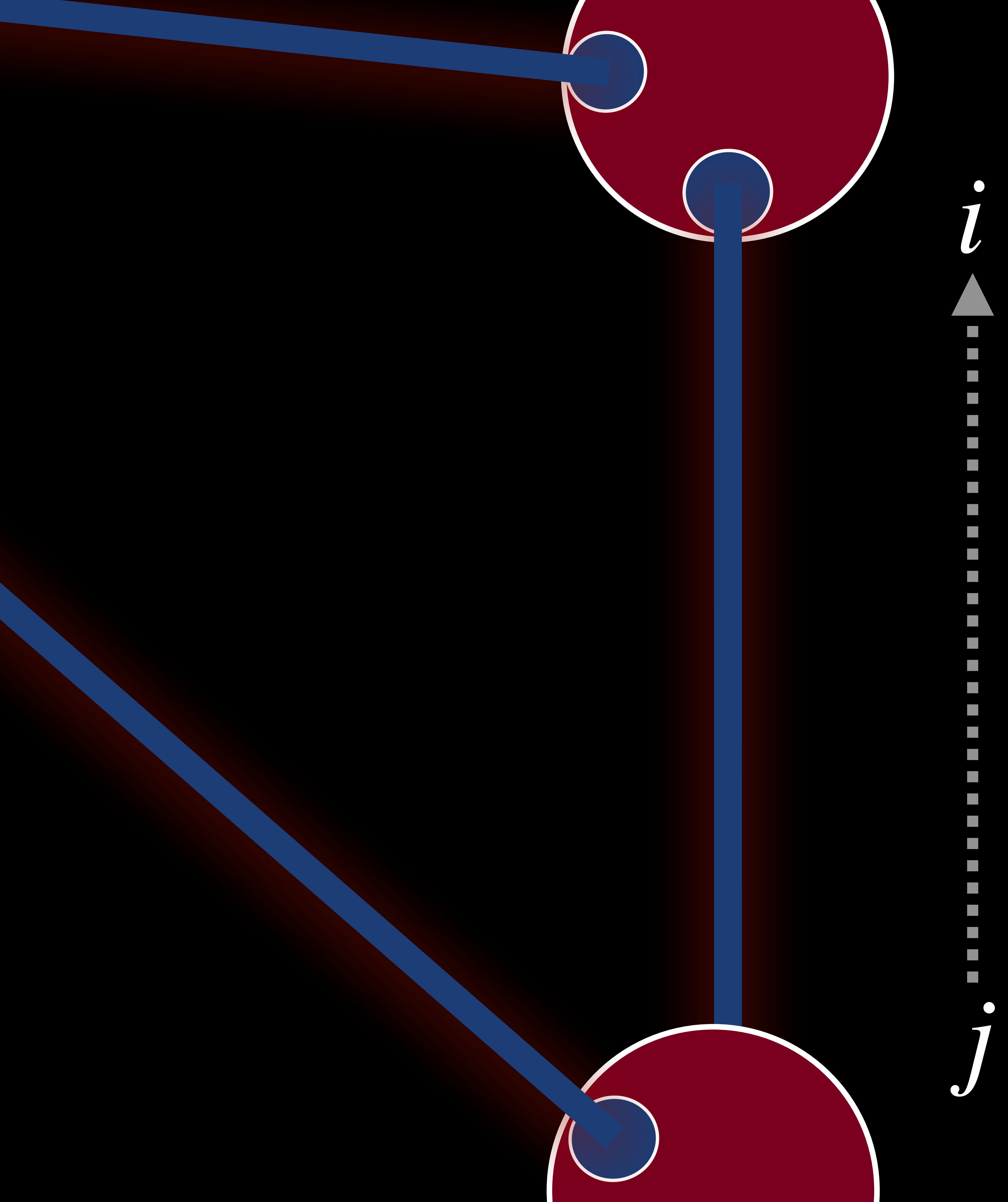
what is flexing filtering?

Noise due to the **non-commutativity** of the **anti-aliasing filters** used onboard and the time-varying **delay operators** of TDI.

The LISA constellation

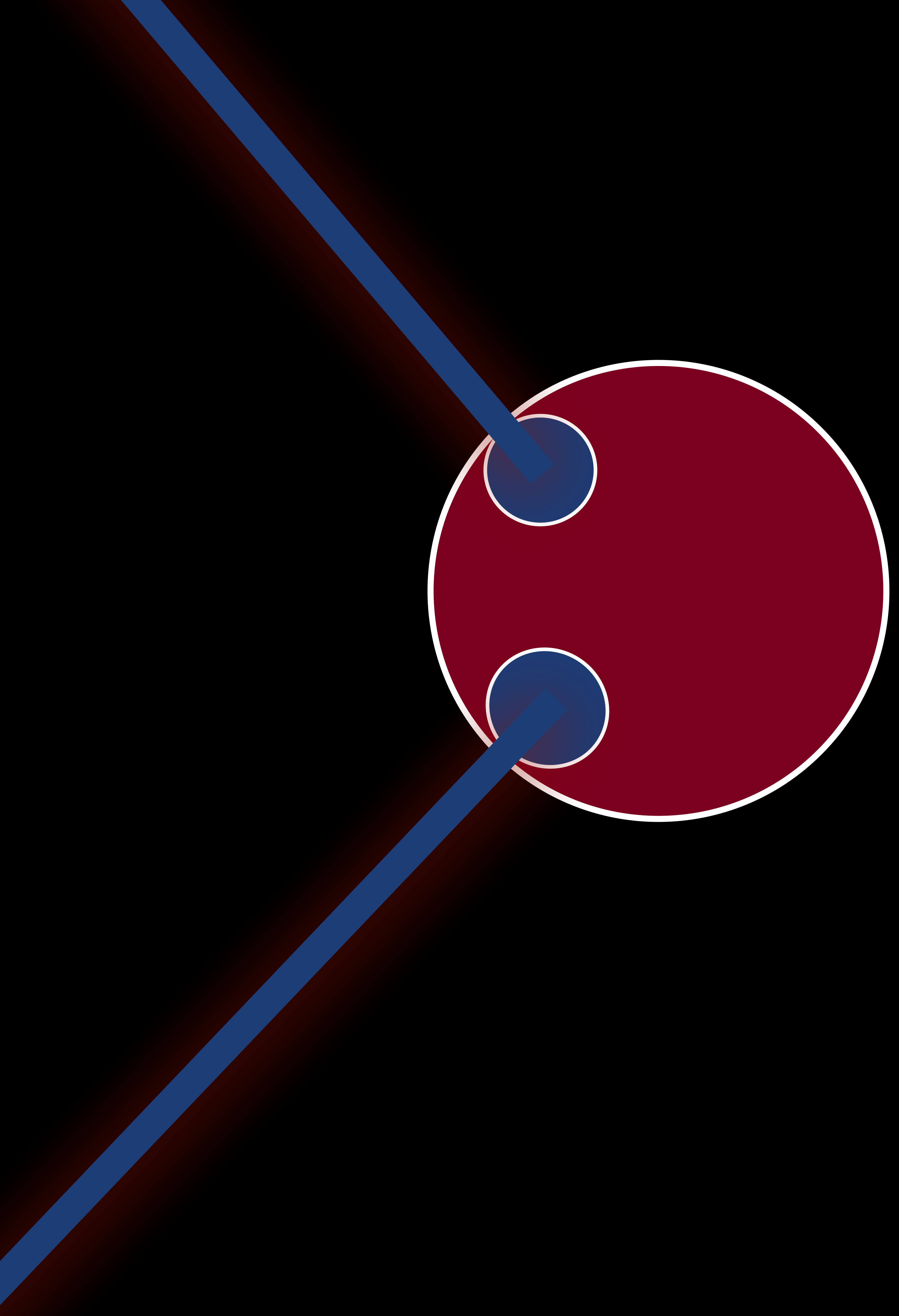


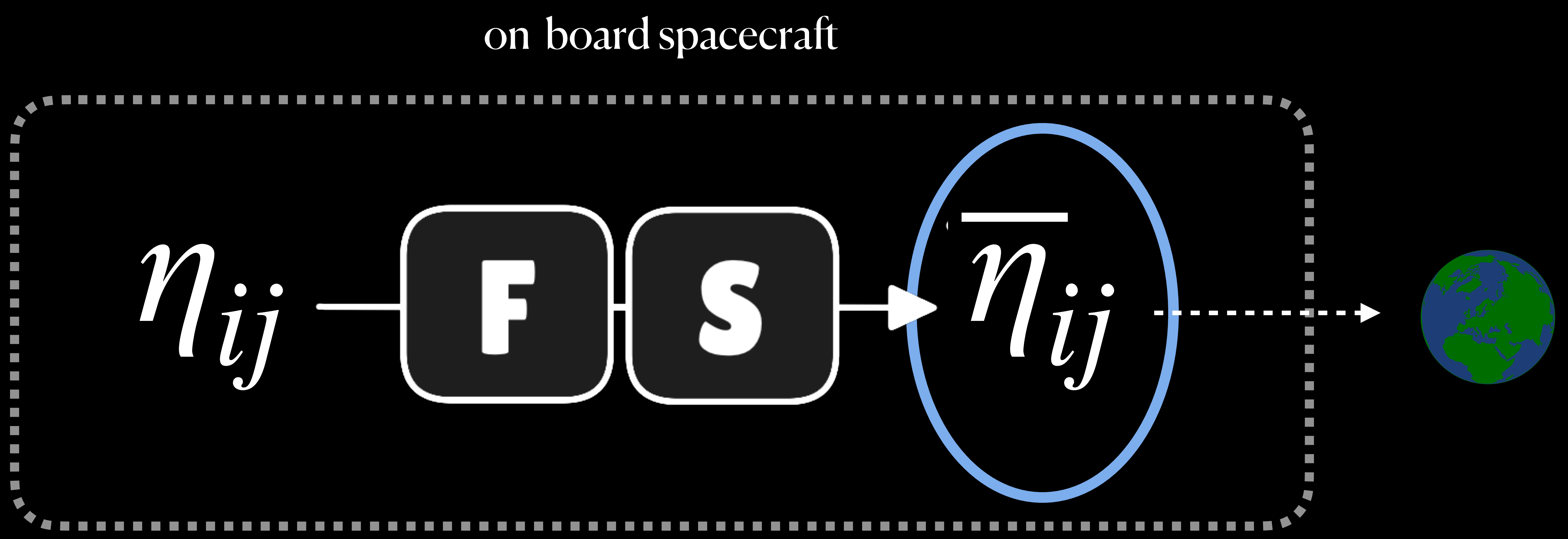
LISA's inter-spacecraft measurement



$$\eta_{ij} = \mathbf{D}_{ij}\,\phi_j - \phi_i$$

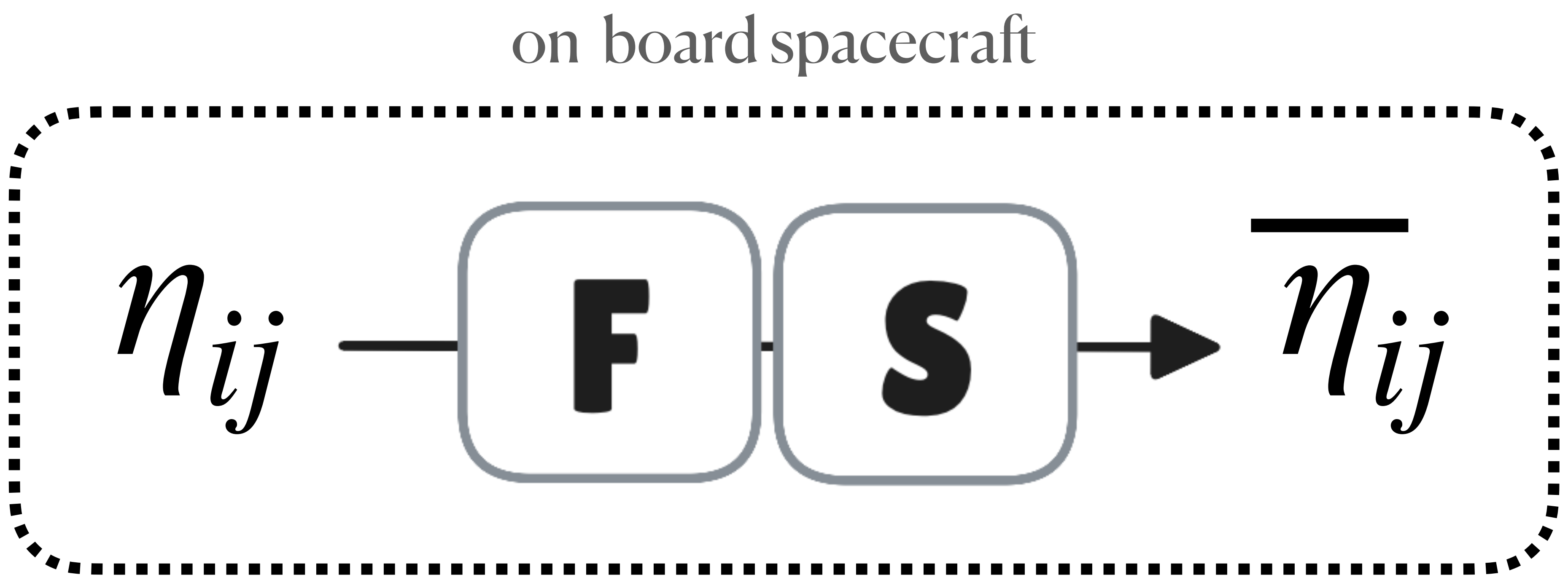
Data transmission to Earth





the starting point of
Time-Delay Interferometry (TDI)

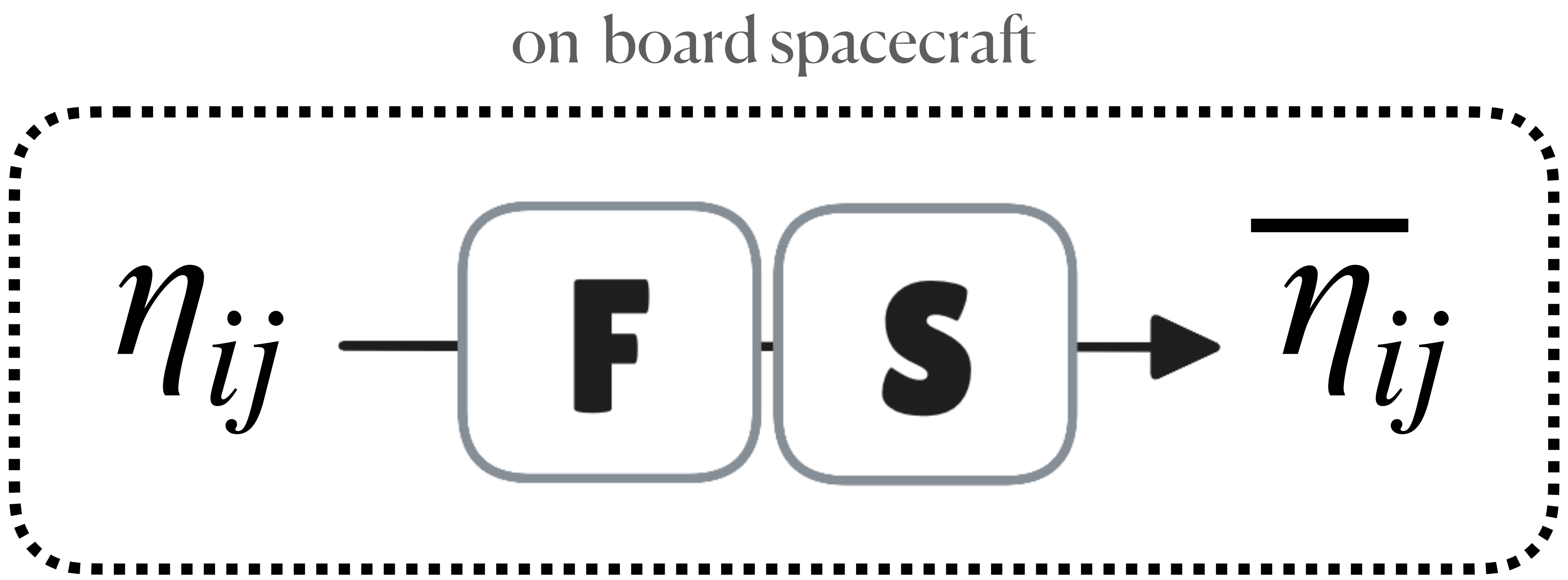
Using **filtered data** $\bar{\eta}$ for TDI results in residual laser noise.



$$\eta_{ij} = \mathbf{D}_{ij}\phi_j - \phi_i$$

$$\bar{\eta}_{ij} = \mathbf{F}\mathbf{D}_{ij}\phi_j - \mathbf{F}\phi_i$$

We identify the noise by re-ordering the filter and delay operator.



$$\eta_{ij} = \mathbf{D}_{ij}\phi_j - \phi_i$$

$$\bar{\eta}_{ij} = \mathbf{D}_{ij}\mathbf{F}\,\phi_j - \mathbf{F}\phi_i + \underbrace{[\mathbf{F}, \mathbf{D}_{ij}]\phi_j}_{\text{flexing filtering}^1\ \text{residual}}$$

[1] Bayle J B, Lilley M, Petiteau A and Halloin H 2019 Phys. Rev. D 99 084023

The amplitude of flexing-filtering residual*
depends on:

1. the delay derivative $\dot{d}$
2. the "flatness" of the filter in-band

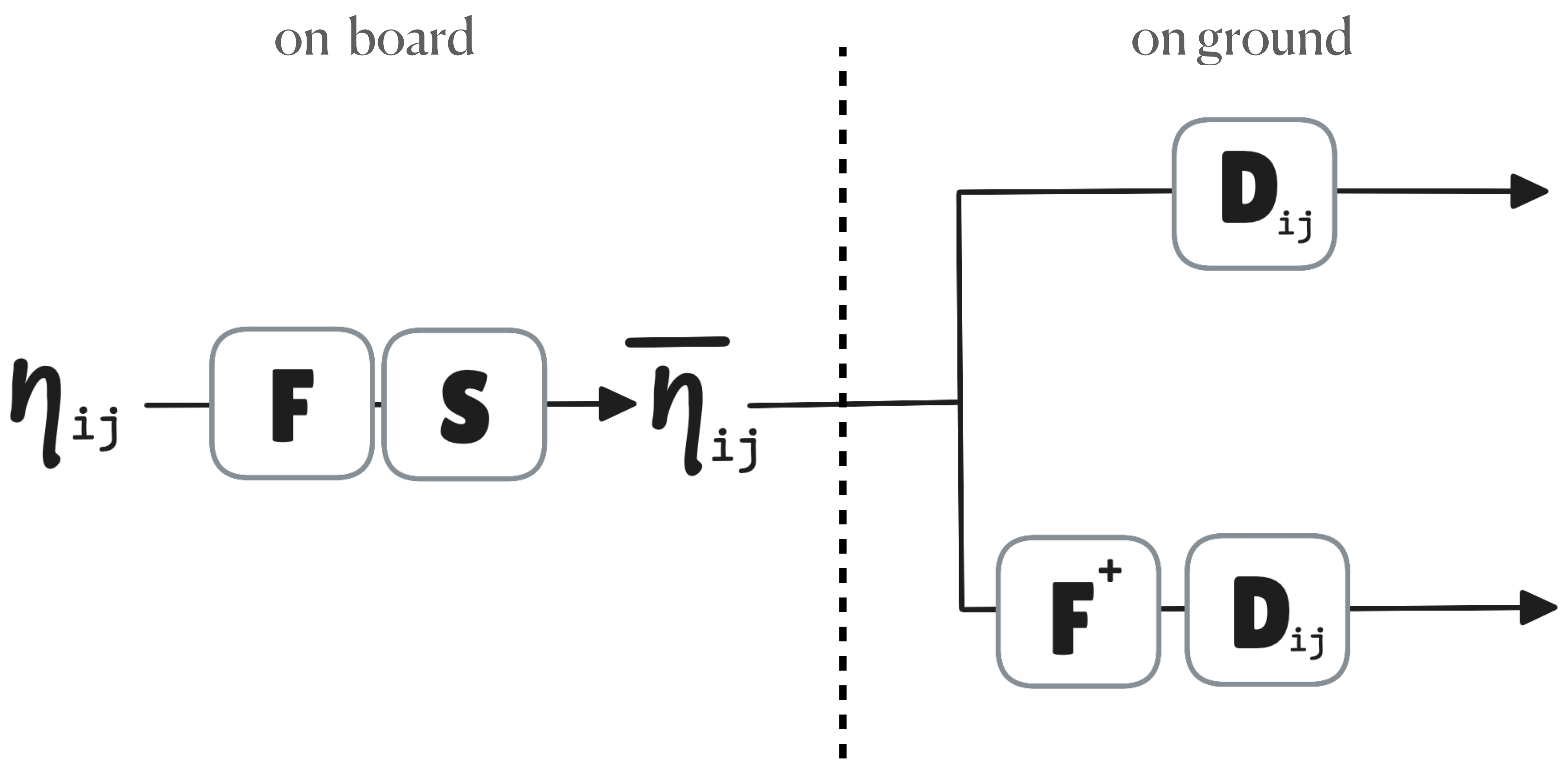
$$\frac{\dot{d}}{2\pi} \cdot \frac{\mathrm{d}\tilde{h}_{\mathbf{F}}(f)}{\mathrm{d}f}$$

* to leading order

The How

how do we fix this at present?

Two approaches exist to correct this noise:



1. **Standard TDI** : using long, flat kaiser filters (145 taps).

2. **TDI with Compensation**[2] : using a quasi-inverse filter $\mathbf{F}^+$ to "lift" non-unity frequency response in-band.

$$\mathbf{F}^+\bar{\eta}_{ij} = \mathbf{F}^+\mathbf{S}\mathbf{F}(\mathbf{D}_{ij}\phi_j - \phi_i) \approx \eta_{ij}$$

[2] Staab M 2023 Ph.D. thesis Gottfried Wilhelm Leibniz University Hannover xiv

The Problem : A flat on-board/on-ground filter chain is computationally **expensive** and causes additional **group delay**.

The Solution : make an optimised design; an anti-aliasing filter with sufficient stop-band attenuation using the least computational power and group delay.

The Idea

a new delay operator !

Modified TDI

We insert the unity operation $\mathbf{1} = \mathbf{F}\mathbf{F}^{-1}$ into the filtered single link measurement and retain the equation's algebraic structure by defining a modified delay operator $\widehat{\mathbf{D}}_{ij}$

$$\bar{\eta}_{ij} = \underbrace{\mathbf{F}\mathbf{D}_{ij}\mathbf{F}^{-1}}_{\text{modified delay operator}}\mathbf{F}\phi_j - \mathbf{F}\phi_i$$

$$= \widehat{\mathbf{D}}_{ij}\bar{\phi}_j - \bar{\phi}_i$$

The modified delay operator is approximated as a sum of the normal delay operator and a small correction scaled by $\dot{d}$

$$
\begin{aligned}
\widehat{\mathbf{D}} &= \mathbf{F}\mathbf{D}\mathbf{F}^{-1} \\
&= \mathbf{D} + [\mathbf{F}, \mathbf{D}]\mathbf{F}^{-1} \\
&\approx \mathbf{D} + \dot{d} \cdot \underbrace{\mathbf{D}\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{G}\mathbf{F}^{-1}}_{\mathbf{H}}
\end{aligned}
$$

$$h_{\mathbf{G}}(\tau) = \tau \cdot h_{\mathbf{F}}(\tau)$$

Performance Review

comparing residual noise in X_2

LISA Instrument[3] parameters

i. three laser lock

ii. white laser noise, ASD of 30 $\mathrm{Hz}/\sqrt{\mathrm{Hz}}$

iii. sampling: 4 Hz for 25000 s

iv. anti-aliasing filter: filter at 4 Hz with **9 taps**

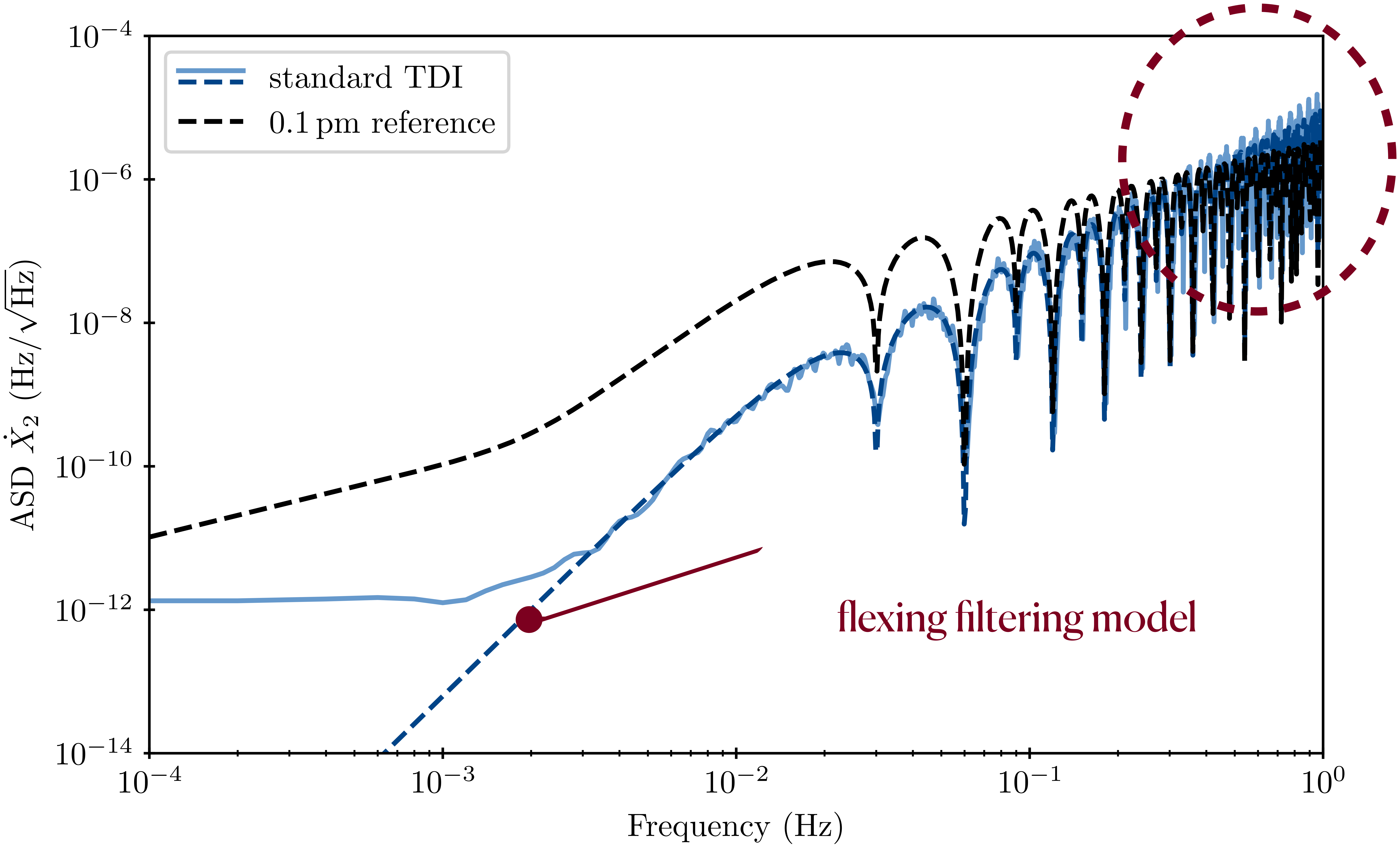
v. interpolation: Lagrange (N = 62)

vi. ESA Trailing Orbits

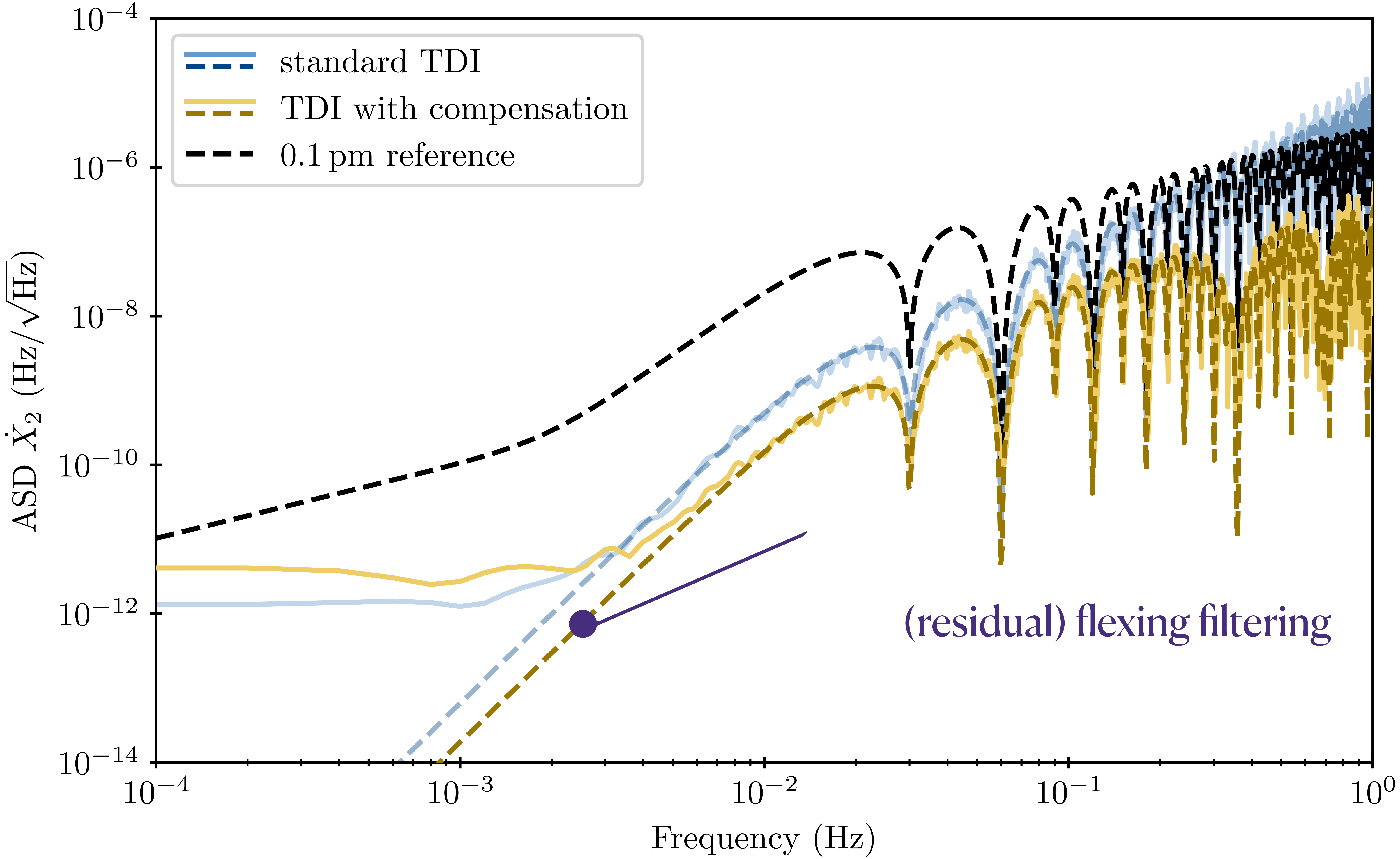
vii. $t_0 = 2.0813 \times 10^9$ s ; to maximize $\dot{d}$

3 Bayle, Jean-Baptiste, Hartwig, Olaf and Staab, Martin — 10.5281/zenodo.13809621

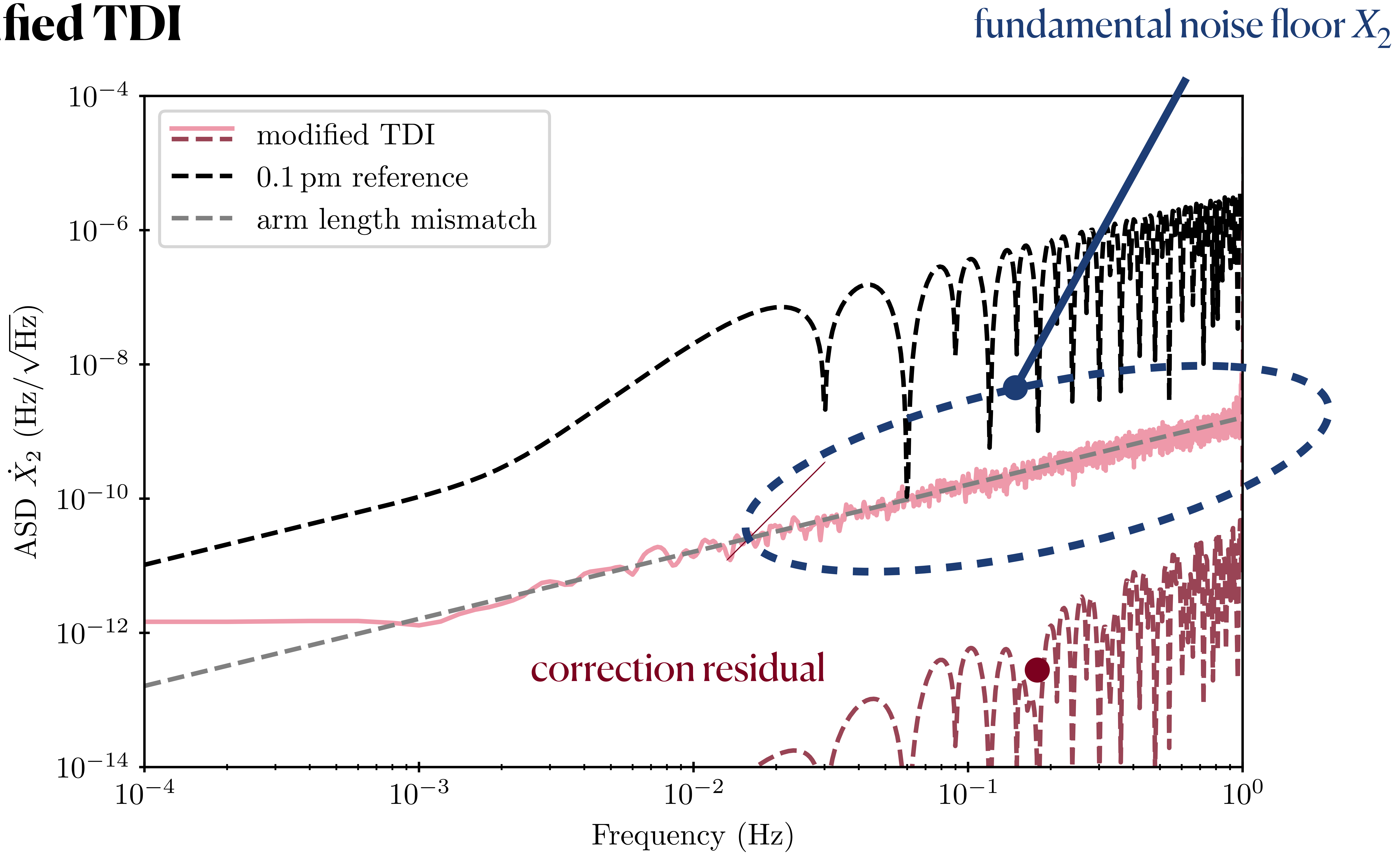
Standard TDI



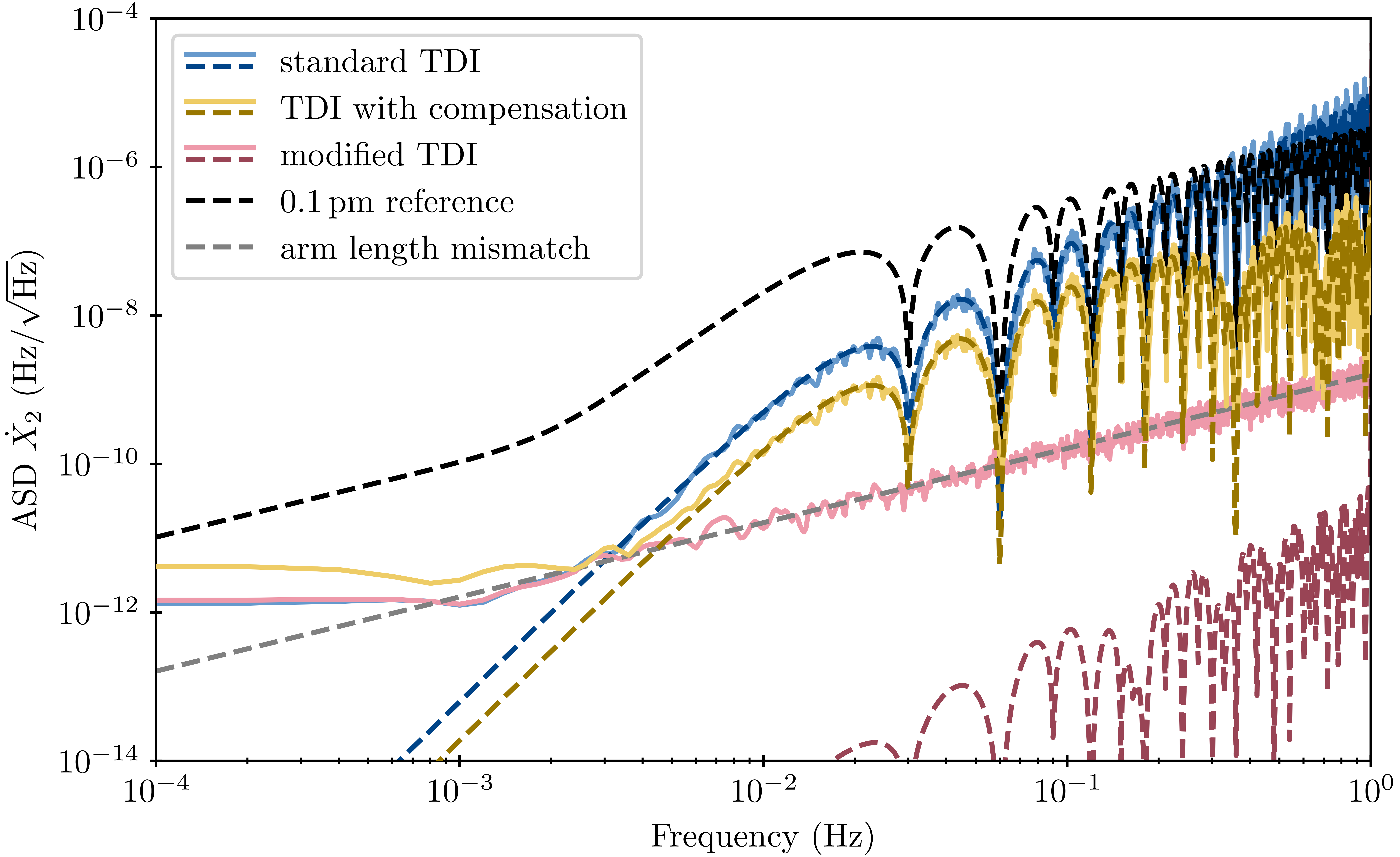
TDI with Compensation



Modified TDI



Modified TDI outperforms the other two topologies



Conclusions

Why use Modified TDI?

Modified TDI allows for **6 order** of noise reduction , contributes **no additional group delay** and keeps on-board **computational cost low** via an inexpensive anti-aliasing filter.

Trade off : Post TDI data is filtered (this can be addressed by a lenient filter in data analysis)

Future Work : other locking configurations, primary noise sources

Thanks :)

If you're interested, you can find our paper on arXiv:2506.04316

backup slides

We need to **approximate the correction H** to apply it on discrete data.

Similar to the pure delay operation
i.e. a discrete convolution between the data and
this approximate correction $\mathscr{H}$.

$$y(nT_s) = \sum_{m=-\infty}^{\infty} x\left((n-m)T_s\right) \cdot h_{\mathscr{H}}\left(mT_s - d\right)$$

The kernel $h_{\mathscr{H}}$ is designed using cosine-sum kernels [3]

$$h_{\mathscr{H}}(t) = \mathrm{rect}\left(\frac{t}{NT_\mathrm{s}}\right)\sum_{n=0}^{N-1} a_n \cdot \cos\left(2\pi f_\mathrm{s}\frac{n}{N}t\right)$$

where the coefficients a_n are approximated using the Parks-McClellan algorithm.

[3] Staab M, Bayle J B, Hartwig O, Hees A, Lilley M, Woan G and Wolf P 2024 Optimal design of interpolation methods for time-delay interferometry (Preprint 2412.14884)

Residual Noise in Modified TDI

Errors between the exact operator $\widehat{\mathbf{D}}$ and the approximate design $\widehat{\mathscr{D}}$ result in noise residues.

$$\underbrace{(\mathbf{D} - \mathscr{D})\phi}_{\text{interpolation error}} + \underbrace{\dot{d} \cdot (\mathbf{H} - \mathscr{H})\phi}_{\text{correction residual}}$$

What contributes to the residual noise?

Topology	Noise source
Standard TDI	flexing filtering $[\mathbf{F}, \mathbf{D}]$
TDI with compensation	(residual) flexing filtering $[\mathbf{F}^{+}\mathbf{F}, \mathbf{D}]$
Modified TDI	correction $(\mathbf{H} - \mathscr{H})$