Bayesian inference on nuclear data and neutron star observations for the nuclear equation of state

(or From crunchy to smooth: nuclear-informed study of the crust-core transition)

Pietro Klausner

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Collaborators

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Introduction

Bayesian inference on nuclear data and neutron star observations for the nuclear equation of state

Bayesian study of NS EoS and predictions on NS properties[1]

The prior distribution is the result of a previous Bayesian analysis, with Skyrme functionals fit on a large set of static and dynamical nuclear experimental observables[2]

Final result informed by both nuclear physics experiments and NS observations

[1] Klausner et al., arxiv 2505.16929

[2] Klausner et al., Phys. Rev. C 111, 014311 (2025)

Background: Skyrme Bayesian analysis (1)

Binding Energies

^{40}Ca, ^{48}Ca, ^{56}Ni, ^{68}Ni, ^{90}Zr, ^{100}Sn, ^{132}Sn, ^{208}Pb

Charge radii

^{40}Ca, ^{48}Ca, ^{90}Zr, ^{132}Sn, ^{208}Pb

Spin-orbit splittings

^{48}Ca $(\nu 2p)$, ^{208}Pb $(\pi 2f)$

Isoscalar Giant Resonances

Monopole

^{90}Zr, ^{208}Pb

Quadrupole

^{208}Pb

Constraints on symmetric matter nEos

Nuclear polarizability

^{48}Ca, ^{208}Pb

EWSR IVGDR

^{208}Pb

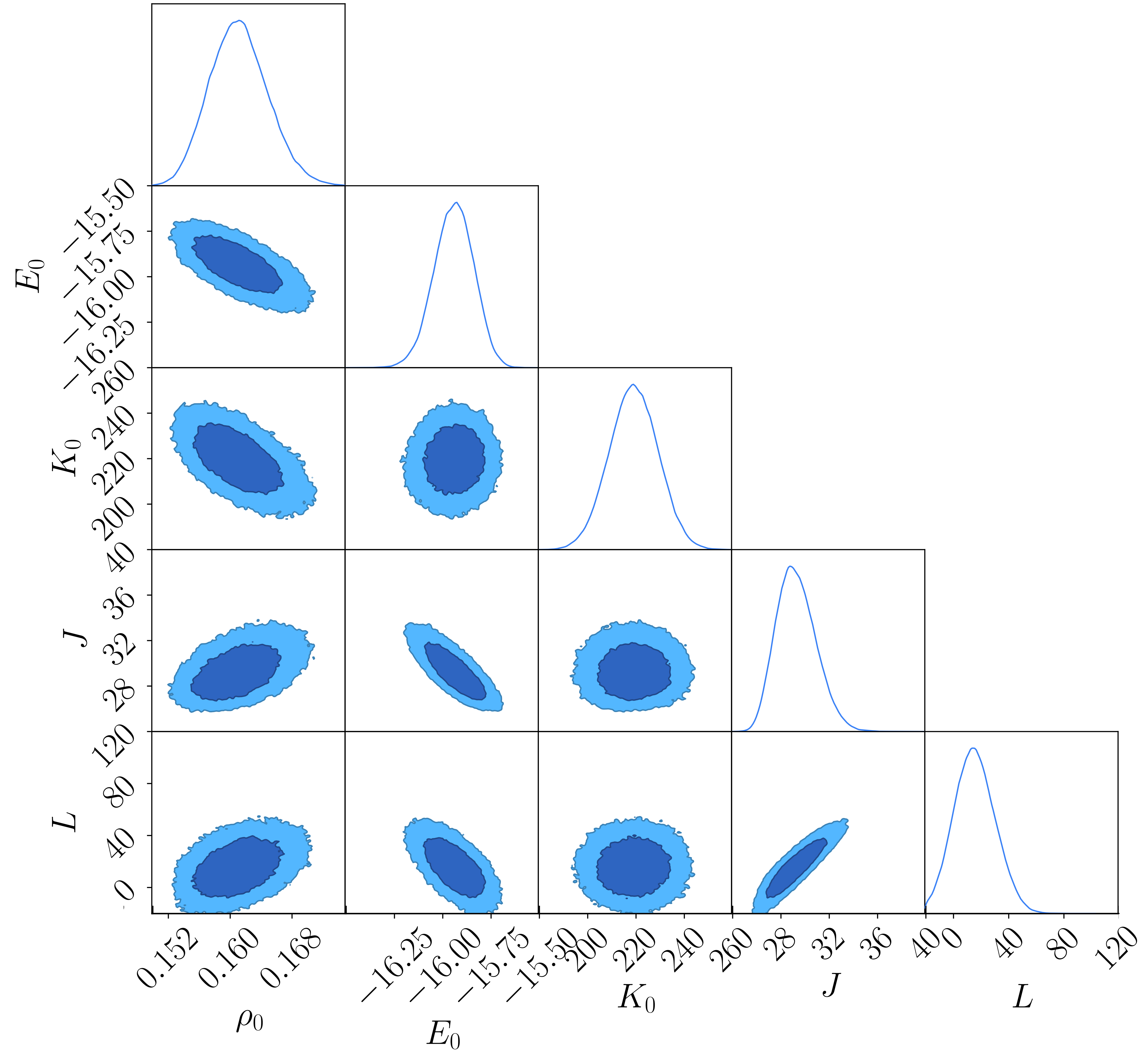
Parity-violating asymmetry

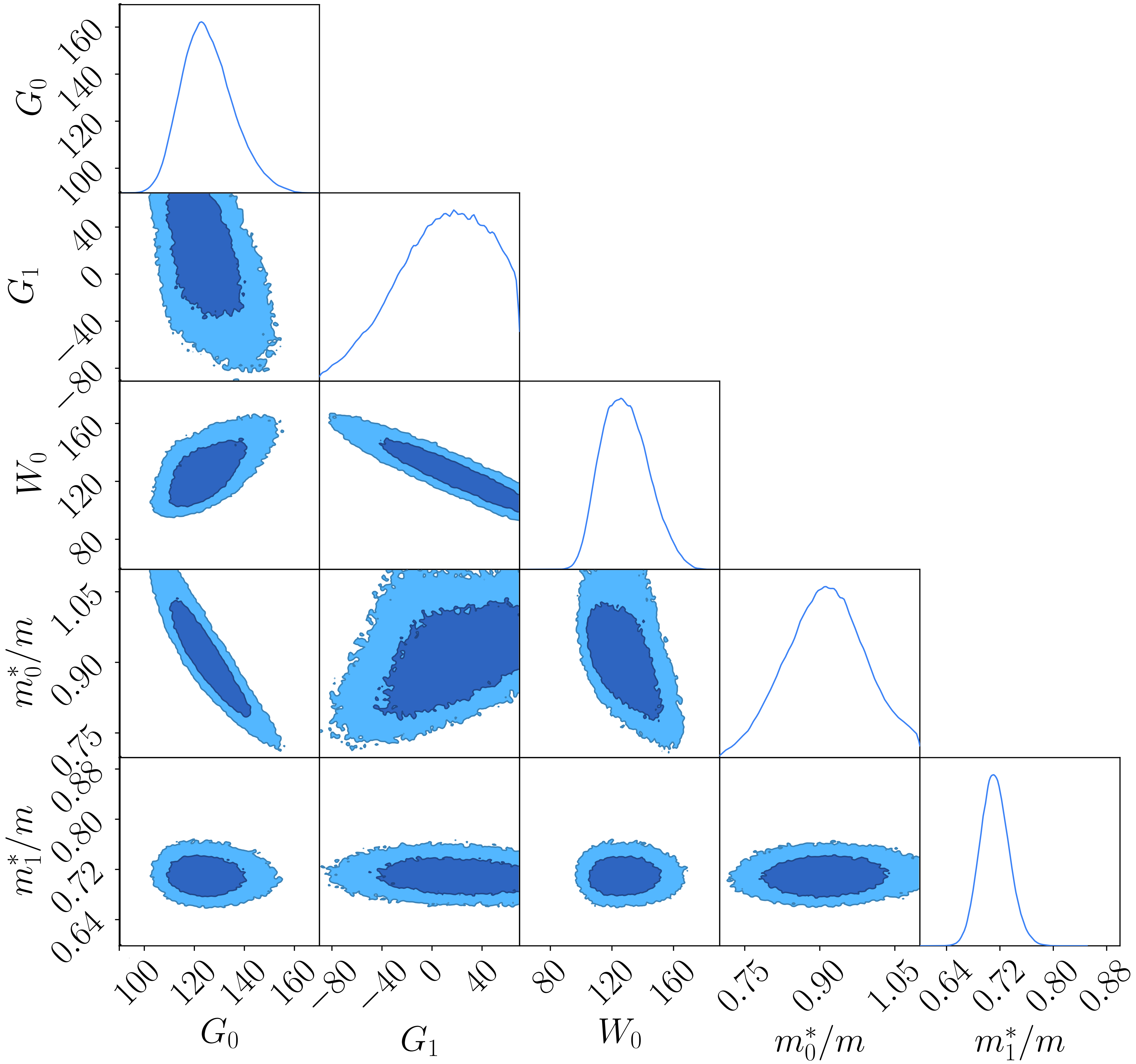
^{48}Ca, ^{208}Pb

(CREX, PREX-II)

Constraints on Symmetry energy

Background: Skyrme Bayesian analysis (2)





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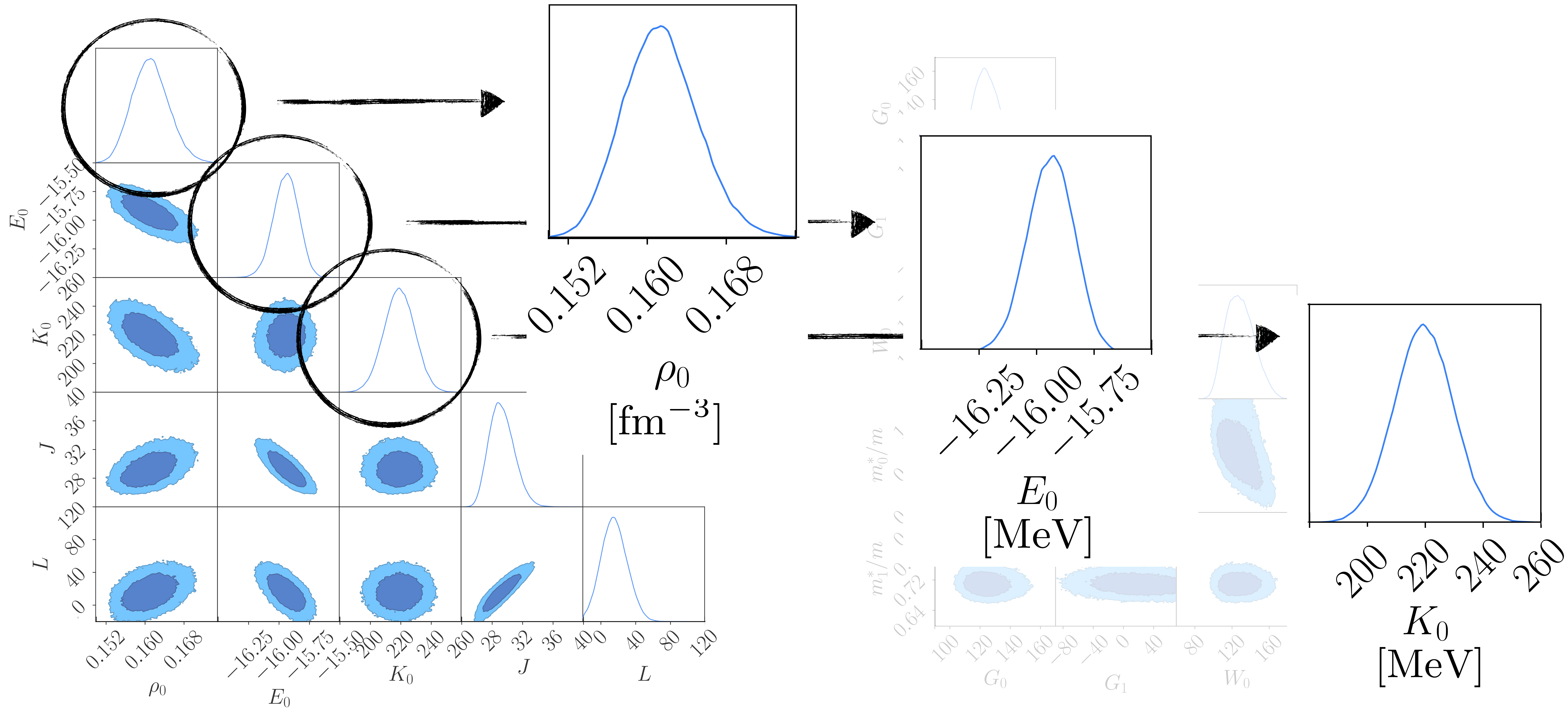

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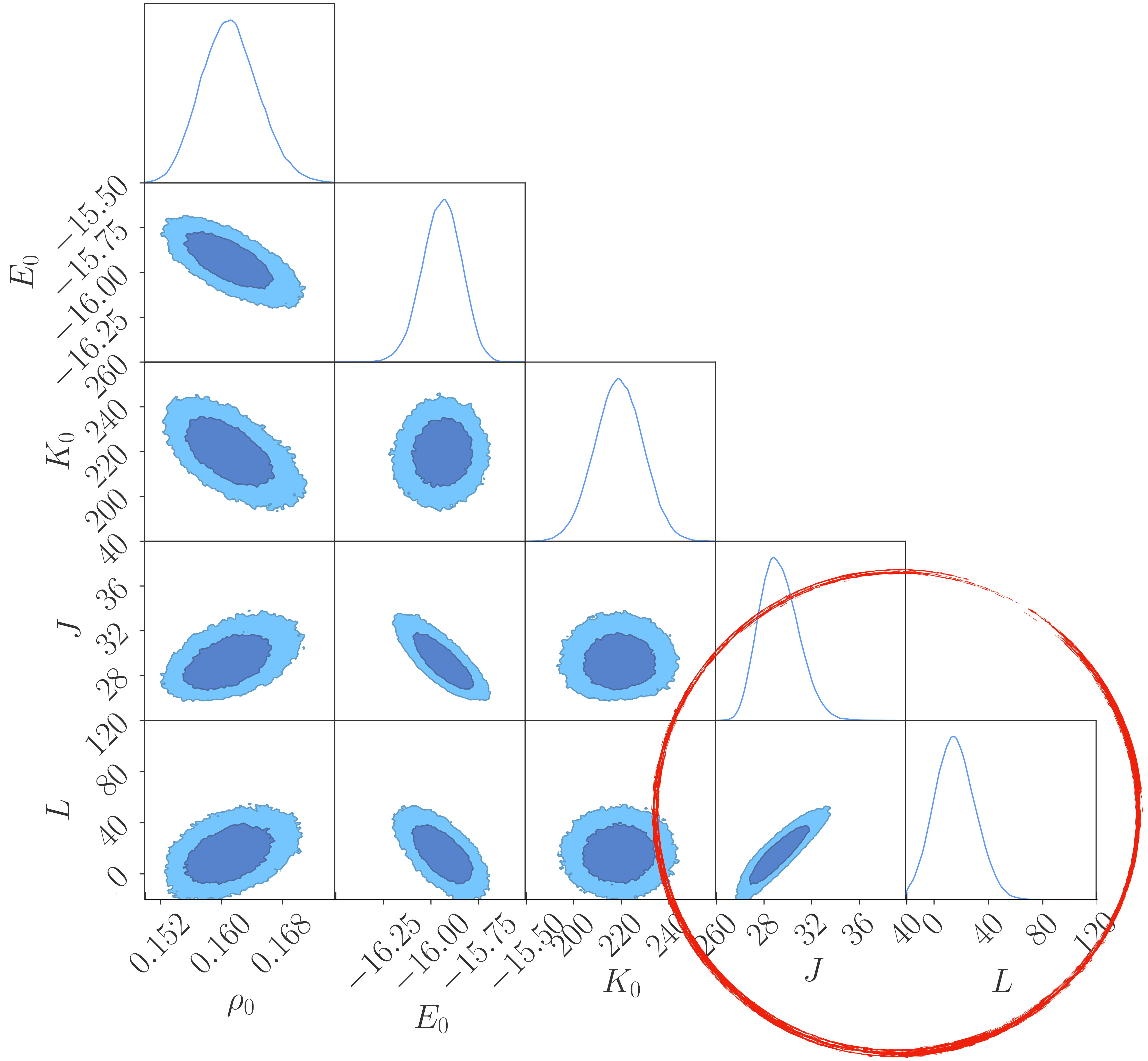

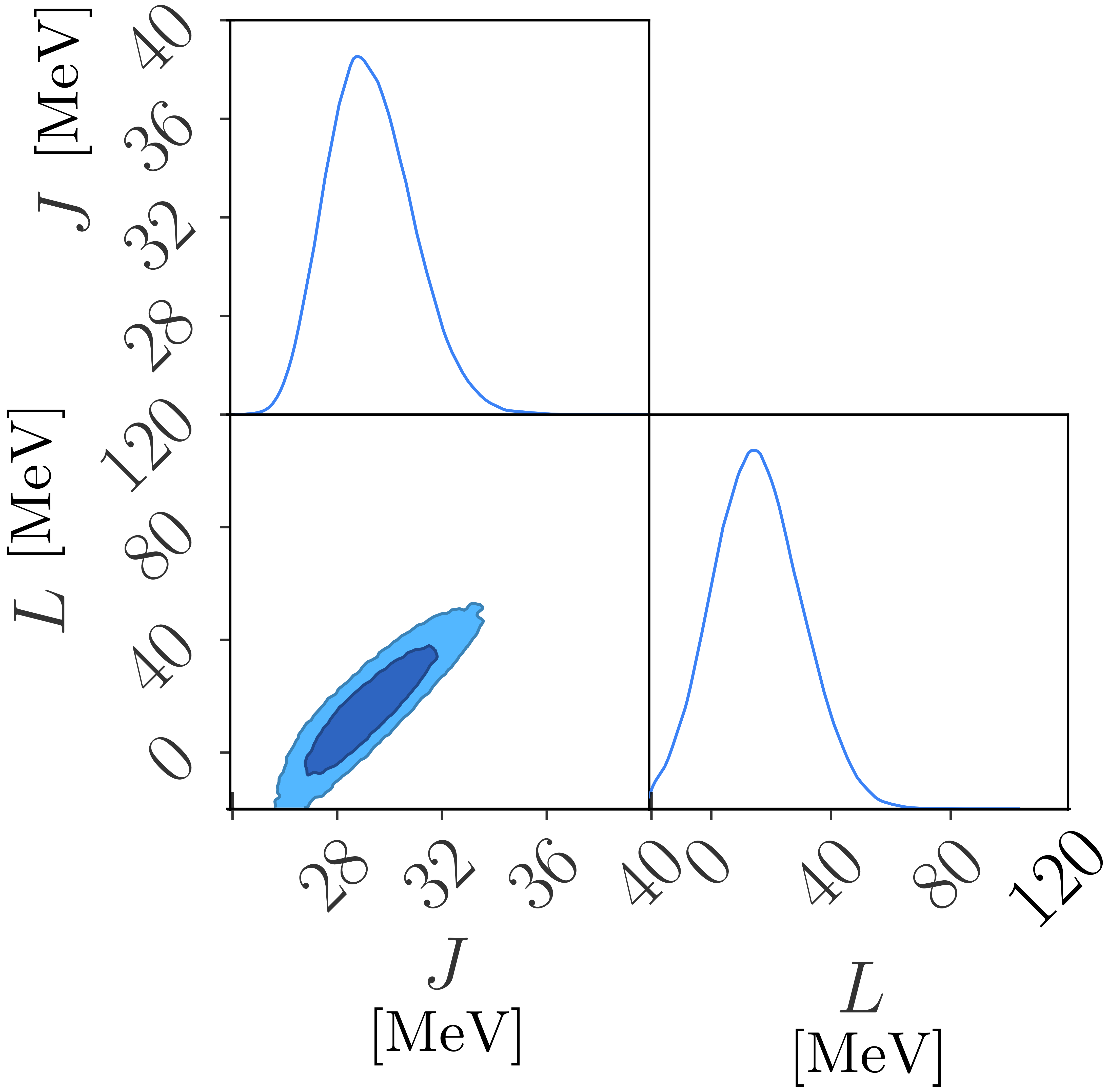

3b

From nuclear experiments to NS properties

Skyrme parametrization

Mapping from Skyrme nEoS to M.M.

M.M.[1]

Inner Crust computed consistently in extended Thomas Fermi; outer crust with BSk24 functional

Neutron star EoS

If stable and causal

Compute NS properties

[1]Margueron et al., Phys. Rev. C **97**, 025805 (2018)

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Mapping of Skyrme into M.M.

Skyrme's parameters[1]

$$n_{sat}, E_{sat}, K_{sat}$$

$$E_{sym}, L_{sym}$$

$$G_0, G_1, w_0, m_0^*/m, m_1^*/m$$

M.M.'s parameters

$$n_{sat}, E_{sat}, K_{sat}, Q_{sat}, Z_{sat}$$

$$E_{sym}, L_{sym}, K_{sym}, Q_{sym}, Z_{sym}$$

$$m_0^*/m, m_1^*/m$$

[1] 1-to-1 correspondence with usual Skyrme's parameters (L.-W. Chen et al. Phys. Rev. C 80, 014322 (2009))

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$$m_0^*/m, m_1^*/m$$

$$K_{sym} = K_{sym}(n_{sat}, E_{sat}, K_{sat}, \ldots)$$

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$E_{sym}, L_{sym}, K_{sym}, Q_{sym}, Z_{sym}, Q^*_{sym}, Z^*_{sym}$

$m_0^*/m, m_1^*/m$

$$K_{sym} = K_{sym}(n_{sat}, E_{sat}, K_{sat}, \dots)$$

Skyrme's formula $n < n_{sat}$

$$Q_{sat} = Q_{sat}(n_{sat}, E_{sat}, \dots) \qquad Z_{sat} = Z_{sat}(n_{sat}, E_{sat}, \dots)$$
$$Q_{sym} = Q_{sym}(n_{sat}, E_{sat}, \dots) \qquad Z_{sym} = Z_{sym}(n_{sat}, E_{sat}, \dots)$$

Randomly extracted $n > n_{sat}$

$Q^*_{sat,sym}, Z^*_{sat,sym}$

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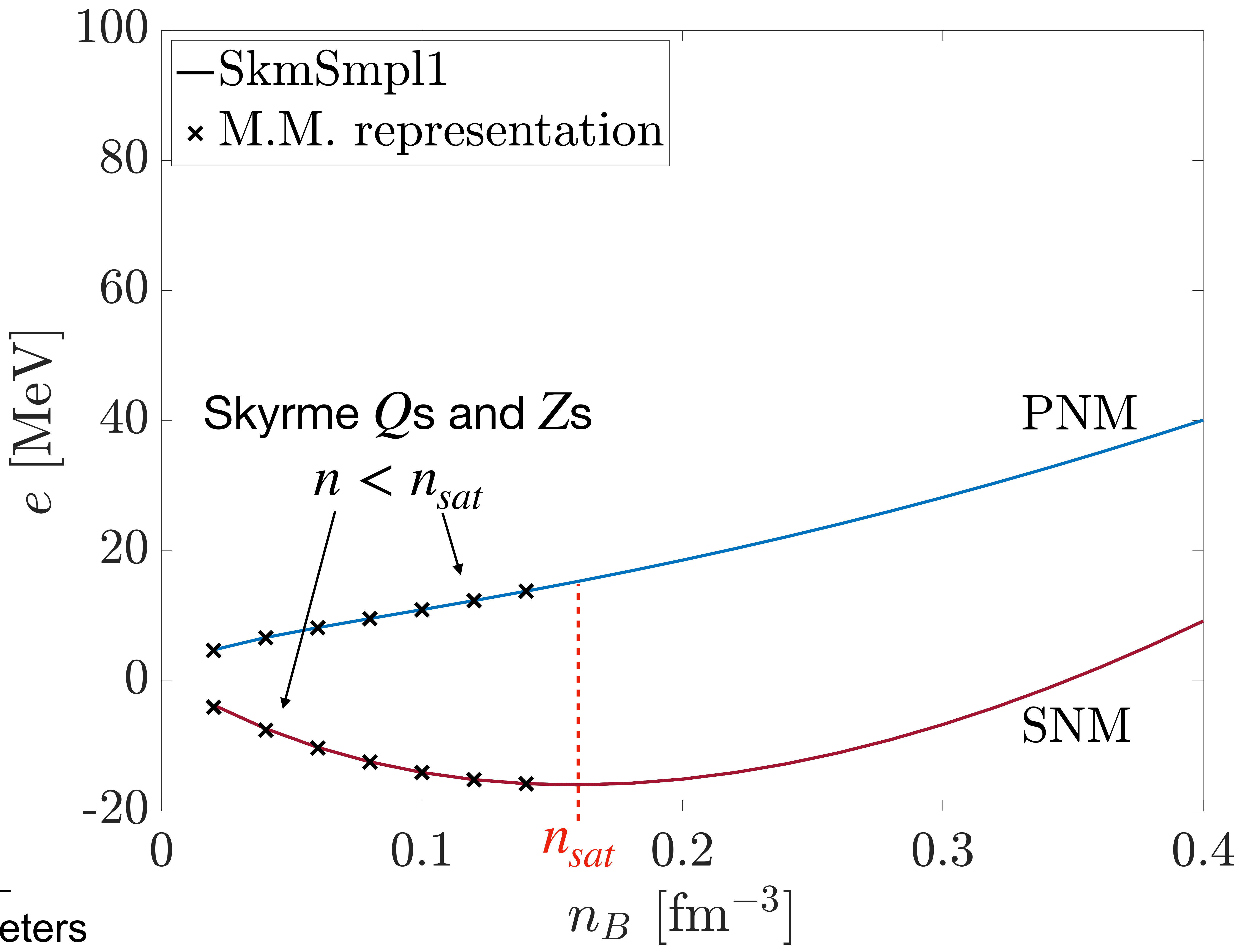
$m_0^*/m, m_1^*/m$



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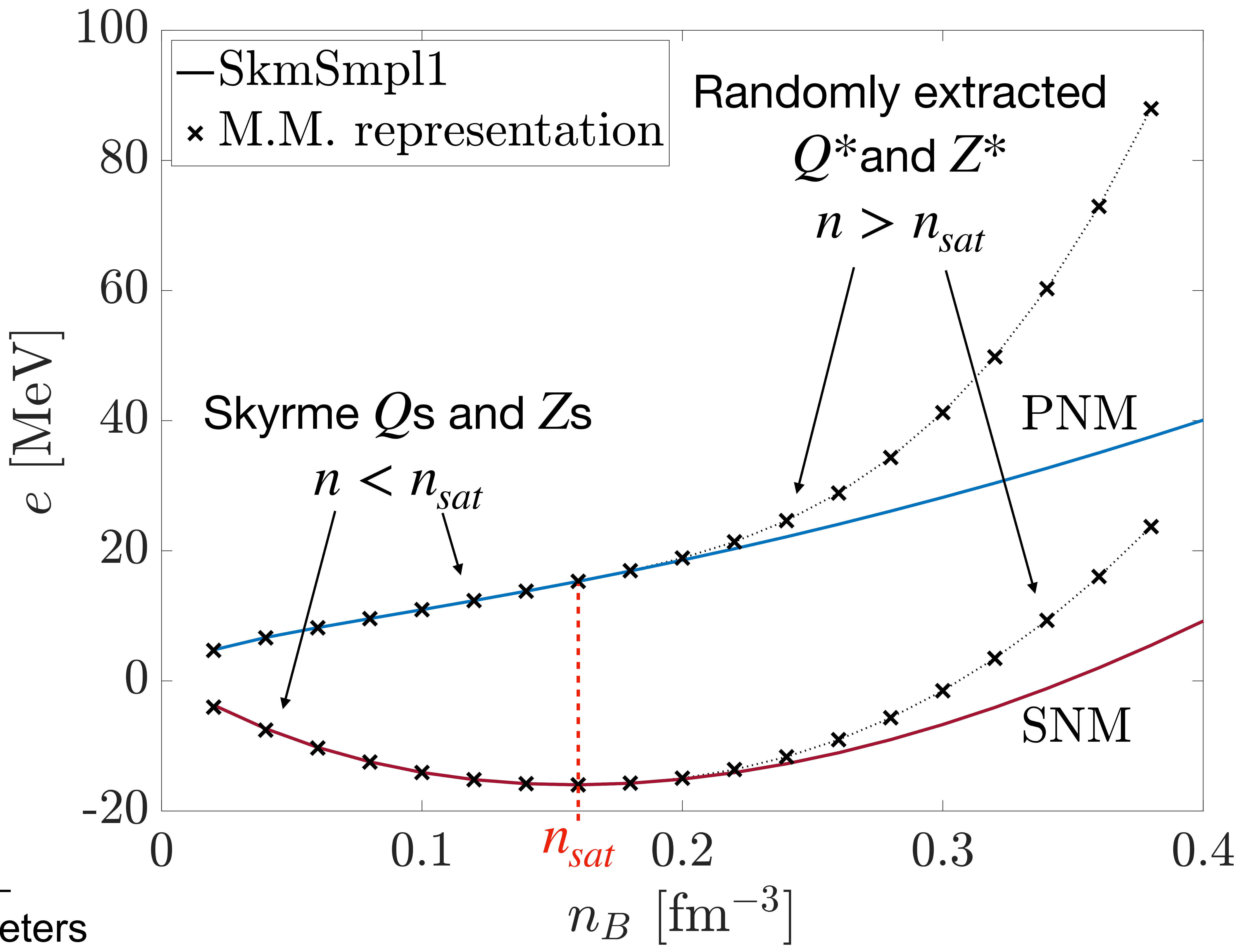
$m_0^*/m, m_1^*/m$



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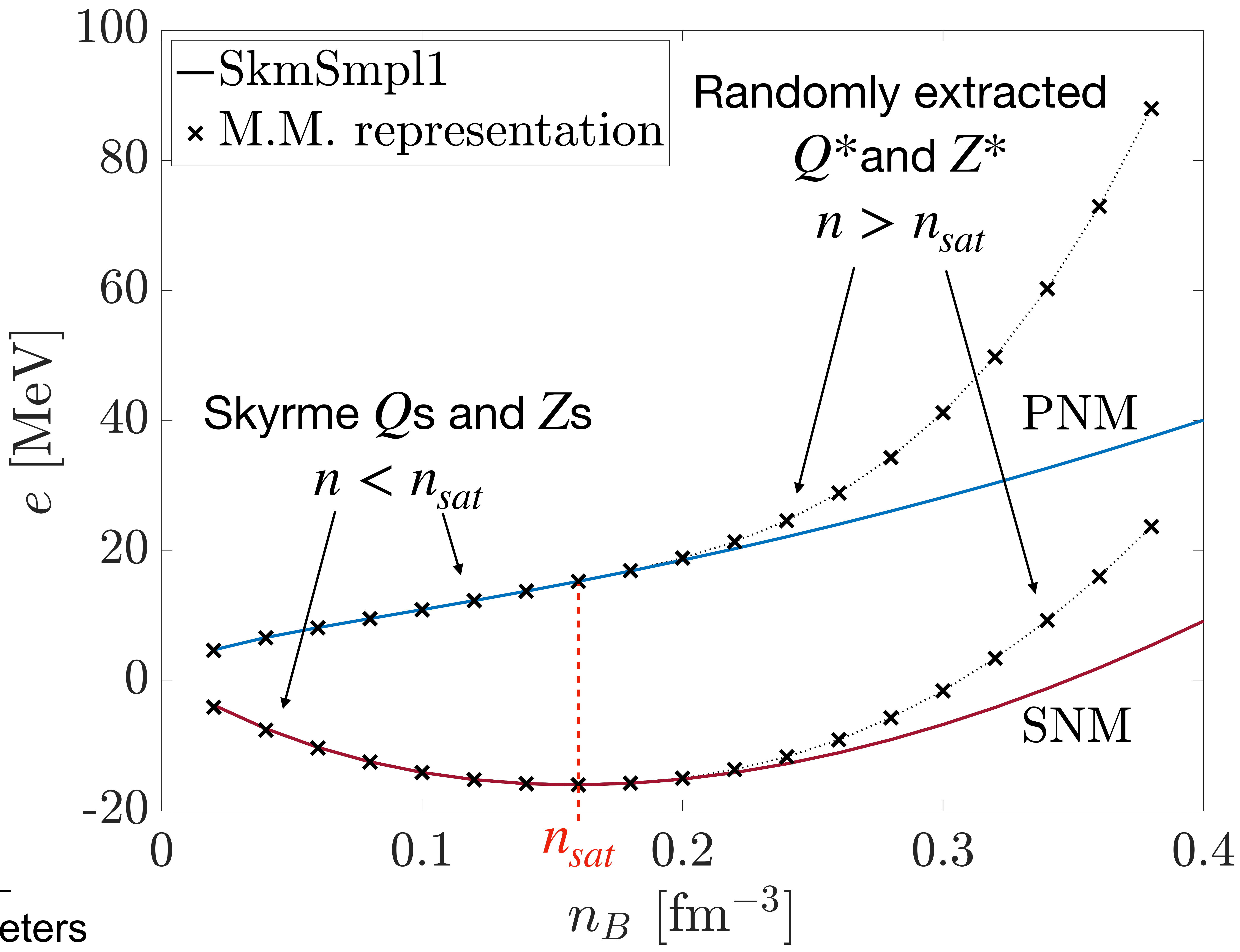

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Compute NS properties

[1]Margueron et al., Phys. Rev. C **97**, 025805 (2018)

Crust composition in simplified ETF

Woods-Saxons's profiles

$$\varepsilon_{WS} = \frac{1}{V_{WS}} \int_0^{R_{WS}} dr\, r^2 \varepsilon_{ETF}^{Sky}(n(r), n_p(r)) + \varepsilon_{e^-}$$

Convergence problems at $n_B \sim 0.06\,\mathrm{fm}^{-3}$

Linear extrapolation of $\Delta\varepsilon$:

$$\Delta\varepsilon := \varepsilon_{WS} - \varepsilon_{hom} \rightarrow \ \Delta\varepsilon(n_{cc}) = 0$$

Approximation checked against:

- CLDM crust
- Dynamical spinodal instability

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Bayesian setup: prior and constraints

Prior distribution

E_{sat}	[MeV]	*
n_{sat}	[fm^{-3}]	*
K_{sat}	[MeV]	*
Q_{sat}	[MeV]	[-2000,2000]
Z_{sat}	[MeV]	[-3000,3000]
E_{sym}	[MeV]	*
L_{sym}	[MeV]	*
Q_{sym}	[MeV]	[-4000,4000]
Z_{sym}	[MeV]	[-5000,5000]
m^*_{IS}	[-]	*
m^*_{IV}	[-]	*
$w0$	[MeV fm^5]	*
G_0	[MeV fm^5]	*
G_1	[MeV fm^5]	*

Observational constraints

- Maximum mass of Neutron Star ($\mathscr{L}_{\mathrm{J0348}}$);
- Tidal deformability results ($\mathscr{L}_{\mathrm{LVC}}$);
- NICER mission mass-radius measurements ($\mathscr{L}_{\mathrm{NICER}}$);
- $\chi-$EFT computations of PNM at low density ($\mathscr{L}_{\chi}$).

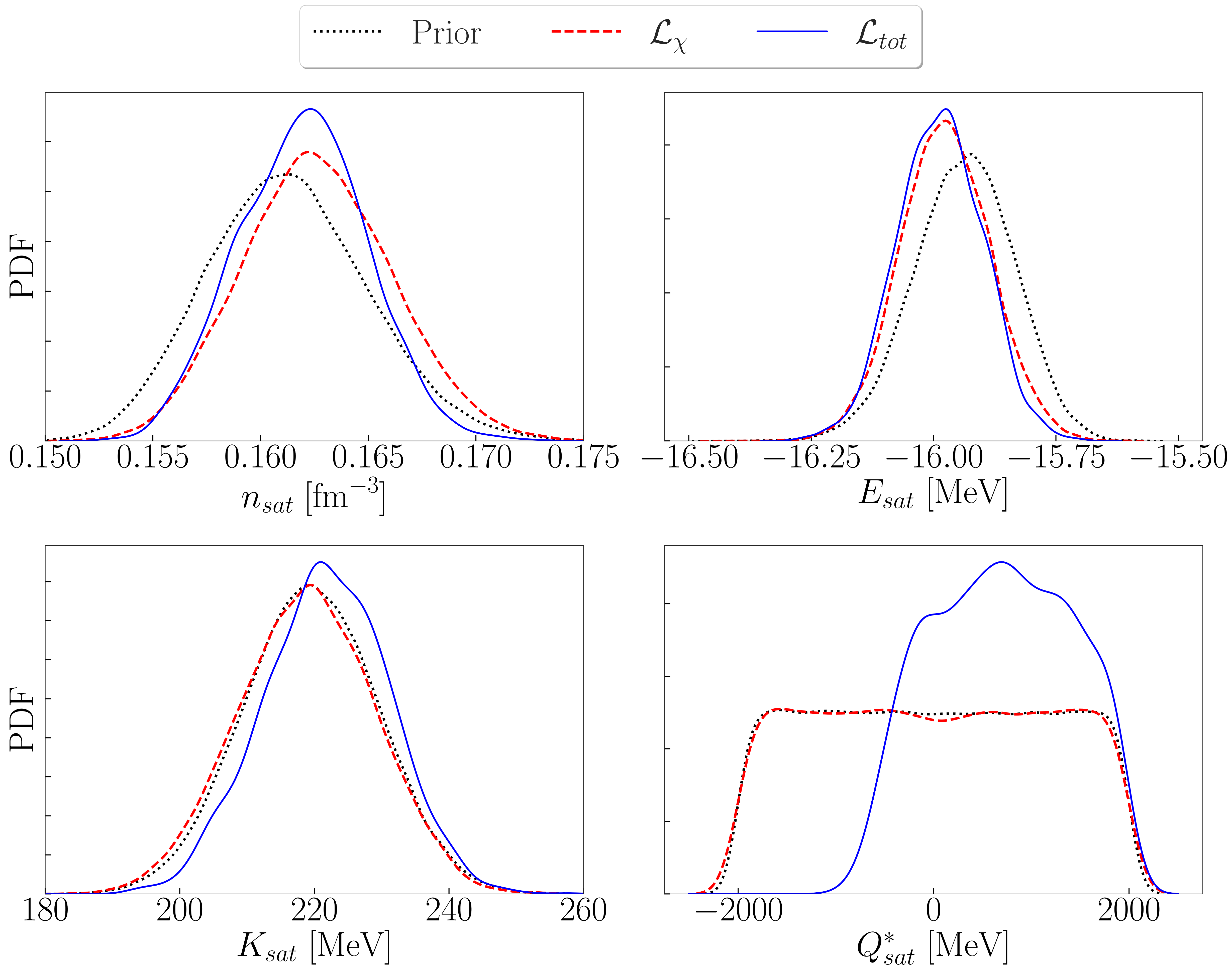
$$\mathscr{L}_{tot} = \prod_i \mathscr{L}_i$$

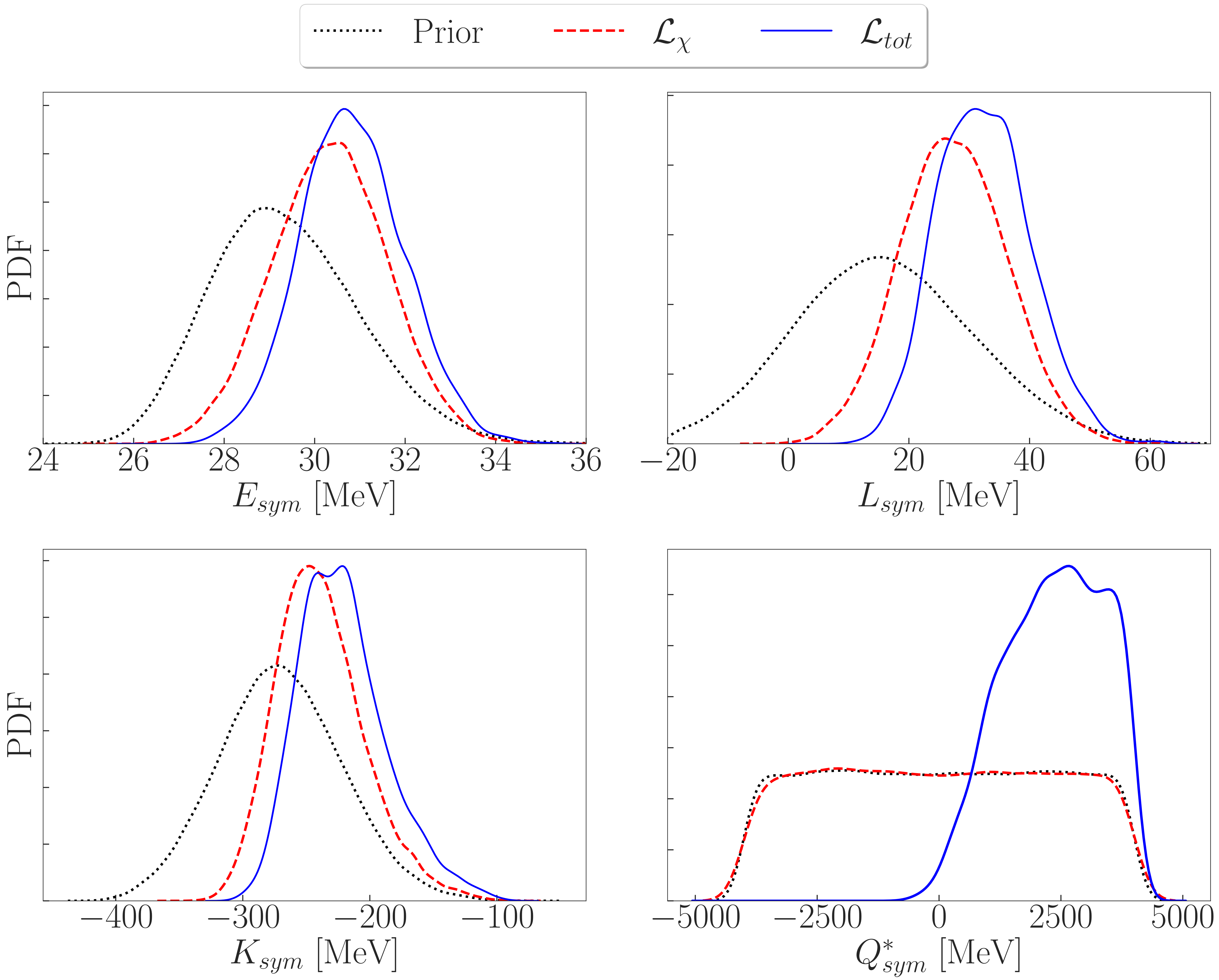
Prior distribution weighted with $\mathscr{L}_{tot}$

$\downarrow$

Posterior distribution

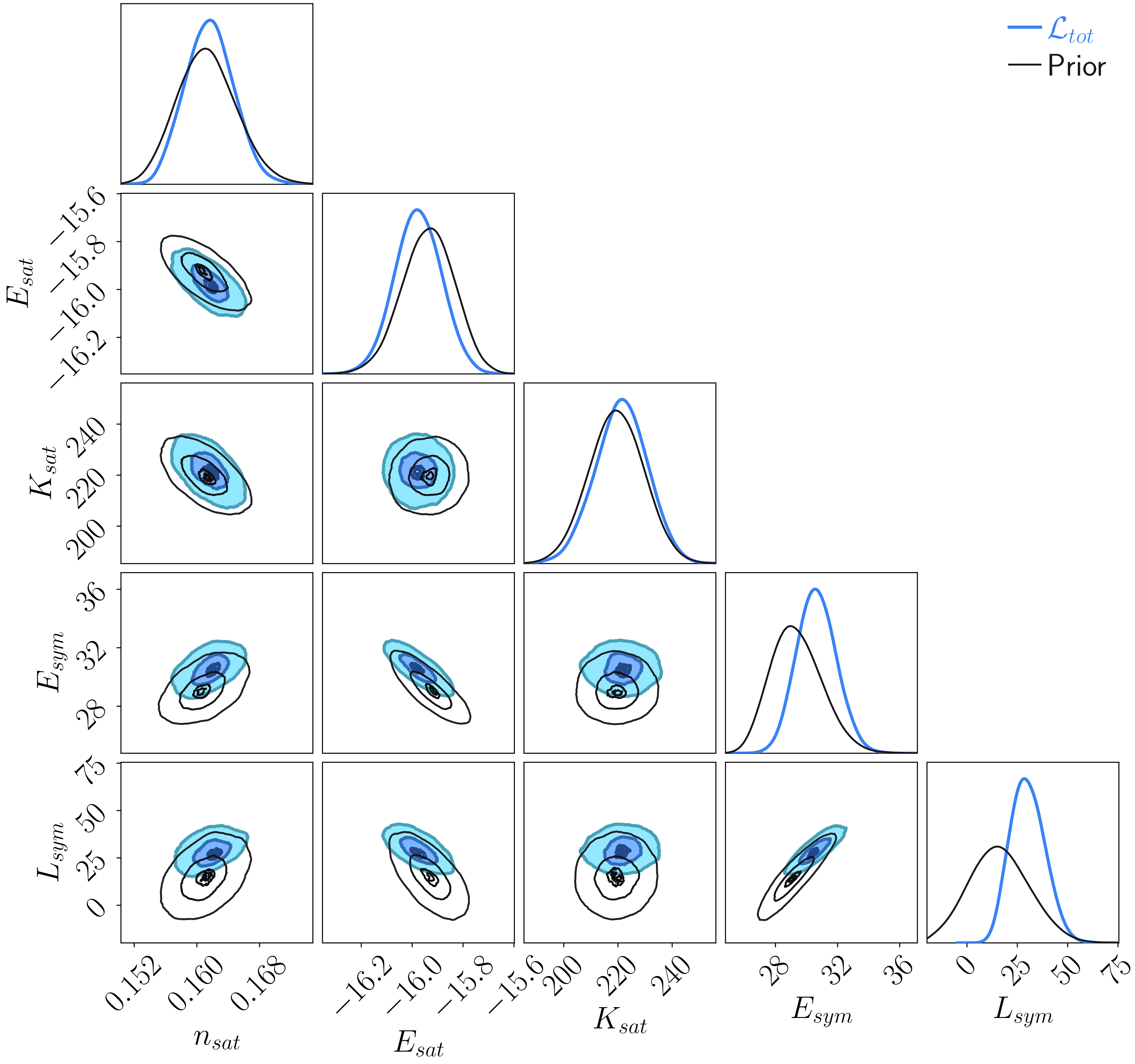
Marginalized Parameters Posterior Distributions

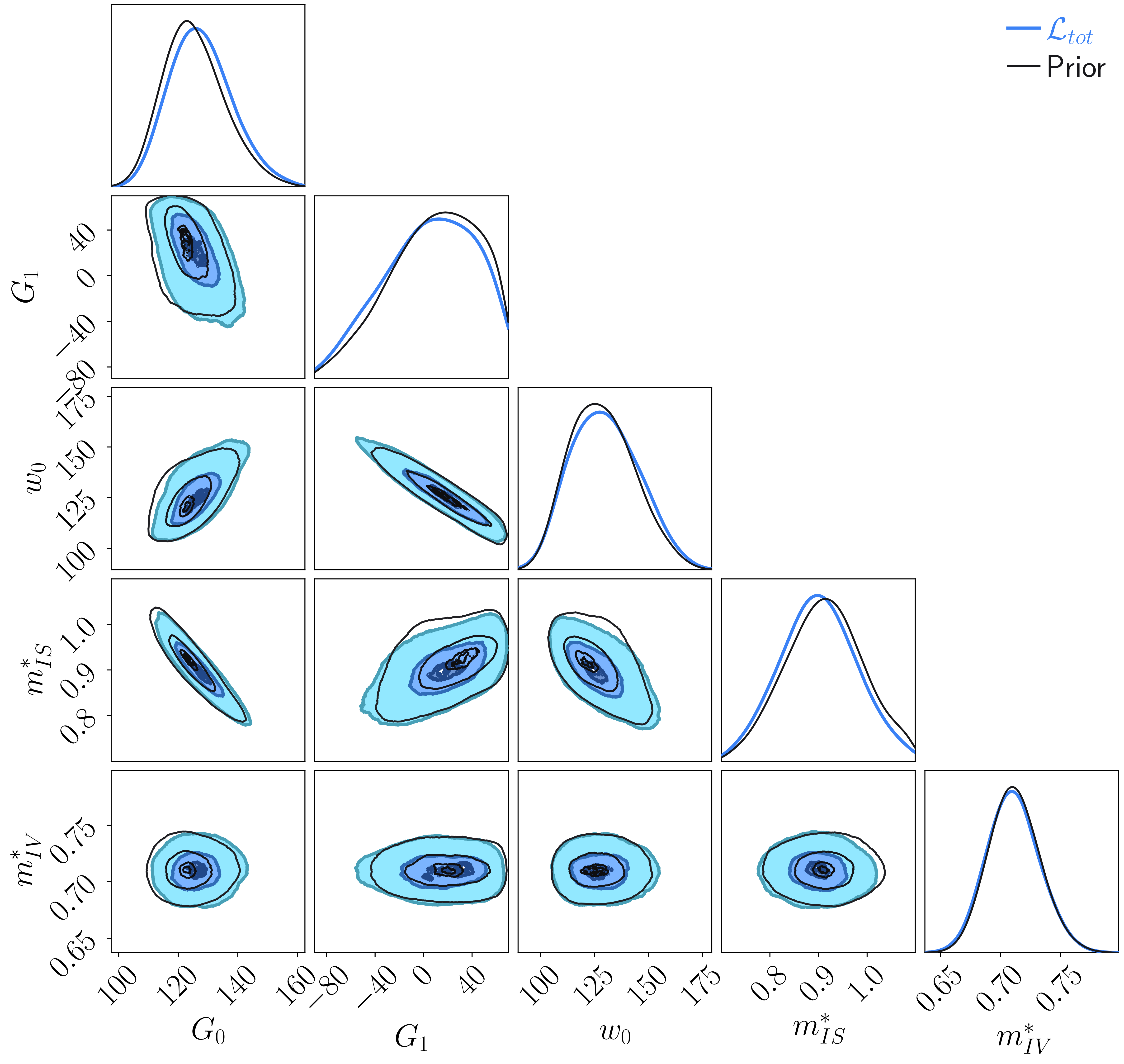




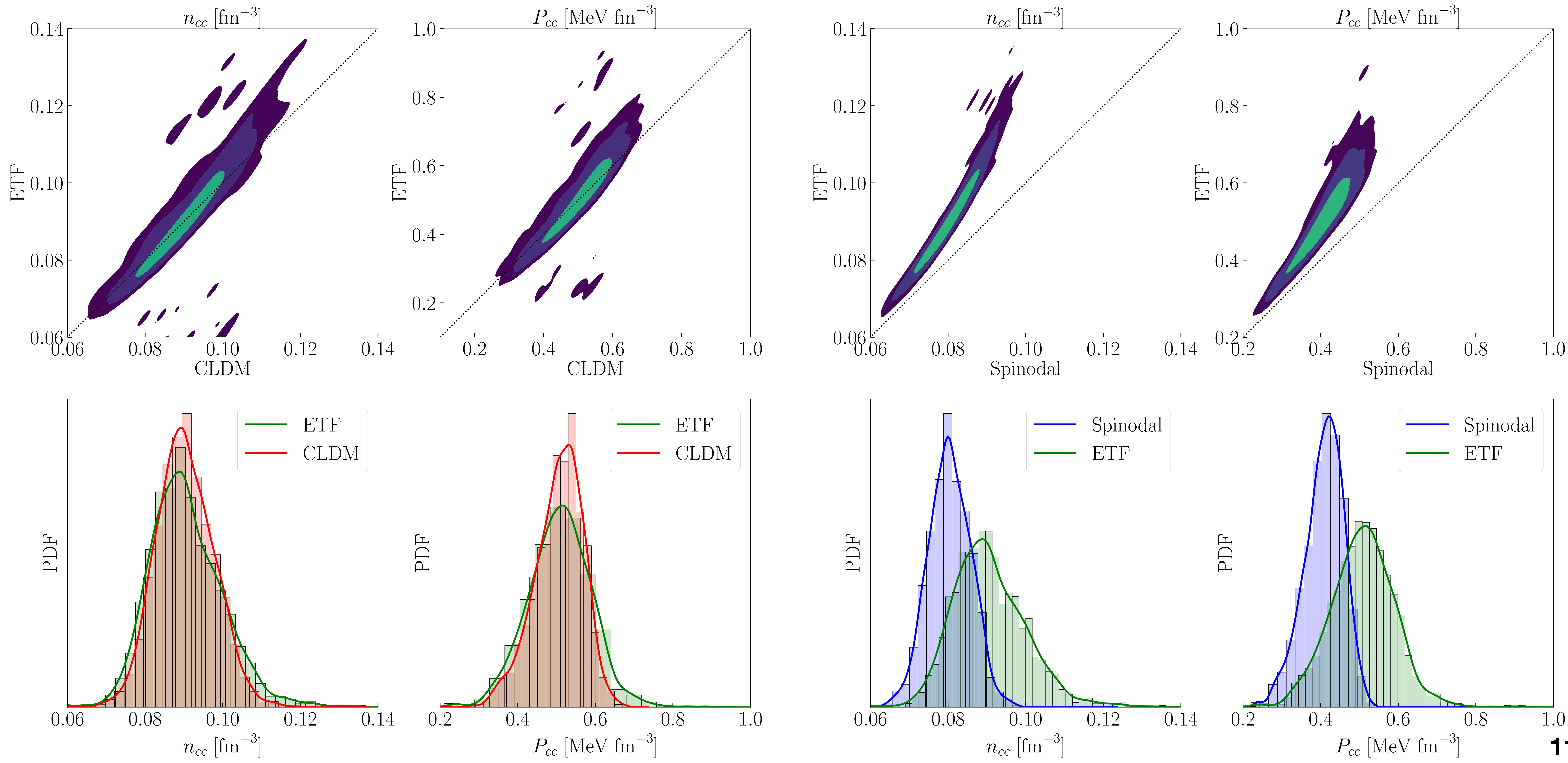
Marginalized Parameters Posterior Distributions

Corner plots

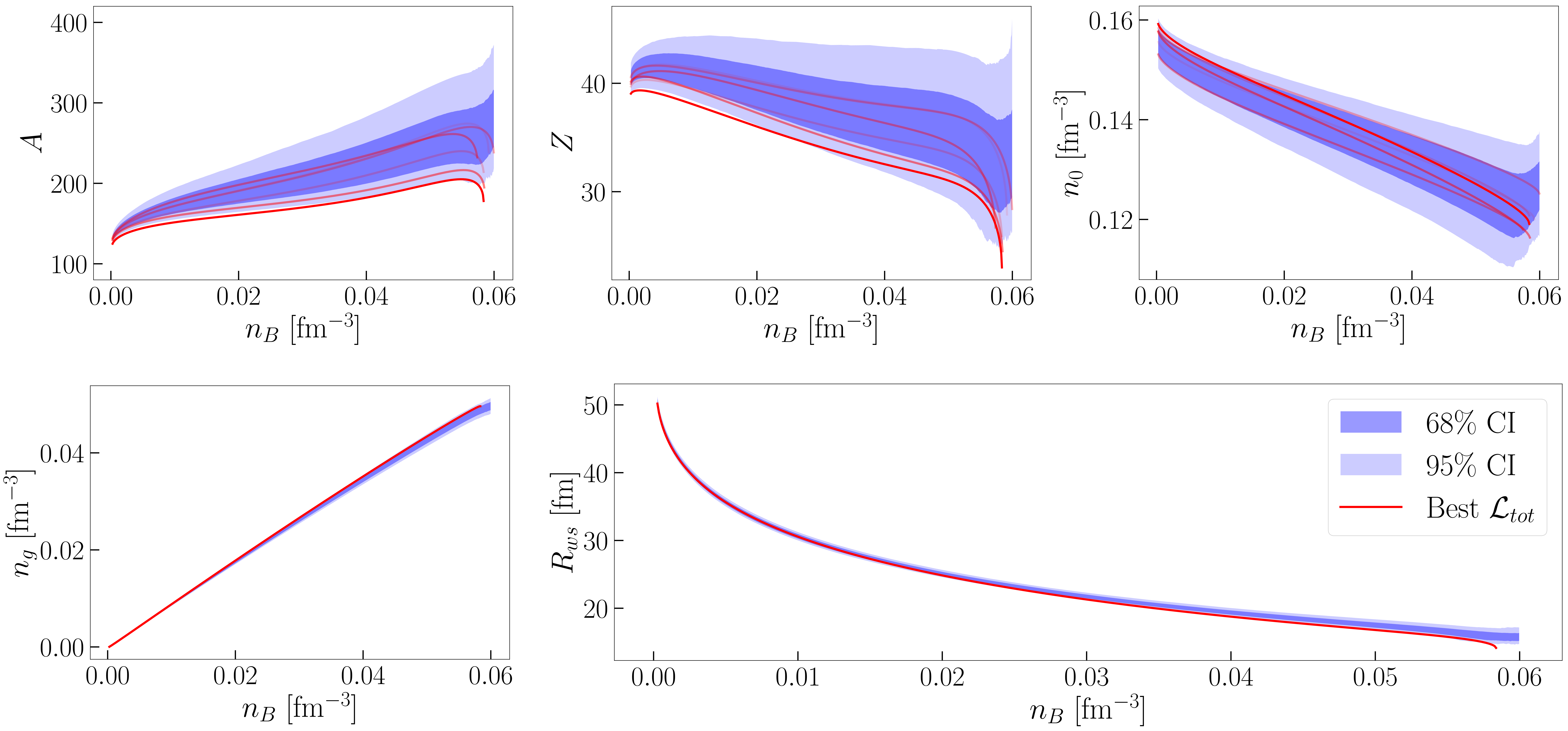




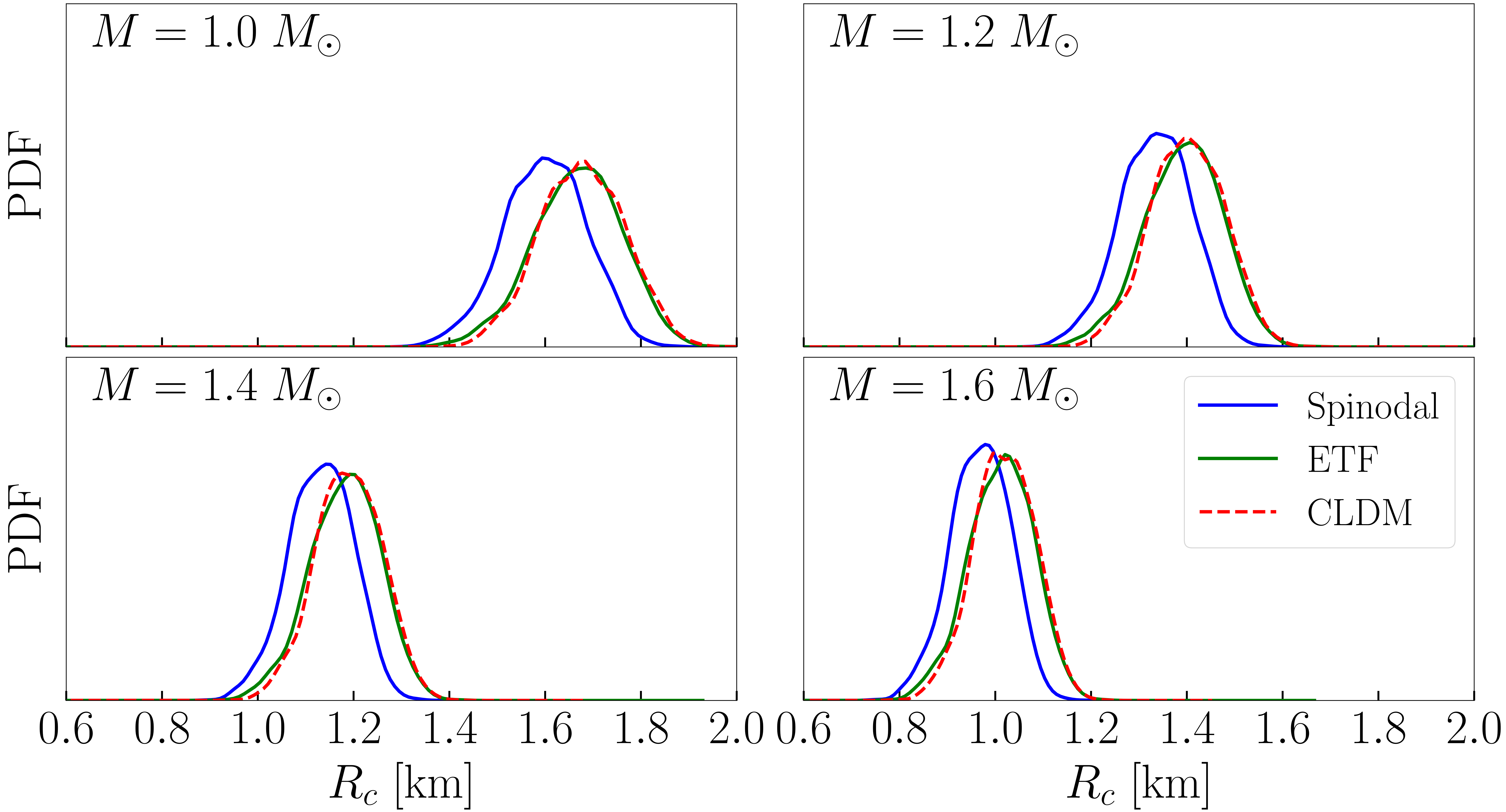
n_{cc} and P_{cc}: ETF, CLDM and Spinodal instability



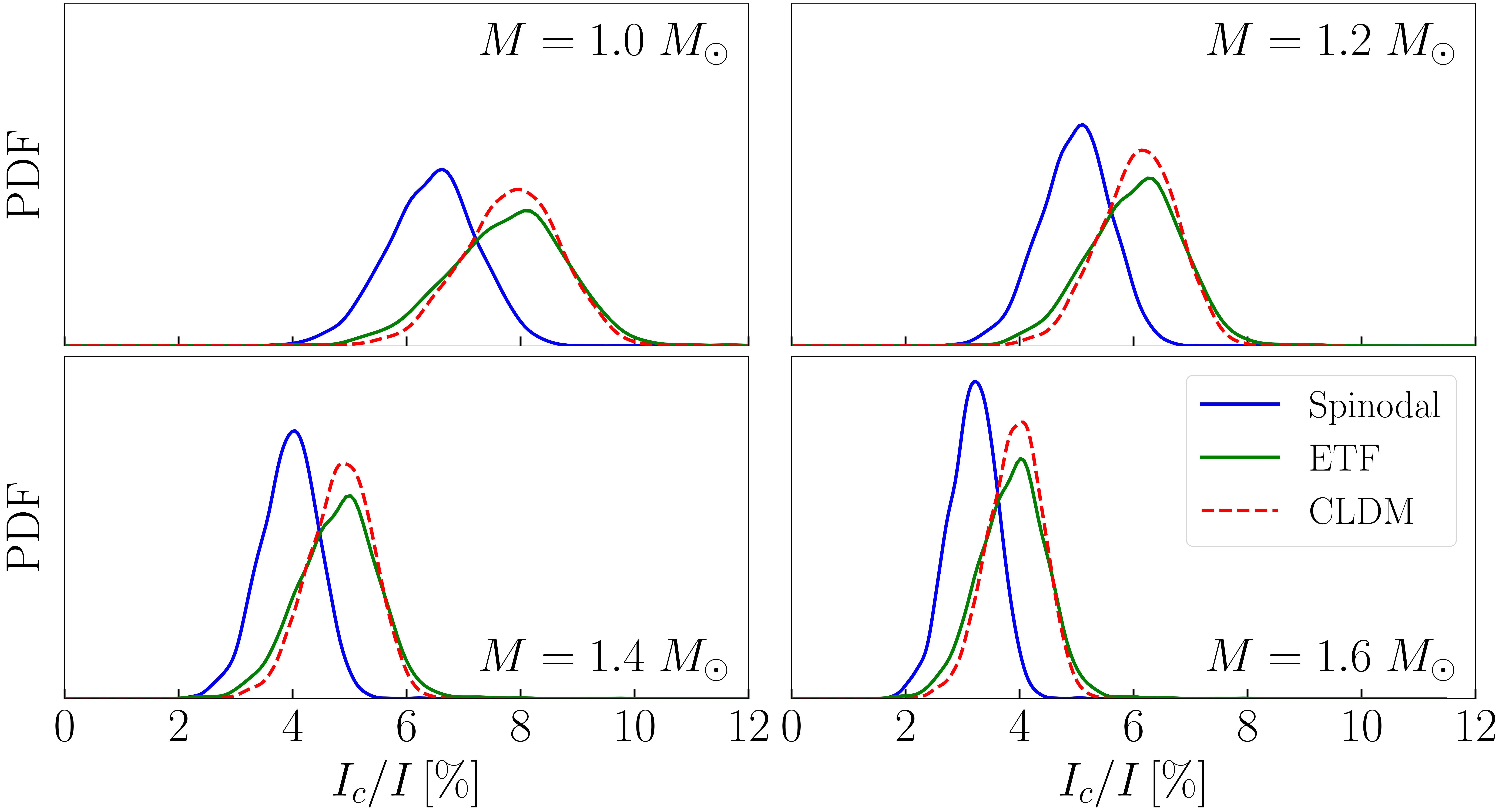
Crust composition



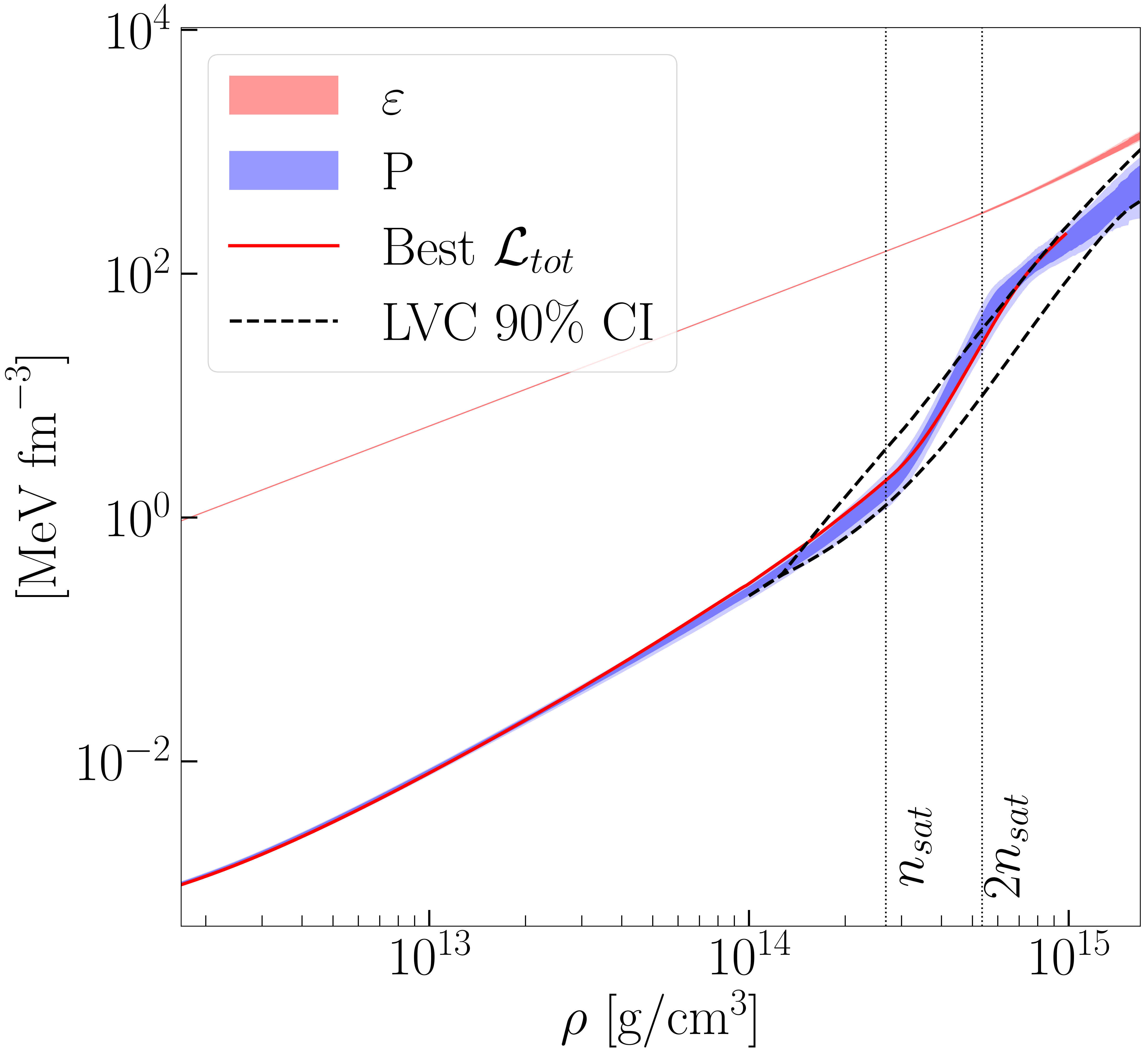
Crust radius

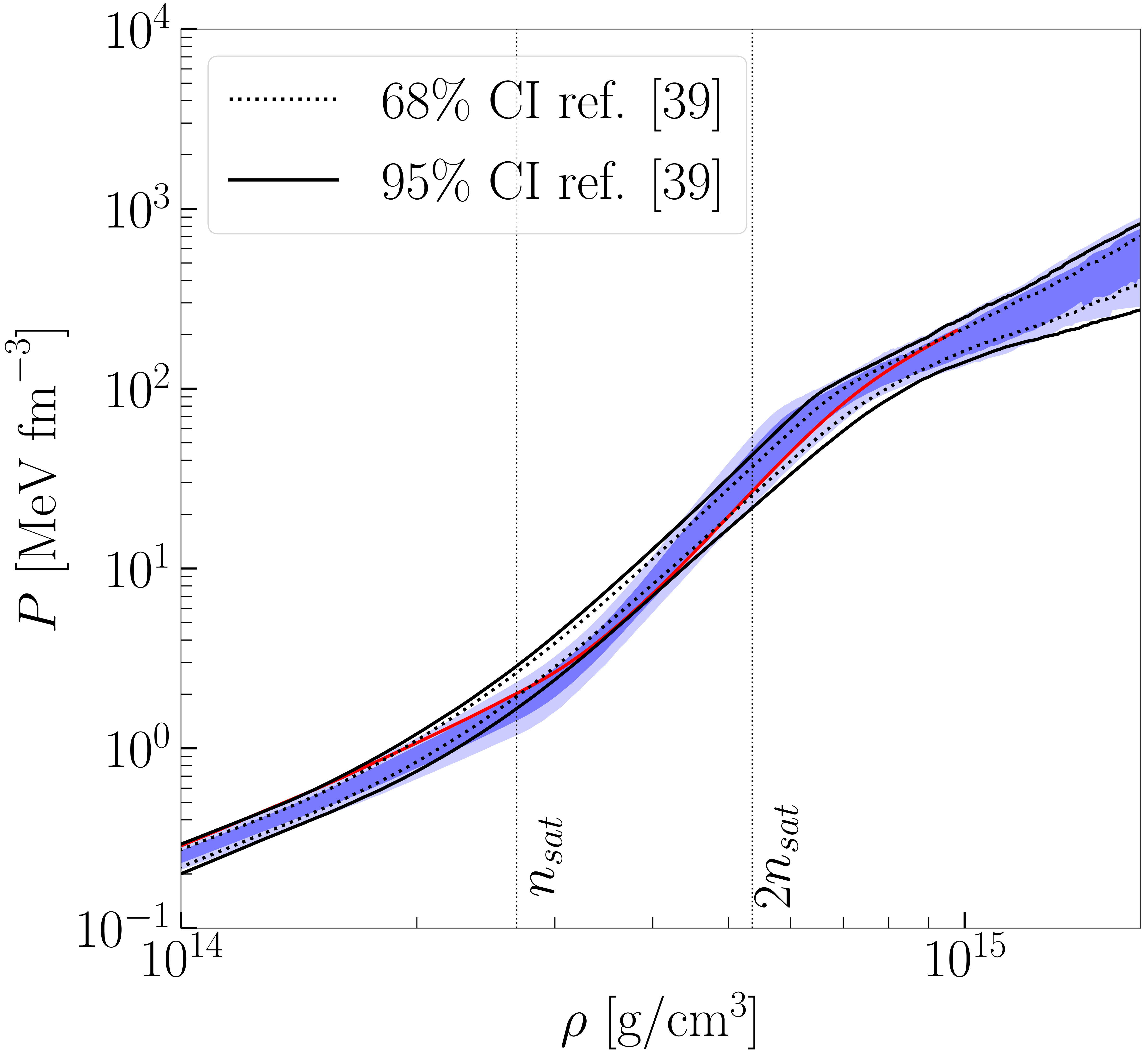


Crust moment of inertia



EoS and symmetry energy





[39] Dinh Thi et al., Universe **2021**, 7, 373

Summary

- Bayesian statistical analysis on nuclear matter parameters with neutron star observations:
  - Prior consistent with nuclear physics experiments, not just a sample of independent nuclear matter parameters
  - NS EoS computed with MM; inner crust with ETF
  - Prediction on different NS properties
  - Effect on structure of the P between n_{sat} and $2n_{sat}$ due to nuclear informed prior

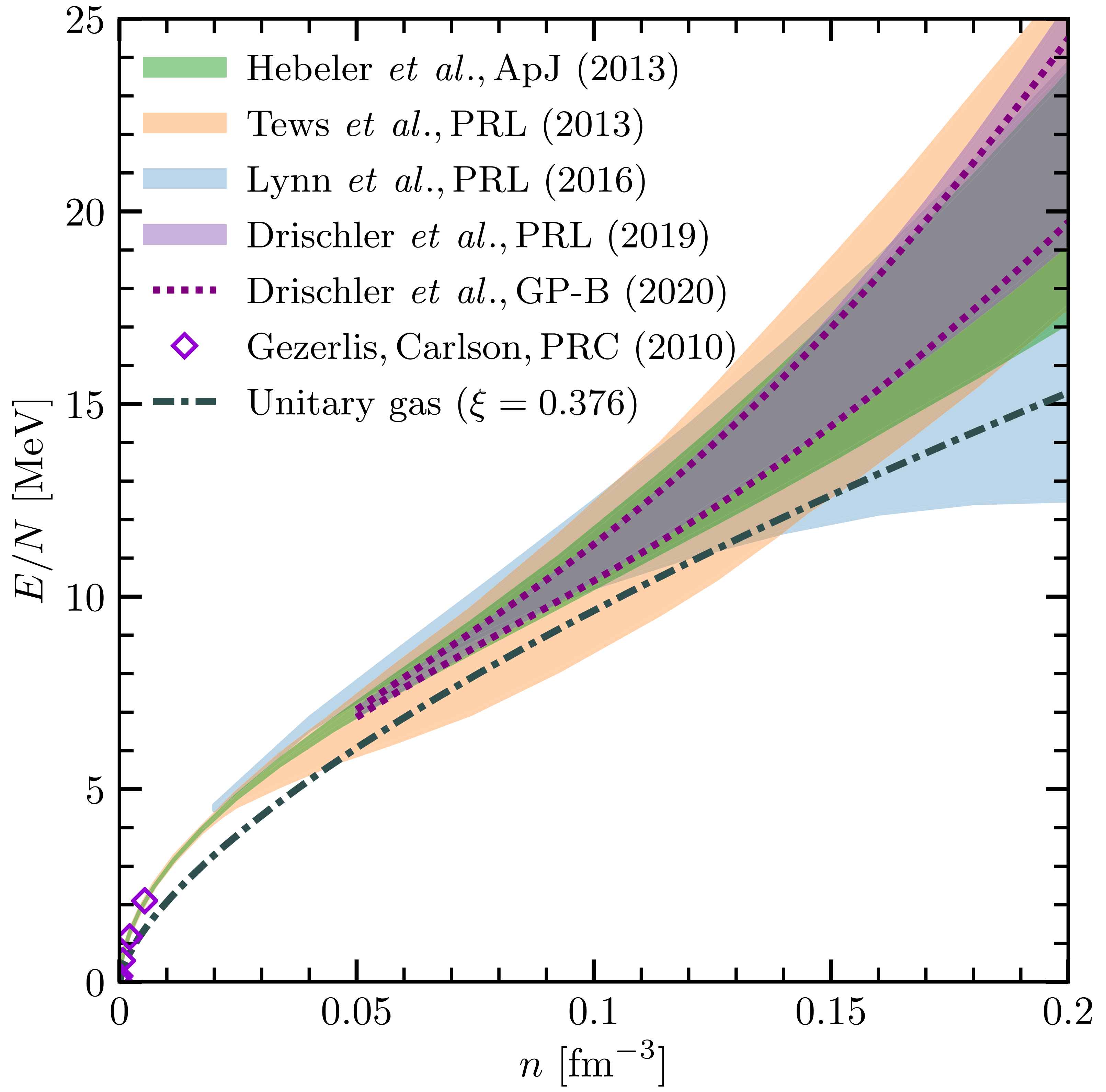
BACKGROUND

Likelihoods

$\mathscr{L}_\chi$:

$$p(e \mid n) = \begin{cases} \exp\left(-\frac{\left(e - e_-(n)\right)^2}{2\sigma_n^2}\right) & \textbf{if } e \in \left(-\infty, e_-(n)\right] \\ 1 & \textbf{if } e \in \left(e_-(n), e_+(n)\right] \\ \exp\left(-\frac{\left(e - e_+(n)\right)^2}{2\sigma_n^2}\right) & \textbf{if } e \in \left(e_+(n), \infty\right) \end{cases}$$

$$\sigma_n = \frac{e_+(n) - e_-(n)}{9\sqrt{2\pi}}$$

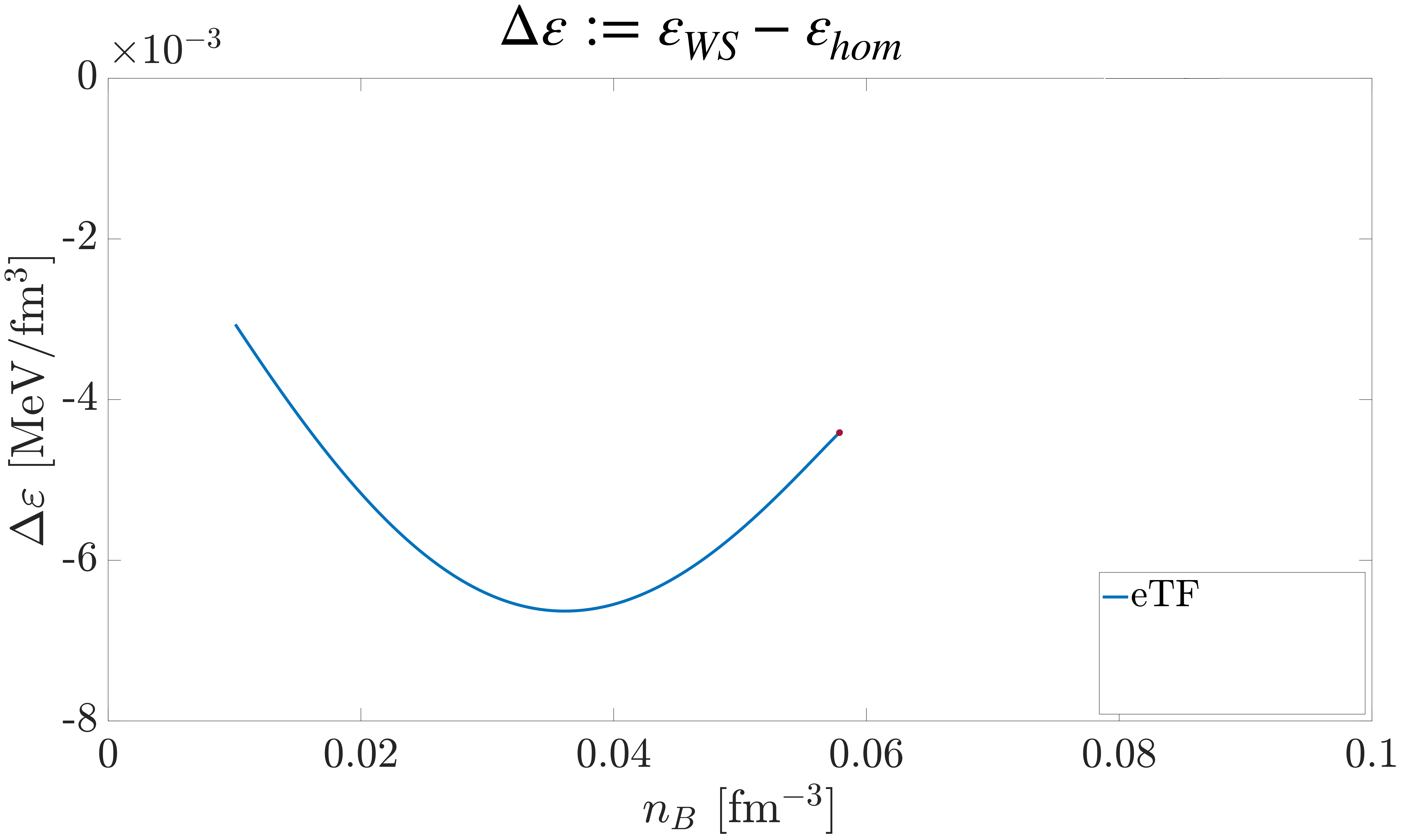


Phys. Rev. C **103**, 025803

$$\mathscr{L}_{NICER} : \int dM\, P_{N19}(M, R(M)) \cdot \int dM\, P_{N21}(M, R(M))$$

$$\mathscr{L}_{LVC} : \int dq\, P(\tilde{\Lambda}(q, m_c), q) \qquad \mathscr{L}_{J0348} : \frac{1}{\sqrt{2\pi}\,\sigma} \int_0^{M_{max}/M_\odot} dx\, \exp\left(-\frac{(x - 2.01)^2}{2\sigma^2}\right)$$

Algorithm breaks at $n_B \sim 0.06\ fm^{-3}$ ...



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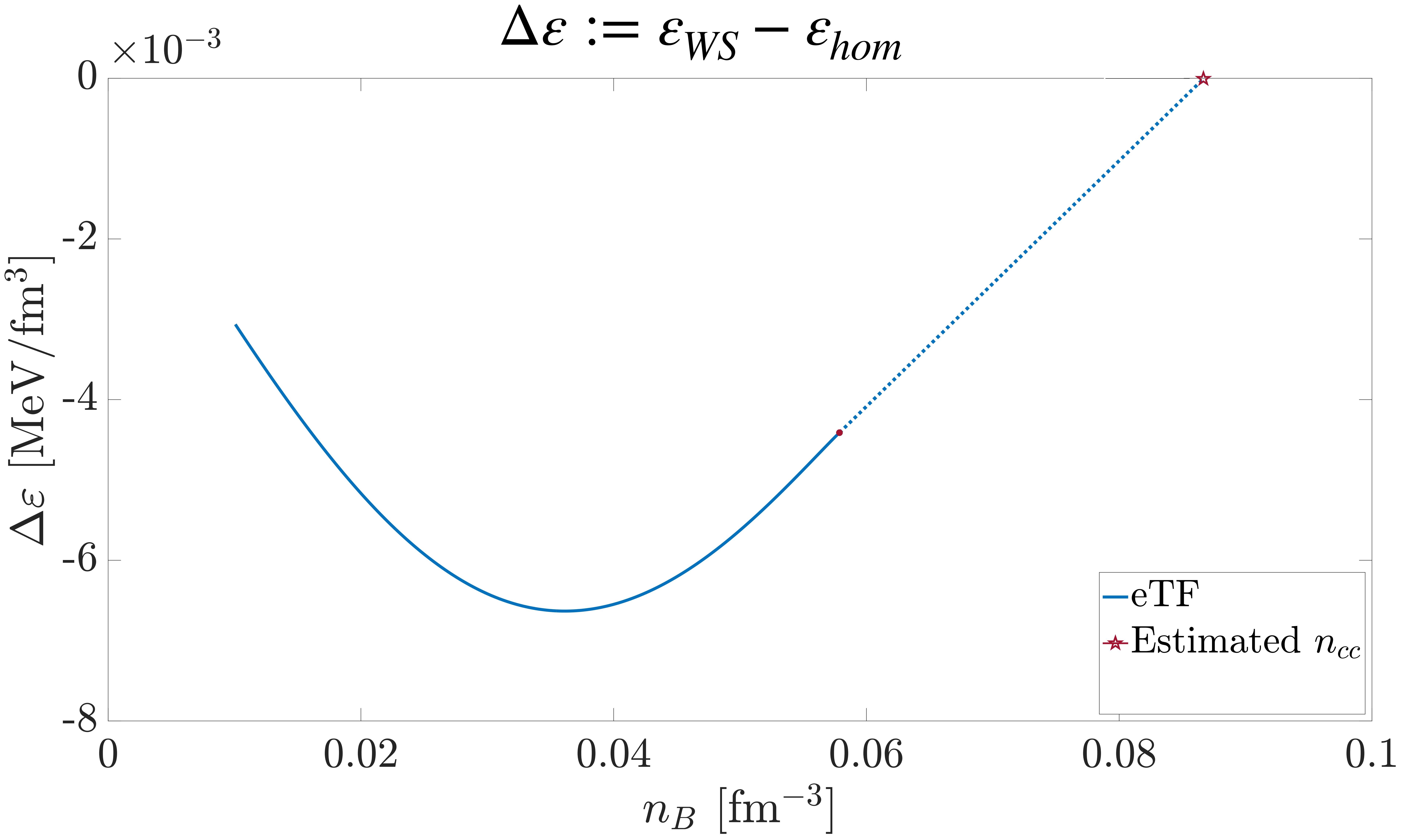

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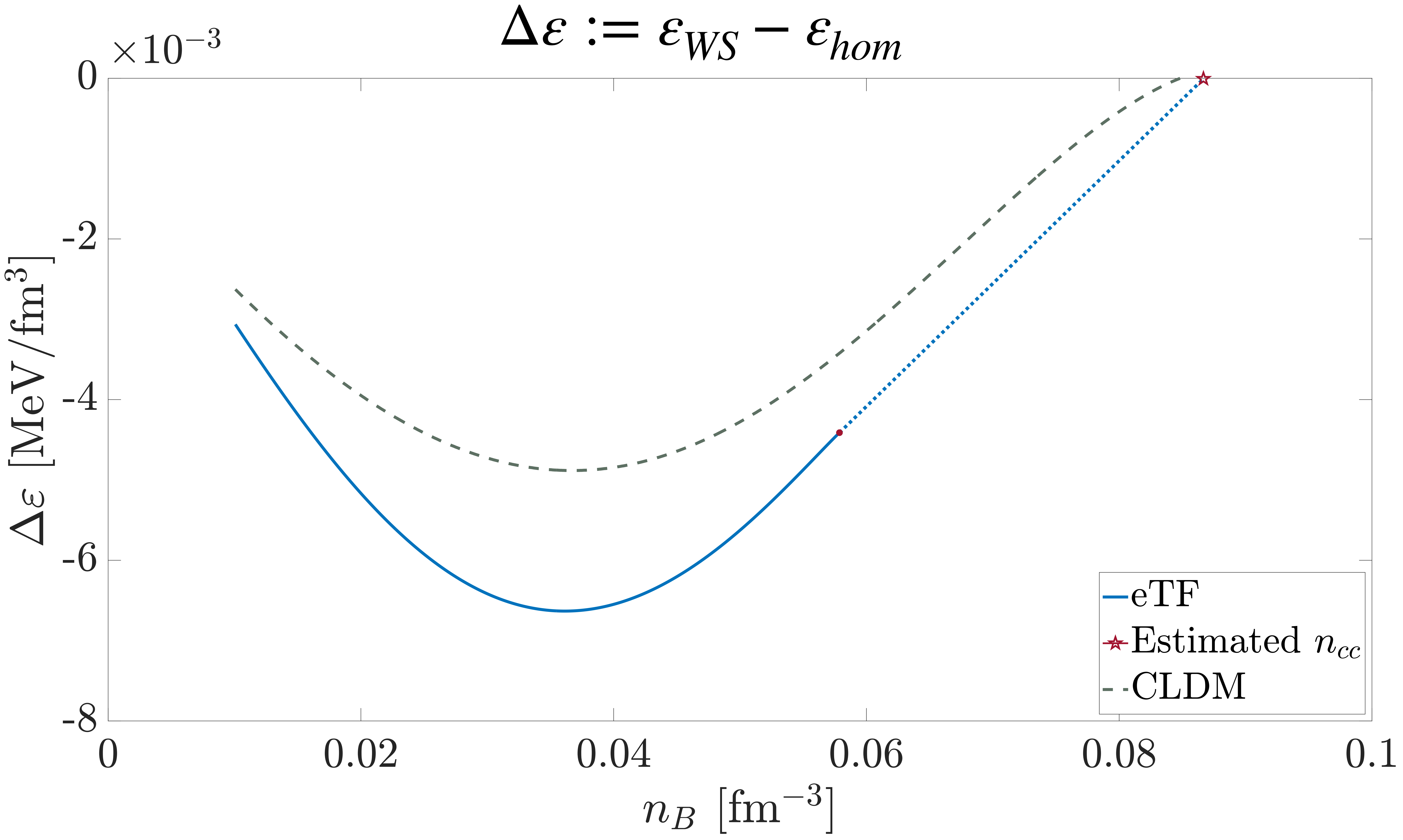
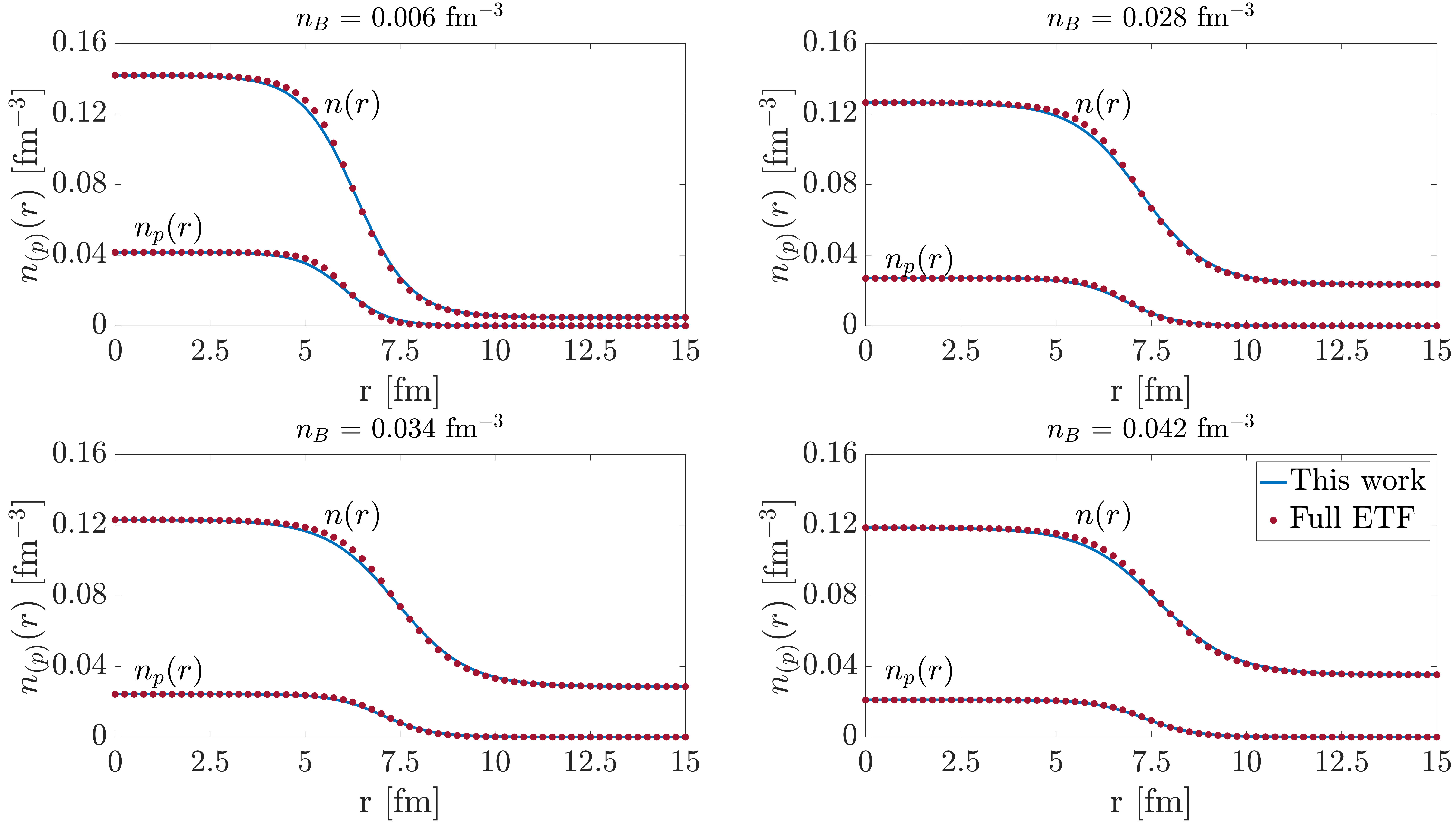
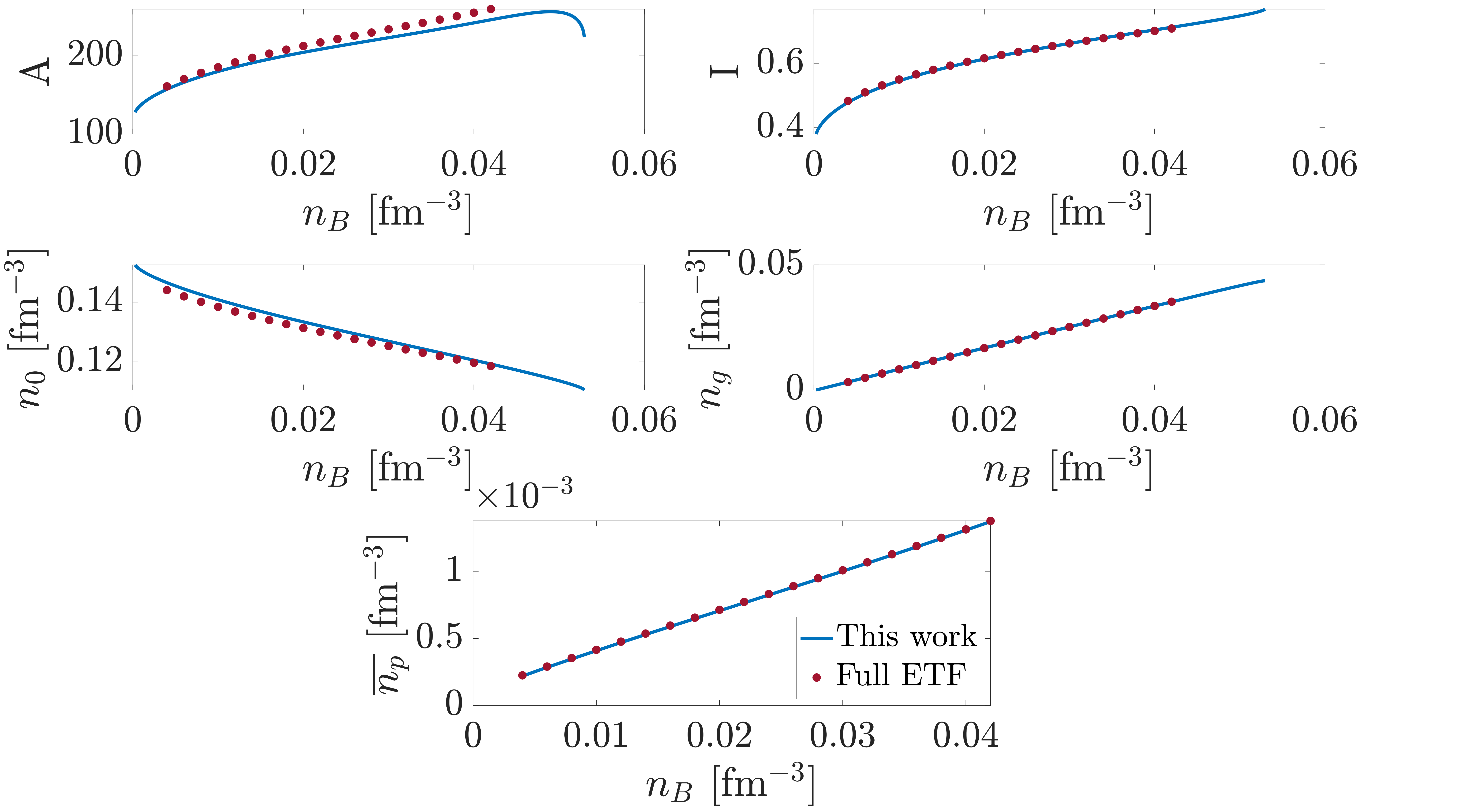


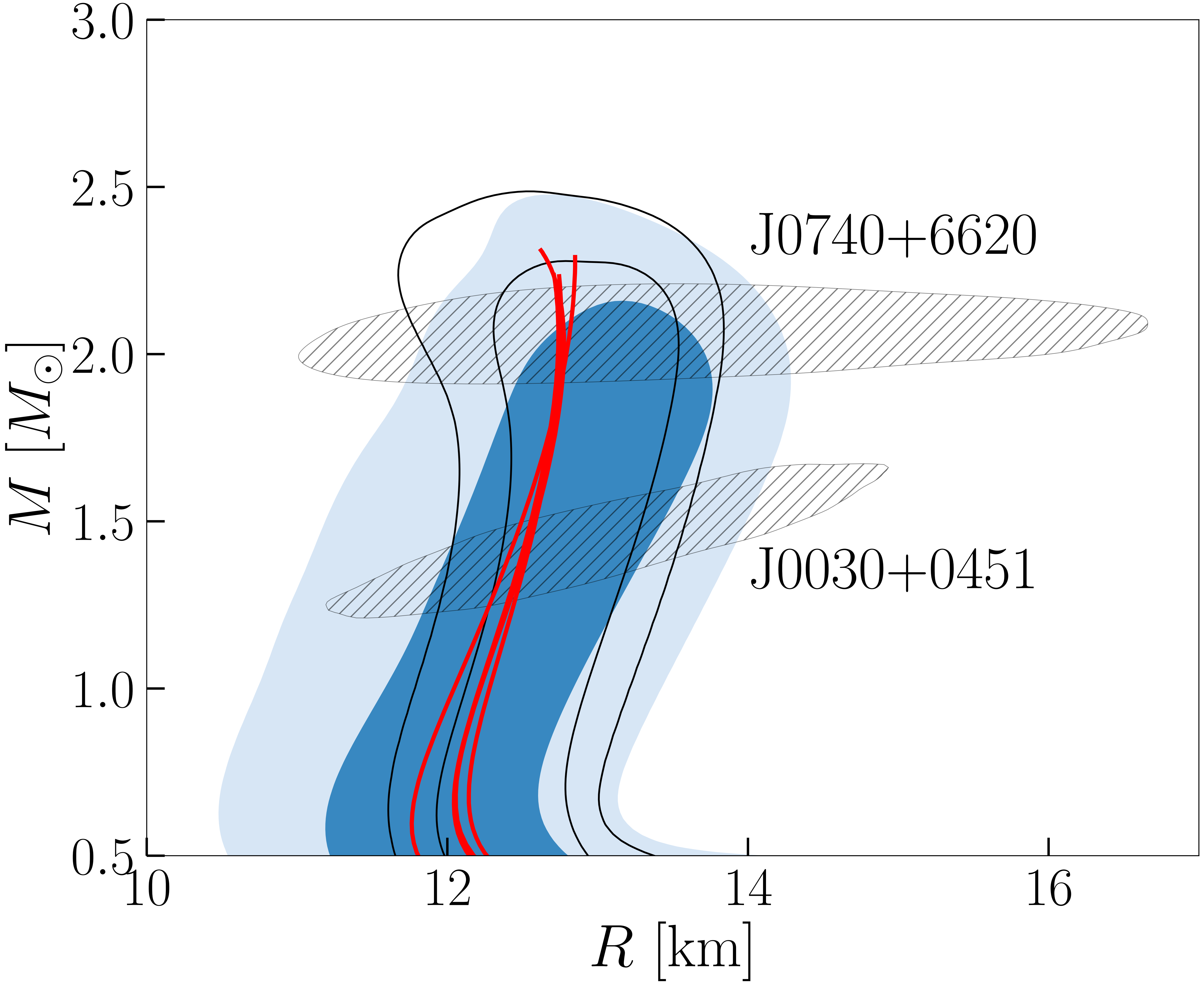
ETF checks (1): density profiles



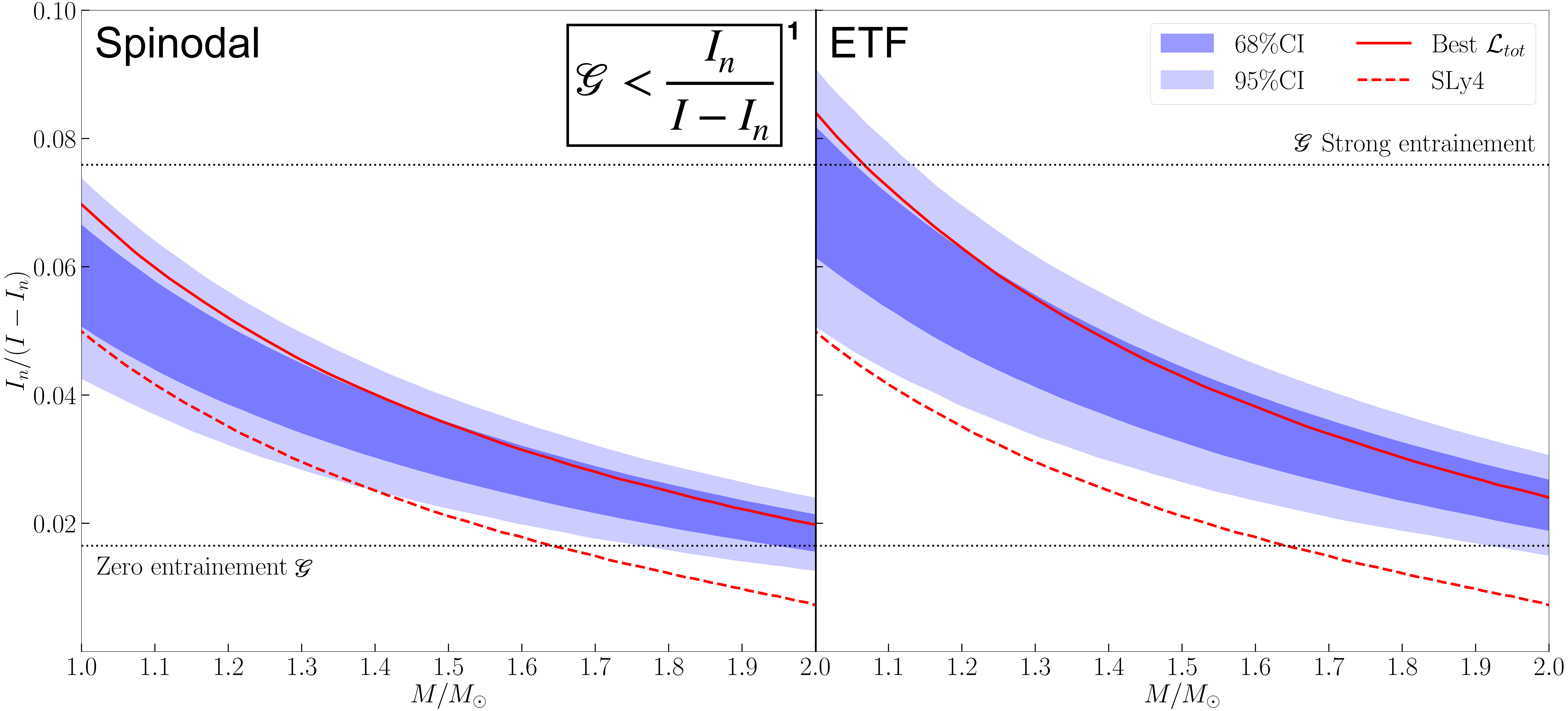
ETF checks (2): crust composition



Mass-Radius plot



Glitch activity $\mathscr{G}$ of Vela



[1] Montoli et al., Universe **2021**, 7, 8