On Soft Contributions to the $B \to \gamma^*$ Form Factors

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Outline

1. **Introduction:** B mesons, LCDAs and soft effects
2. **QCD Factorization framework:** Hadronic tensor, free of kinematic singularities set of form factors and hard-collinear scattering kernels
3. **Light-Cone Sum Rule set-up:** Resonances, dispersive integrals and soft contribution
4. **Conclusions**

Introduction

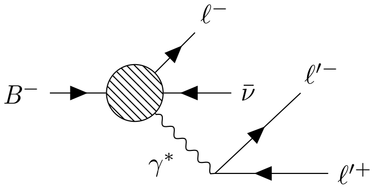
- B *mesons* are **bound states** of the heavy b quark and a light u, d, s or c quark
- provide access to **SM parameters** (and NP):
  - sensitive to CKM matrix elements - Testing the Standard Model
  - study of $B^0 - \overline{B}^0$ oscillations - CP violation
  - rare decays - New Physics
- essential for this: knowledge of the B meson **substructure** (hadronic matrix elements)
- if decays are **factorizable**: LQCD, LCSR, and **QCDF**

QCD Factorization

- if the kinematics allow an **expansion**, QCDF can calculate some hadronic matrix elements relevant to B-meson decays

- calculation depends crucially on some universal hadronic inputs, in particular, the B-meson LCDAs

- for an **energetic photon** ($E_\gamma \gg \Lambda_{\text{QCD}}$), $B \to \gamma^{(*)} \ell^- \bar{\nu}$ is the simplest process that depends on these LCDAs



Leading twist LCDA defined as:

$$\left\langle 0 \middle| \bar{q}_s(tn_-)\,[tn_-,0]\,\not{n}_-\gamma_5 h_v(0) \middle| B_v^- \right\rangle$$
$$= i m_B F_B \int_0^\infty d\omega\ e^{-i\omega t} \phi_+(\omega)$$

Predictions for other processes

- LCDA is **input** for QCDF predictions for non-leptonic (CKM parameters) and rare decays (probe NP)
- LCDA is input for LCSR used to **predict many form factors** for B decays to other mesons
- turn it around: use data on $B^- \to \gamma^* \ell^- \bar{\nu}$ to **extract information** on the LCDAs to predict other quantities
  - **data** depends dominantly on inverse moment of leading LCDA: ${\lambda_B}^{-1} = \int_0^\infty d\omega\ \phi_+(\omega)/\omega$, not well known

- theoretical uncertainty of λ_B is large:
  - 200 MeV from non-leptonic decays [Beneke/Neubert '03 hep-ph/0308039]
  - 460 ± 110 MeV from QCD sum rules [Braun/Ivanov/Korchemsky hep-ph/0309330]
- experimental uncertainty:
  - $\mathcal{BR}\left(B^+ \to \gamma \ell^+ \nu_\ell\right) < 15.6 \times 10^{-6} \implies \lambda_B >$ 300 MeV [BaBar 0907.1681]
  - $\mathcal{BR}\left(B^+ \to \gamma e^+/\mu^+ \nu_\ell\right) < 4.3/3.4 \times 10^{-6} \implies \lambda_B >$ 238 MeV [Belle 1504.05831]

  (Updating this result is a priority at Belle II, and projections look very promising) [Gelb++ '18]

Problem: Soft Contributions

- **soft contributions** are pieces of the $B^- \to \gamma^{(*)}\ell^-\overline{\nu}_\ell$ amplitude that are controlled by long-distance, low-virtuality QCD physics.

- although $B^- \to \gamma^{(*)}\ell^-\overline{\nu}_\ell$ is dominated by **light-like contributions**, soft contributions appear nonetheless

- for instance, ρ and γ^* have the **same quantum numbers** for q^2 close to m_ρ^2
  - resonant enhancement
  - need to have the tail of this soft contribution under control

- constitute hardly-quantifiable **systematic uncertainty**

- **not accessible** within the framework of QCDF

Estimating Soft Contributions

→ light-cone **sum rule** set up [Braun/Khodjamirian 1210.4453]

- **on-shell photon case** has been discussed (2 form factors)

  [Beneke/Rohrwild 1110.3228], [Beneke/Braun/Jib/Wei 1804.04962], [Descotes-Genon/Sachrajda hep-ph/0209216], [Lunghi/Pirjol/Wyler hep-ph/0210091], [Wang/Shen JHEP05(2018)184]

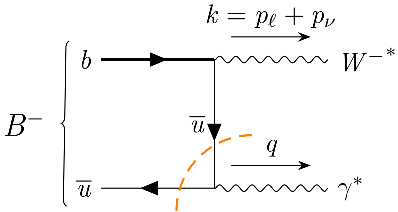
- **off-shell photon case** has been discussed but only estimated from partial hadronic tensor (3 out of 4 form factors)

  [Beneke/Böer/Rigatos/Vos 2102.10060], [Bharucha/Kindra/Mahajan 2102.03193]

→ **general off-shell case** and its **dispersion relations** require all 4 form factors without kinematic singularities [Kürten/Zanke/Kubis/van Dyk 2210.09832]

Factorization Framework and Beyond

We consider the diagram



- γ^* emission from b is m_b power suppressed
- FSR is perturbatively calculable

Up to corrections of $\mathcal{O}(\alpha_e)$, the photoleptonic amplitude reads

$$\mathcal{M}(B^-(p) \to \ell^-(p_\ell)\bar{\nu}_\ell(p_\nu)\gamma^*(q,\varepsilon)) = \frac{G_F V_{ub}}{\sqrt{2}} \left[e Q_B \varepsilon^*_\mu \left(T^{\mu\nu}_{\mathsf{H}}(k,q) + T^{\mu\nu}_{\mathsf{FSR}}(p_\ell, p_\nu) \right) L_\nu \right],$$

where $L^\nu := \langle \ell^- \overline{\nu}_\ell | J^\nu_{\mathsf{L}} | 0 \rangle$.

Hadronic Tensor Splitting

Focus on

$$T_{\mathsf{H}}^{\mu\nu}(k,q) := \int d^4x\; e^{iq\cdot x} \left\langle 0 \middle| T\{J^{\mu}_{\mathsf{em}}(x) J^{\nu}_{\mathrm{H}}(0)\} \middle| B^-(q+k) \right\rangle$$
$$= T^{\mu\nu}_{\mathsf{hom.}} + T^{\mu\nu}_{\mathsf{inhom.}}$$

$T^{\mu\nu}_{\mathsf{hom.}}(k,q) \to$ Lorentz decomposition in terms of $B \to \gamma^*$ form factors $F_1\left(k^2, q^2\right), \ldots, F_4\left(k^2, q^2\right)$.

$q_\mu \left(T^{\mu\nu}_{\mathsf{inhom.}}(k,q) + T^{\mu\nu}_{\mathsf{FSR}}(k,q) \right) \stackrel{!}{=} 0$
$\implies$ Avoid unphysical, kinematic singularities in q^2 in the form factors.

The absence of such **kinematic singularities** is a formal prerequisite for expressing these form factors through hadronic **dispersion relations**

[Kürten/Zanke/Kubis/van Dyk 2210.09832]

Leading Factorizable Contributions

- for $E_\gamma \gg \Lambda_{\text{QCD}}$ in the B-meson rest frame, the time-ordered product in T_{H} is dominated by field configurations at light-like distances, *i.e.*, $x^2 \approx 0$

[Descotes-Genon/Sachrajda hep-ph/0209216]

$$\overbrace{u(x)\bar{u}}(0) = \frac{i}{2\pi^2}\frac{\not{x}}{x^4} - \frac{1}{8\pi^2 x^2}\int_0^1 du \left\{ i x^\rho g \widetilde{G}_{\rho\sigma}(ux)\gamma^\sigma\gamma_5 + (2u-1)x^\rho g G_{\rho\sigma}(ux)\gamma^\sigma \right\} + \dots$$

- for each form factor, the leading power result factorizes:

$$F_i(q^2, k^2) = \frac{ef_B}{m_B}\int_0^\infty d\omega \underbrace{T_i^{\text{tw2}}(\omega, n_+ q, n_- q)}_{\text{scattering kernel}} \underbrace{\phi_+(\omega)}_{\text{LCDA}} + \mathcal{O}\left(\frac{\Lambda_{\text{had}}}{\{n_+ q, m_b\}}\right)^2$$

Hard-Collinear Scattering Kernels

Using HQET + SCET, the hard-collinear scattering kernels T_i^{tw2} are:

$$T_1^{\text{tw2}} = \frac{C_V m_B^2 Q_u J}{n_+ q(\omega - n_- q)}, \quad T_2^{\text{tw2}} = 2\frac{C_V m_B^2 (m_B - n_+ q) Q_u J}{(n_+ q)^2(\omega - n_- q)},$$

$$T_3^{\text{tw2}} = 0 + \mathcal{O}\left(\alpha_s \frac{\Lambda}{m_b}\right), \quad T_4^{\text{tw2}} = \frac{C_V m_B^2 Q_u J}{n_+ q(\omega - n_- q)},$$

- general structure at leading power and to leading order in α_s: the hard-scattering kernels exhibit a **universal ω dependence**
- hard-scattering kernels for the **full basis** of form factors for the first time

Light-Cone Sum Rule Set Up 1/2

1. **Dispersion relation** of form factors:

$$F_i(k^2, q^2) = \underbrace{\frac{f_\rho \mathcal{F}_i^{B\to\rho}(k^2)}{m_\rho^2 - q^2}}_{\text{intermediate on-shell } \rho \text{ meson}} + \underbrace{\frac{1}{\pi}\int_{s_0}^{\infty} ds\, \frac{\mathrm{Im}\left\{F_i(k^2, s)\right\}}{s - q^2}}_{\text{all other intermediate multi-meson states}}$$

2. **QCDF results** as dispersive integrals:

$$F_i^{\mathsf{QCDF}}(k^2, q^2) = \frac{1}{\pi}\int_0^{\infty} ds\, \frac{\mathrm{Im}\left\{F_i^{\mathsf{QCDF}}(k^2, s)\right\}}{s - q^2}$$

3. Local quark-hadron duality **assumption**:

$$\mathrm{Im}\left\{F_i(k^2, s)\right\} \simeq \mathrm{Im}\left\{F_i^{\mathsf{QCDF}}(k^2, s)\right\} \quad \text{for } s > s0$$

Light-Cone Sum Rule Set Up 2/2

4. **Reduce sensitivity** of assumption by Borel transformation → Isolate $\mathcal{F}_i^{B\to\rho}(k^2)$:

$$f_\rho \mathcal{F}_i^{B\to\rho}(k^2) = \frac{1}{\pi}\int_0^{s_0} ds\, e^{-(s-m_\rho^2)/M^2}\, \mathrm{Im}\left\{F_i^{\mathsf{QCDF}}(k^2,s)\right\}$$

5. Write form factor as **leading + soft contribution**:

$$F_i(k^2,q^2) = \frac{1}{\pi}\int_0^{\infty} ds\, \frac{\mathrm{Im}\left\{F_i^{\mathsf{QCDF}}(k^2,s)\right\}}{s-q^2} + \frac{1}{\pi}\int_0^{s_0} ds\, \mathrm{Im}\left\{F_i^{\mathsf{QCDF}}(k^2,s)\right\}\left[\frac{e^{-(s-m_\rho^2)/M^2}}{m_\rho^2-q^2} - \frac{1}{s-q^2}\right]$$

Form Factors = Leading + Soft Contribution

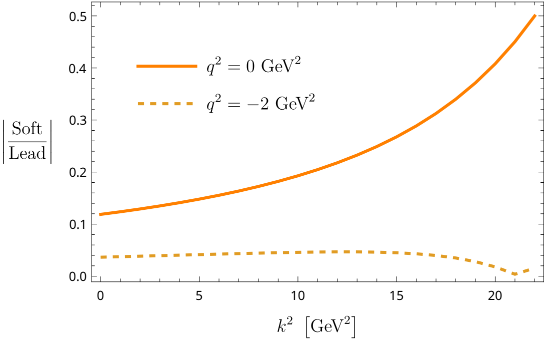
Using the hard-scattering kernels found:

$$F_i(n_+q, n_-q) = \frac{ef_B}{m_B(n_+q)} \int_0^\infty ds\; T_i^{\text{tw2}}\left(\frac{s}{n_+q}, n_+q, n_-q\right) \phi_+\left(\frac{s}{n_+q}\right) + \frac{ef_B}{m_B(n_+q)} \int_0^{s_0} ds\; T_i^{\text{tw2}}\left(\frac{s}{n_+q}, n_+q, n_-q\right) \phi_+\left(\frac{s}{n_+q}\right) \left[\frac{s-(n_+q)(n_-q)}{m_\rho^2-(n_+q)(n_-q)} e^{-(s-m_\rho^2)/M^2} - 1\right]$$

lead + soft

Numerical Results

$$B^- \to \gamma^*(q)[\to \ell^+\ell^-]\, W^*(k)[\to \ell'^- \bar{\nu}_{\ell'}]$$

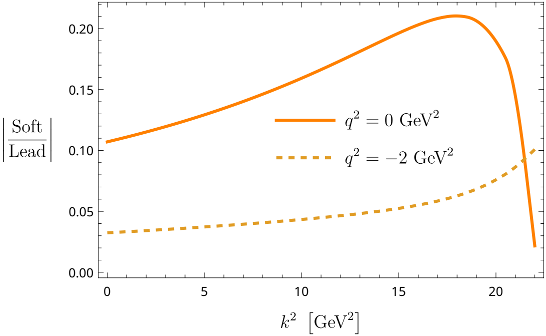


- results obtained from an **exponential model** for the LCDA:
$$\phi_+(\omega) = \frac{\omega}{\lambda_B^2} e^{-\omega/\lambda_B}$$
- soft corrections **under control** over all phase space, <5%
- dispersion relation allows **extrapolation** from negative q^2 to physical positive q^2

where $q^2 = (n_+ q)(n_- q)$ and $k^2 = m_B(m_B - n_+ q)$

Higher-twist contributions

Higher-twist two-particle LCDAs: $g_+^{WW}(\omega)$ and $\phi_-^{t3}(\omega)$.



- **exponential model** for higher-twist LCDAs through relations with $\phi_+(\omega)$
- soft corrections **under control** over all phase space, $<$10%
- dispersion relation allows **extrapolation** from negative q^2 to physical positive q^2

where $q^2 = (n_+ q)(n_- q)$ and $k^2 = m_B(m_B - n_+ q)$

Conclusions

- work with form factors in a basis where they are free of kinematic singularities, needed for sum rule
- hard-scattering kernels exhibit a universal ω dependence
- soft contribution under control over all phase space

- currently working on extending to **three-particle higher twist corrections**
- soon will conduct full **phenomenological analysis** for $B \to \ell \nu_\ell \ell'^+ \ell'^-$

Extra slides

Extra 1

$$T^{\mu\nu}_{\text{hom.}}(k,q) = \frac{1}{m_B}\left[(k\cdot q)g^{\mu\nu} - k^\mu q^\nu\right] F_1\left(k^2,q^2\right) + \frac{1}{m_B}\left[\frac{q^2}{k^2}k^\mu k^\nu - \frac{k\cdot q}{k^2}q^\mu k^\nu + q^\mu q^\nu - q^2 g^{\mu\nu}\right] F_2\left(k^2,q^2\right) + \frac{1}{m_B}\left[\frac{k\cdot q}{k^2}q^\mu k^\nu - \frac{q^2}{k^2}k^\mu k^\nu\right] F_3\left(k^2,q^2\right) + \frac{\mathrm{i}}{m_B}\epsilon^{\mu\nu\rho\sigma}k_\rho q_\sigma F_4\left(k^2,q^2\right)$$

$$F_V = F_4, \quad F_{A_\perp} = \frac{(k\cdot q)F_1(k^2,q^2) - q^2 F_2(k^2,q^2)}{(k+q)\cdot q},$$

$$F_{A_\parallel} = \frac{q^2(F_1(k^2,q^2) + F_2(k^2,q^2))}{(k+q)\cdot q}, \quad F_P = -\frac{(k+q)\cdot q}{k^2}\left(F_2(k^2,q^2) - F_3(k^2,q^2)\right)$$

[1]

[1] M. Beneke, P. B¨oer, P. Rigatos and K. K. Vos, QCD factoriNeverthelesszation of the four-lepton decay $B^- \to \ell^- \bar{\nu}_\ell \ell'^+ \ell'^-$, Eur. Phys. J. C 81 (2021) 638, [2102.10060].

Extra 2

It is easy to check that in the limit $q^2 \to 0$ the form factors become $F_{A_\perp} = F_1(k^2, q^2)$ and $F_{A_\parallel} = 0$. A fourth form factor F_P is not defined in the common decomposition, since it does not contribute in the commonly discussed limit $m_l \to 0$. Nevertheless, it would be expressed in terms of our form factor F_3.

Extra 3

$$-T_1^{\text{2pt,tw3}} = T_2^{\text{2pt,tw3}} = -T_4^{\text{2pt,tw3}} = \frac{m_B^3 Q_u}{2\,(v\cdot q)^2\,(\omega - \hat{q}^2)} \quad (1)$$