Competitive Fierz constraints with electron capture

DUMENIL Victor & HAYEN Leendert

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Contents

Introduction & Context

Method

Results

Conclusion & Outlook

CKM matrix

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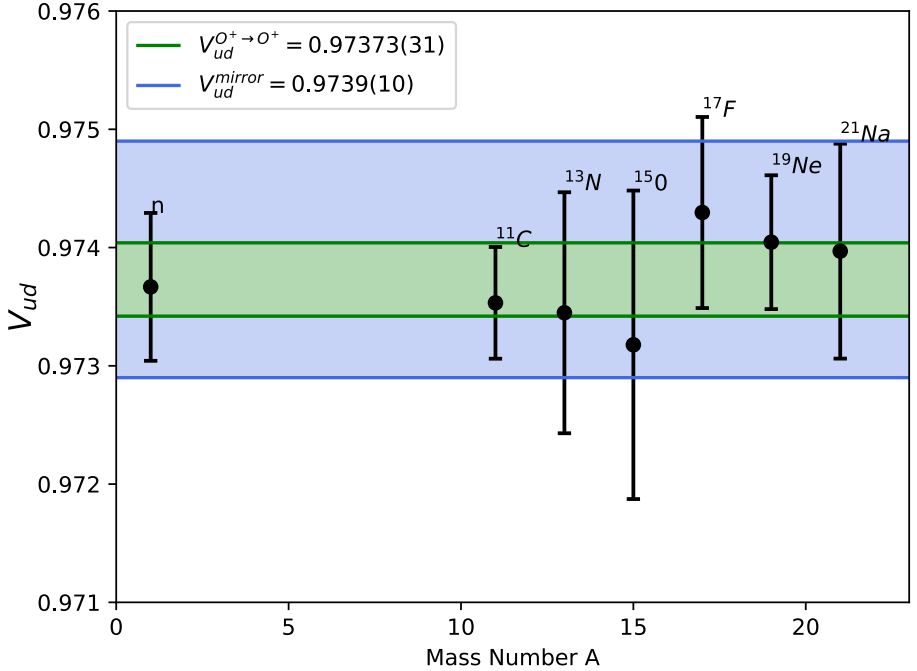
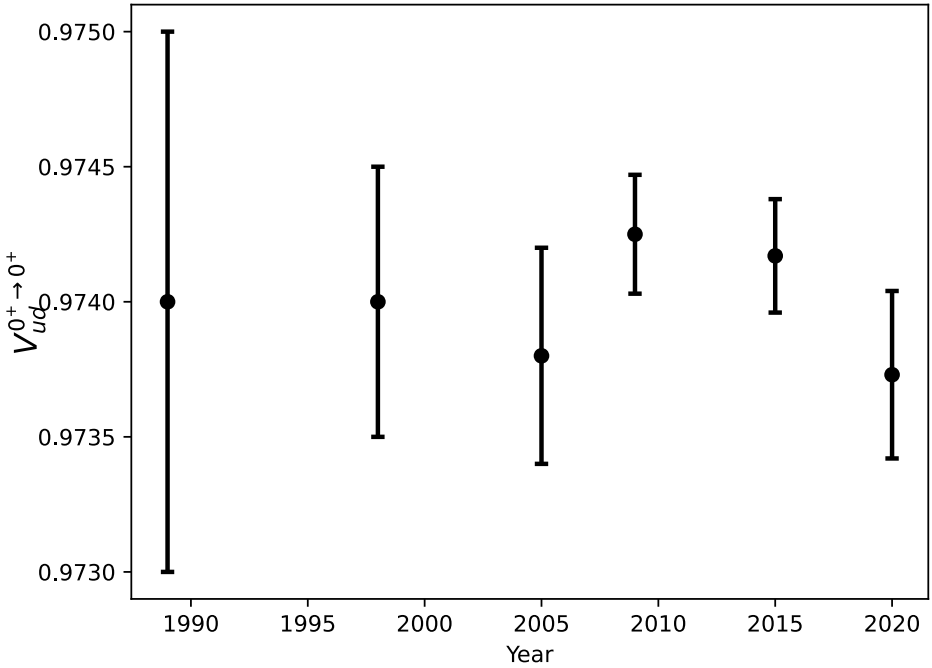
Nuclear β-decay, meson decay (π, K), $|V_{ub}|^2 \sim 10^{-5}$





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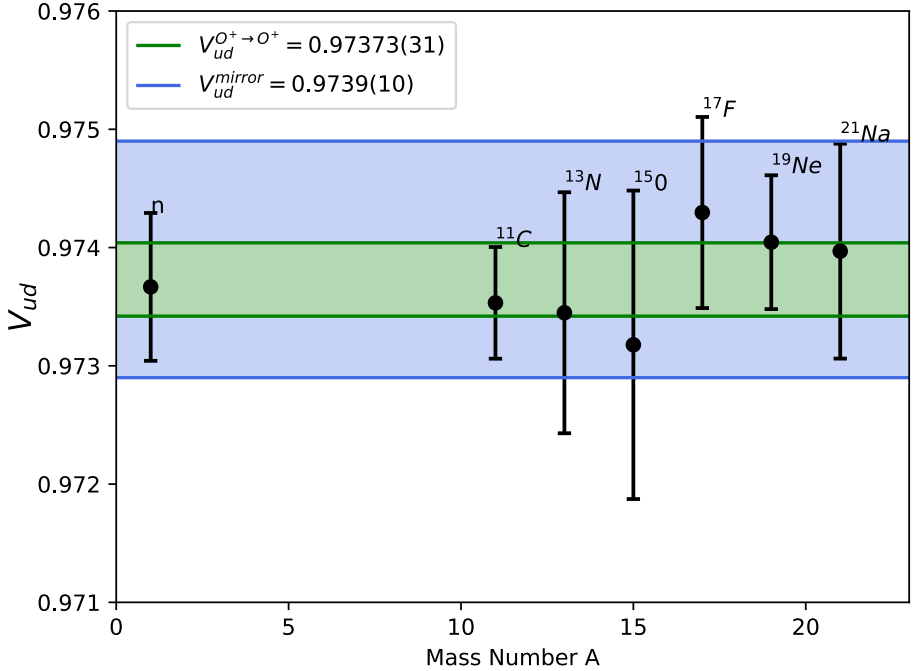
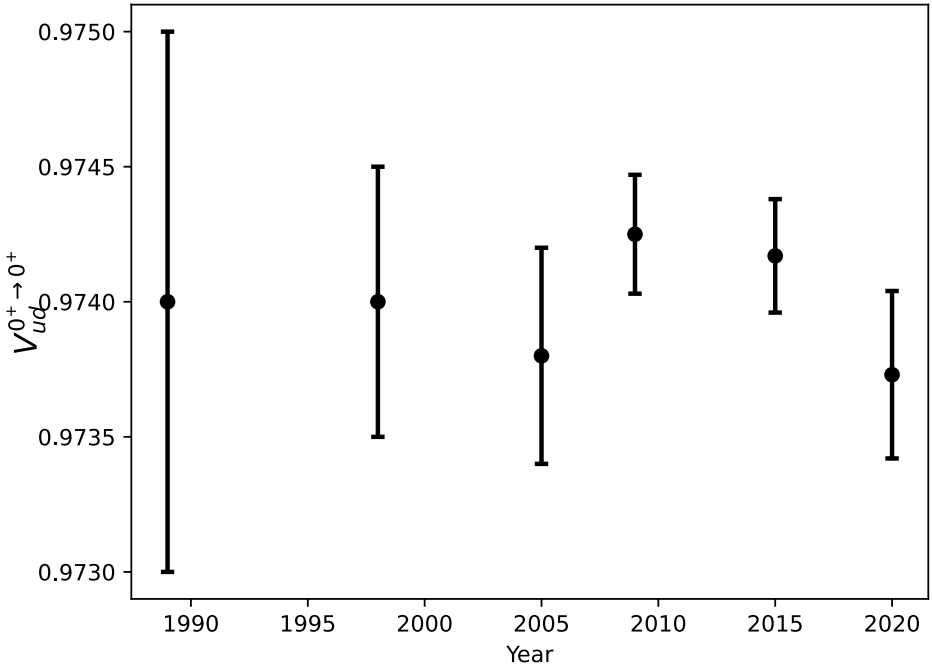
$$|V_{ud}|^2_{0+} + |V_{us}|^2 - 1 = -0.00166(69)$$

The non-unitarity could be explained by a new particle or **exotic currents (fierz term)**





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$$\left. -\epsilon_P \bar{e}\nu_L \cdot \bar{u}\gamma^5 d + \epsilon_T \bar{e}\sigma_{\mu\nu}\nu_L \cdot \bar{u}\sigma^{\mu\nu}(1-\gamma^5)d \right\}$$

at quark level

$V - A$ structure in SM

ϵ_i are proportional to $(M_W/\Lambda_{BSM})^2$ → $\epsilon_i \leq 10^{-4}$ → $\mathbf{\Lambda_{BSM}} \geq$ **15 TeV** assuming natural couplings

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Fierz term : - Interference between the Standard Model & Beyond Standard Model
- Linear dependence on exotic scalar & tensor currents

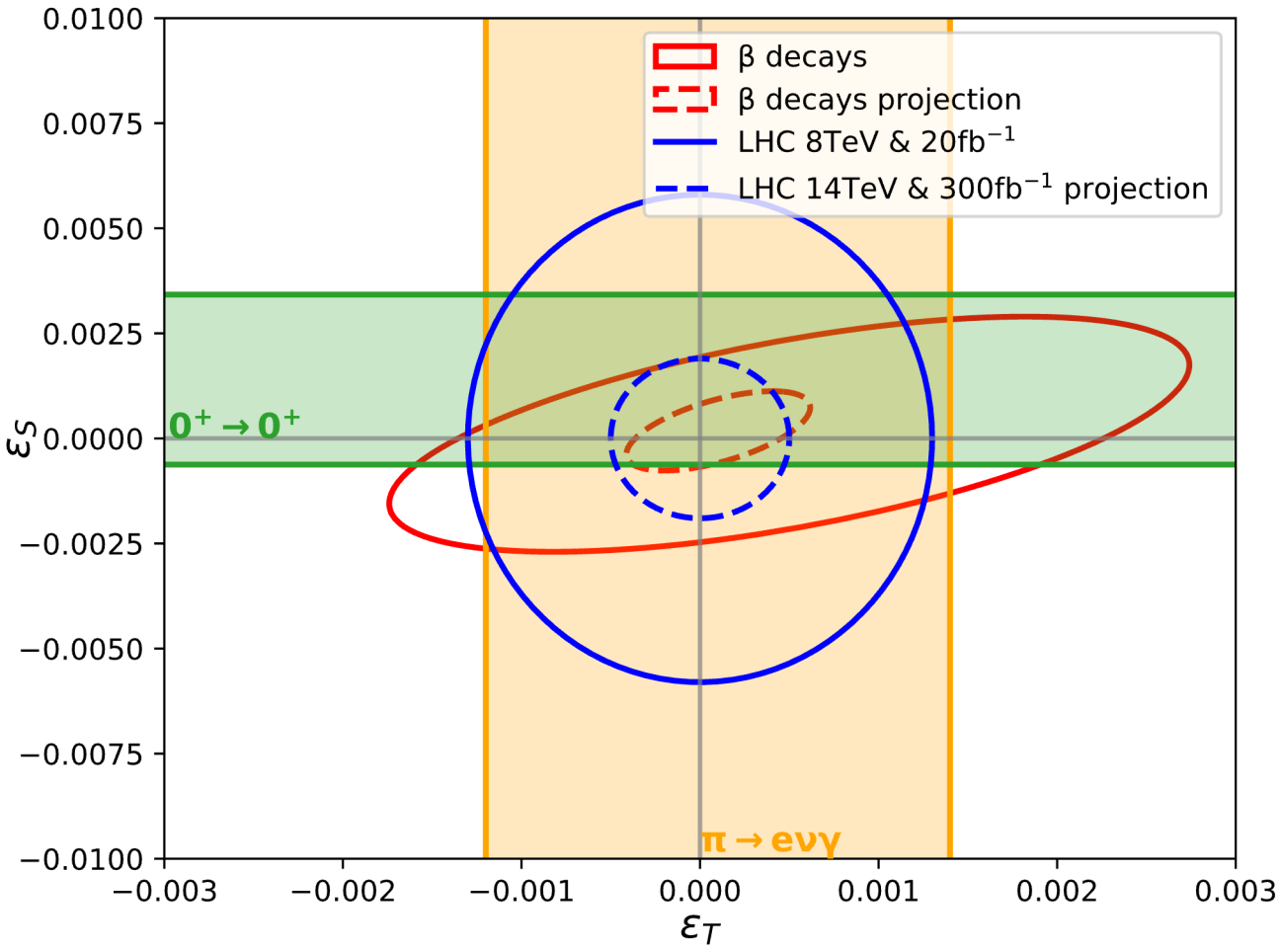
$$b_F = \pm \frac{2\gamma}{1+|\rho|^2} \mathrm{Re}\left(\frac{\epsilon_S g_S}{g_V} + |\rho|^2 \frac{8\epsilon_T g_T}{-2g_A} \right) \rightarrow 0 \text{ in SM}$$

bSTILED : Precision measurement of β-spectrum shape (See R.Garreau talk)

SALER/ASGARD : Measurement of **EC/β+ ratio**

Exotic current constraints

→ Nuclear β-decay is **competitive** with the LHC in the search for exotic currents.



A.Falkowski et al, 2021
M.Gonzalez-Alonso et al, 2019

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EC/β+ branching ratio : **opposite sign** dependence on b_F & probes the **same nuclear matrix elements**
→ **extremely sensitive to new physics**

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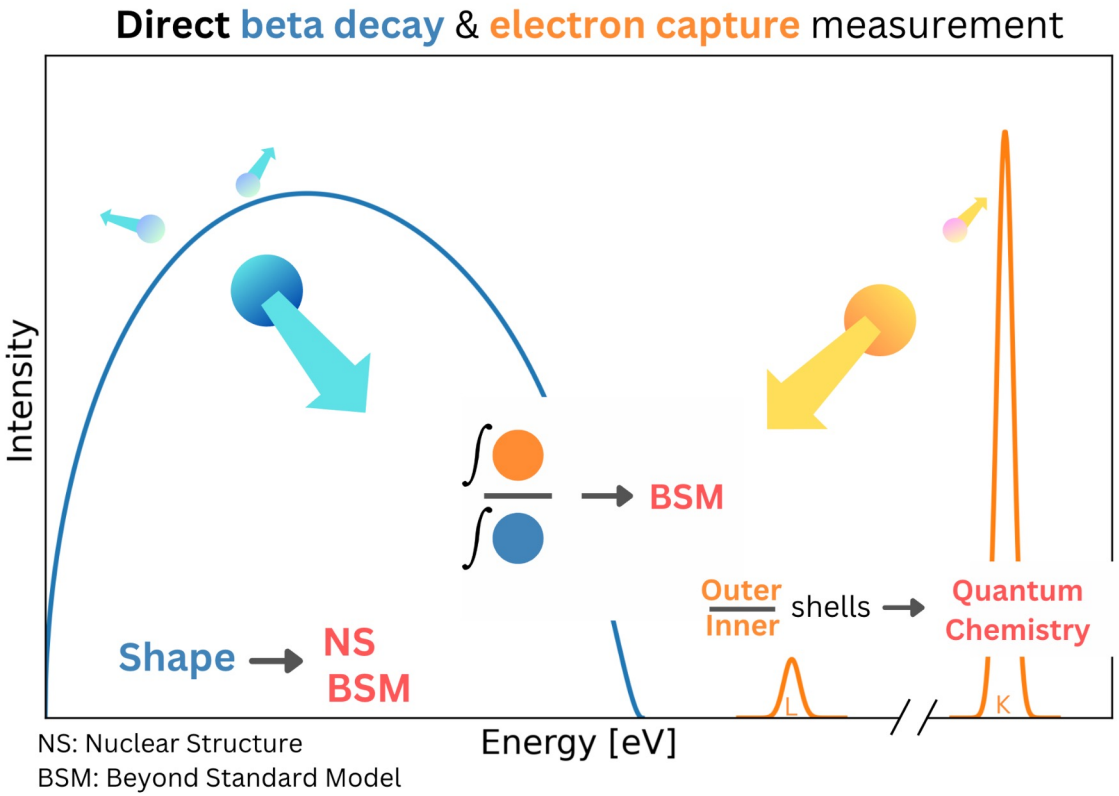

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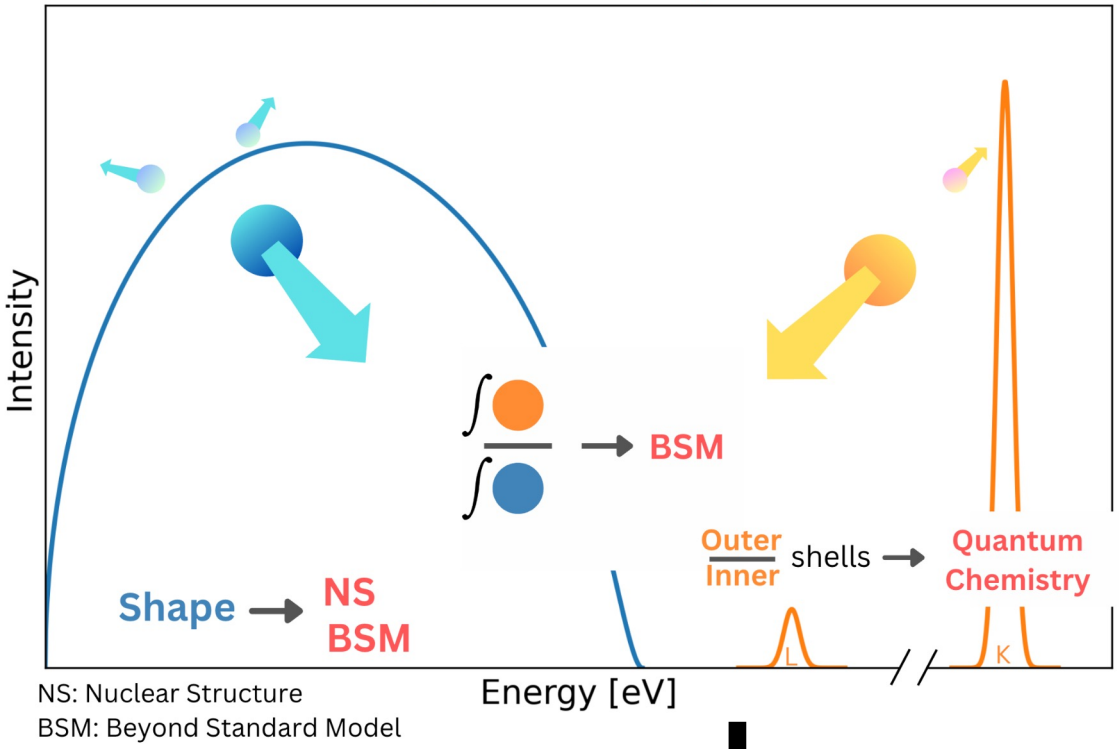
Direct beta decay & electron capture measurement



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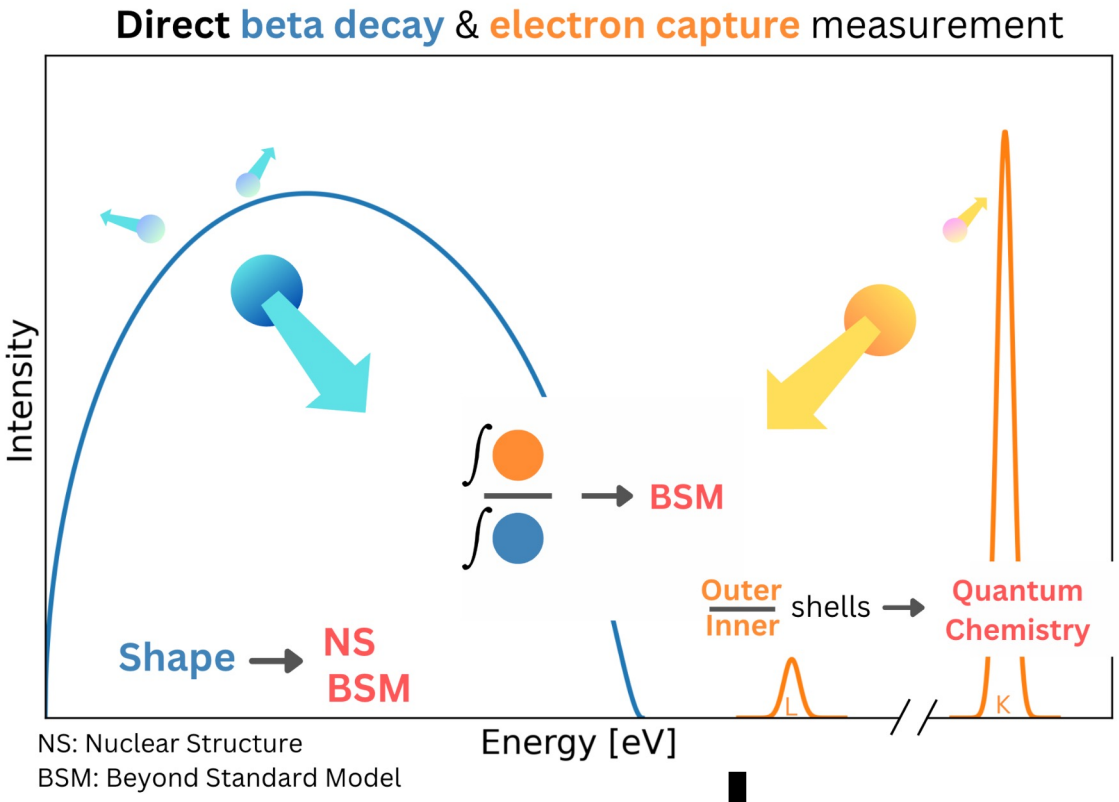

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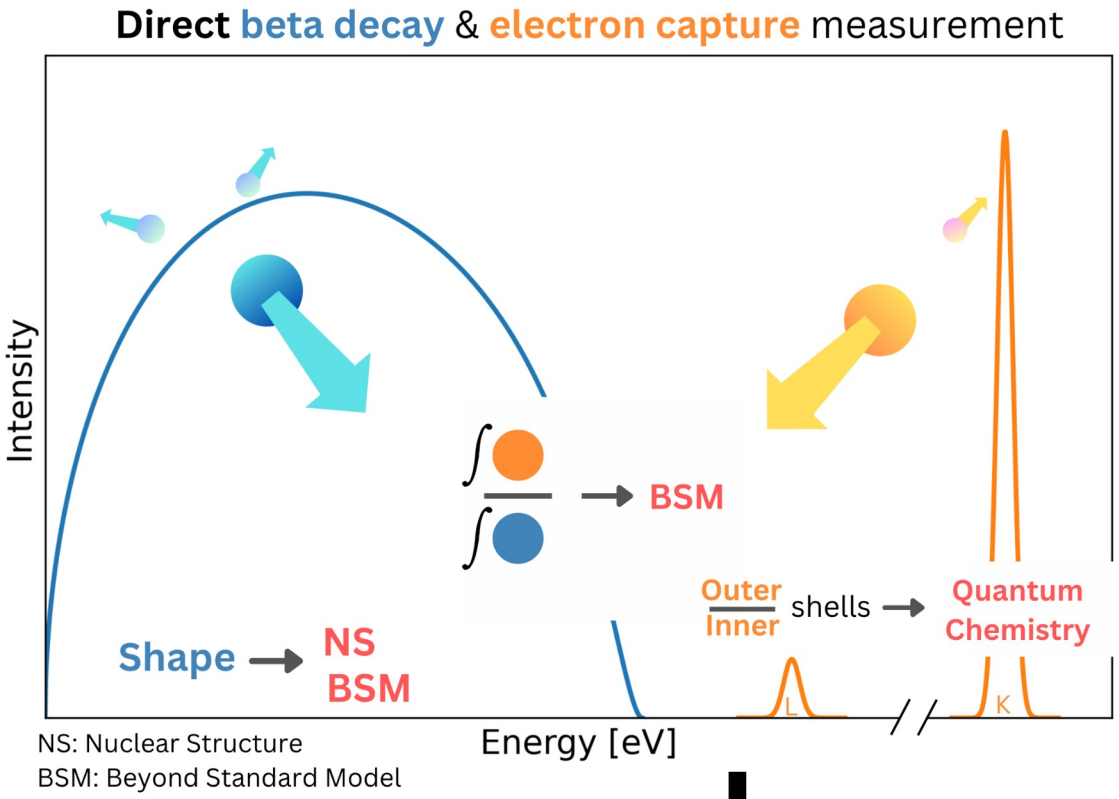

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How to access **nuclear structures** (and δ_{NS}) ?

Branching ratio calculation

*) L.Hayen et al., Rev. Mod. Phys. 90, 015008 (2018)
**) W.Bambynek et al., Rev. Mod. Phys. 49, 77 (1977)

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Shape factor
Phase space
Fermi function
Other corrections (from thecobs/BSG *)

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Shape factor
Neutrino momentum
Amplitude of wave function **
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The shape factor contains all the **nuclear structures information** and will be calculated with **H.Behrens & W.Bühring formalism**

$\boldsymbol{\delta_{NS}}$ can be easily identified by $\delta_{NS} = \frac{C_x}{C_{\beta^+}} - 1$

H.Behrens & W.Bühring formalism

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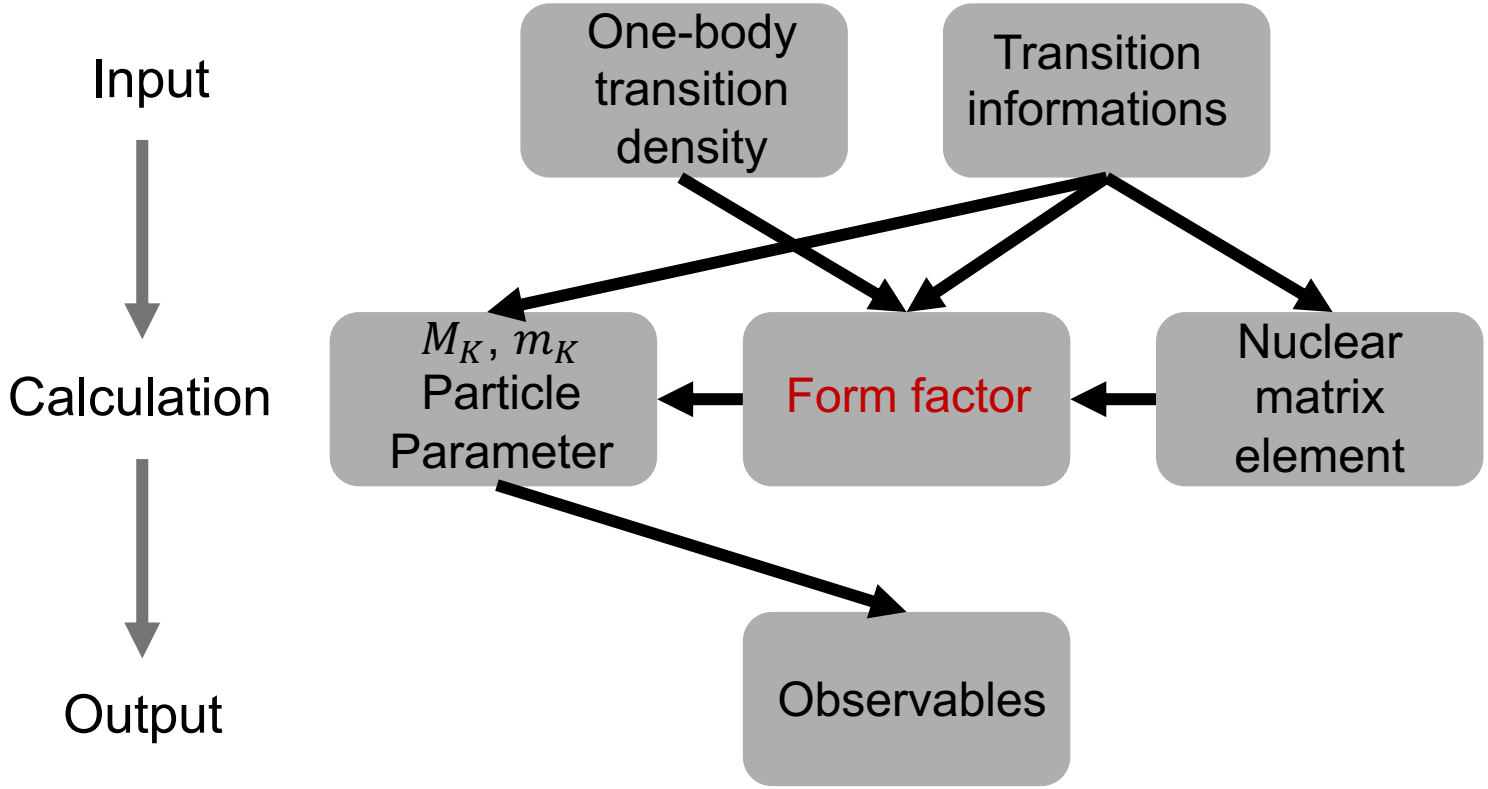
One-body transition density

Behrens & Bühring form factor notation $F_{KLs}^N(k_e, m, n, \rho) = \sum \mathcal{M}_{KLs}^N(k_e, m, n, \rho) \times \left\langle \zeta_f J_f || [c_\alpha^\dagger c_\beta]_\lambda || \zeta_i J_i \right\rangle$

Relativistic electron parameter

Code to reproduce Behrens & Bühring formalism



Possible obervables :

- $\lambda_{EC}/\lambda_{\beta+}$ **ratio**
- Shape factor $C(Z, E)$
- β-spectrum, half-life
- Testing the CVC hypothesis
- g_A quenching
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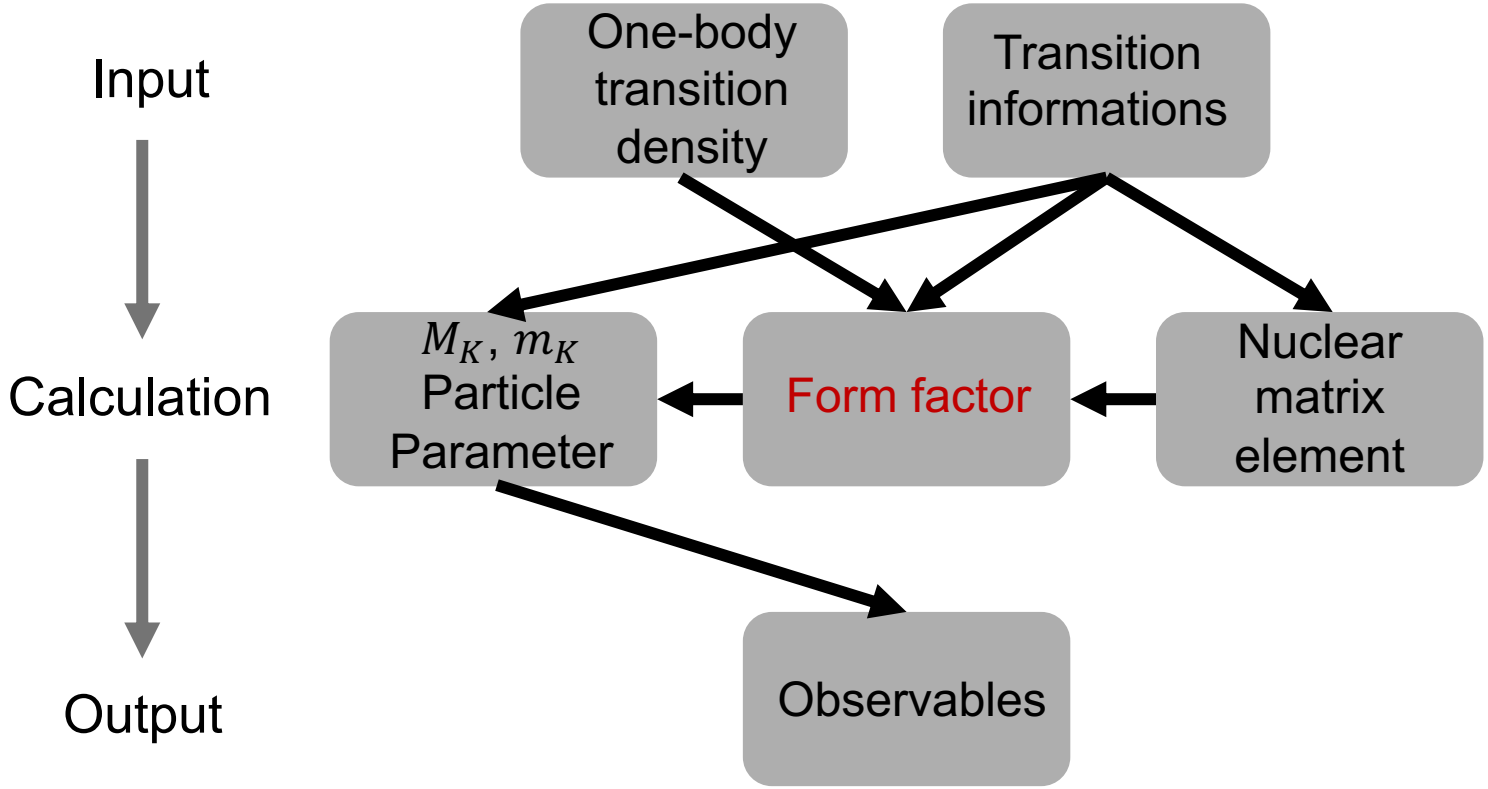

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→ All observables are expressed in terms of form factors for ANY type of transitions

Result for b_F

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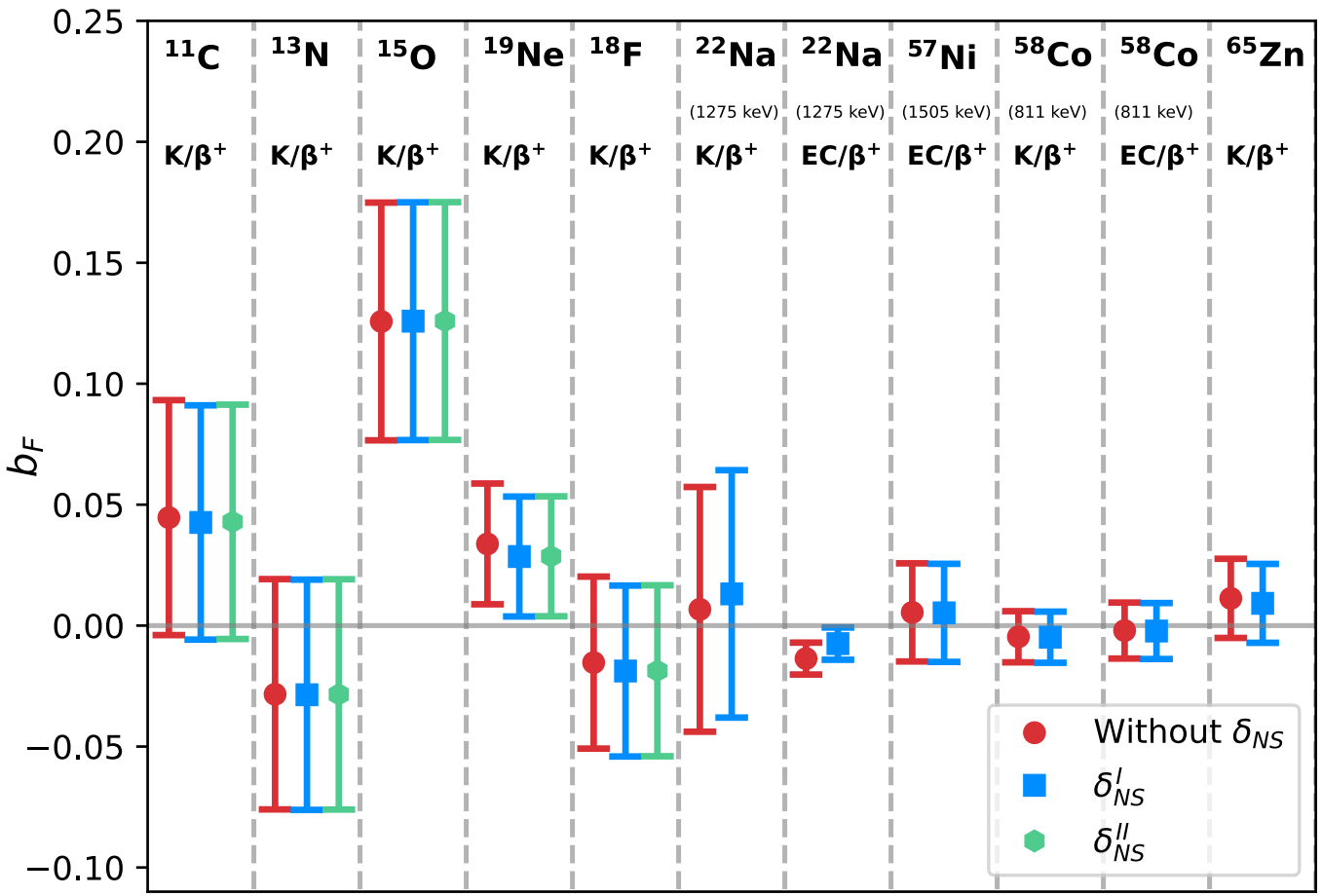
→ **First time** that b_F extraction is done with this 'new way' with 'old' data.



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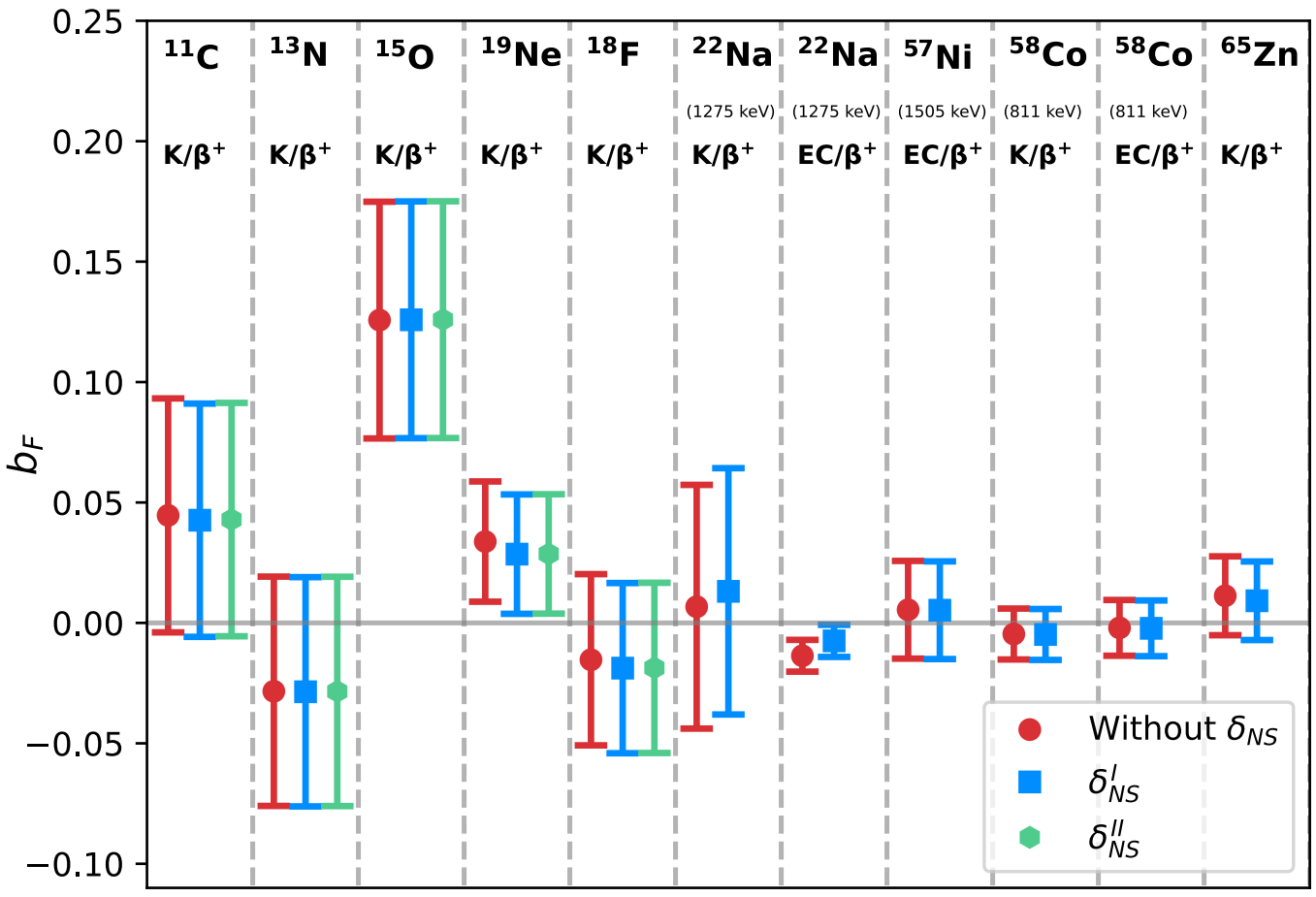
In [2] the experimental uncertainties are larger than [1].
And in [1] the overlap & exchange (atomic) corrections are better estimated.



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Extraction of ε_T by using the most recent contraint on ε_S and g_i from FLAG2024*

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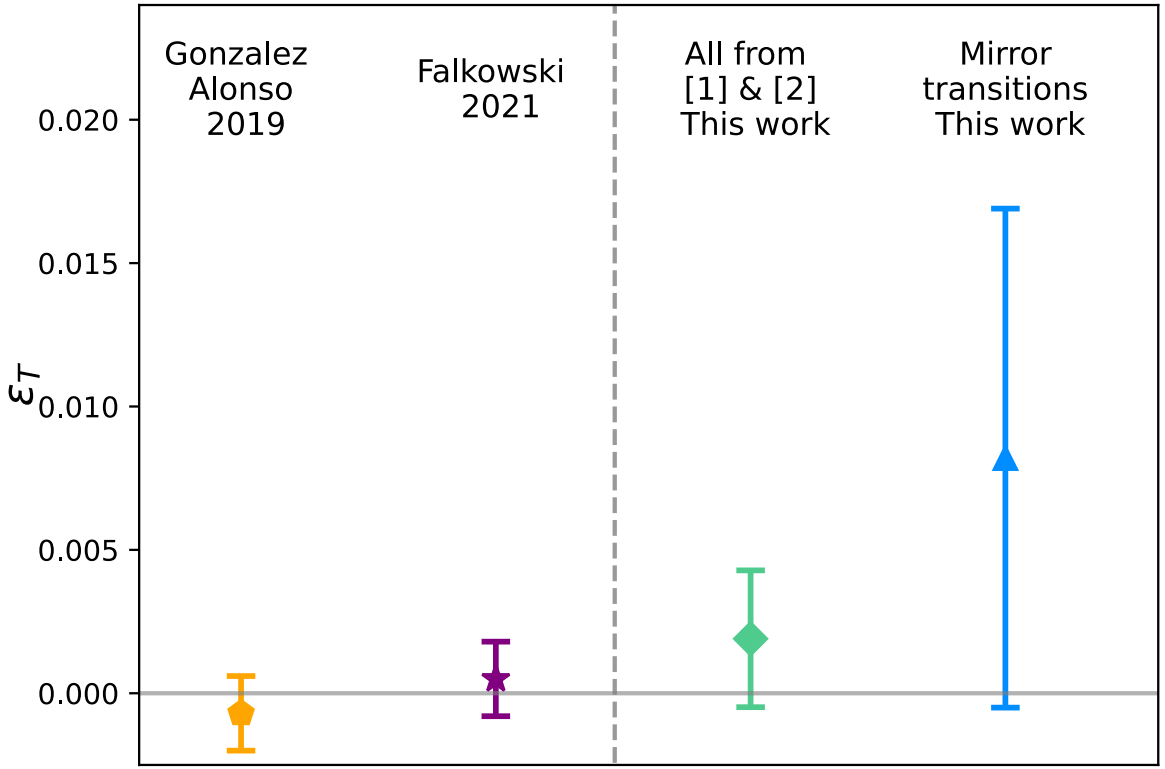

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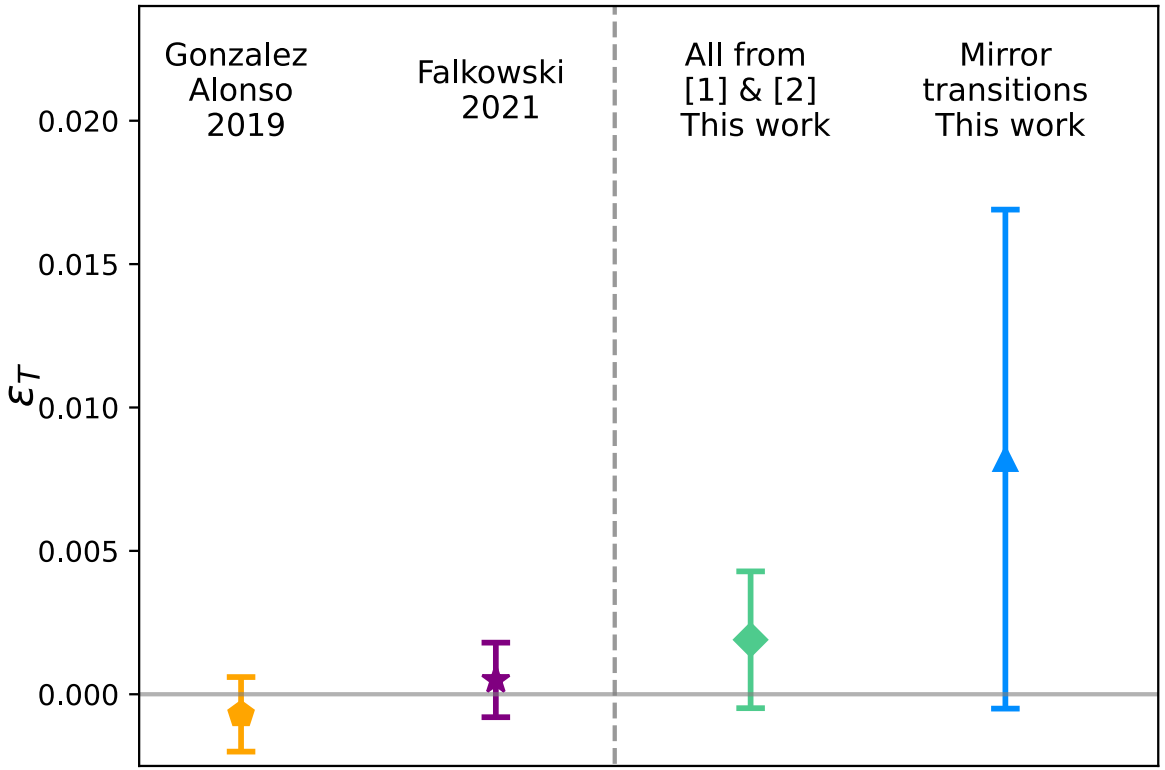

$\boldsymbol{\varepsilon_T^{All} = 0.0019(24)}$ (90%CL)

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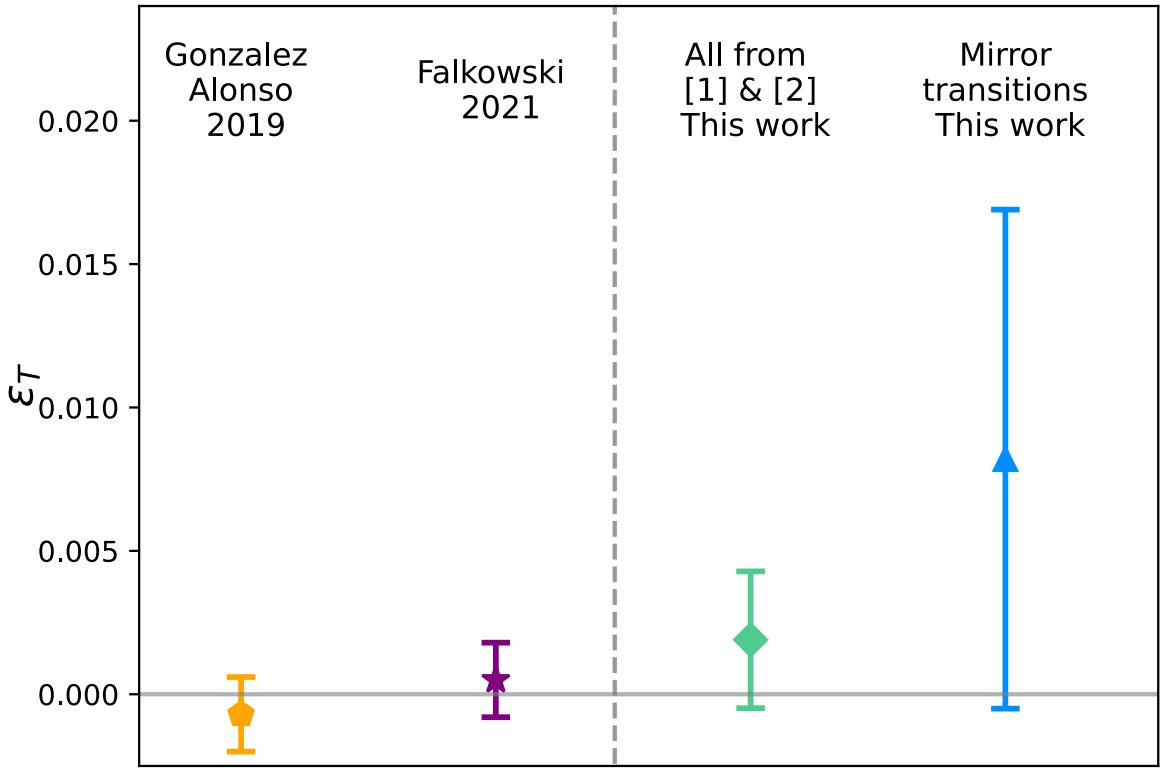

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Outlook

- **ab initio calculations** to appropriately estimate nuclear structure uncertainties in a controlled fashion.
- **add condensed matter theory** to the Behrens & Bühring formalism to take account of superconducting effects.
- Improvement in the estimation of **atomic effects and uncertainties.**
- SALER/ASGARD will reduce the experimental uncertainties.
- Same study for **first forbidden transitions.**

Appendices

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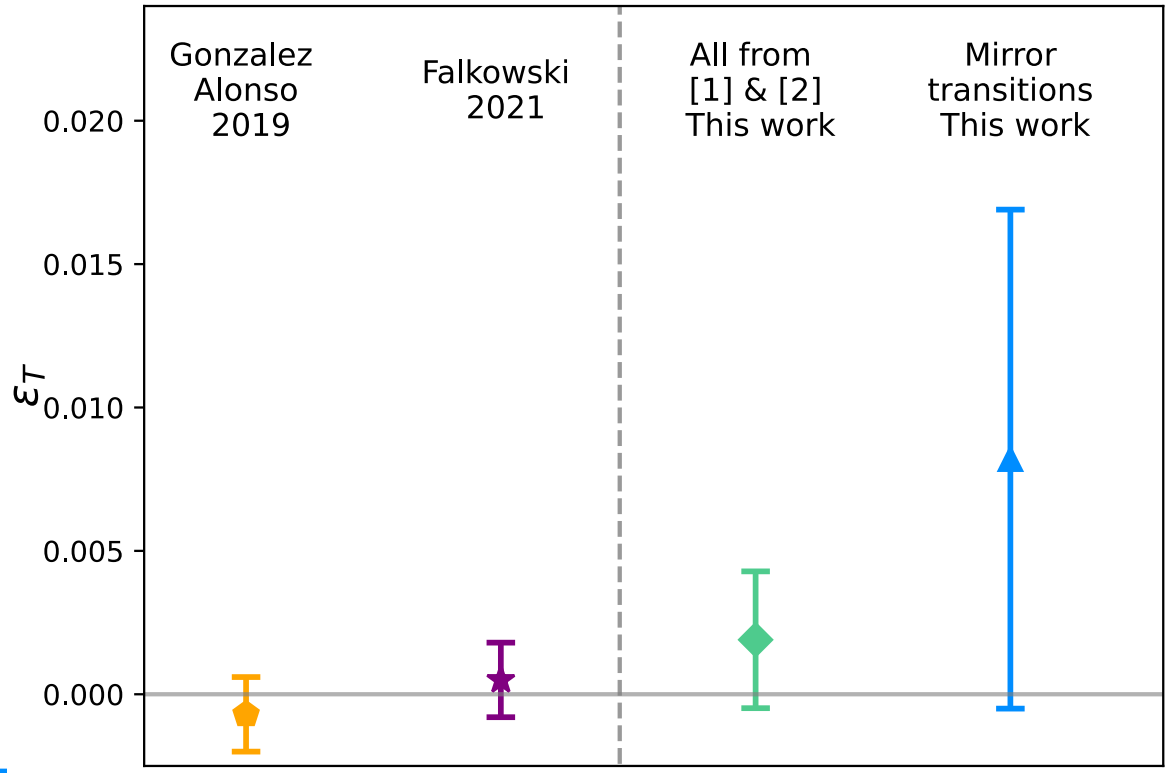

Theoretical uncertainties

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2 different approaches

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Nuclear structure table

[1] X. Mougeot, Applied Radiation and Isotopes, 2019
[2] W.Bambynek et al., Rev. Mod. Phys. 49, 77 (1977)

Isotope	ρ_{exp}	ρ^{I}_{th}	ρ^{II}_{th}	$\delta^{exp}_{NS}(\%)$	$\delta^{I}_{NS}(\%)$	$\delta^{II}_{NS}(\%)$	$(\lambda_{EC}/\lambda_{\beta^+})_{exp}$	$(\lambda_{EC}/\lambda_{\beta^+})_{th}$
$^{11}C^{a,c}$	-0.7544(8)	-0.7728	-0.6927	0.23	0.30	0.26	0.00225(15)	0.00211(6)
$^{22}Na^{b,c}$	-	-	-	-2.0	-1.08	0.32	0.1083(9)	0.1097(9)
[1] $^{57}Ni^{d}$	-	-	-	-	0.03	-	1.460(47)	1.447(11)
$^{58}Co^{d}$	-	-	-	-	0.03	-	5.61(8)	5.63(8)
$^{65}Zn^{e}$	-	-	-	-	0.37	-	30.1(5)	29.6(7)
$^{13}N^{a,c}$	-0.560(1)	-0.5480	-0.5077	0.01	0.03	0.01	0.00168(12)	0.00176(5)
$^{15}O^{a,c}$	0.630(2)	0.5555	0.5185	0.01	0.02	-0.02	0.00107(6)	0.00091(2)
[2] $^{17}F^{b,c}$	1.296(1)	1.1572	1.1376	0.48	0.66	0.64	-	0.00135(2)
$^{18}F^{b,c}$	-	-	-	-	0.61	0.59	0.030(18)	0.0310(5)
$^{19}Ne^{b,c}$	-1.602(1)	-1.6338	-1.6197	0.50	0.70	0.69	0.00096(3)	0.00093(1)

	CKPOT^{a}	USDB^{b}	YSOX^{c}	HO^{d}	JUN45^{e}
Model Space	$1p_{3/2}$, $1p_{1/2}$	$1s_{1/2}$, $1d_{5/2}$ $1d_{3/2}$	$1s_{1/2}$, $1p_{3/2}$, $1p_{1/2}$ $1d_{5/2}$, $1d_{3/2}$	N: $1f_{5/2}$, $2p_{3/2}$, $2p_{1/2}$ P: $1f_{7/2}$	$1f_{7/2}$, $1g_{9/2}$ $2p_{3/2}$, $2p_{3/2}$

W.Bambynek et al., Rev. Mod. Phys. 49, 77 (1977)

Experimental Method

Method	Source	Detectors	Estimated accuracy (%)	Nucleus
Spectroscopy of K events and positrons (no K x-ray escape)	Internal gaseous	pc	6	18F, 58Co(K/B+)
Spectroscopy of K events and positrons (no K x-ray escape)	Internal gaseous	apc	3	11C, 13N, 15O, 19Ne
Spectroscopy of K events and positrons	Internal solid	NaI(Tl)	2	22Na(K/B+), 58Co(K/B+)
Spectroscopy of K Auger electrons and positrons	External solid	NaI(Tl), Si(Li), pc	1	58Co(K/B+)
Measurement of positron-gamma ray coincidences	External solid	Pc, pl, NaI(Tl), Ge(Li)	2.5	22Na(EC/B+), 57Ni, 58Co(EC/B+), 65Zn
Measurement of positron-gamma ray N and positron-gamma ray S coincidences	External solid	Pc, NaI(Tl)	0.3	22Na(EC/B+)
Measurement of gamma ray-(511 keV) beta+ annihilation photon coincidences	External solid	NaI(Tl), Ge(Li)	3	58Co(K/B+)

pc : proportional counter / apc : anticoincidence proportional counter

Experimental Method

Nucleus	References
11C, 13N, 15O, 18F, 19Ne	Bambynek et al 1977
22Na	+ Galan 2009
57Ni	+ Konijin 1958 + Bakhru and Preiss1967
58Co	+ Bé 2016
65Zn	+ Bé et al 2005

Shell Model interactions

CKPOT

A : 8 - 16
Model space : $1p_{3/2}$, $1p_{1/2}$
Inert core : $^{4}_{2}He$
Characteristic : Obtained by adjustment
Coulomb effect : estimated from the observed energy differences of mirror nuclei
Isospin : non-relativistic charge-independent form

YSOX

A : 8 - 30
Model space : $1p_{3/2}$, $1p_{1/2}$, $2s_{1/2}$, $1d_{5/2}$, $1d_{3/2}$
Inert core : $^{4}_{2}He$
Characteristic :Adjust macroscopically, Based on universal monopole interaction (VMU), SFO and SDPF-M
Coulomb effect : like CKPOT
Isospin : Assumed

USDB

A : 16 -
Model space : $1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$
Inert core : $^{16}_{8}O$
Characteristic : Empirical, based on a renormalised G matrix (from the NN potential)
Coulomb effect : Obtained from isobaric analogue states
Isospin : Assumed

HO

A : 48 -
Model space : $2p_{3/2}$, $2p_{1/2}$, $1f_{5/2}$
Inert core : $^{48}_{20}Ca$
Characteristic : Proton-neutron interactions are determined by fitting to the observed spectra
Coulomb effect :
Isospin : Assumed

JUN45

A : 56 -
Model space : $2p_{3/2}$, $2p_{1/2}$, $1f_{5/2}$, $1g_{9/2}$
Inert core : $^{56}_{28}Ni$
Characteristic : Empirical, derived from realistic interaction (Bonn-C potential)
Coulomb effect : via Cole formula
Isospin : Assumed

Nuclear structure uncertainties

$$\delta_{NS} = \delta_{NS}^{exp} + \delta_{NS}^{theo}$$

δ_{NS}^{exp} is calculed with the following from factors

$$^{V}F_{000}^{0}(M_F), ^{A}F_{101}^{0}(M_{GT}), ^{V}F_{111}^{0}$$

Typically for allowed decays the 3 predominant form factors
And are measured experimentally

δ_{NS}^{theo} is calculed theoretically with all other form factors
δ_{NS}^{theo} can be considered as $\delta_{\delta_{NS}^{theo}}$
+ other shell model interactions when it is possible

Radiative corrections in EC/B+ ratio

$$\delta_{\mathrm{RC}}^{\mathrm{EC}} = \frac{\alpha}{2\pi}\left(3\ln\frac{m_p}{m_e} - \frac{27}{4}\right) + \frac{\alpha}{12\pi}(W_0 + 1)^2$$

$$\delta_{\mathrm{RC}}^{\mathrm{tot}} = \frac{\alpha}{2\pi}\left(3\ln\frac{m_p}{m_e} - \frac{27}{4}\right) + \frac{\alpha}{12\pi}(W_0 + 1)^2 - \overline{\delta_{\mathrm{RC}}^{\beta}}$$

Overlap and exchange corrections

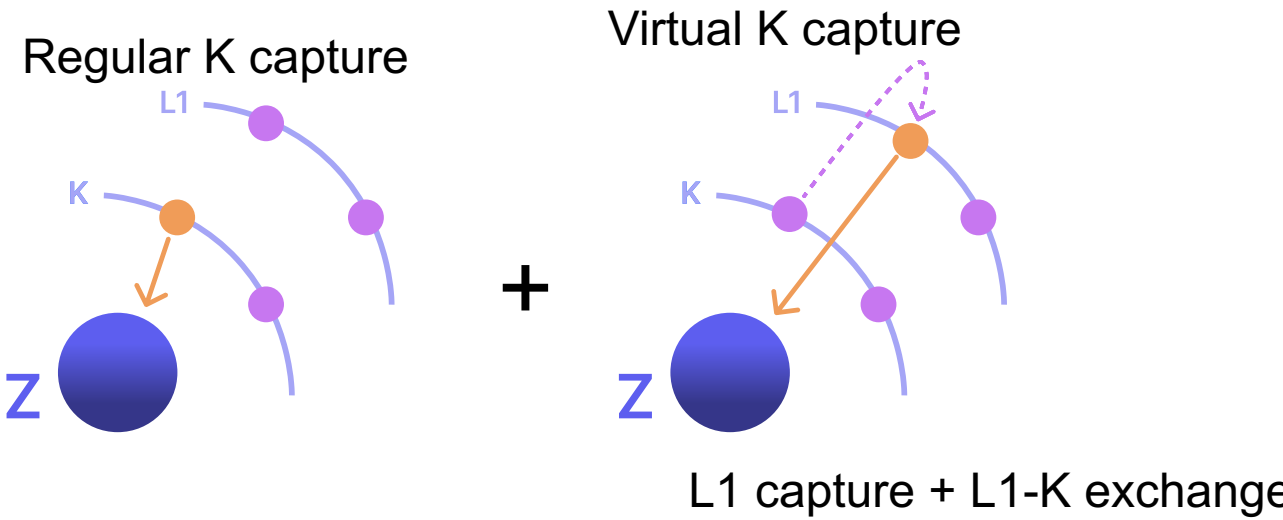
Overlap effect :

Imperfect overlap between initial a final atomic wave functions

→ Variation of nuclear charge

Exchange effect :

K-shell capture contribution



+ M1, …

Overlap and exchange corrections B

$$B_{n\kappa} = \left| \frac{b_{n\kappa}}{\beta_{n\kappa}} \right|^2 \qquad b_{n\kappa} = t_{n\kappa} \underbrace{\left[\prod_{m \neq n} \left\langle (m,\kappa)' | (m,\kappa) \right\rangle \right]}_{\text{Overlap}} \underbrace{\left[\beta_{n\kappa} - \sum_{m \neq n} \beta_{n\kappa} \frac{\left\langle (m,\kappa)' | (n,\kappa) \right\rangle}{\left\langle (m,\kappa)' | (m,\kappa) \right\rangle} \right]}_{\text{Exchange}}$$

Bachall

$$t_{n\kappa} = 1$$

Shake-up and shake-off effects are roughly accounted for, though certain probabilities tend to be **underestimated** while others are **overestimated**.

Vatai

$$t_{n\kappa} = \left\langle (n,\kappa)' | (n,\kappa) \right\rangle^{n_{n\kappa} - 0.5|\kappa|} \left[\prod_{m \neq n} \left\langle (m,\kappa)' | (m,\kappa) \right\rangle^{n_{m\kappa} - 1} \right] \left[\prod_{\substack{m,\mu \\ \mu \neq \kappa}} \left\langle (m,\mu)' | (m,\mu) \right\rangle^{n_{m\mu}} \right]$$

No Shake-up and shake-off effects, but this approach is more comprehensive.

Standard Model (SM)

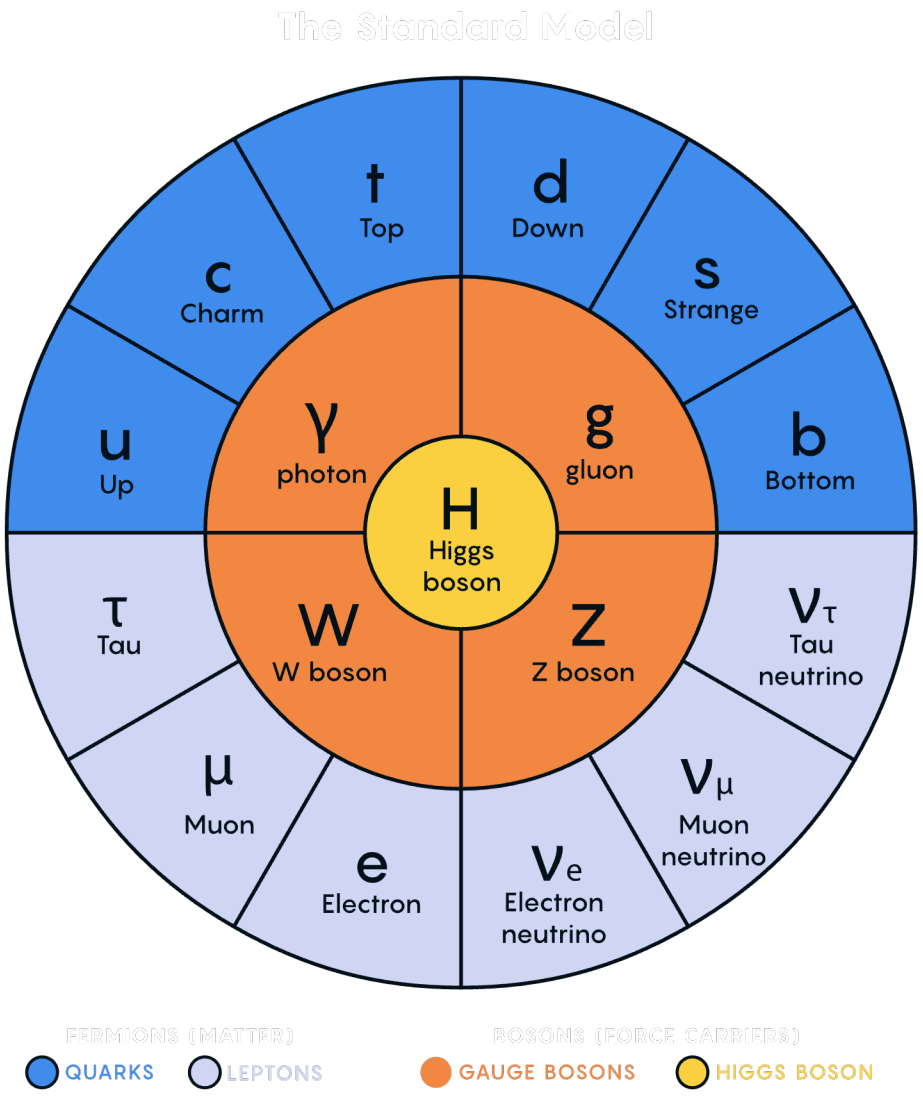
Describes **elementary particles** and **their interactions**

Describes 3 of 4 **fundamental forces**

Strong model, consistent with constaints at **TeV** scale

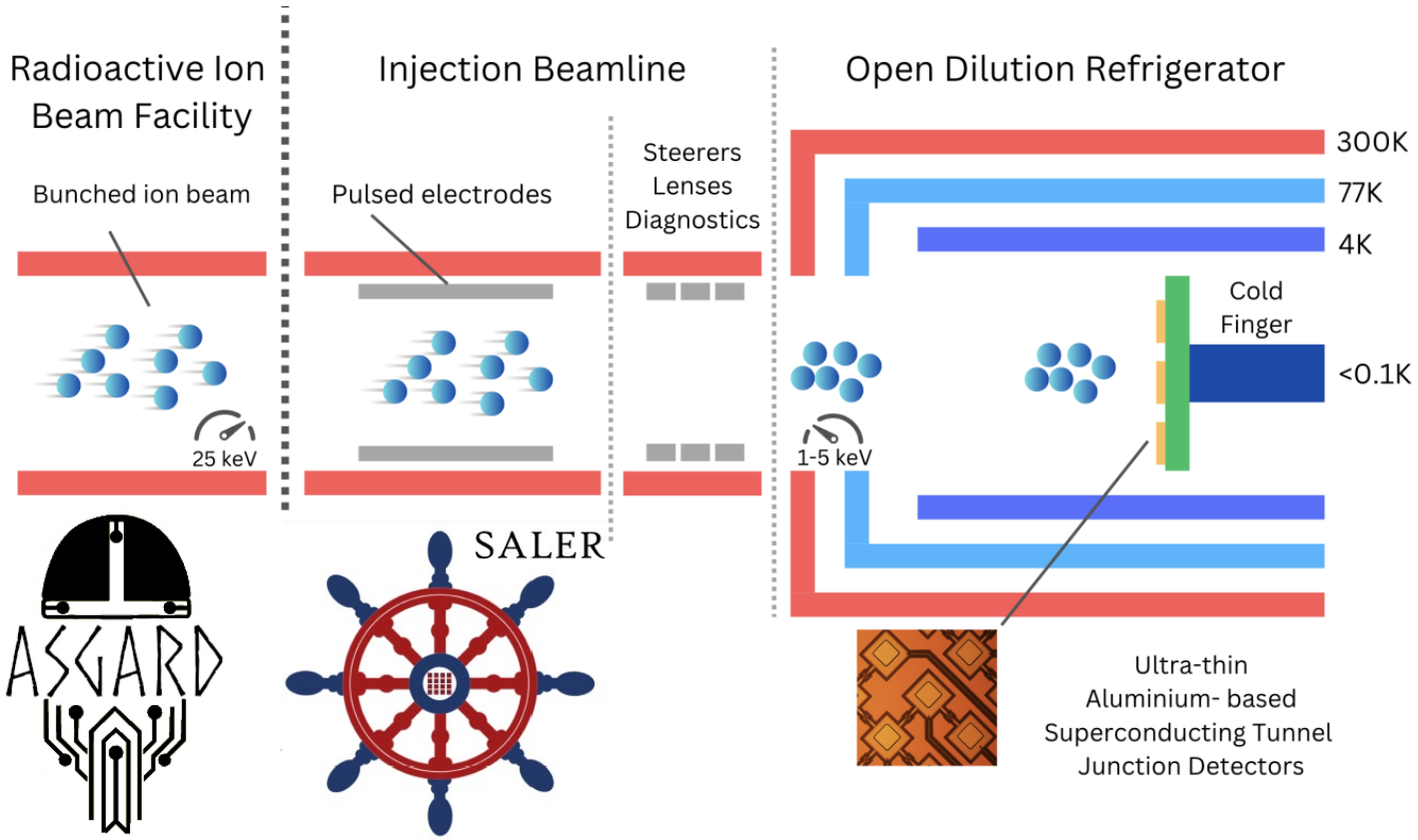
<u>Open questions :</u>

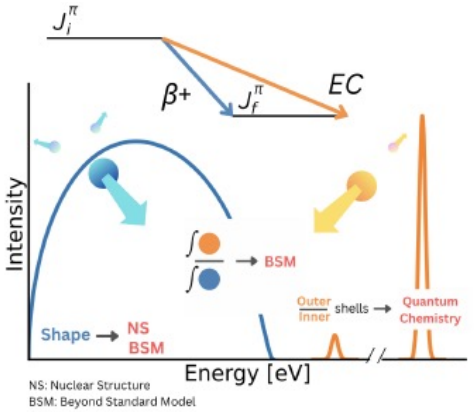
- Gravity
- Dark Matter/Energy
- Asymmetry Matter/Anti-Matter
- Neutrino proprieties (masses, sterile, …)
- Only Vector and Axial current (weak) ?
- CKM unitarity

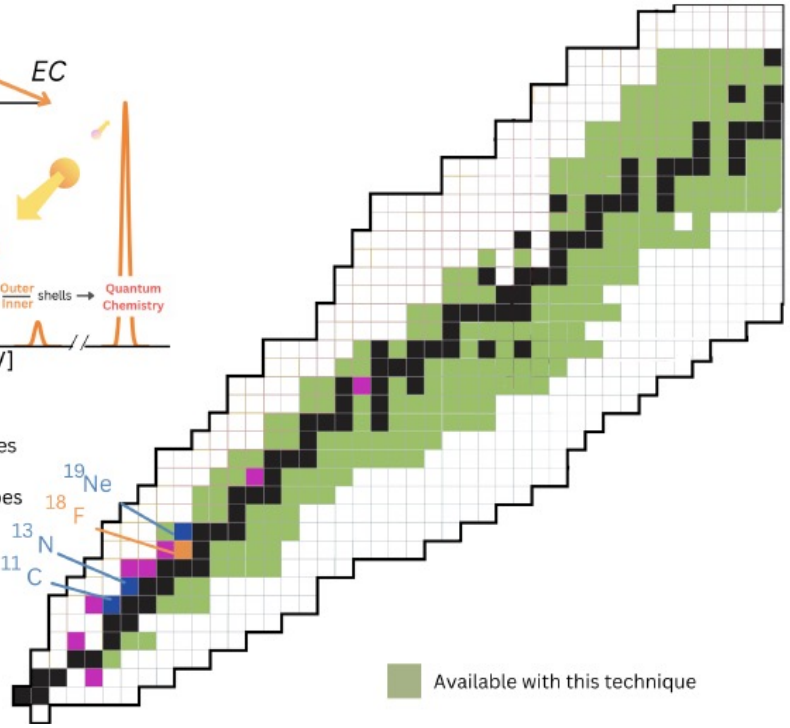


Experimental opportunities with recoil spectroscopy

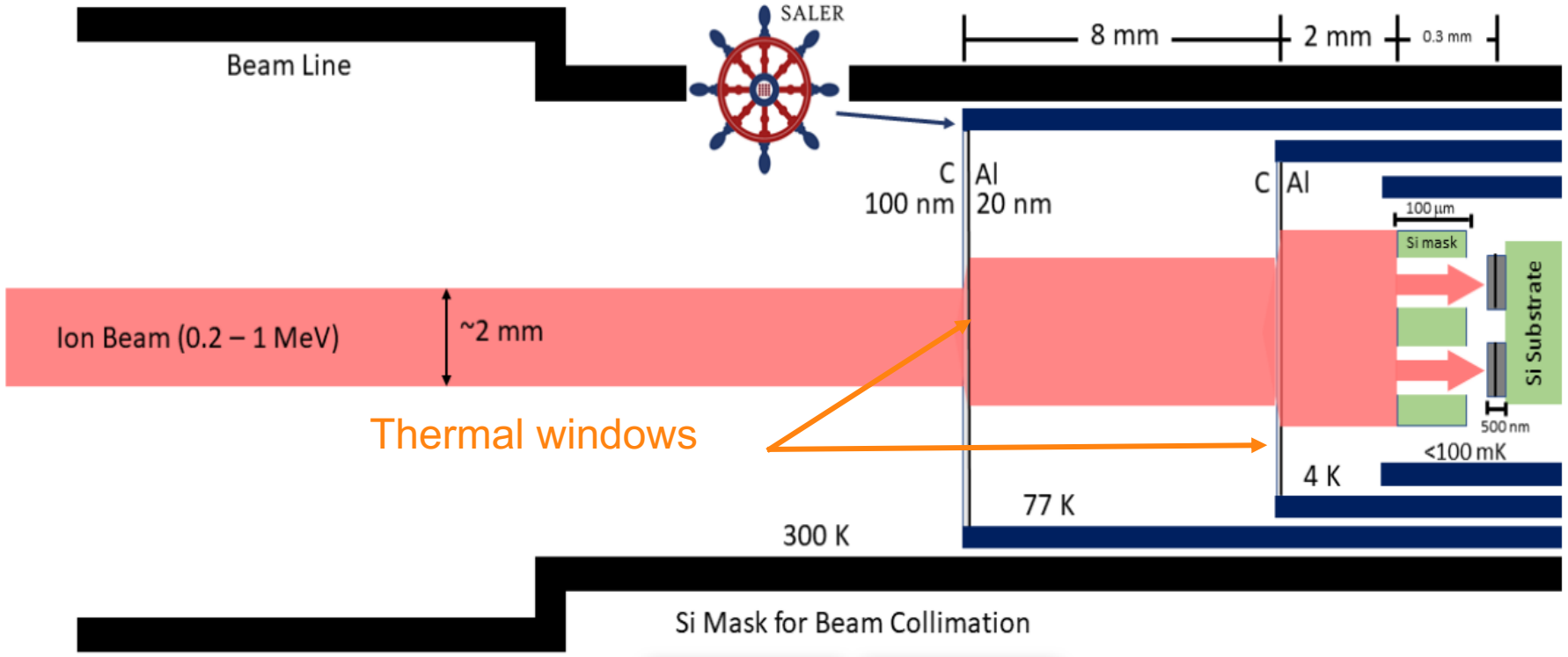
Measurement of recoil energy (<keV) with STJ (~1 eV) → precision spectroscopy
Implementation of nucleus in the STJ
→ BeEST already competitive for sterile neutrino searches (See D.Guillet talk)



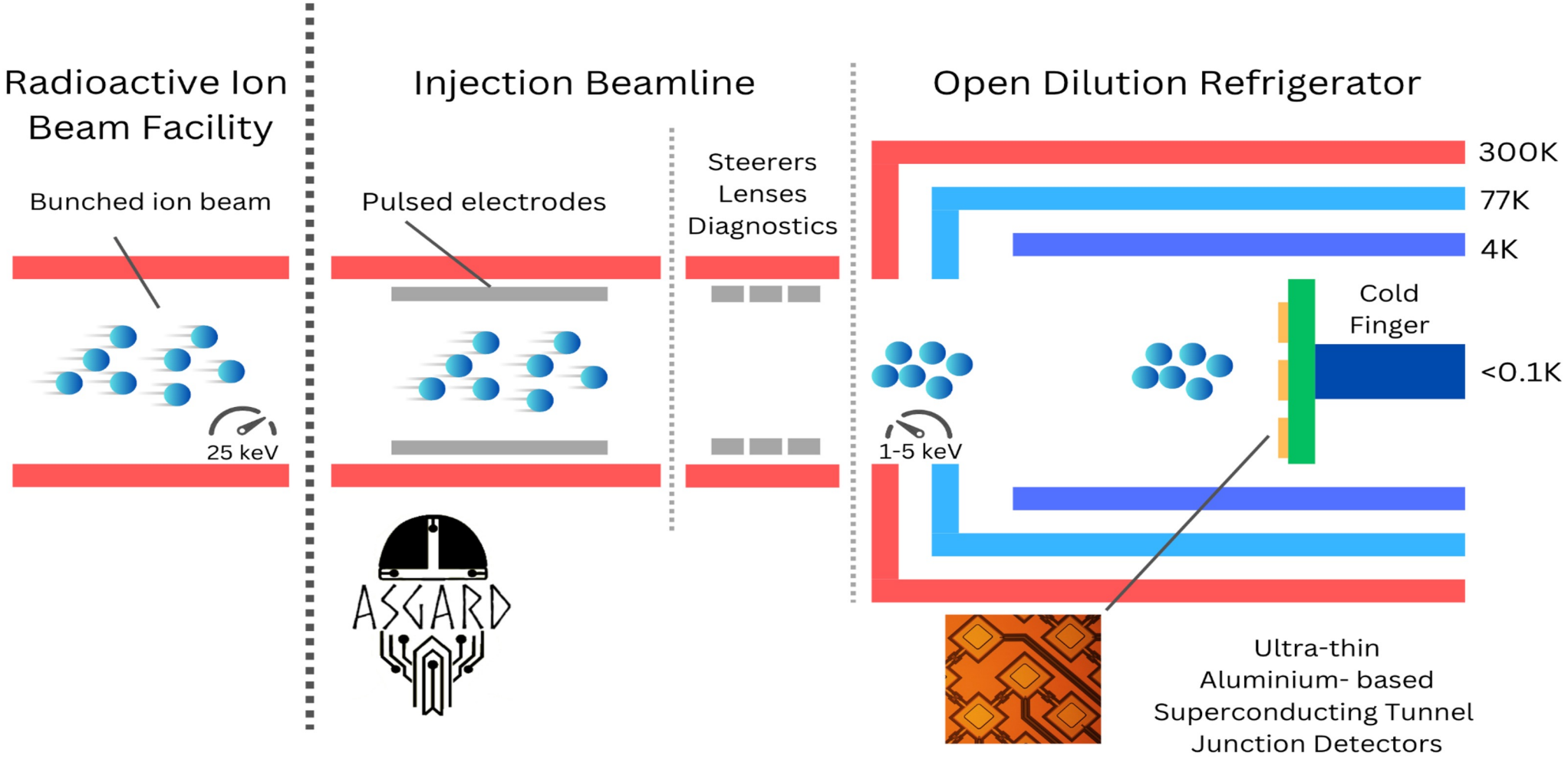




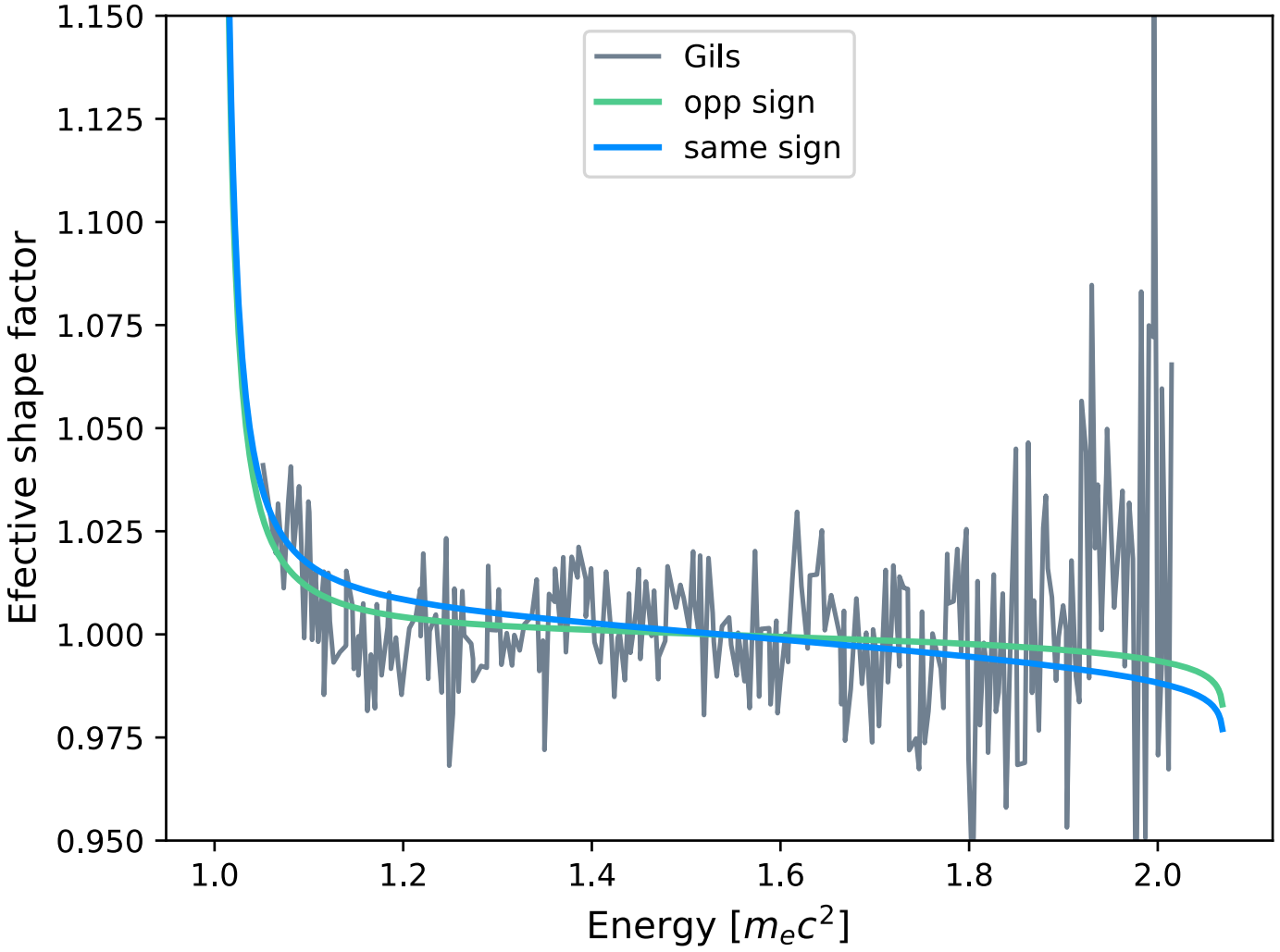
SALER


SALER
Beam Line
8 mm
2 mm
0.3 mm
C
100 nm
Al
20 nm
C
Al
100 µm
Si mask
Si Substrate
Ion Beam (0.2 – 1 MeV)
~2 mm
Thermal windows
500 nm
<100 mK
4 K
77 K
300 K
Si Mask for Beam Collimation

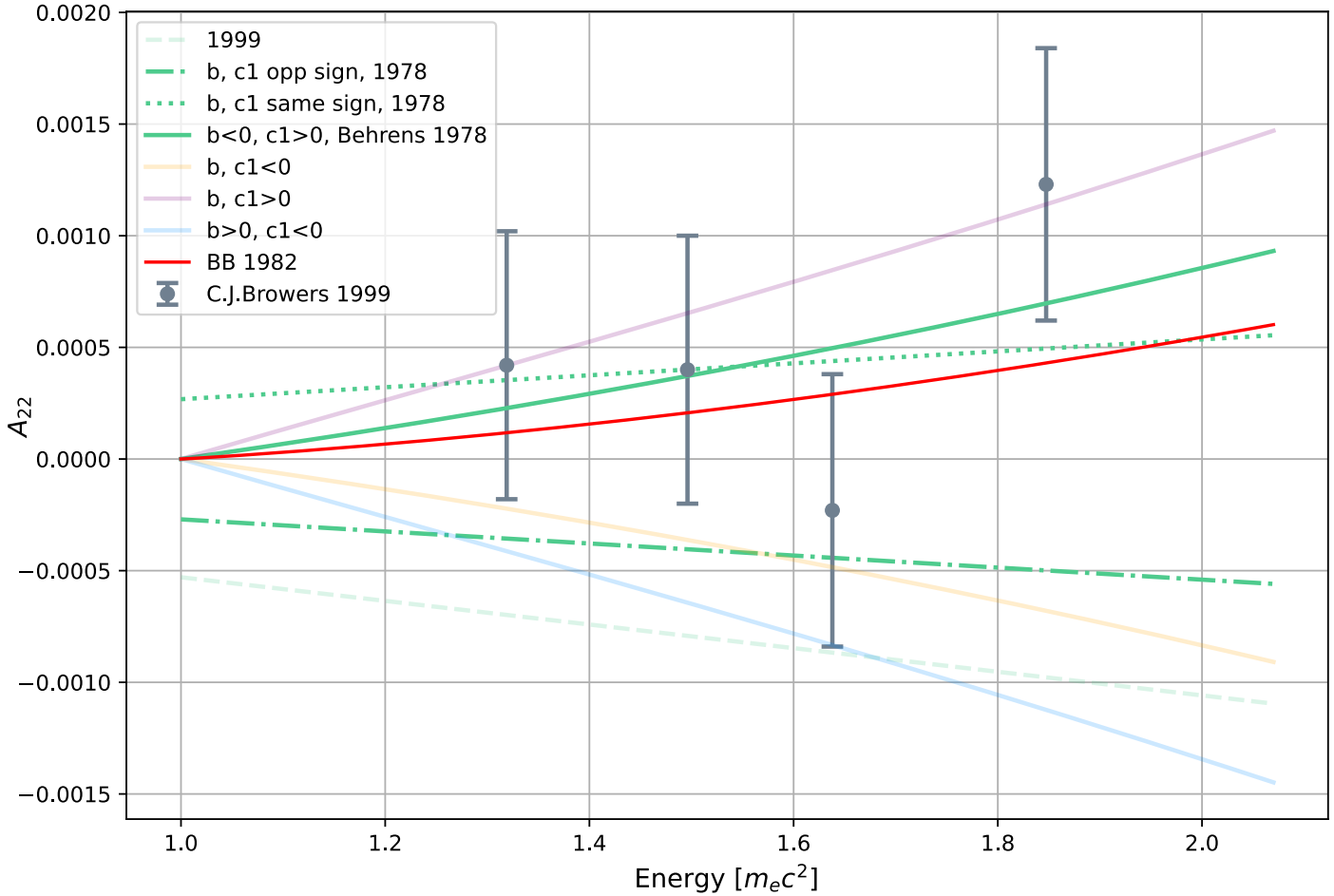
ASGARD


Radioactive Ion Beam Facility
Injection Beamline
Open Dilution Refrigerator
Bunched ion beam
Pulsed electrodes
Steerers
Lenses
Diagnostics
300K
77K
4K
Cold Finger
<0.1K
25 keV
1-5 keV
ASGARD
Ultra-thin Aluminium- based Superconducting Tunnel Junction Detectors

22Na case


Gils
opp sign
same sign
Efective shape factor
1.150
1.125
1.100
1.075
1.050
1.025
1.000
0.975
0.950
1.0
1.2
1.4
1.6
1.8
2.0
Energy [$m_e c^2$]

22Na case


1999
b, c1 opp sign, 1978
b, c1 same sign, 1978
b<0, c1>0, Behrens 1978
b, c1<0
b, c1>0
b>0, c1<0
BB 1982
C.J.Browers 1999
0.0020
0.0015
0.0010
0.0005
0.0000
−0.0005
−0.0010
−0.0015
A_{22}
1.0
1.2
1.4
1.6
1.8
2.0
Energy $[m_e c^2]$

Pion Decay

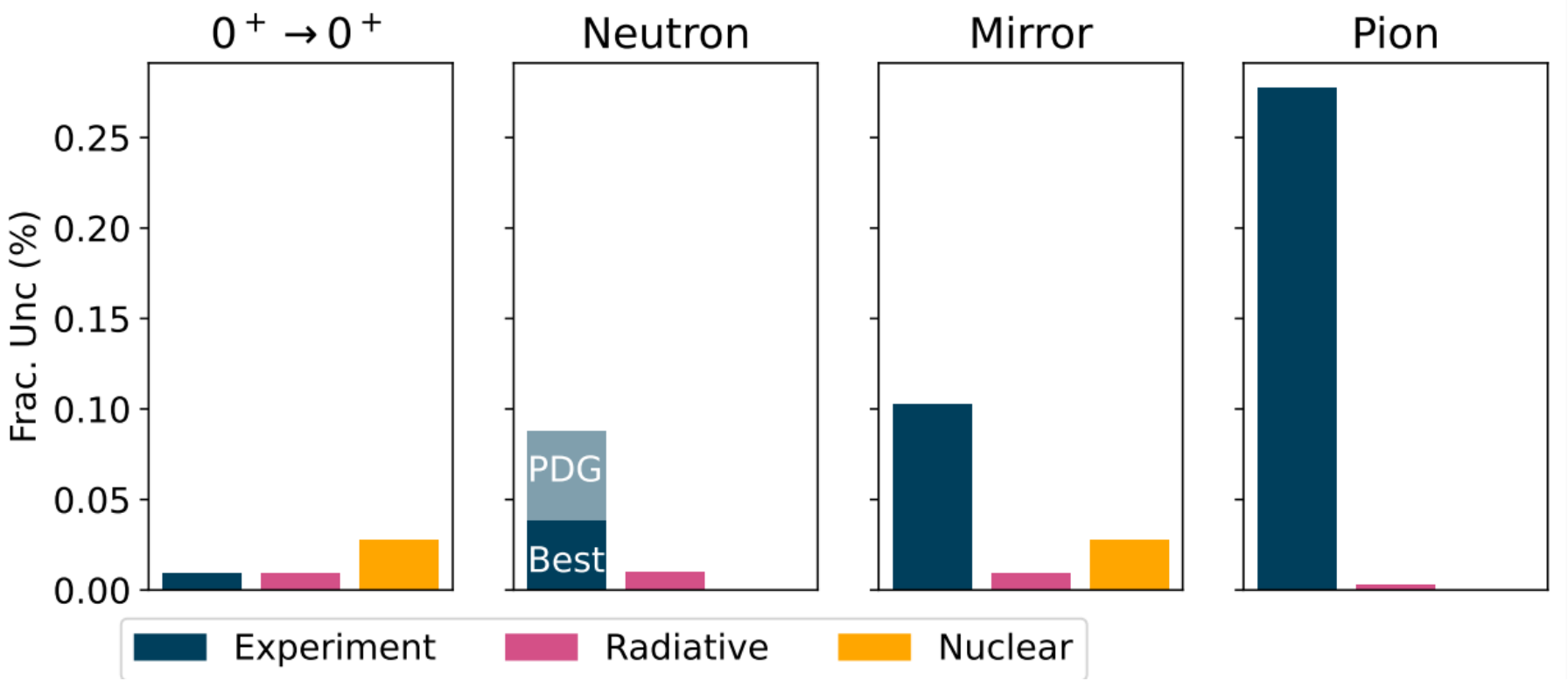
$$\pi^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu} \qquad (99.9877\%)$$

$$\pi^{\pm} \rightarrow e^{\pm} + \nu_{e} \qquad (0.0123\%)$$

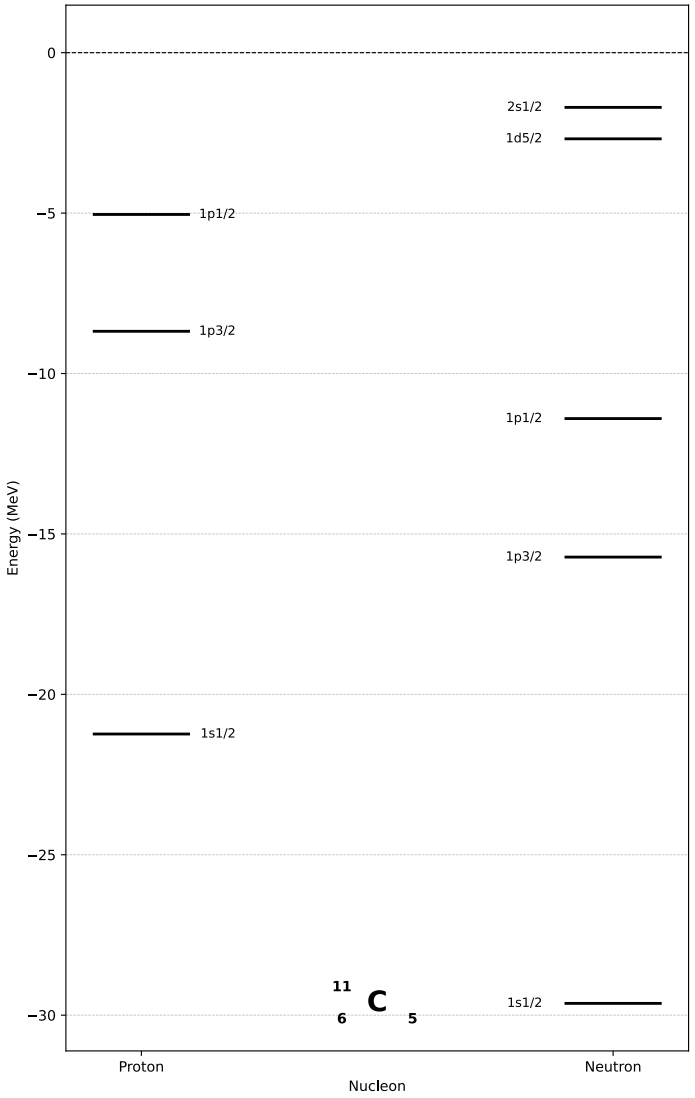
β-decay $\left\{ \pi^{\pm} \rightarrow \pi^{0} + e^{\pm} + \nu_{e} \right.$ $(\ 10^{-6}\%)$

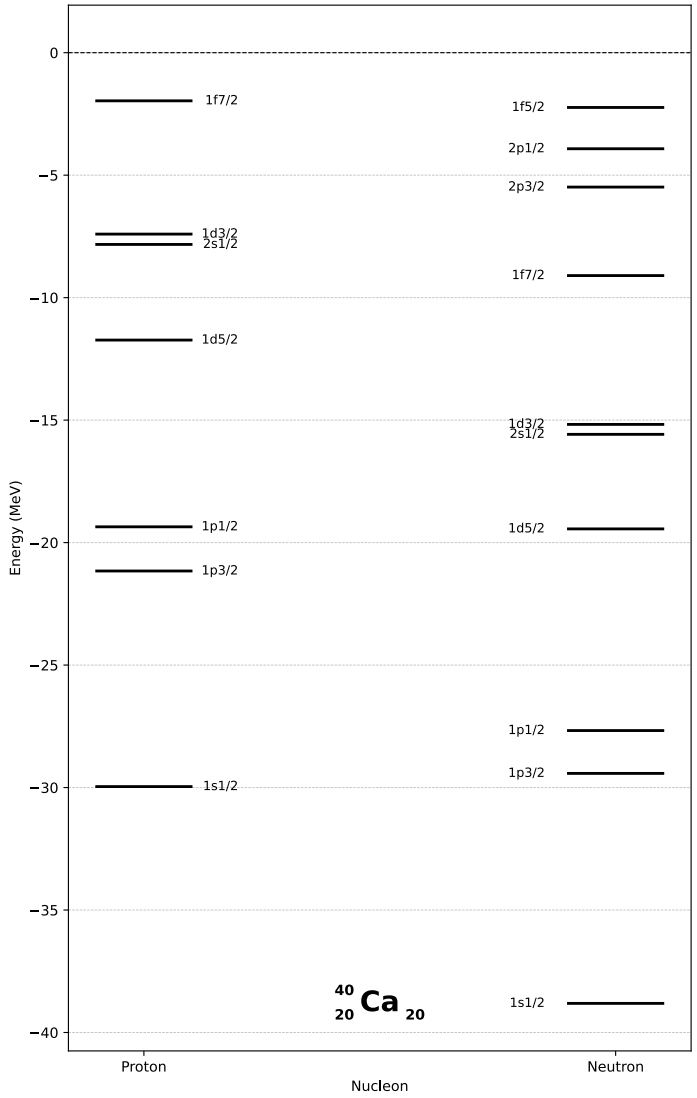
Lack of stat

CKM precision



Naive Shell Model





Shell Model : interactions

$$\mathcal{H} = \sum_{i}^{A} T_i + \sum_{i<j}^{A} V_{ij} + \boxed{(\sum_{i,j,k}^{A} V_{ijk})}$$

3-body term can be omit

$$\mathcal{H} \approx \boxed{\mathcal{H}_{\text{core}}} + \boxed{\mathcal{H}_{\text{valence}}} + \boxed{\mathcal{H}_{\text{core-valence}}}$$

~ constant

Single-particle energy

$$\boxed{\mathcal{H}^{\text{eff}}_{\text{valence}}} = \sum_{i} \epsilon_i a_i^\dagger a_i + \frac{1}{4} \sum_{i,j,k,l} \boxed{< ij|V^{\text{eff}}|kl > a_i^\dagger a_j^\dagger a_k a_l}$$

TBME

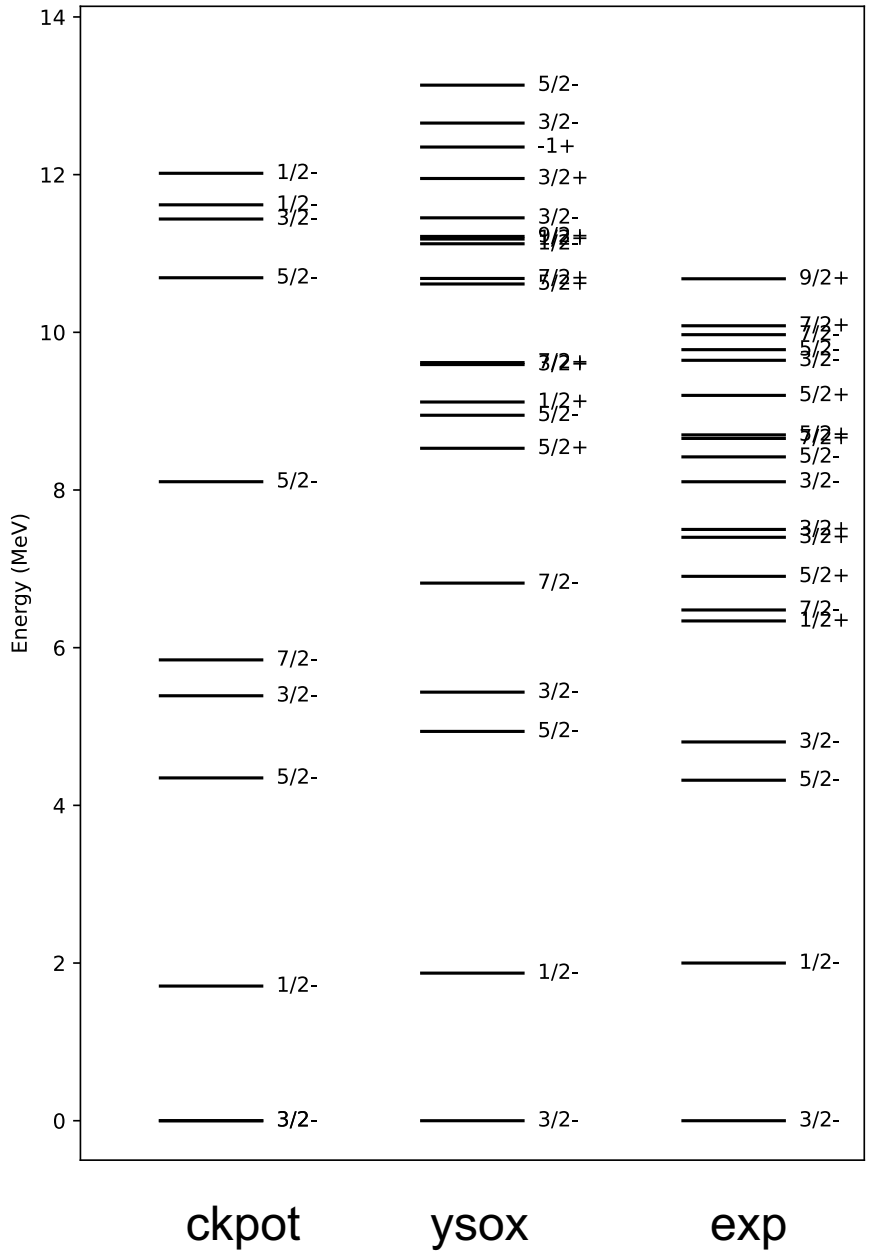
Shell Model : interactions

CKPOT

A : 8 - 16
Model space : $1p_{3/2}$, $1p_{1/2}$
TBME : V^{eff} adjust phenologically
Potential : depend on orbits, angular momentum J, isospin T

YSOX

A : 8 - 30
Model space : $1p_{3/2}$, $1p_{1/2}$, $2s_{1/2}$, $1d_{5/2}$, $1d_{3/2}$
TBME : partially microscopic, partially adjusted
Potential : central, tensor and spin-orbit
+ coulomb correction



Lagrangian

$$\mathcal{L} = \sum [\overline{p}\mathcal{O}_i n] \times [\overline{e}\mathcal{O}_i(C_i + C_i\gamma^5)\nu]$$

$$\begin{aligned}\mathcal{L}_{\text{eff}} \propto & \overline{e}\gamma_\mu\nu_L \cdot \overline{u}\gamma^\mu[C_V - (C_A - 2\epsilon_R)\gamma^5]d + \epsilon_S\overline{e}\nu_L \cdot \overline{u}d \\ & - \epsilon_P\overline{e}\nu_L \cdot \overline{u}\gamma^5 d + \epsilon_T\overline{e}\sigma_{\mu\nu}\nu_L \cdot \overline{u}\sigma^{\mu\nu}(1-\gamma^5)d\end{aligned}$$

Nuclear matrix elements ${}^{V/A}\mathcal{M}^{N}_{KLs}$

Hadronic part :

2 types : - non-relatisvistic → coupling LNWFs
- relativistic → coupling LNWF with SNWF (using CVC theory)

$$\Psi_{nlm} = \begin{pmatrix} g_{nl}(r)\Omega_{lm}(\hat{r}) \\ i f_{nl}(r)\Omega_{-lm}(\hat{r}) \end{pmatrix}$$

$$\begin{pmatrix} m+V & \vec{\sigma}\cdot\vec{p} \\ \vec{\sigma}\cdot\vec{p} & -m+V \end{pmatrix} \Psi_{nlm} = E\Psi_{nlm}$$

$$f_{nl}(r) = \frac{1}{2M}\left[\frac{d}{dr} + \frac{l+1}{r}\right] g_{nl}(r)$$

→ Non-relativistic approximation

$$E \approx m$$

Nuclear matrix elements $^{V/A}\mathcal{M}^{N}_{KLs}$

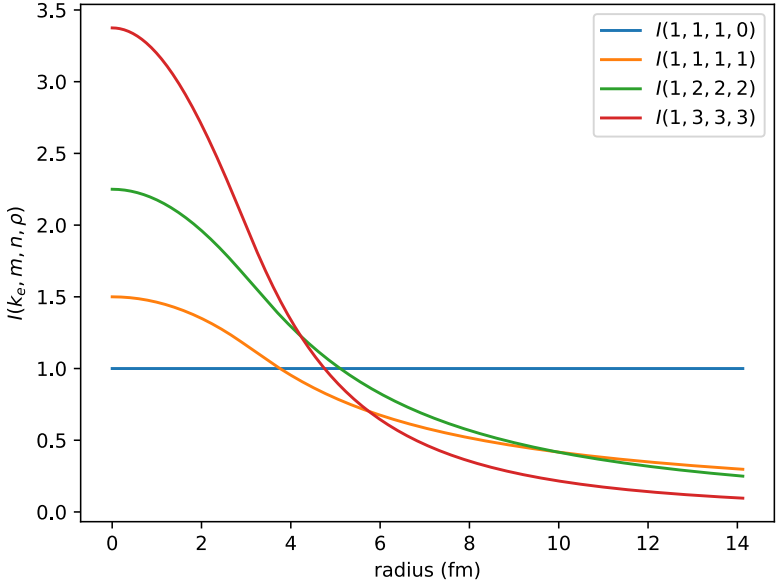
Leptonic part :

$$\begin{Bmatrix} f_\kappa(r) \\ g_\kappa(r) \end{Bmatrix} = \alpha_\kappa \frac{(pr)^{(k-1)}}{2k-1} \sum_{n=0}^{\infty} \begin{Bmatrix} a_{\kappa n} \\ b_{\kappa n} \end{Bmatrix} \left(\frac{r}{R}\right)^n$$

→ Exact solution (for central potential)

Correction on the electron wave central due to central potential

$$I(k, m, n, \rho; r)$$



$$(m_e R), (ER), (\alpha Z)$$

For uniform charge distribution

$$U(r) = \begin{cases} \frac{3}{2} - \frac{1}{2}\left(\frac{r}{R}\right)^2 & \text{if } r \leq R \\ \frac{R}{r} & \text{if } r > R \end{cases}$$

One-body transition density (OBTD)

Theory

Probability of transition of a single nucleon between two nuclear states

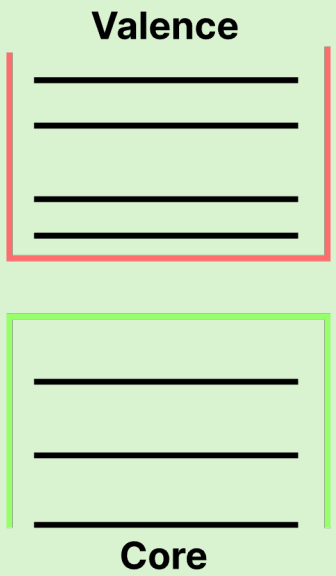
$$\mathrm{OBTD} = \frac{< \zeta_f J_f | [a^\dagger_\alpha \tilde{a}_\beta]_\lambda | \zeta_i J_i >}{\sqrt{2\lambda + 1}}$$

$$\lambda = \Delta J = |J_i - J_f|$$

Model

Provides by **shell model** calculation

Depends :
Interaction
Model space
Valence nucleon configuration



$$|\psi >= |\mathrm{core} > \otimes |\mathrm{valence} >$$

Using

Essential to compute **Form Factors** and evaluate the nuclear structure

Tools :
KSHELL (Python)
NuShellX (Fortran, Windows-only)

GKLs factor

$$G_{KLs}(\kappa_f, \kappa_i) = \sqrt{(2s+1)(2K+1)(2l(\kappa_f)+1)(2l(\kappa_i)+1)(2j_f+1)(2j_i+1)} \times$$

$$\mathbf{i}^{\mathbf{l}(\kappa_\mathbf{f})+\mathbf{l}(\kappa_\mathbf{i})+\mathbf{L}}(-\mathbf{1})^{\mathbf{j_i}-\mathbf{j_f}} C(l(\kappa_f)l(\kappa_i)L;00) \begin{pmatrix} K & s & L \\ j_f & 1/2 & l(\kappa_f) \\ j_i & 1/2 & l(\kappa_i) \end{pmatrix}$$

Biedenharm-Rose

Clebsch-Gordan Coefficient

9j-Wigner symbol

→ Coupling between hadronic and leptonic wave functions

Form Factors

$$< \zeta_f J_f M_f | T_{\lambda\mu} | \zeta_i J_i M_i >= \sum_{\alpha\beta} < \alpha | T_{\lambda\mu} | \beta > < \zeta_f J_f M_f | c^\dagger_\alpha c_\beta | \zeta_i J_i M_i >$$

$$< \zeta_f J_f || T_\lambda | \zeta_i J_i >= \lambda^{-1} \sum_{a,b} < a || T_\lambda || b > < \zeta_f J_f || [c^\dagger_\alpha c_\beta]_\lambda || \zeta_i J_i >$$

$$^{V/A}\mathcal{F}^N_{KLs} = \sum {}^{V/A}\mathcal{M}^N_{KLs} \times \mathrm{OBTD}$$

Form Factors

$$^{V/A}\mathcal{F}^{N}_{KLs} = \sum {}^{V/A}\mathcal{M}^{N}_{KLs} \times \mathrm{OBTD}$$

Nuclear matrix elements

KSHELL
or
NUSHELLX

N : form factor order
K : total angular momentum (rank)
L : orbital multipolarity
s : 0 (spin parallel) or 1 (anti-parallel)

The form factors depend on **induced currents**

$$g_M \approx 0.001$$

$$g_S = g_S = g_T = 0$$

NME and OBTD in the **same NWF basis** (HO, WS, …)
Phase convention (Condon-Shortley or Biedenharm-Rose)

Beta/EC Shape Factor

Theoritical Shape Factor :

Coulomb functions

$$C(E) = \sum_{k_e,k_\nu,K\geq 1} \lambda_{k_e} \left[M_K(k_e,k_\nu)^2 + m_K(k_e,k_\nu)^2 - \frac{2\gamma_{k_e}\mu_{k_e}}{k_e E} M_K(k_e,k_\nu) m_K(k_e,k_\nu) \right]$$
$$+\delta_{\Delta J,0} \left[M_0(1,1)^2 + m_0(1,1)^2 - \frac{2\gamma_{k_e}\mu_{k_e}}{k_e E} M_0(1,1) m_0(1,1) \right]$$

Beta

$$C_x = \sum_{k_e,k_\nu,K\geq 1} \lambda_{k_e} \left[M_K(k_e,k_\nu)^2 + m_K(k_e,k_\nu)^2 + \frac{\kappa_x}{k_x} M_K(k_e,k_\nu) m_K(k_e,k_\nu) \right]$$
$$+\delta_{\Delta J,0} \left[M_0(1,1)^2 + m_0(1,1)^2 - \frac{\kappa_x}{k_x} M_0(1,1) m_0(1,1) \right]$$

EC

$$(\pi_i \pi_f) = (-1)^K \qquad {}^V F^N_{KK0} \quad {}^A F^N_{KK1} \quad {}^V F^N_{KK-11} \quad {}^V F^N_{KK+11}$$
$$(\pi_i \pi_f) = (-1)^{K+1} \qquad {}^A F^N_{KK0} \quad {}^V F^N_{KK1} \quad {}^A F^N_{KK-11} \quad {}^A F^N_{KK+11}$$

Conserved Vector Current (CVC) hypothesis