Neutrino Transport in Dense Media from AdS/CFT

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Introduction

- Gauge/gravity duality (aka Holographic Correspondence):
a way to answer questions in strongly coupled QFTs theories by doing calculations in classical GR
- holographic models can provide a descriptions of many aspects of the non-perturbative physics and can in principle be used to study high-density QCD matter

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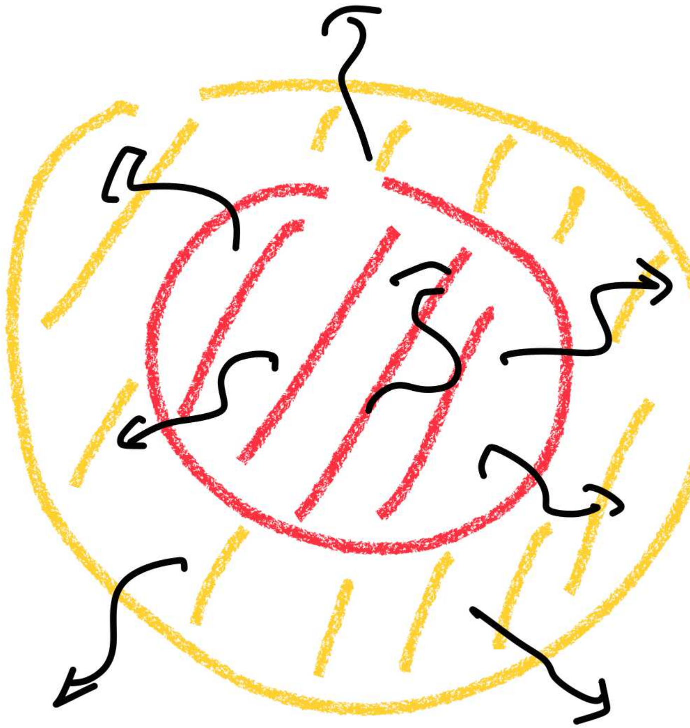
Outline

- What is the gauge/gravity duality?
- Application to Neutrino Diffusion.

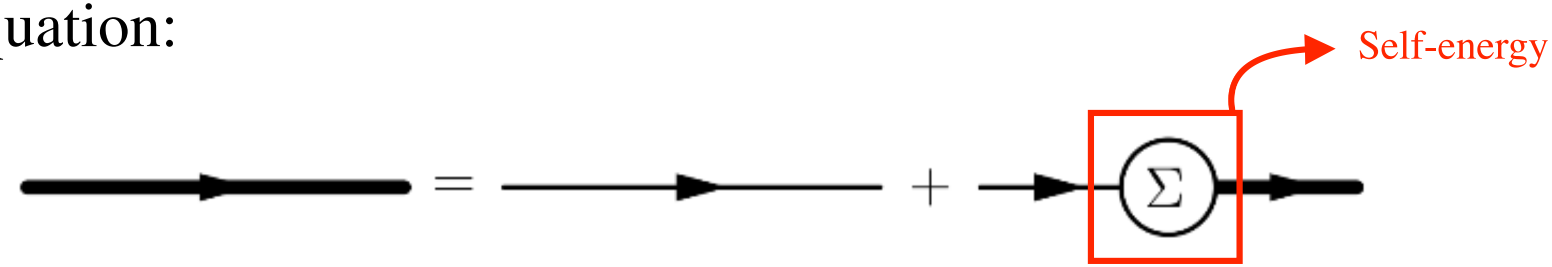
Neutrino transport

- Cooling by neutrino emission is very important in the out-of-equilibrium dynamics of compact objects (proto-NS, supernovae)
- It can be described by diffusion through a dense medium.



Neutrino transport

- Cooling by neutrino emission is very important in the out-of-equilibrium dynamics of compact objects (proto-NS, supernovae)
- It can be described by diffusion through a dense medium.
- Diffusion (via Boltzmann equation) can be phrased in terms of the retarded propagator satisfying in-medium Dyson-Schwinger equation:

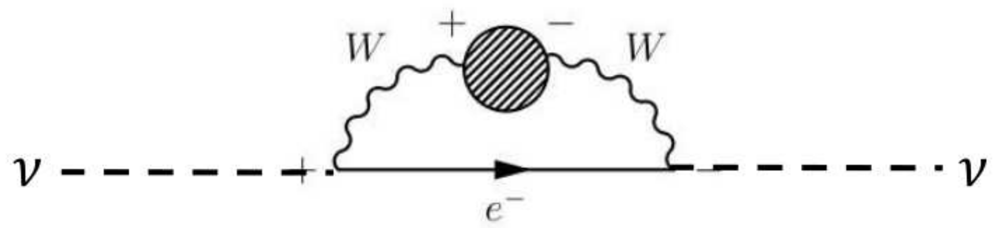


- One of the main problems is the QCD contribution to Σ in the strongly coupled regime

Neutrino transport

To compute the in-medium neutrino diffusion: need
strong-interaction contribution to EW gauge bosons self energies:

$$\Sigma^{\mu\nu}(p) = \Sigma^{\mu\nu}_{EW}(p) + \langle J^{\mu}(p) J^{\nu}(-p) \rangle_{QCD\ medium}$$



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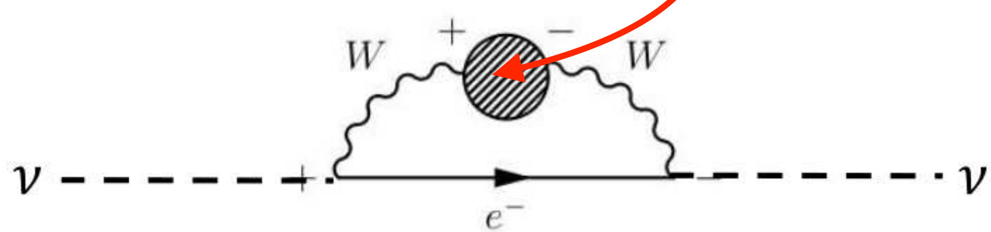

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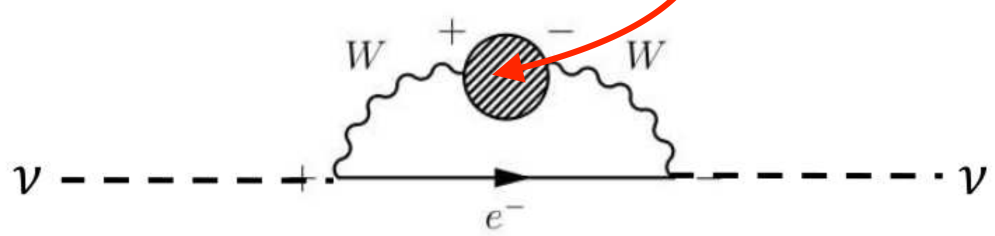
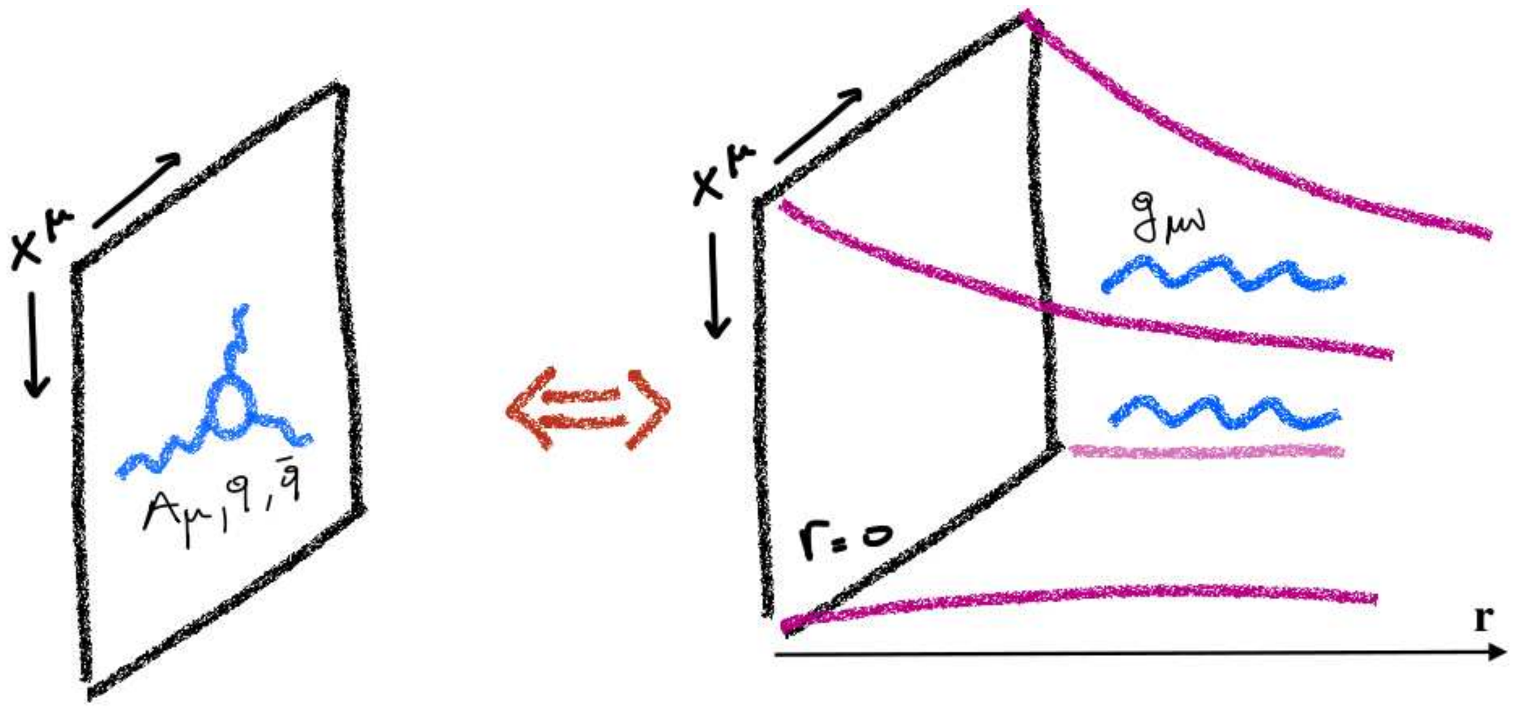


- Can compute real-time $\langle J^\mu J^\nu\rangle_{QCD}$ using holography at finite density and temperature, by a liner perturbation calculation in the bulk.
- Proof of principle calculation in the deconfined phase and in a simplified model Jarvinen, Kiritsis, FN, Préau, '23

The Gauge/Gravity Duality

Conjecture that some 4d quantum field theories have an equivalent description as gravitational theories in higher dimensions



- Well-grounded in the string theory context for SUSY QFTs.
- General features believed to be valid in the absence of SUSY (less under control).

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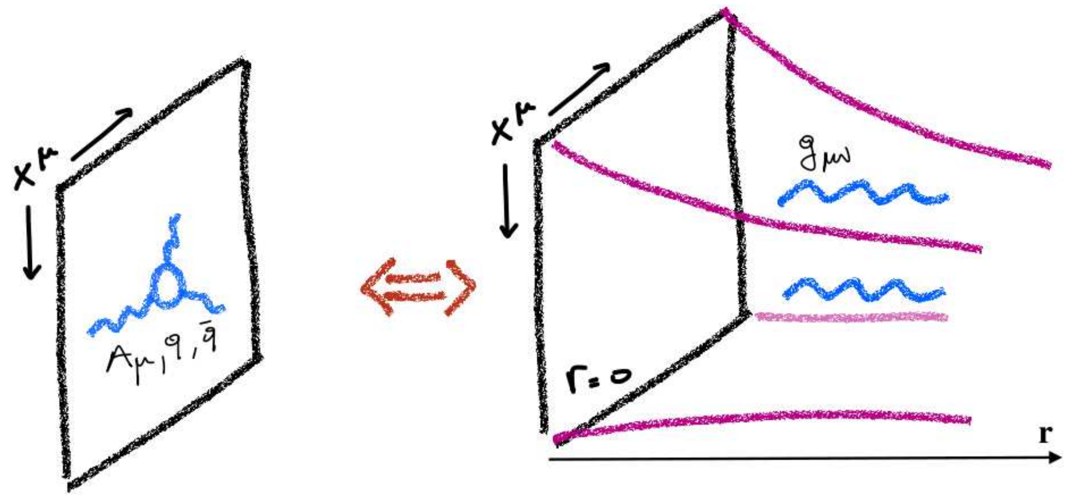
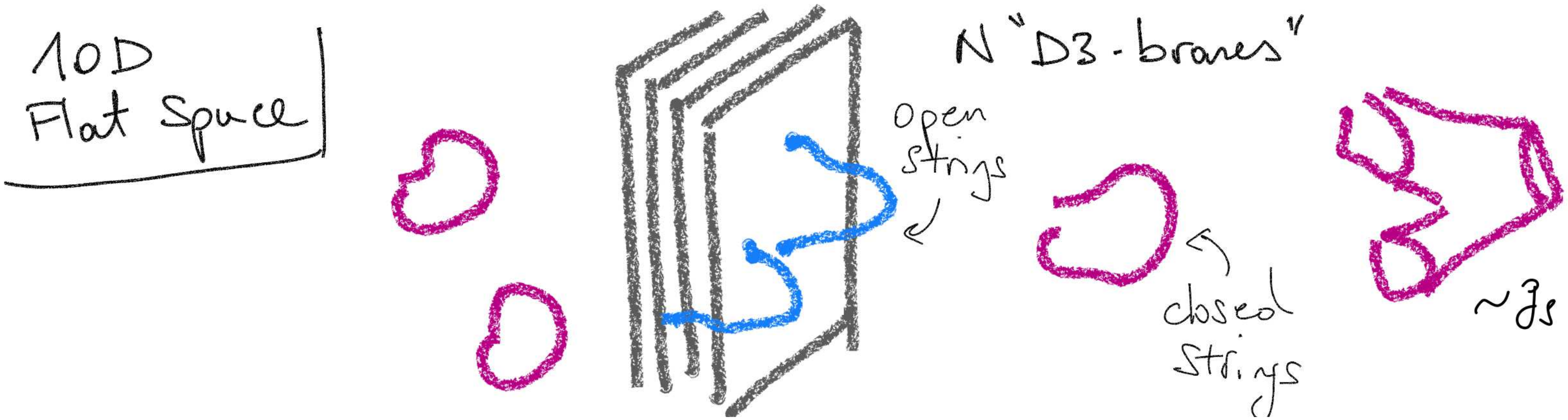


- Equivalent means that the two theories describe the same physics in terms of different degrees of freedom, but arranged in differnt ways.
- Weak QFT coupling: QFT description is perturbative;
- Strong QFT coupling: gravity side captured by classical GR

  (for large N gauge theories)

String Theory Origin



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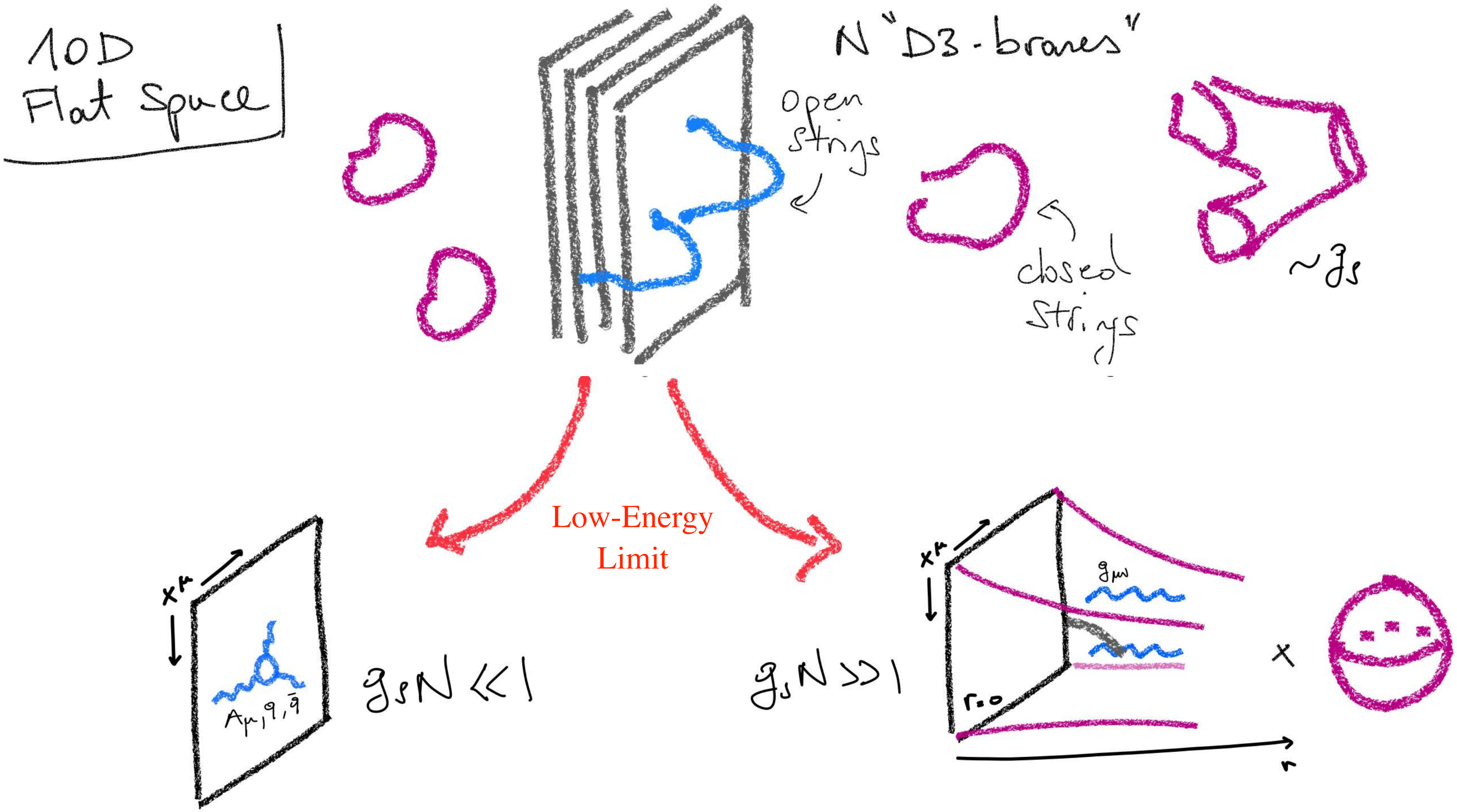
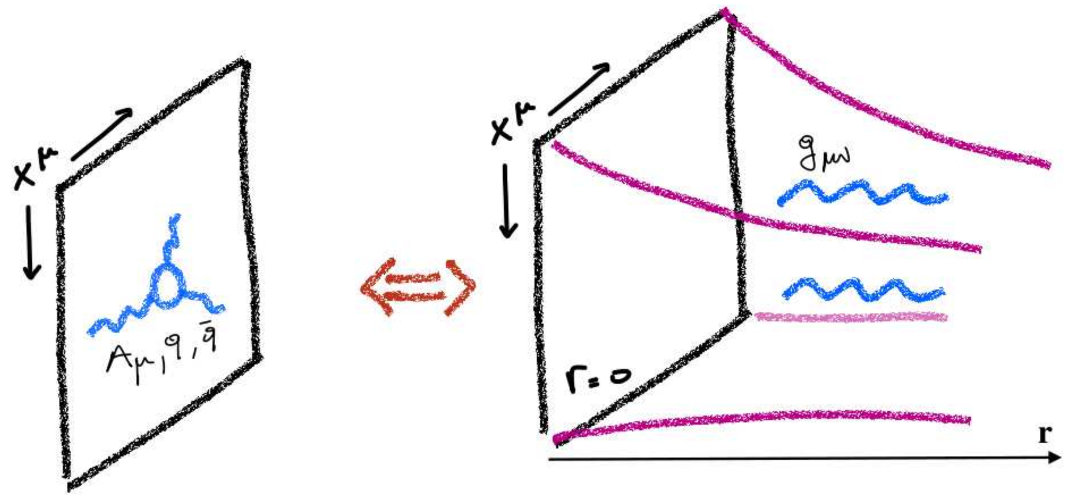


$SU(N)$ (Supersymmetric) Yang-Mills

Gravity on $AdS_5 \times S^5$

Anti-de Sitter Space



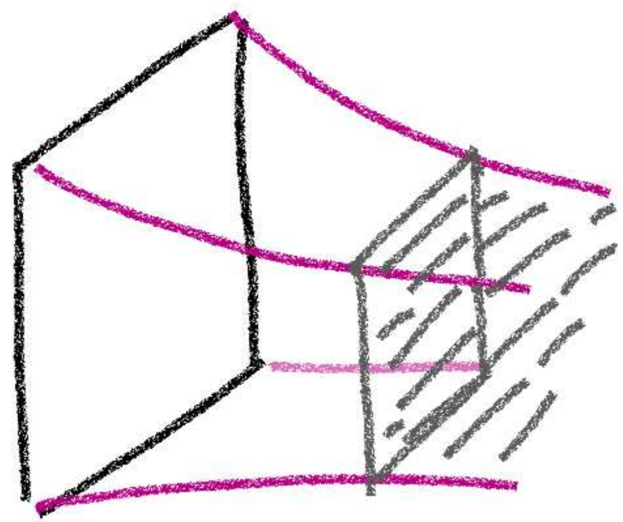
- QFT is conformal $\Leftrightarrow$ Gravity side is AdS spacetime:

$$ds^2 = \frac{\ell^2}{r^2}(dr^2 + dx_\mu^2)$$

- $r = 0$: *boundary* of AdS = spacetime where the QFT lives (hence *holography*).
- Broken conformal invariance $\Leftrightarrow$ AdS deformed in the interior.

Hot and dense thermodynamics states

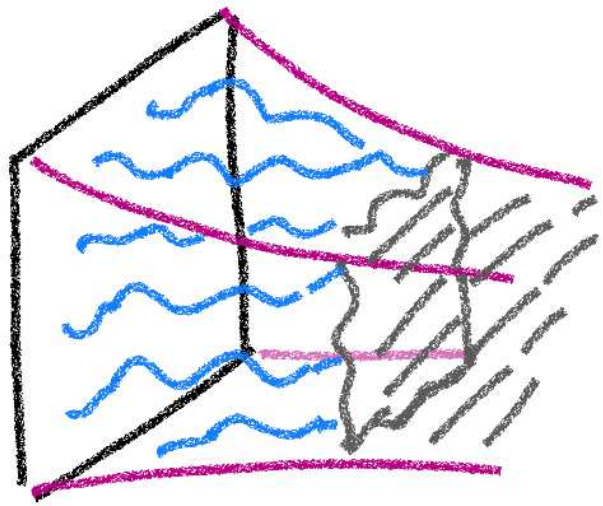
- Finite $T \Leftrightarrow$ 5D Black Hole geometry



- EoS obtained from 5D Black Hole Thermodynamics (standard GR)

Beyond thermodynamics

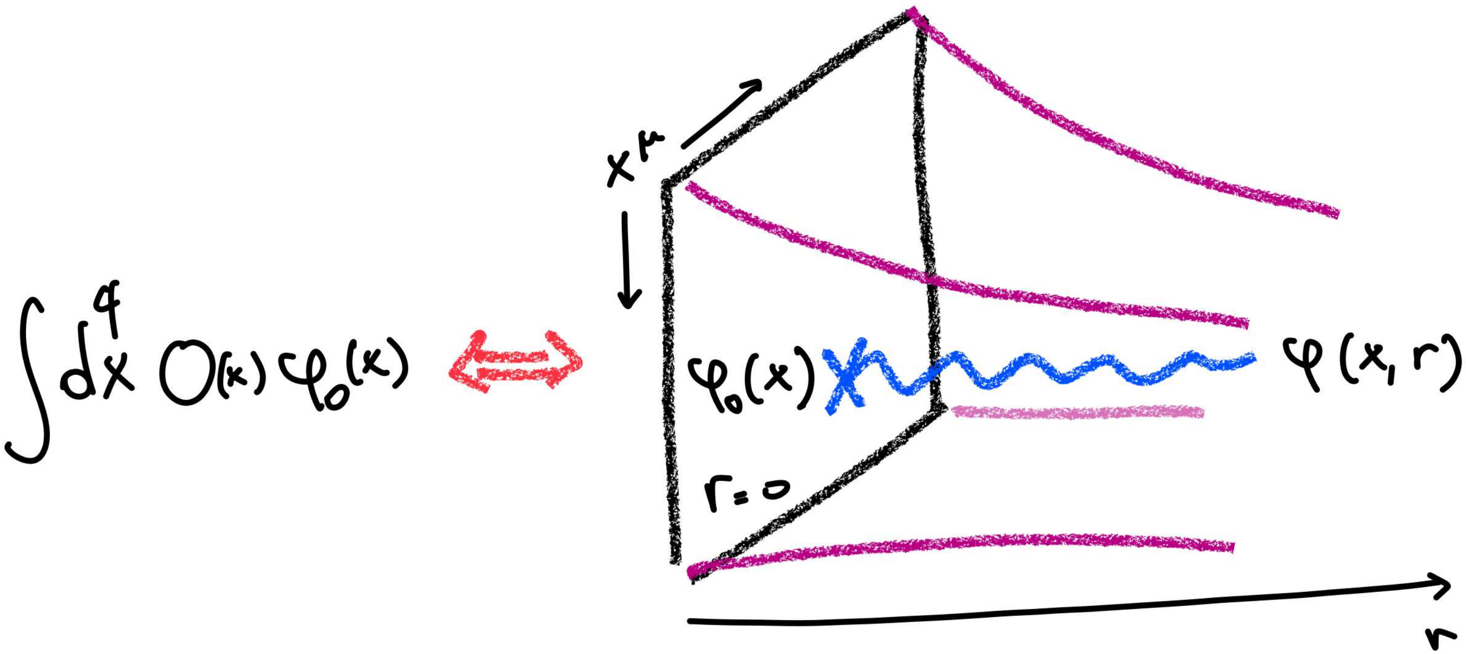
- Out-of-equilibrium evolution can be obtained by evolving bulk state
- Linear hydro $\leftrightarrow$ linear perturbations around BHs in GR



- Can compute transport coefficients (viscosities) entering non-ideal hydro by a simple linearized GR calculation.
- QFT: transport encoded in real-time correlators $\langle O(x,t)O(x',t')\rangle$. How is this computed on the gravity side?

Field/Operator correspondence

- QFT operator $O(x) \Leftrightarrow$ Bulk field $\Phi(x, r)$.
- $\Phi_0(x) = \Phi(x, 0)$ is a source for $O(x)$ in the QFT:



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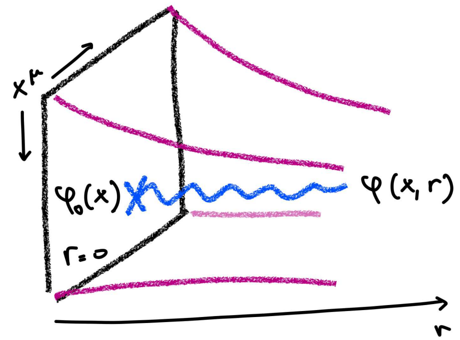

Dimension Δ of O determined by the mass of Φ:
$m^2 = \Delta(\Delta - d)$.

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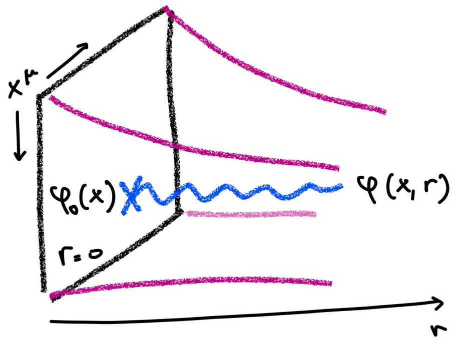

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in the large-N limit:

$$\mathcal{Z}_{QFT}[\Phi_0(x)] = \exp iS_{cl}[\Phi_0(x)]$$

$S_{cl}[\Phi_0]$: classical bulk action evaluated on the solution of the field equations with fixed boundary condition $\Phi_0(x)$.

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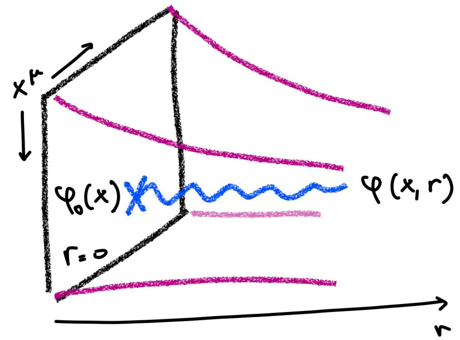

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$$\langle O(x_1) \ldots O(x_n) \rangle = \frac{\delta}{\delta \Phi_0(x_1)} \cdots \frac{\delta}{\delta \Phi_0(x_n)} S_{cl}[\Phi_0]$$

Holographic 2-point function

- Take a free scalar on AdS

$$S = \int_0^{+\infty} dr \int d^4x \sqrt{g} \left(g^{MN} \partial_M \Phi \partial_N \Phi - m^2 \Phi^2 \right) \qquad m^2 = \Delta(\Delta - d)$$

- Solve field equation with boundary condition at $r = 0$

$$(\Box - m^2)\Phi = 0, \qquad \Phi(x, r) \to \Phi_0(x) r^{(d-\Delta)} \quad r \to 0$$

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Independent of boundary cond.

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$$S_{on-shell} \propto \int d^4x d^4y \, \Phi_0(x) \left[(\partial_r K)\Big|_{r=0} (x - y) \right] \Phi_0(y)$$

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<O(x)O(y)>

Holographic models for QCD

- Dictionary:

4D Operator		5D Bulk field
TrF^2	$\Leftrightarrow$	Φ
$T_{\mu\nu}$ Stress tensor	$\Leftrightarrow$	$g_{\mu\nu}$ bulk metric
J_L^μ, J_R^μ $U(N_f)_L \times U(N_f)_R$ flavor currents	$\Leftrightarrow$	A_L^μ, A_R^μ $U(N_f)_L \times U(N_f)_R$ gauge fields
$\psi^i \psi_j$ quark bilinear	$\Leftrightarrow$	T^i_j matrix of scalars

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- Bottom-up: Einstein-Scalar-Yang-Mills action depending on *phenomenological potentials* (functions of the scalars)
- State of the art: V-QCD model Järvinen, Kiritsis '11

Minimal Holographic Neutrino Transport

Goal: compute correlators of non-abelian chiral current operators.

- QFT: Operators $J_\mu^{a,L} = \bar{q}^L \gamma_\mu T^a q^L$, $J_\mu^{a,R} = \bar{q}^R \gamma_\mu T^a q^R$

  $\leftrightarrow$

  Bulk: $U(N_f)_L \times U(N_f)_R$ gauge fields $A_M^{a,L}$, $A_M^{a,R}$

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- Minimal bulk theory: 5d Einstein Gravity + $U(N_f)_L \times U(N_f)_R$ 5d Yang-Mills theory:

$$S_{5d} = M^3 N_c^2 \int d^5x \sqrt{g} \left(R + \frac{12}{\ell^2} + \right.$$

$$\left. + \kappa \frac{N_f}{N_c} Tr \left[F^{(L)}_{MN} F^{MN\,(L)} + F^{(R)}_{MN} F^{MN\,(R)} \right] \right)$$

$$F^{(L)} = dA^{(L)} - A^{(L)} \wedge A^{(L)}, \quad F^{(R)} = dA^{(R)} - A^{(R)} \wedge A^{(R)}$$

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Color sector

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Flavor sector

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Charged Black Hole Solutions

- Look for state at finite temperature and baryon density
- Baryon number $\leftrightarrow$ $U(1)_V$ gauge field $A_0^{(L)} = A_0^{(R)} = V_0(r)\mathbf{1}$
- Solution with non-trivial abelian vector charge and finite temperature: Charged AdS Black hole

$$ds_5^2 = \frac{\ell^2}{r^2}\left(-f(r)dt^2 + \frac{dr^2}{f(r)} + d\vec{x}^2\right), \quad V_0 = 2\mu\left(1 - \frac{r^2}{r_H^2}\right)$$

$$f(r) = 1 - (3 - 2\pi T r_H)\left(\frac{r}{r_H}\right)^4 + 2\left(1 - \pi T r_H\right)\left(\frac{r}{r_H}\right)^6$$

- μ = baryon chemical potential; $f(r) = 0$ at horizon $r = r_H$; $f'(r_H) \propto 1/T$
- Two parameters: μ, T; Choose $T/\mu \ll 1$ (near-extremal BH).

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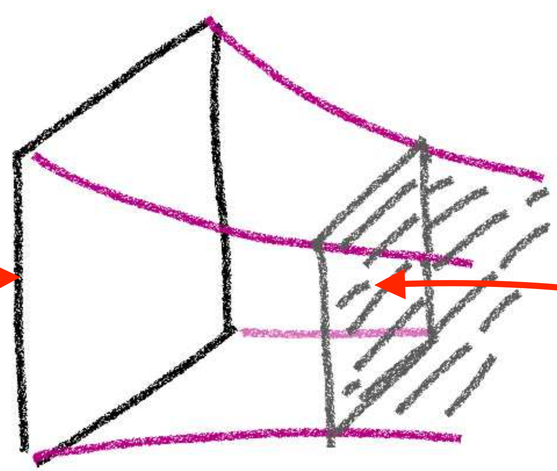
AdS boundary



r=rH
BH Horizon

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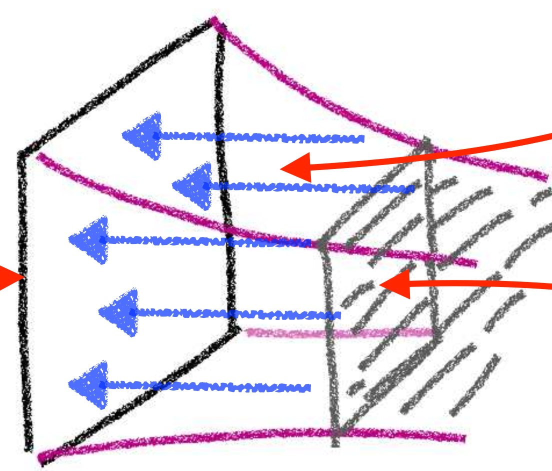
Electric Field



r=0
AdS boundary

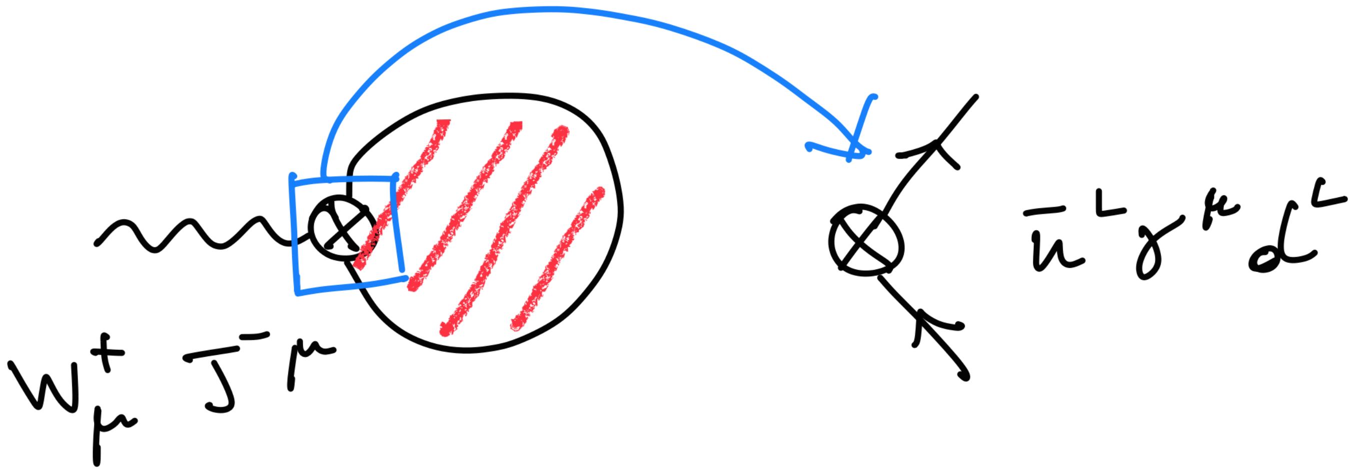
r=rH
BH Horizon

Pros and Cons of minimal model

- Pros: simplicity:
  - Minimal number of fields
  - Minimal number of adjustable parameters
  - Background solution known analytically
- Cons: Not realistic:
  - QFT is conformal (vacuum state at $T = \mu = 0$ is AdS): no QCD running coupling.
  - Thermal state is in deconfined phase (albeit strongly coupled)
  - Missing low-dimension operators (gluon fields, quark bilinears)
- Good testing ground for non-abelian current correlator computation, computation can be carried over to a more realistic model.

Charged Current Operator/Field

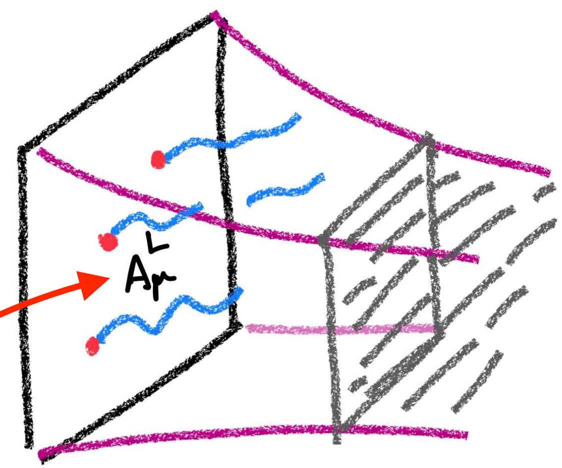
- Set $N_f = 2$, $q_i = (u, d)$, bulk gauge group $U(2)_L \times U(2)_R$



$$\bar{u}^L \gamma^\mu d = (\bar{u}^L \ \bar{d}^L)\, \gamma^\mu \underbrace{\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}}_{\frac{\sigma^1}{2} + i\frac{\sigma^2}{2}} \begin{pmatrix} u^L \\ d^L \end{pmatrix}$$

$J^{L,-}_\mu = J^{1,L} + iJ^{2,L} \Rightarrow$ turn on bulk field $A^{L,1}_\mu(x,r)$, $A^{L,2}_\mu(x,r)$

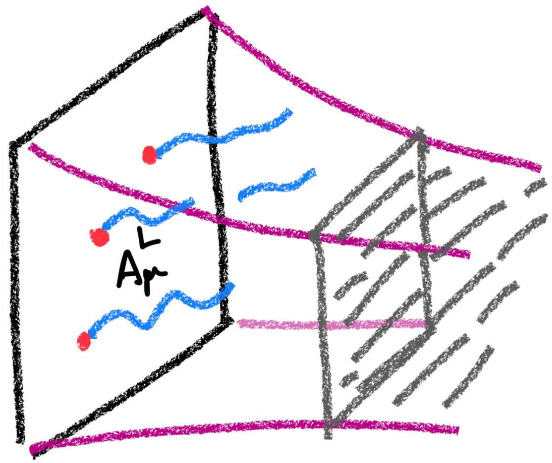
Charged Current Operator/Field



- Solve 5d field equations for non-abelian gauge field fluctuations $\boxed{A_\mu^{L,1}, A_\mu^{L,2}}$ added to charged BH background $\nabla^M F_{MN}^{(L)} = 0$
- obtain 2-point function: $\langle J_\mu^- J_\nu^+ \rangle_{\mathrm{R}}(\omega, \vec{k})$.

Retarded correlator $\longleftrightarrow$ Infalling solution at BH horizon

Charged Current Operator/Field



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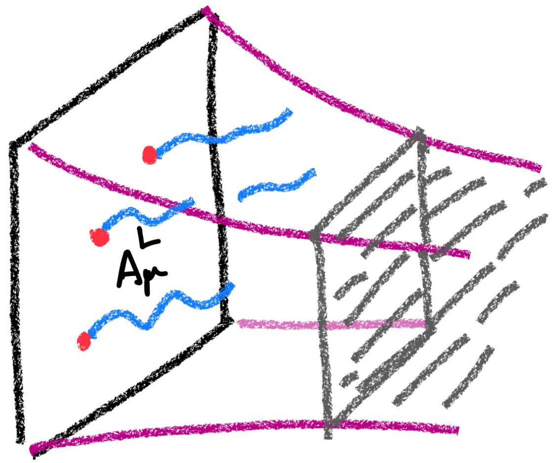
$$\langle J_\mu^- J_\nu^+ \rangle_{\mathrm{R}} = \left(P_{\mu\nu}^{\perp} \Pi^{\perp}(\omega, \vec{k}) + P_{\mu\nu}^{\|} \Pi^{\|}(\omega, \vec{k}) \right)$$

Transverse / Longitudinal Projectors

Charged Current Operator/Field



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Polarisation functions

Hydrodynamic limit

- In the limit $\omega \to 0, \vec{k} \to 0$, 2-point function becomes universal:

$$\Pi^{\perp}(\omega, \vec{k}) = \sigma\omega, \qquad \Pi^{\|}(\omega, \vec{k}) = \sigma \frac{\omega^2 - k^2}{\omega + iDk^2}$$

Conductivity

Diffusion constant

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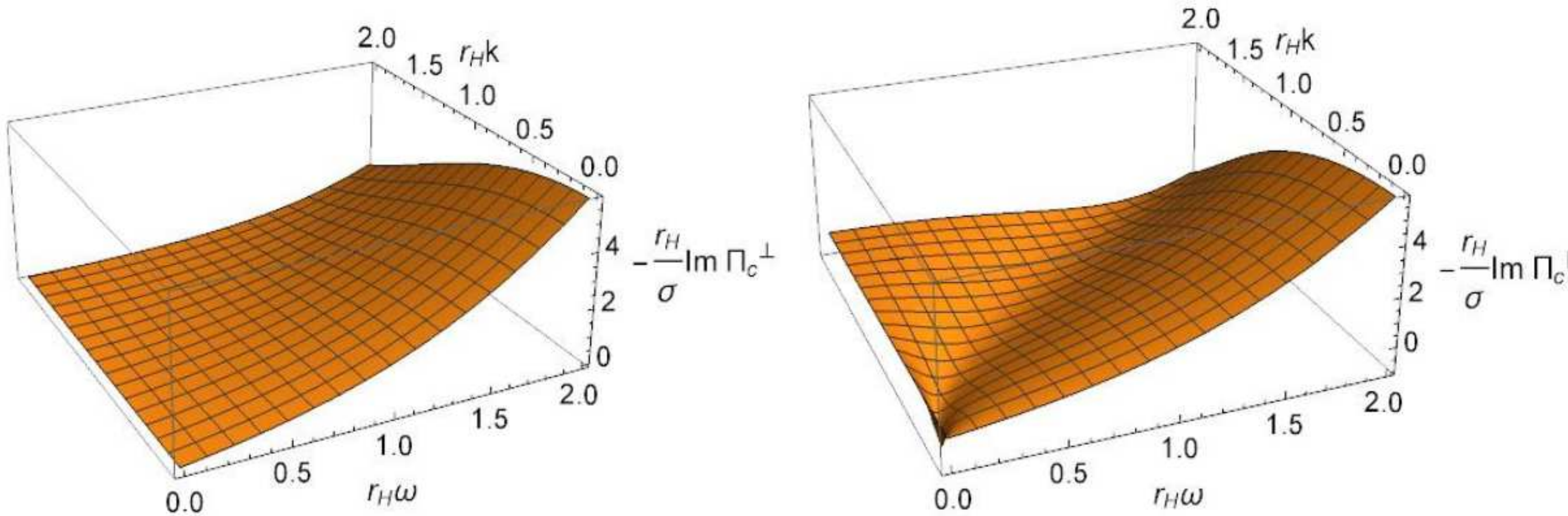
Conductivity

Diffusion constant

- Normally hydrodynamic limit breaks down as $T \to 0$. However the charged AdS-BH is special: hints of hydro up $\omega, |\vec{k}| \sim \mu$, even for $T/\mu \ll 1$.
- Due to non-hydro poles having small residues for $T/\mu \to 0$ (Davison,Parnachev '13; Arean, Davison, Goutéraux, Suzuki '21).

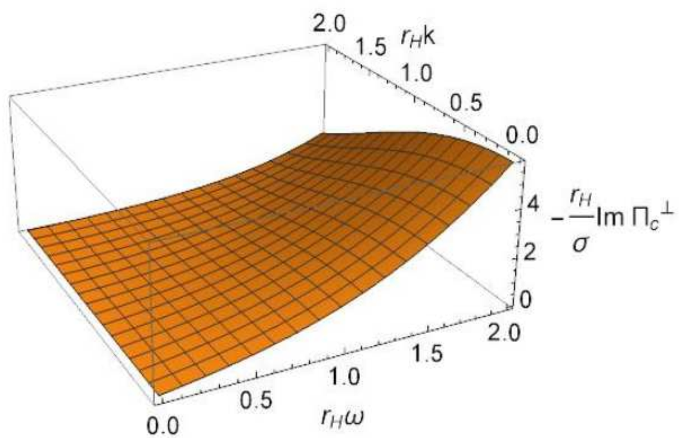
Numerical computation

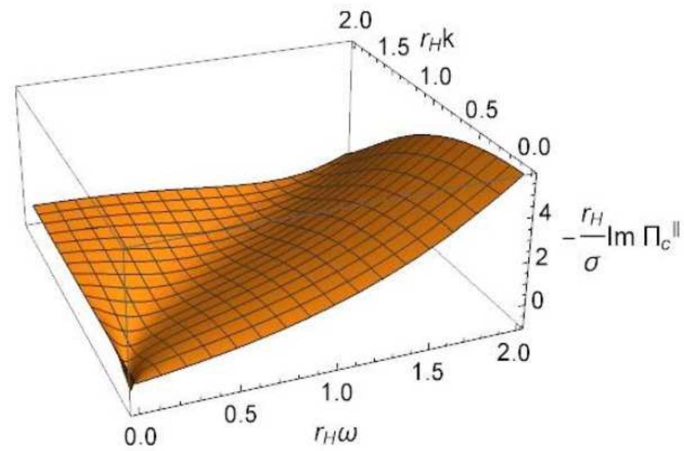
- Fix $N_c = 3$, $N_f = 2$;
- Fix parameters in the action $M_p\ell$, κ to match UV perturbative limit
- Obtain polarization functions numerically



$$\frac{\mu_q}{T} \simeq 65$$

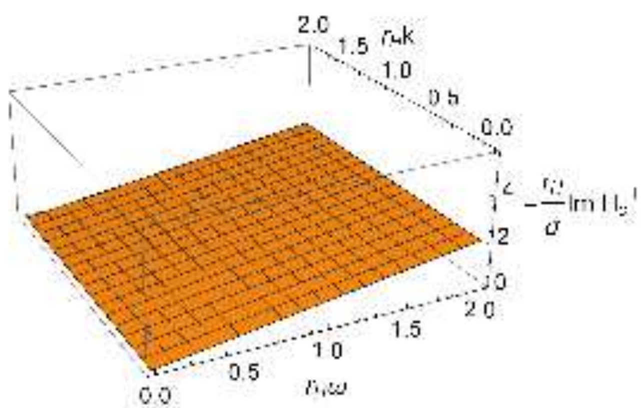
Comparing with Hydro

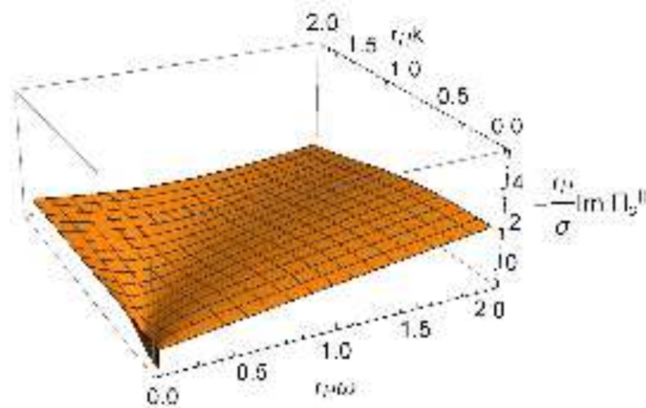




Full Correlator

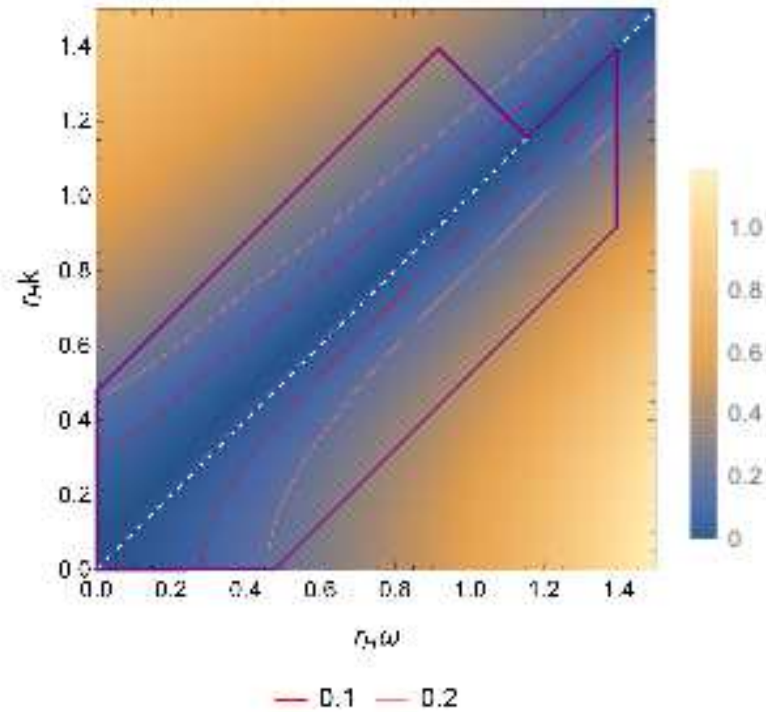
$$\frac{\mu_q}{T} \simeq 65$$

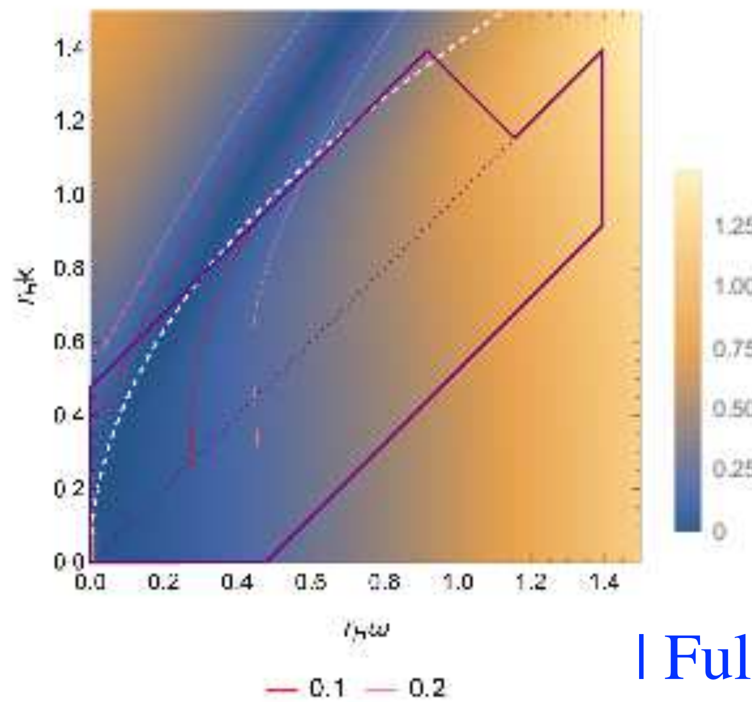




Hydro Approximation

Comparing with Hydro

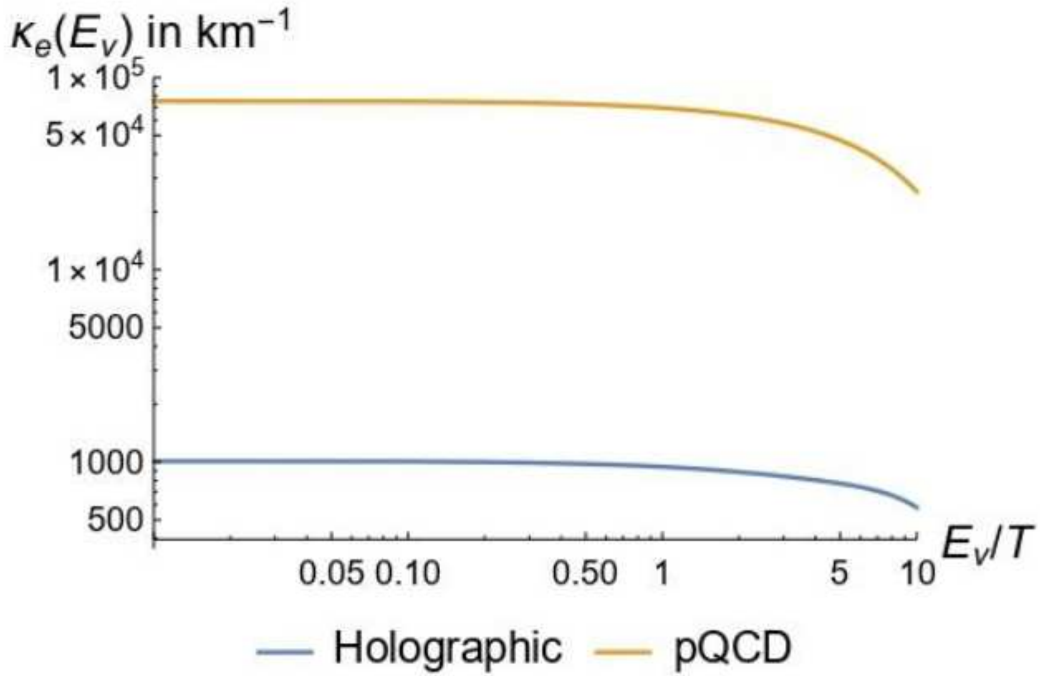




$$\frac{|\,\text{Full - Hydro}\,|}{\text{Hydro}}$$

Neutrino Opacities

- Opacity $\kappa(E_\nu)$: relevant diffusion coefficient entering Boltzmann equation (obtained from integrated current-current correlator)



$$T = 10\text{ MeV},$$
$$n_B = n_s = 0.11\text{ fm}^{-3}$$

Conclusion and future goals

- Holography can compute out-of-equilibrium current correlators in a strongly coupled dense medium, needed for neutrino transport
- So far:
  - conformal toy model
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- So far:
  - conformal toy model
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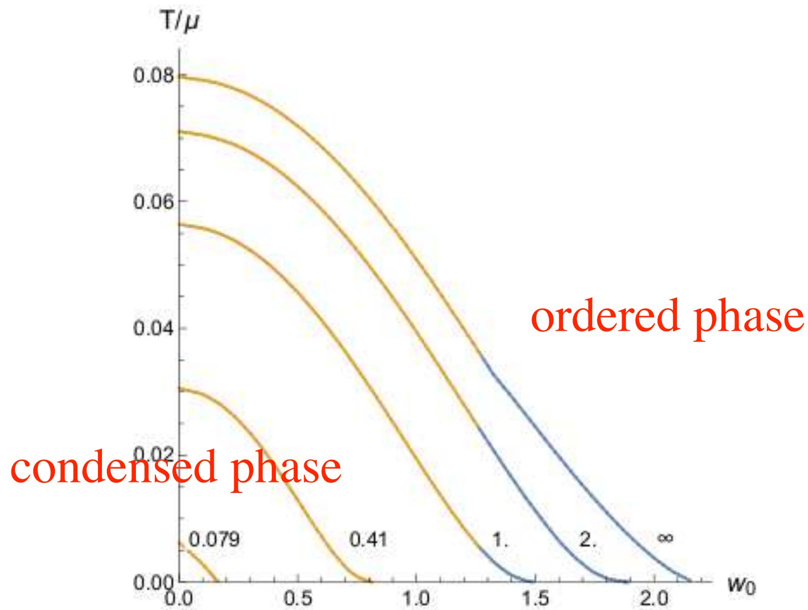
- Future:
  - Include isospin chemical potential (different phases may appear) (in progress)
  - Extend the computation to neutral current contribution (this requires including bulk metric fluctuations) (in progress)
  - Work in a realistic holographic QCD model such as V-QCD
  - Do the computation in the confined (nucleonic) phase (under construction)

Exotic phases

- Holography predicts *exotic phases* in the presnce of both baryon and isospin chemical potential: condensation of a vector order parameter

  Jarvinen, Kiritsis, FN, Préau, '24



- Do these phases have a place in the NS phase diagram?

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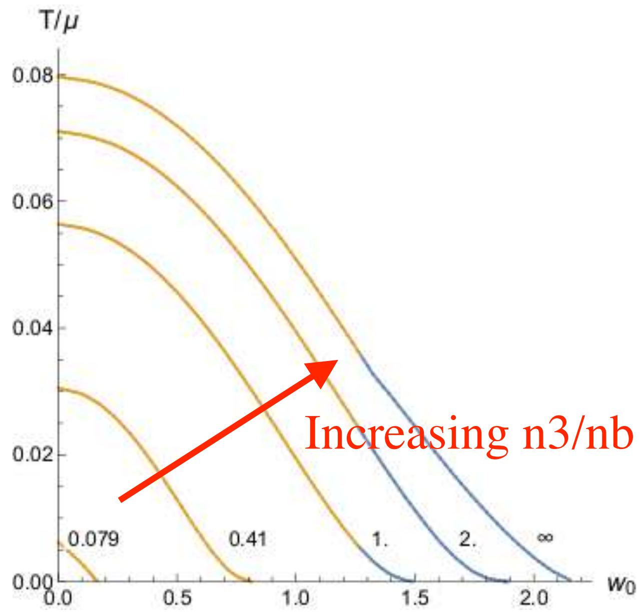

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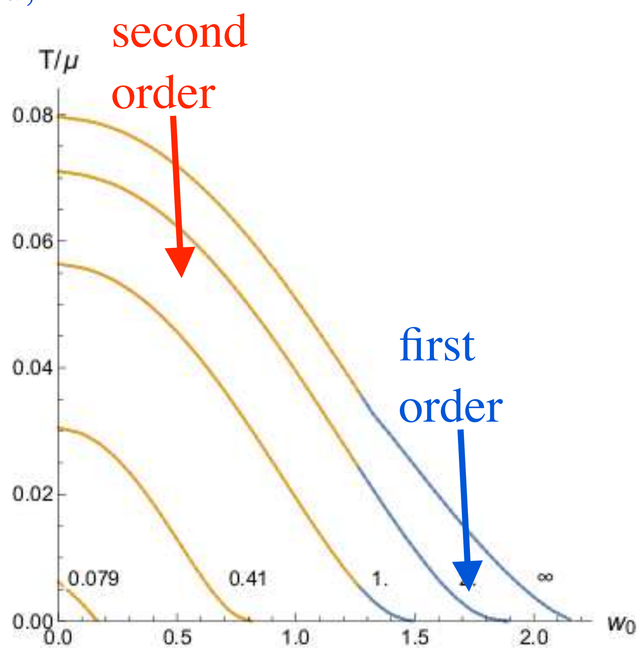

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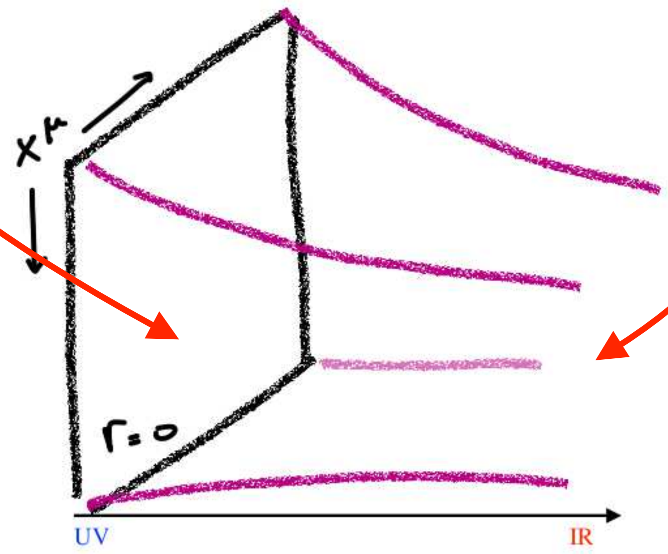
Confinement

- UV of the QFT $\Leftrightarrow$ near-boundary region
- IR of the QFT $\Leftrightarrow$ interior



Conformal (AdS space)

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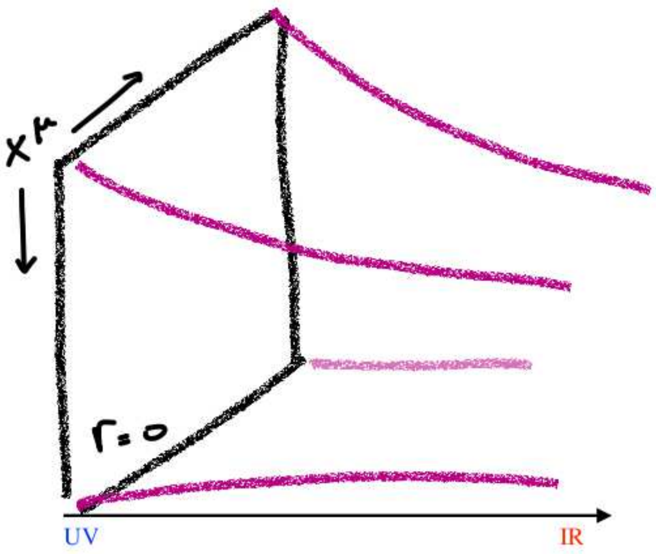

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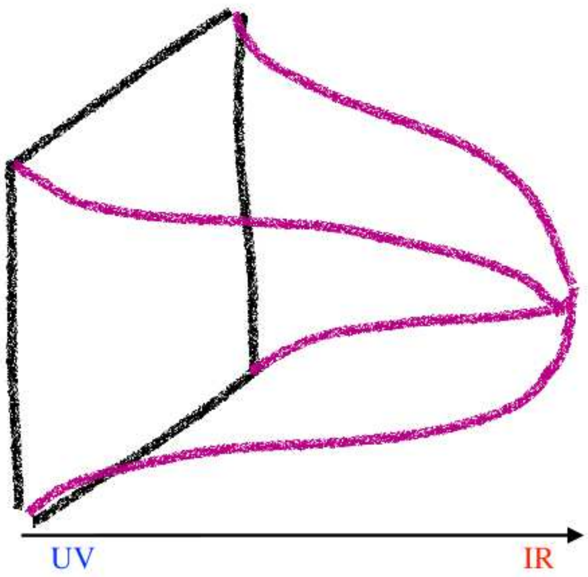

Confining

- Confinement is associated to properties of the interior geometry.
- Interior dynamically determined by bulk EOM.

Minimal holographic YM

- The bulk theory is five-dimensional (x^μ + RG coordinate r)
- Include only lowest dimension YM operators ($\Delta = 4$)

4D Operator		Bulk field	Coupling
TrF^2	$\Leftrightarrow$	Φ	$N\int e^{-\Phi}\, TrF^2$
$T_{\mu\nu}$	$\Leftrightarrow$	$g_{\mu\nu}$	$\int g_{\mu\nu}T^{\mu\nu}$

$\lambda = N g_{YM}^2 = e^{\Phi}$ (finite in the large N limit).

- Breaking of conformal symmetry, mass gap, confinement, and all non-perturbative dynamics driven by the dilaton dynamics (aka the Yang-Mills coupling).
- (Eventually: add axion field $a \Rightarrow TrF\tilde{F}$)

5-D Eistein-Dilaton Theory

Gursoy, Kiritsis, FN, 2007

Bulk dynamics described by a 2-derivative action:

$$S_c = -M_p^3 N_c^2 \int d^5x \sqrt{-g} \left[R + \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} - V(\lambda) \right]$$

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- IR: λ large, $e^A \to 0;$ $\qquad V(\lambda) \sim \lambda^{4/3}(\log\lambda)^{1/2}$
- Features: asymptotic freedom, confinement, discrete linear glueball spectrum, correct thermodynamics and phase diagram

Five dimensional setup: Yang-Mills

The Poincaré-invariant vacuum solution has the general form:

$$ds^2 = e^{2A(r)}(dr^2 + dx_\mu dx^\mu), \quad \lambda = \lambda(r), \quad 0 < r < +\infty$$

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- $\lambda(r) \propto$ running 't Hooft coupling
- $A(r), \lambda(r)$ determined by solving bulk Einstein's equations.

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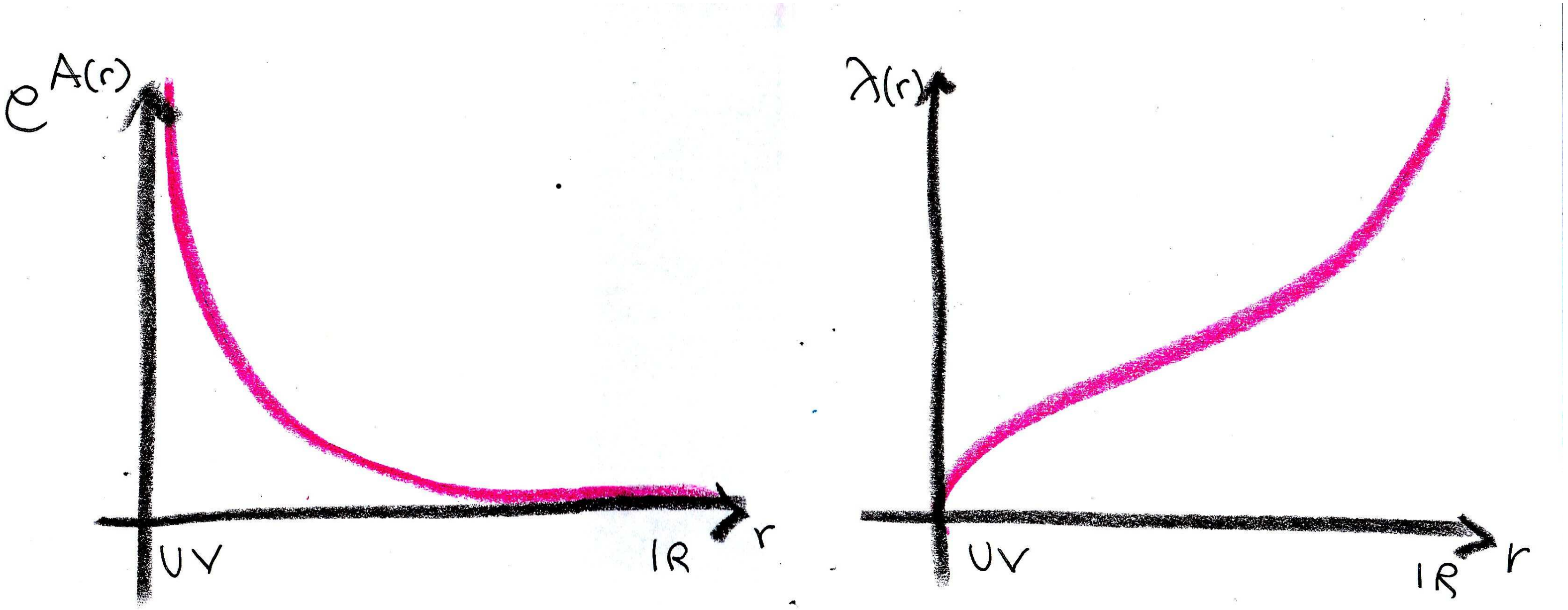
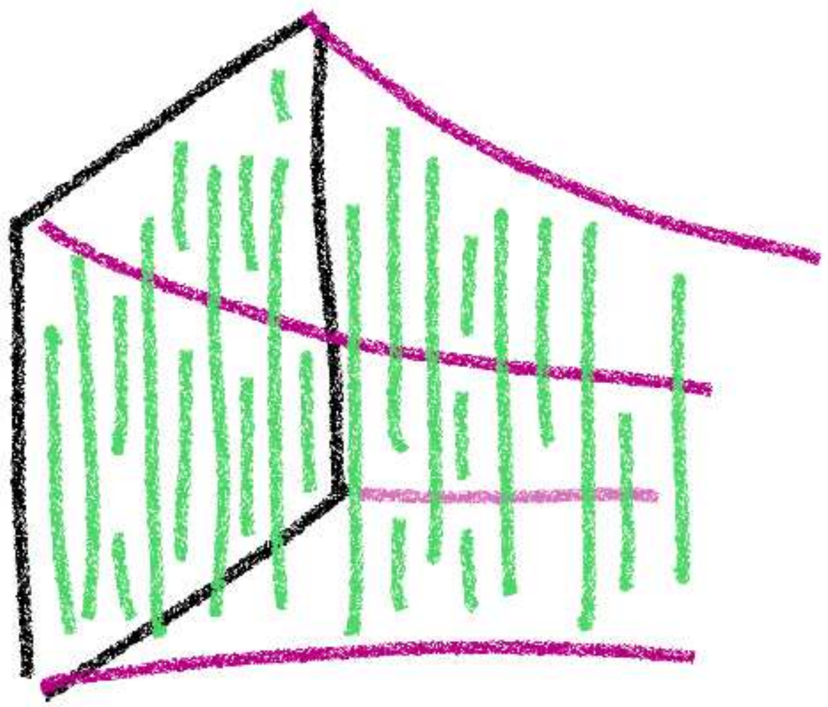


Adding Flavor: V-QCD

Jarvinen, Kiritsis 2011

N_f quark flavors $\Leftrightarrow$ N_f space-filling branes-antibranes.



Adding Flavor: V-QCD

Jarvinen, Kiritsis 2011

Flavor brane worldvolume fields:

- $U(N_f)_L \times U(N_f)_R$ gauge fields

$$A_B^{a;L}, A_B^{a;R} \quad \Leftrightarrow \quad J_\mu^{a;L,R} \equiv \bar{q}^i \gamma_\mu \, (\tau^a)_i^j \, (1 \pm \gamma_5) q_j$$

$a = 1 \ldots N_f^2, \; i, j = 1 \ldots N_f$

$U_B(1)$ current $\Leftrightarrow$ abelian vector $A_\mu^{(L)} + A_\mu^{(R)}$

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$$S_{VQCD} = S_c + S_{DBI} + S_{CS}$$

Action: DBI term

$$S_{DBI} = -M_p^3 N_c Tr \int d^5x\, V_f(\lambda, \mathcal{T}^\dagger \mathcal{T}) \left[\sqrt{-\det \mathbf{A}^{(L)}} + \sqrt{-\det \mathbf{A}^{(R)}}\right]$$

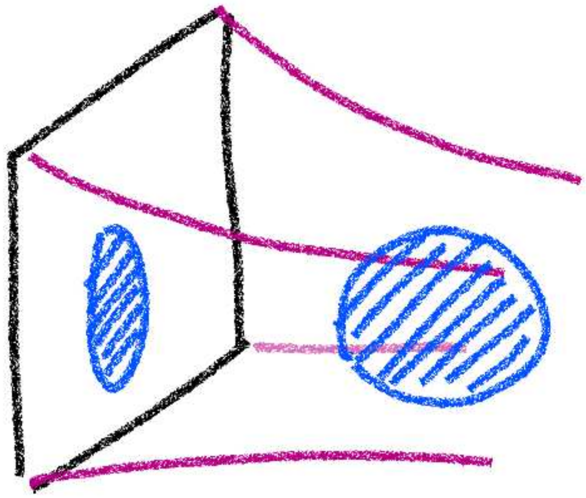
$$\mathbf{A}_{ab} = g_{ab} + w(\lambda, \mathcal{T}^\dagger \mathcal{T}) F_{ab} + \kappa(\lambda, \mathcal{T}^\dagger \mathcal{T}) (D_a \mathcal{T})^\dagger D_b \mathcal{T} \ + \ h.c.$$

To quadratic order:

$$S_{DBI} \simeq M_p^3 N_c \int d^5x\, V_f\, w^2 \left(Tr F_L^2 + Tr F_R^2 \right)$$

Baryonic phase

- Single baryon $\Leftrightarrow$ bulk instanton of non-abelian gauge fields.



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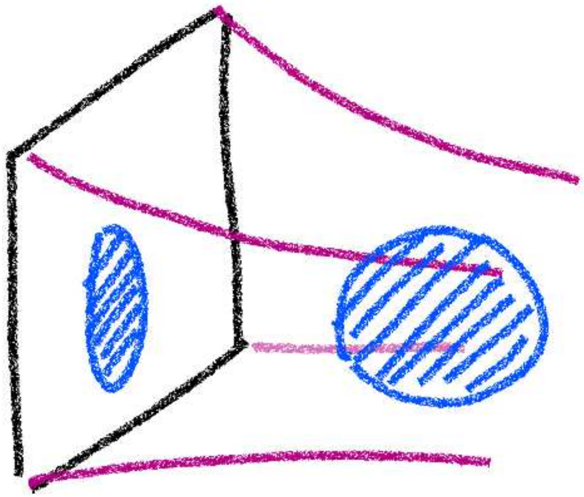
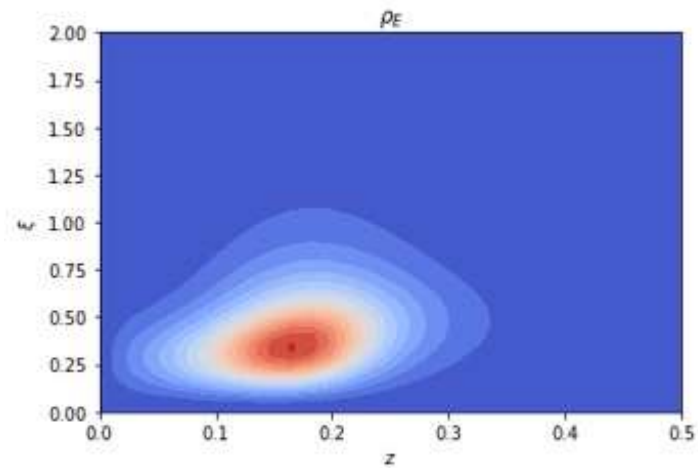


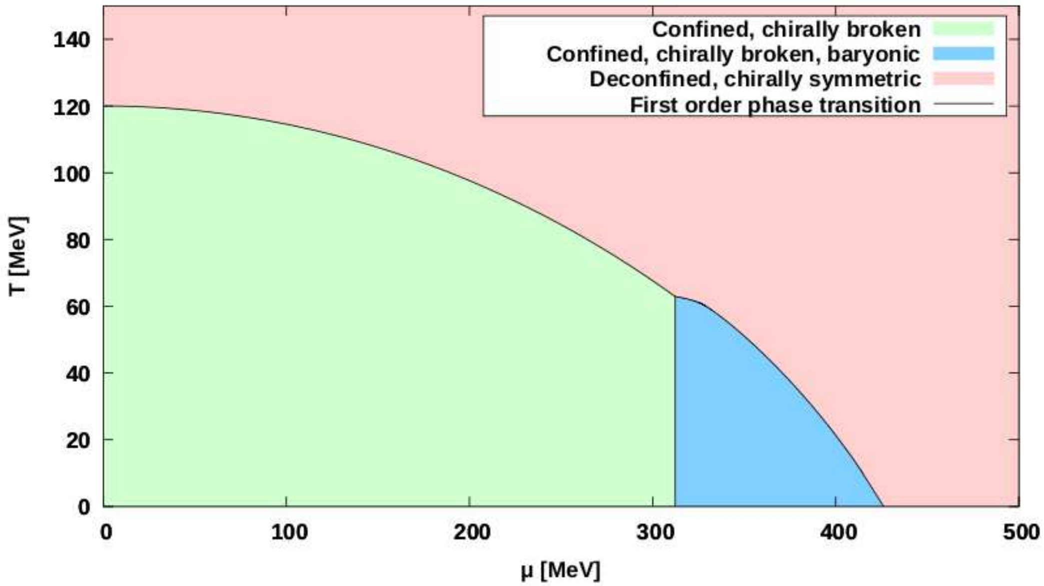


- V-QCD Baryon constructed numerically Järvinen, Kiritsis, FN, Préau, '22
- In progress: multi-baryon fluid/solid (not a black hole)

Effective/hybrid models

- Effective holographic description of baryonic matter in V-QCD

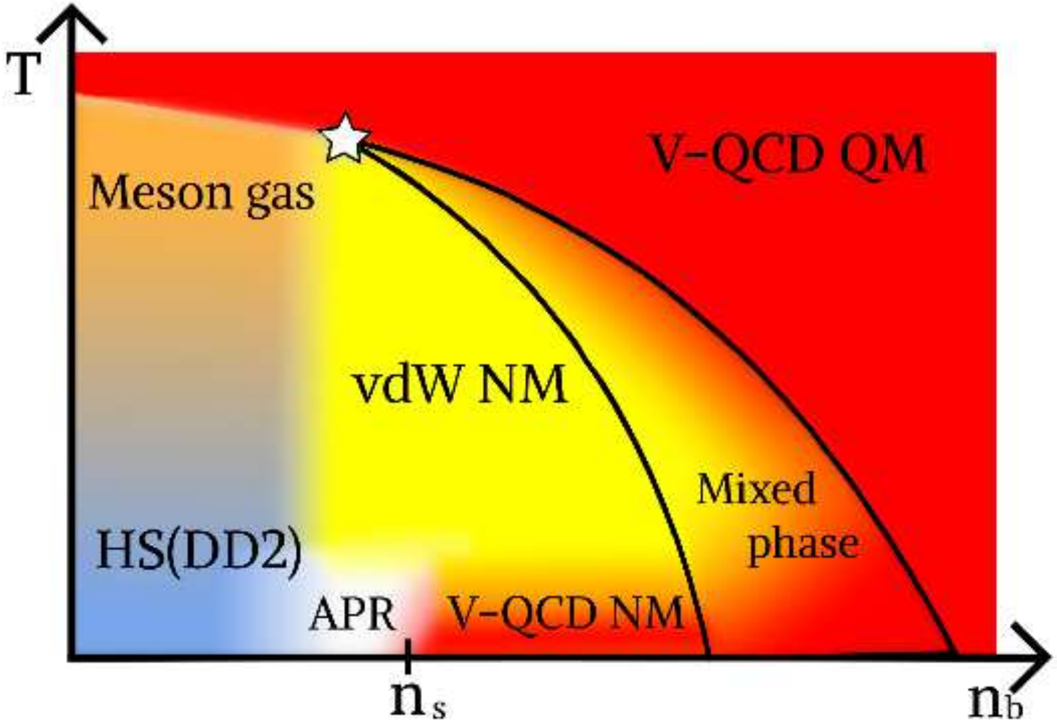
  Ishii, Järvinen, Nijs, '19.



Baryon distribution realized in the bulk as a homogeneous thin layer.

Effective/hybrid models

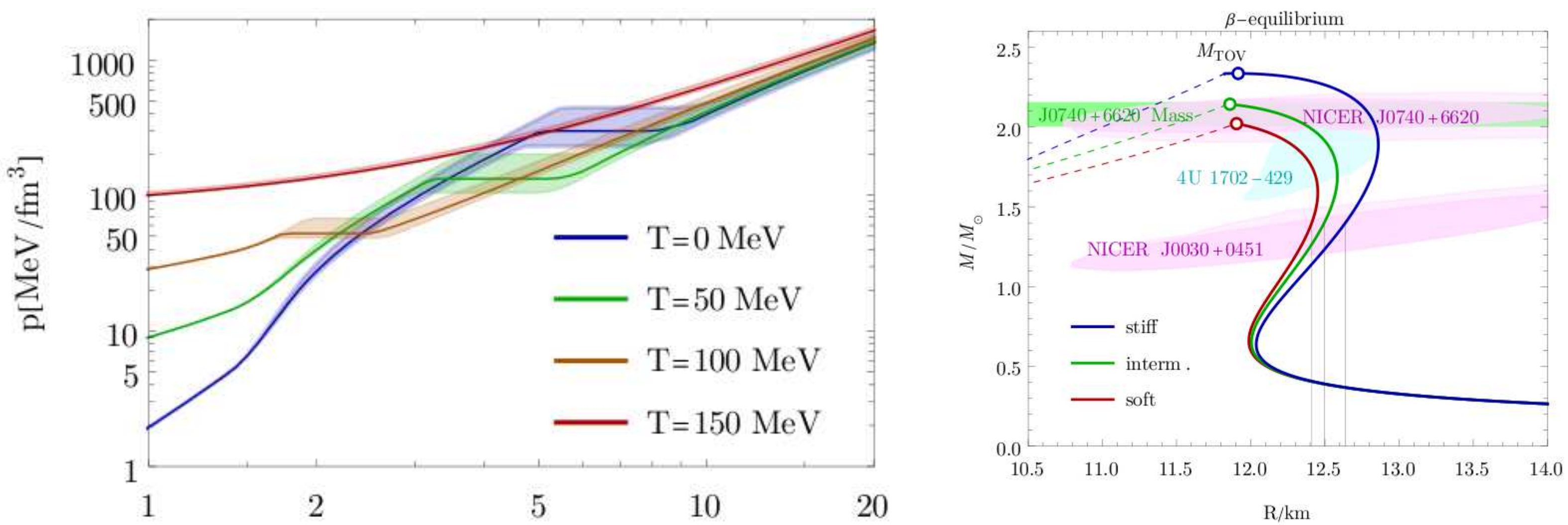
- State of the art Hybrid model Demircik, Ecker, Järvinen, '21



Baryonic phase EoS at finite T described by a VdW model

Neutron Star EoS

- Zero and finite temperature EoS from low to high density (hadronic to deconfined) Demircik, Ecker, Järvinen, '21.



- Static EoS too stiff to support deconfined core. However...