Discussion session: strategies for a "causal" metamodel?

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Metamodel representation of the nucleonic EoS

Starting point:

Homogeneous isotropic nucleonic matter

Only strong nuclear interaction

An energy density written as $\epsilon(n_n, n_p)$

Metamodel representation:

Any parametrization $\epsilon_X(n_n, n_p)$ of $\epsilon(n_n, n_p)$ such that there exists a choice of parameters X for which $\epsilon_X(n_n, n_p) \sim \epsilon(n_n, n_p)$ up to a desired level of accuracy

Metamodel representation of the nucleonic EoS

Which are the best uses for a metamodel?

EoS reconstruction

Bayesian inference

Numerical relativity simulation

Assessing uncertainty from astro observations

Can we decipher the composition?

Metamodel representation of the nucleonic EoS

The quality and usability of a metamodel scheme depend on the possibility of easily implementing, ideally even by construction and exactly, several desired properties, such as:

Low or high density limits e.g. YGLO or pQCD

Stability and causality

Reproducing selected features of known EOSs

The number and nature of parameters X

Low computational cost

Possibility to include other species

A possible choice of the energy density

Starting ansatz:

$$\epsilon(n,\delta,n_\mu) = \boxed{\epsilon_k(n,\delta,n_\mu)} + n\left[\boxed{e_0(n)} + \delta^2 \boxed{e_2(n)} + \delta^4 \boxed{e_4(n)}\right]$$

Quartic correction for the PNM

$$e_4(n) = A\left(n/n_0\right)^B$$

free fermi gas energy density for $npe\mu$ matter

Nucleonic Potential (per baryon)

$$e_0(x) = V_0(x) + \frac{h_0 + h_1 x + h_2 x^2 + h_3 x^{(2+q_0)}}{(1+a_0 x)(1+b_0 x)(1+c_0 x)}$$

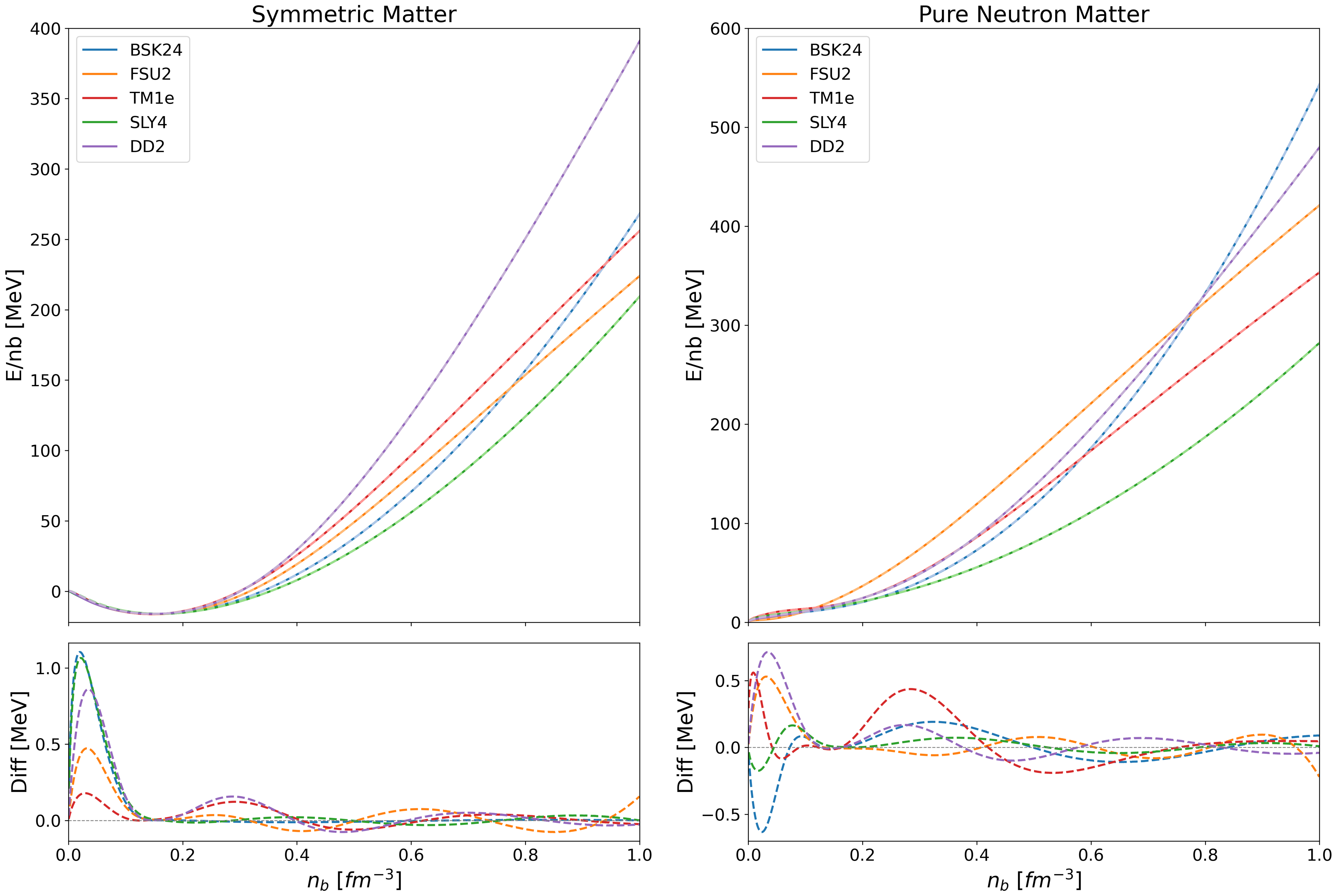
Fitting existing EOS

The desire of low dimensionality in parameters can be relaxed (until the algorithm converge)

How and what quantities to fit?

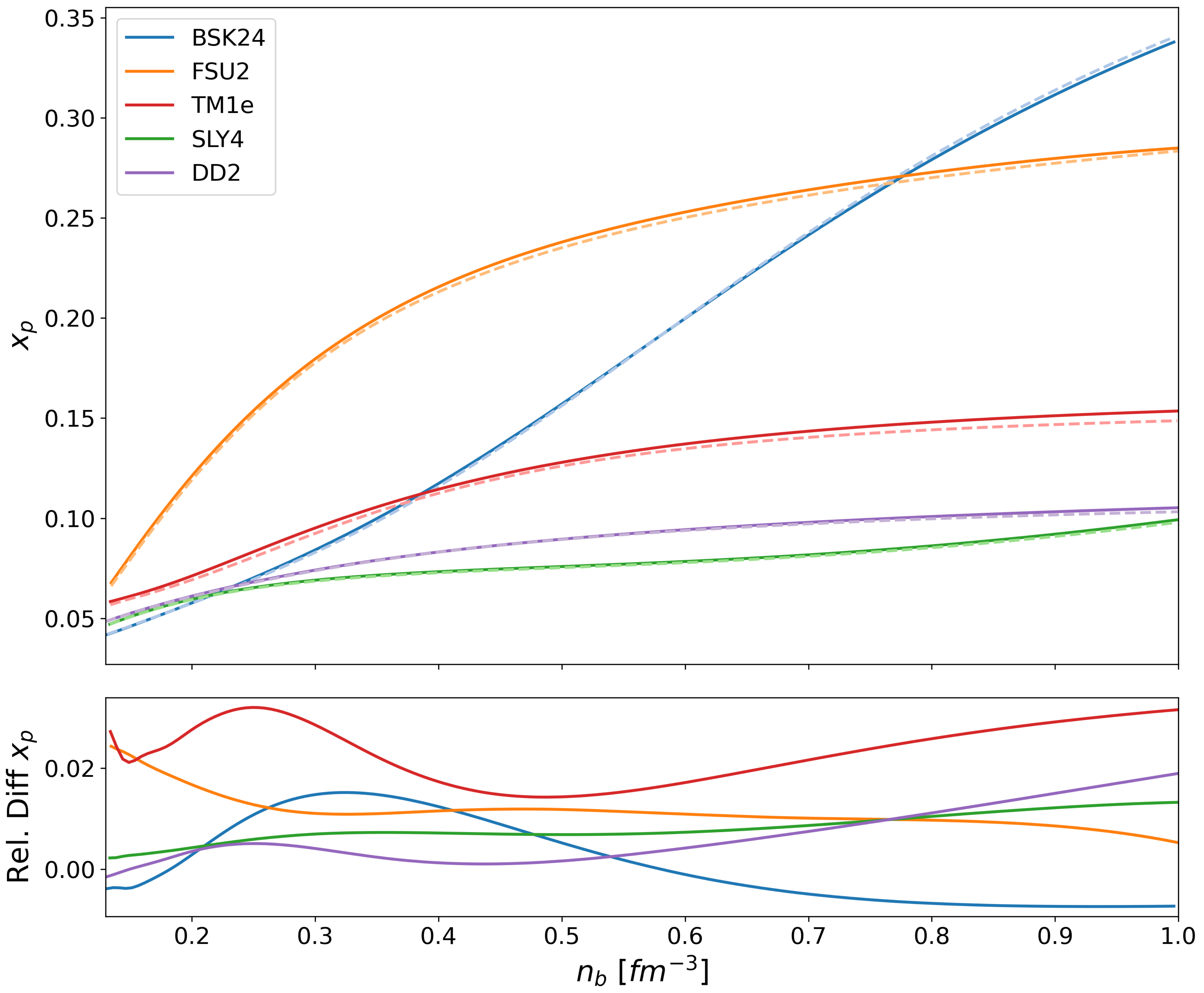
Do the parameters are degenerate?

How the uncertainty in the EoS propagate in the different scenarios?

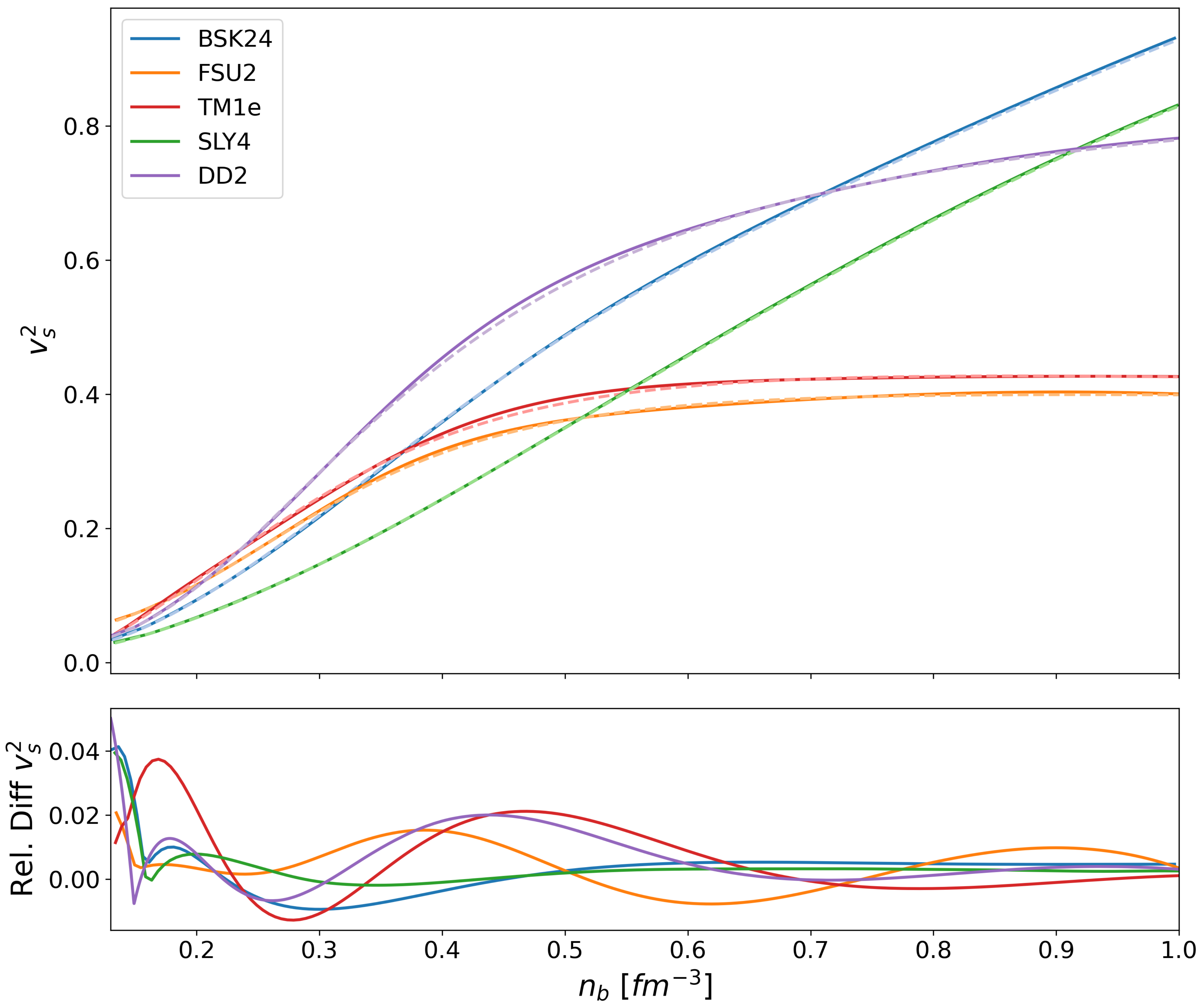


FIT: composition and vs2

Proton fraction



Speed of sound



Bayes inference: filters and likelihood zoo

[Phys. Rev. X 15, 021014]

$$\mathscr{M} : \mathbf{X} \to \{\epsilon(n_B), P(n_B), \delta(n_B), v_\beta(n_B), v_{FR}(n_B), \dots\}$$

$$\mathscr{L}(\mathbf{X}) = \prod_j \mathscr{L}_j(\mathbf{X}) = \prod_j p\left(D_j \,|\, \mathscr{M}(\mathbf{X})\right)$$

Nuclear

pQCD

χ_{EFT}

Nuclei information: NMP, AME masses, more?

Heavy Ion collision

Radio and X-Ray

Heavy pulsar radio timing

Black widow pulsar

NICER M-R

Kilonovae and gamma ray bursts

Gravitational wave

GW170817

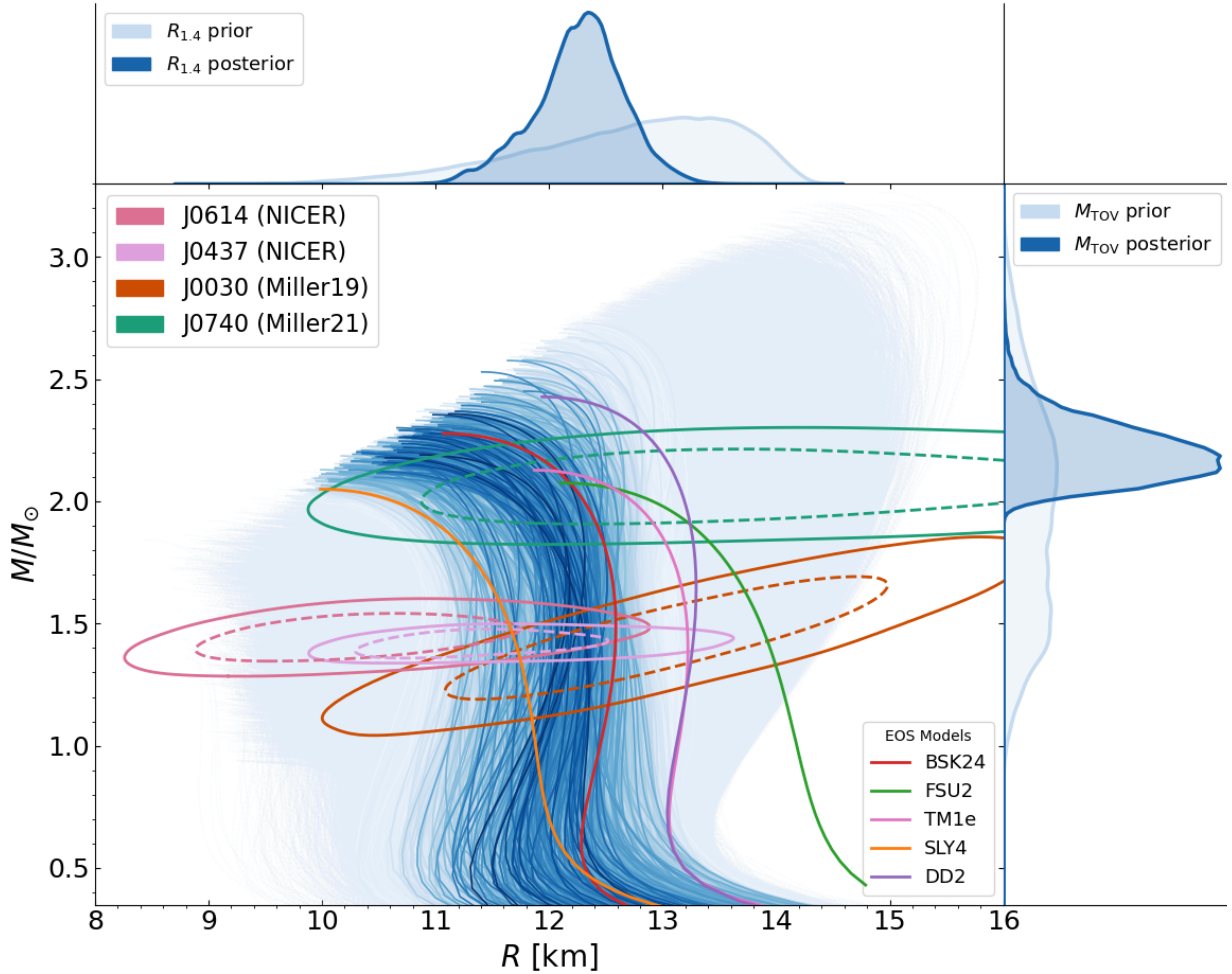
GW190425

Bayes inference

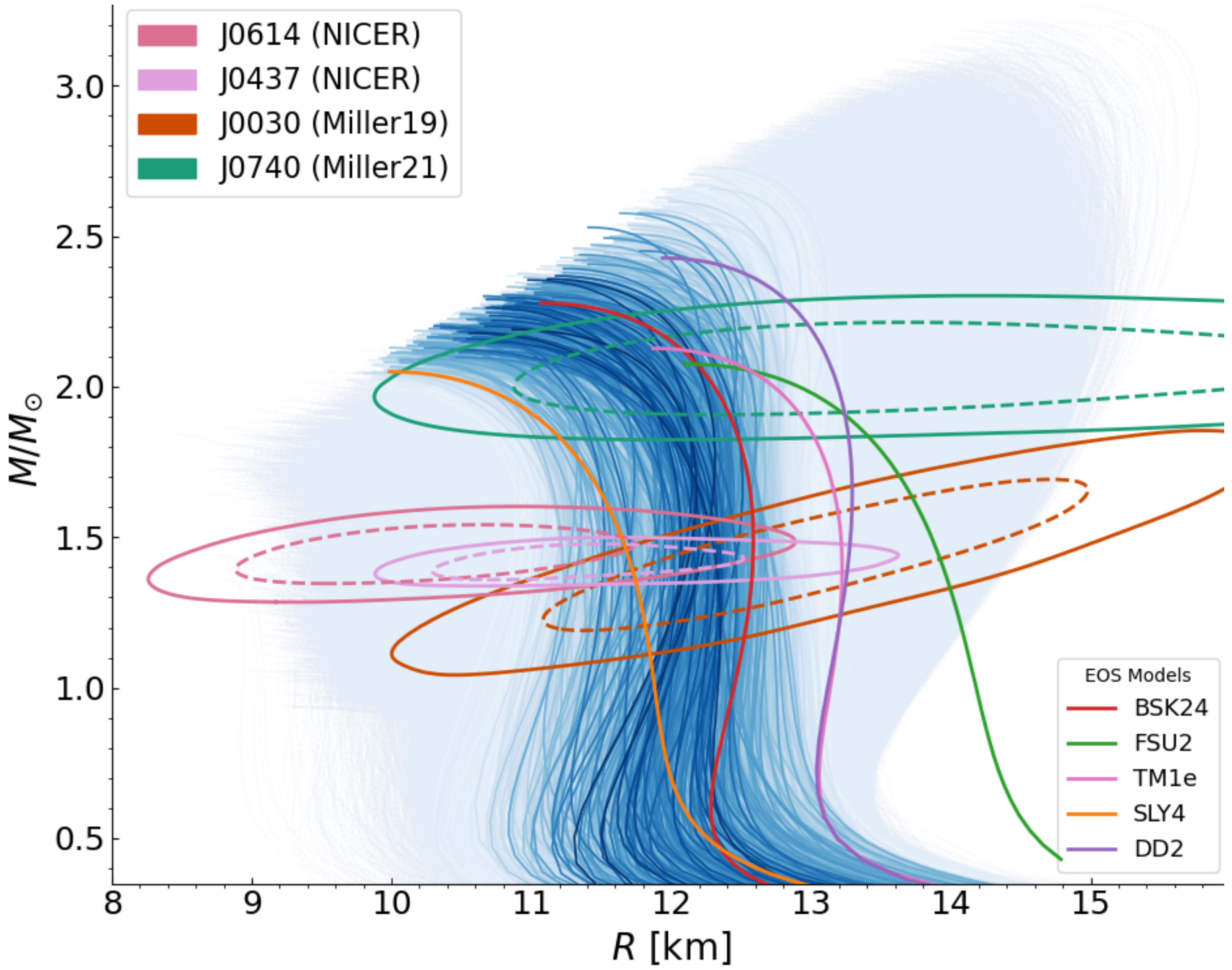
How to best control the underlining hypothesis and/or biases of the chosen model and prior?

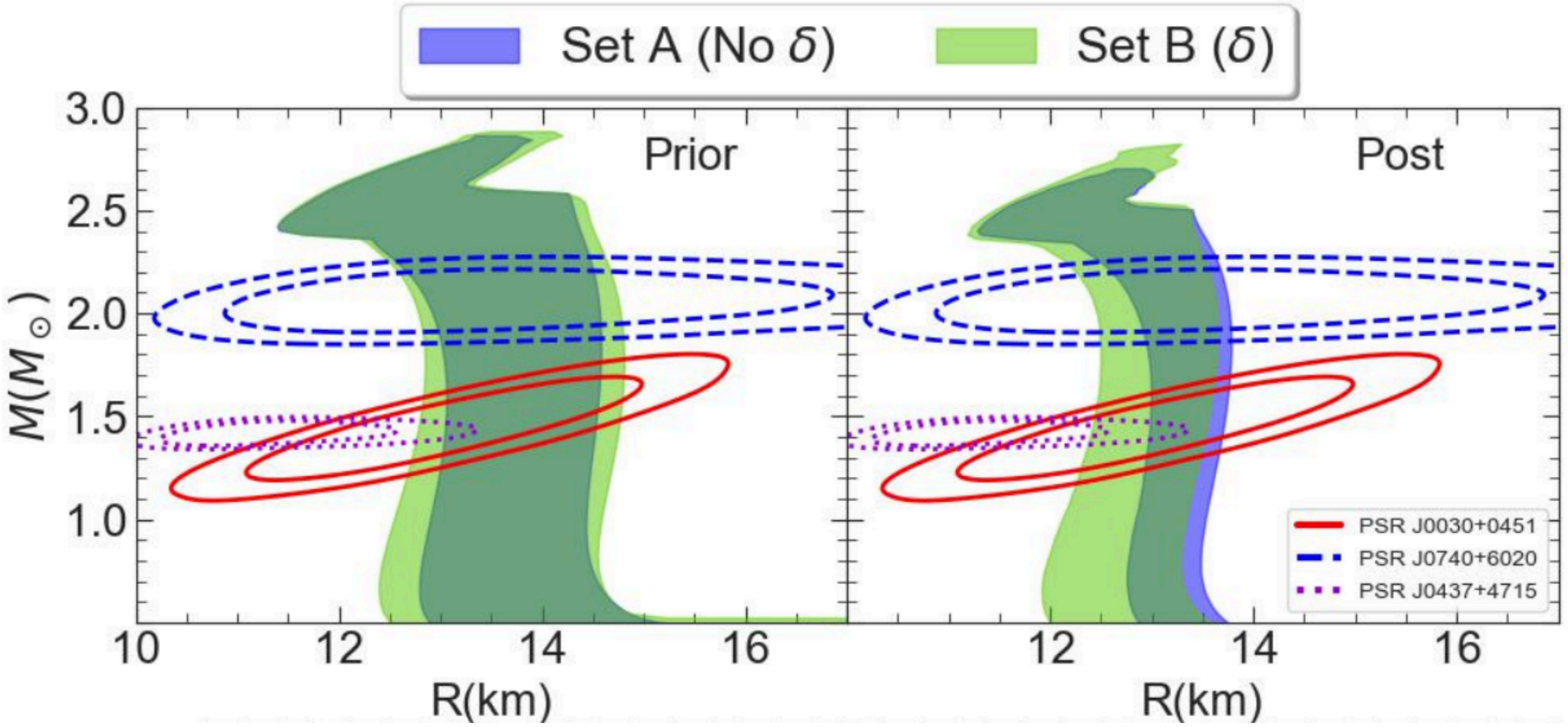
The efficient exploration of the parameters space is crucial

Which quantities can be really trusted when predicted by an inference?



An (almost) flexible causal model



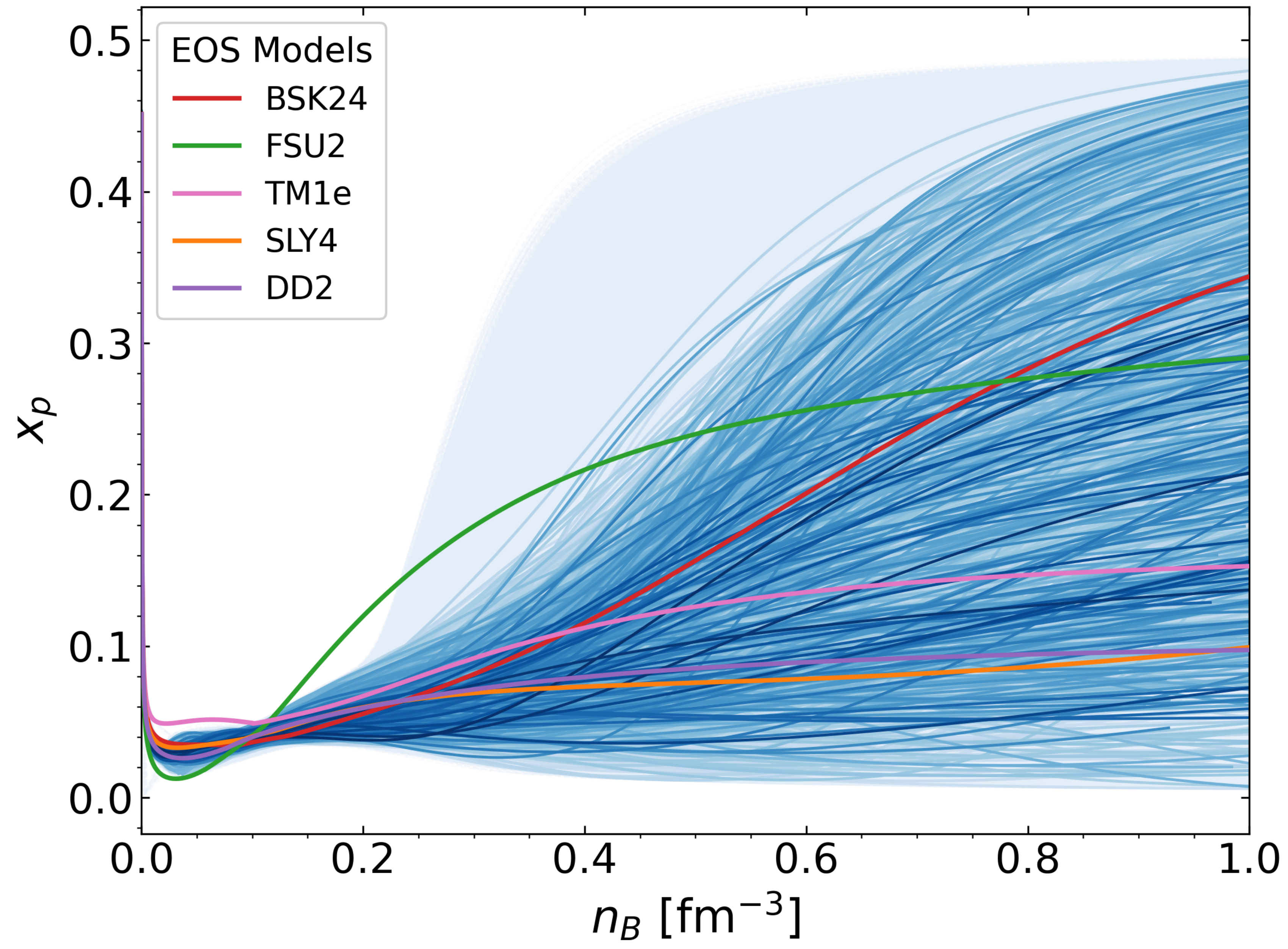


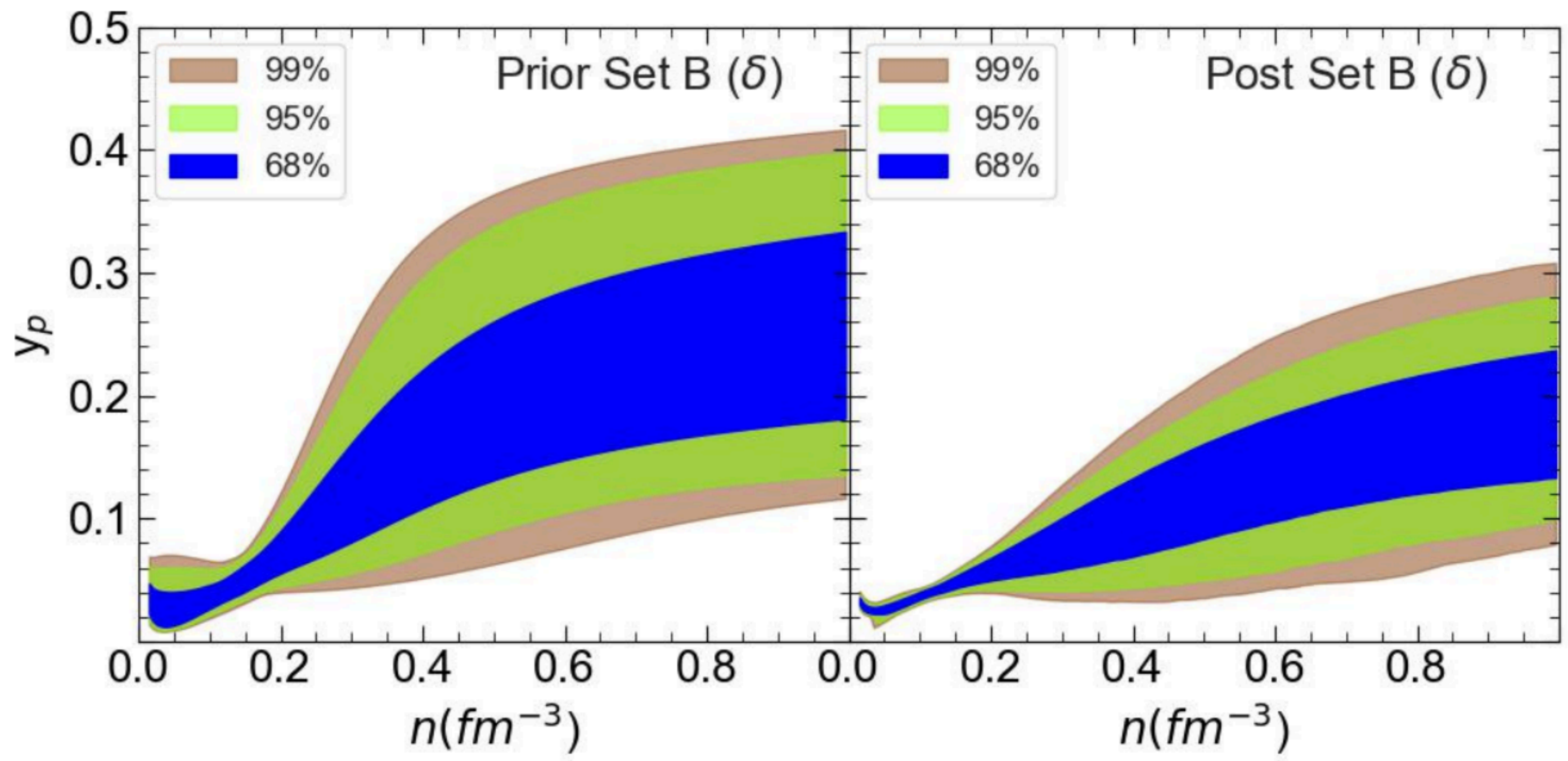
Each choice has advantages and disadvantages

→ Relativistic: causality is included in the base package, but the internal correlations are very complex

→ Non Relativistic: easier mapping between parameters and physical properties = "almost" agnostic model, but causality to be purchased separately

An (almost) flexible causal model





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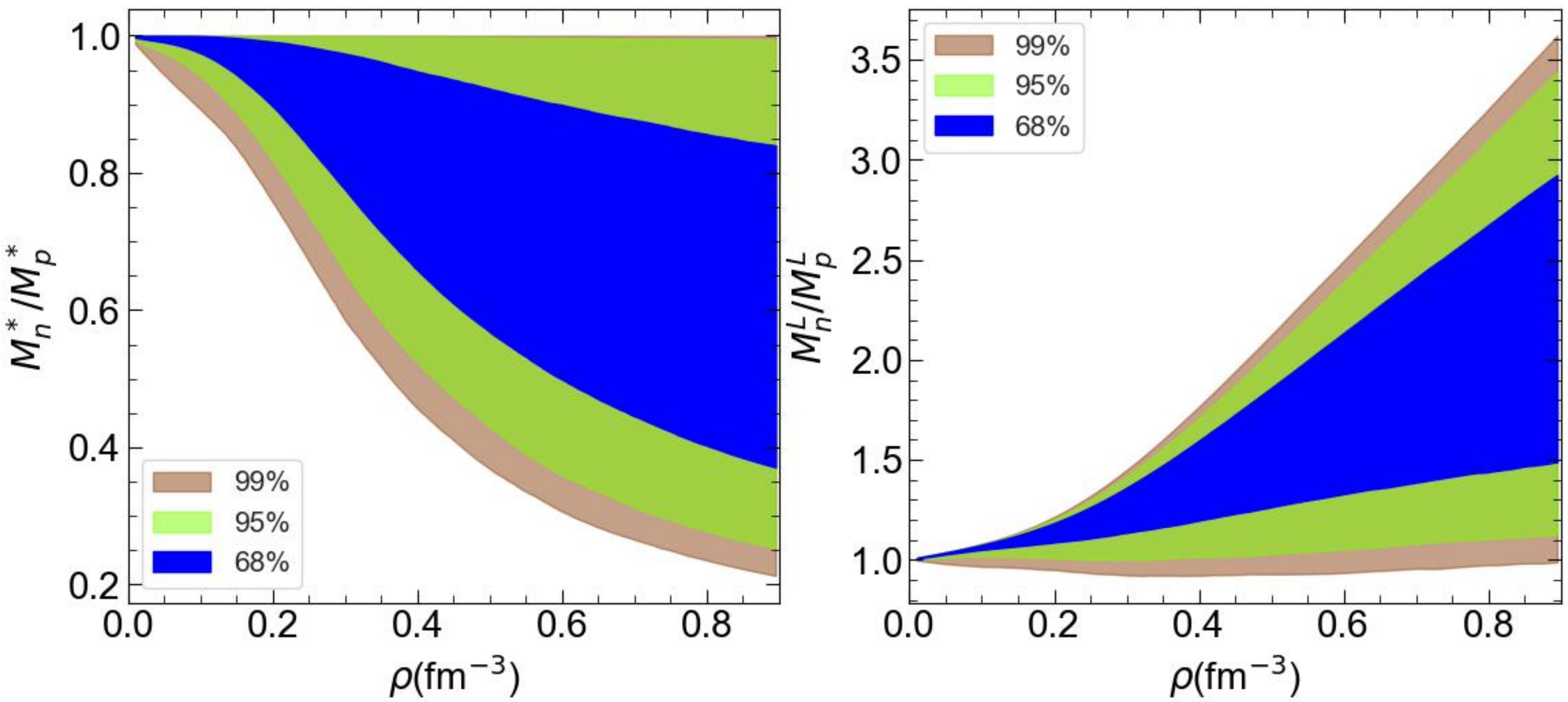
Effective mass extravaganza



At finite temperature, a key role is played by the nucleon effective mass… but which one?

Non Relativistic -> Landau Mass

Relativistic -> Dirac Mass

In asymmetric matter, these two masses show opposite sign of the neutron-proton mass splitting

Flexibility can't save us now. Are we doomed?

Stay tuned !

Summary of questions

– The "perfect" meta model doesn't exist

Which compromises can we take for each meta model scenario of use?

– There are many observations

How to choose which one to include in the inferences?

– Which low or high density limit can be important to implement?

– Tradeoff between an improved crust and a lightweight model?

– Hyperons, quarks, phase transition?

– Finite temperature extension?

Effective mass, RMF vs non relativistic

BACKUP SLIDES

Nucleonic potentials

$$e_0(x) = V_0(x) + \boxed{\frac{h_0 + h_1 x + h_2 x^2 + h_3 x^{(2+q_0)}}{(1 + a_0 x)(1 + b_0 x)(1 + c_0 x)}}$$

$$V_0(x) = \frac{s_0\, x^3}{1 + w_0(3x+1)^{3+g_0}}$$

$h_{0,1,2}$ are fixed through a simple mapping with the NMP up to second order

h_3 and q_0 controls the stiffness/softness at high density

a_0, b_0 and c_0 balance the numerator for causality

Nucleonic potentials

$$e_0(x) = V_0(x) + \frac{h_0 + h_1 x + h_2 x^2 + h_3 x^{(2+q_0)}}{(1 + a_0 x)(1 + b_0 x)(1 + c_0 x)}$$

$$V_0(x) = \frac{s_0\, x^3}{1 + w_0 (3x+1)^{3+g_0}}$$

s_0 is fixed by the request of the energy vanishing at $n = 0$

x is cubic to not modify the mapping with NMP up to second order

g_0 takes care of causality while w_0 tunes the dominant range of the correction

e_2 is built with the same structure

Quartic correction in δ for the PNM

E_{sym} / J, L_{sym} and K_{sym} are usually defined starting from:

$$\left.\frac{\partial^2 e}{\partial \delta^2}\right|_{x=0,\boxed{\delta=0}}$$

By fixing the "sym" parameters of a given EoS with the quadratic expansion in δ, we would not reproduce the PNM ($\delta = 1$) around saturation density

We introduce a term in the energy density to correct this behavior:

$$\boxed{e_4(n) = A\left(n/n_0\right)^B}$$

Fitting existing EoS

Test the flexibility of the model to reproduce β-equilibrated EoS

Constrain the space of the unphysical parameters

We have chosen: Sly4, BSK24, DD2, FSU2 and TM1e

Procedures

- Fix the NMP up to second order from COMPOSE
- Fit e_4 in the interval [0.12,0.2] fm^{-3}on PNM
- Keep e_4 fixed and fit e_0 at least up to n_{tov} on SM
- Repeat the same for e_2 on PNM while keeping both e_0 and e_4 fixed

FIT: results for SNM and PNM

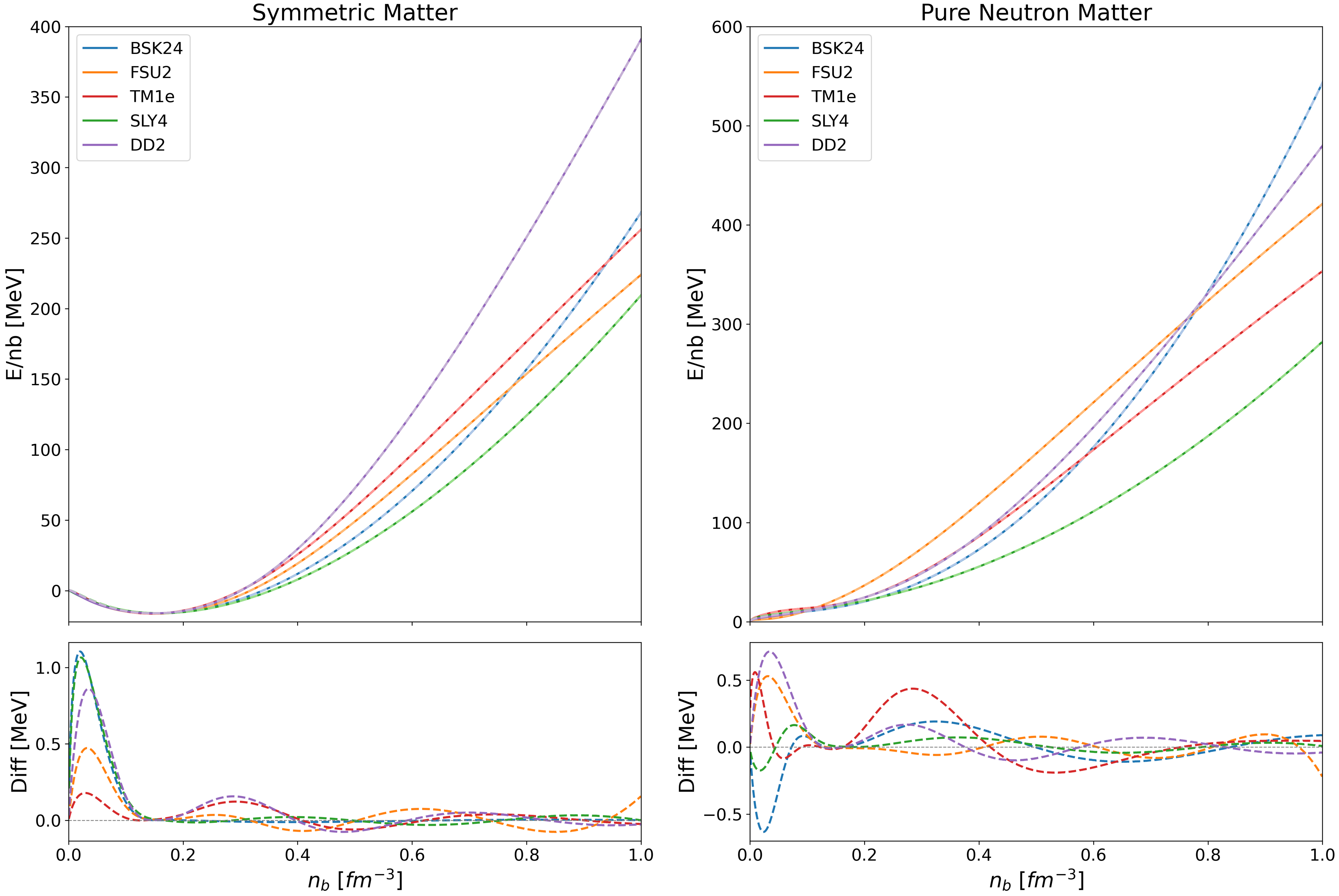
We found satisfying results with

$a_{0,2} = b_{0,2}$ and $g_{0,2} = 0$

Less Paramaters!

Before saturation the accuracy is limited

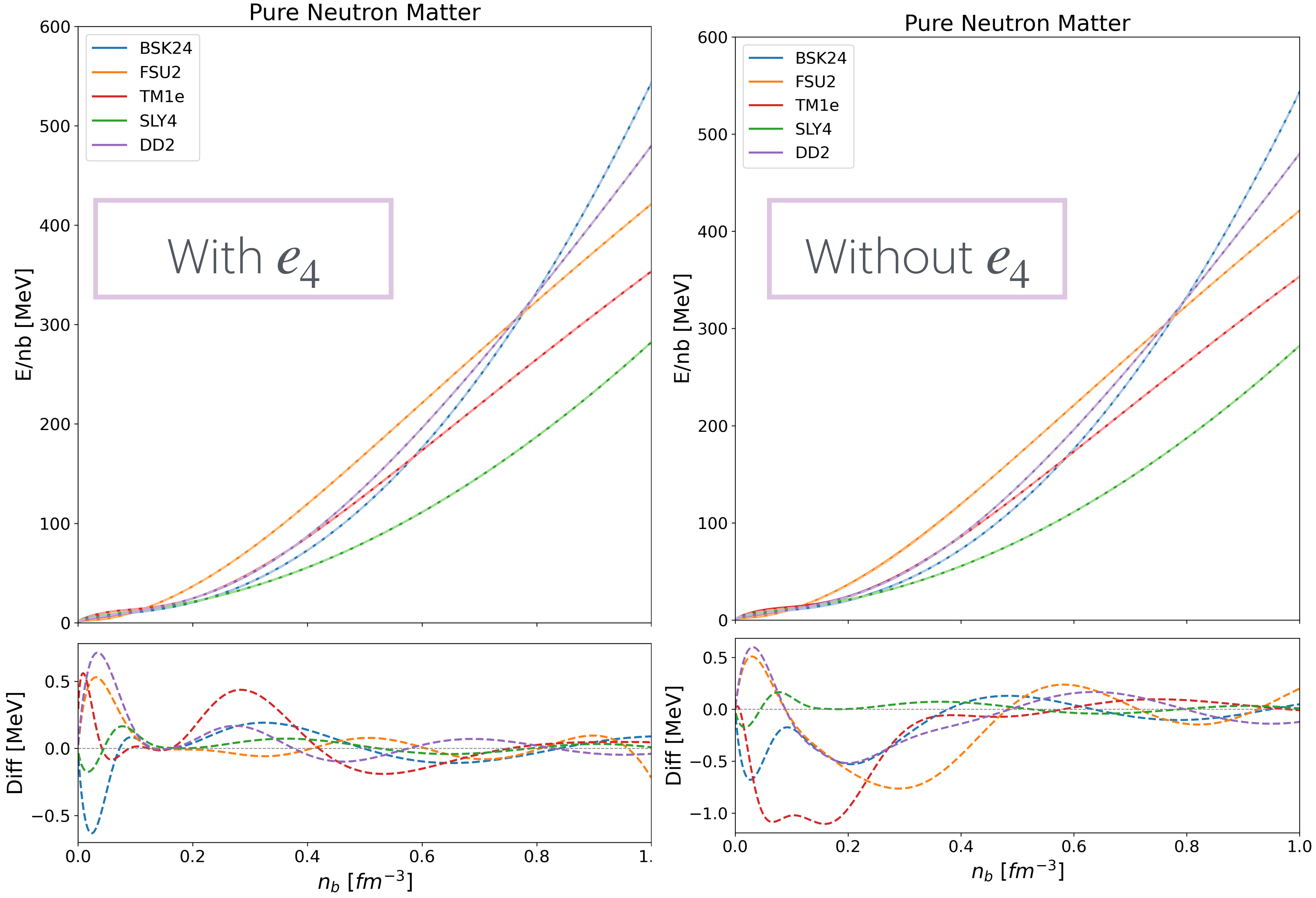
Focus on astrophysics



Fit without the quartic δ correction

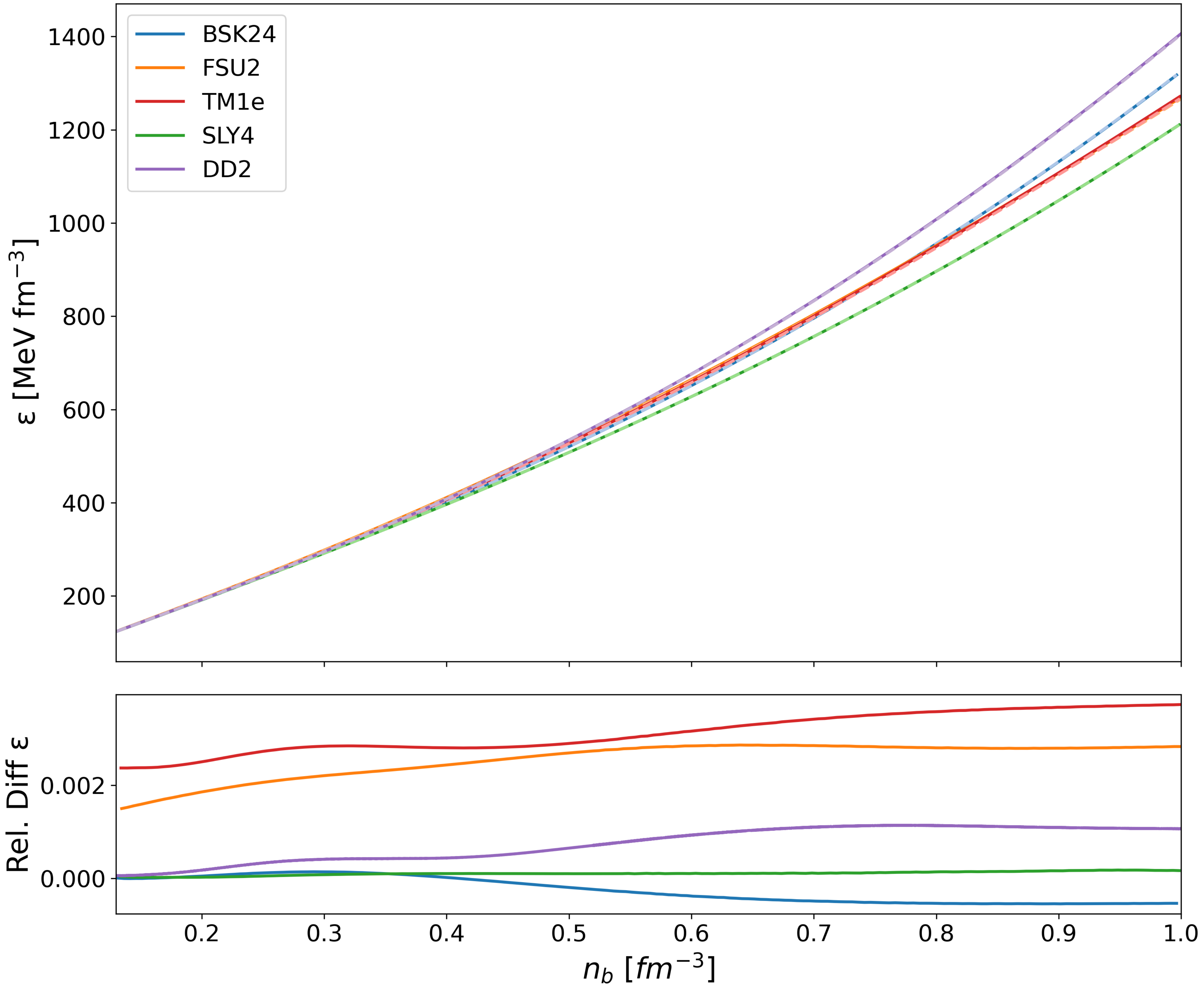
Without the correction around saturation the PNM fit exhibit a $\sim 0.5/1$ MeV of difference

The overall accuracy doesn't change

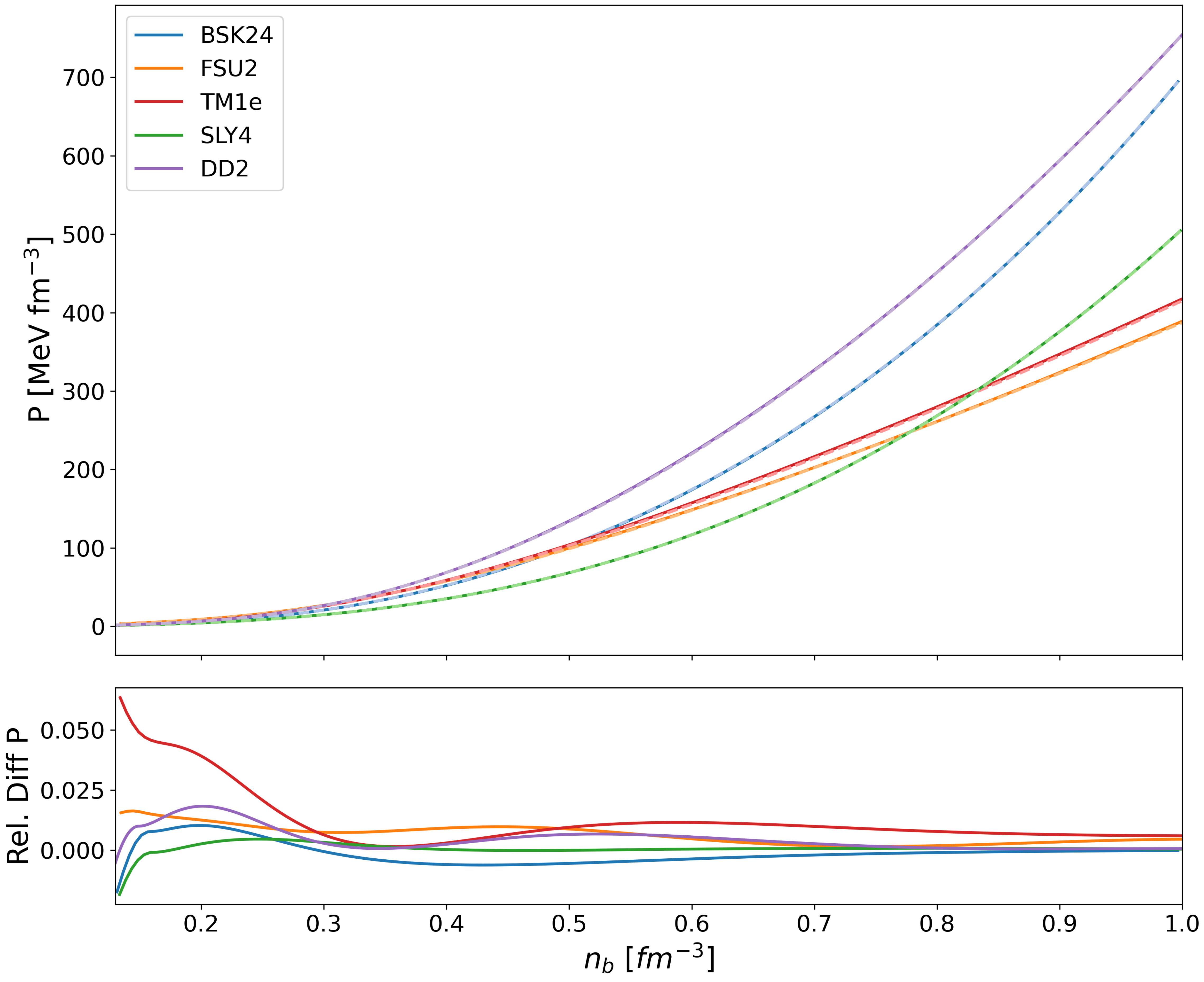


FIT: energy density and pressure

Energy density



Pressure



Bayes inference

$$\mathscr{M} : \mathbf{X} \to \{\epsilon(n_B), P(n_B), \delta(n_B), v_\beta(n_B), v_{FR}(n_B), \ldots\}$$

$$\mathscr{L}(\mathbf{X}) = \prod_j \mathscr{L}_j(\mathbf{X}) = \prod_j p\left(D_j \,|\, \mathscr{M}(\mathbf{X})\right)$$

Informed prior sampling the χ_{EFT} band[1] of PNM energy with a metropolis MCMC

At this stage we have 10^9 models

We extract 10^5 models that pass through the remaining filter:

- AME2020 nuclear masses table
- Maximum observed NS mass from radio-timing of PSRJ0348 and PSRJ0740
- Tidal deformability from GW170817 event detected by Ligo/Virgo collaboration
- NICER+XMN M-R measurements of PSRJ0030, PSRJ0347, PSRJ0614 and

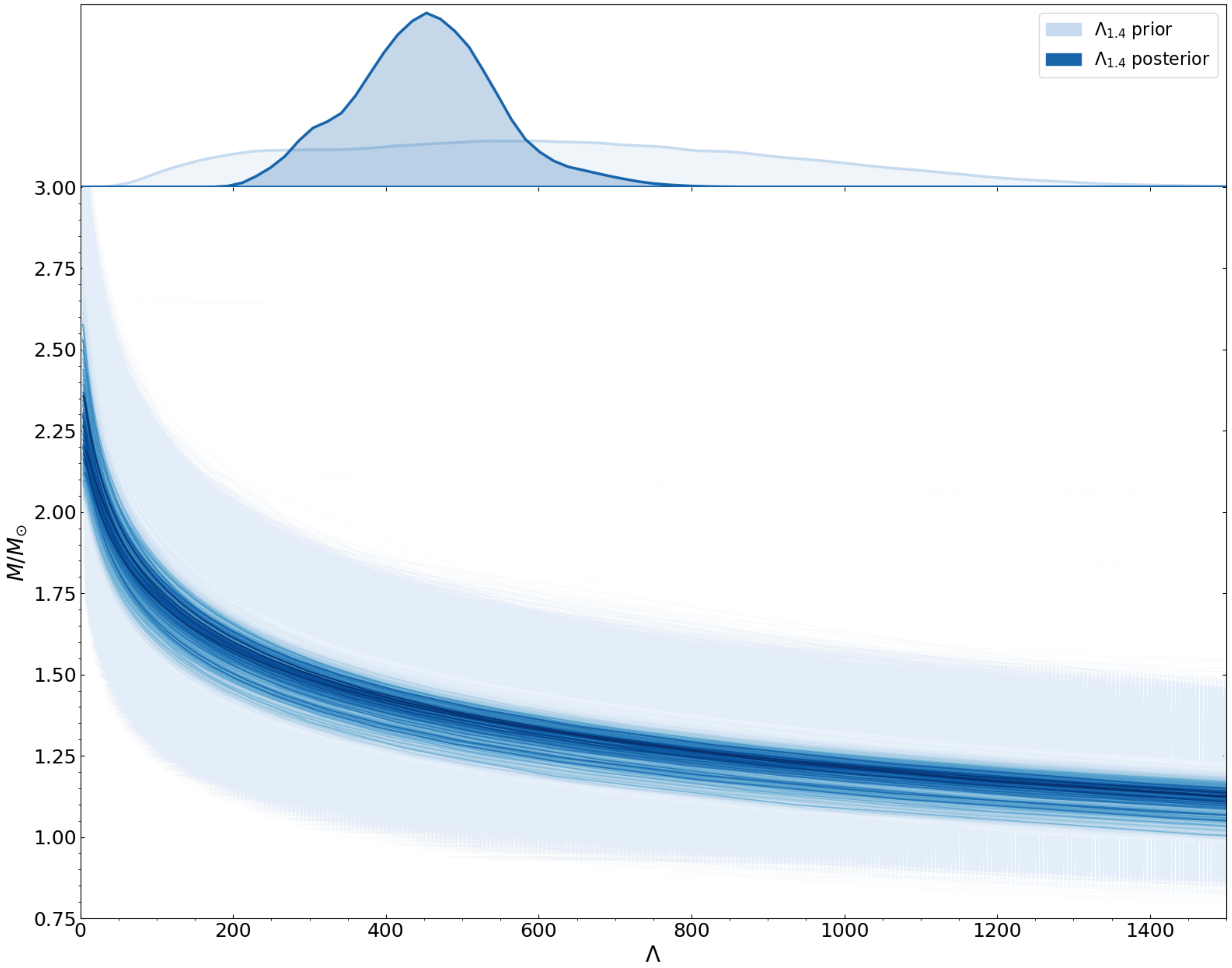
[1] Huth et al, 2021, Phys. Rev. C, 103, 025803

Tidal deformability

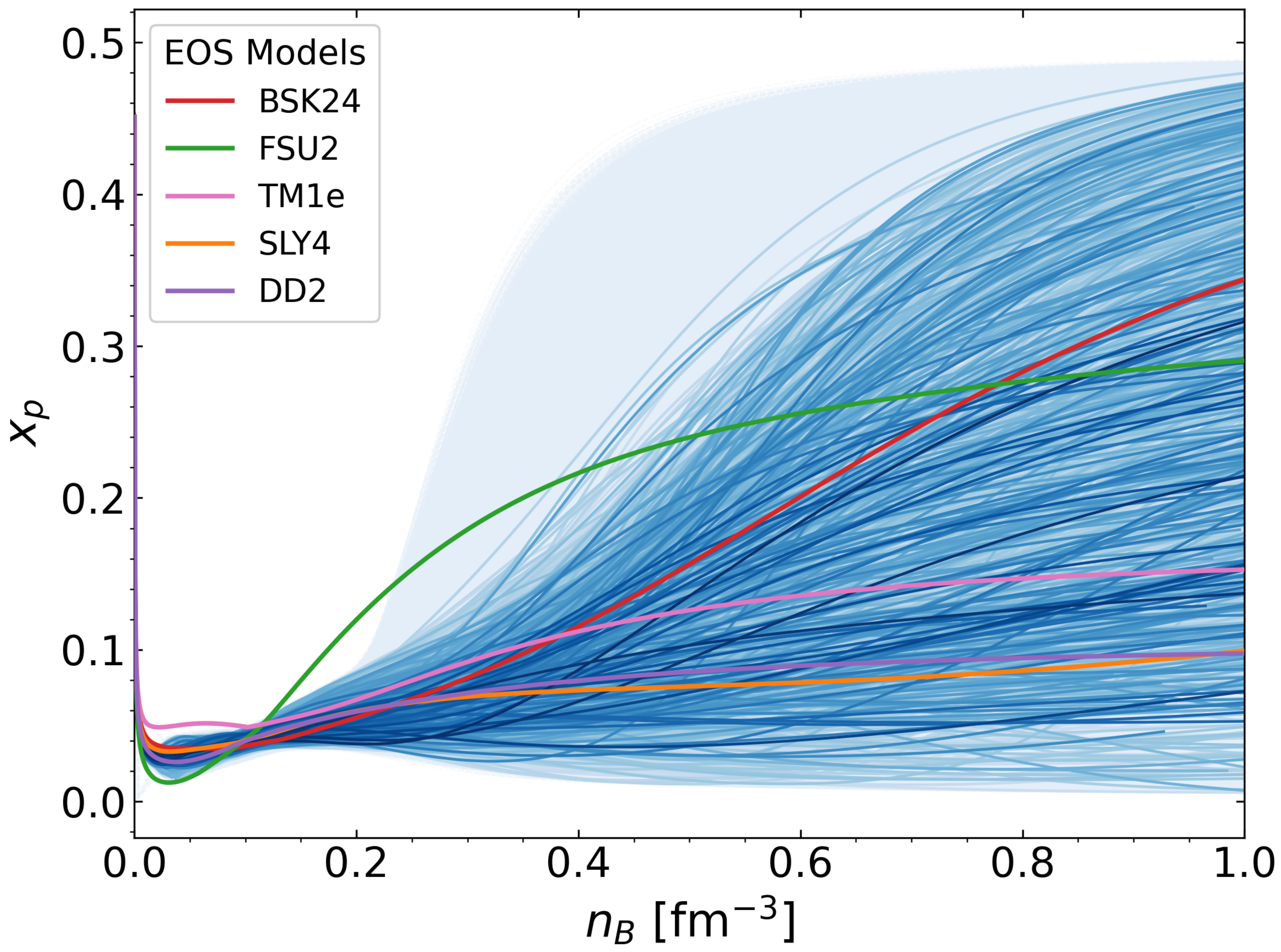
We cover a wide range of masses, radius and tidal

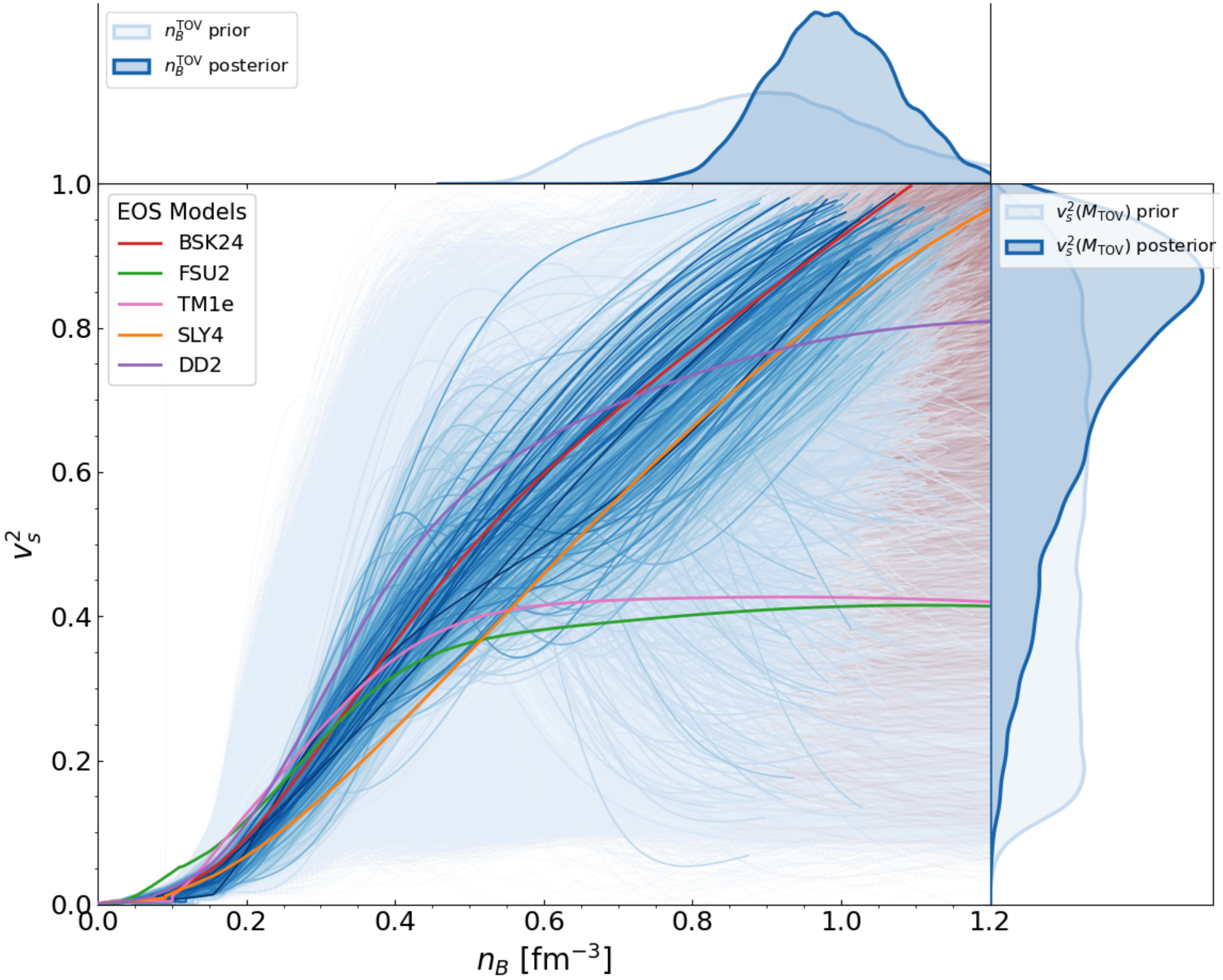
The two newest nicer data suggest a soft EoS

$\Lambda_{1.4} > 800$ is disfavored

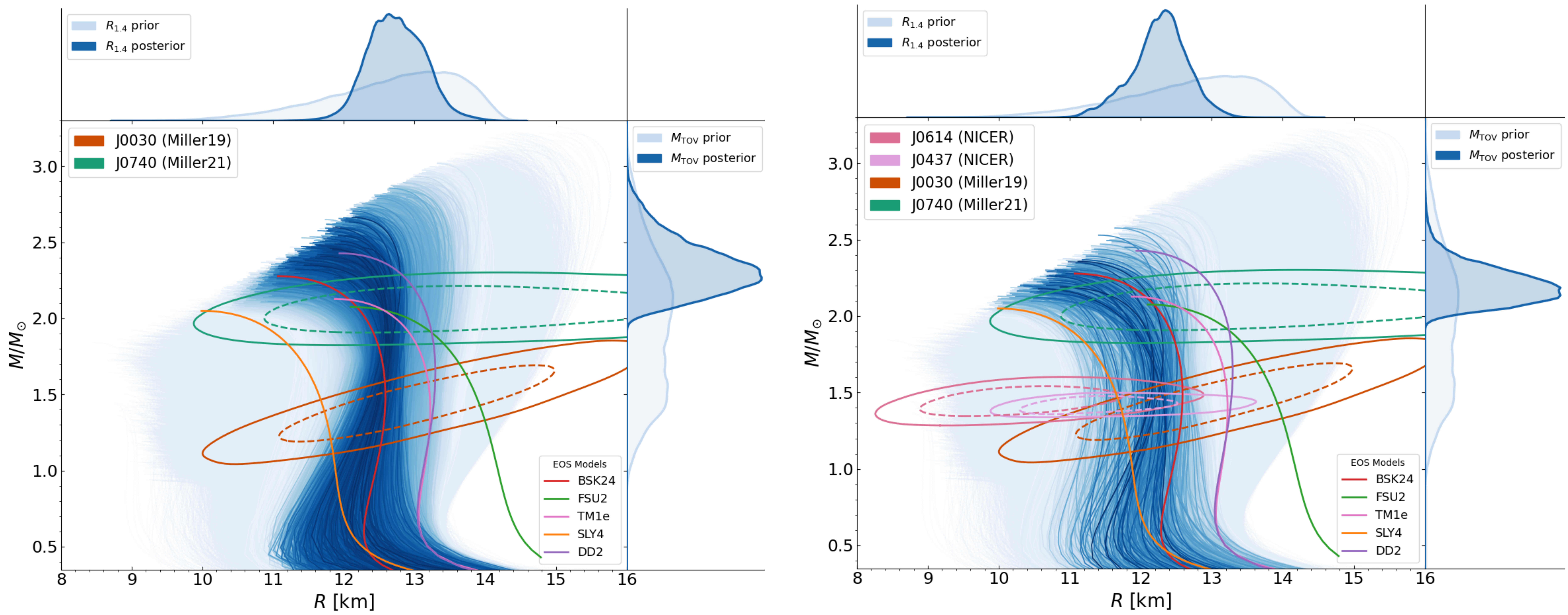


Composition and speed of sound

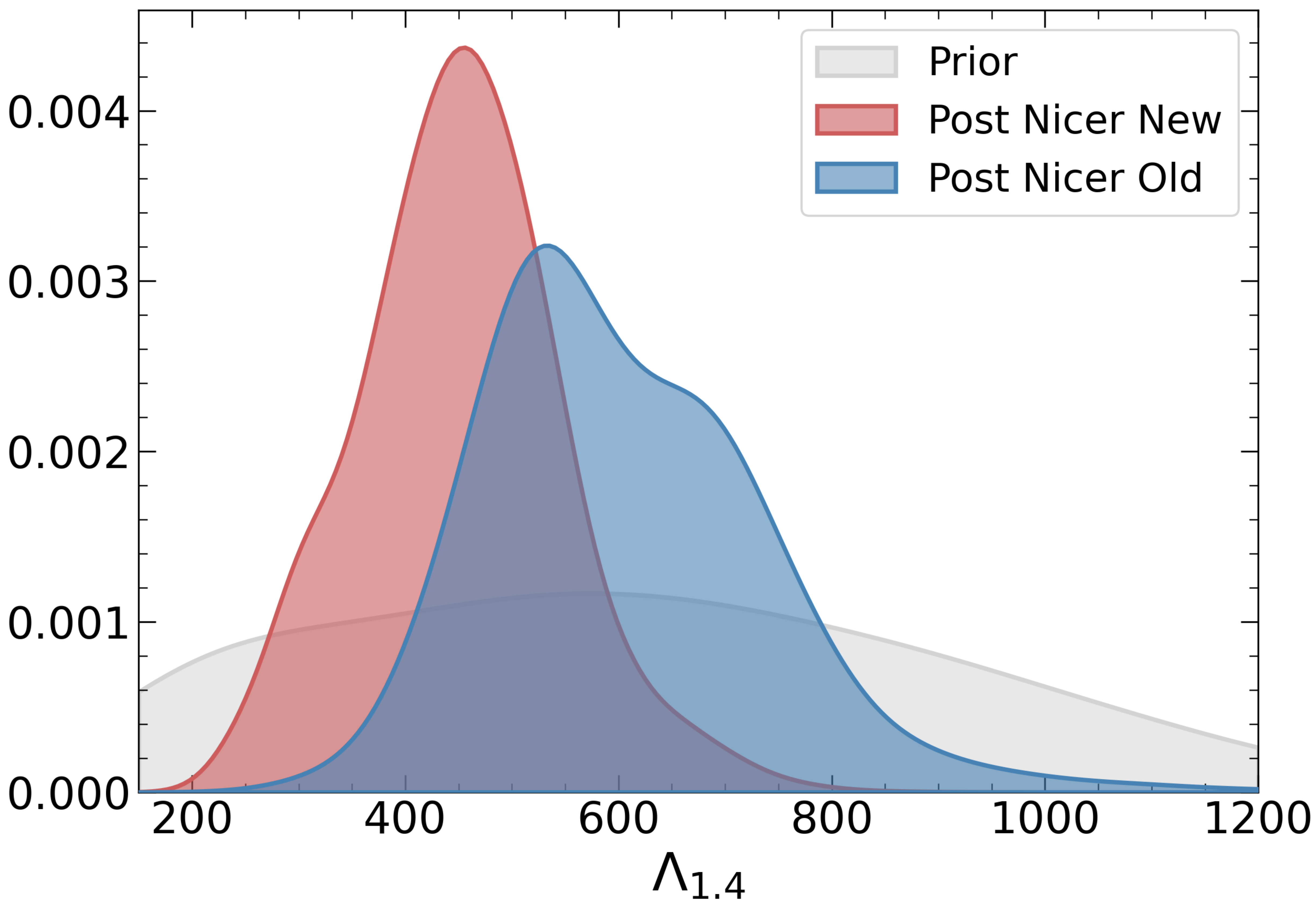


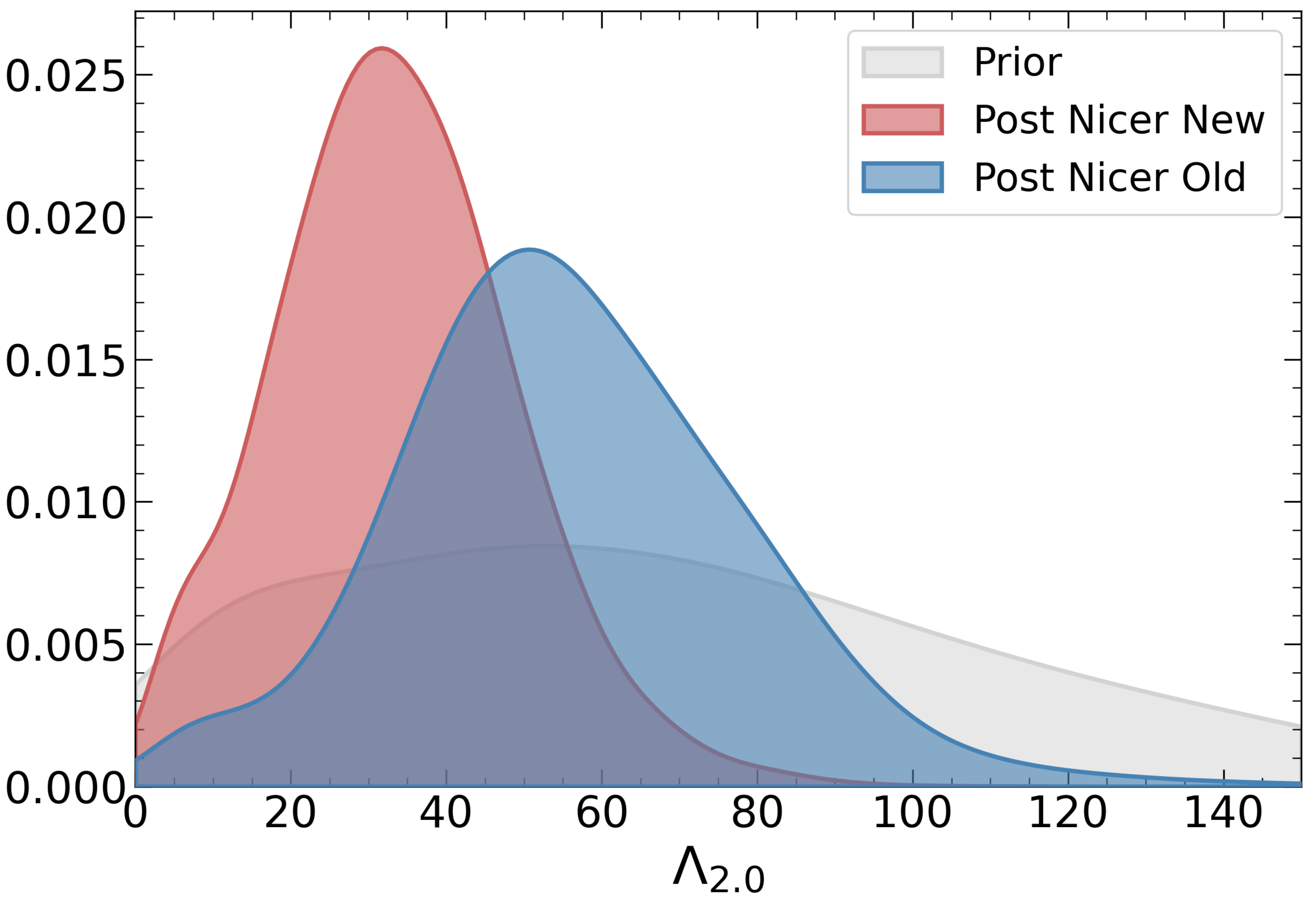


Nicer old vs nicer new: Mass-Radius



Nicer old vs nicer new: Tidal deformability





Nicer old vs nicer new: M_{tov}

The two latest Nicer measures prefer soft EoS

We can see the effects on the PDF of M_{TOV} which is peaked on lower masses