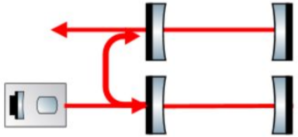
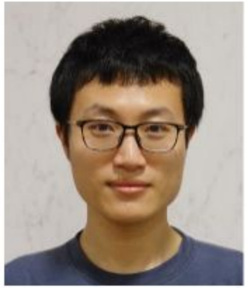
Two filter cavities vs coupled filter cavity for Frequency Dependent Squeezing



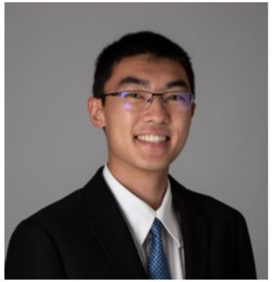

Isander-Louis Ahrend (laboratoire astroparticules cosmologie)

Eleonora Capocasa

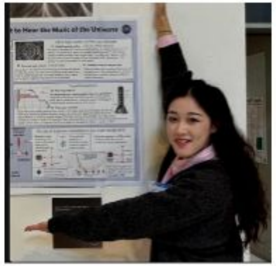
Yuhang Zhao



Jacques Ding



Fangfei Liu

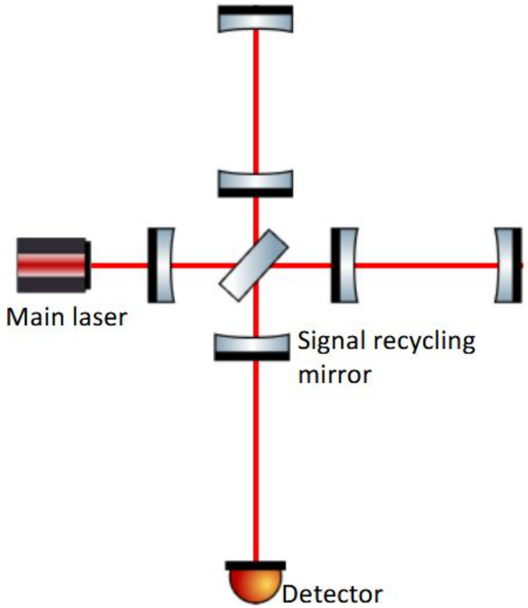


Matteo Barsuglia

Université Paris Cité

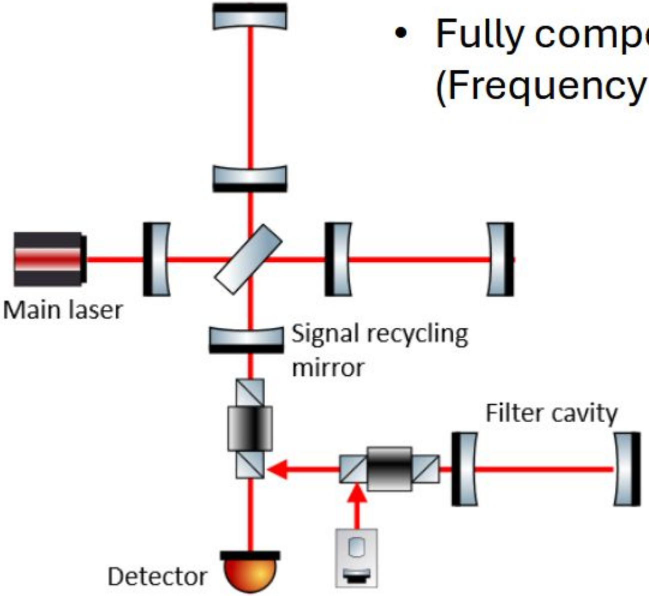
https://quantum-fresco.in2p3.fr/

Quantum noise in a tuned interferometer

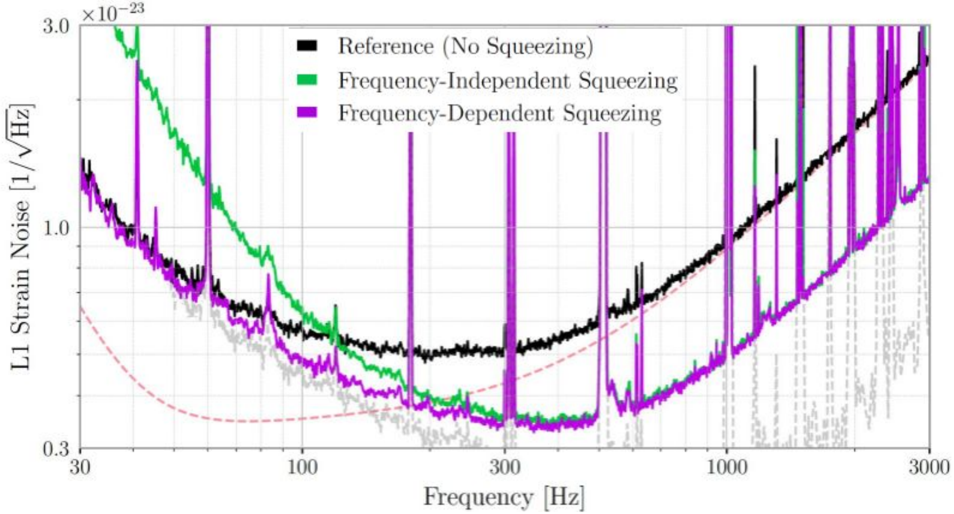


- Current interferometers operate in broadband configuration = tuned signal recycling
- Interferometer rotates quantum noise quadratures (radiation pressure & shot noise)

Quantum noise in a tuned interferometer



- Fully compensate the rotation using a single filter cavity (Frequency Dependent squeezing)



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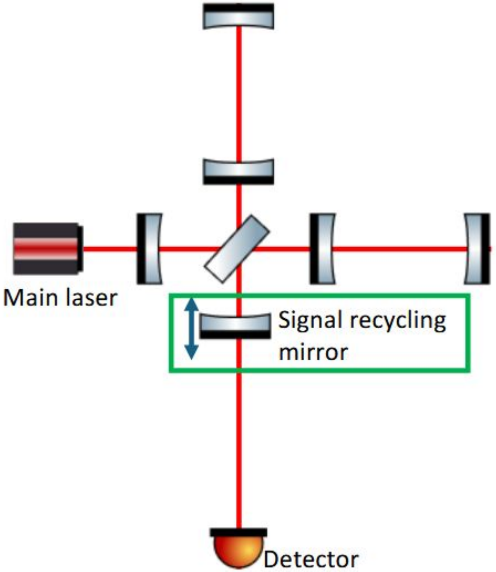
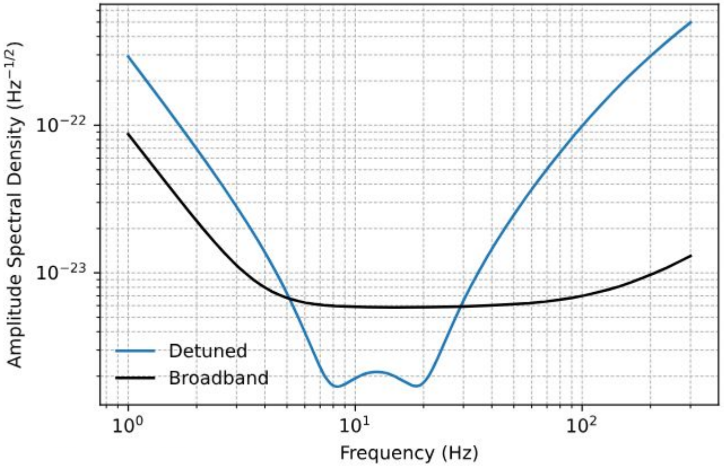
Quantum noise in a detuned interferometer



- Einstein Telescope Low Frequency for this talk (not official numbers):
  - 10 km arms, 45° BS angle
  - 1550 nm
  - 18 kW in arms
  - Detuned SR



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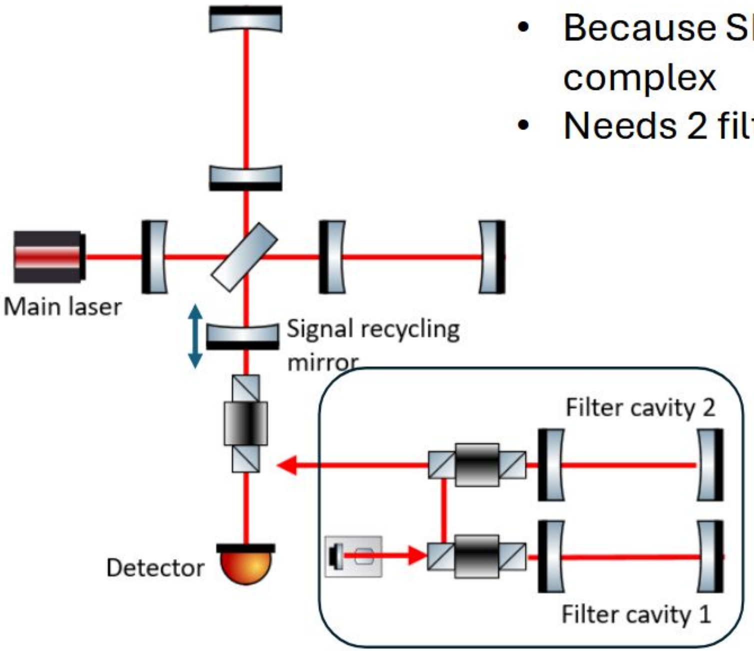
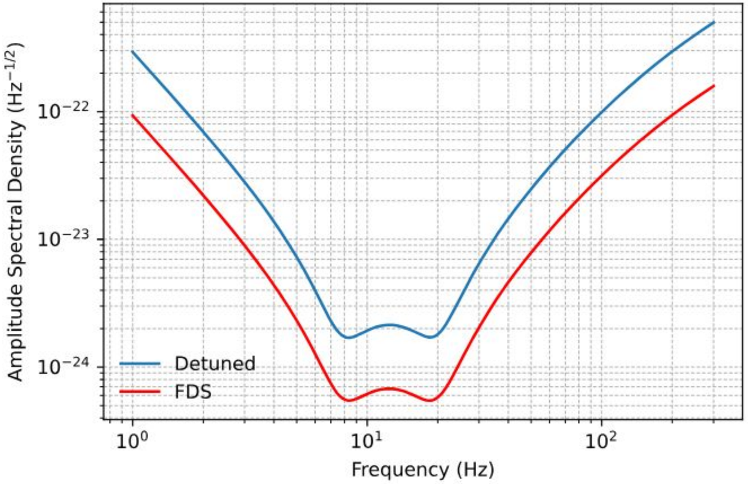
Frequency dependent squeezing for ET-LF



- Because SR detuned, quadrature rotation more complex
- Needs 2 filter cavities for FDS



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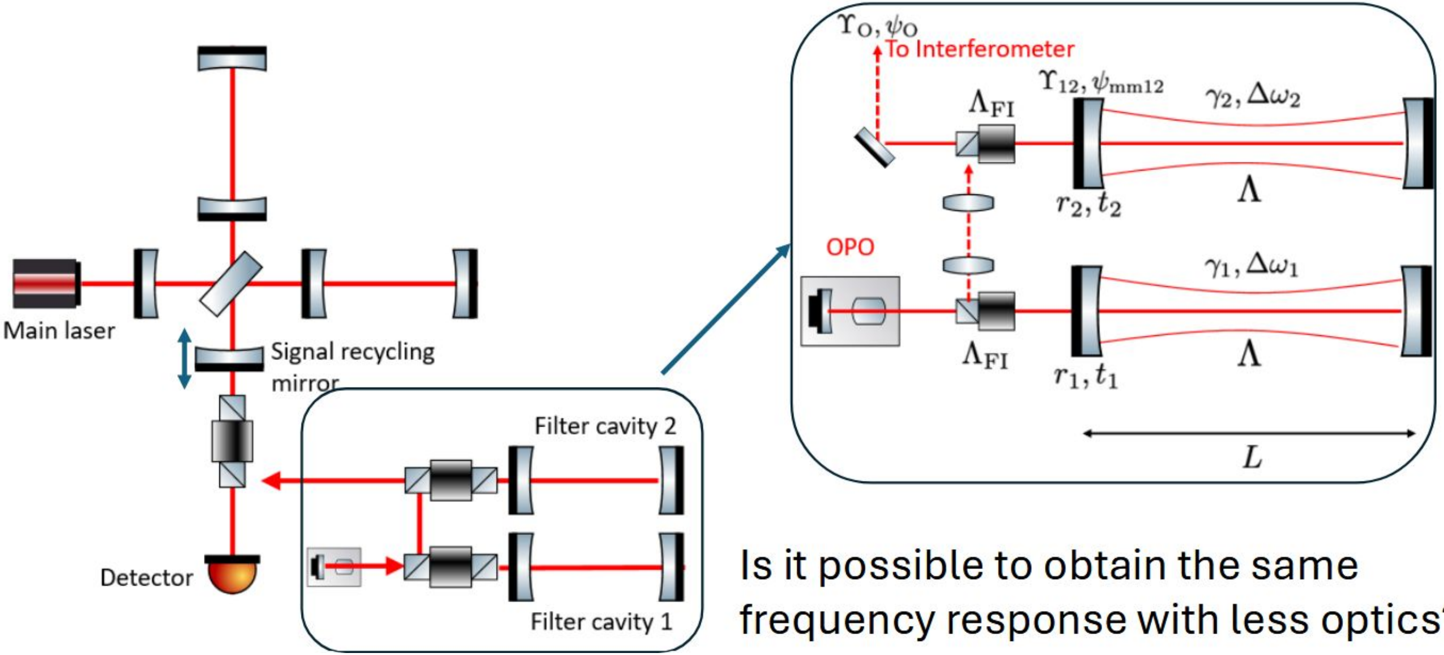
Frequency dependent squeezing for ET-LF



Is it possible to obtain the same frequency response with less optics?

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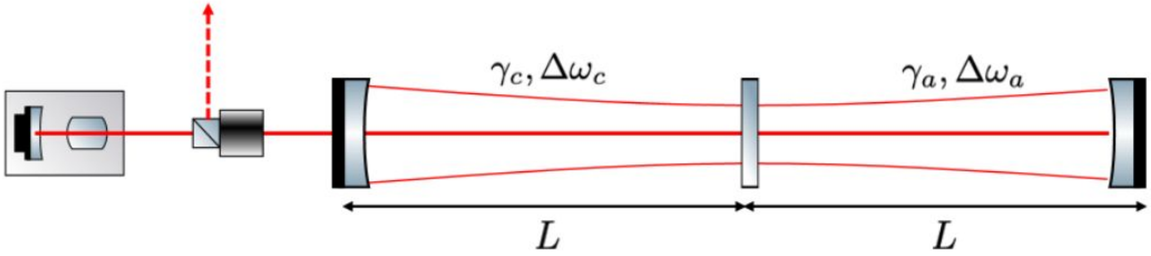
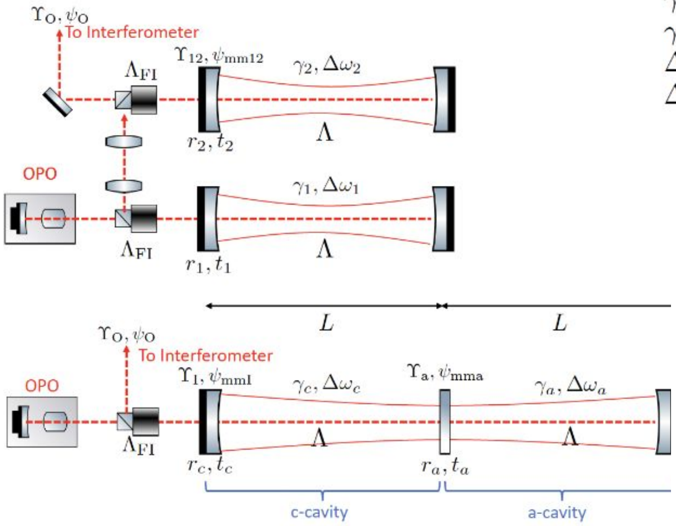
Coupled filter cavity



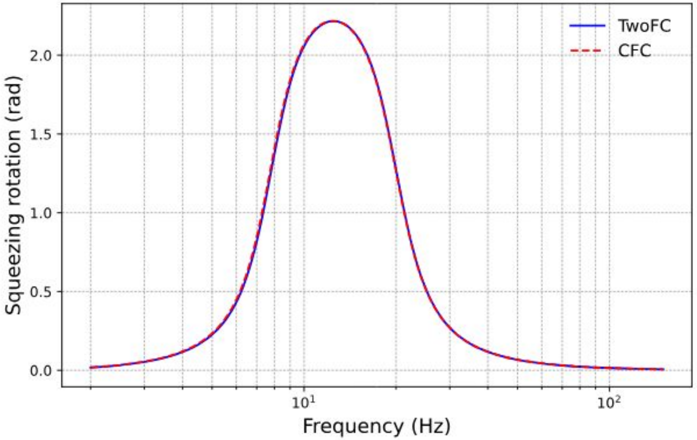
- One less Faraday
- One less mirror
- Less mode matching optics
- Same total footprint

Initially studied in Phys. Rev. D **101**, 082002 and Phys. Rev. D **110**, 082006, but no full quantum degradation analysis and issue of middle mirror transmissivity

Equivalence between two filter cavities and coupled filter cavity



γ_1	1st FC linewidth	$2\pi \times (4.26)$ rad/s
γ_2	2nd FC linewidth	$2\pi \times (1.65)$ rad/s
$\Delta\omega_1$	1st FC detuning	$2\pi \times 19.51$ rad/s
$\Delta\omega_2$	2nd FC detuning	$2\pi \times (-7.65)$ rad/s



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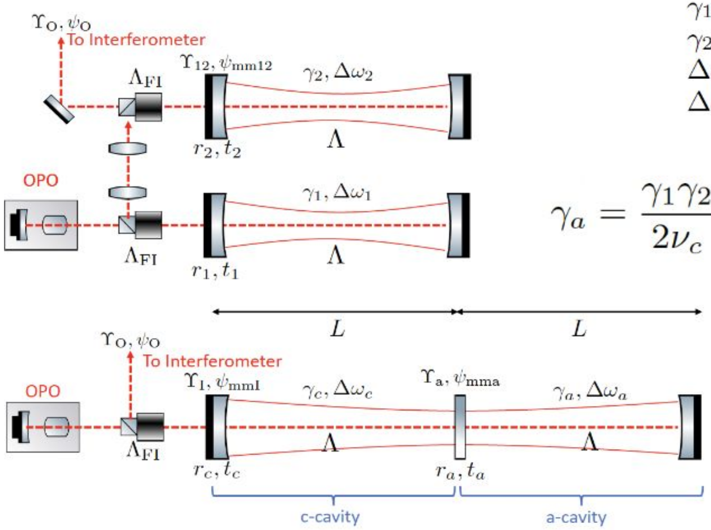
Equivalence between two filter cavities and coupled filter cavity



γ_1	1st FC linewidth	$2\pi \times (4.26)$ rad/s
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$$\gamma_a = \frac{\gamma_1\gamma_2}{2\nu_c}\left[1 + \left(\frac{\Delta\omega_1 - \Delta\omega_2}{\gamma_1 + \gamma_2}\right)^2\right] = \frac{cT_a}{4L} \qquad T_a \propto L^2$$

for $L = 1$ km, $T_a = 0.27$ ppm (!)

for $L = 5$ km, $T_a = 6.75$ ppm

- Compatible with current (future?) coatings
- **CFC only feasible for long enough cavities**

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Squeezing degradation

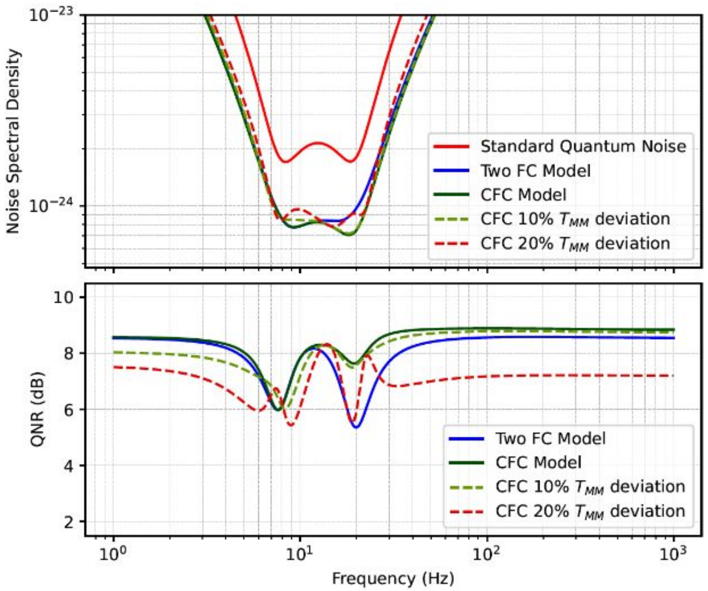
Physical Review D 104, 062006 (2021)
Physical Review D 90, 062006 (2014)
(...)

- Squeezing degradation sources:
  - **Loss**: coupling to vacuum
  - **Mode mismatch**: possible coupling between squeezing and antisqueezing
  - **Phase noise**: Technical, also couples squeezing to antisqueezing
- Figures of merit: $\bar{S} = e^{-2r}$
  - Efficiency: $\bar{S} = \eta e^{-2r} + 1 - \eta$
  - Dephasing: $\bar{S} = (1 - \Xi)e^{-2r} + \Xi\, e^{2r}$
  - Misphasing: $\bar{S} = e^{-2r}\cos^2\Delta\theta_D + e^{2r}\sin^2\Delta\theta_D$

$$\bar{S}[\Omega] = \eta[\Omega]\{[(1 - \Xi[\Omega])e^{-2r} + \Xi[\Omega]e^{2r}]\cos^2(\Delta\theta_D[\Omega]) + [(1 - \Xi[\Omega])e^{2r} + \Xi[\Omega]e^{-2r}]\sin^2(\Delta\theta_D[\Omega])\} + 1 - \eta[\Omega]$$

Example for misphasing: error on middle mirror transmissivity

- What if the middle mirror has 10% or 20% manufacturing error on transmission value?
- The optical system has no "imperfections" but squeezing is still degraded
    - Rotation no longer matches the interferometer's
    - Can be partially compensated using other degrees of freedom (detunings)



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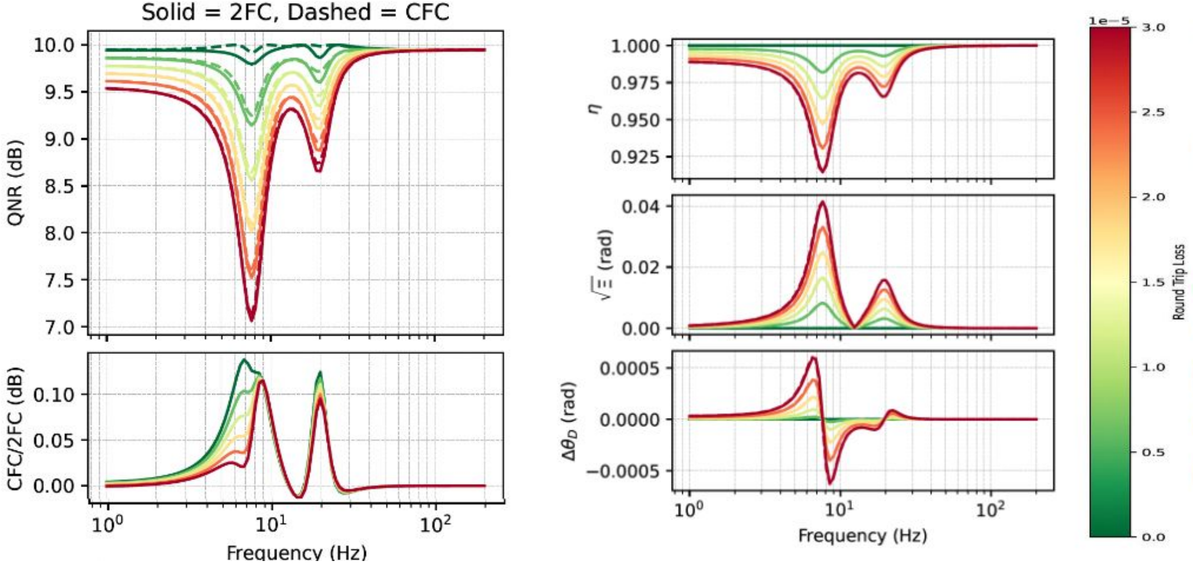
Comparing Round Trip Loss

- Mathematically, if all cavity losses equal, the **2FC-CFC equivalence still holds**



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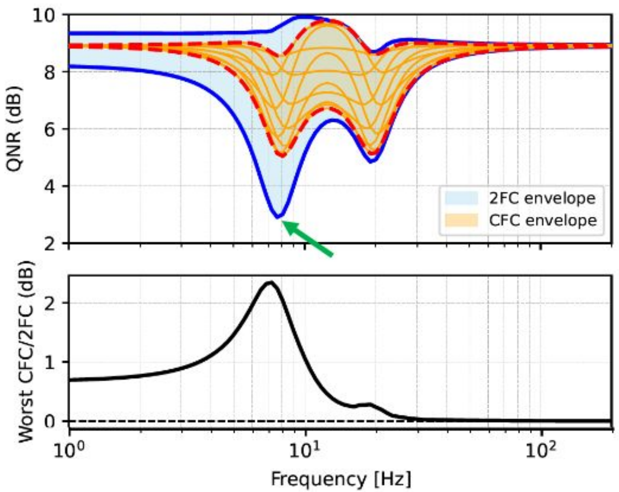
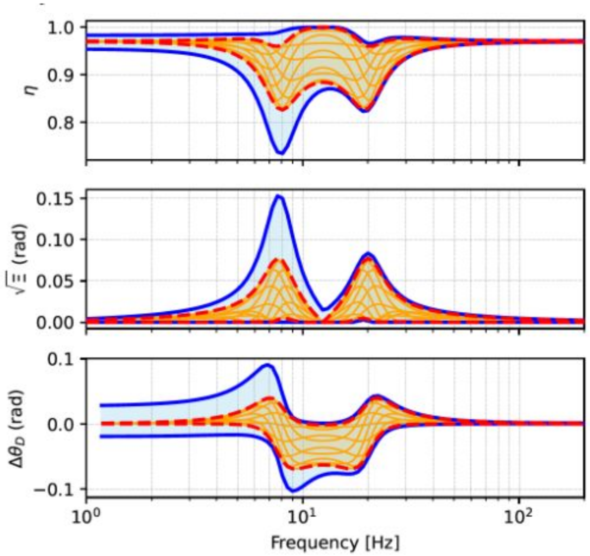
Comparing mode-mismatch

- 4% MM input, 3% output, 1% between cavities (2FC only)





- Significant dephasing at resonance frequencies
- **1% extra MM ⇒ 2 dB lost**
- CFC better on this set of MM params but hard to generalize (how to measure intra-cavity mismatch?)

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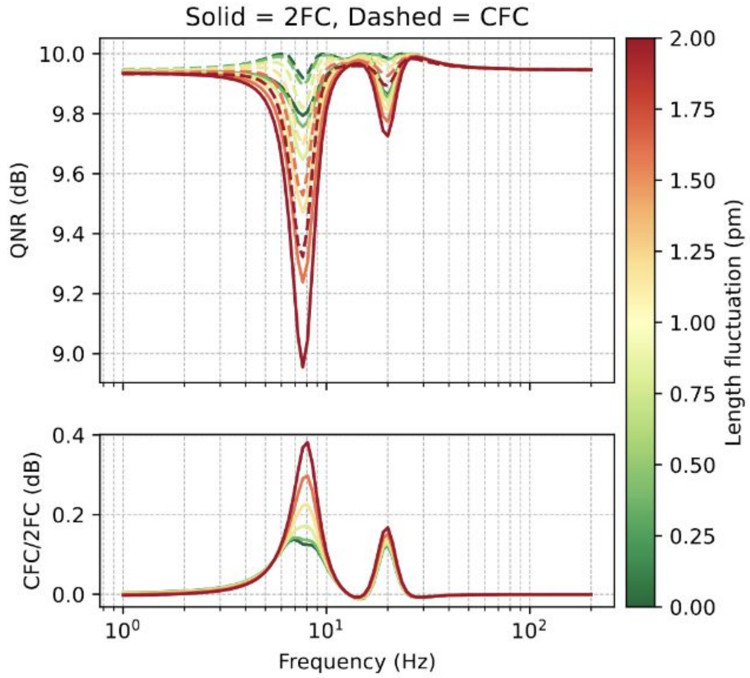
Comparing length noise



- Cavity length fluctuations due to imperfect controls
- Assume same length noise (decorrelated) for all cavities
- Realistic ~pm for single filter cavities (Virgo/LIGO)
- **CFC somewhat better**
- But actual control of coupled cavity to be further investigated.

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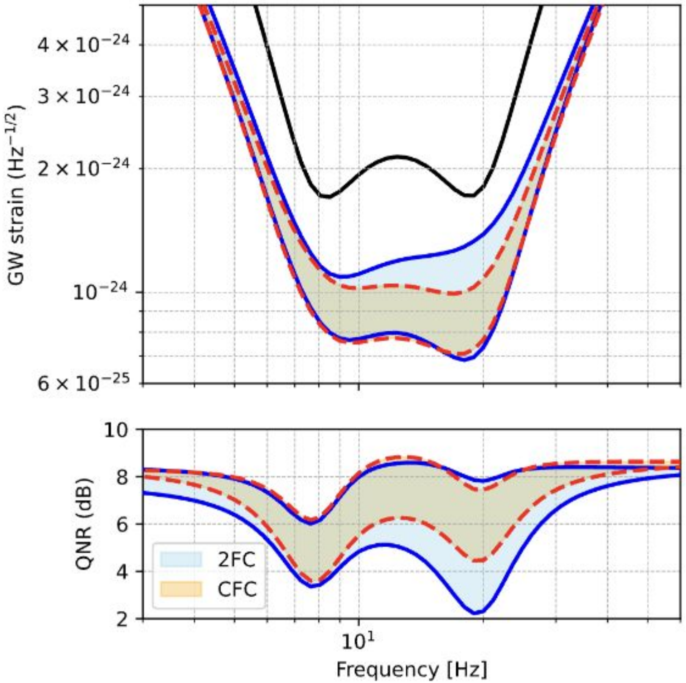
Full budget on FDS

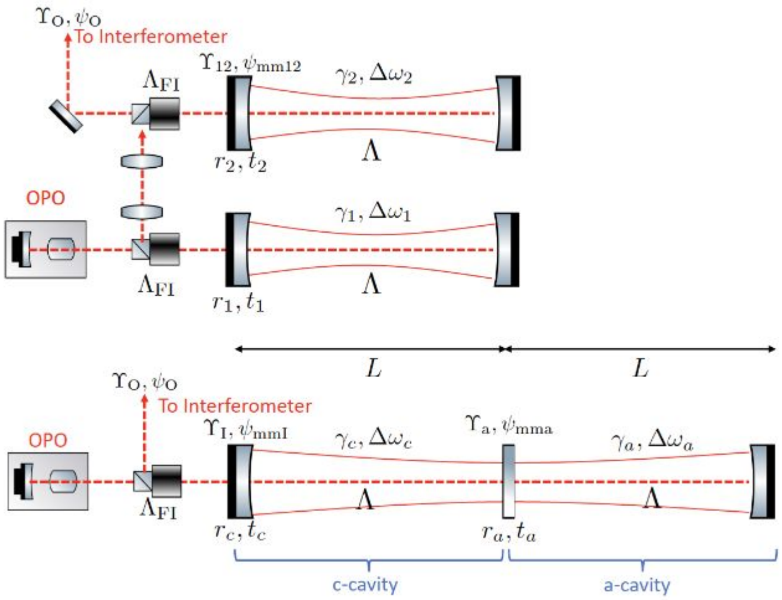


- Add all loss sources in the FDS system (lossless ITF for now)
- Add extra Faraday isolator losses to 2FC

- Degradation phenomenon dominated by mode matching

- **CFC more robust to degradation**

Takeaways



- Theoretical equivalence between 2FC and CFC
  - Also holds when losses are considered
- Constraint on middle mirror transmission can be attained if cavities are ~5 km long each
- Non-linear addition of mode mismatches
- Under some simplifying hypotheses (how valid?), CFC does better than 2FC
- Controls of CFC need to be further investigated

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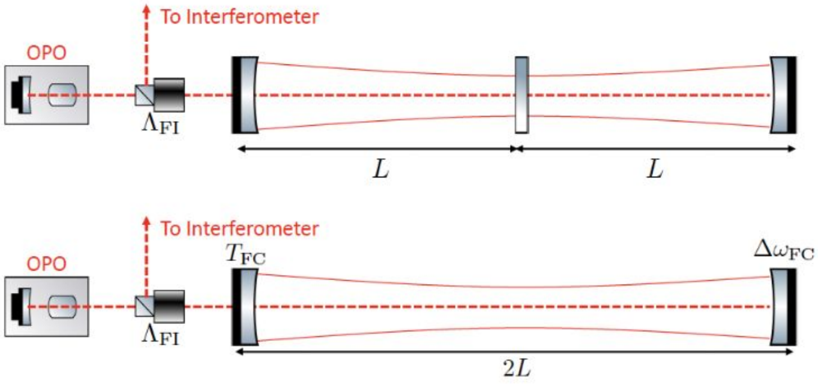
?

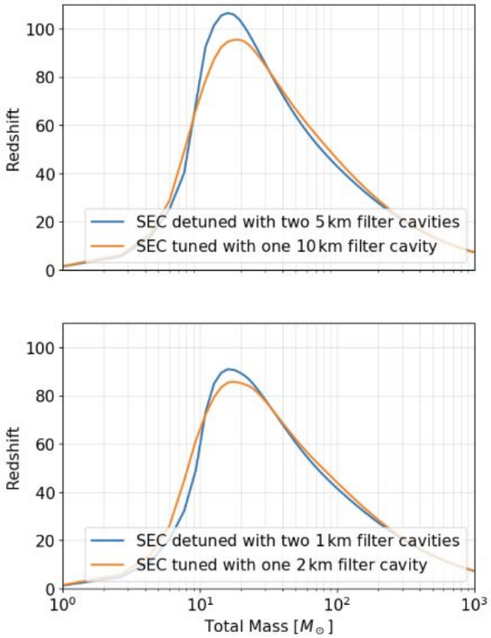
The link

https://arxiv.org/abs/2506.02222

Path to CFC: tuned ET with single FC



Parameter	Physical meaning	Value
T_{SRM}	Transmissivity of signal recycling mirror	50 %
T_{FC}^{2km}	2km FC input mirror transmissivity	0.08 %
$\Delta\omega_{FC}^{2km}$	2km FC detuning	4.78 Hz
T_{FC}^{5km}	10km FC input mirror transmissivity	0.40 %
$\Delta\omega_{FC}^{10km}$	10km FC detuning	4.78 Hz
r_{tuned}	Injected squeezing	19 dB



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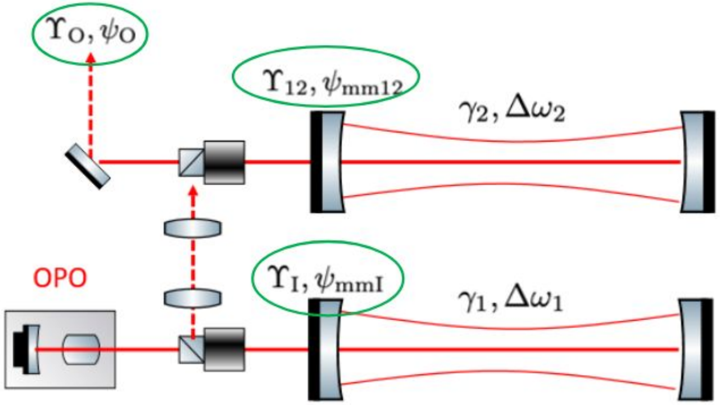
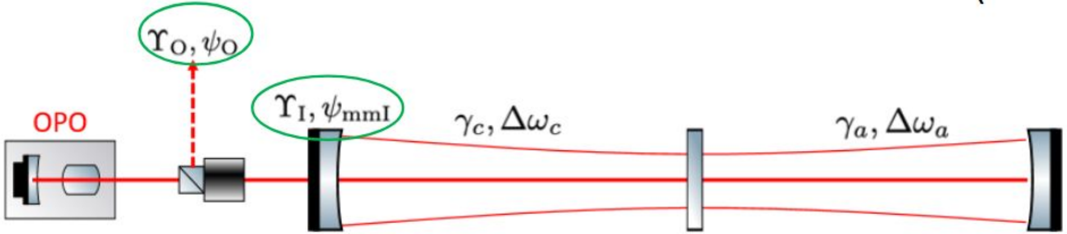
Comparing mode-mismatch



Simplifying hypotheses:

- Mode mismatch coming from free space optics (clipping, uncompensated astigmatism)
- No internal mode mismatch in CFC (symmetry considerations)
- 4% MM input, 3% output, 1% between cavities (2FC only)

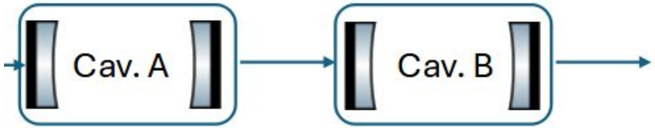


Addition of mode mismatches

- Assume two consecutive mode mismatches: $\hat{a}_{00}^{\text{out}} = \sqrt{1-\Upsilon}\,\hat{a}_{00}^{\text{in}} + \sqrt{\Upsilon}\,e^{i\psi_{mm}}\,\hat{a}_{mm}^{\text{in}}$

Amplitude of mismatch

Phase of mismatch (e.g. waist size, waist position...)



- Not simply a sum of mismatches

$$\Upsilon' = \Upsilon_A + \Upsilon_B - 2\Upsilon_A\Upsilon_B + 2\sqrt{\Upsilon_A(1-\Upsilon_B)\Upsilon_B(1-\Upsilon_A)}\,\cos(\psi_A - \psi_B)$$

- Exemple: $1\% + 3\% \in [0.5\%\,,7\%]$

Cascading (coherent) mismatches easily destroys squeezing

Substrate losses

- 5 cm middle mirror
- 2 ppm/cm losses (conservative)

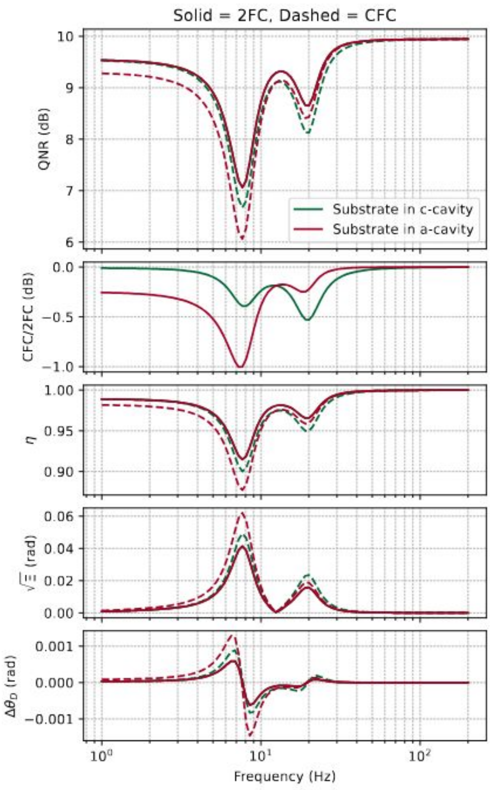


Table of values

Parameter	Physical meaning	Value
λ	Laser wavelength	1550 nm
I_{arm}	Power in the arms	18 kW
T_{arm}	ITM transmission	0.007
L_{arm}	Interferometer arm length	10 km
m_{ETM}	Mass of test-mass	211.3 kg
T_{SRM}	SRM transmission	0.2
L_{SEC}	SEC length	100 m
ϕ_{SEC}	SEC detuning	$\frac{\pi-0.75}{2}$ rad
θ_{HD}	Homodyne angle	-0.27 rad
$\delta\theta_{\mathrm{HD}}$	HD RMS phase noise	10 mrad (typ.)
γ_1	1st FC linewidth	$2\pi \times (4.26)$ rad/s
γ_2	2nd FC linewidth	$2\pi \times (1.65)$ rad/s
$\Delta\omega_1$	1st FC detuning	$2\pi \times 19.51$ rad/s
$\Delta\omega_2$	2nd FC detuning	$2\pi \times (-7.65)$ rad/s
Λ	Round Trip Loss	30 ppm (typ.)
δL	FC RMS length noise	1 pm (typ.)
Υ_I	Input mode mismatch	4%
Υ_O	Output mode mismatch	3%
Υ_{12}	Mode mismatch FC1-FC2	1%
Υ_a	Internal mismatch (CFC)	$\sim 0\%$
Λ_{FI}	FI loss (double pass)	1%
$L_{\mathrm{FC1,2}}$	Length of each FC	5 km
L_{CFC}	Total length of CFC	10 km
T_1	1st FC input transmission	$6.9 \cdot 10^{-4}$
T_2	2nd FC input transmission	$1.8 \cdot 10^{-3}$
T_a	CFC middle transmission	$6.75 \cdot 10^{-6}$
T_c	CFC input transmission	$2.47 \cdot 10^{-3}$
$L_{\mathrm{FC1,2}}$	Length of each FC	1 km
L_{CFC}	Total length of CFC	2 km
T_1	1st FC input transmission	$1.4 \cdot 10^{-4}$
T_2	2nd FC input transmission	$3.6 \cdot 10^{-4}$
T_a	CFC middle transmission	$2.7 \cdot 10^{-7}$
T_c	CFC input transmission	$4.95 \cdot 10^{-4}$