Status of V_{ub} and V_{cb} measurements at LHCb

GDR Intensity Frontier Annual Workshop
13/11/2025

Alois Caillet (LPNHE)

The CKM Matrix

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- Describes the quark **flavour changing transition couplings**

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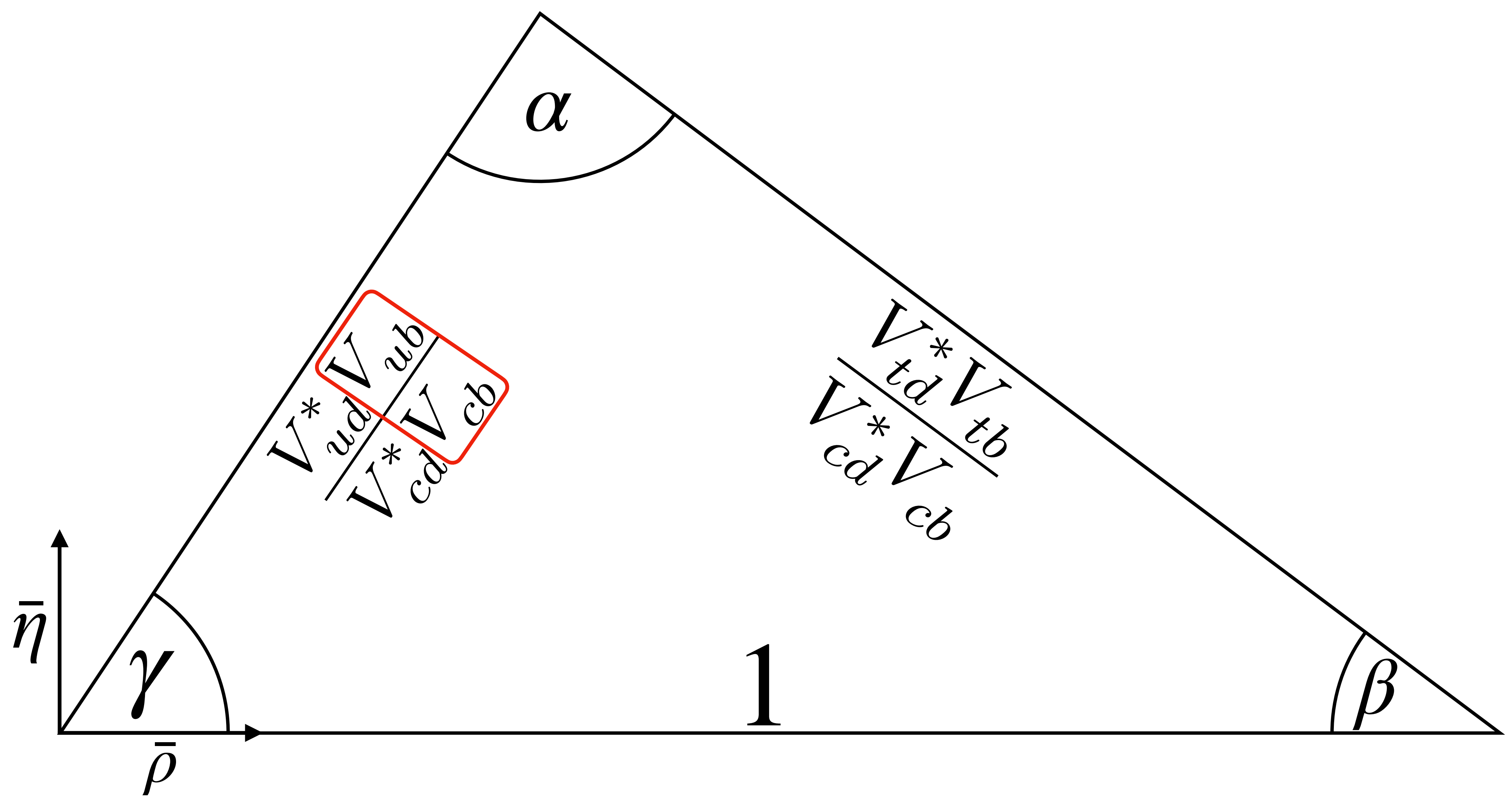
- Describes the quark **flavour changing transition couplings**

$$\Rightarrow \; V_{ud}V_{ub}^{*} + V_{cd}V_{cb}^{*} + V_{td}V_{tb}^{*} = 0$$

- Its **unitarity** translates into the **unitarity triangle**



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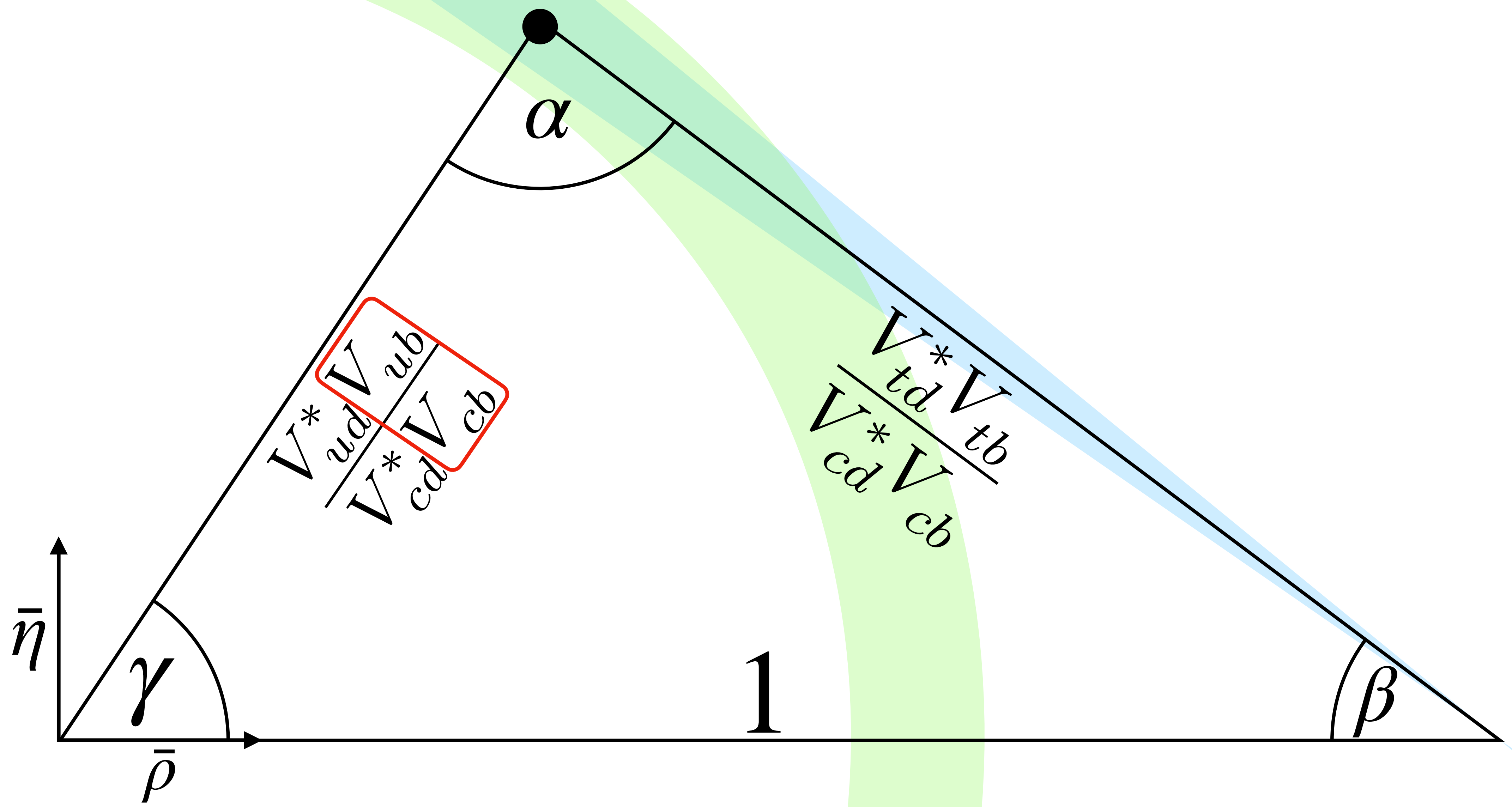
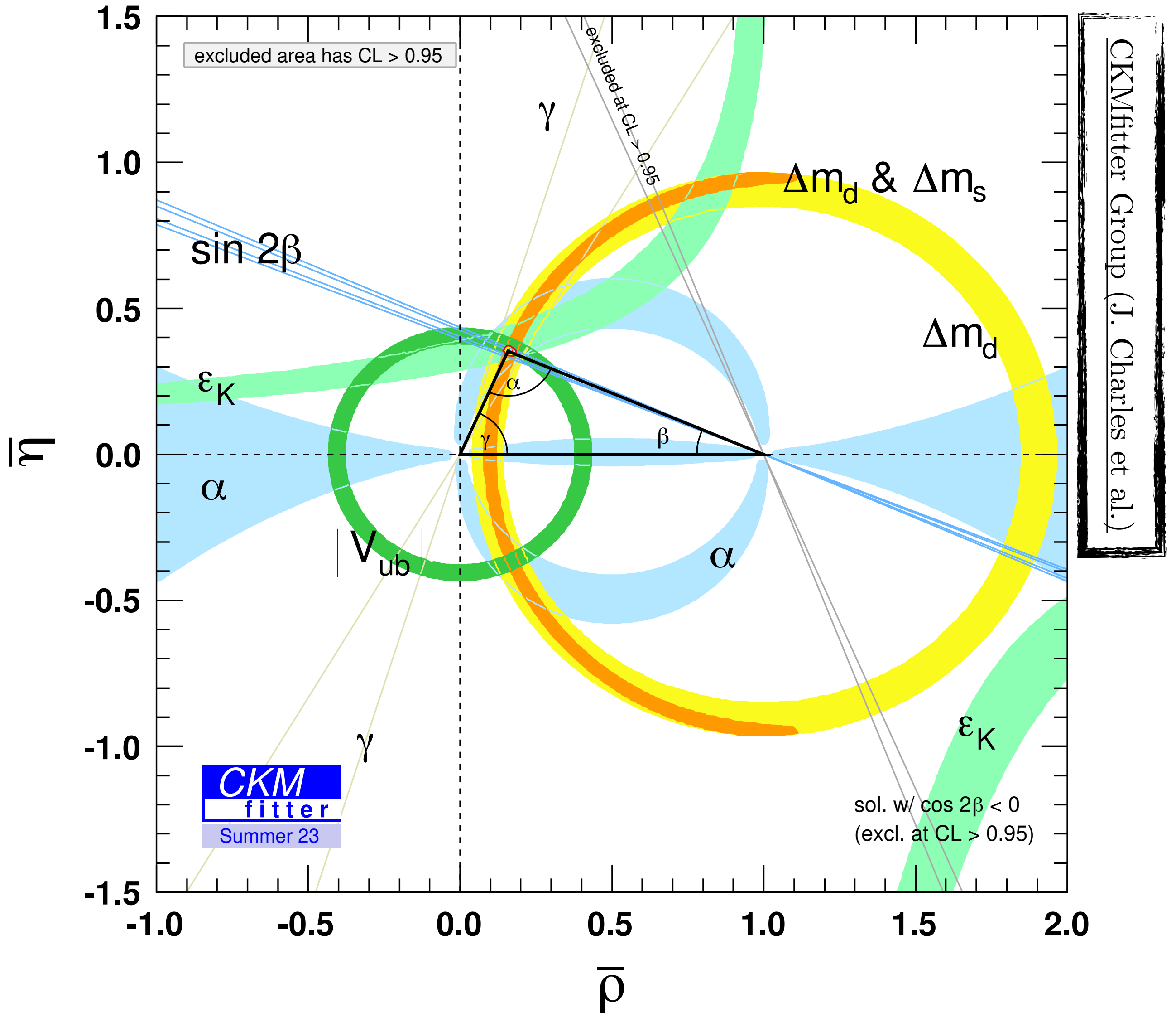
- V_{ub} and V_{cb} both connect β and its opposite side



Motivation



✦ <u>These measurements can allow:</u>

- Constraining the **Unitarity Triangle** of the CKM matrix

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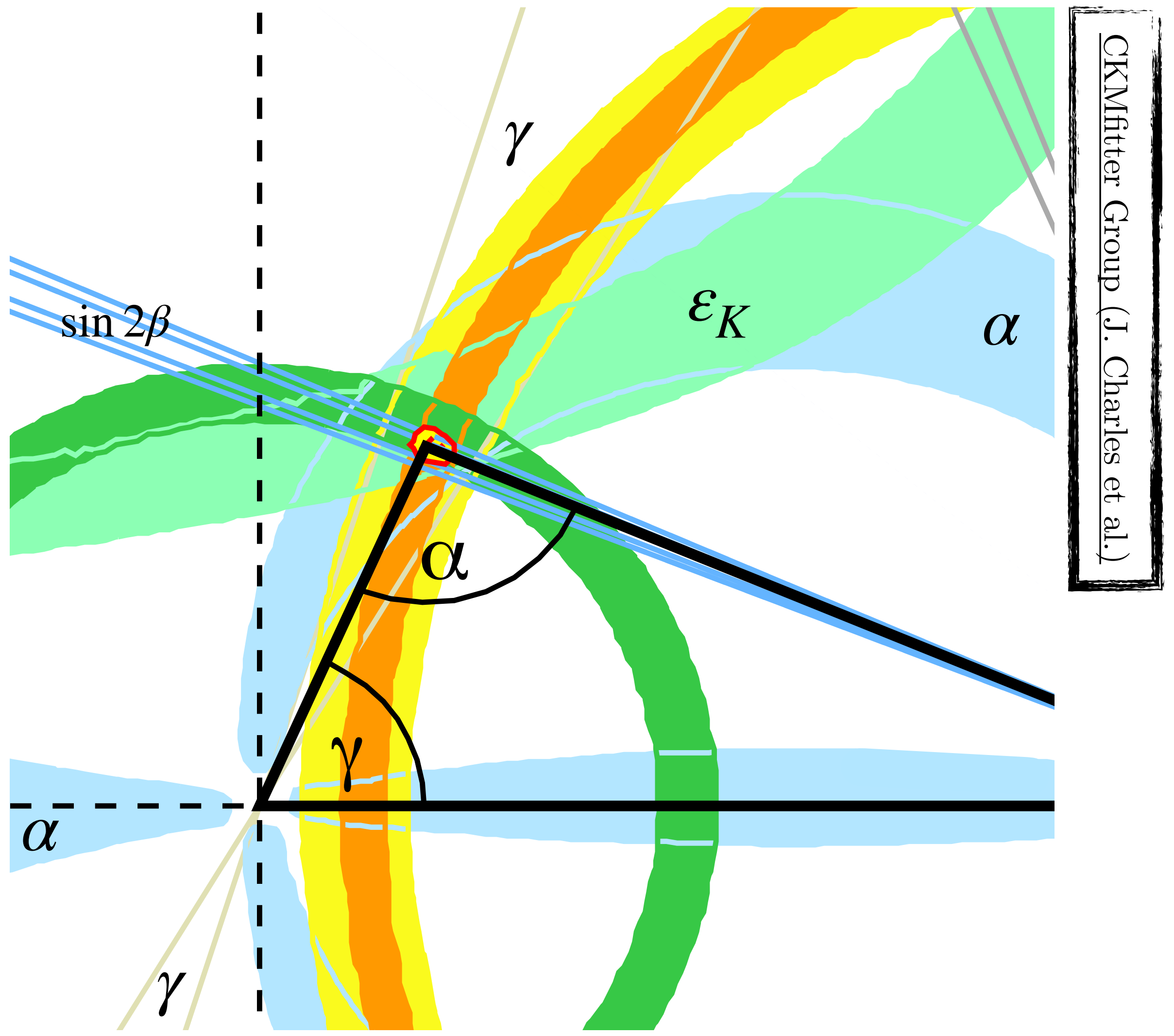

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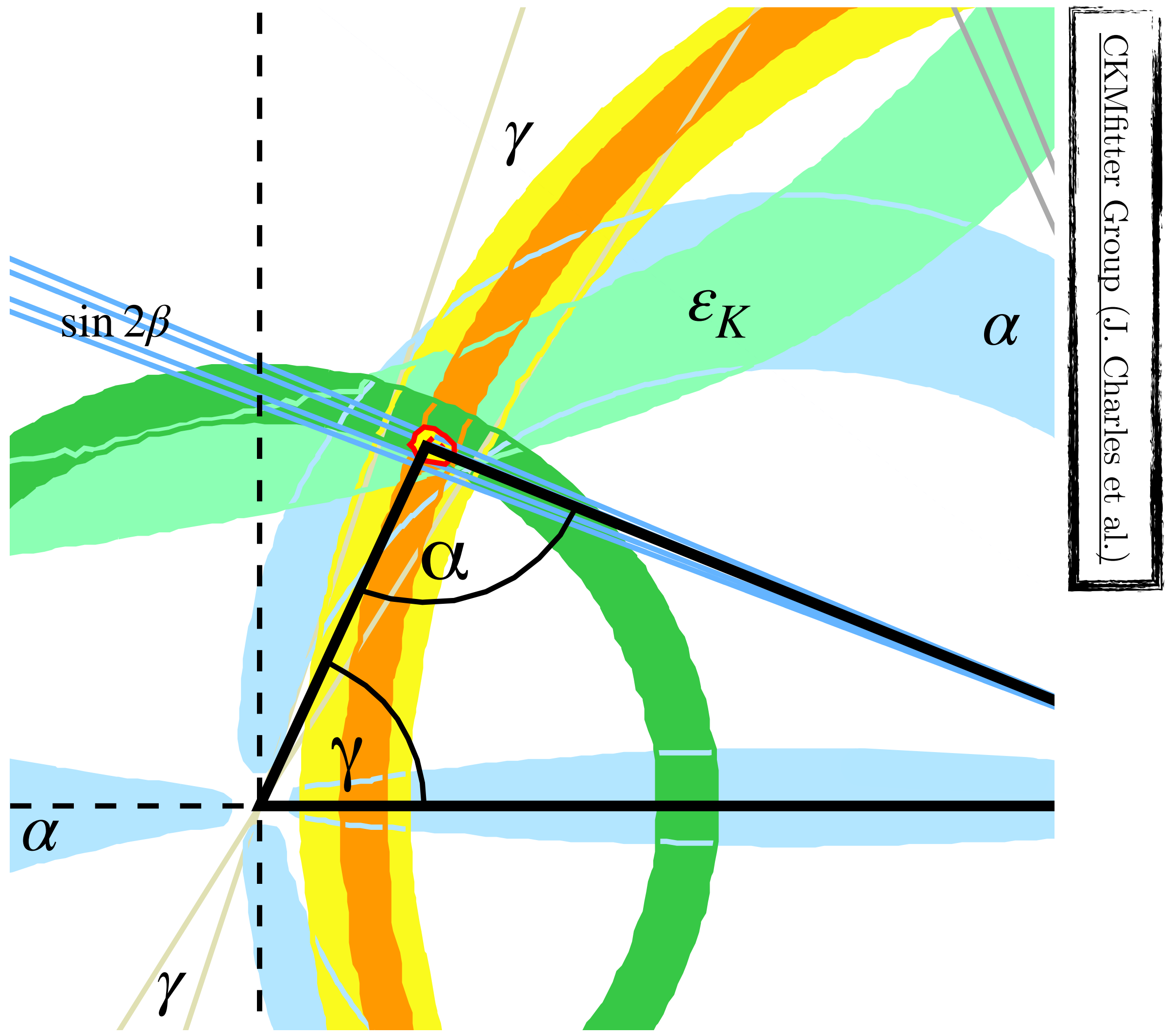

- Improving **precision** on **not well known** CKM elements

$$|V_{CKM}| = \begin{bmatrix} 0.97435 \pm 0.00016 & 0.22500 \pm 0.00067 & \boxed{0.00369 \pm 0.00011} \\ 0.22486 \pm 0.00067 & 0.97349 \pm 0.00016 & \boxed{0.04182^{+0.00085}_{-0.00074}} \\ 0.00857^{+0.00020}_{-0.00018} & 0.04110^{+0.00083}_{-0.00072} & 0.999118^{+0.000031}_{-0.000036} \end{bmatrix}$$

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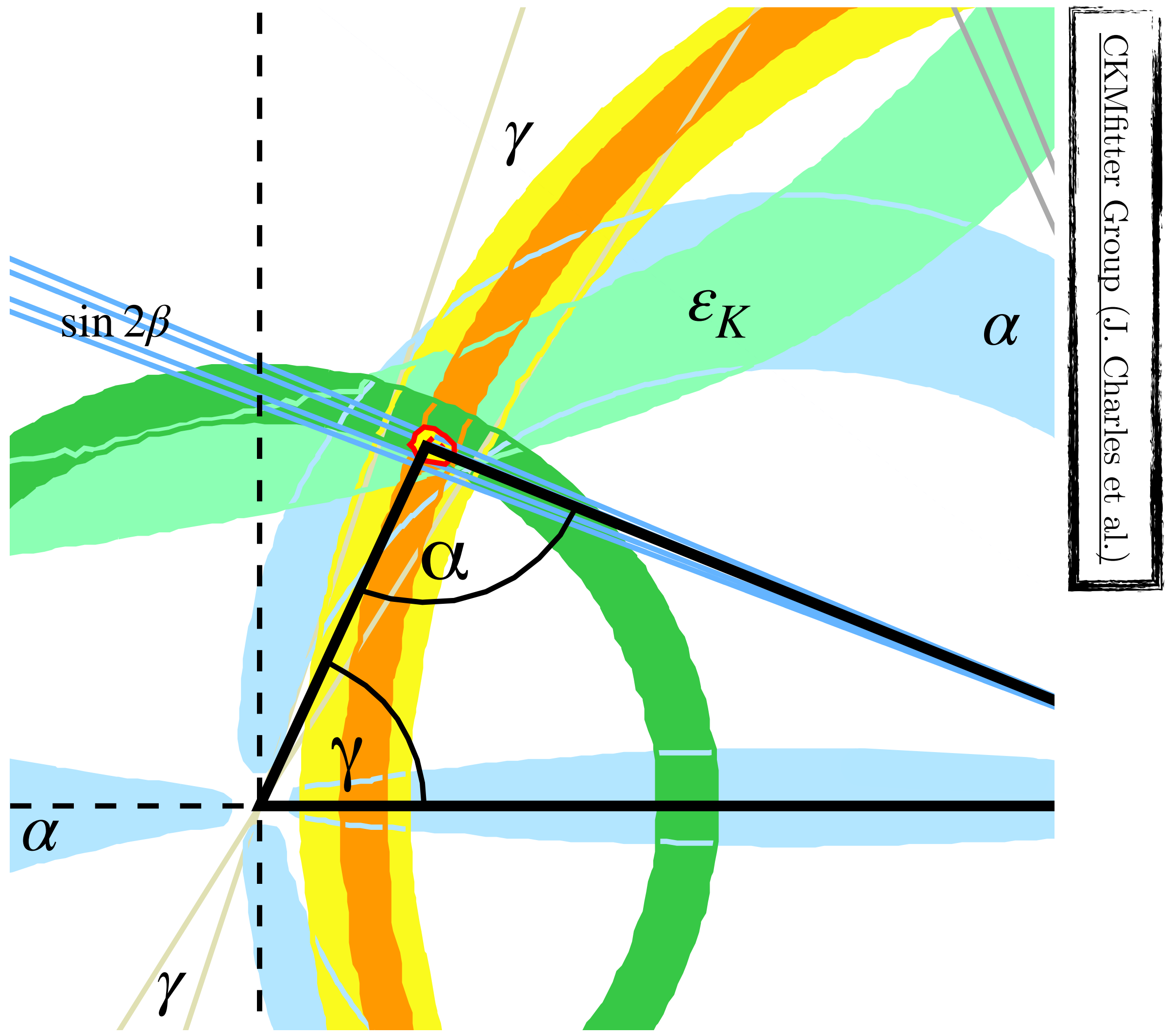

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- ◉ Probing **New Physics**

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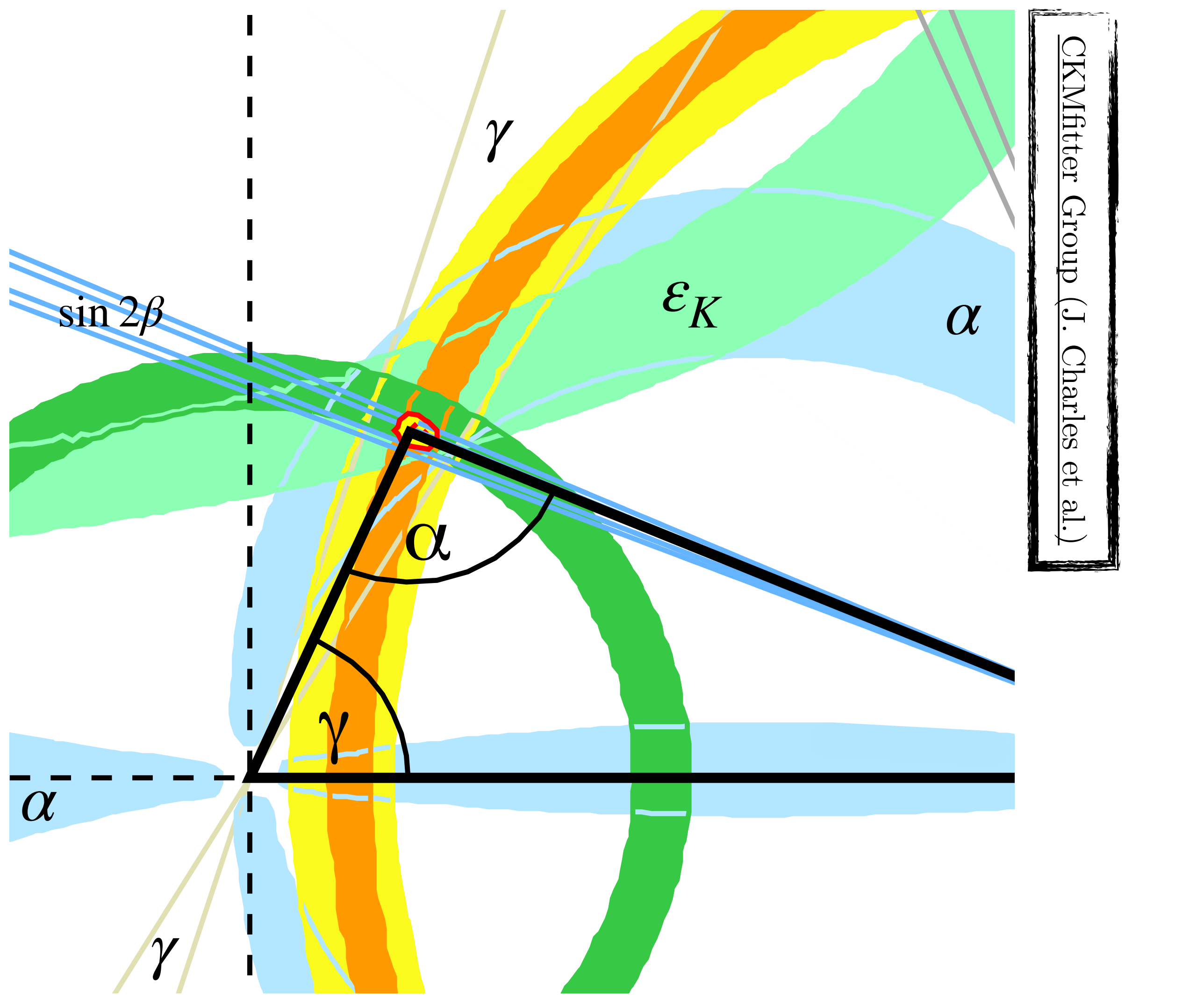

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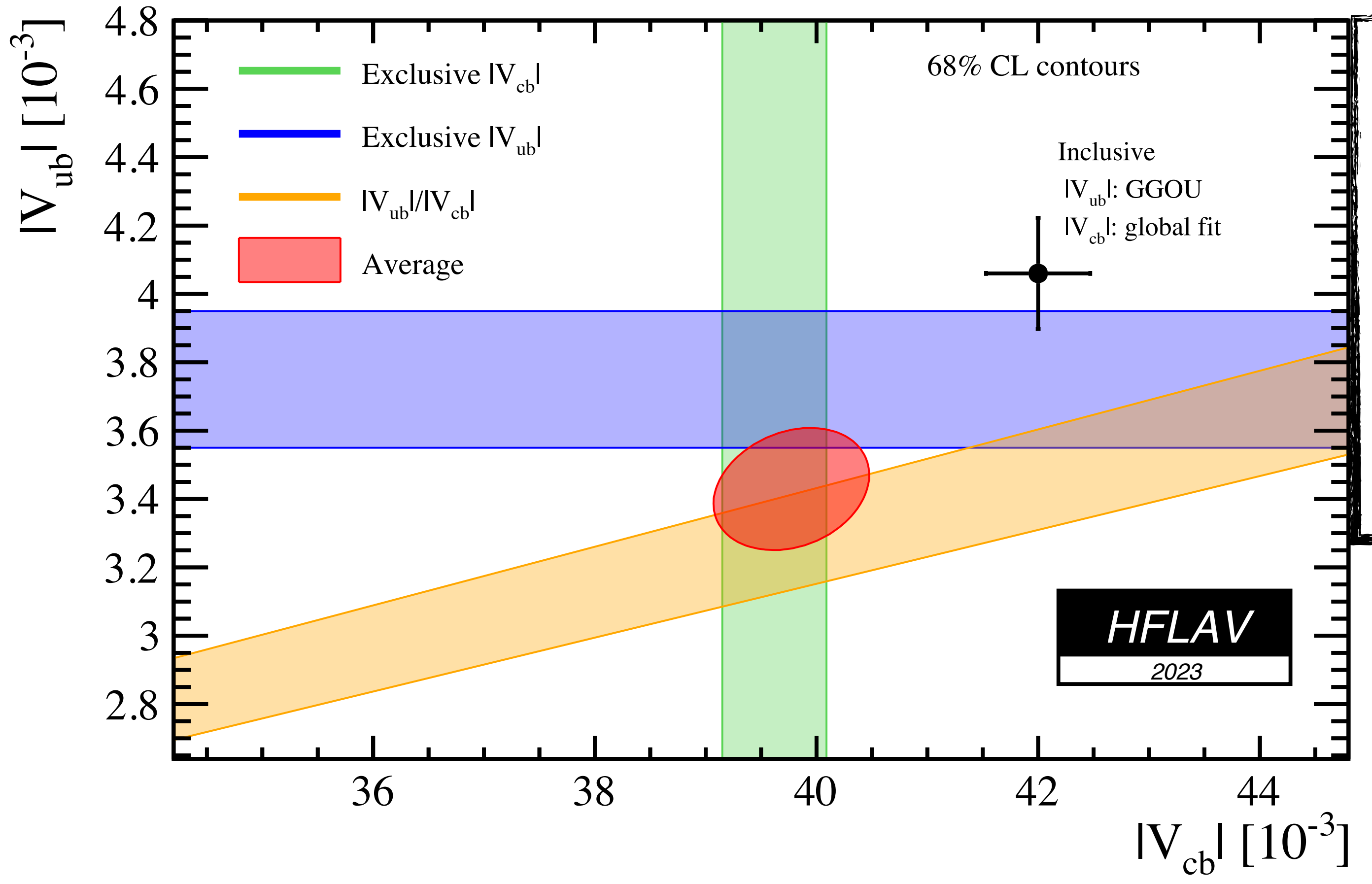
- ◉ Helping to resolve **tension** between **exclusive** and **inclusive** measurement



arXiv:2411.18639 [hep-ex]

Motivation

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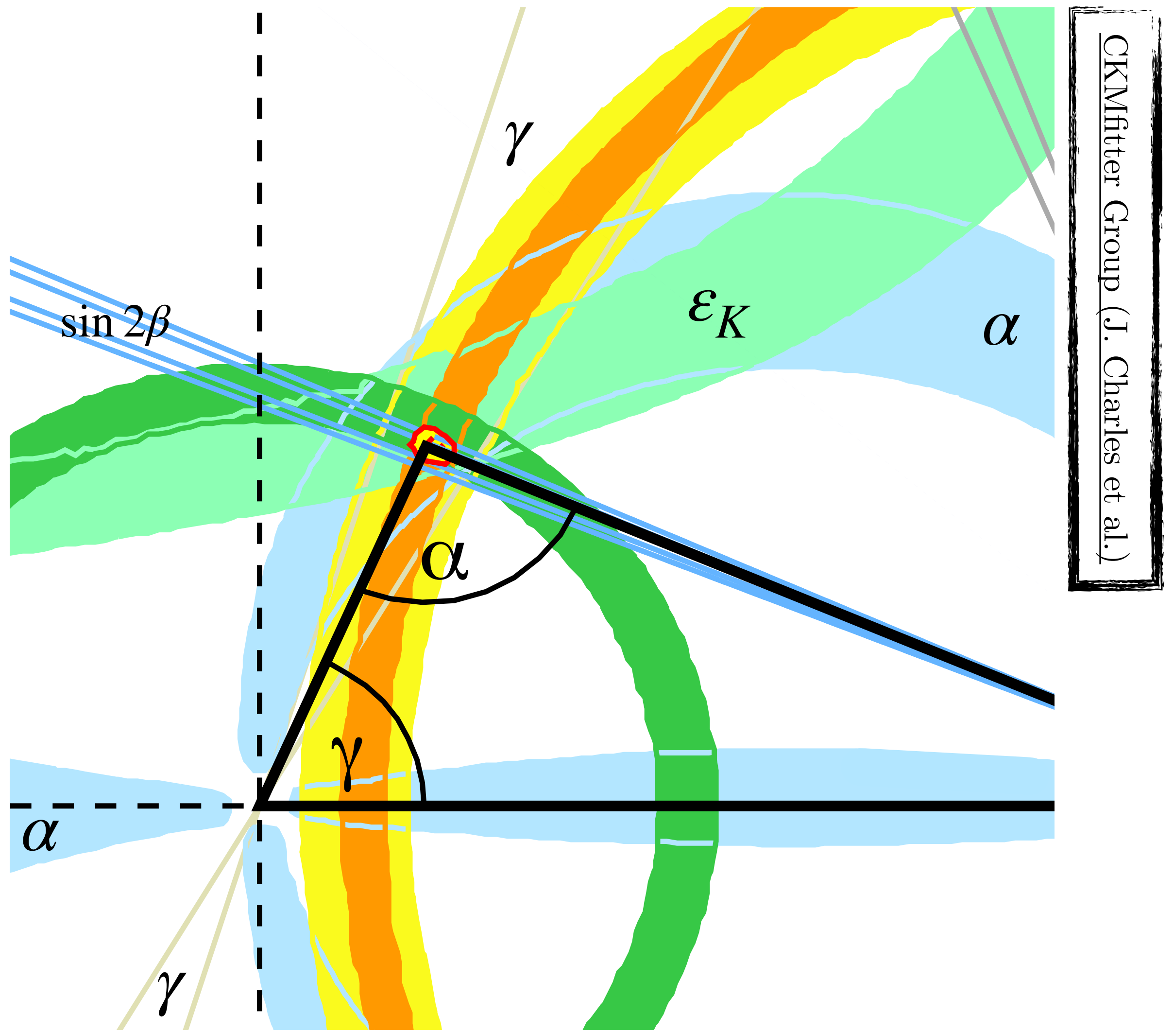

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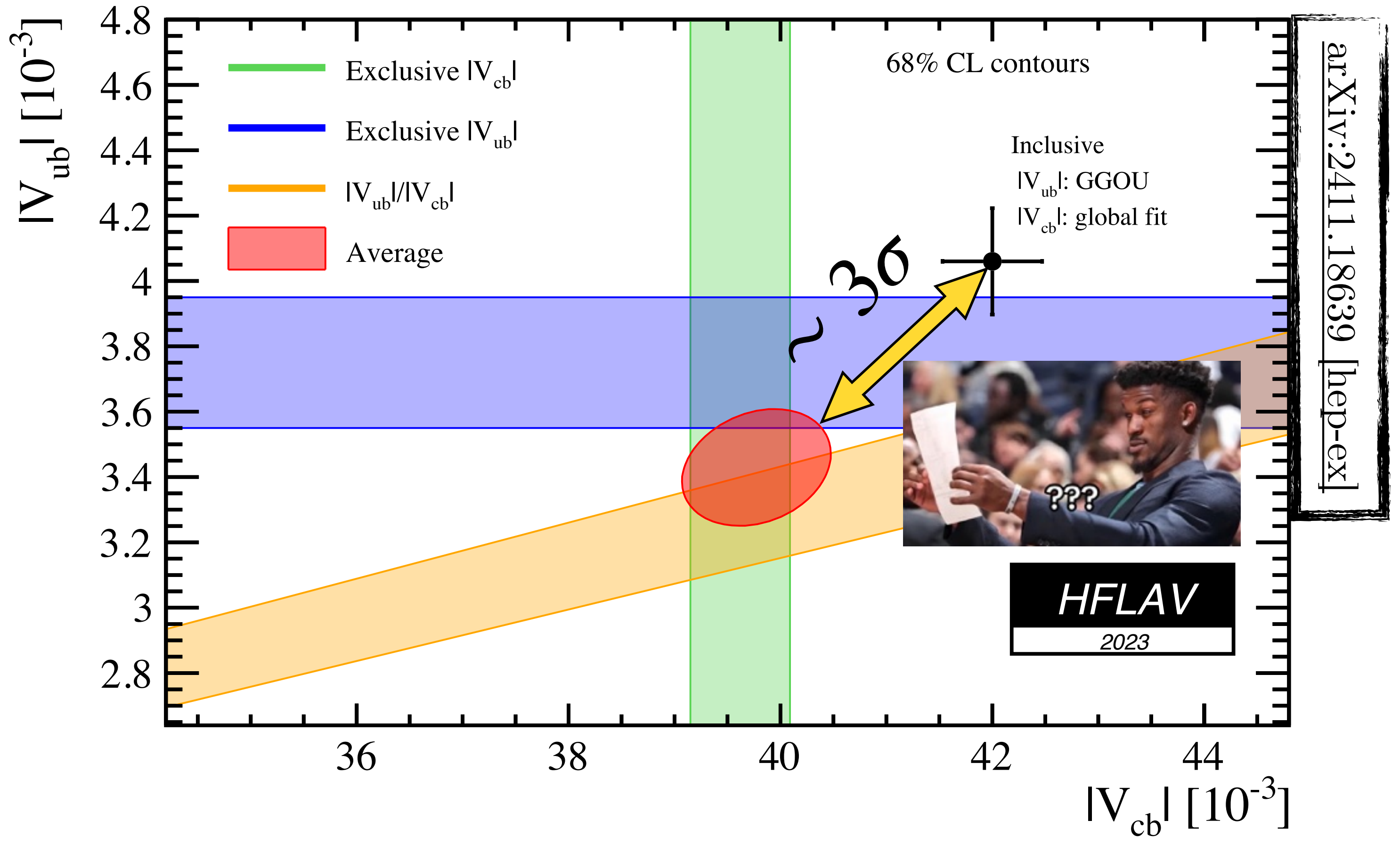
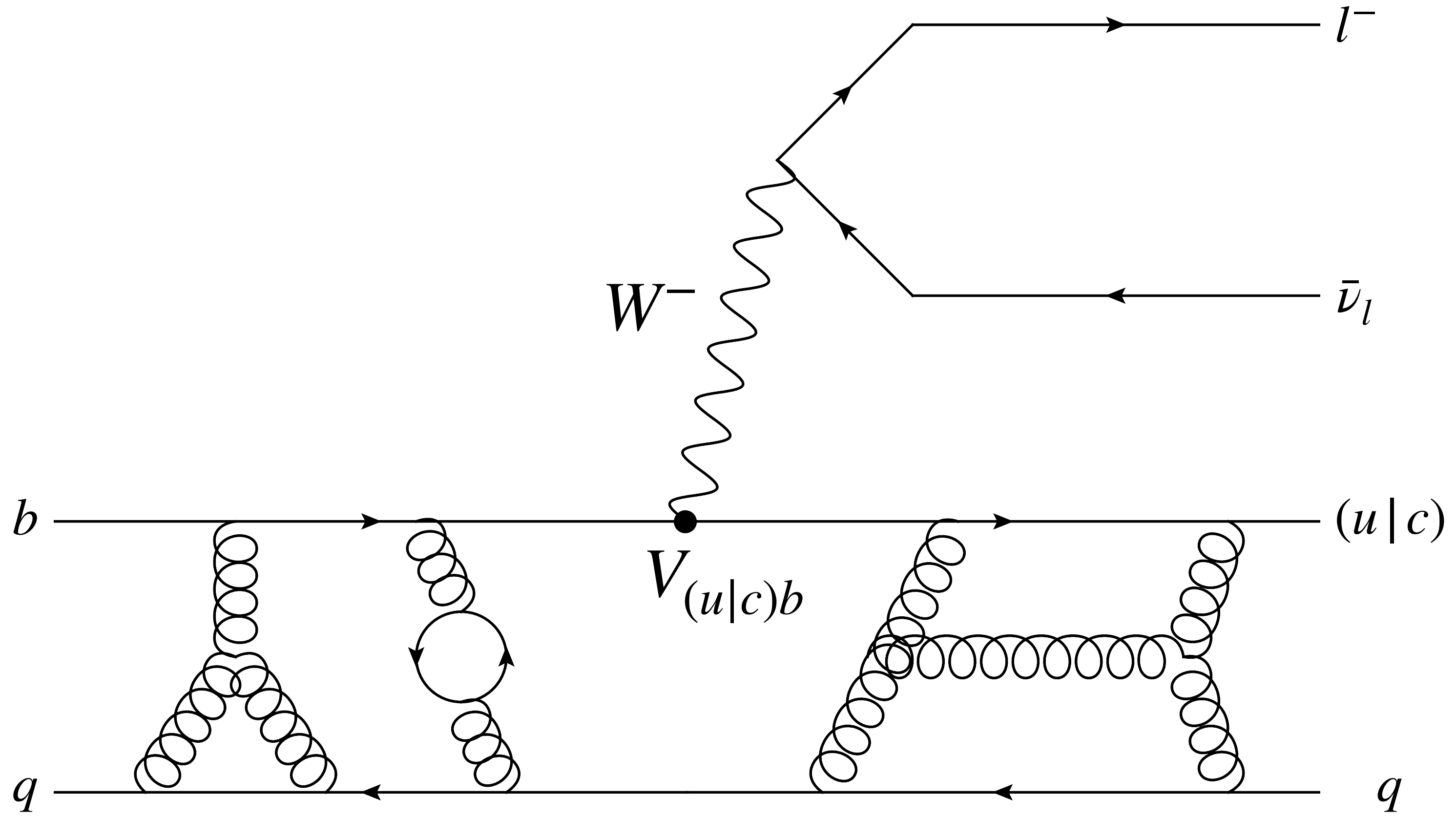


Measurement strategy for $|V_{ub}|$ and $|V_{cb}|$

Semi-Leptonic Decays



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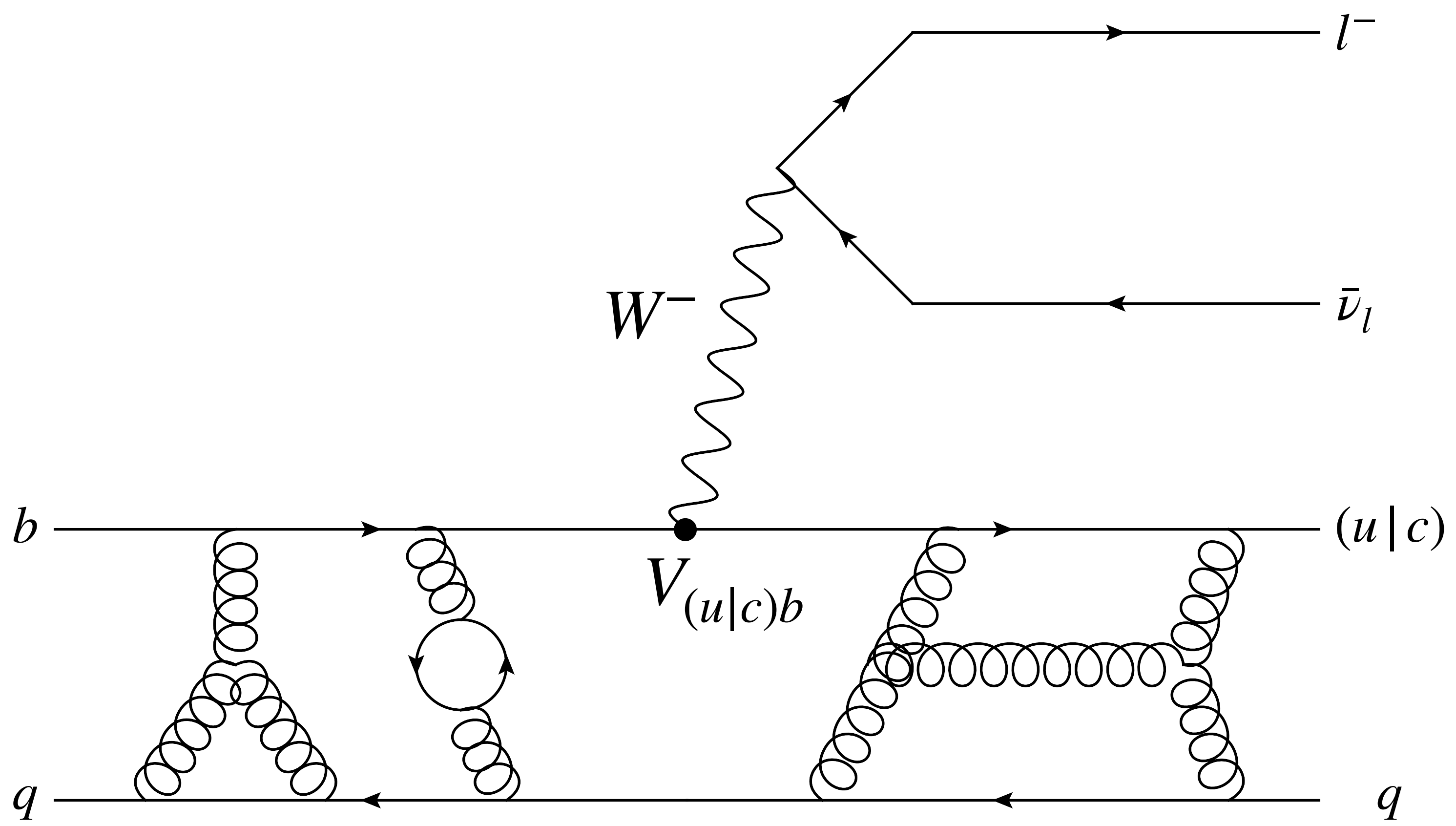

✦ Inclusive decays :

- Final hadronic states not reconstructed : $B \longrightarrow X_{(u|c)}\, l\, \bar{\nu}_l$
- Experimentally challenging and theoretically cleaner than exclusives
- $BR = |V_{(u|c)b}|^2 [\, \Gamma(b \rightarrow (u|c)\, l\, \bar{\nu}_l) + 1/m_{c,b} + \alpha_s + \ldots \,]$

Heavy quark expansion

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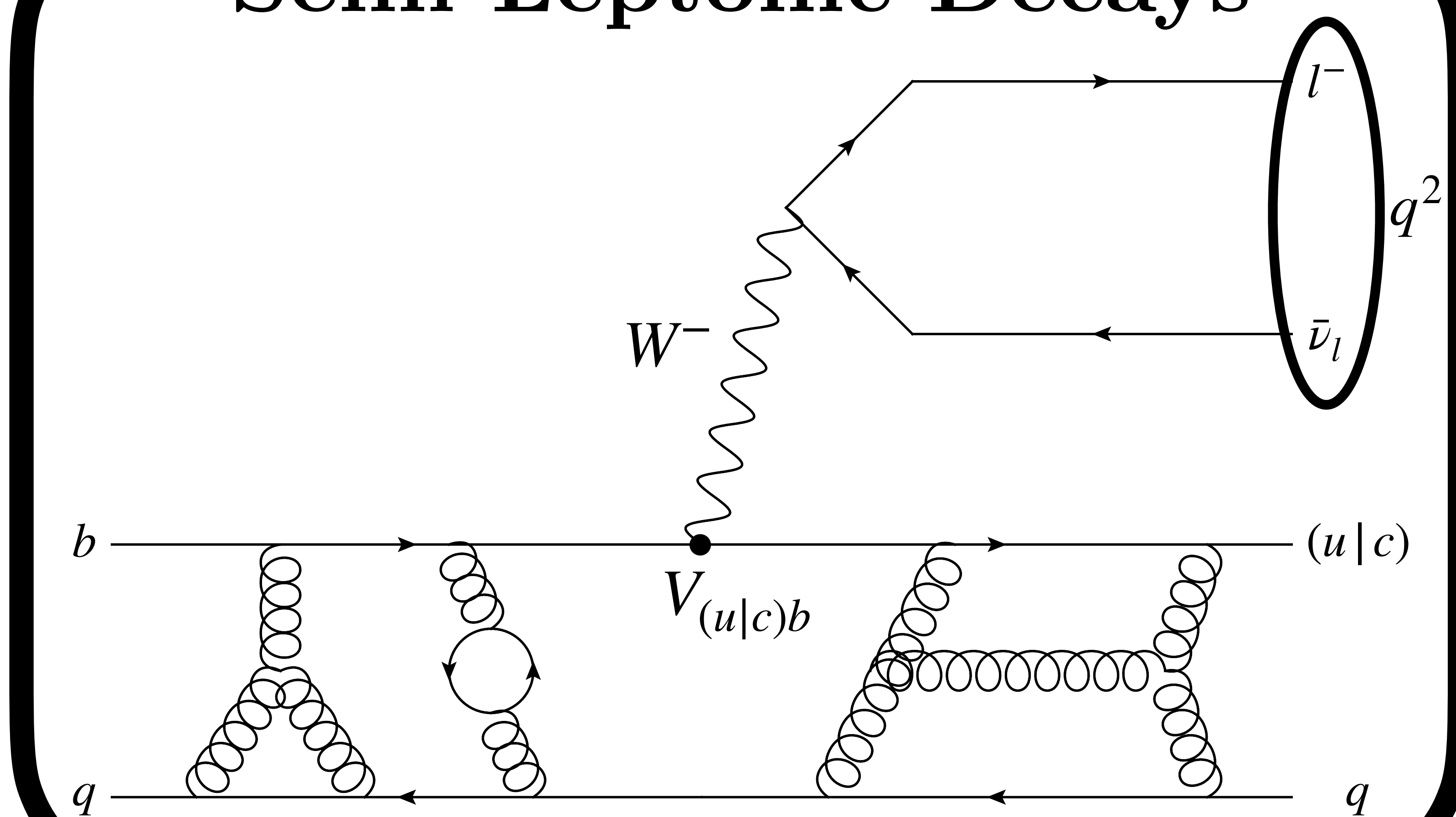

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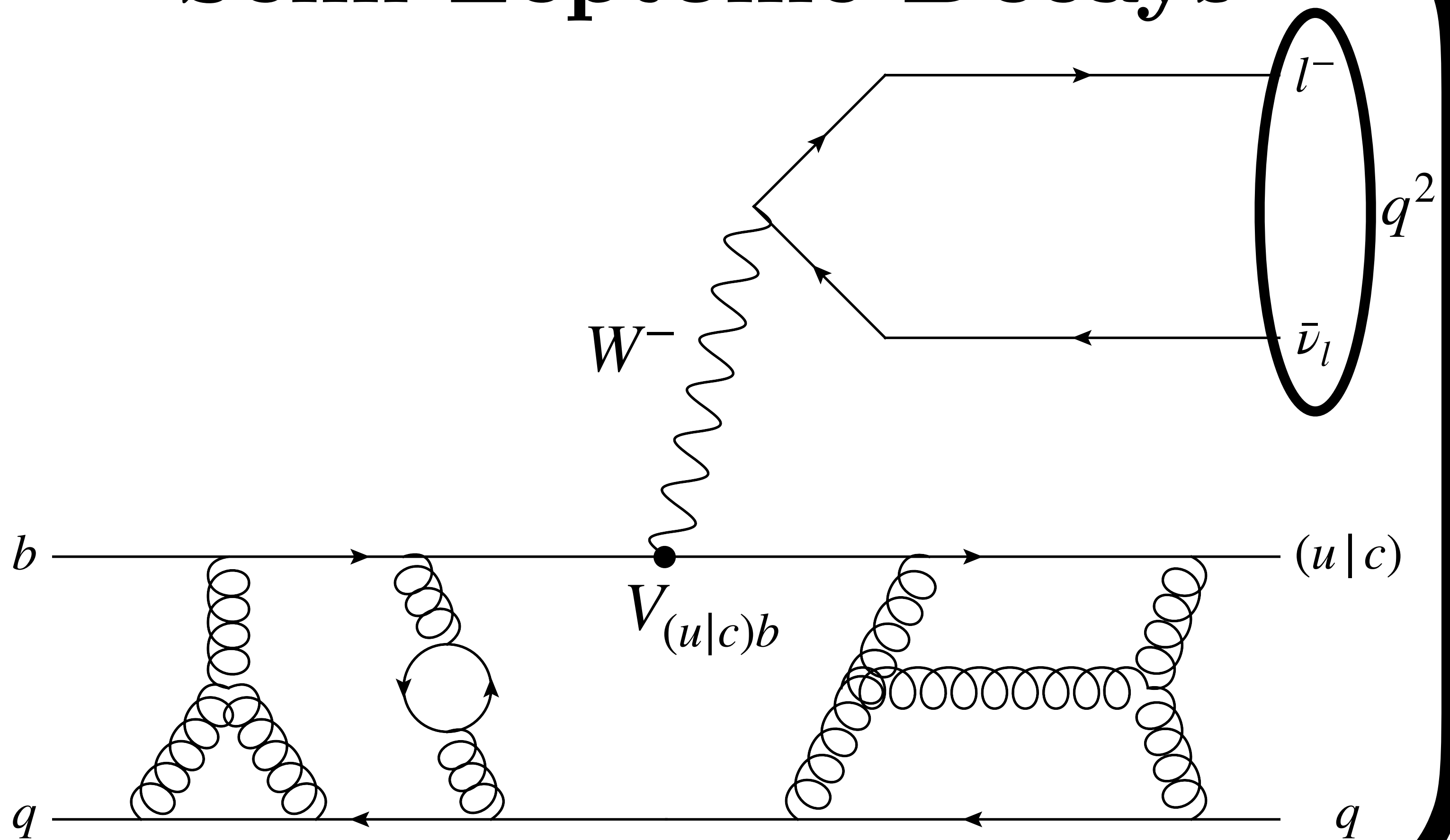

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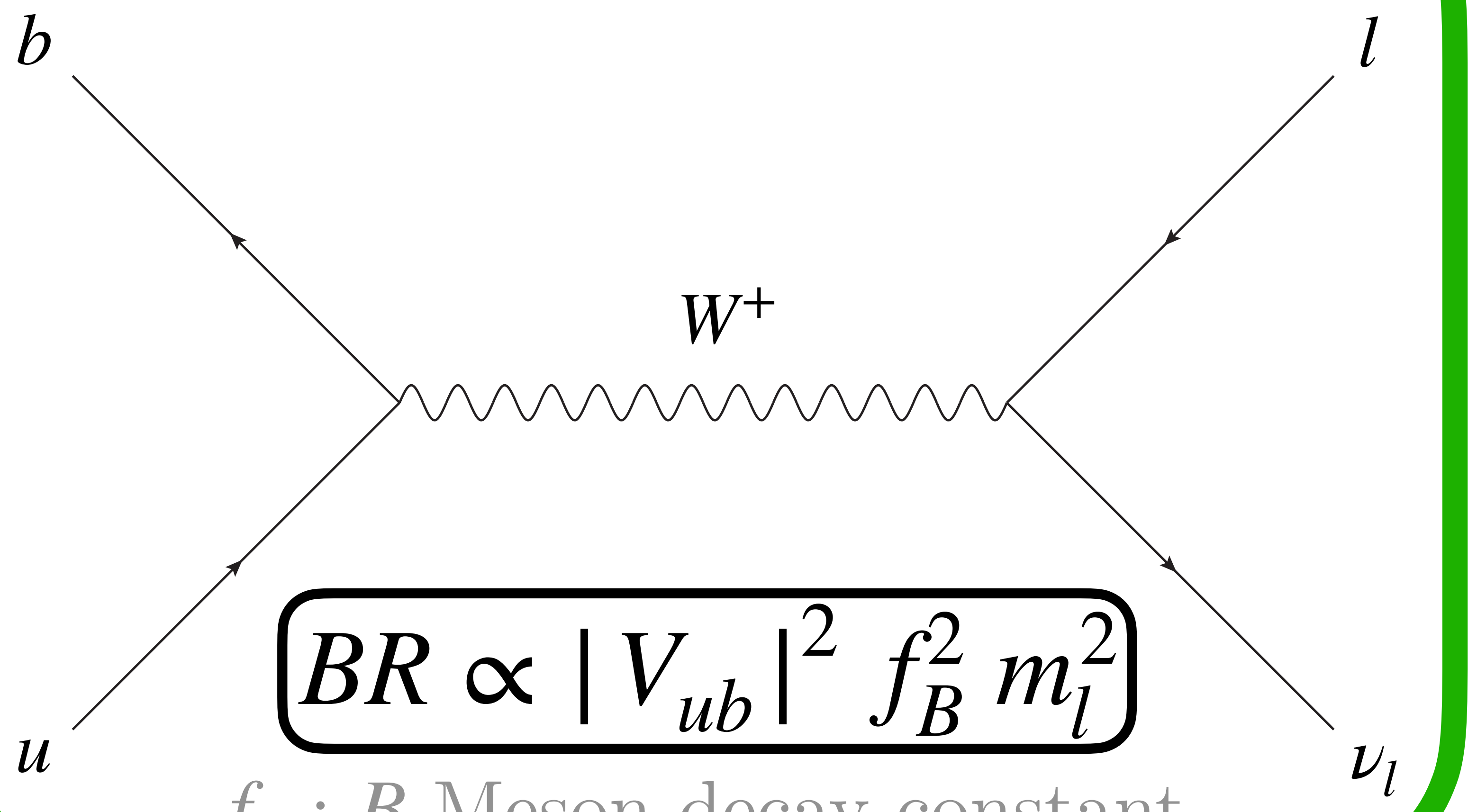

$\boxed{BR \propto |V_{ub}|^2\, f_B^2\, m_l^2}$

f_B : B-Meson decay constant

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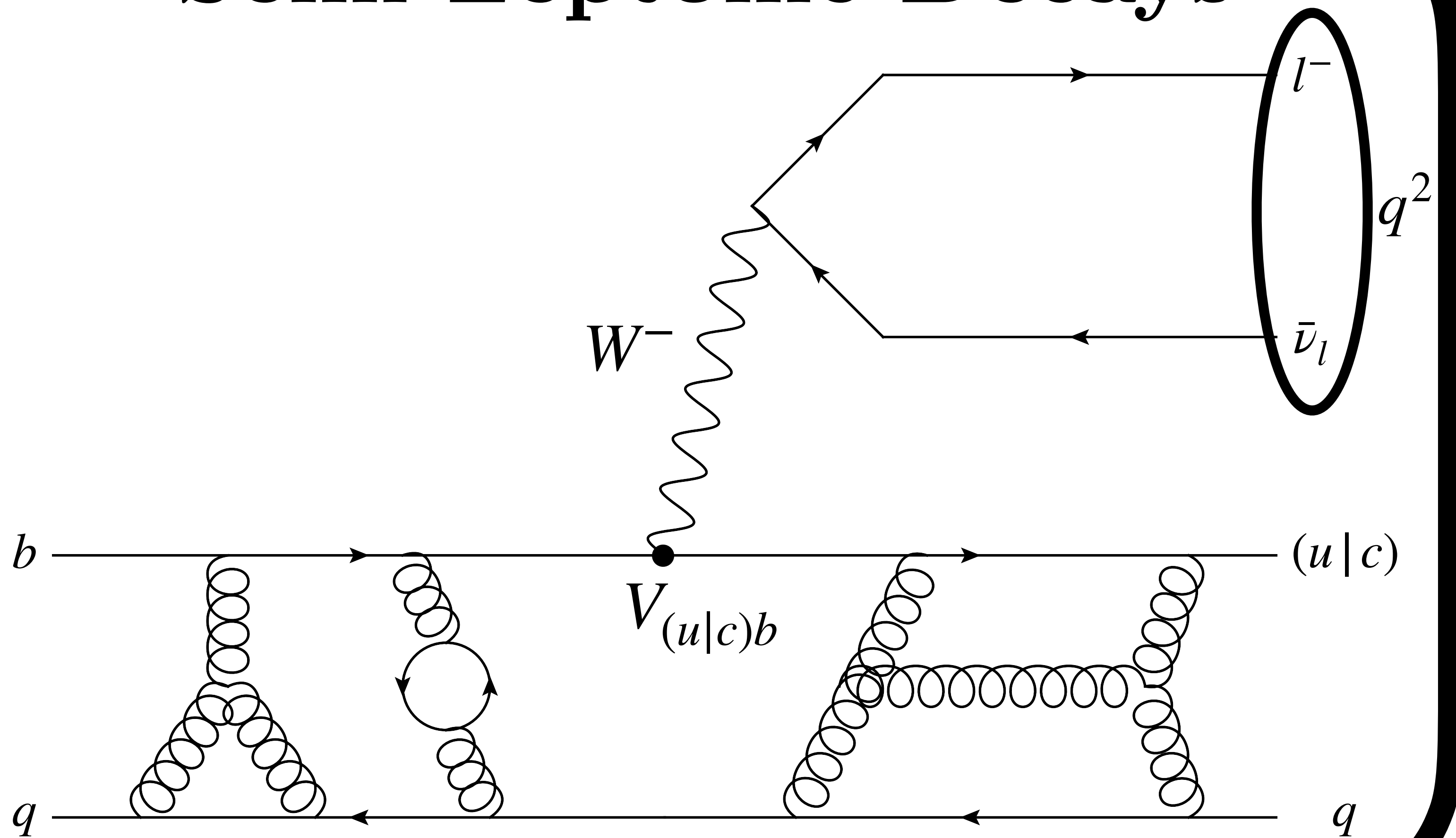

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…clusives

… $1/m_{c,b} + \alpha_s + \ldots$]

Heavy quark expansion

B FACTORIES

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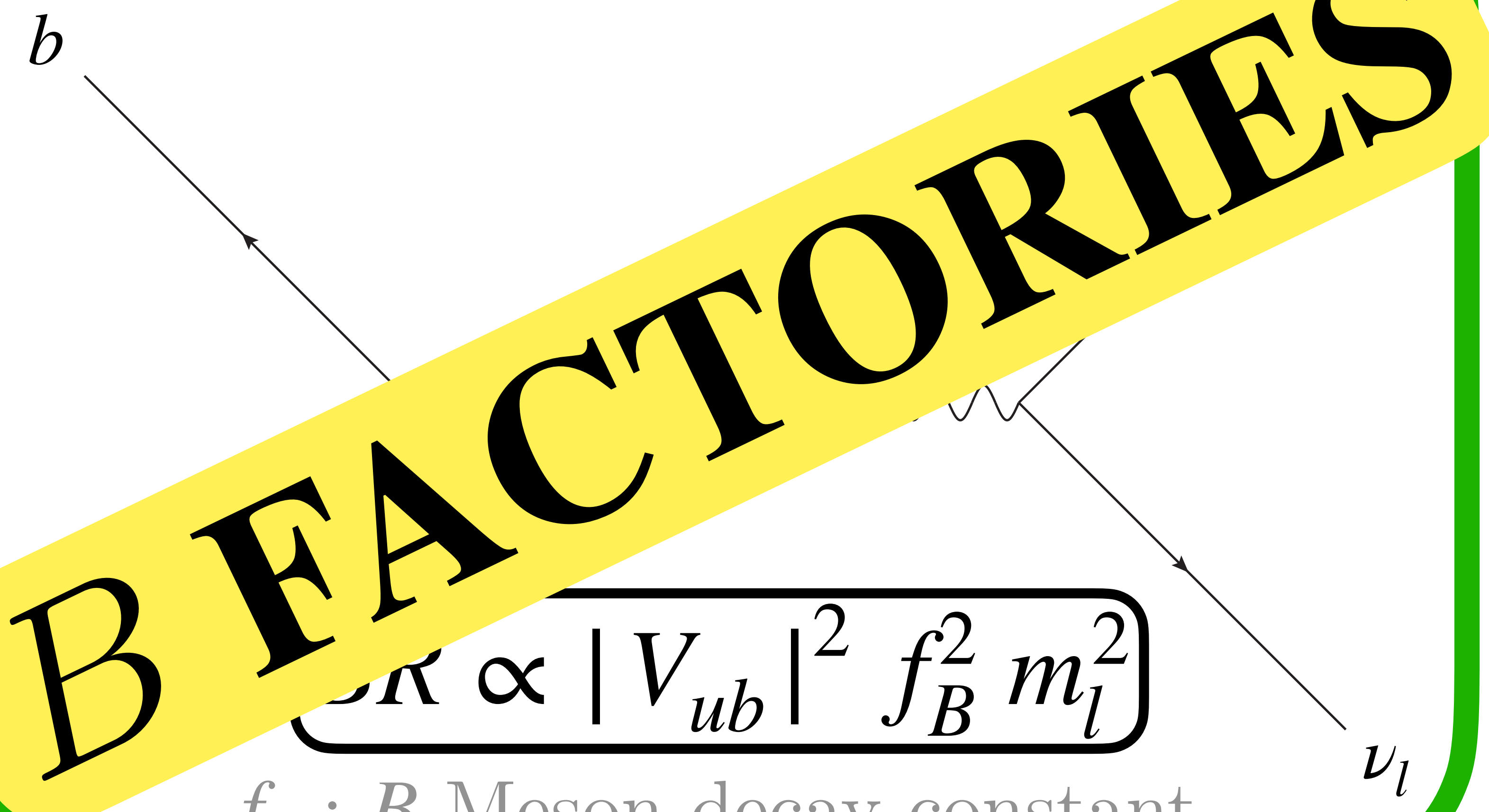

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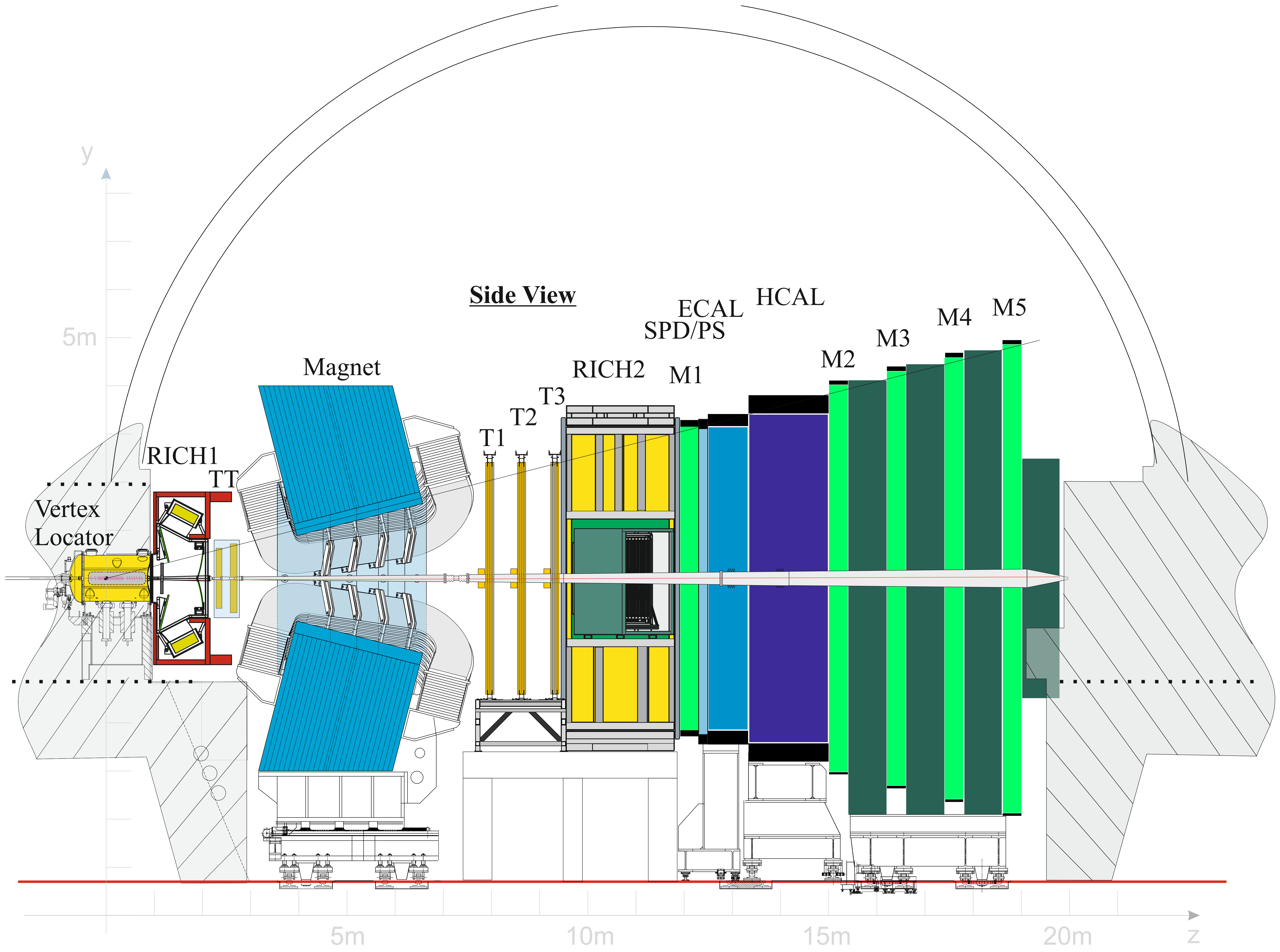
$|V_{ub}|$ and $|V_{cb}|$ measurement @ LHCb



Int. J. Mod. Phys. A 30 (2015), 1530022

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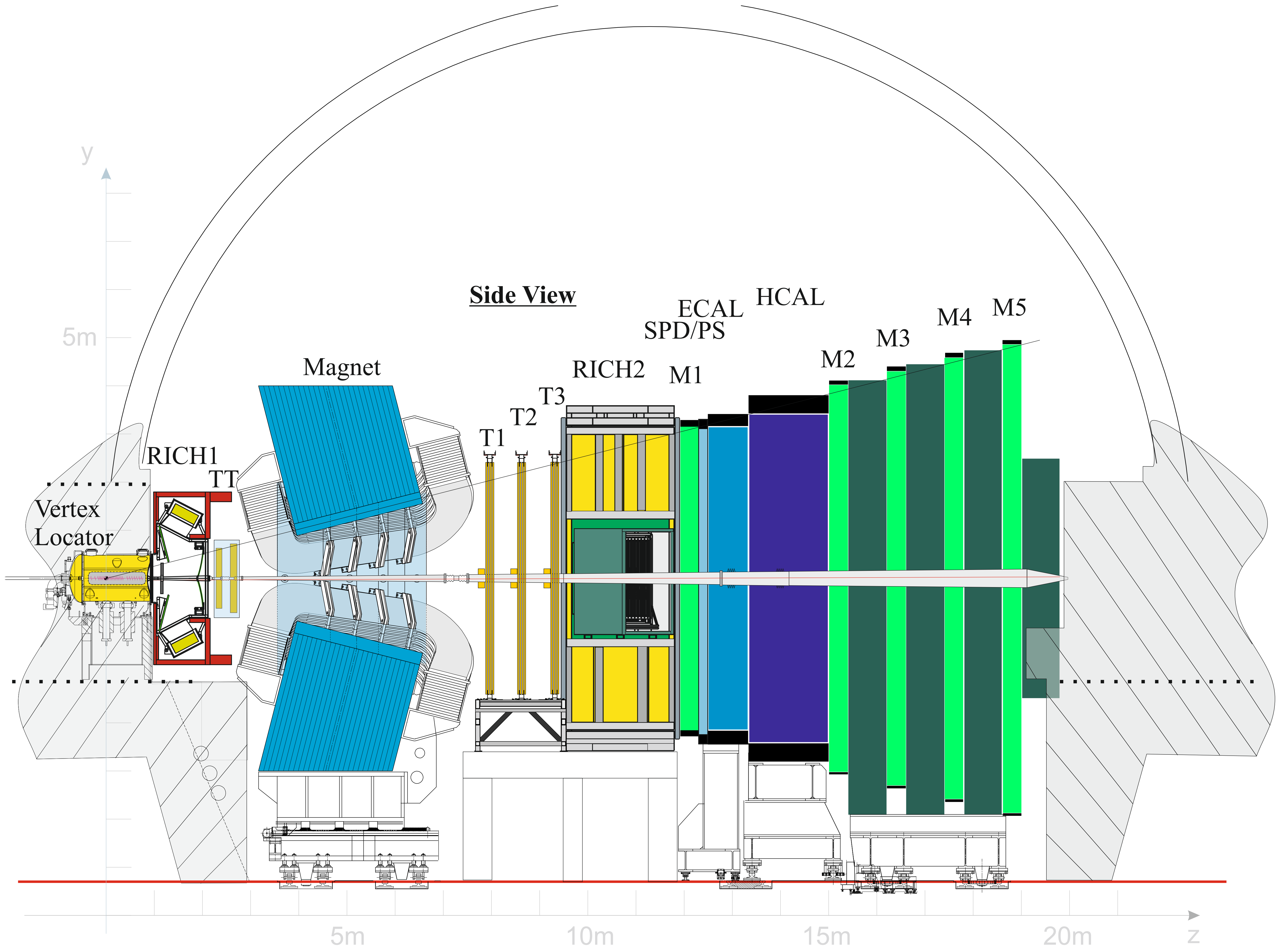

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✓ Large samples of $B^{\pm,0}$ mesons and heavier b -hadrons like B_s^0, B_c and Λ_b^0

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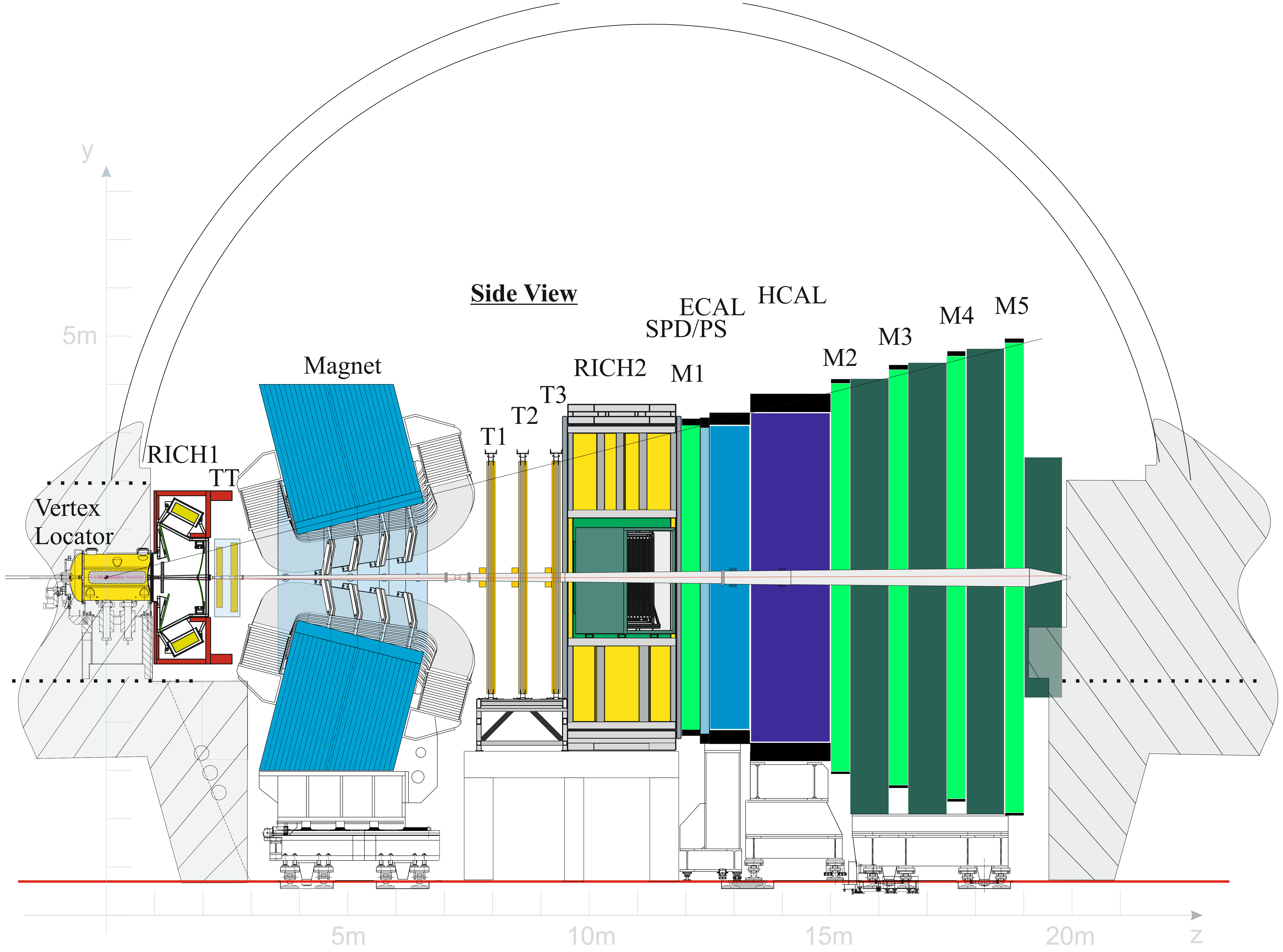

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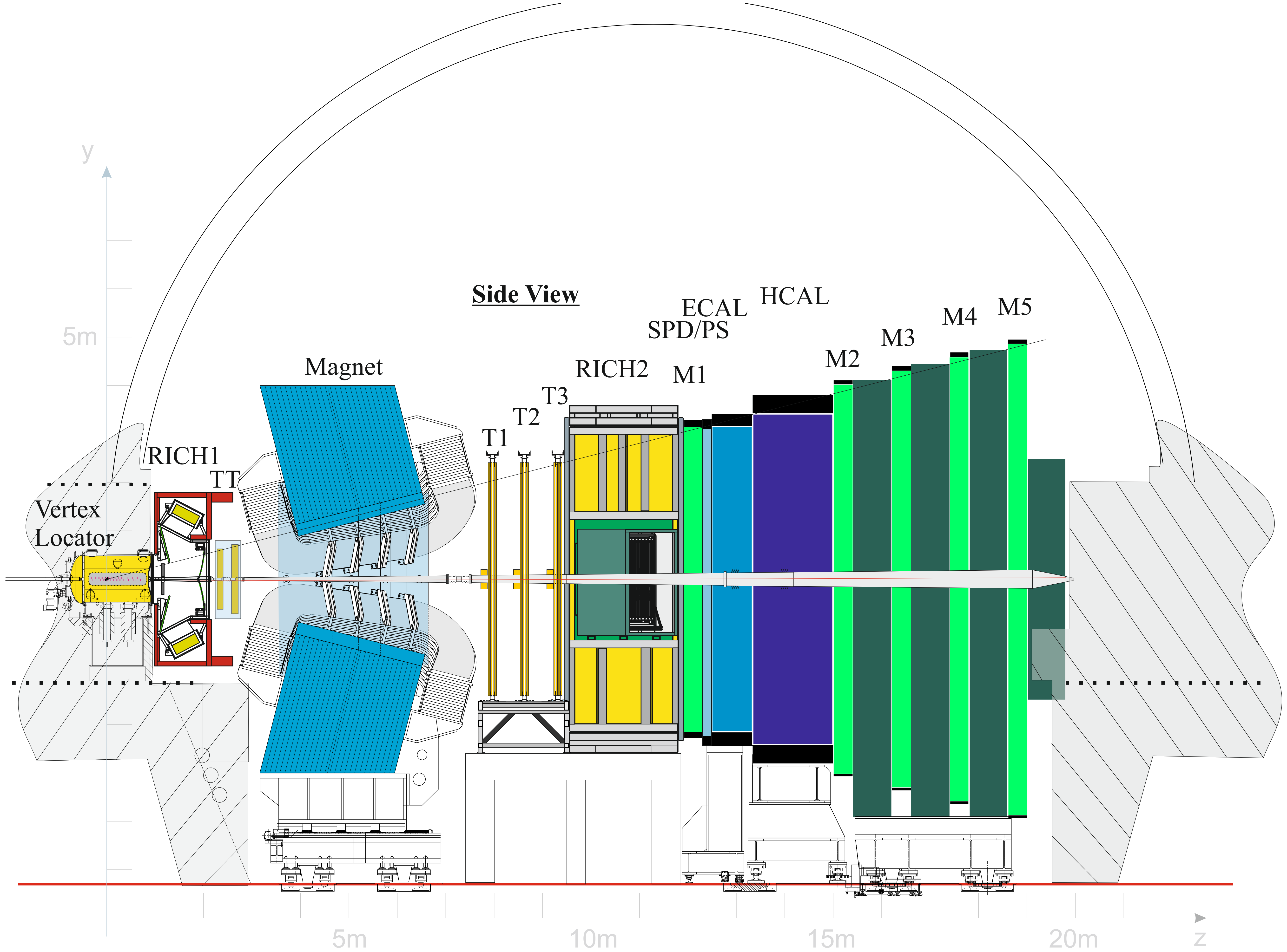

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✗ Dirty hadronic environment and unreconstructed ν → **large background**

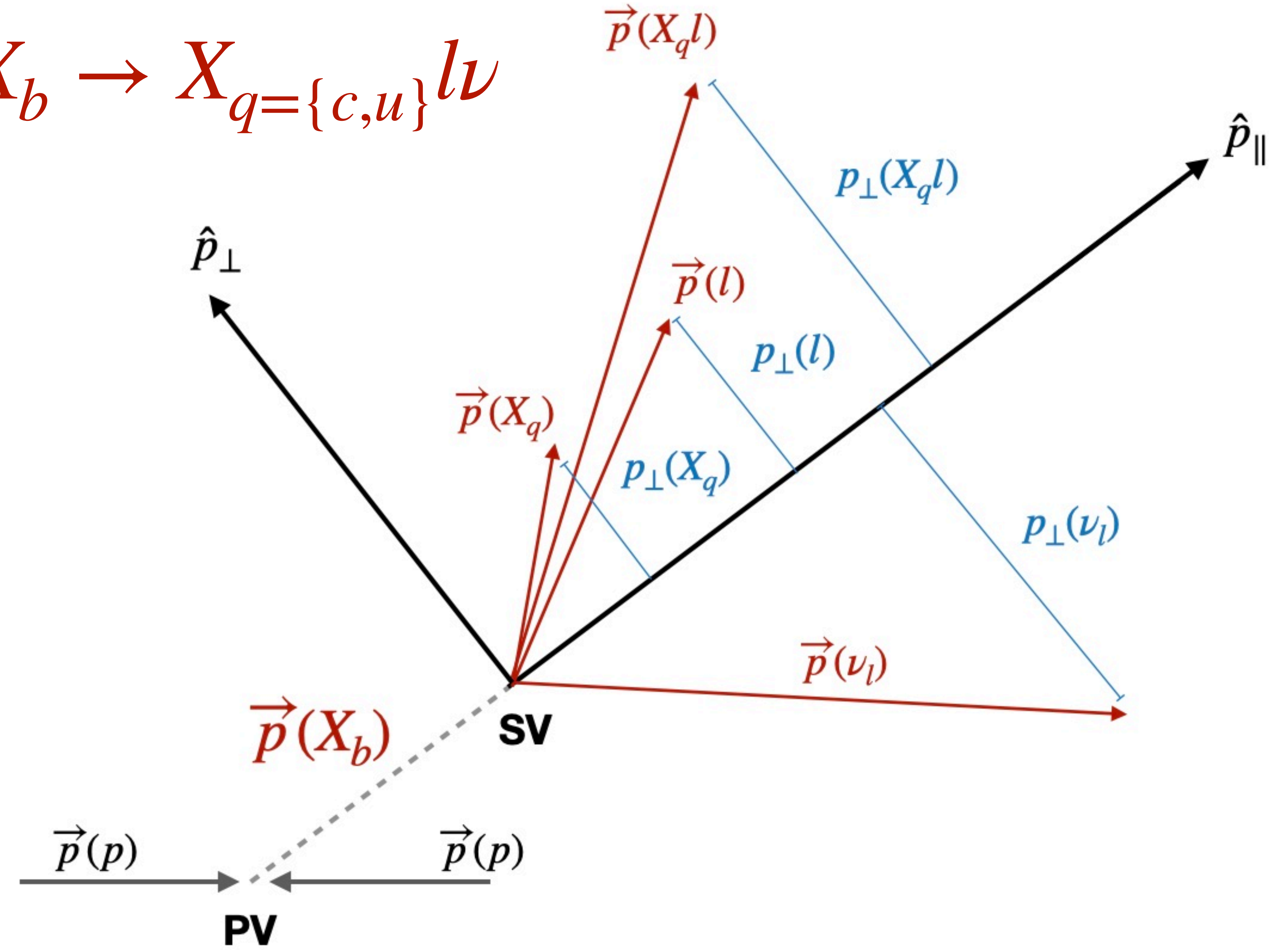
✗ Luminosity and $b\bar{b}$ production rate cannot be determined precisely ⇒ **large uncertainty of measured absolutes BFs**

Semileptonic analysis @ LHCb

X_b **mass** obtained with a reconstruction technique using the **visible decay products** $X_{(u|c)}l$ and **kinematic constraints**

$$\Rightarrow M_{corr}(X_b) = \sqrt{M(X_{(u|c)}l)^2 + p_\perp(X_{(u|c)}l)^2} + p_\perp(X_{(u|c)}l)$$

$X_b \to X_{q=\{c,u\}} l\nu$



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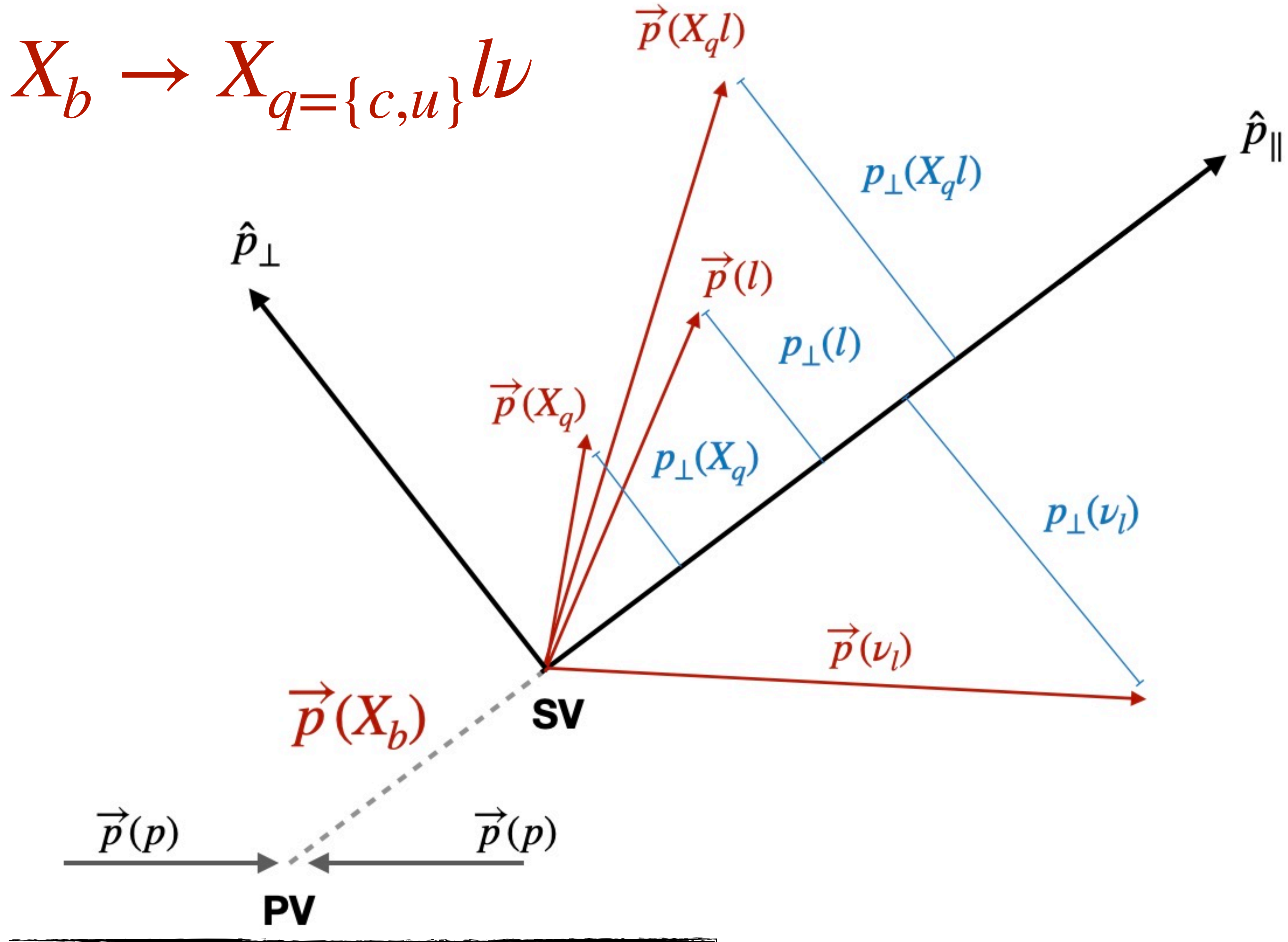
Two fold ambiguity wrt q^2 leads to a **degraded resolution**

$\Rightarrow$ **Regression algorithm** and **unfolding** needed to access to the true q^2



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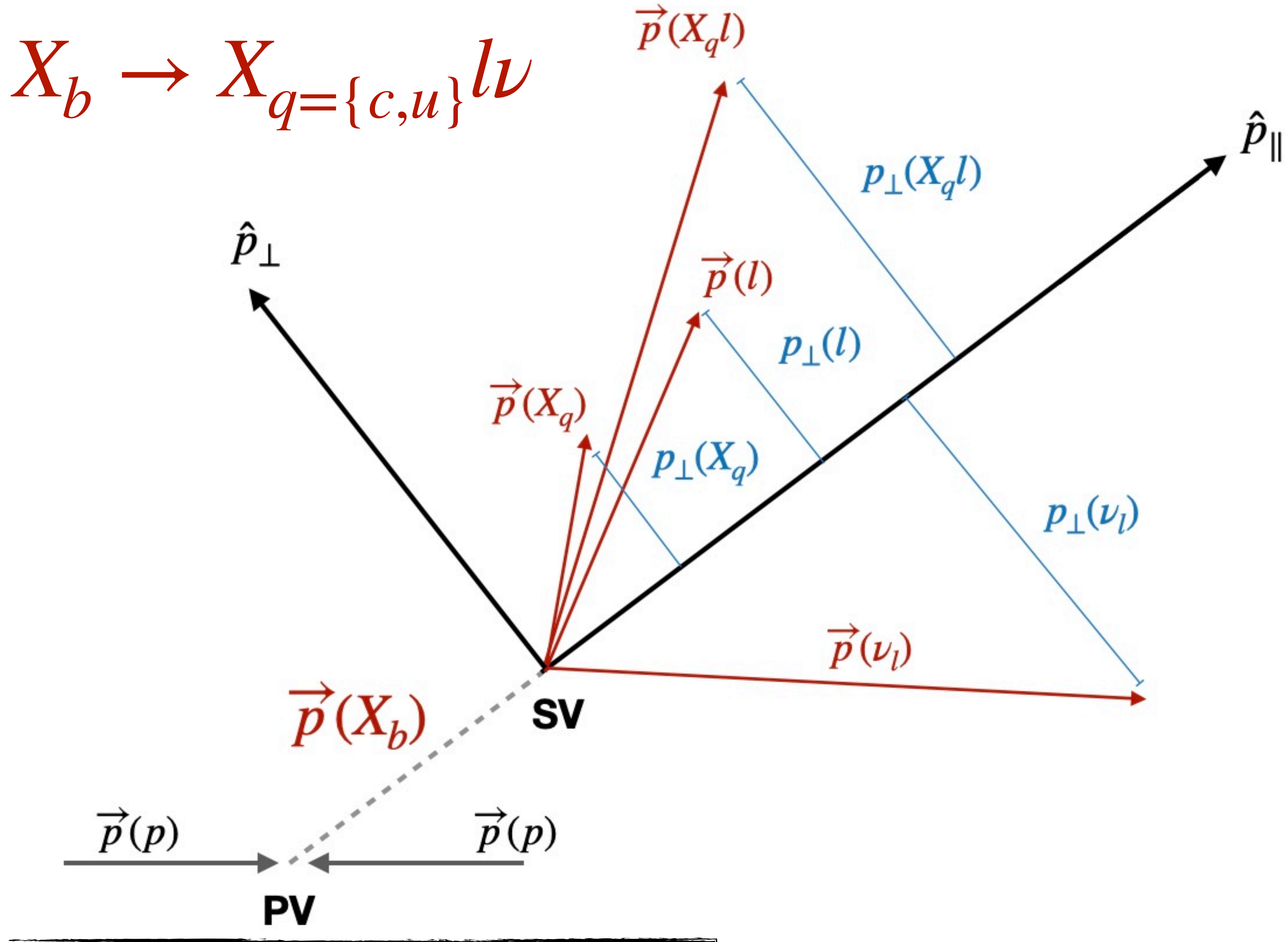
Normalisation decays used to cancel the $b\bar{b}$-production uncertainties and reduces the systematic ones $\Rightarrow$ Imported input : BFs Normalisation, fragmentation fraction, ...



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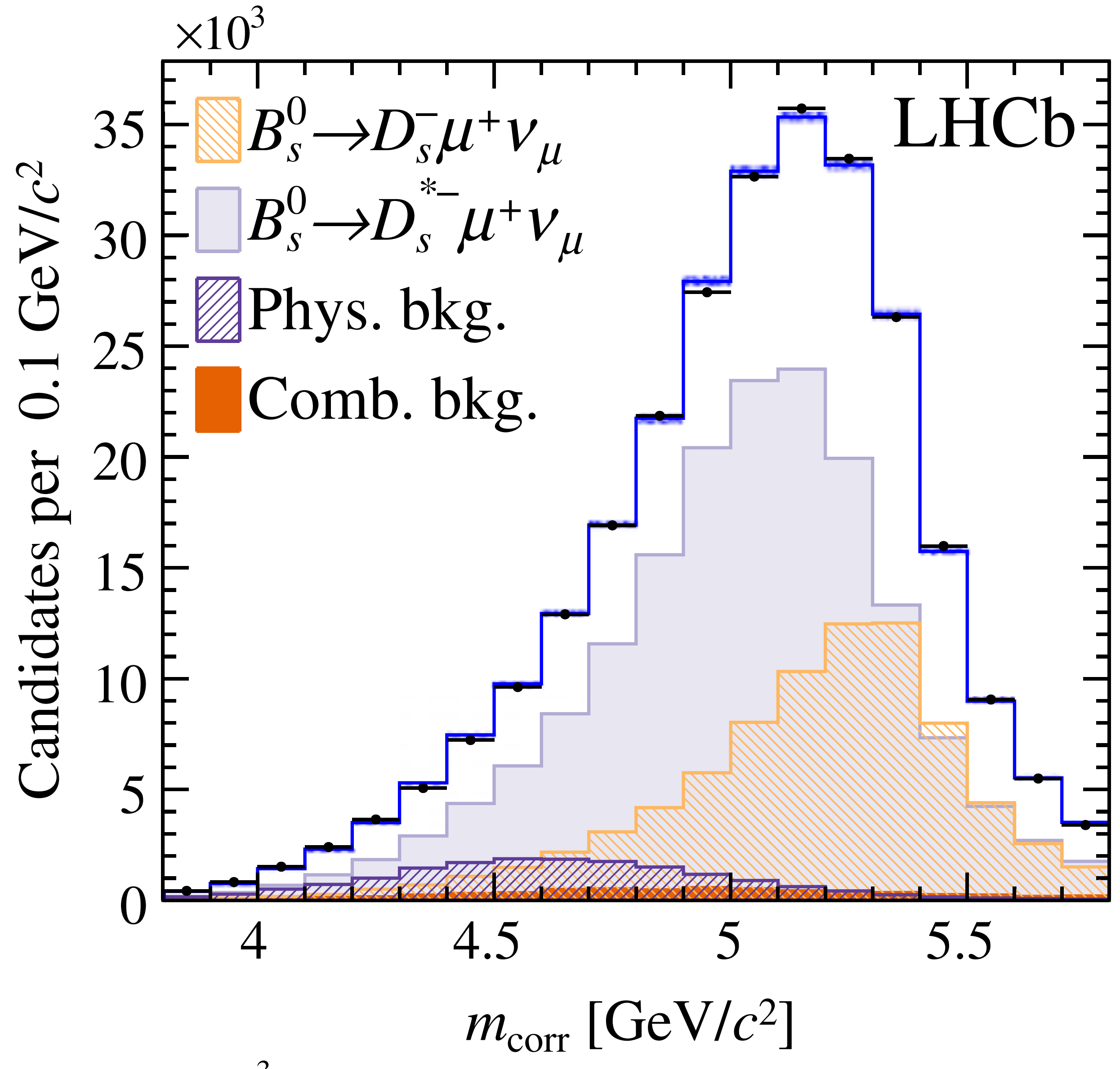
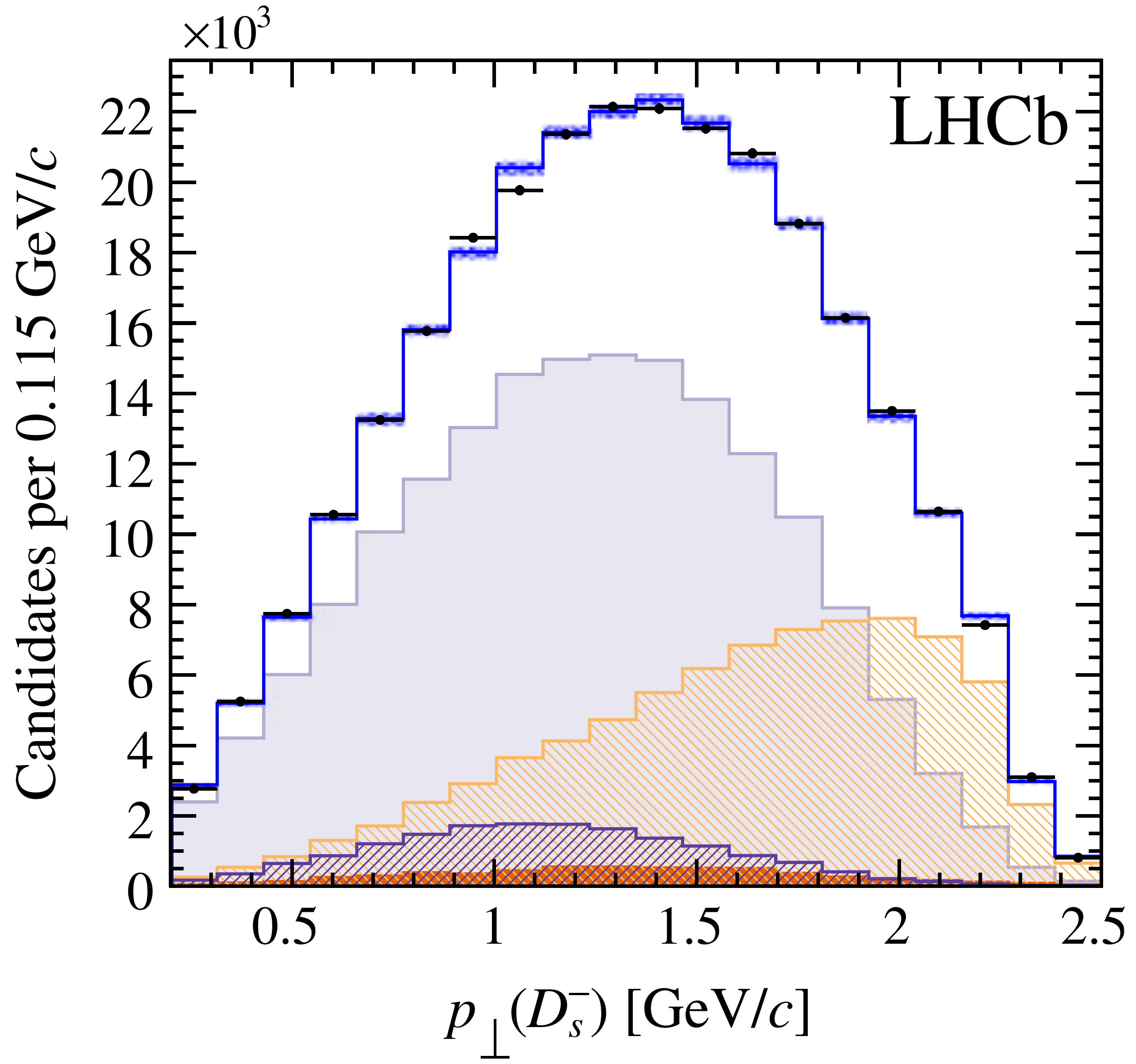
Published $|V_{(u|c)b}|$ measurements at LHCb

$|V_{cb}|$ from $B_s^0 \to D_s^{(*)-}(\to [KK]_\phi \pi^-)\,\mu\,\nu_\mu$ decays

- First $|V_{cb}|$ measurement from B_s^0 decays
- Dataset : $1\ \text{fb}^{-1}$ @ $\sqrt{s} = 7$ TeV + $2\ \text{fb}^{-1}$ @ $\sqrt{s} = 8$ TeV
- Normalisation mode : $B^0 \to D^{(*)-}(\to [KK]_\phi \pi^-)\,\mu\,\nu_\mu$
- Signal extraction : 2D fit wrt $M_{corr}(B_s^0)$ and $p_\perp(D_s^-)$



Phys. Rev. D 101, 072004



Results :

$$|V_{cb}|_{\text{CLN}} = (\,41.4 \pm 0.6_{\text{stat}} \pm 0.9_{\text{sys}} \pm 1.2_{\text{ext}}\,) \times 10^{-3}$$

$$|V_{cb}|_{\text{BGL}} = (\,42.3 \pm 0.8_{\text{stat}} \pm 0.9_{\text{sys}} \pm 1.2_{\text{ext}}\,) \times 10^{-3}$$

CLN and BGL in mutual agreement

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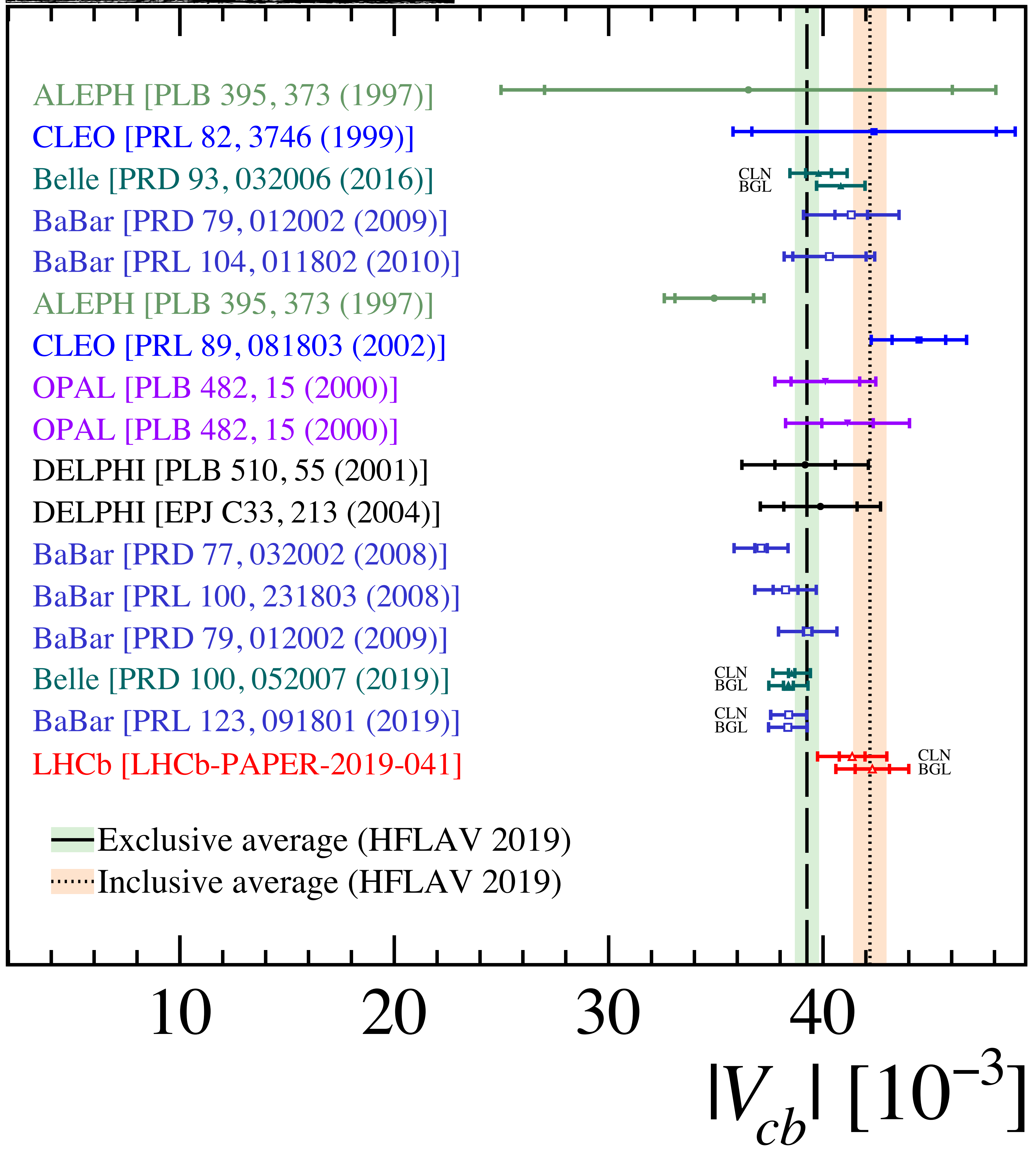
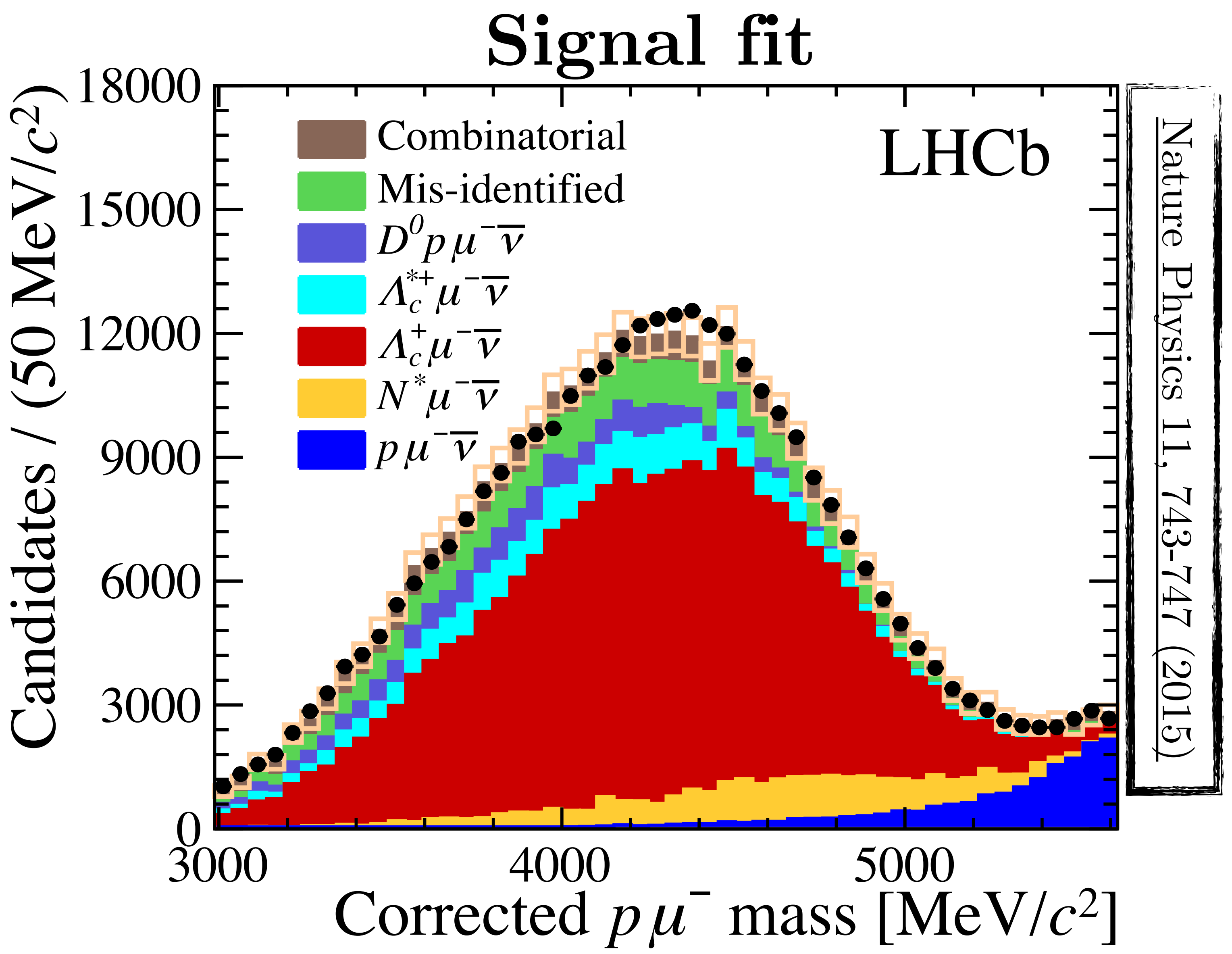


Agreement with inclusives and exclusives measurement

$|V_{ub}|$ from $\Lambda_b^0 \to p\,\mu\,\nu_\mu$ decays

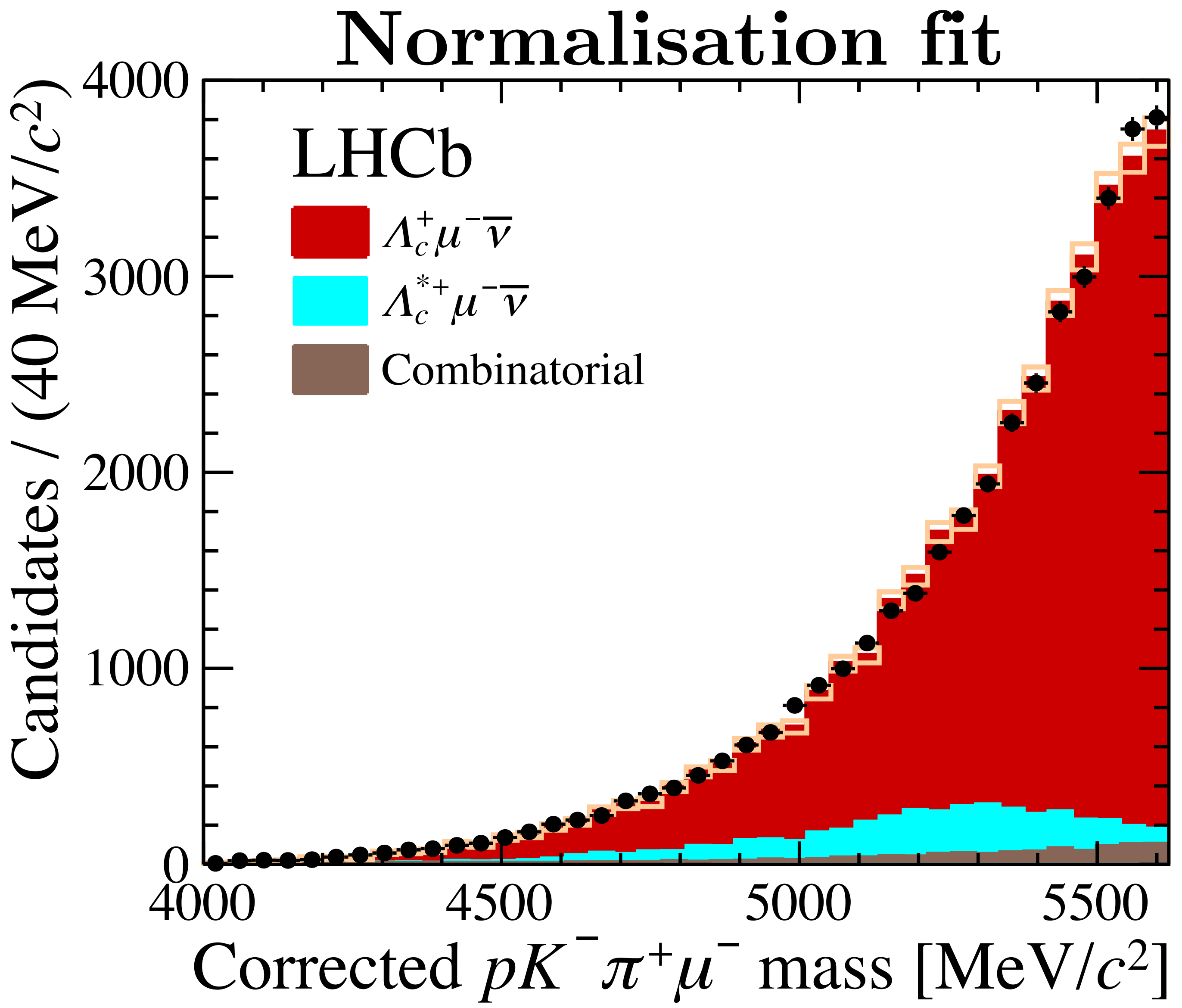
- <u>Dataset :</u> 2 fb^{-1} @ $\sqrt{s} = 8$ TeV
- <u>Normalisation mode :</u> $\Lambda_b^0 \to \Lambda_c^+(\to p\,K^-\pi^+)\,\mu\,\nu_\mu$
- <u>Signal extraction :</u> Fit on $M_{corr}(B_s^0)$ wrt the :
  - $p\,\mu$ system
  - $p\,K^-\pi^+\mu^-$ system but only for the normalization mode



Results :

$$\frac{|V_{ub}|}{|V_{cb}|} = 0.083 \pm 0.004_{\text{exp}}\, 0.004_{\text{theo}}$$

$\to V_{cb}$ coming from the normalization mode



Form Factors have been estimated by <u>Lattice QCD</u>

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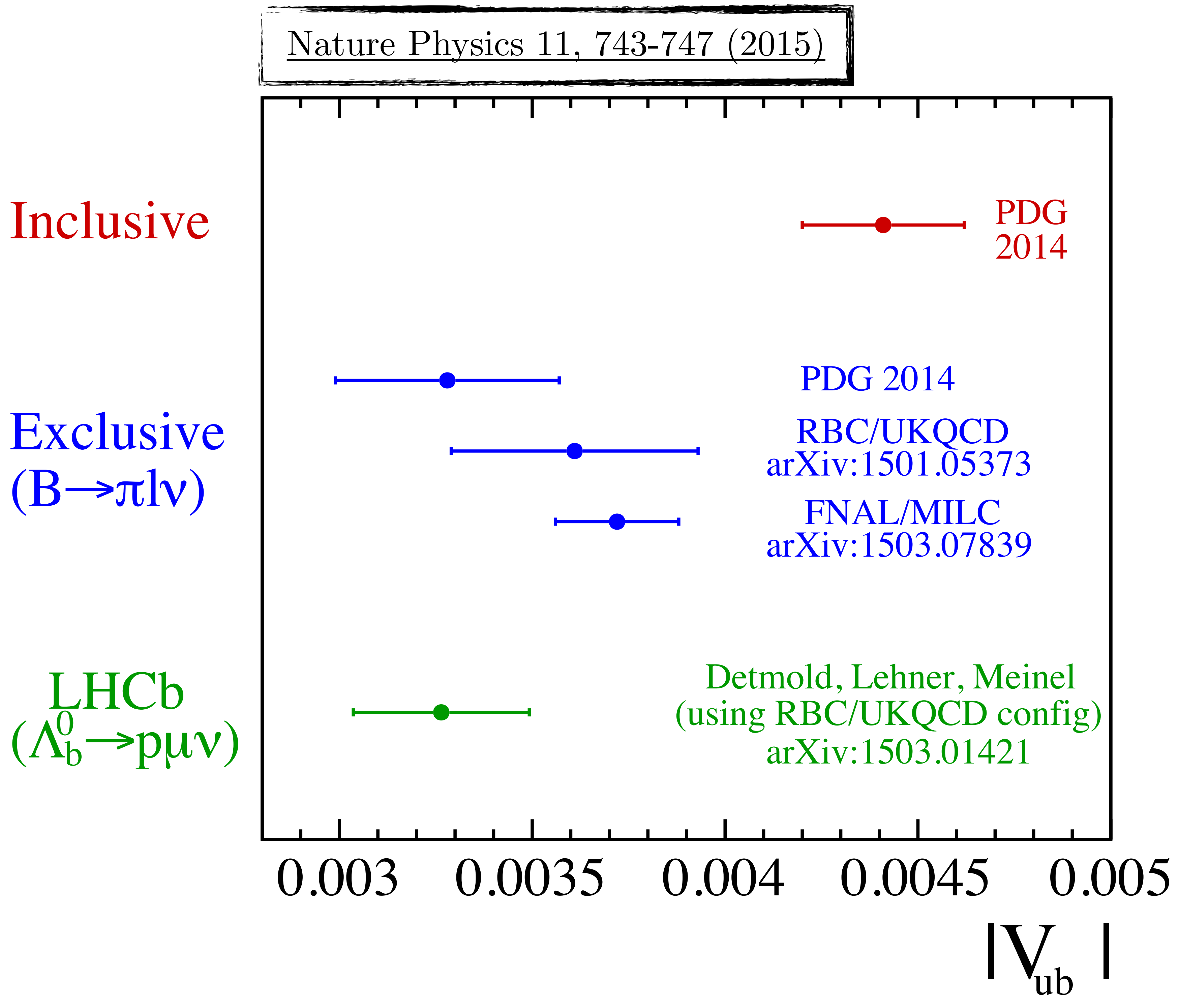


Results :

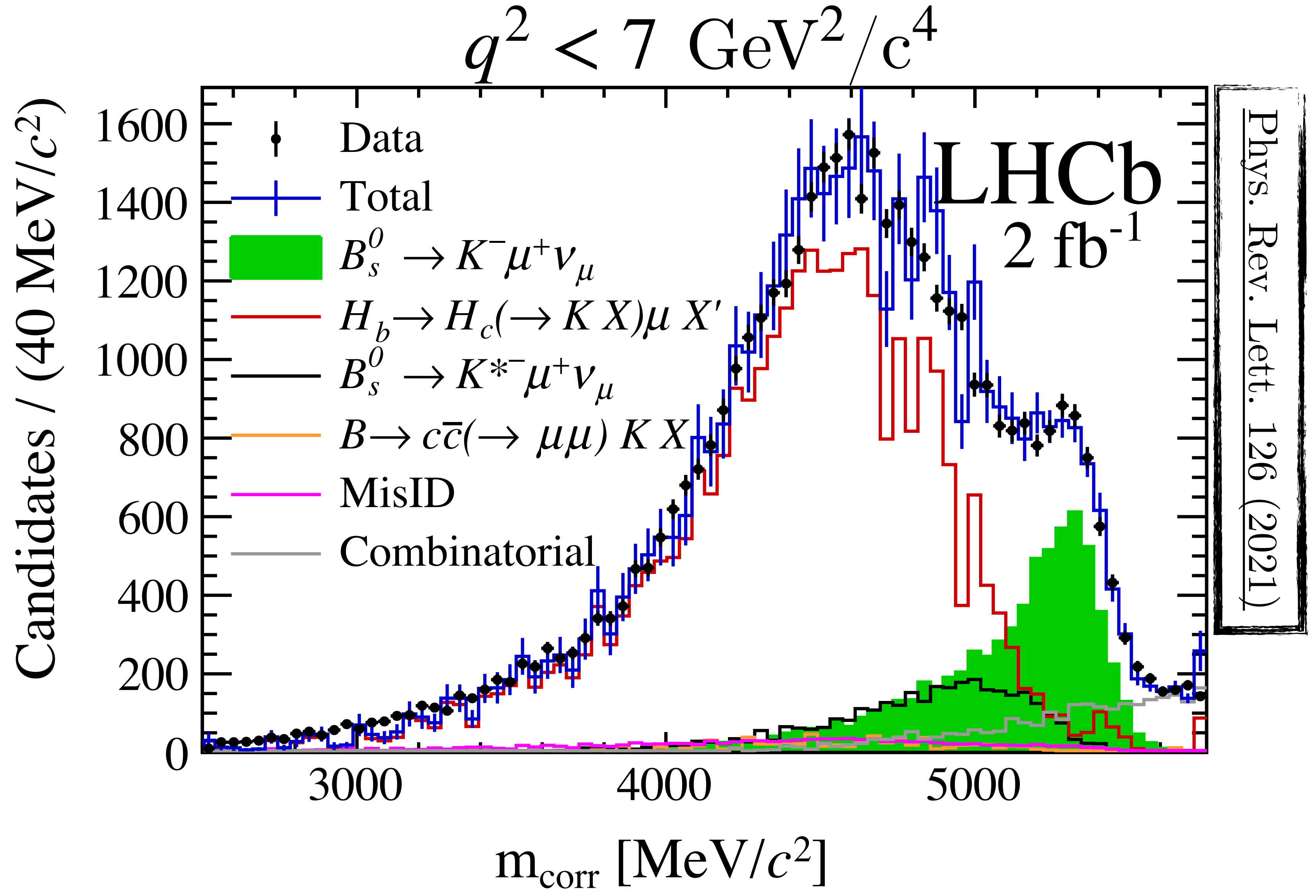
$$|V_{ub}| = (\, 3.27 \pm 0.15_{\text{exp}} \pm 0.16_{\text{theo}} \pm 0.06_{\text{norm}}\,) \times 10^{-3}$$

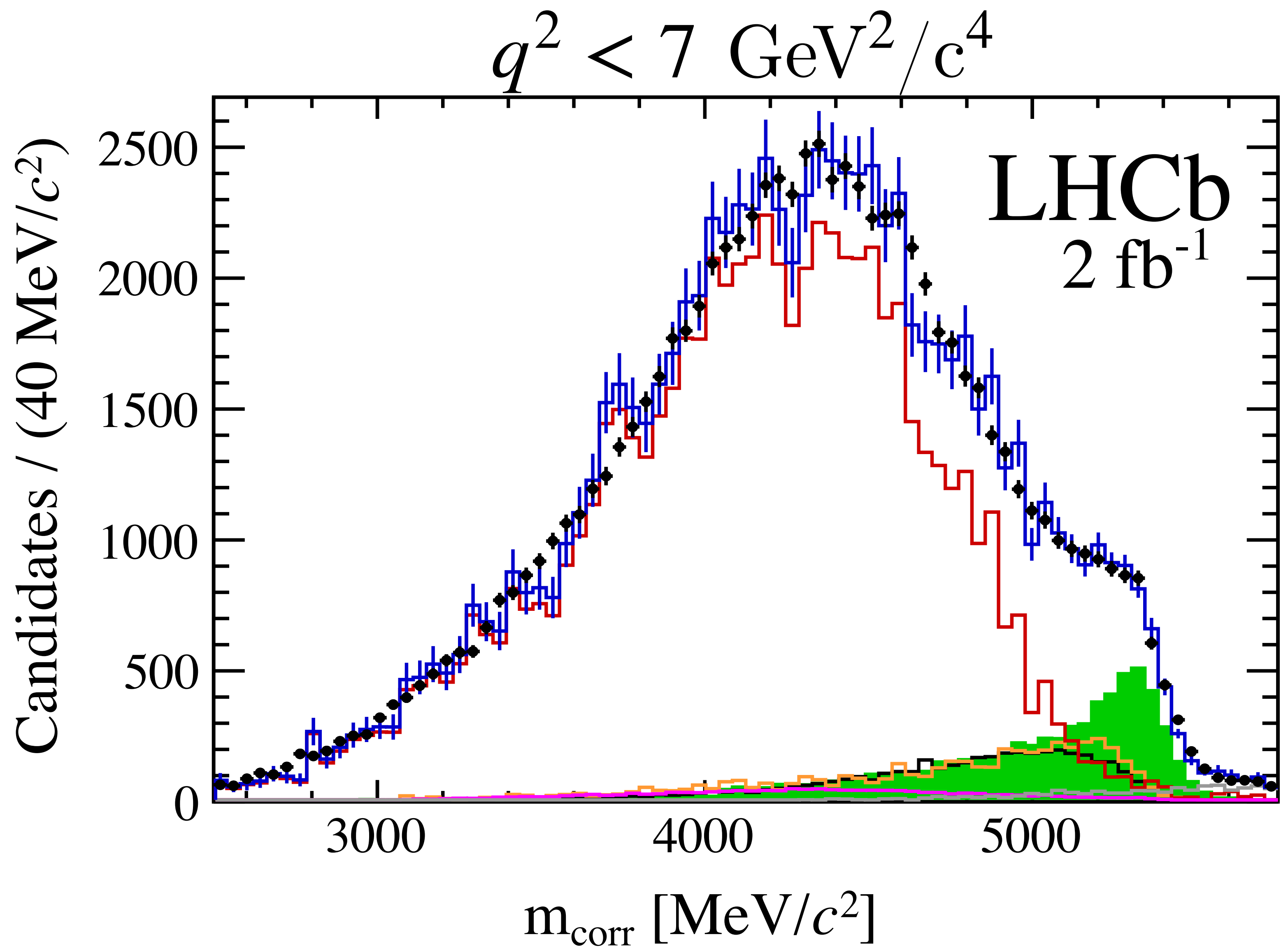
→ Using the world average $|V_{cb}| = (\, 39.5 \pm 0.8\,) \times 10^{-3}$ using exclusive decays

Not in agreement with inclusives measurement

$|V_{ub}|/|V_{cb}|$ from $B_s^0 \to K\,\mu\,\nu_\mu$ decays

- First observation of the $B_s^0 \to K\,\mu\,\nu_\mu$ decay
- Dataset : 2 fb^{-1} @ $\sqrt{s} = 8$ TeV
- Normalisation mode : $B_s^0 \to D_s^-(\to KK\pi^-)\,\mu^+\nu_\mu$
- Signal extraction : $M_{corr}(B_s^0)$ in two q^2 regions $\lessgtr$ 7 $\mathrm{GeV}^2/\mathrm{c}^4$





Results :

$$|V_{ub}|/|V_{cb}|_{\mathrm{LCSR},\,q^2<7\,\mathrm{GeV}^2} = 0.0607 \pm 0.0015_{\mathrm{stat}} \pm 0.0013_{\mathrm{sys}} \pm 0.0008_{\mathrm{norm}} \pm 0.0030_{\mathrm{theo}}$$

$$|V_{ub}|/|V_{cb}|_{\mathrm{LQCD},\,q^2>7\,\mathrm{GeV}^2} = 0.0946 \pm 0.0030_{\mathrm{stat}} \pm {}^{+0.0024}_{-0.0025}{}_{\mathrm{sys}} \pm 0.0013_{\mathrm{norm}} \pm 0.0068_{\mathrm{theo}}$$

Form Factors has been estimated by <u>LCSR</u> at low q^2 and <u>LQCD</u> at high q^2

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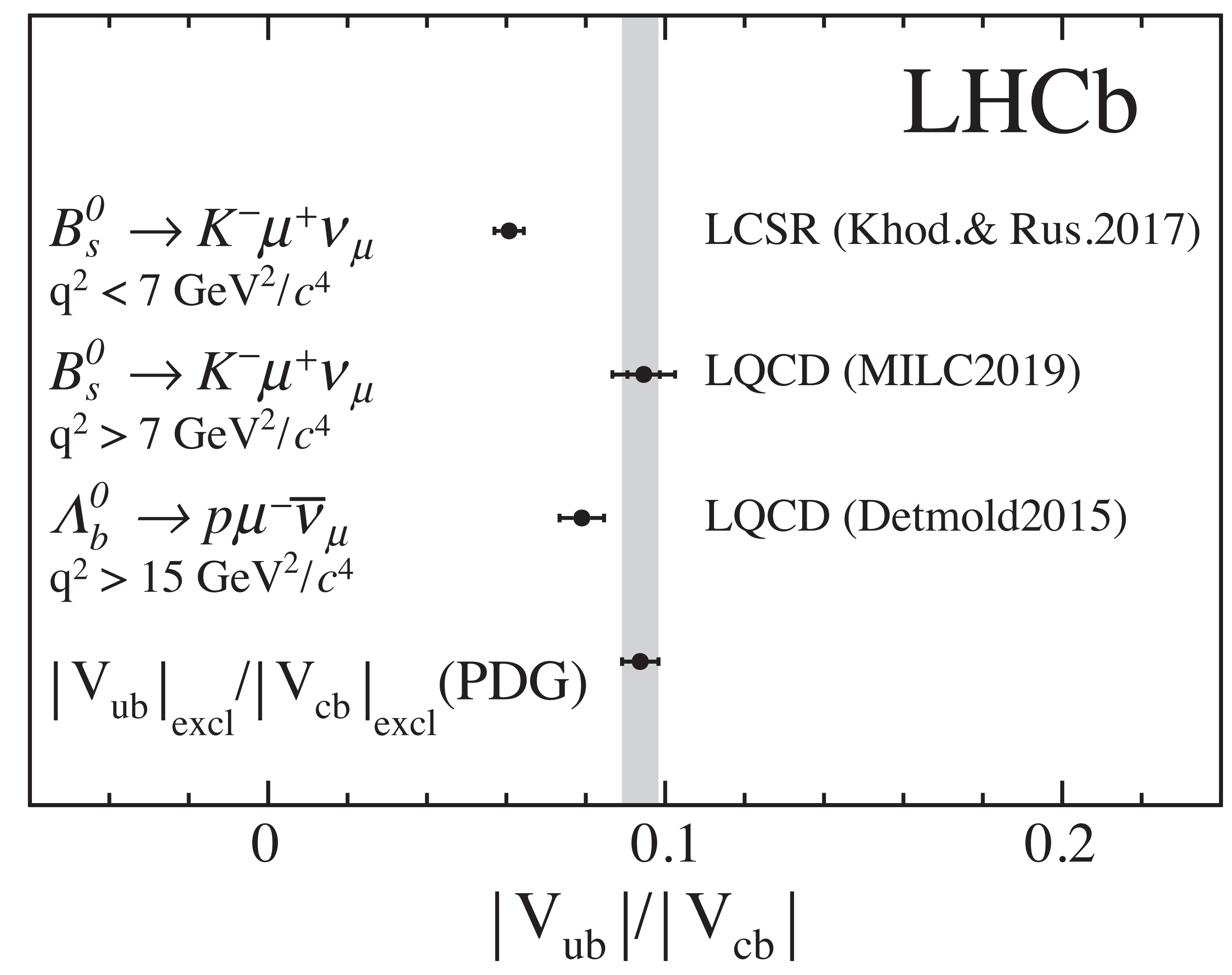
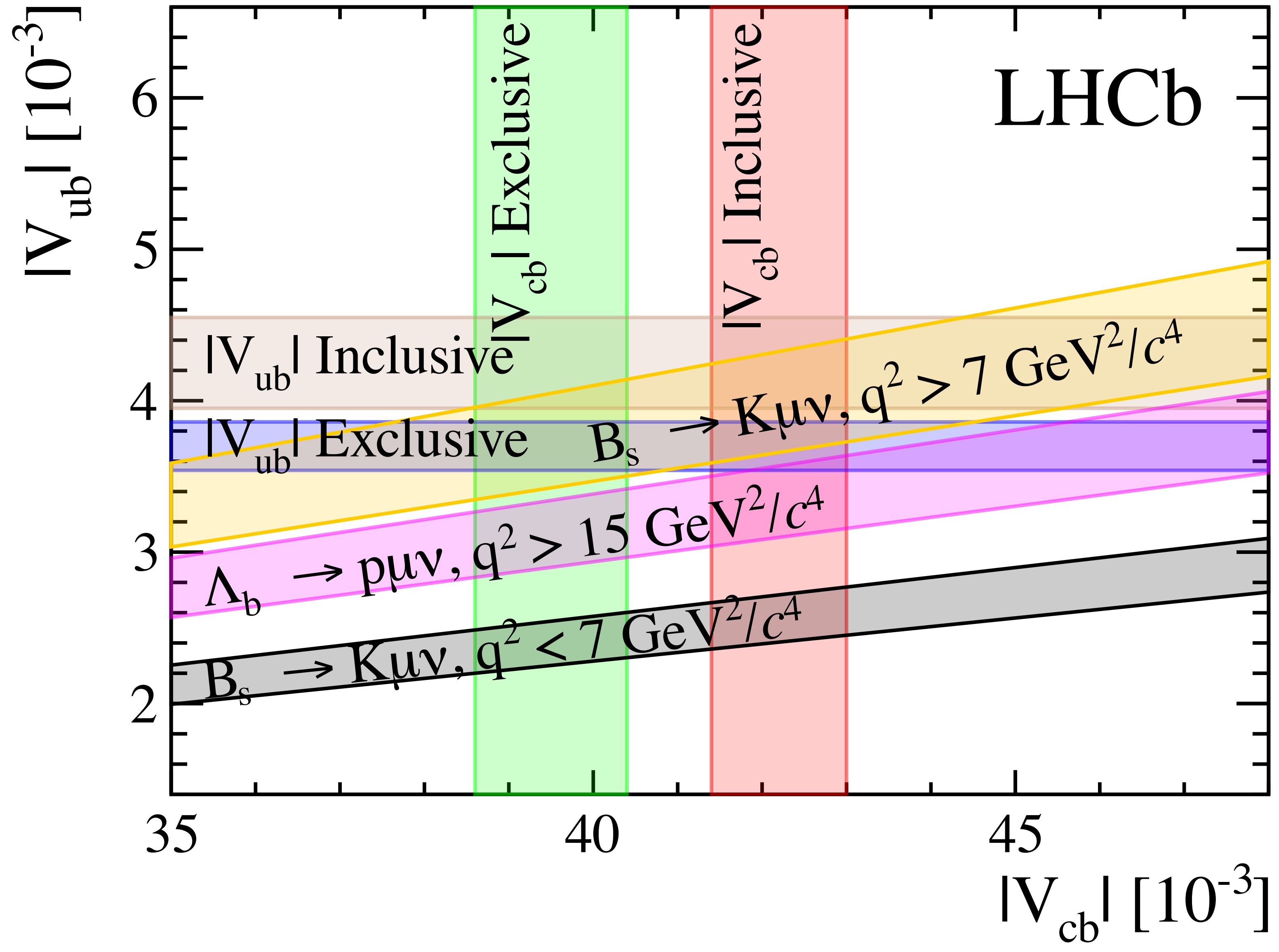


Phys. Rev. Lett. 126 (2021)



Results :

$$|V_{ub}|/|V_{cb}|_{\mathrm{LCSR},\,q^2<7\,\mathrm{GeV}^2} = 0.0607 \pm 0.0015_{\mathrm{stat}} \pm 0.0013_{\mathrm{sys}} \pm 0.0008_{\mathrm{norm}} \pm 0.0030_{\mathrm{theo}}$$

$$|V_{ub}|/|V_{cb}|_{\mathrm{LQCD},\,q^2>7\,\mathrm{GeV}^2} = 0.0946 \pm 0.0030_{\mathrm{stat}} \pm {}^{+0.0024}_{-0.0025}{}_{\mathrm{sys}} \pm 0.0013_{\mathrm{norm}} \pm 0.0068_{\mathrm{theo}}$$

$|V_{ub}|$ at low q^2 not in agreement with the incl/excl averages

Future $|V_{(u|c)b}|$ measurements at LHCb

Future Run 2 $|V_{(u|c)b}|$ measurements at LHCb

- Extracting $|V_{ub}|$ and a part of the $B^+ \to \rho^0$ BCL Form Factors from $B^+ \to \rho^0 \mu\, \nu_\mu$ in 10 q^2 bins

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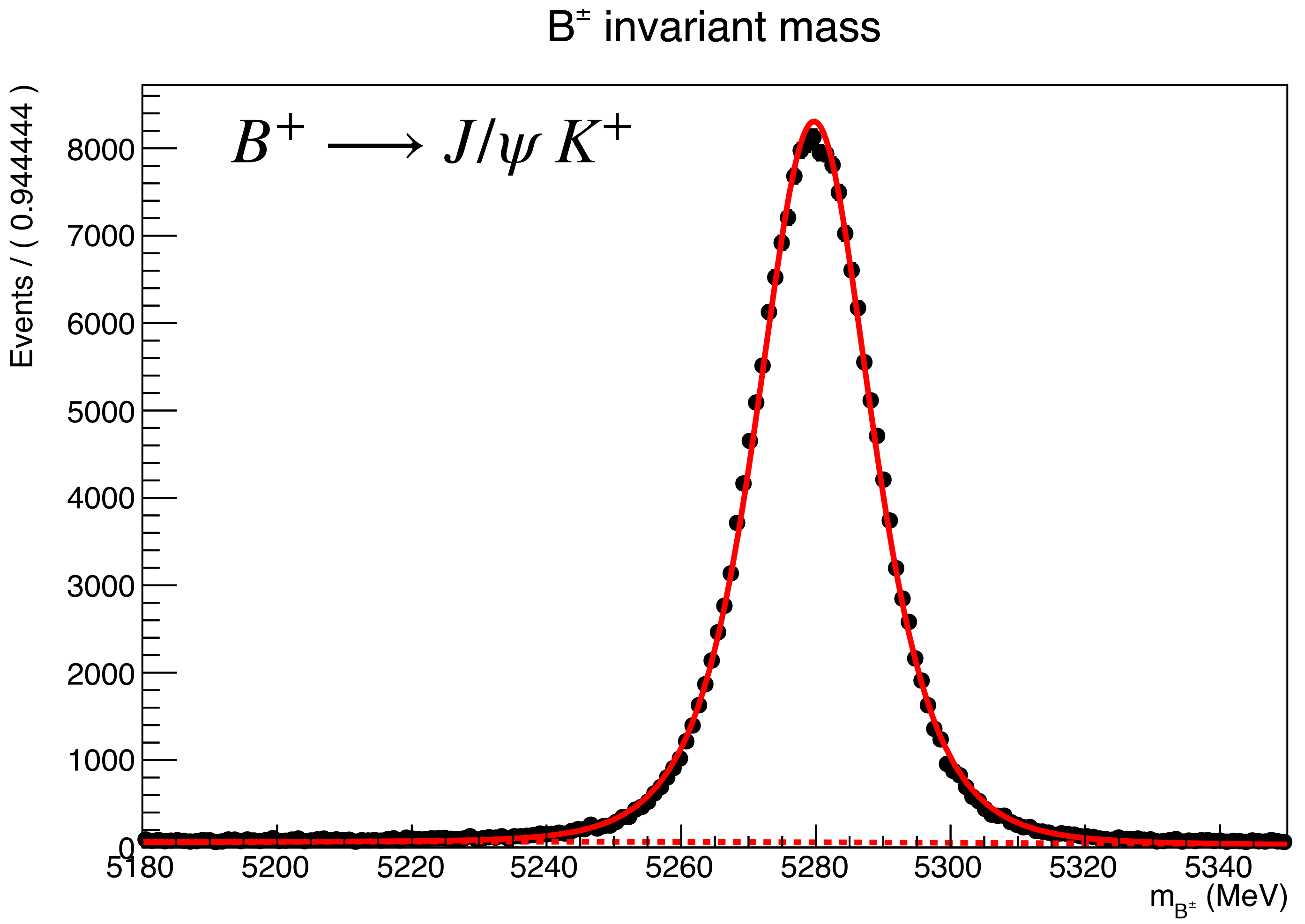
- Extracting $|V_{cb}|$ from $\Lambda_b^0 \to \Lambda_c^+\, \mu\, \nu_\mu$

  $\to$ First determination of $|V_{cb}|$ from a baryonic semileptonic decay

Status of $|V_{ub}|$ measurements from $B_s^0 \to K \mu \nu_\mu$ Run 2 dataset

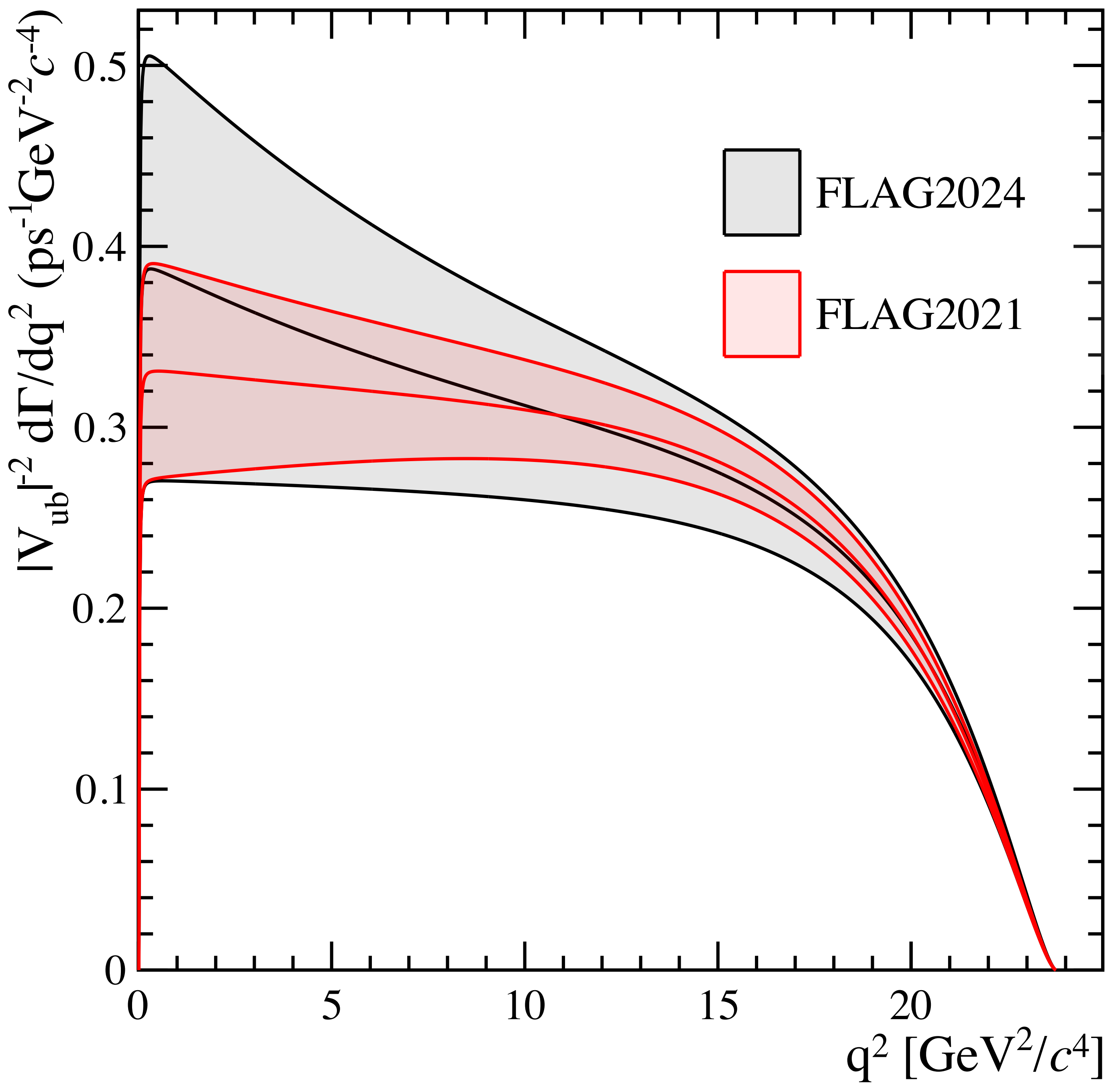
Analysis overview



- **Goal :** Determination of $\dfrac{d\Gamma}{dq^2}$ and $|V_{ub}|$ from $B_s^0 \to K^-\mu^+\nu_\mu$

- **Normalisation channel :** $B^+ \longrightarrow J/\psi\, K^+$
  - $\Rightarrow$ Absolute measurement of $|V_{ub}|$ & high statistics

- **Dataset :** Full Run 2 ~ 5.67 fb^{-1} (factor 6 increase wrt Run1 analysis)
  - $\Rightarrow$ Possibility to increase the number of q^2 bins from 2 (Run1) to ~ 8 - 10

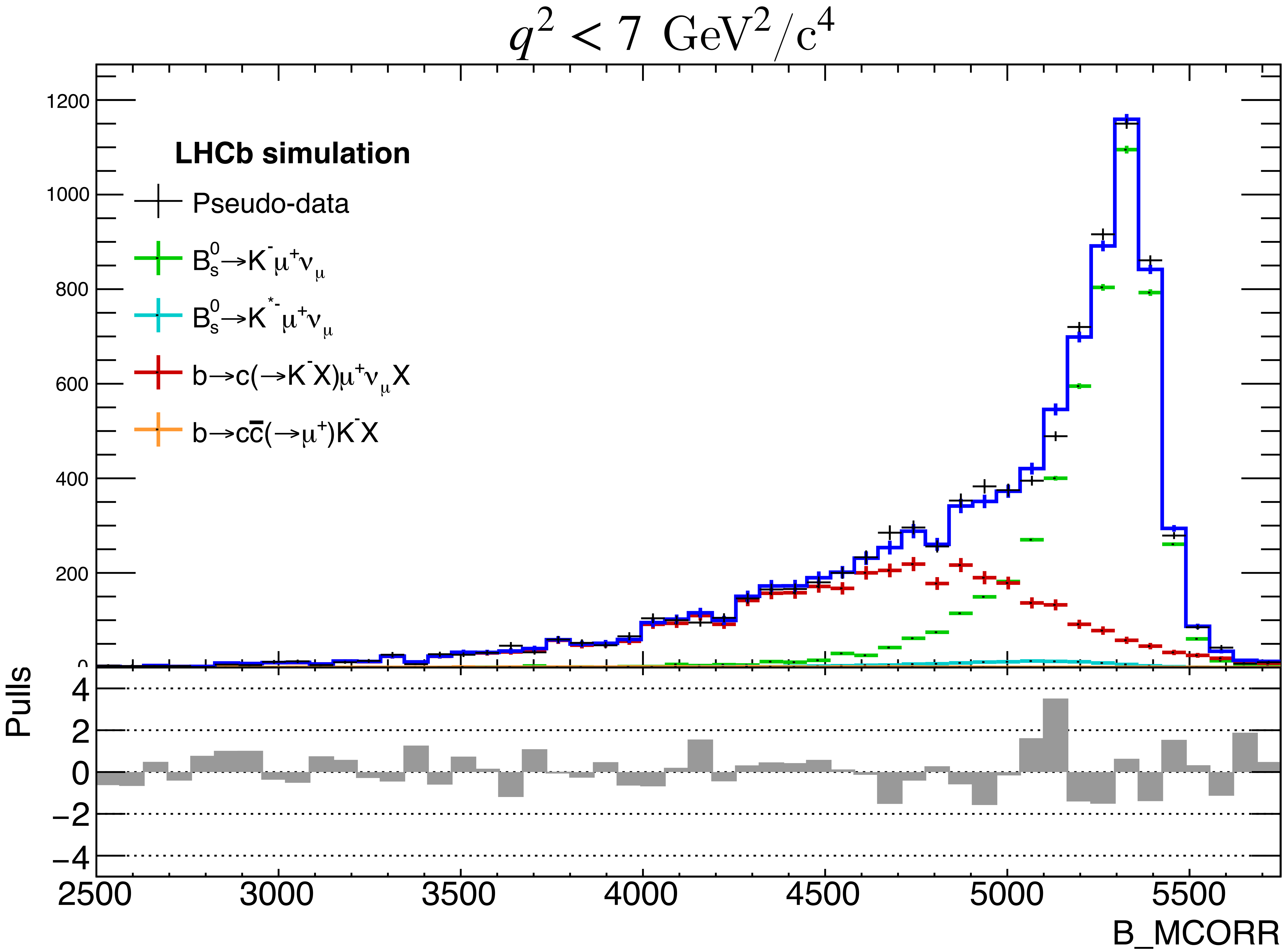


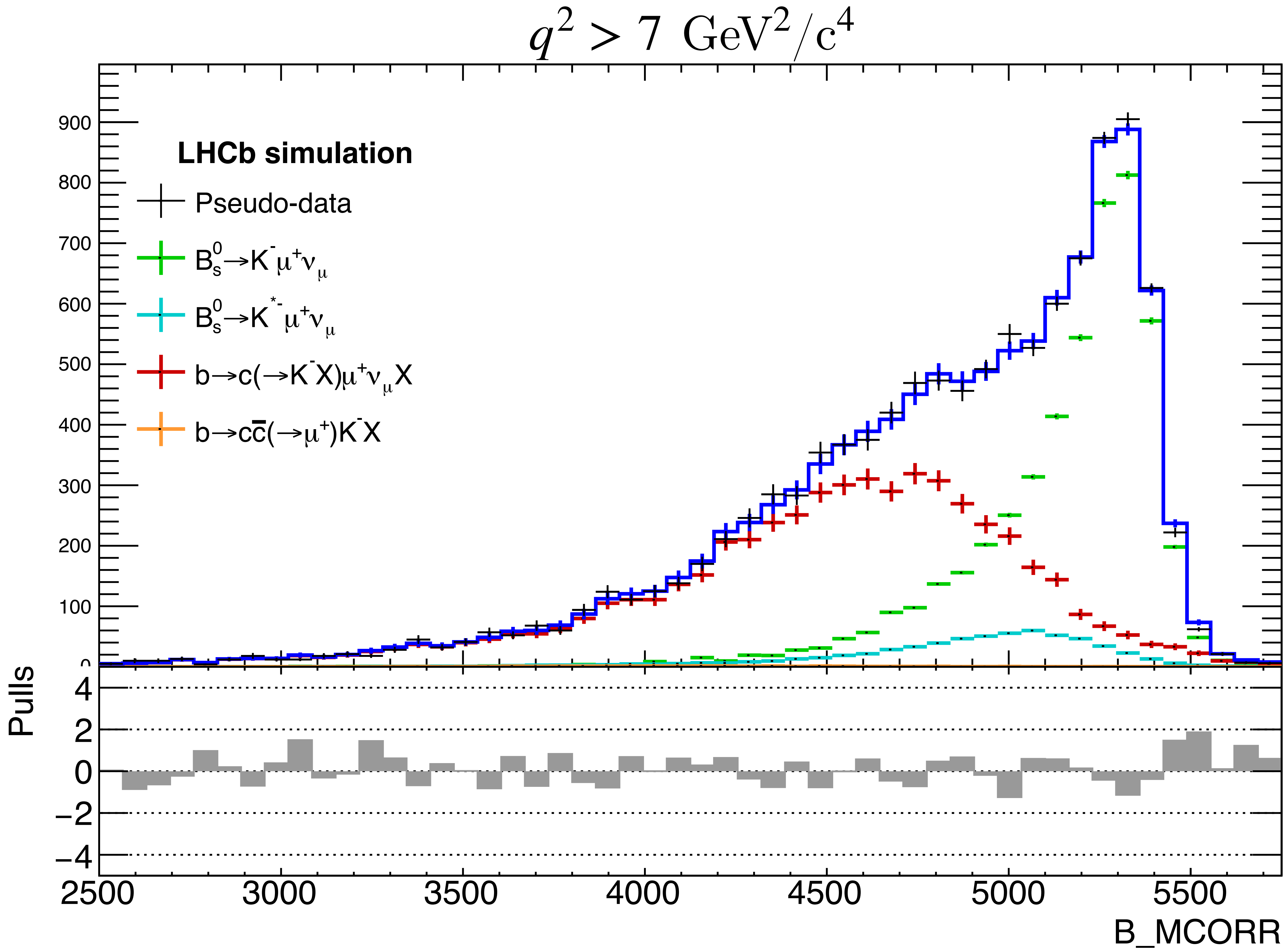
- **FF parameterization :**
  - $\Rightarrow$ FF schemes taken from FLAG 2024 and test others when available

- Based on Run1 strategy, working to optimise the various steps

Current M_{corr} plots ?

Toys





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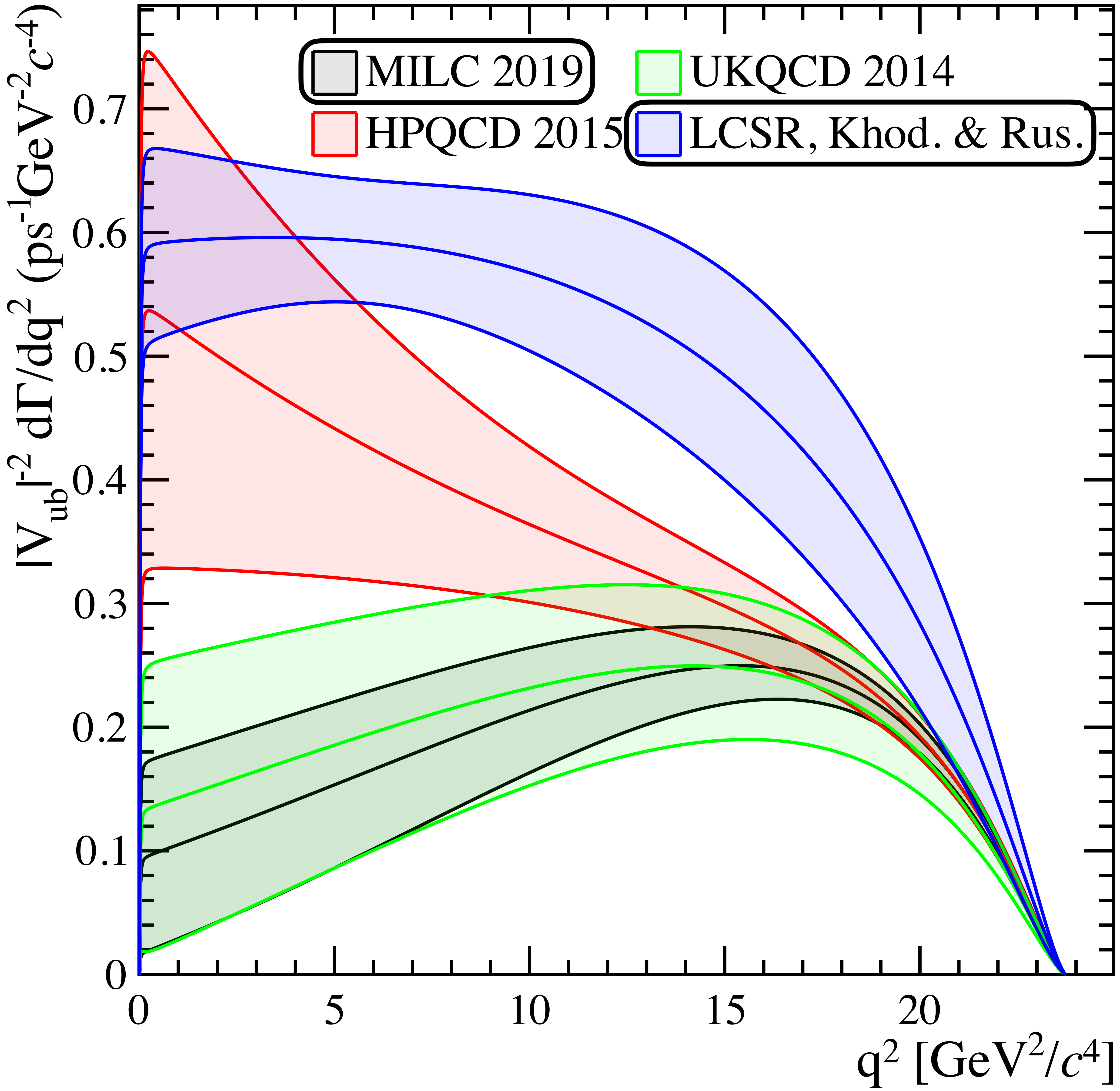
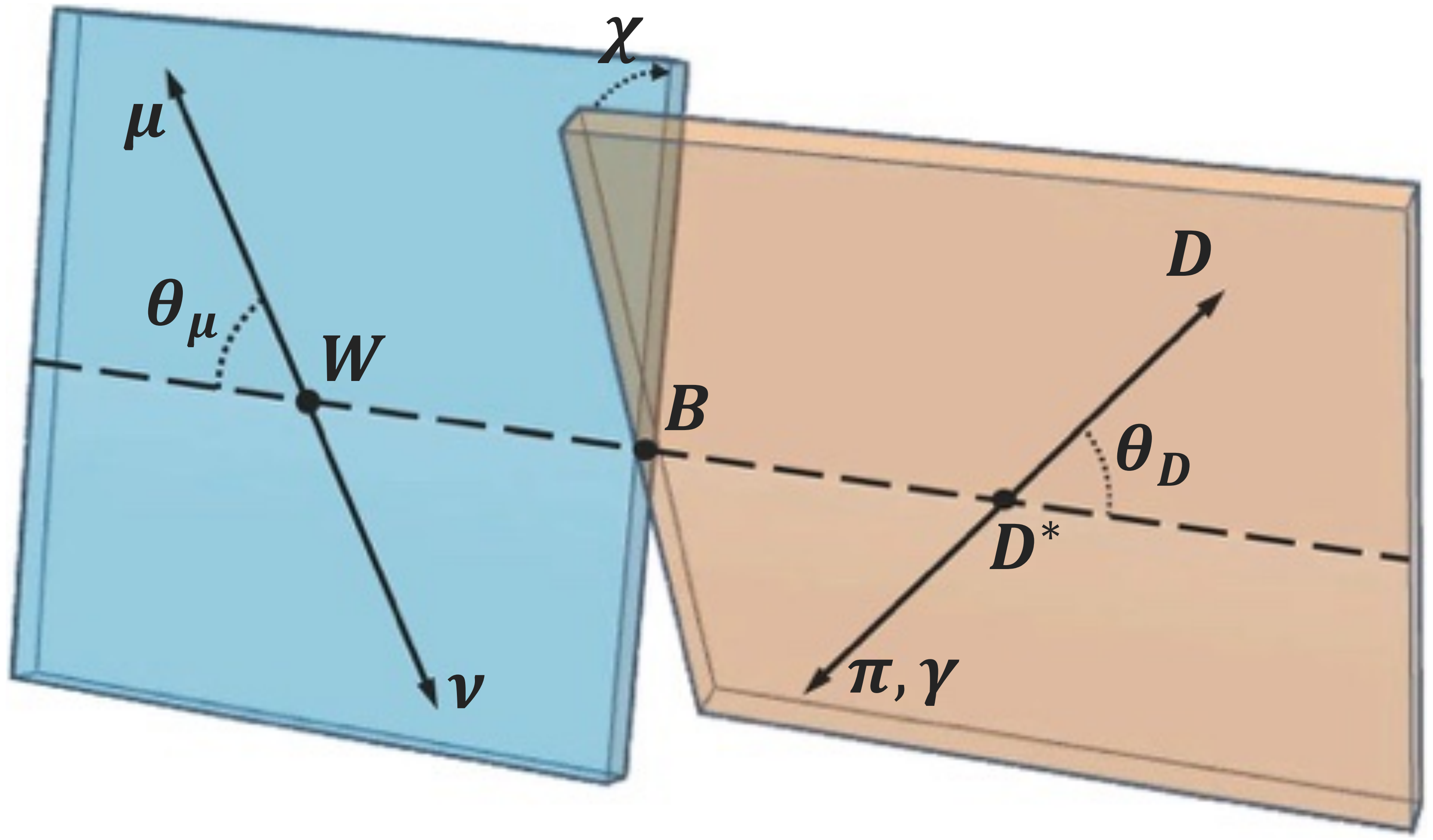
- Waiting for Belle 2 new results !

BACK-UP

Form Factors used in $B_s^0 \to K\,\mu\,\nu_\mu$ from Run 1



$B \to D^{*-} \mu \nu_\mu$ analysis theory : CLN



$$\frac{d^4\Gamma(B \to D^*\mu\nu)}{dw\, d\cos\theta_\mu\, d\cos\theta_D\, d\chi} = \frac{3m_B^3 m_{D^*}^2 G_F^2}{16(4\pi)^4}\eta_{EW}^2 |V_{cb}|^2 |\mathcal{A}(w,\theta_\mu,\theta_D,\chi)|^2$$

$$|\mathcal{A}(w,\theta_\mu,\theta_D,\chi)|^2 = \sum_i^6 \mathcal{H}_i(w) k_i(\theta_\mu,\theta_D,\chi)$$

$$w = v_B \cdot v_{D^*} = (m_B^2 + m_{D^*}^2 - q^2)/(2m_B m_{D^*})$$

$$H_{\pm/0}(w) = 2\frac{\sqrt{m_B m_{D^*}}}{m_B + m_{D^*}}(1-r^2)(w+1)(w^2-1)^{1/4} \times h_{A_1}(w)\tilde{H}_{\pm/0}(w),$$

with $r = m_{D^*}/m_B$ and

$$\tilde{H}_\pm(w) = \frac{\sqrt{1-2wr+r^2}}{1-r}\left[1 \mp \sqrt{\frac{w-1}{w+1}}R_1(w)\right],$$

$$\tilde{H}_0(w) = 1 + \frac{(w-1)(1-R_2(w))}{1-r}.$$

$$h_{A_1}(w) = h_{A_1}(1)[1 - 8\rho^2 z + (53\rho^2 - 15)z^2 - (231\rho^2 - 91)z^3],$$

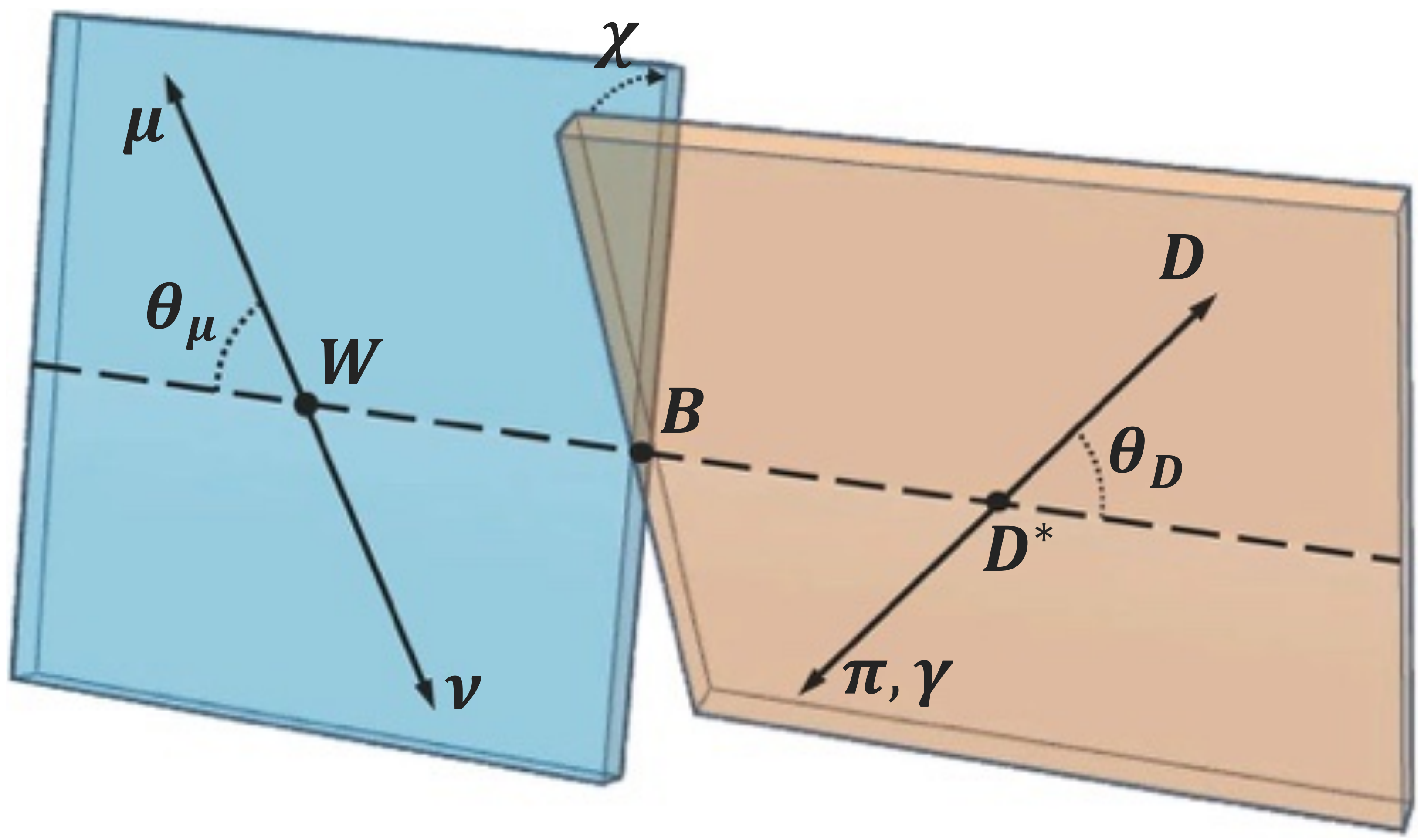
$$R_1(w) = R_1(1) - 0.12(w-1) + 0.05(w-1)^2$$

$$R_2(w) = R_2(1) - 0.11(w-1) - 0.06(w-1)^2$$

$$z \equiv \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} + \sqrt{2}}$$

		$k_i(\theta_\mu,\theta_D,\chi)$	
i	$\mathcal{H}_i(w)$	$D^* \to D\gamma$	$D^* \to D\pi^0$
1	H_+^2	$\frac{1}{2}(1+\cos^2\theta_D)(1-\cos\theta_\mu)^2$	$\sin^2\theta_D(1-\cos\theta_\mu)^2$
2	H_-^2	$\frac{1}{2}(1+\cos^2\theta_D)(1+\cos\theta_\mu)^2$	$\sin^2\theta_D(1+\cos\theta_\mu)^2$
3	H_0^2	$2\sin^2\theta_D\sin^2\theta_\mu$	$4\cos^2\theta_D\sin^2\theta_\mu$
4	H_+H_-	$\sin^2\theta_D\sin^2\theta_\mu\cos 2\chi$	$-2\sin^2\theta_D\sin^2\theta_\mu\cos 2\chi$
5	H_+H_0	$\sin 2\theta_D\sin\theta_\mu(1-\cos\theta_\mu)\cos\chi$	$-2\sin 2\theta_D\sin\theta_\mu(1-\cos\theta_\mu)\cos\chi$
6	H_-H_0	$-\sin 2\theta_D\sin\theta_\mu(1+\cos\theta_\mu)\cos\chi$	$2\sin 2\theta_D\sin\theta_\mu(1+\cos\theta_\mu)\cos\chi$

$B \to D^{*-}\,\mu\,\nu_\mu$ analysis theory : BGL



$$\frac{\mathrm{d}^4\Gamma(B\to D^*\mu\nu)}{\mathrm{d}w\,\mathrm{d}\cos\theta_\mu\,\mathrm{d}\cos\theta_D\,\mathrm{d}\chi} = \frac{3m_B^3 m_{D^*}^2 G_\mathrm{F}^2}{16(4\pi)^4}\eta_\mathrm{EW}^2|V_{cb}|^2|\mathcal{A}(w,\theta_\mu,\theta_D,\chi)|^2$$

$$|\mathcal{A}(w,\theta_\mu,\theta_D,\chi)|^2 = \sum_i^6 \mathcal{H}_i(w)k_i(\theta_\mu,\theta_D,\chi)$$

$$w = v_B\cdot v_{D^*} = (m_B^2+m_{D^*}^2-q^2)/(2m_Bm_{D^*})$$

$$H_{\pm/0}(w) = 2\frac{\sqrt{m_Bm_{D^*}}}{m_B+m_{D^*}}(1-r^2)(w+1)(w^2-1)^{1/4} \times h_{A_1}(w)\tilde{H}_{\pm/0}(w),$$

with $r = m_{D^*}/m_B$ and

$$\tilde{H}_\pm(w) = \frac{\sqrt{1-2wr+r^2}}{1-r}\left[1\mp\sqrt{\frac{w-1}{w+1}}R_1(w)\right],$$

$$\tilde{H}_0(w) = 1+\frac{(w-1)(1-R_2(w))}{1-r}.$$

$$h_{A_1}(w) = \frac{f(w)}{\sqrt{m_Bm_{D^*}}(1+w)},$$

$$R_1(w) = (w+1)m_Bm_{D^*}\frac{g(w)}{f(w)},$$

$$R_2(w) = \frac{w-r}{w-1} - \frac{\mathcal{F}_1(w)}{m_B(w-1)f(w)}$$

$$f(z) = \frac{1}{P_{1^+}(z)\phi_f(z)}\sum_{n=0}^\infty b_nz^n \qquad \phi_f(z) = \frac{4r}{m_B^2}\sqrt{\frac{n_I}{3\pi\tilde{\chi}_{1^+}(0)}}\frac{(1+z)\sqrt{(1-z)^3}}{[(1+r)(1-z)+2\sqrt{r}(1+z)]^4},$$

$$g(z) = \frac{1}{P_{1^-}(z)\phi_g(z)}\sum_{n=0}^\infty a_nz^n \qquad \phi_g(z) = 16r^2\sqrt{\frac{n_I}{3\pi\tilde{\chi}_{1^-}(0)}} \times\frac{(1+z)^2}{\sqrt{(1-z)}[(1+r)(1-z)+2\sqrt{r}(1+z)]^4},$$

$$\mathcal{F}_1(z) = \frac{1}{P_{1^+}(z)\phi_{\mathcal{F}_1}(z)}\sum_{n=0}^\infty c_nz^n \qquad \phi_{\mathcal{F}_1}(z) = \frac{4r}{m_B^3}\sqrt{\frac{n_I}{6\pi\tilde{\chi}_{1^+}(0)}}\frac{(1+z)\sqrt{(1-z)^5}}{[(1+r)(1-z)+2\sqrt{r}(1+z)]^5}$$

$$P_{1^\pm}(z) = C_{1^\pm}\prod_{k=1}^{\mathrm{poles}}\frac{z-z_k}{1-zz_k} \qquad z_k = \frac{\sqrt{t_+-m_k^2}-\sqrt{t_+-t_-}}{\sqrt{t_+-m_k^2}+\sqrt{t_+-t_-}}$$

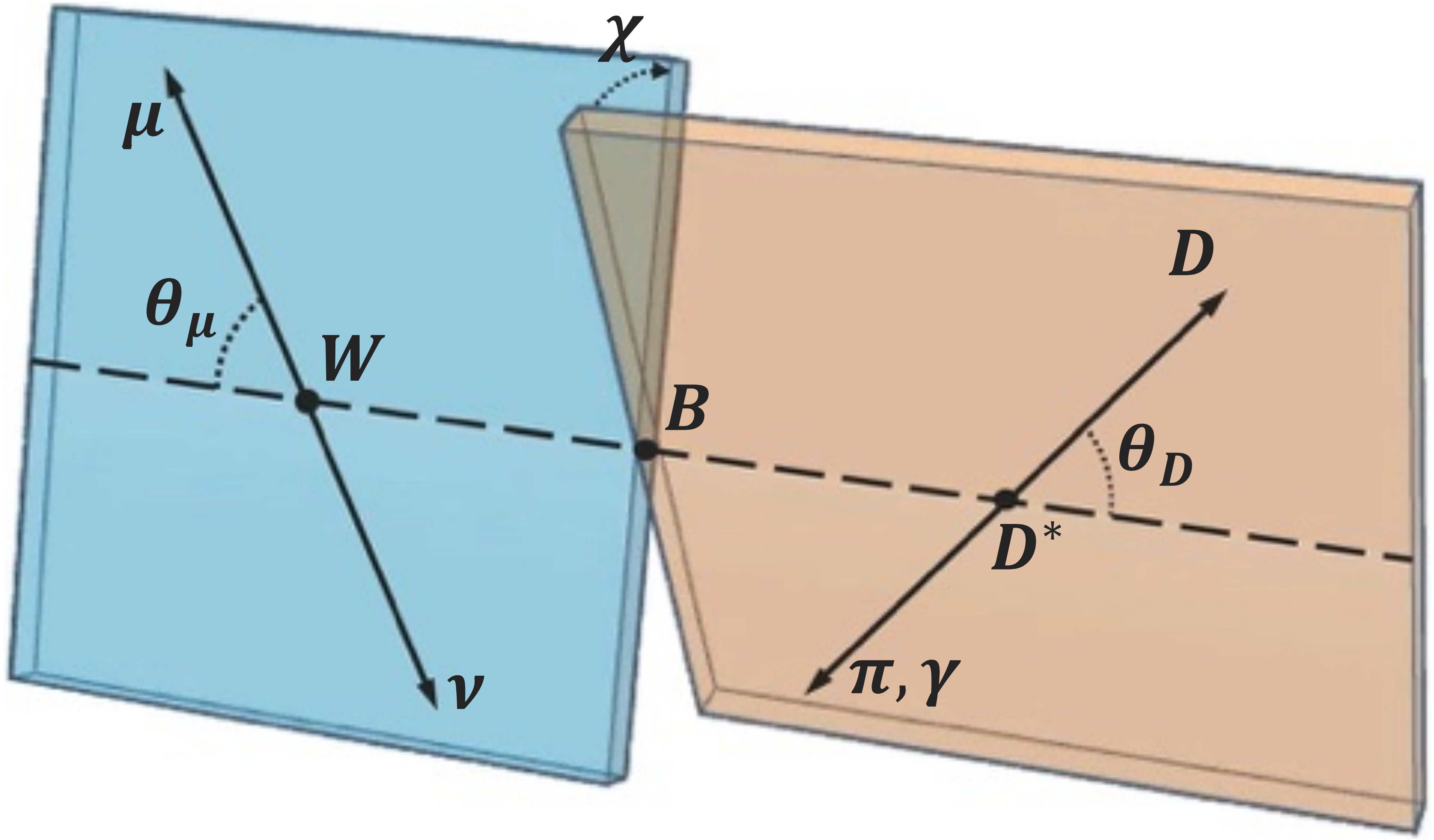
$$t_\pm = (m_B\pm m_{D^*})^2 \qquad n_I = 2.6$$

$$\sum_{n=0}^\infty a_n^2 \le 1, \qquad \sum_{n=0}^\infty(b_n^2+c_n^2)\le 1$$

$$b_0 = 2\sqrt{m_Bm_{D^*}}P_{1^+}(0)\phi_f(0)h_{A_1}$$

$$c_0 = (m_B-m_{D^*})\frac{\phi_{\mathcal{F}_1}(0)}{\phi_f(0)}b_0$$

$B \to D^- \mu\, \nu_\mu$ analysis theory : CLN

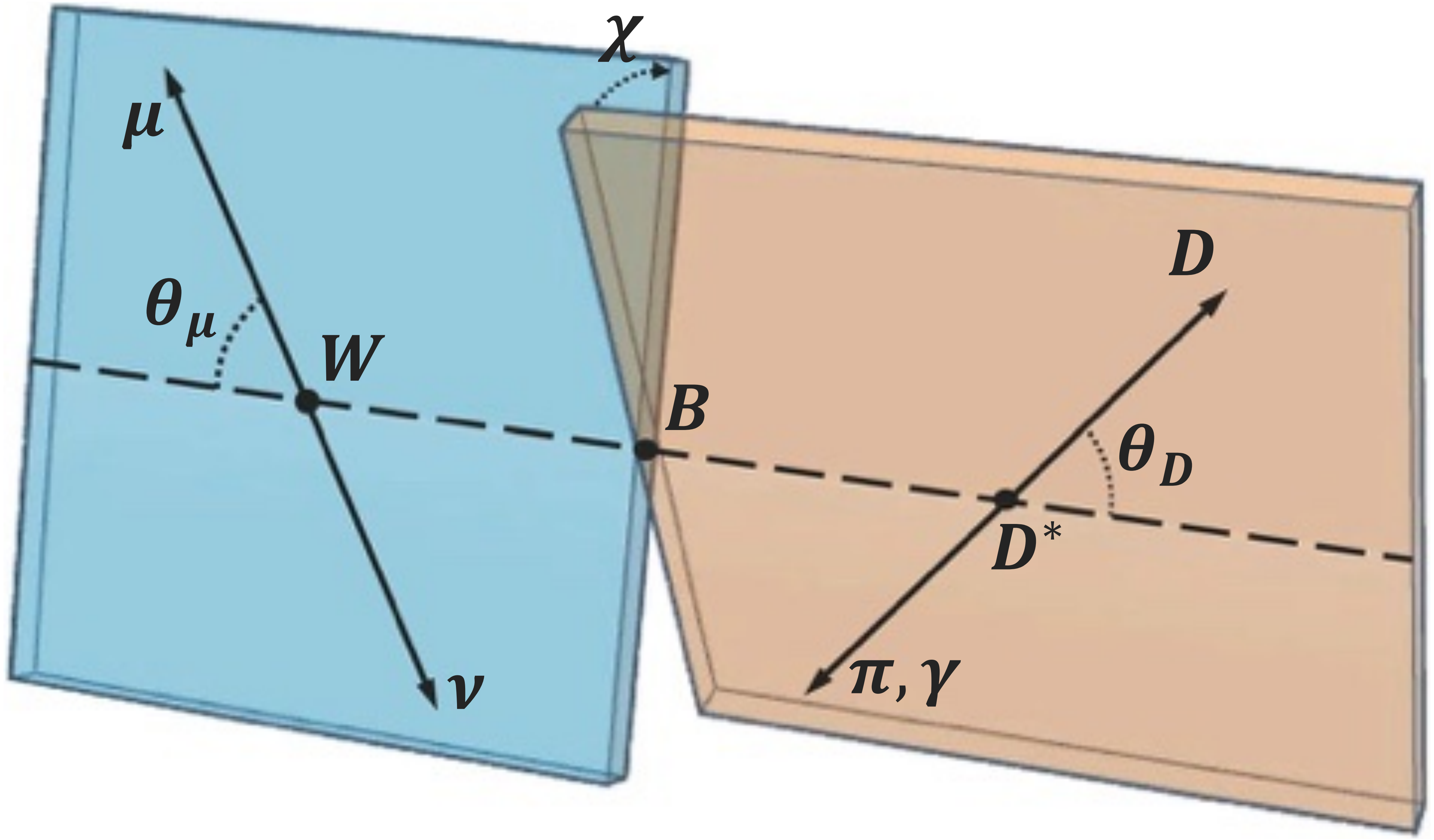


$$\frac{\mathrm{d}\Gamma(B \to D\mu\nu)}{\mathrm{d}w} = \frac{G_F^2 m_D^3}{48\pi^3}(m_B + m_D)^2 \eta_{\mathrm{EW}}^2 \times |V_{cb}|^2 (w^2 - 1)^{3/2} |\mathcal{G}(w)|^2$$

$$\mathcal{G}(z) = \mathcal{G}(0)[1 - 8\rho^2 z + (51\rho^2 - 10)z^2 - (252\rho^2 - 84)z^3]$$

$$w = v_B \cdot v_{D^*} = (m_B^2 + m_{D^*}^2 - q^2)/(2m_B m_{D^*})$$

$B \to D^- \, \mu \, \nu_\mu$ analysis theory : BGL



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$$|\mathcal{G}(z)|^2 = \frac{4r}{(1+r)^2}|f_+(z)|^2 \qquad r = m_D/m_B$$

$$f_+(z) = \frac{1}{P_{1^-}(z)\phi(z)} \sum_{n=0}^{\infty} d_n z^n \qquad \sum_{n=0}^{\infty} d_n^2 \leq 1$$

$$d_0 = \frac{1+r}{2\sqrt{r}}\mathcal{G}(0)P_{1^-}(0)\phi(0)$$