How do hadrons propagate in a constant magnetic field?

Karim Benakli

(CNRS - Sorbonne Université, Paris)

Based on: K.B. To appear

- K.B., Wenqi Ke, Bruno Le Floch ’24
- K.B., Wenqi Ke, Cassiano Daniel ’22 ’23
- K.B., Nathan Berkovits, Cassiano Daniel, Matheus Lize ‘21

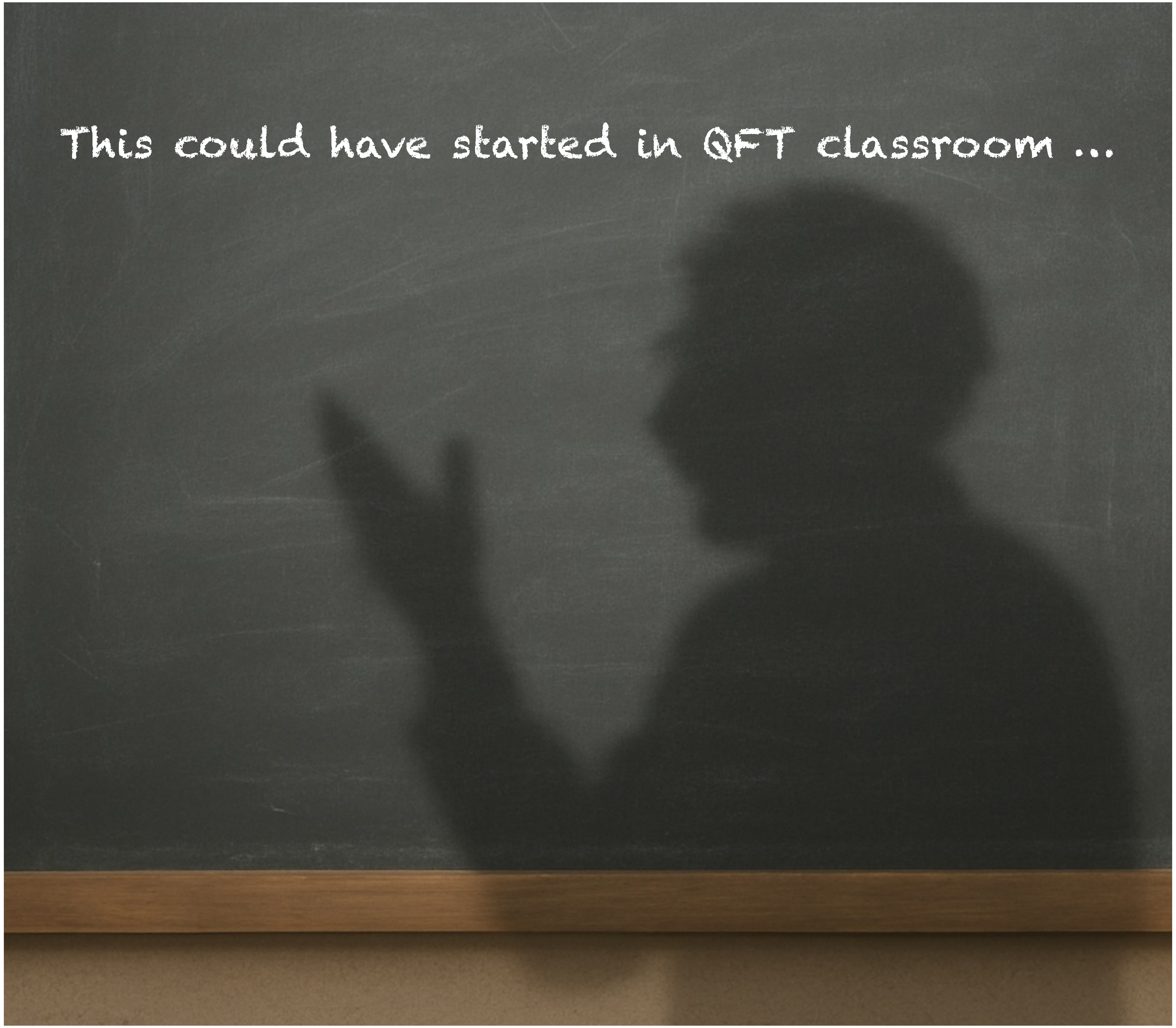
Gdr Inf annual meeting, November 12-14, 2025

Plan

- Identifying the problem – following its historical development as a guide

- Progress so far: what has been accomplished, and what remains open

- The current state of the field


This could have started in QFT classroom ...

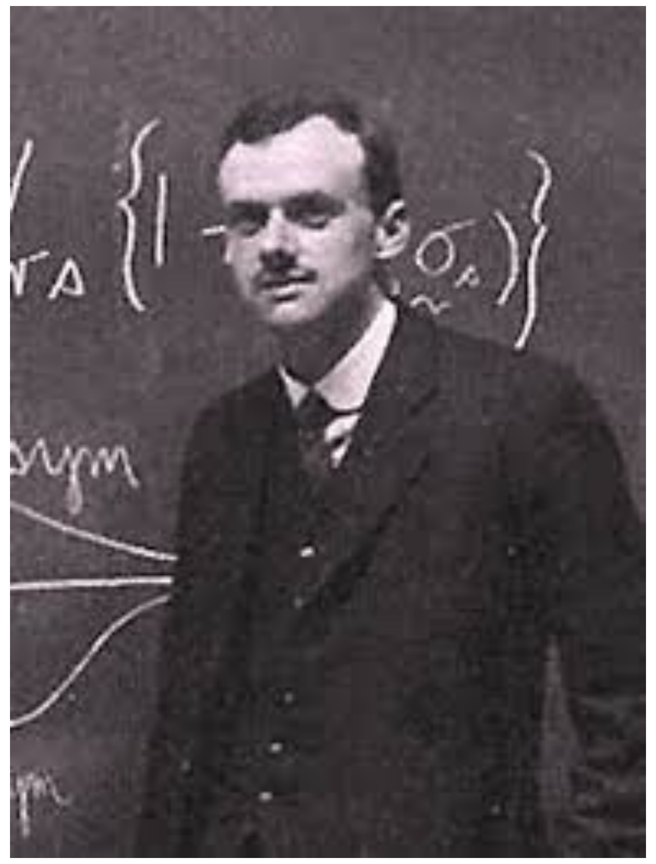

SPIN
MAGNETIC FIELD
ZEEMAN
EFFECT
MAGNETIC FIELD? THEN CHANGE DERIVATIVES TO COVARIANT ONES!
IS IT THE RECIPE FOR ALL FIELDS?
I DIDN'T SEE 3/2,2... IN TEXTBOOOKS
IT MUST BE TRUE
I'LL GO AND CHECK THAT!
NO ANSWER IN MY BOOKS!

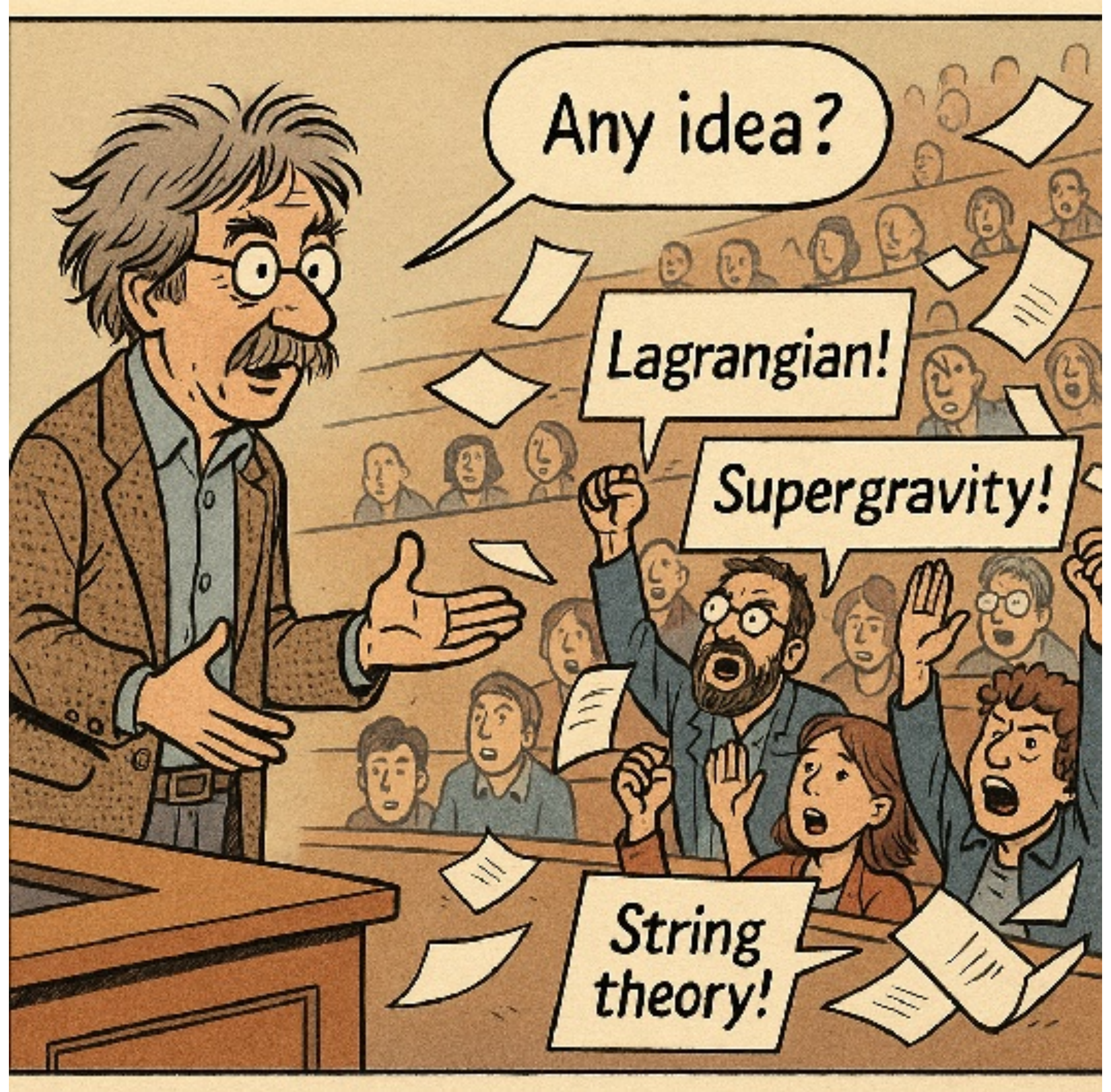

FORMULAS FOR HIGHER SPIN FIELDS 3/2, 2...?
I WILL ASK DIRAC, FIERZ, PAULI ... THEY MUST KNOW
YOU'VE GOT TO BE KIDDING ME!
LET'S SEE WHAT MY COLLEAGUES THINK ABOUT THIS!
UNIVERSITY
Any idea?
Lagrangian!
Supergravity!
String theory!
WE SHALL TRY ALL THESE!!!

The true story is not much different...

The true story is not much different...

It started with Dirac in 1936 ...





Dirac, Fierz, Pauli, Federbusch,Velo, Zwanziger,
Itzykson, Voros, Madore, …
Ferrara, Telegdi, Argyres, Nappi,
Porrati, Rahman, Sagnotti, Deser,
K.B., Berkovits, Ke …

1936

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Relativistic Wave Equations

By P. A. M. DIRAC, F.R.S., St. John's College, Cambridge

(*Received March 25, 1936*)

448 P. A. M. Dirac

The elementary particles known to present-day physics, the electron, positron, neutron, and proton, each have a spin of a half, and thus the work of the present paper will have no immediate physical application. All the same, it is desirable to have the equations ready for a possible future discovery of an elementary particle with a spin greater than a half, or for approximate application to composite particles. Further, the underlying theory is of considerable mathematical interest.

Dirac 1936:

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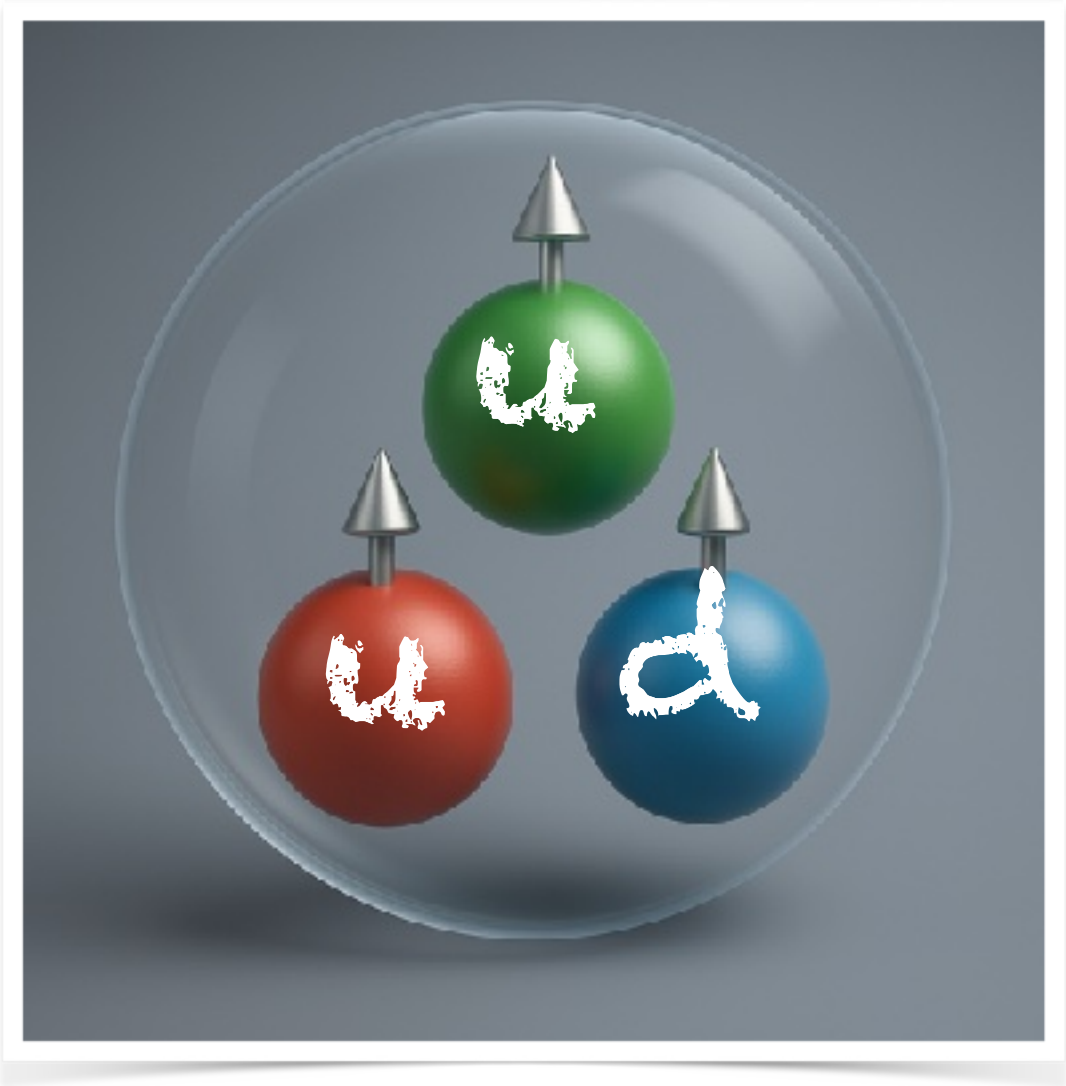
Further, the underlying theory is of considerable <u>mathematical interest.</u> »

Proton

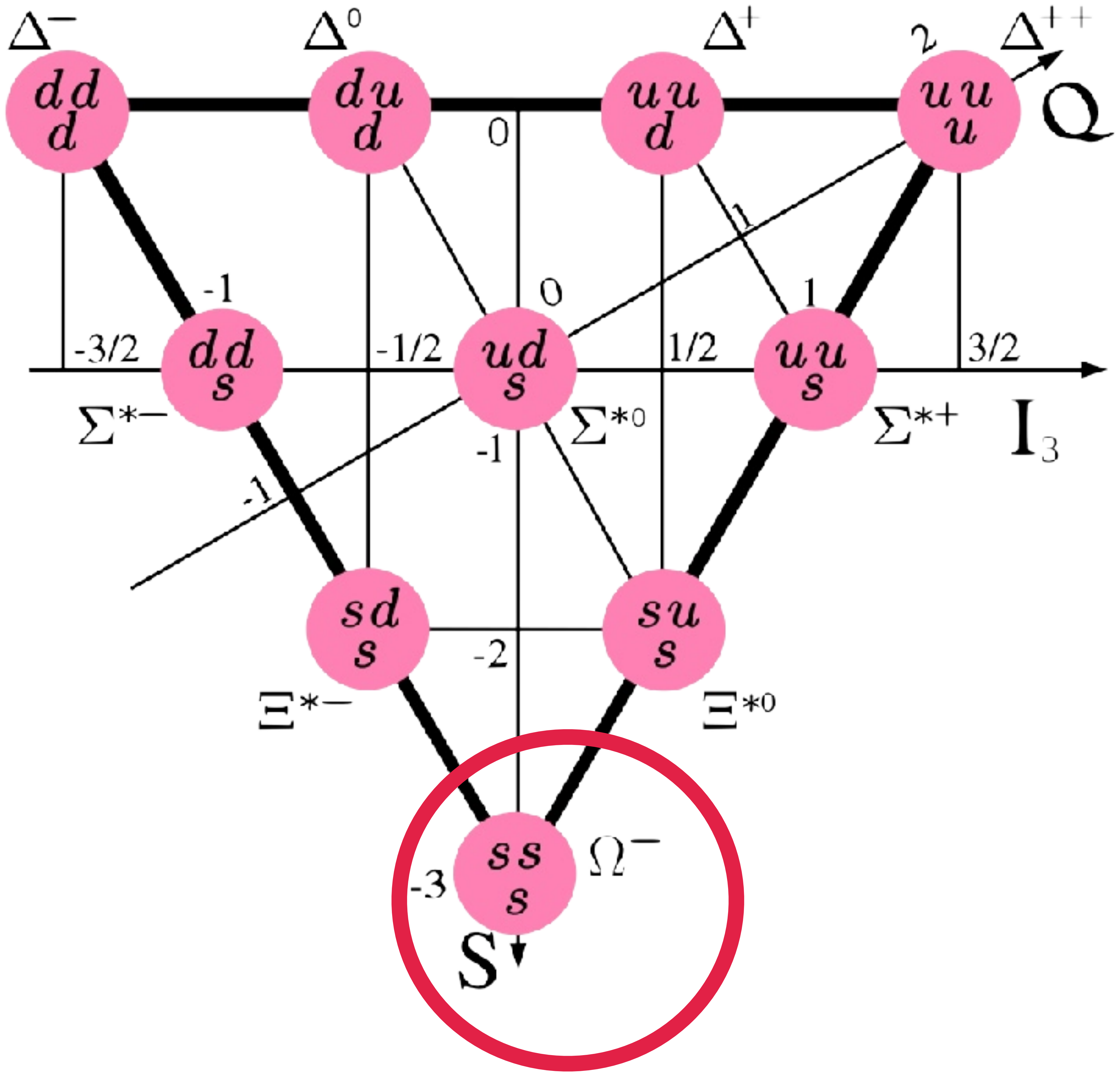


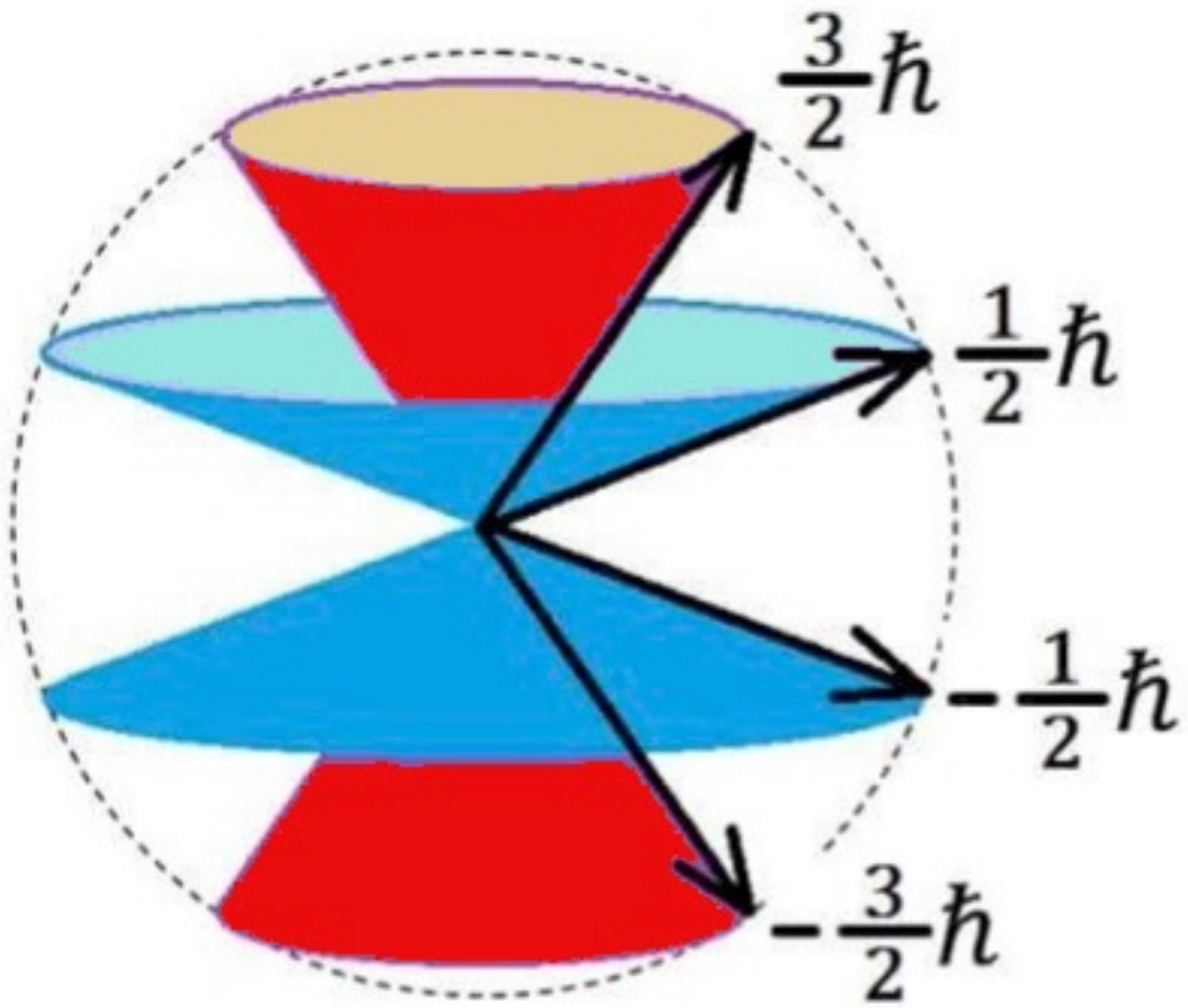
Delta +

Δ^{+} (Delta plus) ($\approx\Delta(1232)$ resonance) ~1232 MeV spin 3/2

Liftetime~1.6×10^{-23} s (approx width ~120 MeV $\rightarrow$ lifetime $\hbar/\Gamma$)

The decuplet spin 3/2

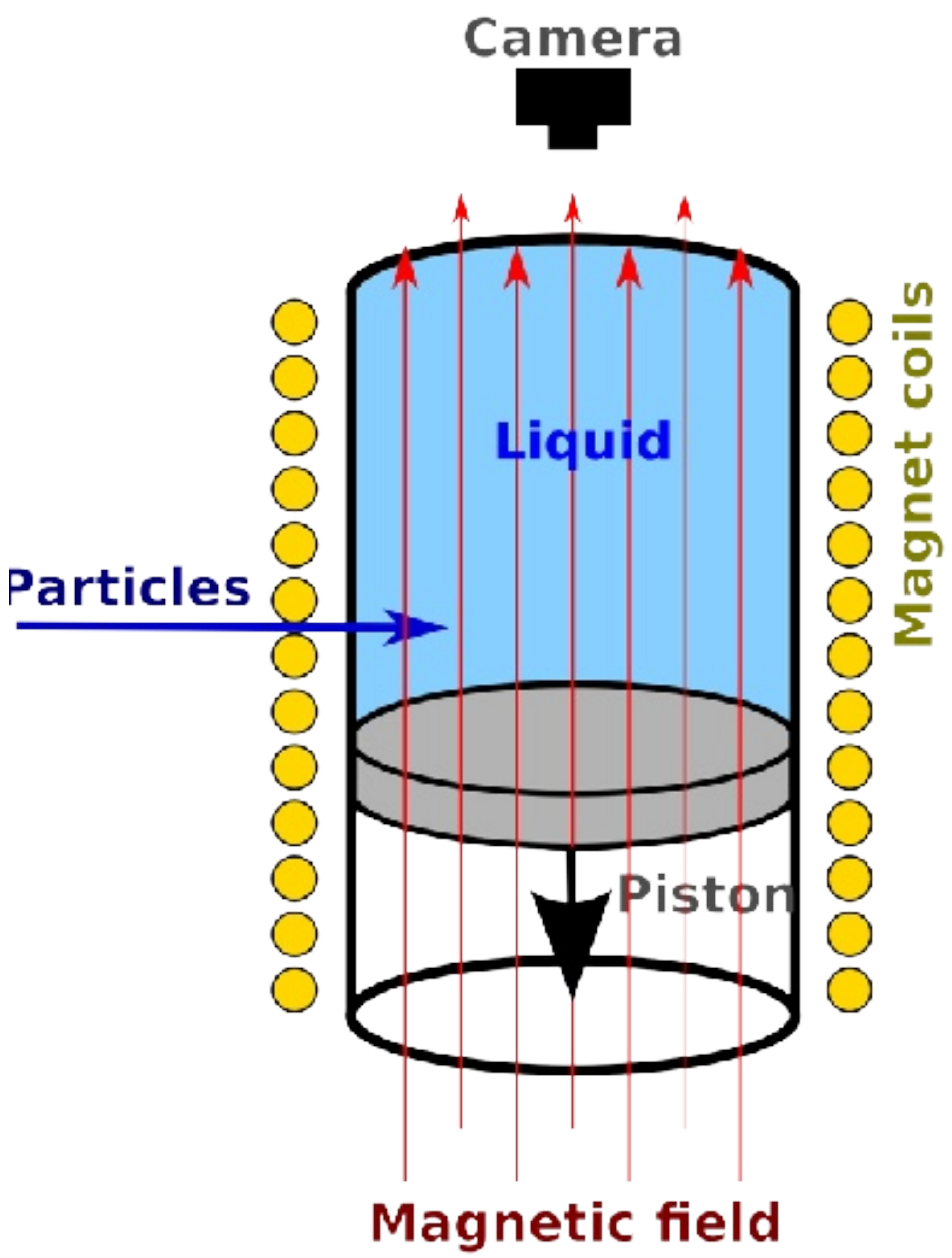


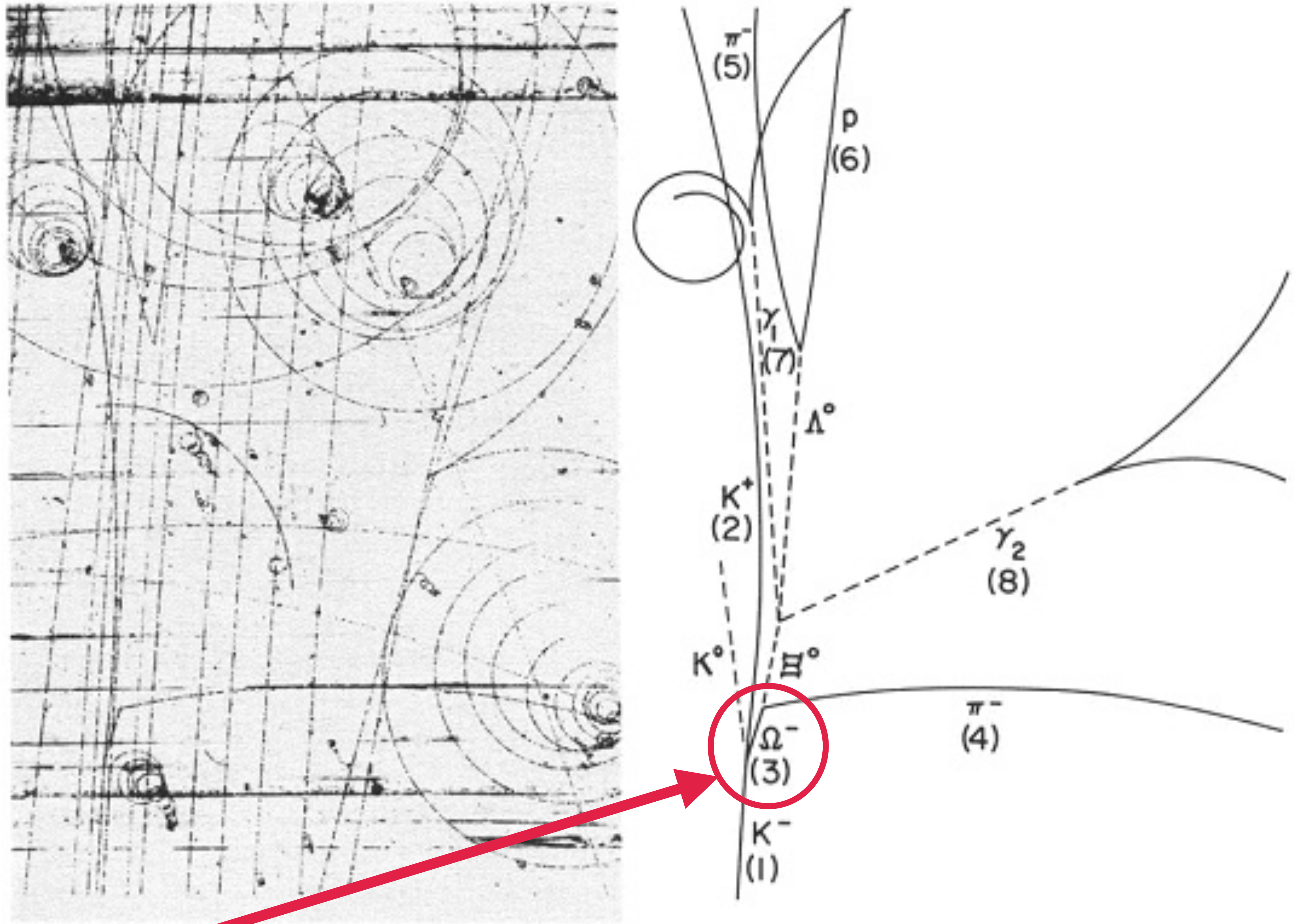


spin 3/2

In 1964, the discovery of the Ω baryon in a magnetic field

15-foot





$$K^- + p \to \Omega^- + K^- + K^0$$
$$\mapsto \Xi^0 + \pi^+$$
$$\mapsto \Lambda^0 + \pi^0$$
$$\mapsto \gamma + \gamma$$
$$\mapsto e^+e^-$$
$$\mapsto e^+e^-$$
$$\mapsto \pi^- p$$

In 1964, the Omega Hyperon was discovered at Brookhaven in a bubble chamber. It was predicted in 1962 by M. Gell-mann and Y. Ne'eman to have spin 3/2.

In 2006, it was shown to have spin 3/2.

Measurement of the Spin of the Omega-Minus Hyperon,
by the BaBar collaboration at SLAC (B. Aubert et al.), Phys. Rev. Lett. 97, 112001 (2006), and hep-ex/0606039.

Charged particles : just use covariant derivatives

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Relativistic Wave Equations

By P. A. M. Dirac, F.R.S., St. John's College, Cambridge

(Received March 25, 1936)

ion. This equivalence between our equations (40) and the usual electron quations persists when there is an electromagnetic field present, provided he effect of the field on equations (40) is the usual one of requiring p to be eplaced by $p + eA$, A being the vector potential. Thus our equation

On relativistic wave equations for particles of arbitrary spin in an electromagnetic field

By M. Fierz and W. Pauli

Physikalisches Institut der Eidgenössischen Technischen Hochschule, Zürich

(*Communicated by P. A. M. Dirac, F.R.S.—Received* 31 *May* 1939)

1. Introduction

The investigations of Dirac (1936) on relativistic wave equations for particles with arbitrary spin have recently been followed up by one of us (Fierz, 1939, referred to as (A)) It was there found possible to set up a scheme of second quantization in the absence of an external field, and to derive expressions for the current vector and the energy-momentum tensor. These considerations will be extended in the present paper to the case when there is an external electromagnetic field, but we shall in the first instance disregard the second quantization and confine ourselves to a *c*-number theory.

The difficulty of this problem is illustrated by the fact that the most immediate method of taking into account the effect of the electromagnetic field, proposed by Dirac (1936), leads to inconsistent equations as soon as the spin is greater than 1. To make this clear we consider Dirac's equations for a particle of spin 3/2, which in the force-free case run as follows:

For Fierz and Pauli,
the Lagrangian
in absence of external field
was a not an issue.

External field
was THE problem

Massive spin 2: Fierz-Pauli (1939)

A massive spin 2 particle has 5 degrees of freedom: helicities -2, -1, 0, +1,+2
Represented by a symmetric tensor h_{mn} that satisfies

Equation of motion **10 d.o.f** → $(\partial^r \partial_r - M^2) h_{mn} = 0$

Constraints **-5 d.o.f**
- -4 d.o.f → $\partial^m h_{mn} = 0$
- -1 d.o.f → $h \equiv h^m_m = 0$

These can be obtained from

Massive spin 2: minimal coupling ?

A massive spin 2 particle has 5 degrees of freedom: helicities -2, -1, 0, +1,+2
Represented by a symmetric tensor h_{mn} that satisfies

Equation of motion **10 d.o.f** $\longrightarrow$ $(D^r D_r - M^2)h_{mn} = 0$

Constraints **-5 d.o.f**
- -4 d.o.f $\longrightarrow$ $D^m h_{mn} = 0$
- -1 d.o.f $\longrightarrow$ $h \equiv h^m_m = 0$

$$[D^m, D^r D_r - M^2]h_{mn} = 0 \quad \Rightarrow \quad iQ\, F^{mr}\, D_r h_{mn} = 0$$

A new constraint !

Pathological theory: the d.o.f.'s number is different when a constant magnetic field is switched on.

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These can be obtained from

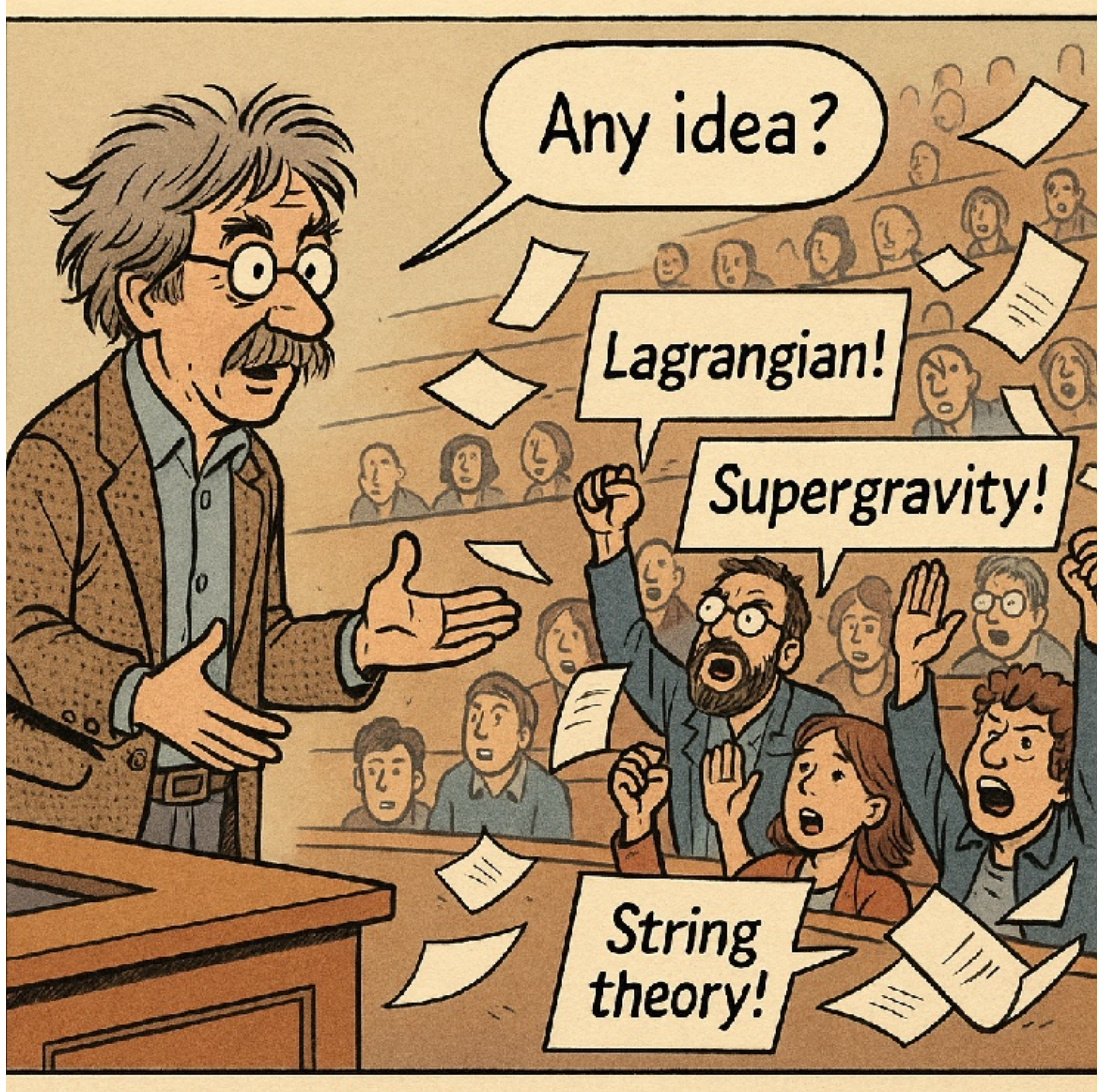
$$\mathcal{L}_{FP} = \boxed{\frac{1}{2}h^{mn}\partial^2 h_{mn} - \frac{1}{2}h\partial^2 h + h_{mn}\partial^m\partial^n h + \partial^n h_{mn}\partial_k h^{mk}} \; \boxed{-\frac{1}{2}M^2\left(h^{mn}h_{mn} - h^2\right)}$$

Linear expansion of Einstein-Hilbert

Mass terms:
FP combination allows to get the constraints

1939

Fierz and Pauli propose the Lagrangian route …



The hunt for a Lagrangian is launched …

Known Lagrangians lead to unphysical modes: ghosts

Until 1960

Minimal Electromagnetic Coupling for Spin Two Particles (*).

P. FEDERBUSH

Department of Physics and Laboratory for Nuclear Science
Massachusetts Institute of Technology - Cambridge, Mass.

(ricevuto il 24 Ottobre 1960)

Summary. — It is noted that if the same Lagrangian that describes the linearized theory of general relativity is extended to charged massed spin 2 particles the coupling cannot be minimal electromagnetic.

It has the property of leading to a theory of massless particles like gravitons when $m = 0$. (The gauge invariance this implies probably gives a unique specification of this Lagrangian.) When $m \neq 0$ the equations describe spin 2 particles with the correct ~~number of propagating solutions, five for particle and five for antiparticle. However, if the field is coupled to an external electromagnetic field minimally (*i.e.* by the prescription $\partial_n \to \partial_n + ieA_n$)~~ in general the number of propagating fields becomes 12 rather than 10. The type of calculations necessary to reach this conclusion are described in ref. [1]. By the addition of a term

$$i\frac{e}{2}\tilde{A}^{\mu\nu}F_{\mu}{}^{\beta}A_{\beta\nu}\,,$$

to the Lagrangian the correct number of canonical variables is restored, although other inconsistencies may remain.

In conclusion we note that not only is the minimal electromagnetic coupling not unique, since it depends on the choice of the free Lagrangian, but in at least two theories, the usual spin $\frac{3}{2}$ and spin 2 theories, it leads to inconsistencies. The type of inconsistency induced in spin 2 theory is suggestive of the difficulties that one encounters in the effort to avoid subsidiary conditions; but the lack of a renormalizable theory of these spins keeps the question hypothetical.

The Federbush Lagrangian

One adds to the Lagrangian gyromagnetic coupling $-i\ 2\ g\ Q\ \bar{h}_{mn}\ F^{nk}\ h_k{}^m$

Gyromagnetic ratio

E.O.M's + constraints lead to

$$\boxed{\frac{3}{2}M^4 h = -i(2g - 1)iQ\,F^{mn}\,D_n D^p h_{pm} + \cdots}$$

Remember:

Equation of motion **10 d.o.f** $(\partial^r\partial_r - M^2)h_{mn} = 0$

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E.O.M's + constraints lead to

$$\boxed{\frac{3}{2}M^4 h = -i(2g - 1)iQ\, F^{mn}\, D_n D^p h_{pm} + \cdots}$$

No ghost in the Federbush Lagrangian : $h \equiv h^m_m = 0$

$\Rightarrow \qquad g = 1/2$

But Velo-Zwanziger in 1969 found

One considers minimal couplings and adds to the Lagrangian gyromagnetic couplings

Extract: E.O.M's + constraints

Using the method of characteristics, one can show that for a magnetic field, that the vector n_μ along the normal to the characteristic hypersurfaces has components:

$$\boxed{\frac{n_0^2}{|\vec{n}|^2} = \frac{1}{1-(\frac{3e}{2m^2})^2\vec{B}^2}}$$

Superluminal propagation

Velo - Zwanziger '69

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Summary. — It is noted that if the same Lagrangian that describes the linearized theory of general relativity is extended to charged massed spin 2 particles the coupling cannot be minimal electromagnetic.

Solves one issue: * No more ghosts

This has two issues: * Superluminal Propagation
* Gyromagnteic ratio = 1/2

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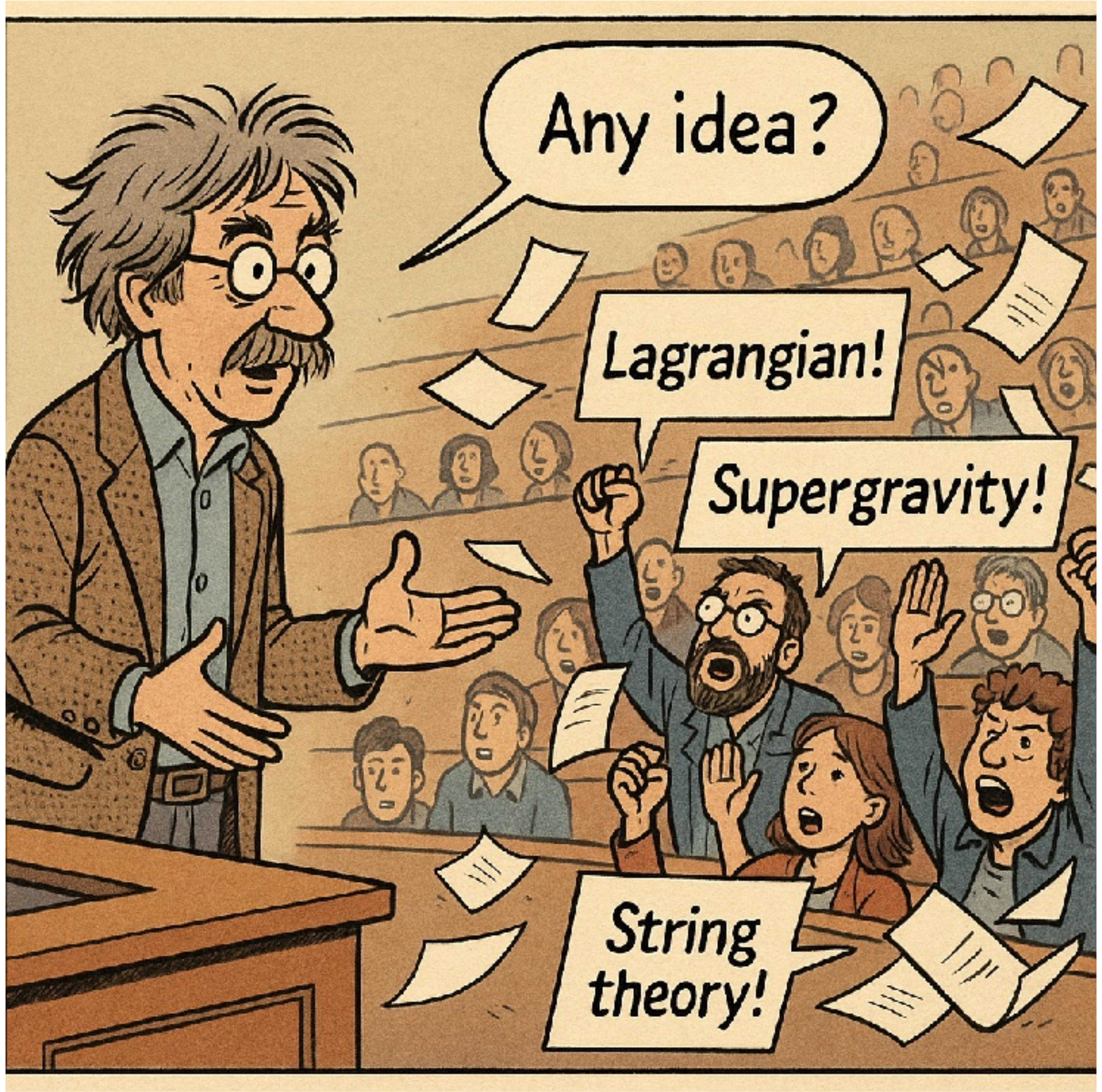
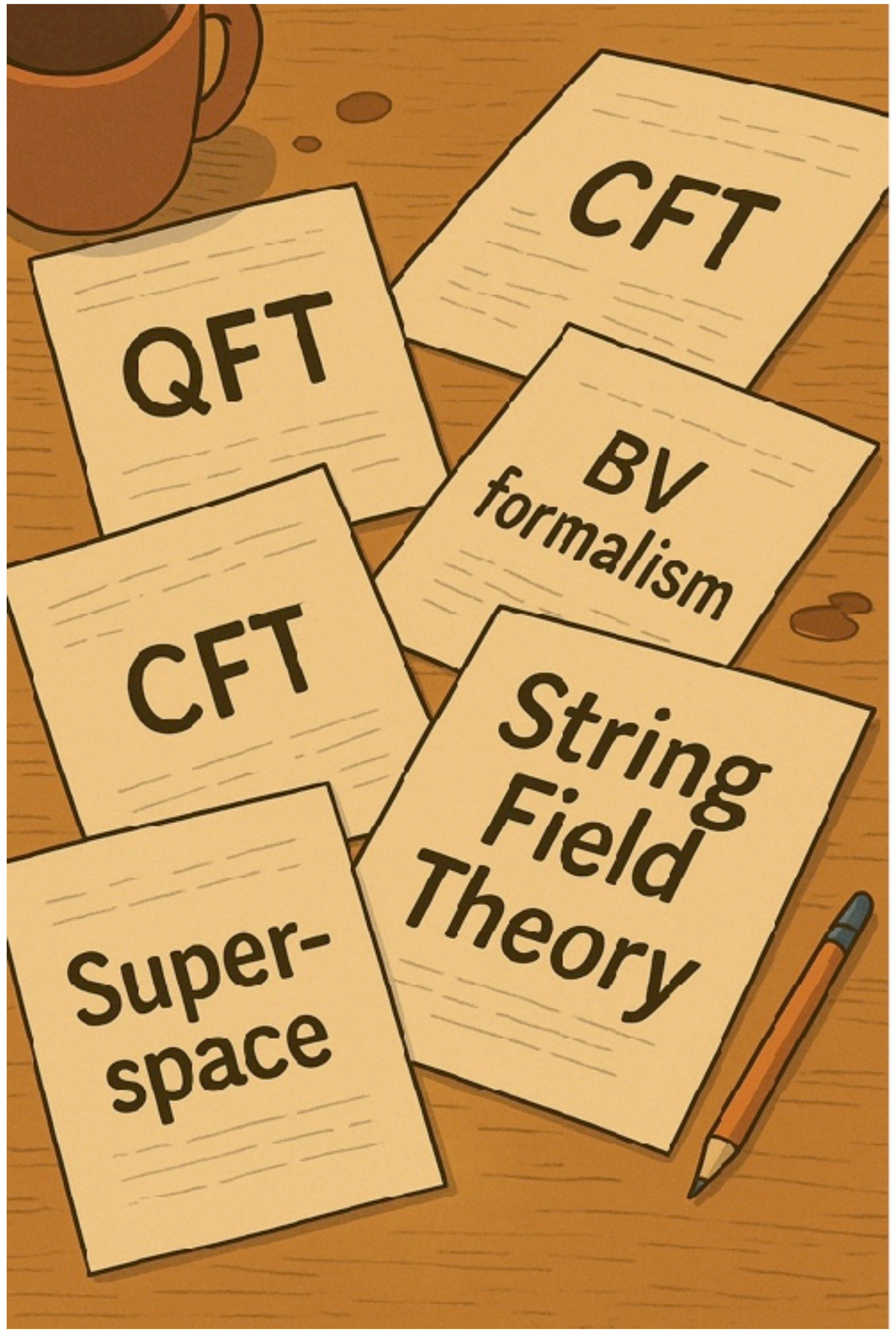
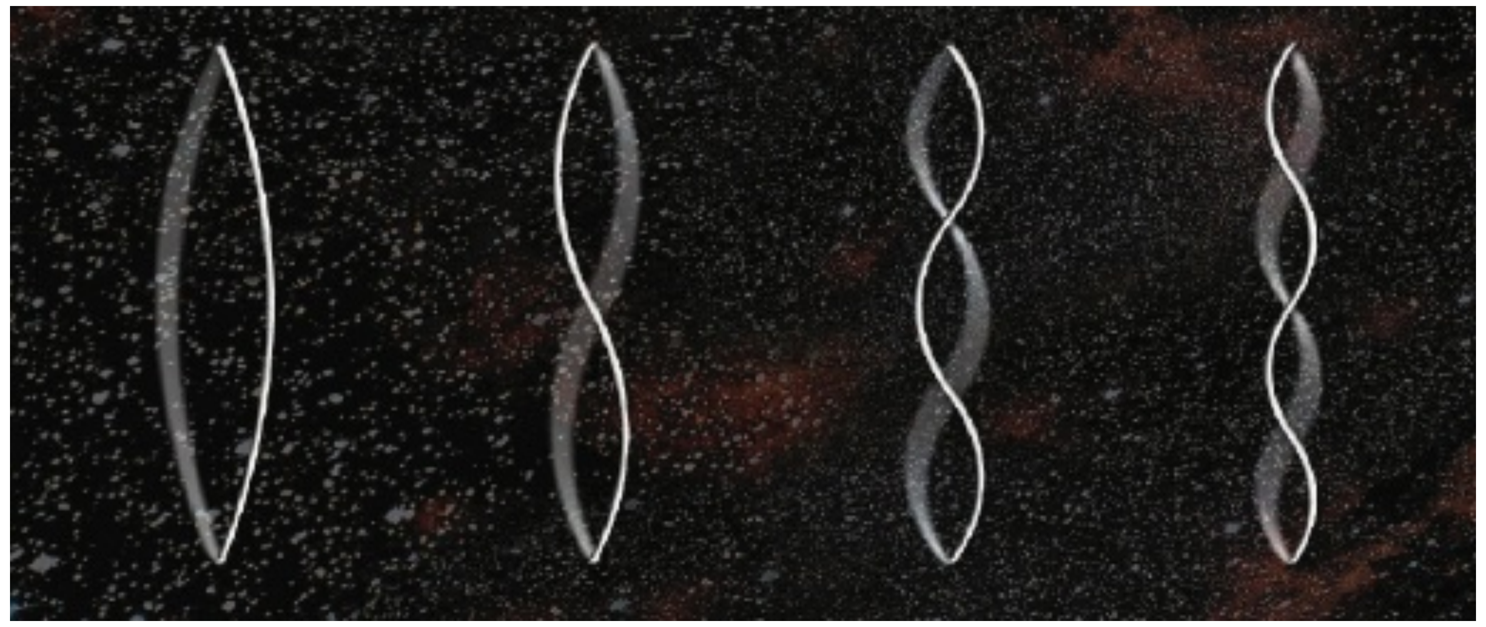

Any idea?
Lagrangian!
Supergravity!
String theory!


WE SHALL TRY ALL THESE!!!

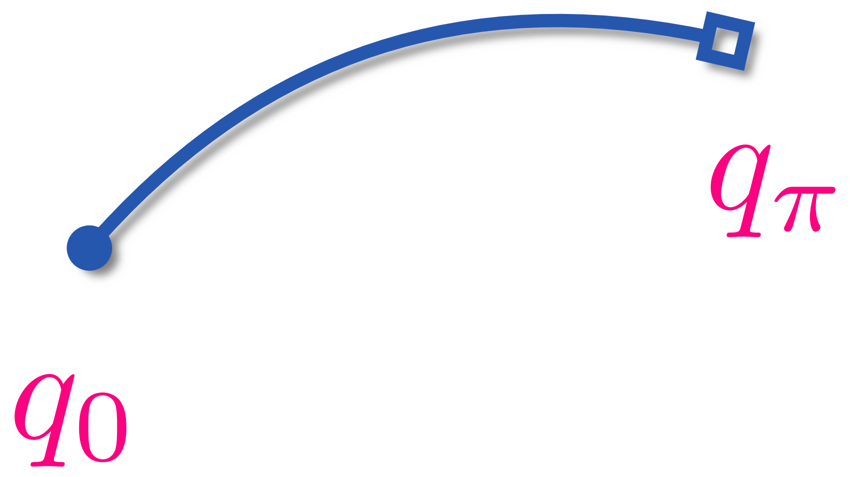

CFT
QFT
BV formalism
CFT
String Field Theory
Super-space

Open strings massive modes

Open string



Open string



Different oscillator modes $\longrightarrow$ Different Particles

Boundary charges $\longrightarrow$ $Q = q_0 \; + \; q_\pi$

First massive string

$$|\Psi>_{(N=2)} = B_m \, a_2^{m\dagger}|0> \; + h_{mn} \, a_1^{m\dagger} a_1^{n\dagger}|0>$$

Vector *Spin 2 state*

Put in an electromagnetic background

(Abouelsaood, Callan, Nappi, Yost '87)

Argyres-Nappi Lagrangian: Complicated

B_m is a Stückelberg field for h_{mn} $\Rightarrow$ ***Only*** h_{mn} remains

Argyres-Nappi Lagrangian for charged spin-2

$$\mathcal{H}_{mn} = (\eta_{mk} - \mathrm{i}\epsilon_{mk})(\eta_{nl} - \mathrm{i}\epsilon_{nl})\, h^{kl}$$

$$\boxed{\begin{aligned}\mathcal{L}_{AN} =& \bar{\mathcal{H}}_{mk}\mathfrak{D}^2 h_n{}^k - \bar{\mathcal{H}}\mathfrak{D}^2\mathcal{H} - \bar{\mathcal{H}}_{mn}\left\{\mathfrak{D}^m\mathfrak{D}^k\left[(1+\mathrm{i}\epsilon)\, h\right]_k{}^n - \frac{1}{2}\mathfrak{D}^m\mathfrak{D}^n\mathcal{H} + (m \leftrightarrow n)\right\} + \bar{\mathcal{H}}\mathfrak{D}^m\mathfrak{D}^n\mathcal{H}_{mn} \\ & - M^2\left(\bar{\mathcal{H}}_{mk}h_n{}^k - \bar{\mathcal{H}}\mathcal{H}\right) - 2i\bar{\mathcal{H}}_{mn}\left(\epsilon^{mk}h_k{}^n - h^m{}_k\epsilon^{kn}\right)\end{aligned}}$$

EOM's and constraints of the spin-2

$$\boxed{\begin{aligned}\left(\mathfrak{D}^2 - 2\right)\mathcal{H}_{mn} - 2\mathrm{i}\left(\epsilon_{km}\mathcal{H}^k{}_n + \epsilon_{kn}\mathcal{H}^k{}_m\right) &= 0 \\ \mathfrak{D}^n\mathcal{H}_{mn} &= 0 \\ \mathcal{H} &= 0\end{aligned}}$$

Where:

$$\epsilon = \frac{1}{\pi}(\mathrm{arctanh}(2\pi\alpha' q_0 F) + \mathrm{arctanh}(2\pi\alpha' q_\pi F)) \qquad \mathcal{M}\mathcal{M}^T = \frac{\varepsilon}{QF}, \qquad Q = q_0 + q_\pi \qquad -i\mathcal{M}\mathcal{D} \equiv -i\mathfrak{D},$$

Remarks about the Argyres-Nappi Lagrangian

- The A-N. Lagrangian $\longrightarrow$ Causal and solves the Velo-Zwanziger problem!

- Expanding in « powers of the electromagnetic field strength »:

No ghost the Federbush Lagrangian $\longrightarrow$ $-ie\ \bar{h}_{mn}\ F^{nk}\ h_k{}^m \Rightarrow g = 1/2$

Causality $\longrightarrow$ Argyres-Nappi Lagrangian $\longrightarrow$ $-i4e\ \bar{h}_{mn}\ F^{nk}\ h_k{}^m \Rightarrow g = 2$

Expansion of the kinetic term leads to an infinite series of terms !

- **BUT:** The A-N Lagrangian is consistent **ONLY in D=26** dimensions.

Spin 2 equations : Quite simple ?

$$[(D^2 - M^2)\eta_{mn} + \mathrm{i}(\varepsilon_{mn} - \varepsilon_{nm})]H^{mn} = 0$$

$$H^m_m = 0 \longleftarrow \text{Trace constraint}$$

$$D_m H^{mn} = 0 \longleftarrow \text{Divergence constraint}$$

These could (nearly) have been guessed :

- The equation of motion is the generalization of Proca equation in a electromagnetic field
- The constraint are those you can get from partial to covariant derivative

Difficult to guess the spin 3/2

Charged spin 3/2 case

Massive Spin 3/2 Electrodynamics

S. Deser ♯, V. Pascalutsa ♭ and A. Waldron ♯

♯ Physics Department, Brandeis University, Waltham, MA 02454, USA
deser,wally@brandeis.edu

♭ Department of Physics, Flinders University, Bedford Park, SA 5042, Australia
phvvp@flinders.edu.au

(August 6, 2018)

Introduction

Gauge interactions of massive (let alone massless) relativistic higher spin fields constitute an ancient and difficult subject. Whatever the formal problems these models encounter, *effective* higher spin theories must be constructible since approximately localised higher spin particles exist. Such models should achieve low energy consistency, and share some of the physical properties described by their lower spin hadronic physics counterparts.

Conclusion

Our study of causality showed that no model maintaining the correct DOF avoids sharing the pathology of the minimal one. In fact this result applies to

Neutral Massive spin 3/2

Fierz and Pauli system of equations for spin 3/2:

$$(i\gamma^\mu \partial_\mu - m)\psi_\nu = 0$$ ← Dirac equation for fermions

$$\gamma^\mu \psi_\mu = 0$$ ← Trace constraint

$$\partial^\mu \psi_\mu = 0$$ ← Divergence constraint

The trace constraint for charged spin 3/2

When the spin 3/2 is charged and propagates in a (constant) electromagnetic field, one try different Lagrangian and typically gets to a constraint of the from:

$$\gamma^\mu R^\nu_\mu \psi_\nu = 0$$

Now if the determinant $detR = 0$ then, ψ_0 , which is supposed to be a Lagrange multiplier, will propagate.

Ex: Deser-Pascalutsa-Waldron ‘00

This leads to superluminal propagation (acausality).

Violation of causality: use spin 3/2 states to send messages to your past!

"When have I met that girl?"

Impose the trace constraint

For the spin 3/2 is charged and propagates in a (constant) electromagnetic field, **one imposes**

$$\boxed{\gamma^m \Psi_m = 0}$$

This was proposed by Porrati - Rahman '09. They took an ansatz with a Lagrangian of a non-minimal form and try to see what would be the form of the Lagrangian and the corresponding equations of motion. The conclusion was:

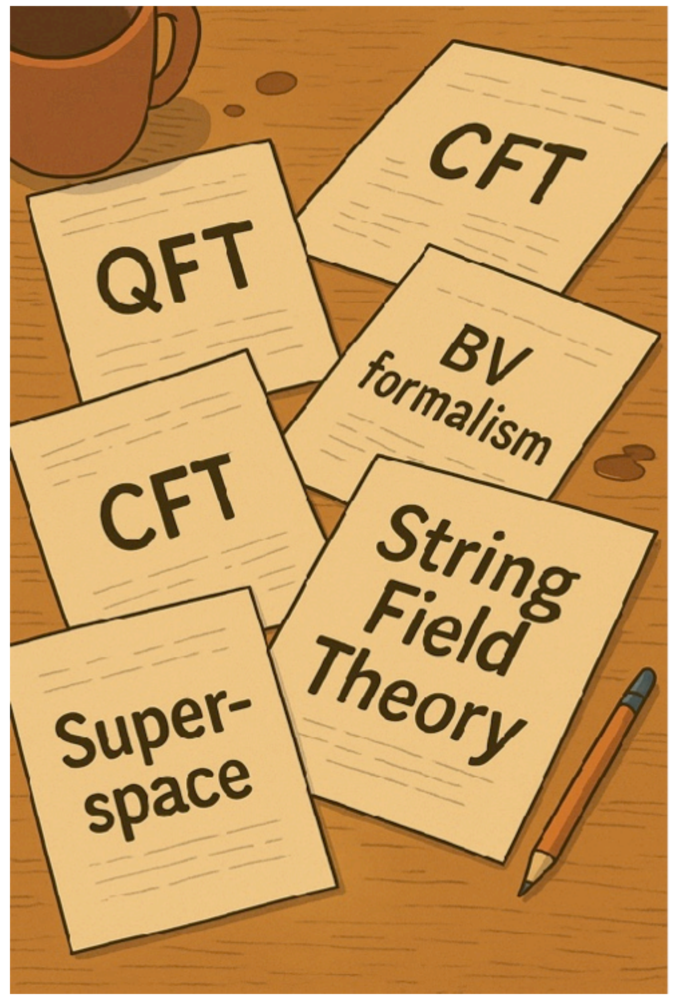
- There is a solution which is an infinite expansion in $\frac{e^2 F\tilde{F}}{m^4}$ $\frac{e^2 F^2}{m^4}$
- Some consistency condition giving the corrections of (n+1)th-order to the nth-order to preserve causality.
- The expansion in infinite in both the Lagrangian and the equations of motion

Spin 3/2 non-causal ?

Porrati and Rahman : Proof that a causal solution

can be obtained à priori

but with some **iterative implicit Lagrangian**



Explicit equations ?

[KB, N. Berkovits, C. Daniel, M. Lize '21]

Physical states

The string field Φ will describe on-shell:

12 bosonic d.o.f.

5 polarisations of massive spin-2

3 polarisations of massive spin-1

2 x 2 spin-0

12 fermionic d.o.f.

2 x Dirac massive spin-3/2

2 x Dirac spin-1/2

KB, Berkovits, Daniel, Matheus '21

Superspace Lagrangian

[KB, N. Berkovits, C. Daniel, M. Lize '21]

- The action of the BRST operator leads to (compact form):

$$
\begin{aligned}
S = -\frac{1}{16}\int d^4x\, p_0^2 \bar{p}_0^2 \Big\{ & V_n^\dagger (\eta^{nm} - i\epsilon^{nm}) \Big[-\left\{d_0^2, \bar{d}_0^2\right\} V_m + 16\Pi_0^n \Pi_{n0} V_m - 32\left(\eta_{mp} - i\epsilon_{mp}\right) V^p \\
& - 32\left(\left(\partial\bar{\theta}_0 \bar{d}_0\right) V_m + \left(\partial\theta_0 d_0\right) V_m\right) + 8\bar{\sigma}_m^{\dot{\alpha}\alpha}\left(d_{\alpha 0}\bar{U}_{2\dot{\alpha}} - \bar{d}_{\dot{\alpha}0} U_{1\alpha}\right) + 32\Pi_{m0} B \\
& + 24\bar{\sigma}_m^{\dot{\alpha}\alpha}\left[\bar{d}_{\dot{\alpha}0}, d_{\alpha 0}\right] C\Big] + U_2^\alpha \Big[-8\sigma^n_{\alpha\dot{\alpha}}\left(\eta_{nm} - i\epsilon_{nm}\right)\bar{d}_0^{\dot{\alpha}} V^m + 4\bar{d}_{\dot{\alpha}0} d_{\alpha 0}\bar{U}_2^{\dot{\alpha}} - 4\bar{d}_0^2 U_{1\alpha} \\
& + d_{\alpha 0}\bar{d}_0^2(-2iB + 18C) + \partial\theta_{\alpha 0}(-32iB - 96C) - 48i\Pi_{\alpha\dot{\alpha}0}\bar{d}_0^{\dot{\alpha}} C\Big] \\
& - \bar{U}_{1\dot{\alpha}}\Big[-8\bar{\sigma}^{n\dot{\alpha}\alpha}\left(\eta_{nm} - i\epsilon_{nm}\right) d_{\alpha 0} V^m + 4 d_0^2 \bar{U}_2^{\dot{\alpha}} - 4 d_0^\alpha \bar{d}_0^{\dot{\alpha}} U_{1\alpha} - \bar{d}_0^{\dot{\alpha}} d_0^2 (2iB + 18C) \\
& + \partial\bar{\theta}_0^{\dot{\alpha}}(-32iB + 96C) + 48i\Pi_0^{\dot{\alpha}\alpha} d_{\alpha 0} C\Big] + B^\dagger \Big[-32\Pi_0^n\left(\eta_{nm} - i\epsilon_{nm}\right) V^m \\
& + \left(\left\{d_0^2, \bar{d}_0^2\right\} - 64\right) B + 3i\left[d_0^2, \bar{d}_0^2\right] C - i\left(2d_0^2 \bar{d}_{\dot{\alpha}0} + 32\partial\bar{\theta}_{\dot{\alpha}0}\right)\bar{U}_2^{\dot{\alpha}} + i\left(2\bar{d}_0^2 d_0^\alpha + 32\partial\theta_0^\alpha\right) U_{1\alpha}\Big] \\
& + 3C^\dagger \Big[-8\bar{\sigma}^{n\dot{\alpha}\alpha}\left[d_{\alpha 0}, \bar{d}_{\dot{\alpha}0}\right]\left(\eta_{nm} - i\epsilon_{nm}\right) V^m - \left(6 d_0^\alpha \bar{d}_0^2 + 8i\Pi_0^{\dot{\alpha}\alpha}\bar{d}_{\dot{\alpha}0}\right) U_{1\alpha} \\
& - \left(6\bar{d}_{\dot{\alpha}0} d_0^2 + 8i\Pi_{\alpha\dot{\alpha}0} d_0^\alpha\right)\bar{U}_2^{\dot{\alpha}} - \left[d_0^2, \bar{d}_0^2\right] iB \\
& - \left(-11\left\{d_0^2, \bar{d}_0^2\right\} + 128\Pi_0^n\Pi_{n0} - 256\partial\bar{\theta}_{\dot{\alpha}0}\bar{d}_0^{\dot{\alpha}} - 256\partial\theta_0^\alpha d_{\alpha 0} - 64\right) C\Big]\Big\}
\end{aligned}
$$

Simplified $\longrightarrow$ **[KB, Wenqi Ke, C. Daniel '21]**

The charged massive spin 3/2 equation

g=2

$$(i\not{D} - M)\Psi_m = i\,\frac{e}{M}\,F_{mn}\,(1-\kappa\,\gamma_5)\,\Psi^n, \qquad \kappa = \pm 1$$

$$D^m\Psi_m = \frac{e}{2M}\left(F^{mn} - i\,\widetilde{F}^{\,mn}\gamma_5\right)\gamma_n\,\Psi_m.$$

Divergence constraint

$$\gamma^m\Psi_m = 0,$$

Trace constraint

KB-Berkovits-Daniel-Lize ’21,
KB-Daniel-Ke ‘22

Generalisation for arbirary spin >1

g=2

The bosonic fields of spin $s+1$ equations of motion and constraints:

$$\boxed{\begin{aligned} &\left(D^2 - M^2\right) h_{mn} = 2\mathrm{i}\epsilon_{k(m_1} h^k{}_{n_1 n_2, \cdots, n_s)} \\ &D^m h_{m n_1 n_2, \cdots, n_s} = 0, \\ &h^m{}_{m n_2, \cdots, n_s} = 0 \end{aligned}}$$

Divergence constraint

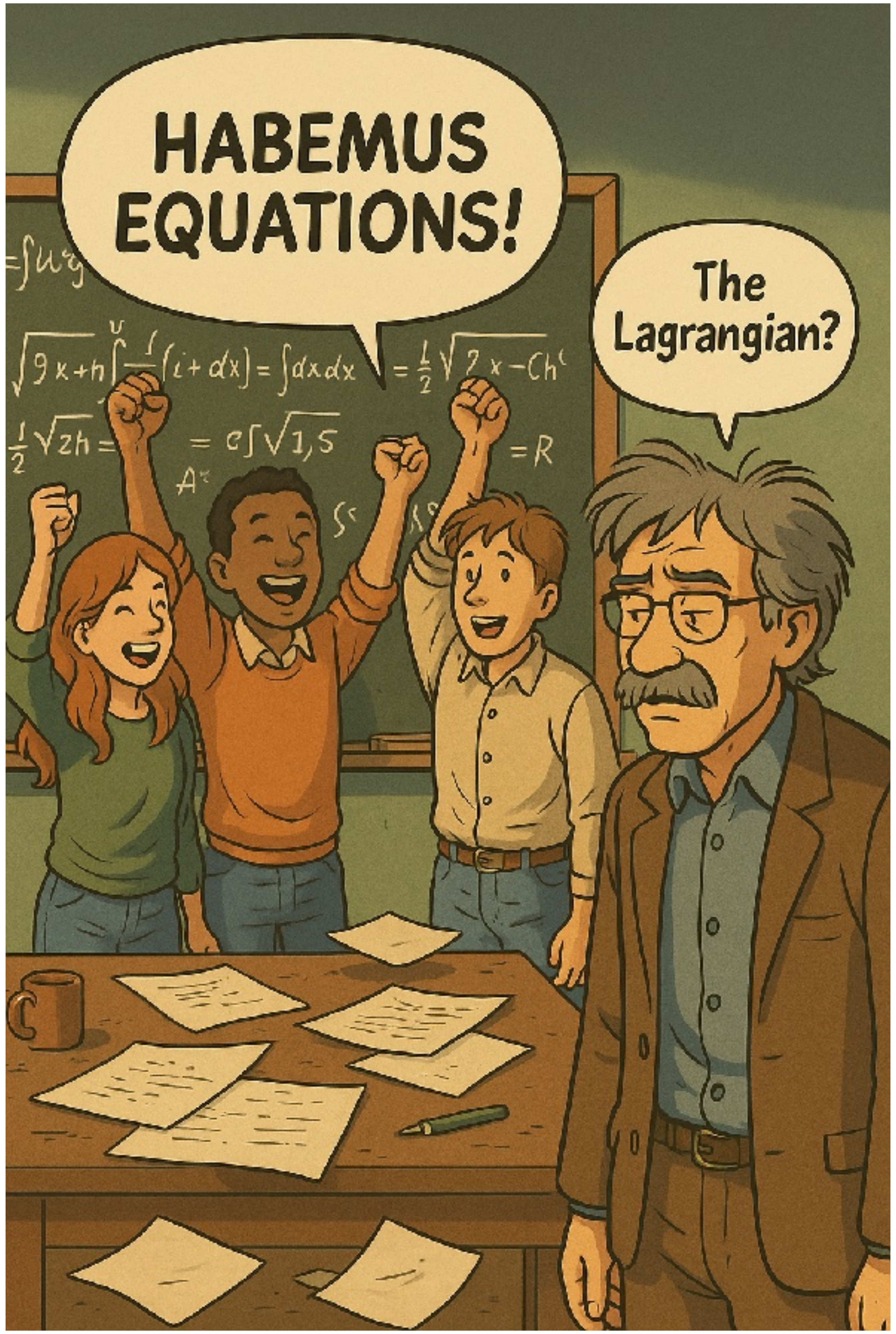
Trace constraint

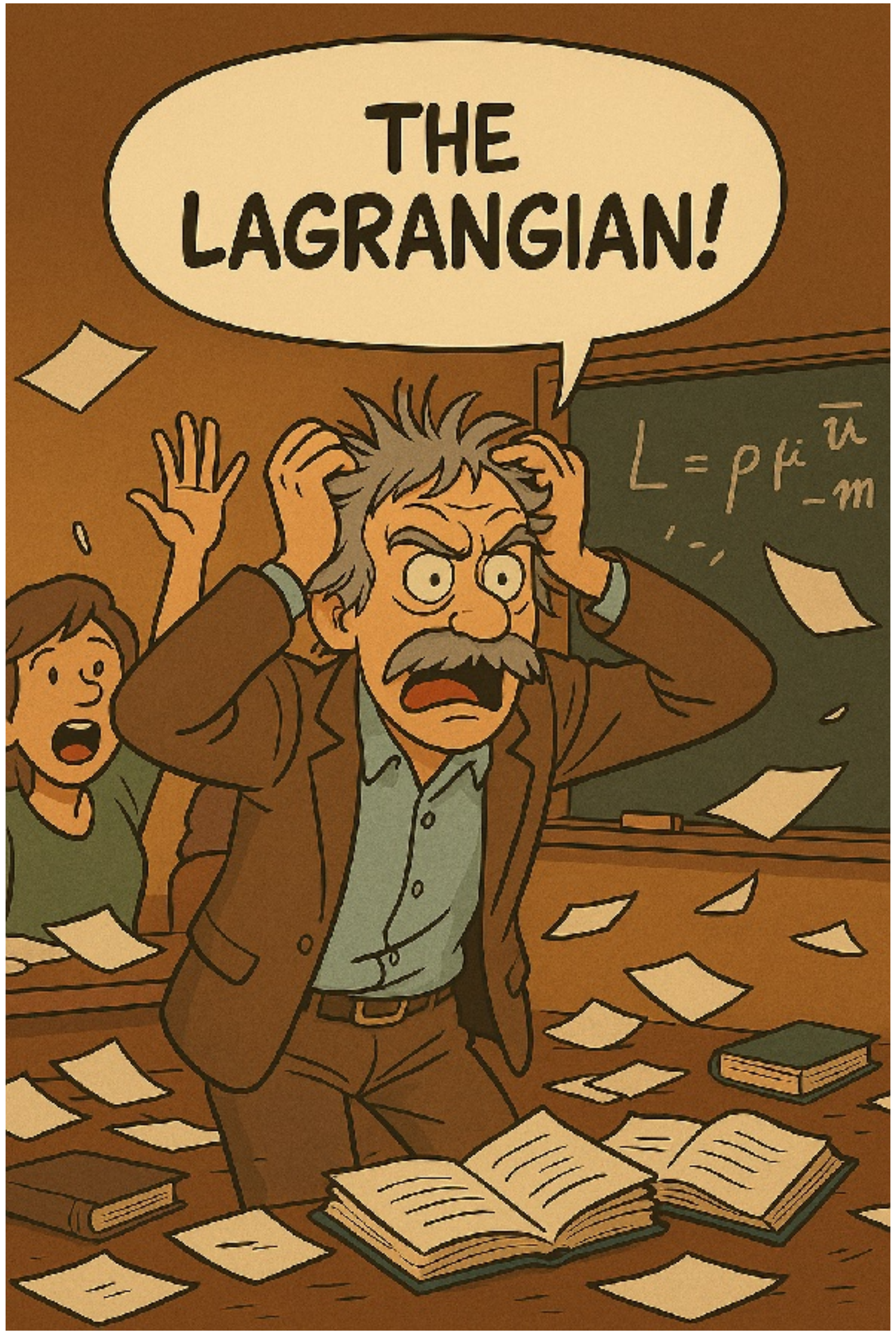
For the fermion of spin $s+1/2$, we get:

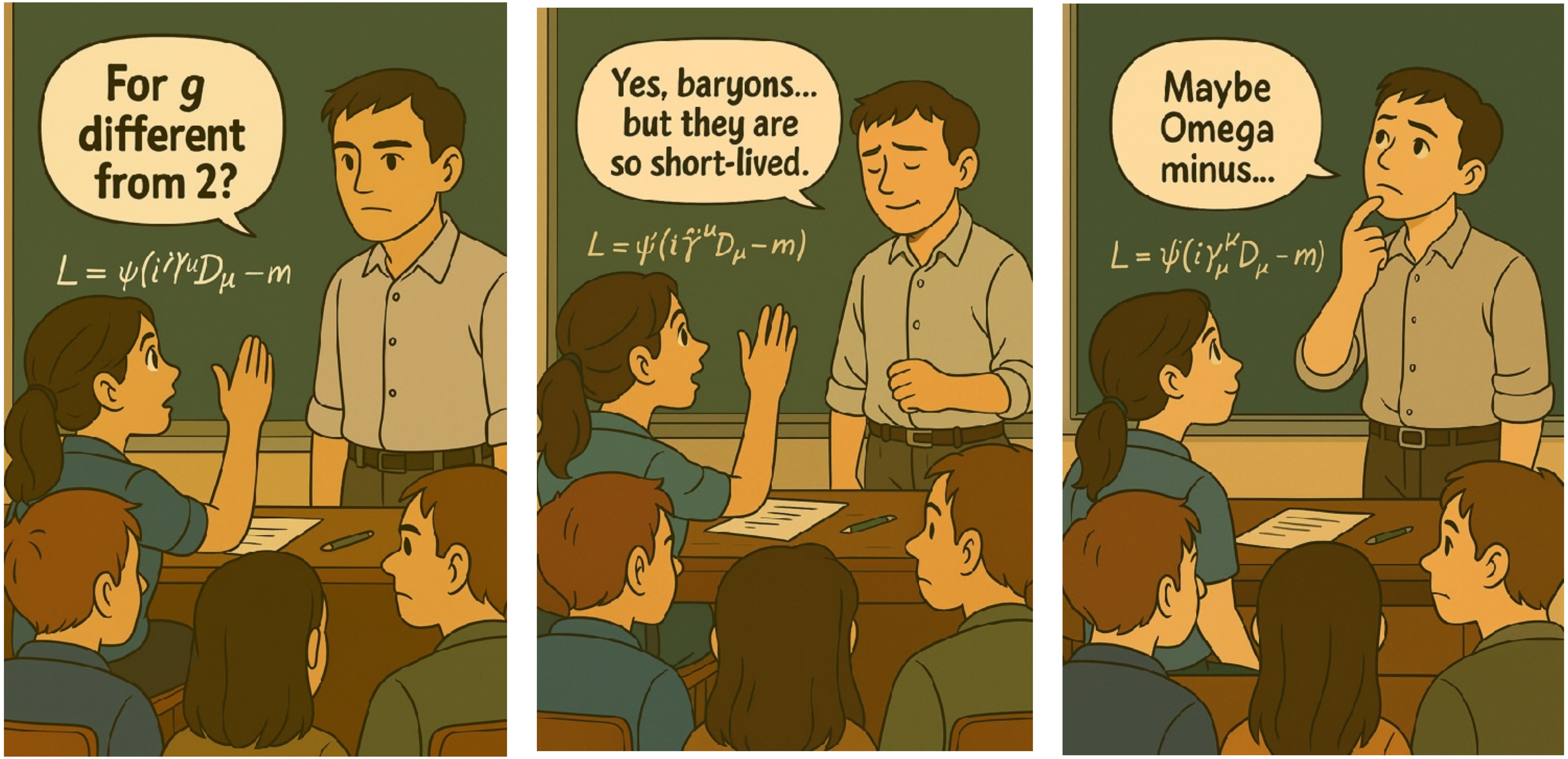
$$\boxed{\begin{aligned} &\left(\mathrm{i}\not{D} + M\right) \Psi_{n_1 n_2, \cdots, n_s} = -\mathrm{i}\frac{2}{M}\epsilon_{m(n_1} \Psi^{n_1}_{L n_2, \cdots, n_s)} \\ &\left[D^m + \frac{1}{2M}\left(\epsilon^{m(n_1} + \mathrm{i}\tilde{\epsilon}^{m(n_1}\right)\gamma_m\right] \Psi_{n_1 n_2, \cdots, n_s)} = 0 \\ &\gamma^m \Psi_{m n_2, \cdots, n_s} = 0 \end{aligned}}$$

Divergence constraint

Trace constraint


HABEMUS EQUATIONS!
The Lagrangian?


THE LAGRANGIAN!


For g different from 2?
Yes, baryons... but they are so short-lived.
Maybe Omega minus...

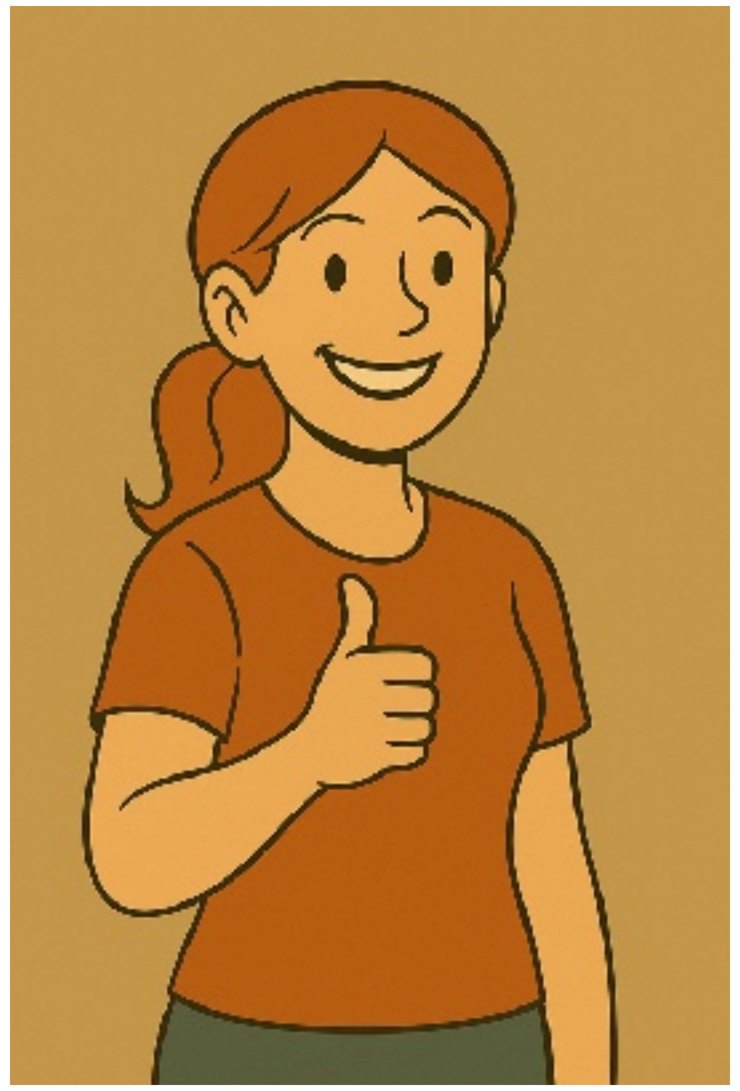
Case g$\neq$ 2

Why Study Equations of Motion for Spin > 1 ?

SM world physics

1. Fundamental & Mathematical Motivation:

- Higher–spin fields require nontrivial constraints (tracelessness, divergences, γ-trace…).
- **Interacting theories raise deep issues: causality, hyperbolicity, consistency.**
- A long-standing **theoretical frontier** in QFT.



2. Higher–Spin Baryons in the Standard Model

•Several baryons have **spin 3/2**, e.g. **Δ, Ω^-**.

•Their effective EoMs already show **sensitivity to gyromagnetic couplings**.

•Laboratory for studying **spin-3/2 consistency** in a real theory (QCD).

Why Study Equations of Motion for Spin > 1 ?

BSM world physics

🟩 3. Beyond the Standard Model: New Higher–Spin Particles

•BSM frameworks might have **spin > 1 states**.

(**Example:** charged **spin-3/2** dark-matter candidates (Meissner–Nicolai model)).

•These require **controlled, consistent** equations of motion to be viable.

🟧 4. Composite Higher–Spin States Usually Have g ≠ 2

•Unlike fundament elementary states, composites have **anomalous gyromagnetic ratios**.

•**Non-minimal EM couplings** can trigger consistency problems

•Understanding EoMs helps determine **phenomenological viability**.

Charged massive spin-2 with g≠2

EFT expansions

- Equation of motion:

$$\beta = 2(\gamma - 1)$$

$$\begin{aligned} E_{\mu\nu}[h;F] = (D^2 - M^2)h_{\mu\nu} &- 2ie\gamma\big(F_\mu{}^\rho h_{\rho\nu} - h_\mu{}^\rho F_{\rho\nu}\big) \\ &+ \frac{ie\beta}{M^2}\big(F_\mu{}^\rho D^2 h_{\rho\nu} + F_\nu{}^\rho D^2 h_{\rho\mu}\big) \\ &+ \frac{e^2}{M^2}\Big[c_1(\gamma)\big(F_{\mu\rho}F^{\rho\sigma}h_{\sigma\nu} + F_{\nu\rho}F^{\rho\sigma}h_{\mu\sigma}\big) + c_2(\gamma)\left(F_{\rho\sigma}F^{\rho\sigma}\right)h_{\mu\nu}\Big] + \mathcal{O}(F^3) = 0, \end{aligned}$$

with the constraints

$$h^\mu{}_\mu = 0, \qquad D^\mu h_{\mu\nu} \simeq 0$$

Conclusions

The true story is not much different…

Dirac indeed proposed to generalize the covariant derivative to all spins — an idea as bold as it was elegant.

Fierz and Pauli soon discovered that it doesn’t quite work: the equations become inconsistent. Proposed the Lagrangian route

Velo and Zwanziger found out that the Lagrangian “solution” violated causality.

Then came new attempts: alternative Lagrangians, supergravity, and string theory…

They solved part of the spin-2 case.

We solved the corresponding spin-3/2 case, and I’m now extending it to gyromagnetic ratios $\neq 2$.

Conclusion …


SPIN
MAGNETIC FIELD
ZEEMAN EFFECT
MAGNETIC FIELD? THEN CHANGE DERIVATIVES TO COVARIANT ONES!
IS IT THE RECIPE FOR ALL FIELDS?
THE STORY IS DIFFERENT... LOOK IN THE NEW TEXTBOOKS!

The hunt for a Lagrangian continues …

Thank you