Equation of State in the era of new nuclear physics and multi-messenger constraints

Chiranjib Mondal

Modélisation des Astres Compacts
Caen
July 10, 2025

Neutron star matter

Hydrostatic equilibrium

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Hydrostatic equilibrium

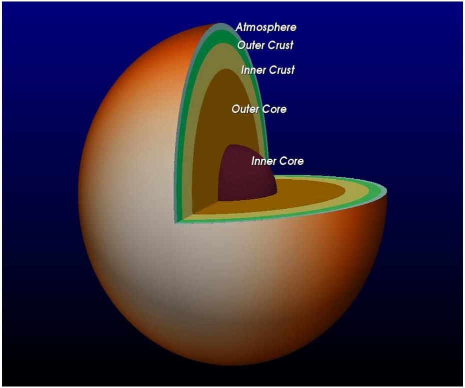
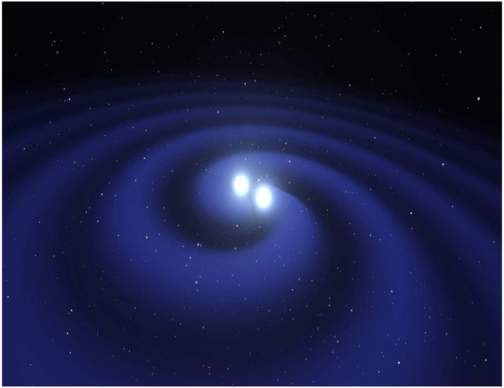


Fig courtesy: C. Gonzalez-Boquera

- Cold catalyzed matter in full thermodynamic equilibrium at T=0. (Isolated case)

Neutron star matter

Hydrostatic equilibrium and beyond



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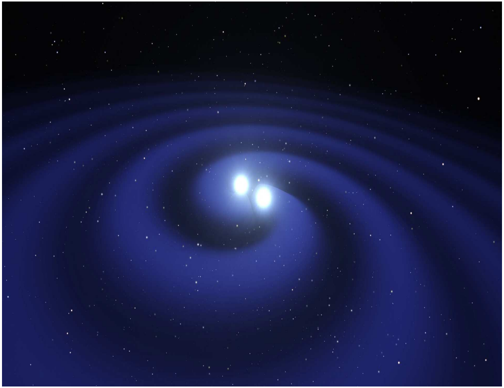

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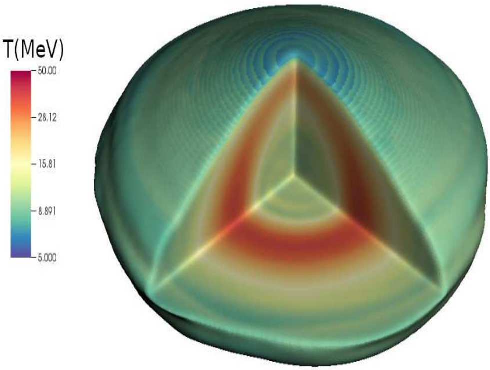

Fig courtesy:
Perego *et. al.*, EPJA 55, 124 (2019)

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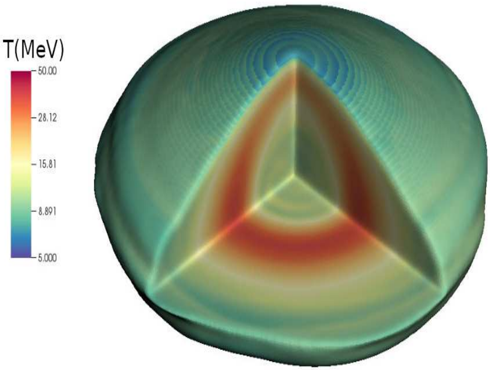

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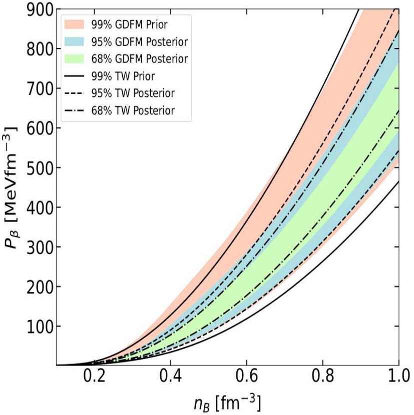
- We need $\mathrm{P}(\mathrm{n}, \mathrm{T}, \mathrm{x_p})$.

EoS

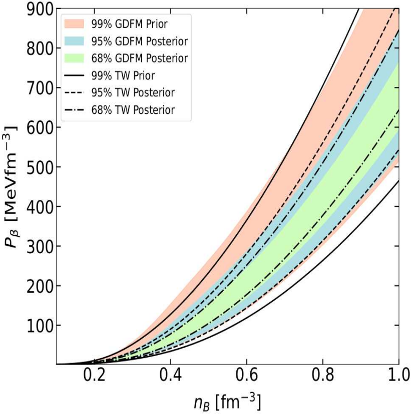


P. Char and CM, Phys.Rev.D 111, 103024 (2025).

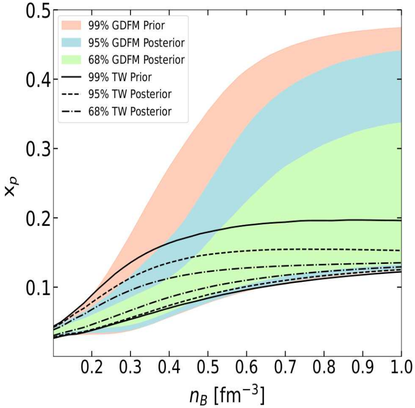
RMF metamodelling

The question of composition

EoS



Composition



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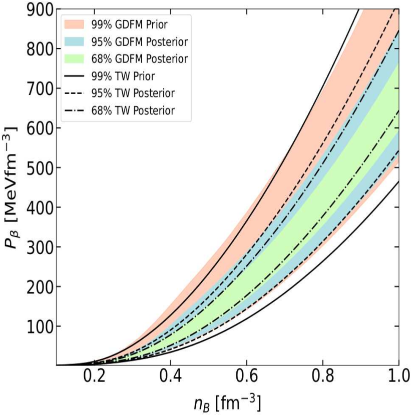

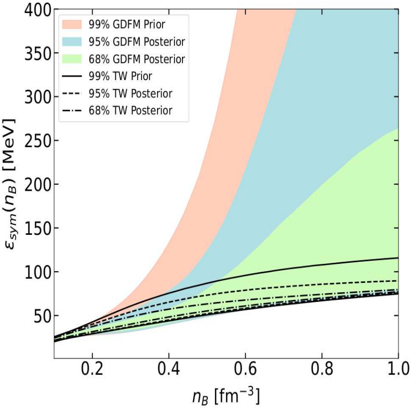

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The question of composition

From single loud detection by ET (phase transition or PT)

- Low density phase: nucleonic meta-modeling
- High density phase: constant sound speed

Different choices for $\mathrm{q}, \mathcal{M}_{\mathrm{c}}, \mathcal{D}_{\mathrm{L}}$ and PT injection models.

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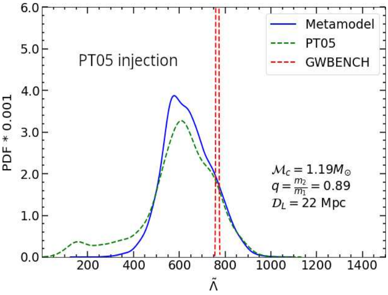

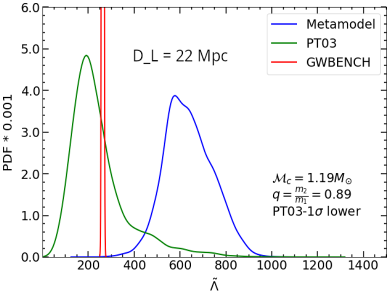

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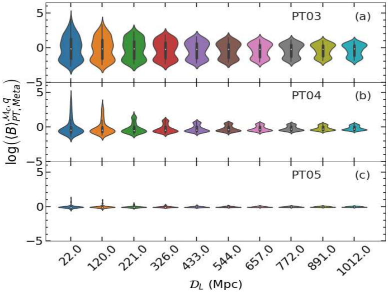

$$\boxed{B_{\rm pt,meta}(\tilde{\Lambda}) = \frac{p_{\rm pt}(\tilde{\Lambda})}{p_{\rm meta}(\tilde{\Lambda})}}$$

$$\boxed{\log\left(\langle B \rangle_{\rm pt,meta}\right) = \int d\tilde{\Lambda}\ p_{\rm GW}(\tilde{\Lambda}) \log\left[\frac{p_{\rm pt}(\tilde{\Lambda})}{p_{\rm meta}(\tilde{\Lambda})}\right]}.$$

Bayesian studies with whole mass table?

Machine learning masses?

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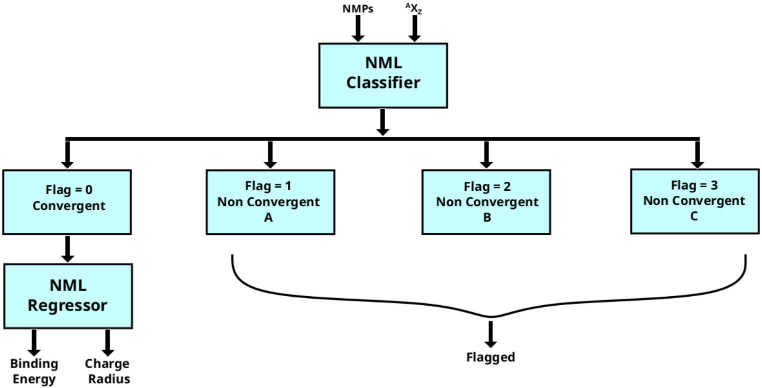
The idea



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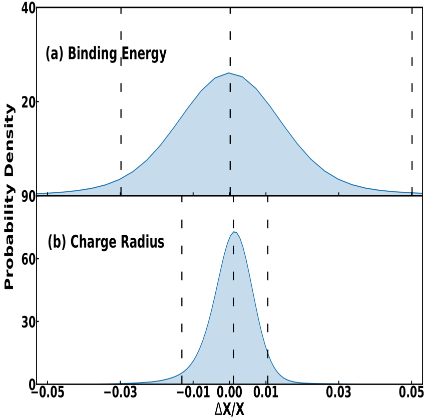
Training



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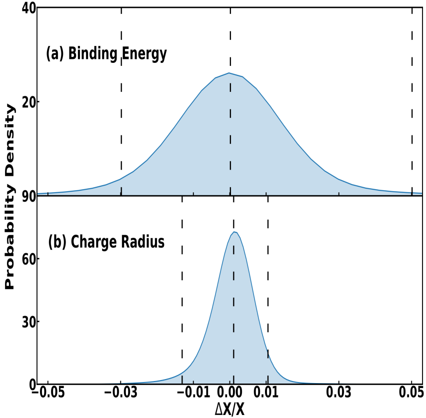
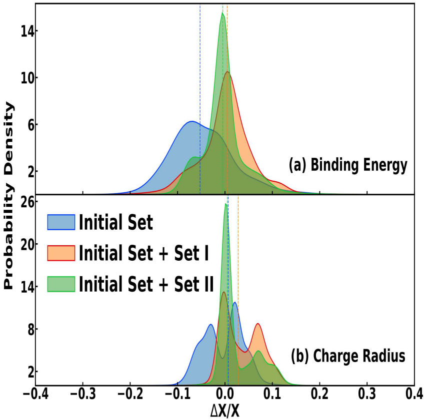


Prediction



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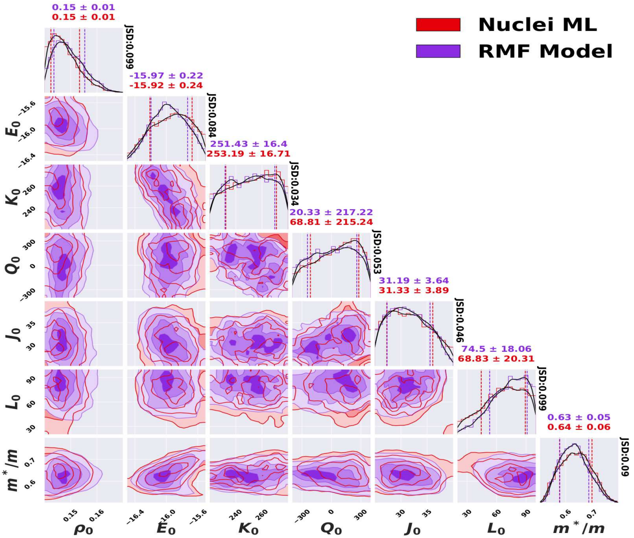
Bayesian studies with whole mass table?

First proof-of-principle results



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Relevance of temperature

Merger simulations

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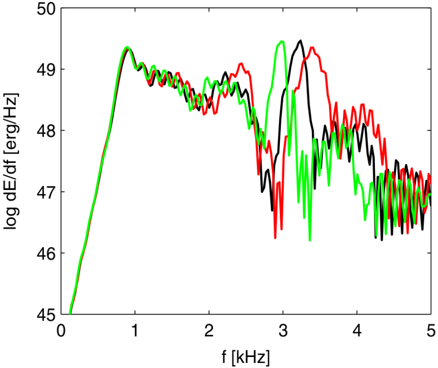
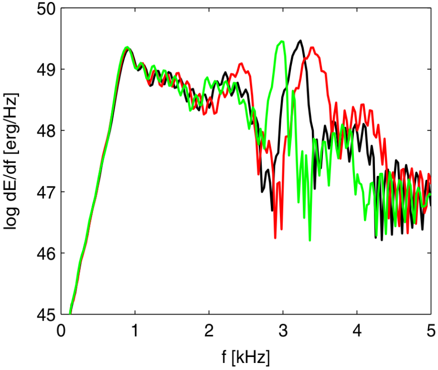


Fig Courtesy:
Bauswein *et. al.* PRD 82, 084043 (2010)

- Luminosity spectra for $M_1 = M_2 = 1.35 M_\odot$ binary system



- Luminosity spectra for $M_1 = M_2 = 1.35M_\odot$ binary system
- Ideal gas index: $\Gamma_{th}(n, T, x_p) = \frac{P_{th}}{n\mathcal{E}_{th}} + 1$.
- Red:$\Gamma_{th} = 1.5$; Green:$\Gamma_{th} = 2$; **Black**: Full temp dep.
  The frequencies can vary from 50-250 Hz.

Usage of a common framework

State-of-the-art BSkG3 model

- End-to-end NS merger simulations =>
  Hydrodynamics, Nucleosynthesis, radiative transfer
  [See Just *et. al.* ApJL 951, 12 (2023), MNRAS 510, 2804 (2022), MNRAS 510, 2820 (2022)]

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Main Features:

- Finite Nuclei: HFB equations on the grid, fitted on whole mass table.
  $\sigma_{\mathrm{M}}^{rms} = 0.631$ MeV
  $\sigma_{r_{\mathrm{ch}}} = 0.0237$ fm
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- Neutron stars:
  Fulfill the observational constraint on massive pulsars.

Energy Density Functional

Brussels Skyrme model

- At baryon density n, asymmetry $\delta\left(=\frac{\mathrm{n_n}-\mathrm{n_p}}{\mathrm{n}}\right)$, temperature T,

$$
\begin{aligned}
&\mathcal{F} \equiv \mathcal{E} - \mathrm{T}\mathcal{S}, \\
&\text{where } \mathcal{E} = \sum_{\mathrm{q}} \frac{\hbar^2}{2\mathrm{M}_\mathrm{q}^*}\tau_\mathrm{q} + \frac{1}{8}\mathrm{t}_0\left\{3-(2\mathrm{x}_0+1)\delta^2\right\}\mathrm{n}^2 \\
&+\frac{1}{48}\mathrm{t}_3\left\{3-(2\mathrm{x}_3+1)\delta^2\right\}\mathrm{n}^{\alpha+2}. \\
&\text{with } \frac{\hbar^2}{2\mathrm{M}_\mathrm{q}^*} = \frac{\hbar^2}{2\mathrm{M}_\mathrm{q}} + f\left(\mathrm{n},\delta\right)
\end{aligned}
$$

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$$\text{with } \frac{\hbar^2}{2\mathrm{M_q^*}} = \frac{\hbar^2}{2\mathrm{M_q}} + f(\mathrm{n}, \delta)$$

- The density and kinetic density involves Fermi integrals (q=n,p):

$$\mathrm{n_q} = \frac{1}{2\pi^2}\left(\frac{2\mathrm{M_q^*}}{\hbar^2}\right)^{\frac{3}{2}}\mathrm{T}^{\frac{3}{2}}\mathrm{I}_{\frac{1}{2}}(\nu_{\mathrm{q}}); \quad \tau_{\mathrm{q}} = \frac{1}{2\pi^2}\left(\frac{2\mathrm{M_q^*}}{\hbar^2}\right)^{\frac{5}{2}}\mathrm{T}^{\frac{5}{2}}\mathrm{I}_{\frac{3}{2}}(\nu_{\mathrm{q}})$$

where, $\mathrm{I}_\sigma(\nu_{\mathrm{q}}) = \int_0^\infty \frac{x^\sigma}{1+\exp(x-\nu_{\mathrm{q}})}dx$

WS cell at finite temperature

Thomas Fermi (fast approximation of HFB)

Form of the density:

$$\mathrm{n_q(r) = n_{B,q} + \frac{n_{0,q}}{1+\exp\left\{\left(\frac{C_q-R_{WS}}{r-R_{WS}}\right)^2-1\right\}\exp\left(\frac{r-C_q}{a_q}\right)}}$$

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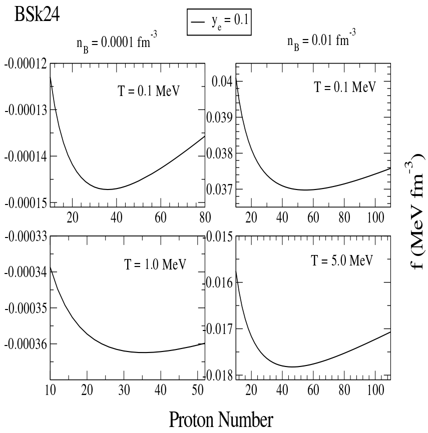
Chemical potential for damped profile [Chamel, **Shchechilin**, and Chugunov, PRC 111, 015805 (2025).]:

$$\tilde{\mu}_q = \frac{3}{R_{WS}^3}\int_0^{R_{WS}} dr \cdot r^2 \frac{\partial \mathcal{F}(r)}{\partial n_q(r)},$$

$$n_q = \frac{1}{2\pi^2}\left(\frac{2M_q^*}{\hbar^2}\right)^{\frac{3}{2}} T^{\frac{3}{2}} I_{\frac{1}{2}}(\nu_q); \quad \tau_q = \frac{1}{2\pi^2}\left(\frac{2M_q^*}{\hbar^2}\right)^{\frac{5}{2}} T^{\frac{5}{2}} I_{\frac{3}{2}}(\nu_q);$$

$$S_q = \frac{5}{3T}\frac{\hbar^2}{2M_q^*}\tau_q - \nu_q n_q.$$

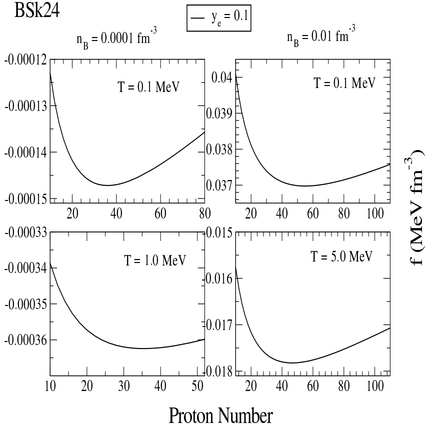
Given ρ, T, y_e

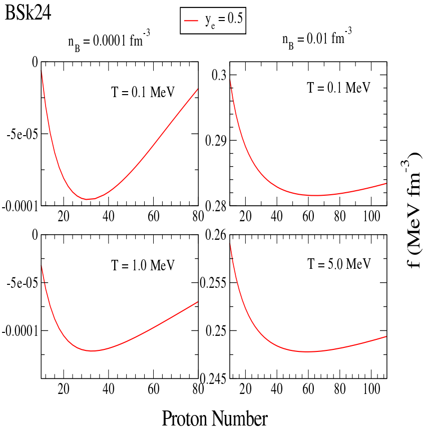


Clusters at finite temperature

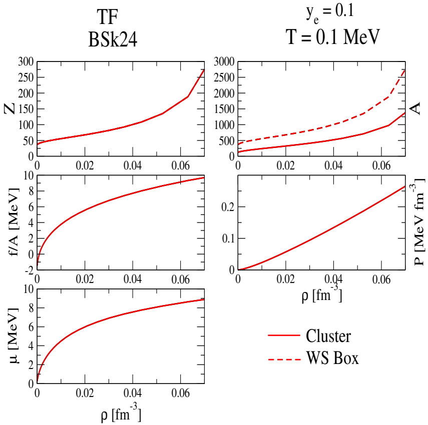
Full grid

Given ρ, T, y_e





ρ-variation



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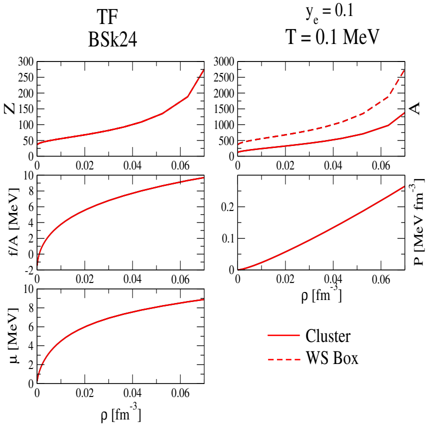
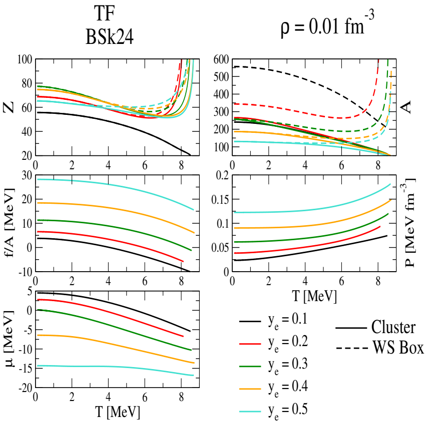


T-variation



Prospects

- Unified equation of state at zero and finite temperature.
- Systematically ETF, ETF+SI, Pairing for inner crust......
- Merger simulations using BSkG models are being explored.