On the contribution from the light quarks to $H \to \gamma\gamma$ and $H \to \gamma Z$

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based on work with Baiyi QIAN, 2507.22551

PROLOG

Light quark contributions to Higgs decays

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(Dated: February 5, 2025)

The literature establishes that the light fermions contributions to the decays $H \to Z\gamma$ and $H \to \gamma\gamma$ are negligible since their coupling with the Higgs is proportional to m_f. In the present letter, we show that although such a conclusion is true for leptons, the light quark contributions are zero when we consider their non-perturbative effects.

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A. I. Hernández-Juárez, R. Gaitán, and R. Martinez, arXiv:2502.01668 [hep-ph]

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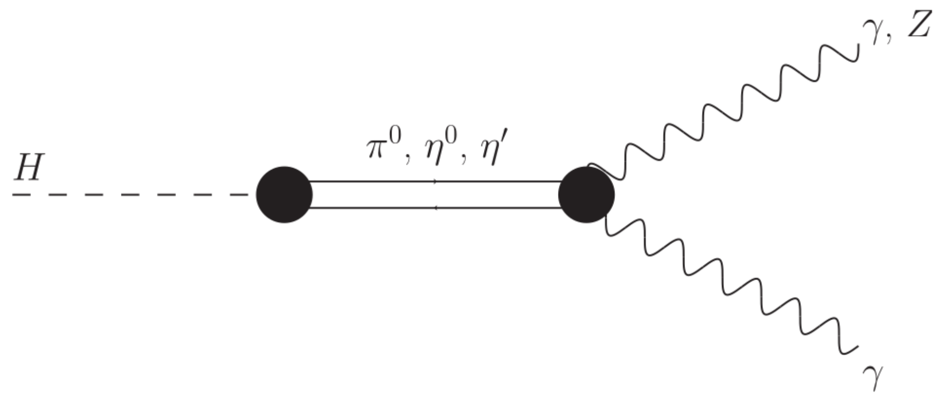

FIG. 3. $H \to V\gamma$ ($V = Z, \gamma$) decay mediated by the π^0, η^0 and η' mesons.

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From the trivial vanishing of the contributions from π^0, η and η' mesons to the amplitudes for the $H \to \gamma\gamma$ and $H \to Z\gamma$ decays, the authors of arXiv:2502.01668 conclude that

the light fermions do not contribute to the $H \to \gamma\gamma$ decay.

The contributins [to $H \to \gamma\gamma$ and $H \to Z\gamma$] from the up, down and strange quarks are zero...

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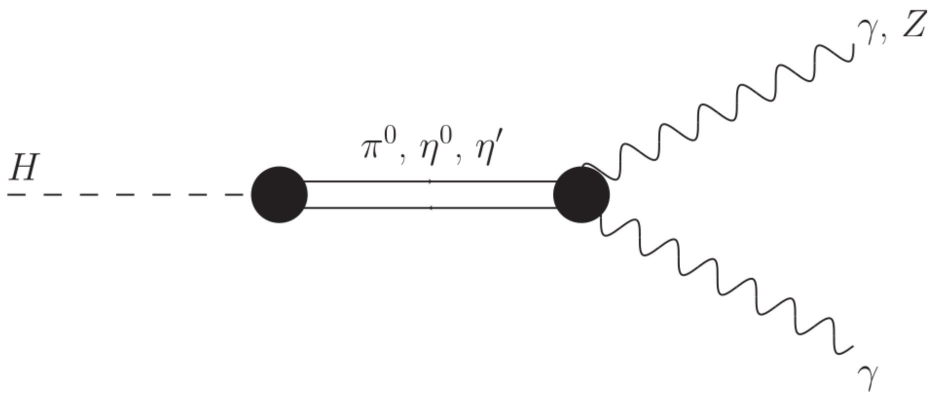

FIG. 3. $H \to V\gamma$ ($V = Z, \gamma$) decay mediated by the π^0, η^0 and η' mesons.

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The contributins [to $H \to \gamma\gamma$ and $H \to Z\gamma$] from the up, down and strange quarks are zero... [?!]

OUTLINE

- Introduction

- A few things (I think) I understand

- Things I would like to understand better

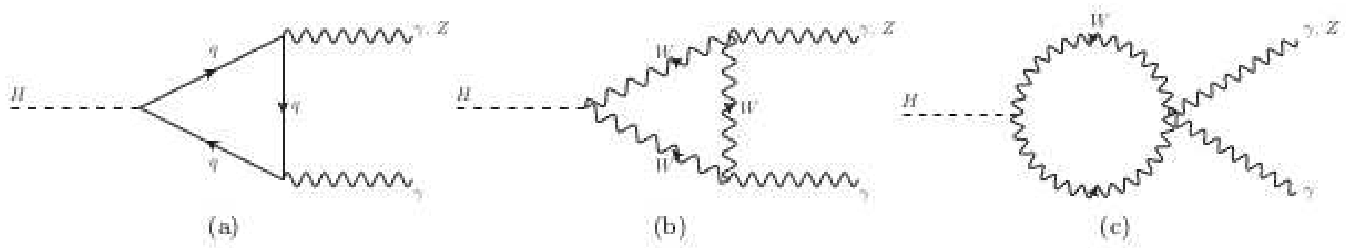
- Summary — Conclusion

Introduction

$H \to \gamma\gamma$ discovery channel of the BEH scalar boson at LHC

$H \to \gamma\gamma$ important channel for experimental determination of M_H

$H \to \gamma\gamma$ and $H \to \gamma Z$ are arise only at loop level in the SM (interesting for BSM physics)



Effective description through dimension five terms

$$\mathcal{L}_{\text{eff}} \supset -\frac{1}{4} C_{H\gamma\gamma} H F^{\mu\nu} F_{\mu\nu} - \frac{1}{2} C_{H\gamma Z} H F^{\mu\nu} Z_{\mu\nu}$$

$$C_{H\gamma X} = C^{\text{SM}}_{H\gamma X} + C^{\text{BSM}}_{H\gamma X} \qquad C^{\text{SM}}_{H\gamma X} = C^{(W)}_{H\gamma X} + \sum_f C^{(f)}_{H\gamma X} \quad X = \gamma, Z$$

in perturbation theory $C^{(f)}_{H\gamma X} \sim \mathcal{O}(m_f^2)$

perturbation theory appropriate for the leptons...

... and to some extent for t (pole mass) or b and c [$m_{b,c} = \overline{m}_{b,c}(m_{b,c})$]

see e.g. A. Djouadi, Phys. Rept. 457, 1 (2008) [arXiv:hep-ph/0503172] in perturbation theory $C_{H\gamma X}^{(f)} \sim \mathcal{O}(m_f^2)$

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what about the light quarks, u, d, s?

	pQCD
$C^{(t)}_{H\gamma\gamma}$	$-8.66 \cdot 10^{-6}$
$C^{(b)}_{H\gamma\gamma}$	$(1.13 - 1.49i) \cdot 10^{-7}$
$C^{(c)}_{H\gamma\gamma}$	$(9.22 - 7.57i) \cdot 10^{-8}$
$C^{(s)}_{H\gamma\gamma}$	$(3.35 - 1.56i) \cdot 10^{-10}$
$C^{(d)}_{H\gamma\gamma}$	$(1.77 - 0.57i) \cdot 10^{-12}$
$C^{(u)}_{H\gamma\gamma}$	$(1.81 - 0.53i) \cdot 10^{-12}$
$C^{(t)}_{H\gamma Z}$	$-3.30 \cdot 10^{-6}$
$C^{(b)}_{H\gamma Z}$	$(6.82 - 3.83i) \cdot 10^{-8}$
$C^{(c)}_{H\gamma Z}$	$(1.05 - 0.41i) \cdot 10^{-8}$
$C^{(s)}_{H\gamma Z}$	$(7.93 - 1.89i) \cdot 10^{-11}$
$C^{(d)}_{H\gamma Z}$	$(2.94 - 0.48i) \cdot 10^{-13}$
$C^{(u)}_{H\gamma Z}$	$(8.06 - 1.22i) \cdot 10^{-14}$

units: GeV^{-1}

$$C^{(W)}_{H\gamma\gamma} \sim -4.5 \cdot C^{(t)}_{H\gamma\gamma} \qquad C^{(W)}_{H\gamma Z} \sim -18 \cdot C^{(t)}_{H\gamma Z}$$

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since pQCD not appropriate for light quarks, then how could one compute their contribution non perturbatively?

A few things (I think) I understand

starting point provided by the SM Lagrangian

$$\mathcal{L}_{\text{int}}^{\text{SM}} = gM_W H(W^+ \cdot W^-) - \frac{g}{2M_W} H J_{(H)} - eA_\mu J^\mu_{(\gamma)} - \frac{g}{2c_w} Z_\mu J^\mu_{(Z)} + \cdots,$$

$$J_{(H)} = \sum_f m_f \bar{\psi}_f \psi_f, \quad J^\mu_{(\gamma)} = \sum_f e_f \bar{\psi}_f \gamma^\mu \psi_f, \quad J^\mu_{(Z)} = \sum_f \bar{\psi}_f \gamma^\mu (\mathrm{v}_f - \mathrm{a}_f \gamma_5) \psi_f,$$

in the absence of EW corrections the decay amplitudes

$$\mathcal{A}_{\gamma\gamma}^{(f)\lambda_1\lambda_2} = ge^2 \frac{m_f}{2M_W} \Gamma_{\gamma\gamma}^{(f)\mu\nu}(q_1, q_2) \epsilon_\mu^{\lambda_1}(q_1)^* \epsilon_\nu^{\lambda_2}(q_2)^* \Big|_{q_1^2 = q_2^2 = 0, q_3^2 = M_H^2},$$

$$\mathcal{A}_{\gamma Z}^{(f)\lambda_1\lambda_2} = g^2 e \frac{m_f}{4c_{\mathrm{w}} M_W} \Gamma_{\gamma Z}^{(f)\mu\nu}(q_1, q_2) \epsilon_\mu^{\lambda_1}(q_1)^* \epsilon_\nu^{\lambda_2}(q_2)^* \Big|_{q_1^2 = 0, q_2^2 = M_Z^2, q_3^2 = M_H^2},$$

involve the connected three-point functions

$$\Gamma_{\gamma X}^{(f)\mu\nu}(q_1, q_2) = \int d^4x \, e^{iq_1 \cdot x} \int d^4y \, e^{iq_2 \cdot y} \langle 0 | T\{[\bar{\psi}_f \psi_f](0) J^\mu_{(\gamma)}(x) J^\nu_{(X)}(y)\} | 0 \rangle_0. \quad X = \gamma, Z.$$

computed in free-field theory (for leptons) or in pure QCD (for quarks)

when computed perturbatively

$$\Gamma_{\gamma X}^{(f)\mu\nu}(q_1, q_2) \sim \mathcal{O}(m_f) \qquad \mathcal{A}_{\gamma X}^{(f)\lambda_1\lambda_2}, C_{H\gamma X}^{(f)} \sim \mathcal{O}(m_f^2)$$

from now on, concentrate on the light quarks

$$J^{\rm s}_{(H)} = \frac{{\rm tr}(\mathcal{M})}{3}\bar\psi\psi,\ \ J^{\rm ns}_{(H)} = \bar\psi\ \overline{\mathcal{M}}\ \psi,\ \ J^{\mu}_{(\gamma)} = \bar\psi\mathcal{Q}\gamma^\mu\psi,\ \ J^{\mu}_{(Z)} = \bar\psi\mathcal{Q}_Z\gamma^\mu\psi{-}2s_{\rm w}^2J^\mu_{(\gamma)} = -\frac{1}{6}\bar\psi\gamma^\mu\psi{+}(1{-}2s_{\rm w}^2)\bar\psi\mathcal{Q}\gamma^\mu\psi$$

$$\psi \equiv \begin{pmatrix} u \\ d \\ c \end{pmatrix},\ \ \mathcal{M} = {\rm diag}(m_u, m_d, m_s),\quad \overline{\mathcal{M}}\equiv \mathcal{M}{-}\frac{1}{3}{\rm tr}(\mathcal{M}),\ \ \mathcal{Q} = {\rm diag}\left(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}\right),\ \ \mathcal{Q}_Z = {\rm diag}\left(\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right)$$

take the chiral limit $m_u, m_d, m_s \to 0$

only three QCD three-point functions to consider [$q_3 \equiv q_1 + q_2$]

$$\Gamma^{\mu\nu}_{H_{\rm ns}\gamma\gamma}(q_1,q_2),\ \ \Gamma^{\mu\nu}_{H_{\rm s}\gamma\gamma}(q_1,q_2),\ \ \Gamma^{\mu\nu}_{H_{\rm ns}\gamma Z_{\rm s}}(q_1,q_2) \qquad \{q_1^\mu\,;\,q_2^\nu\}\Gamma^{\mu\nu}(q_1,q_2) = \{0\,;0\}$$

$$\begin{aligned}
\Gamma^{\mu\nu}_{H_{\rm ns}\gamma\gamma}(q_1,q_2) &= 4{\rm tr}(\overline{\mathcal{M}}\,\mathcal{Q}\mathcal{Q}) \times \left[\mathcal{F}_{\gamma\gamma}(q_1^2,q_2^2,q_3^2)P^{\mu\nu}(q_1,q_2) + \mathcal{G}_{\gamma\gamma}(q_1^2,q_2^2,q_3^2)Q^{\mu\nu}(q_1,q_2)\right], \\
\Gamma^{\mu\nu}_{H_{\rm s}\gamma\gamma}(q_1,q_2) &= \frac{4}{3}\,{\rm tr}(\mathcal{M}){\rm tr}(\mathcal{Q}\mathcal{Q}) \times \left[\tilde{\mathcal{F}}_{\gamma\gamma}(q_1^2,q_2^2,q_3^2)P^{\mu\nu}(q_1,q_2) + \tilde{\mathcal{G}}_{\gamma\gamma}(q_1^2,q_2^2,q_3^2)Q^{\mu\nu}(q_1,q_2)\right], \\
\Gamma^{\mu\nu}_{H_{\rm ns}\gamma Z_{\rm s}}(q_1,q_2) &= -\frac{2}{3}{\rm tr}(\overline{\mathcal{M}}\,\mathcal{Q}) \times \left[\mathcal{F}_{\gamma Z}(q_1^2,q_2^2,q_3^2)P^{\mu\nu}(q_1,q_2) + \mathcal{G}_{\gamma Z}(q_1^2,q_2^2,q_3^2)Q^{\mu\nu}(q_1,q_2)\right].
\end{aligned}$$

$$P^{\mu\nu}(q_1,q_2) = q_1^\nu q_2^\mu - (q_1\cdot q_2)\eta^{\mu\nu},\ \ Q^{\mu\nu}(q_1,q_2) = q_1^2 q_2^\mu q_2^\nu + q_2^2 q_1^\mu q_1^\nu - (q_1\cdot q_2)q_1^\mu q_2^\nu - q_1^2 q_2^2\eta^{\mu\nu}$$

note: $\epsilon_\mu^{\lambda_1}(q_1)^* Q^{\mu\nu}(q_1,q_2)$ vanishes for $q_1^2 = 0$

expressions for the effective couplings (terms linear in the light-quark masses)

$$C^{(u)}_{H\gamma\gamma} = -\frac{ge^2}{2}\frac{m_u}{M_W}\, e_u^2 \times 2\left[\mathcal{F}_{\gamma\gamma}(0,0,M_H^2) + \tilde{\mathcal{F}}_{\gamma\gamma}(0,0,M_H^2)\right]$$

$$C^{(d)}_{H\gamma\gamma} = \frac{m_d}{m_s}\, C^{(s)}_{H\gamma\gamma} = -\frac{ge^2}{2}\frac{m_d}{M_W}\, e_d^2 \times 4\left[-\mathcal{F}_{\gamma\gamma}(0,0,M_H^2) + 2\tilde{\mathcal{F}}_{\gamma\gamma}(0,0,M_H^2)\right]$$

$$C^{(u)}_{H\gamma Z} = -\frac{g^2 e}{4}\frac{m_u}{c_{\rm w} M_W}\, e_u \times \frac{2}{3}\left\{-\mathcal{F}_{\gamma Z}(0,M_Z^2,M_H^2) + 2(1-2s_{\rm w}^2)\left[\mathcal{F}_{\gamma\gamma}(0,M_Z^2,M_H^2) + \tilde{\mathcal{F}}_{\gamma\gamma}(0,M_Z^2,M_H^2)\right]\right\}$$

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note: the differences $\mathcal{F}_{\gamma\gamma}(q_1^2,q_2^2,q_3^2) - \tilde{\mathcal{F}}_{\gamma\gamma}(q_1^2,q_2^2,q_3^2)$ and $\mathcal{F}_{\gamma\gamma}(q_1^2,q_2^2,q_3^2) - \mathcal{F}_{\gamma Z}(q_1^2,q_2^2,q_3^2)$ are OZI suppressed

$$\mathcal{F}_{\gamma\gamma}(q_1^2,q_2^2,q_3^2) - \tilde{\mathcal{F}}_{\gamma\gamma}(q_1^2,q_2^2,q_3^2),\ \mathcal{F}_{\gamma\gamma}(q_1^2,q_2^2,q_3^2) - \mathcal{F}_{\gamma Z}(q_1^2,q_2^2,q_3^2) \sim \mathcal{O}(1/N_c)$$

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$$C^{(u)}_{H\gamma Z} = -\frac{g^2e}{4}\frac{m_u}{c_{\rm w}M_W}\,e_u\times\frac{2}{3}\left\{-\mathcal{F}_{\gamma Z}(0,M_Z^2,M_H^2)+2(1-2s_{\rm w}^2)\left[\mathcal{F}_{\gamma\gamma}(0,M_Z^2,M_H^2)+\tilde{\mathcal{F}}_{\gamma\gamma}(0,M_Z^2,M_H^2)\right]\right\}$$

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note: the differences $\mathcal{F}_{\gamma\gamma}(q_1^2,q_2^2,q_3^2)-\tilde{\mathcal{F}}_{\gamma\gamma}(q_1^2,q_2^2,q_3^2)$ and $\mathcal{F}_{\gamma\gamma}(q_1^2,q_2^2,q_3^2)-\mathcal{F}_{\gamma Z}(q_1^2,q_2^2,q_3^2)$ are OZI suppressed

$$\mathcal{F}_{\gamma\gamma}(q_1^2,q_2^2,q_3^2)-\tilde{\mathcal{F}}_{\gamma\gamma}(q_1^2,q_2^2,q_3^2),\ \mathcal{F}_{\gamma\gamma}(q_1^2,q_2^2,q_3^2)-\mathcal{F}_{\gamma Z}(q_1^2,q_2^2,q_3^2)\sim\mathcal{O}(1/N_c)$$

the three-point functions under considerations are singlets of flavour $SU(3)_V$ (eightfold way) but are *not* singlets of the full flavour group $SU(3)_L\times SU(3)_R$ in the chiral limit!

this means that when computed perturbatively in QCD, these functions vanish at each loop order!

[in the chiral limit]

what about full QCD?

let's concentrate on $\mathcal{F}_{\gamma\gamma}(q_1^2, q_2^2, q_3^2)$ and consider

$$\Gamma_{\mu\nu}^{abc}(q_1, q_2) = \int d^4x\, e^{iq_1\cdot x} \int d^4y\, e^{iq_2\cdot y} \langle 0|T\{S^a(0)V_\mu^b(x)V_\nu^c(y)\}|0\rangle$$
$$= d^{abc}\left[\mathcal{F}(q_1^2, q_2^2, q_3^2)P_{\mu\nu}(q_1, q_2) + \mathcal{G}(q_1^2, q_2^2, q_3^2)Q_{\mu\nu}(q_1, q_2)\right]$$

defined in terms of the colour-singlet and flavour-octet quark bilinears

$$S^a = \bar\psi\frac{\lambda^a}{2}\psi, \quad V_\mu^a = \bar\psi\frac{\lambda^a}{2}\gamma_\mu\psi,$$

from the OPE one obtains, in the chiral limit

$$\lim_{\lambda\to\infty} \lambda^2\Gamma_{\mu\nu}^{abc}(\lambda q_1, \lambda q_2) = -d^{abc}\frac{\langle\bar q q\rangle_0}{2q_1^2q_2^2q_3^2}\left\{\left[(q_1^2+q_2^2)[1+\mathcal{O}(\alpha_s)]+q_3^2[1+\mathcal{O}(\alpha_s)]\right]P_{\mu\nu}(q_1, q_2)+2[1+\mathcal{O}(\alpha_s)]Q_{\mu\nu}(q_1, q_2)\right\}+\cdots$$

when the momenta q_1, q_2 and q_3 are all in the deep-Euclidean region, with

$$\langle\bar q q\rangle_0 \equiv \langle\bar u u\rangle_0 = \langle\bar d d\rangle_0 = \langle\bar s s\rangle_0$$

the single-flavour quark condensate in the chiral limit B. Moussallam and J. Stern, [arXiv:hep-ph/9404353] or, when only q_2 and q_3 become large in the Euclidian region,

$$\lim_{\lambda\to\infty} \lambda\Gamma^{abc}_{\mu\nu}(q_1, \lambda q_2) = d^{abc}\,\Pi_{VT}(q_1^2)\frac{1}{q_2^2}[1+\mathcal{O}(\alpha_s)]P_{\mu\nu}(q_1,q_2)+\cdots,$$

where $\Pi_{VT}(q^2)$ is given by the vector-tensor correlator in the chiral limit

$$\frac{i}{2}\int d^4x\, e^{iq\cdot x}\langle 0|T\Big\{V^a_\mu(x)\Big(\bar\psi\frac{\lambda^b}{2}[\gamma_\nu,\gamma_\rho]\psi\Big)(0)\Big\}|0\rangle = \delta^{ab}\Pi_{VT}(q^2)(q_\nu\eta_{\mu\rho}-q_\rho\eta_{\mu\nu})$$

with

$$\Pi_{VT}(0) \propto \langle \bar q q\rangle_0 \times \chi_q$$

B. L. Ioffe and A. V. Smilga, Nucl. Phys. B 232, 109 (1984)

in chiral perturbation theory one finds

$$\mathcal{F}(0,0,0)\neq 0$$

even in the chiral limit

M. K., B. Moussallam and J. Stern, Nucl. Phys. B 429, 125 (1994) or, when only q_2 and q_3 become large in the Euclidian region,

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ample evidence, from all scales, that in the chiral limit

$$\mathcal{F}_{\gamma\gamma}(0,0,0)\neq 0, \quad \tilde{\mathcal{F}}_{\gamma\gamma}(0,0,0)\neq 0, \quad \mathcal{F}_{\gamma Z}(0,0,0)\neq 0$$

i.e. that $C^{(q)}_{H\gamma X}=\mathcal{O}(m_q) \quad q=u,d,s \quad X=\gamma,Z$

Things I would like to understand better

how to evaluate the effective couplings for the light quarks in the non-perturbative regime of QCD

difficult task, take the limit $N_c \to \infty$ as a guide

then $\mathcal{F}(q_1^2, q_2^2, q_3^2) = \mathcal{F}_{\gamma\gamma}(q_1^2, q_2^2, q_3^2) = \mathcal{F}_{\gamma Z}(q_1^2, q_2^2, q_3^2) = \tilde{\mathcal{F}}_{\gamma\gamma}(q_1^2, q_2^2, q_3^2)$

furthermore $[m_u = m_d \equiv \hat{m}]$

$$\mathcal{F}(0,0,0) = \frac{9}{5}\,\frac{\tilde{c}}{m_s - \hat{m}} + \mathcal{O}(m_q) \quad \tilde{c} = \frac{5}{16\pi\alpha^2}\left[\frac{\Gamma_{\rho\to e^+e^-}}{M_\rho} - 3\frac{\Gamma_{\omega\to e^+e^-}}{M_\omega} - 3\frac{\Gamma_{\phi\to e^+e^-}}{M_\phi}\right] = 4.6(0.8)\cdot 10^{-3}$$

M. K., B. Moussallam and J. Stern, Nucl. Phys. B 429, 125 (1994)

and

$$\Pi(0) = -\frac{8}{3}(L_{10} + 2H_1) - \frac{4}{9}\mathcal{F}(0,0,0)(5\hat{m} + m_s) + \mathcal{O}(m_q^2)$$

with

$$(q^\mu q^\nu - \eta^{\mu\nu} q^2)\Pi(q^2) = i\int d^4x e^{iq\cdot x}\,\langle 0|T\{J^\mu_{(\gamma)}(x)J^\nu_{(\gamma)}(0)\}|0\rangle$$

implying that well-known low-energy theorems

M. A. Shifman, A. I. Vainshtein, M. B. Voloshin and V. I. Zakharov, Sov. J. Nucl. Phys. 30, 711 (1979)

$$\lim_{M_H\to 0}\left[C^{(u)}_{H\gamma\gamma} + C^{(d)}_{H\gamma\gamma}\right] = \frac{ge^2}{2M_W}\times\hat{m}\frac{\partial}{\partial\hat{m}}\Pi(0) + \mathcal{O}(m_q^2) \qquad \lim_{M_H\to 0} C^{(s)}_{H\gamma\gamma} = \frac{ge^2}{2M_W}\times m_s\frac{\partial}{\partial m_s}\Pi(0) + \mathcal{O}(m_q^2)$$

are satisfied (can be extended to $N_c = 3$, i.e. chiral logs, with some additional work) in the large-N_c limit, $\mathcal{F}(q_1^2, q_2^2, q_3^2)$ is an infinite series of zero-width bound-state poles in each channel

G. 't Hooft, Nucl. Phys. B **72**, 461 (1974) E. Witten, Nucl. Phys. B **160**, 57 (1979)

lowest-meson-dominance (LMD) approximation: in each channel, the lowest-lying resonance dominates

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M. F. L. Golterman and S. Peris, Phys. Rev. D **61**, 034018 (2000)

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$$\mathcal{F}_{\mathrm{LMD}}(q_1^2, q_2^2, q_3^2) = \frac{a + b\,(q_1^2 + q_2^2) + c\,q_3^2}{(q_1^2 - M_V^2)(q_2^2 - M_V^2)(q_3^2 - M_S^2)} \qquad b = c = -\frac{1}{2}\,\langle \bar{q}q \rangle_0 \qquad a = -\frac{9}{5} M_V^4 M_S^2 \frac{\tilde{c}}{m_s - \hat{m}}$$

note: $C_{H\gamma X}^{(q)}$ depends only on RG-invariant quantities

$$\frac{m_q}{m_s - \hat{m}} \quad \text{or} \quad m_q \langle \bar{q}q \rangle_0$$

	pQCD	LMD
$C^{(t)}_{H\gamma\gamma}$	$-8.66 \cdot 10^{-6}$	$-$
$C^{(b)}_{H\gamma\gamma}$	$(1.13 - 1.49i) \cdot 10^{-7}$	$-$
$C^{(c)}_{H\gamma\gamma}$	$(9.22 - 7.57i) \cdot 10^{-8}$	$-$
$C^{(s)}_{H\gamma\gamma}$	$(3.35 - 1.56i) \cdot 10^{-10}$	$-4.41 \cdot 10^{-7}$
$C^{(d)}_{H\gamma\gamma}$	$(1.77 - 0.57i) \cdot 10^{-12}$	$-2.23 \cdot 10^{-8}$
$C^{(u)}_{H\gamma\gamma}$	$(1.81 - 0.53i) \cdot 10^{-12}$	$-4.18 \cdot 10^{-8}$
$C^{(t)}_{H\gamma Z}$	$-3.30 \cdot 10^{-6}$	$-$
$C^{(b)}_{H\gamma Z}$	$(6.82 - 3.83i) \cdot 10^{-8}$	$-$
$C^{(c)}_{H\gamma Z}$	$(1.05 - 0.41i) \cdot 10^{-8}$	$-$
$C^{(s)}_{H\gamma Z}$	$(7.93 - 1.89i) \cdot 10^{-11}$	$6.20 \cdot 10^{-11}$
$C^{(d)}_{H\gamma Z}$	$(2.94 - 0.48i) \cdot 10^{-13}$	$3.14 \cdot 10^{-12}$
$C^{(u)}_{H\gamma Z}$	$(8.06 - 1.22i) \cdot 10^{-14}$	$1.70 \cdot 10^{-12}$

units: GeV^{-1} precision: $\sim \mathcal{O}(1/N_c) \sim 30\%$

increase of the absolute value of the sum $C^{(u)}_{H\gamma\gamma} + C^{(d)}_{H\gamma\gamma} + C^{(s)}_{H\gamma\gamma}$

by at least three orders of magnitude as compared to the lowest-order perturbative evaluation

$$\Gamma(H \to \gamma\gamma)_{\mathrm{LMD}} = 0.97 \cdot \Gamma(H \to \gamma\gamma)_{\mathrm{pQCD}}$$

	pQCD	LMD
$C^{(t)}_{H\gamma\gamma}$	$-8.66 \cdot 10^{-6}$	$-$
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units: GeV^{-1} precision: $\sim \mathcal{O}(1/N_c) \sim 30\%$

no such enhancemant in $C^{(q)}_{H\gamma Z}$ (feature specific to LMD approximation)

$$\mathcal{F}_{\rm LMD}(0, M_Z^2, M_H^2) \sim \frac{1 + \frac{M_Z^2}{M_H^2}}{1 - \frac{M_Z^2}{M_V^2}} \times \mathcal{F}_{\rm LMD}(0,0,M_H^2) \sim -10^{-4} \times \mathcal{F}_{\rm LMD}(0,0,M_H^2)$$

$$\Gamma(H \to \gamma Z)_{\rm LMD} \sim \Gamma(H \to \gamma Z)_{\rm pQCD}$$

	pQCD	LMD
$C^{(t)}_{H\gamma\gamma}$	$-8.66 \cdot 10^{-6}$	$-$
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units: GeV^{-1} precision: $\sim \mathcal{O}(1/N_c) \sim 30\%$

the coefficients $C^{(q)}_{H\gamma X}$ are purely real in this description (only poles in the large-N_c limit)

imaginary parts require multi-particle states (i.e. branch points and cuts)

Summary — Conclusion

precision on $\Gamma(H \to \gamma\gamma)$ is expected to increase in the future

$$\frac{\Delta \mathrm{Br}\,(H \to \gamma\gamma)}{\mathrm{Br}\,(H \to \gamma\gamma)}\Big|_{\mathrm{PDG24}} = 8\% \quad \longrightarrow \quad \sim 1\%$$

the contribution from the light quarks is potentially less suppressed than expected from its perturbative evaluation $C^{(q)}_{H\gamma X}\big|_{\mathrm{pQCD}} \sim \mathcal{O}(m_q^2) \quad \longrightarrow \quad C^{(q)}_{H\gamma X}\big|_{\mathrm{QCD}} \sim \mathcal{O}(m_q)$

this is an exact result from QCD!

the non-perturbative evaluation of $C^{(q)}_{H\gamma X}$ represents a daunting task

several very different scales involved

$$m_\gamma = 0 \quad \Lambda_{\mathrm{had}} \sim 1\ \mathrm{GeV} \quad M_H \sim 125\ \mathrm{GeV}\ \ (M_Z)$$

LMD approximation to the large-N_c limit of QCD indicates that large enhancements are possible

but remains to be confirmed by a more elaborate calculation

should also include imaginary parts

$$\mathrm{Im}\,\mathcal{A}(H \to \gamma\gamma) = \sum_n\!\!\!\!\!\!\int d\Phi_n \langle H|n\rangle\langle n|\gamma\gamma\rangle \quad |\mathcal{A}(H \to \gamma\gamma)|^2 \geq [\mathrm{Im}\,\mathcal{A}(H \to \gamma\gamma)]^2$$

can $\varepsilon + \varepsilon + \varepsilon + \cdots$ become sizeable?