



Oscillation Modes of Neutron Stars in Binary Inspirals: Dynamical Tides and Gravitational Waves

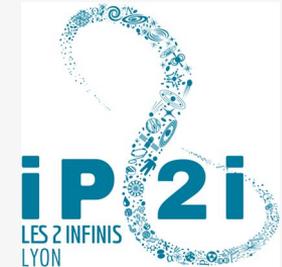
Bikram Keshari Pradhan

In Collaboration with Tathagata Ghosh, Dhruv Pathak and Prof. Debarati Chatterjee

Inter-University Centre for Astronomy and Astrophysics (IUCAA), Pune

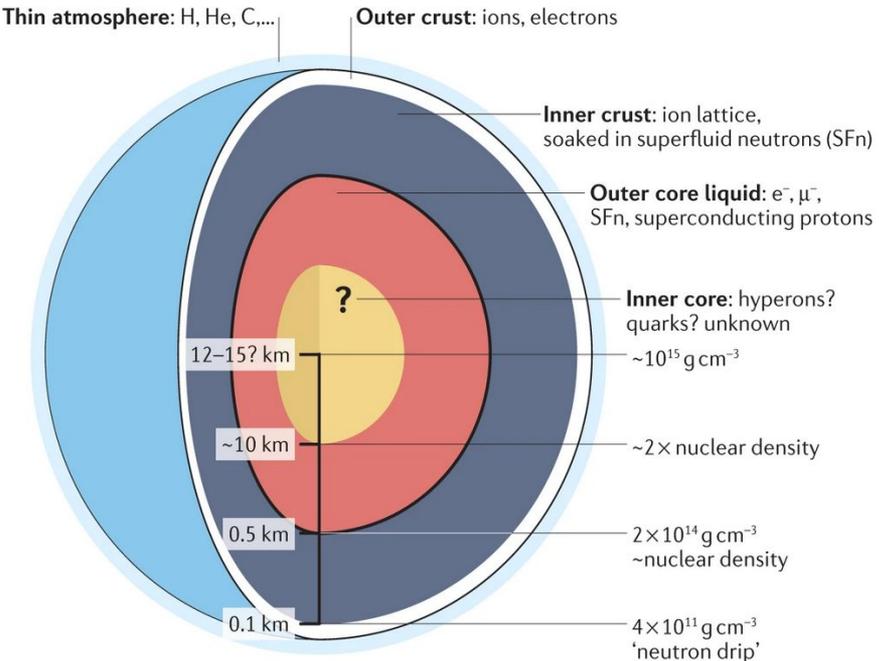
❖ [Astrophys.J. 966 \(2024\) 1, 79.](#)

Modélisation des Astres Compacts - Caen 2025

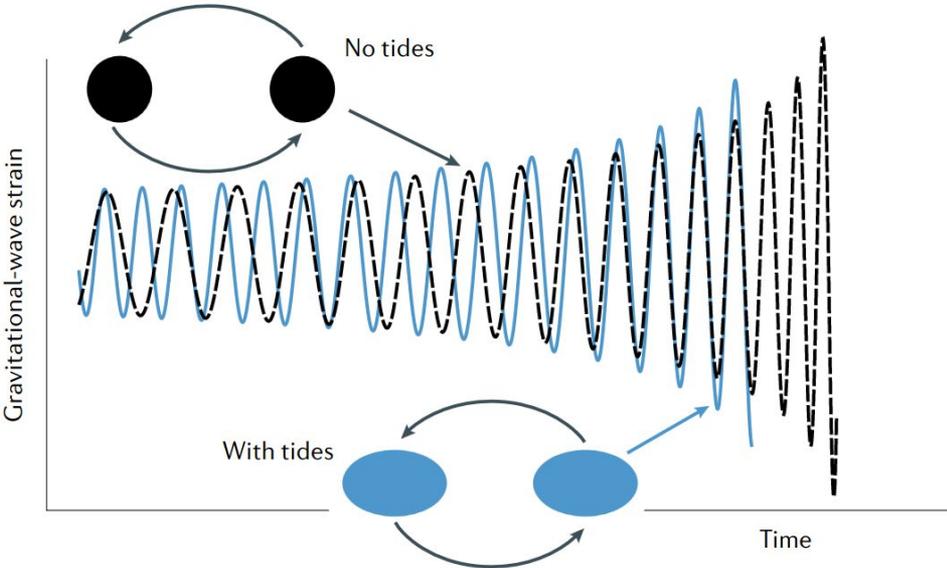


Introduction: Neutron Stars and GW

- Observed with electromagnetic, GW detectors.
- Binary GW events: GW170817, GW190425, GW190814



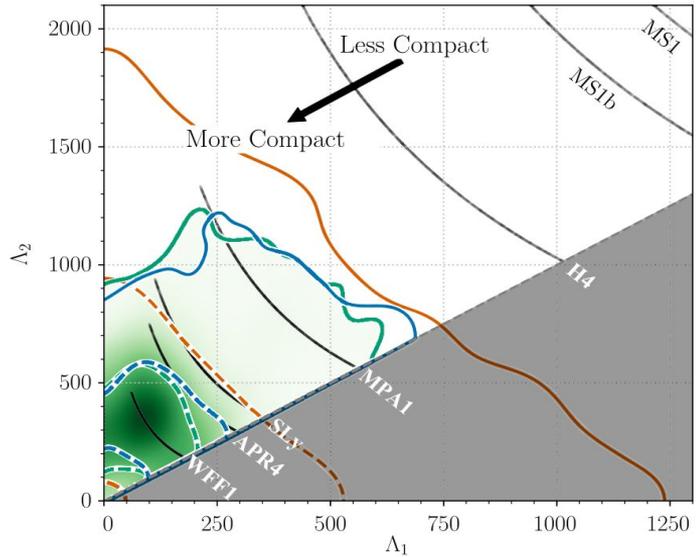
The figure illustrates the thin atmosphere, the outer and inner crust, and the outer and inner core, with the respective densities at different depths. Adapted with permission from NASA, NICER Team.



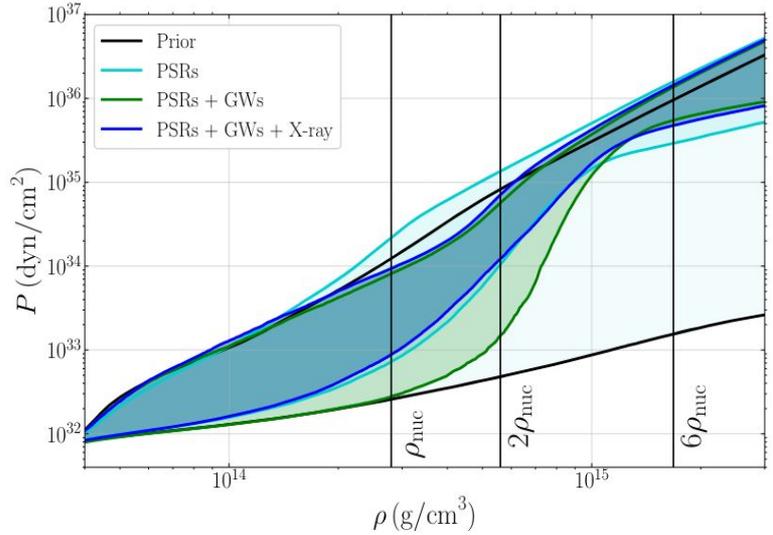
Credit: Zack Carson, Thesis

Introduction: Neutron Stars and GW

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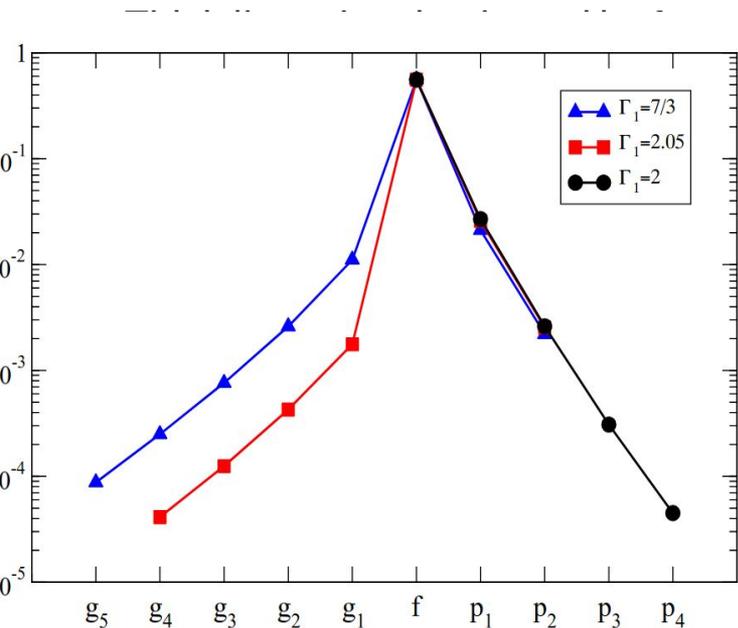


PC: B. P. Abbott *et al.*, *Phys. Rev. Lett.* 121, 161101 (2018), LVC



P. Landry *et al.*, [Phys. Rev. D. 101, 123007 \(2020\)](#).

Neutron Star in Binary System and Oscillation modes



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[MNRAS, 270, 611](#))

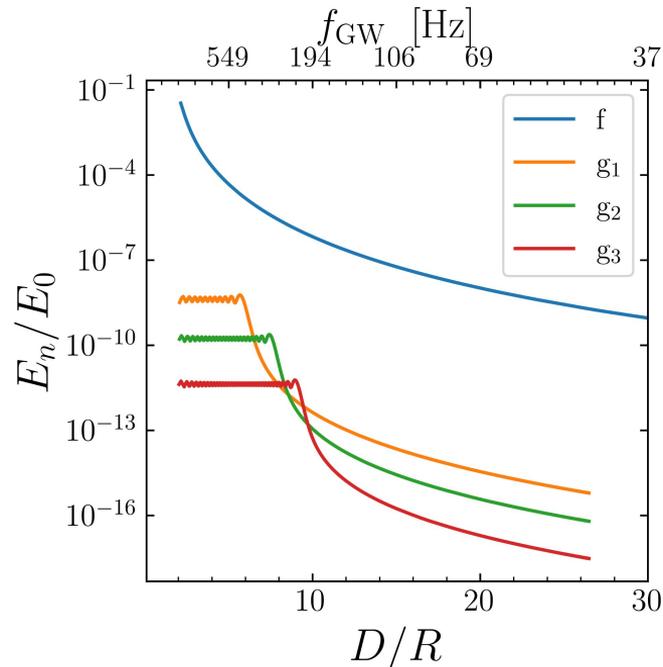
$$\frac{V_{lm}}{-1} Q_{nl} e^{-im\Phi(t)}$$

$$] = \sum_n E_n(t)$$

$$Q_{ij} Q_{ij} \Big] - \frac{1}{2} Q_{ij} \mathcal{E}_{ij}$$

$$Q_{ij} = -\lambda_2 \mathcal{E}_{ij}$$

Flanagan and Hinderer, [PRD 77 \(2008\) 021502](#)



Andersson & Pnigouras, 2020, [PRD101, 083001](#)

- Adiabatic limit: $1/\omega_f \ll 1/\omega_{\text{GW}}$.
- Finite frequency corrections to f-mode tidal correction : dynamical f-mode tide (P. Schmidt, & T. Hinderer 2019, PRD, 100,021501(R))

Neutron Star in Binary System and Oscillation modes

- Tidal distortion dominated by f-mode.

$$\vec{\xi}(\mathbf{r}, t) = \sum_n a_n(t) \vec{\xi}_n(r)$$

(Lai D., 1994, [MNRAS, 270, 611](#))

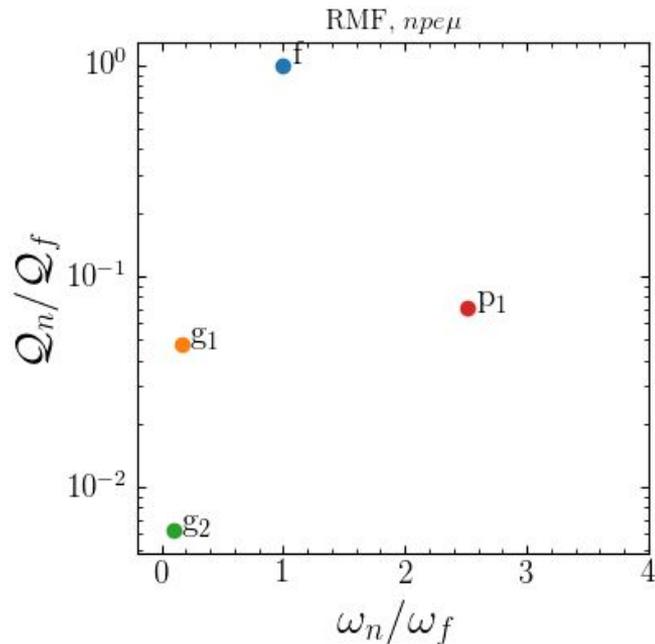
$$\ddot{a}_n + \omega_n^2 a_n = - \int d^3x \rho \vec{\xi}_{nl}^* \cdot \nabla U = \frac{M_B W_{lm}}{D^{l+1}} Q_{nl} e^{-im\Phi(t)}$$

$$E_{tide}(t) = \frac{1}{2} \sum_n \left[|\dot{a}_n(t)|^2 + \omega_n^2 |a_n(t)|^2 \right] = \sum_n E_n(t)$$

$$\mathcal{L}_N = \frac{1}{4\lambda_{2,A}\omega_f^2} \left[\frac{dQ_{ij}}{dt} \frac{dQ_{ij}}{dt} - \omega_f^2 Q_{ij} Q_{ij} \right] - \frac{1}{2} Q_{ij} \mathcal{E}_{ij}$$

$$Q_{ij} = -\lambda_2 \mathcal{E}_{ij}$$

Flanagan and Hinderer, [PRD 77 \(2008\) 021502](#)



- Adiabatic limit: $1/\omega_f \ll 1/\omega_{GW}$.
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Neutron Star in Binary System and Oscillation modes

- Tidal distortion dominated by f-mode.

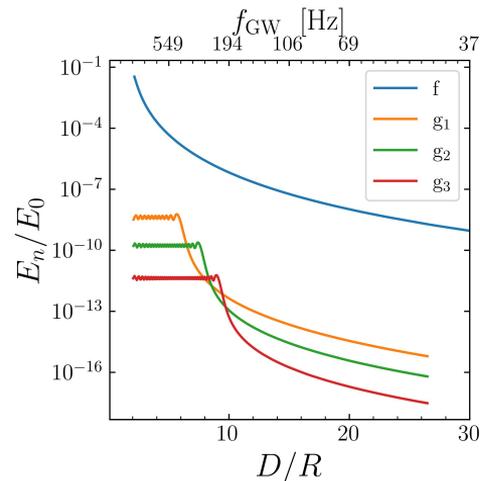
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(Lai D., 1994, [MNRAS, 270, 611](#))

$$E_{\text{tide}}(t) = \frac{1}{2} \sum_n \left[|\dot{a}_n(t)|^2 + \omega_n^2 |a_n(t)|^2 \right] = \sum_n E_n(t)$$

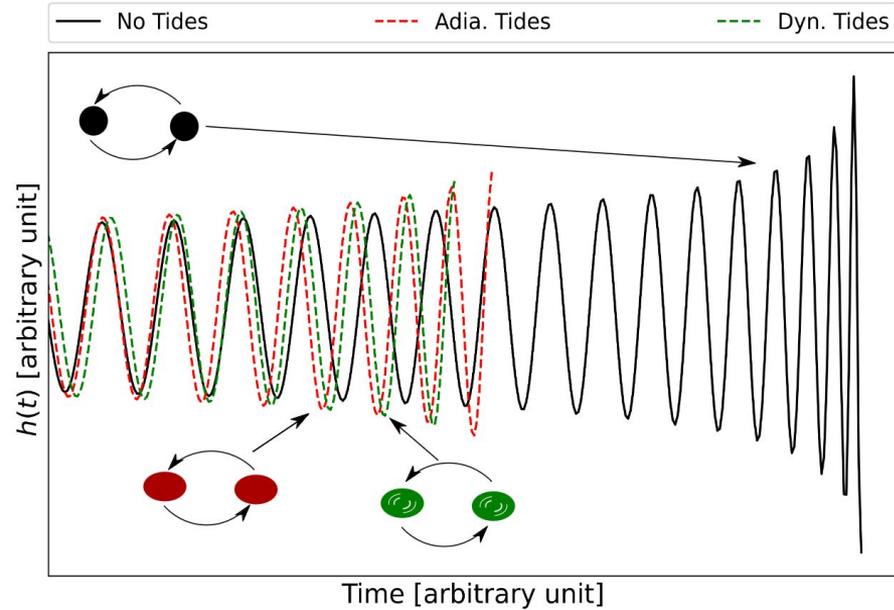
$$\mathcal{L}_N = \frac{1}{4\lambda_{2,A}\omega_f^2} \left[\frac{dQ_{ij}}{dt} \frac{dQ_{ij}}{dt} - \omega_f^2 Q_{ij} Q_{ij} \right] - \frac{1}{2} Q_{ij} \mathcal{E}_{ij}$$

$$Q_{ij} = -\lambda_2 \mathcal{E}_{ij} \quad \text{Flanagan and Hinderer, [PRD 77 \(2008\) 021502](#)}$$



- Complete GW: Orbital dynamics+internal dynamics (L_N)
- f-mode tidal energy is dominated so other modes are ignored
- Adiabatic limit: $f > 1$ kHz. Implies $1/\omega_f \ll 1/\omega_{\text{GW}}$.
- Dynamical tide: Finite freq. Correction
- Nuclear EoS depends on the inferred Tidal parameters and Hence on the GW waveform model

Dynamical tides in Binary Neutron Star System

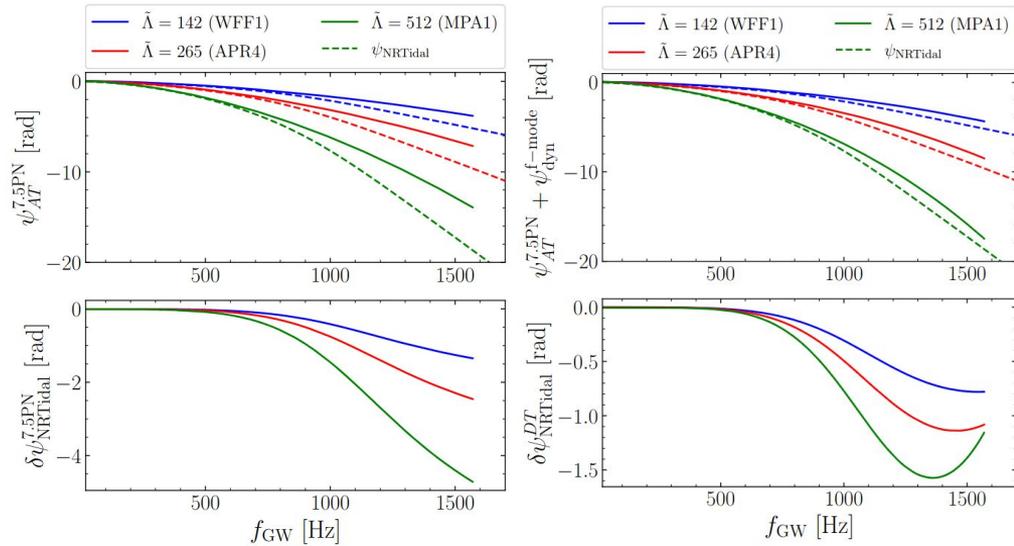


- GW waveform model used : Taylor F2
- F-mode dyn. correction, P. Schmidt, & T. Hinderer 2019, PRD, 100,021501(R),

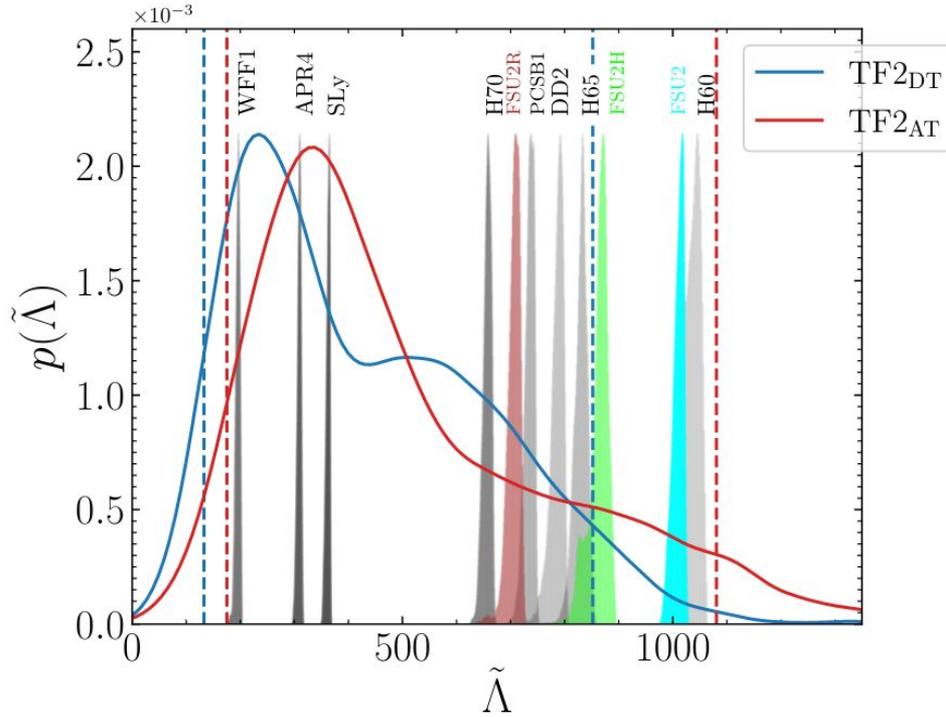
$$h(f) = A(f)e^{i(\Psi_{pp} + \Psi_{AT} + \Psi_{dyn})}$$

$$\Psi_{AT}(\Lambda_2, \dots) \text{ OR, } \Psi_{AT}(\tilde{\Lambda}, \dots)$$

$$\Psi_{dyn}(\Lambda_2, m\omega_2)$$



Results: GW170817



- Inclusion of f-mode dynamical phase lowers the 90% upper bound of $\tilde{\Lambda}$ by $\sim 15\%$.
- Λ_3 and Σ_2 or the choice of multipole Love relation has no significant effect

Results: GW170817, Nuclear Physics and f-mode dynamical tide

NS EOS Modelling: Relativistic Mean Field (RMF) model

$$\mathcal{L} = \sum_B \bar{\psi}_B (i\gamma^\mu \partial_\mu - m_B + g_{\sigma B} \sigma - g_{\omega B} \gamma_\mu \omega^\mu - g_{\rho B} \gamma_\mu \vec{I}_B \cdot \vec{\rho}^\mu) \psi_B + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - U_\sigma$$

$$+ \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} - \frac{1}{4} (\vec{\rho}_{\mu\nu} \cdot \vec{\rho}^{\mu\nu} - 2m_\rho^2 \vec{\rho}_\mu \cdot \vec{\rho}^\mu) + \Lambda_\omega (g_\rho^2 \vec{\rho}_\mu \cdot \vec{\rho}^\mu) (g_\omega^2 \omega_\mu \omega^\mu)$$

$$+ \mathcal{L}_\ell$$

$$U_\sigma = \frac{1}{3} b m_p (g_\sigma \sigma)^3 + \frac{1}{4} c (g_\sigma \sigma)^4 \quad \mathcal{L}_\ell = \sum_{\ell=\{e^-, \mu^-\}} \bar{\psi}_\ell (i\gamma^\mu \partial_\mu - m_\ell) \psi_\ell$$

➤ Coupling parameters are fixed to nuclear saturation parameters

Iso-Scalar Couplings

$$\square \quad n_0, E_{\text{sat}}, m^* = m_N - g_\sigma \sigma, K$$

Iso-Vector Couplings

$$\square \quad J, L$$

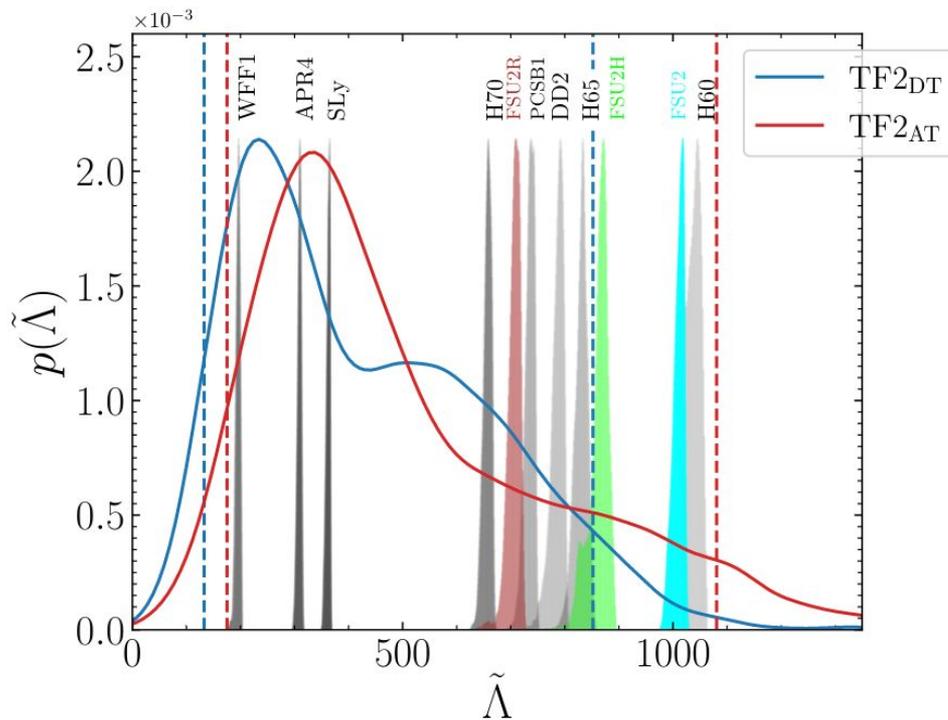
$$\epsilon/n = \epsilon_{IS} + \epsilon_{IV} \delta^2$$

$$\epsilon_{IS} = \epsilon_{\text{sat}} + \frac{K}{2!} x^2 + \dots$$

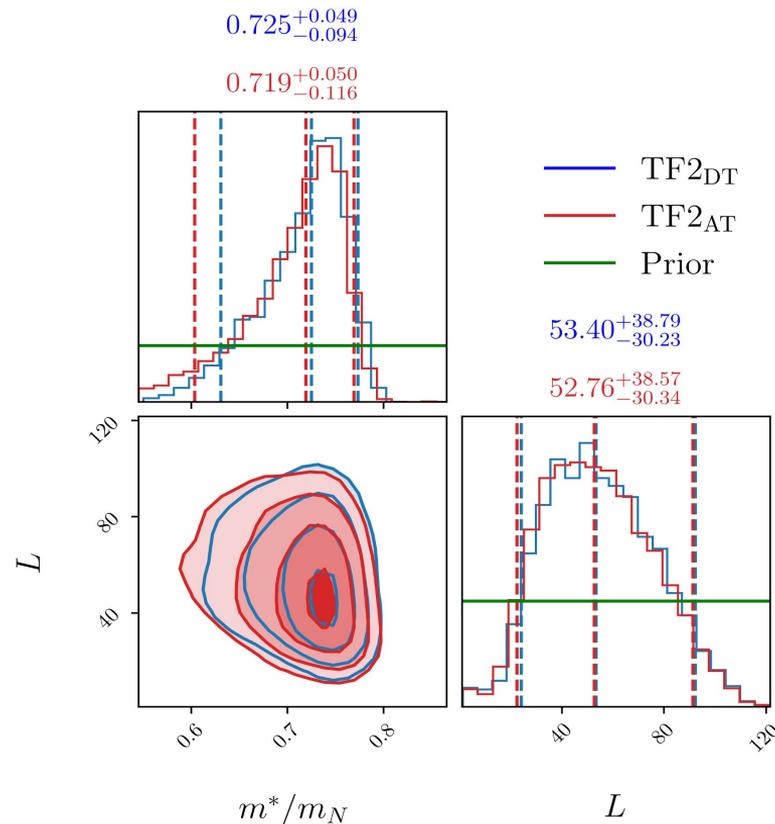
$$\epsilon_{IV} = J_{\text{sym}} + L_{\text{sym}} x + \dots$$

$$\text{where, } x = \frac{n_b - n_0}{3n_0} \ \& \ \delta = \frac{n_n - n_p}{n}$$

GW170817, Nuclear Physics and f-mode dynamical tide

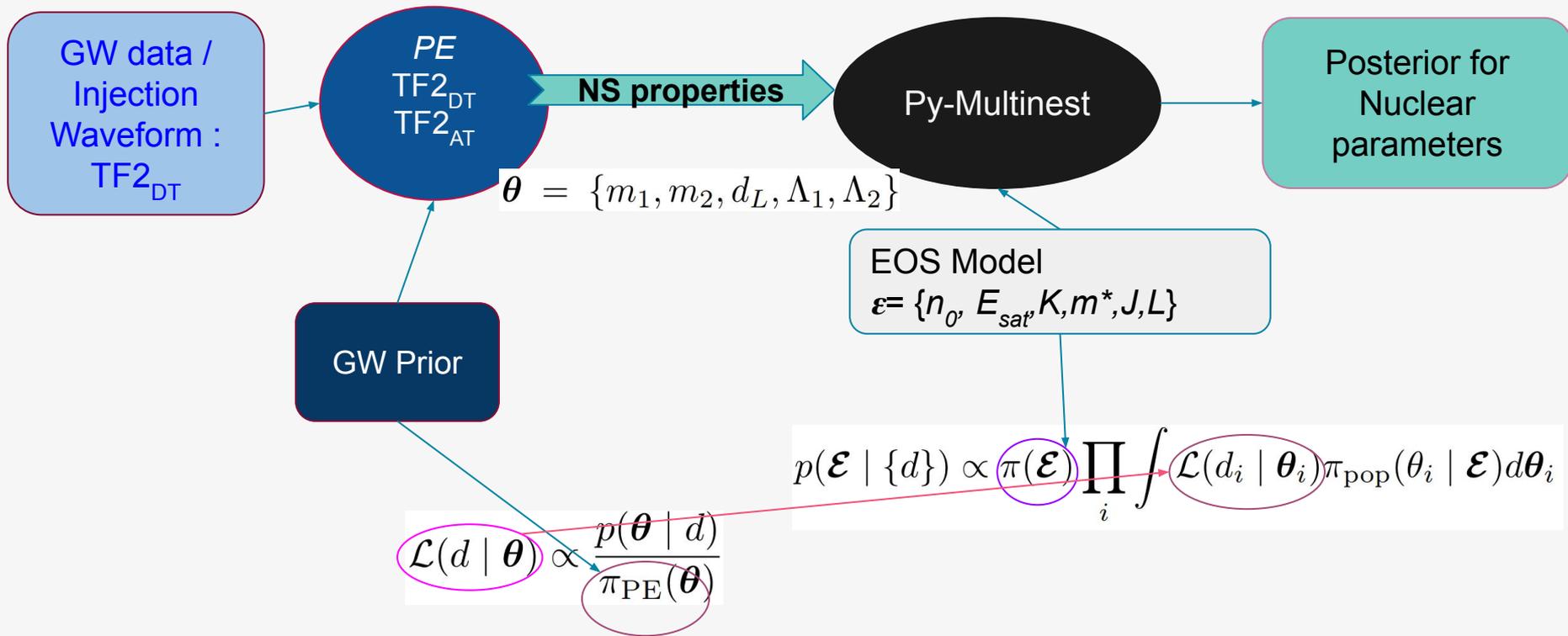


- Only m^* and L get constrained.
- No significant effect of dynamical tide on nuclear parameter.



❖ Pradhan et al., *Astrophys.J.* 966 (2024) 1, 79.

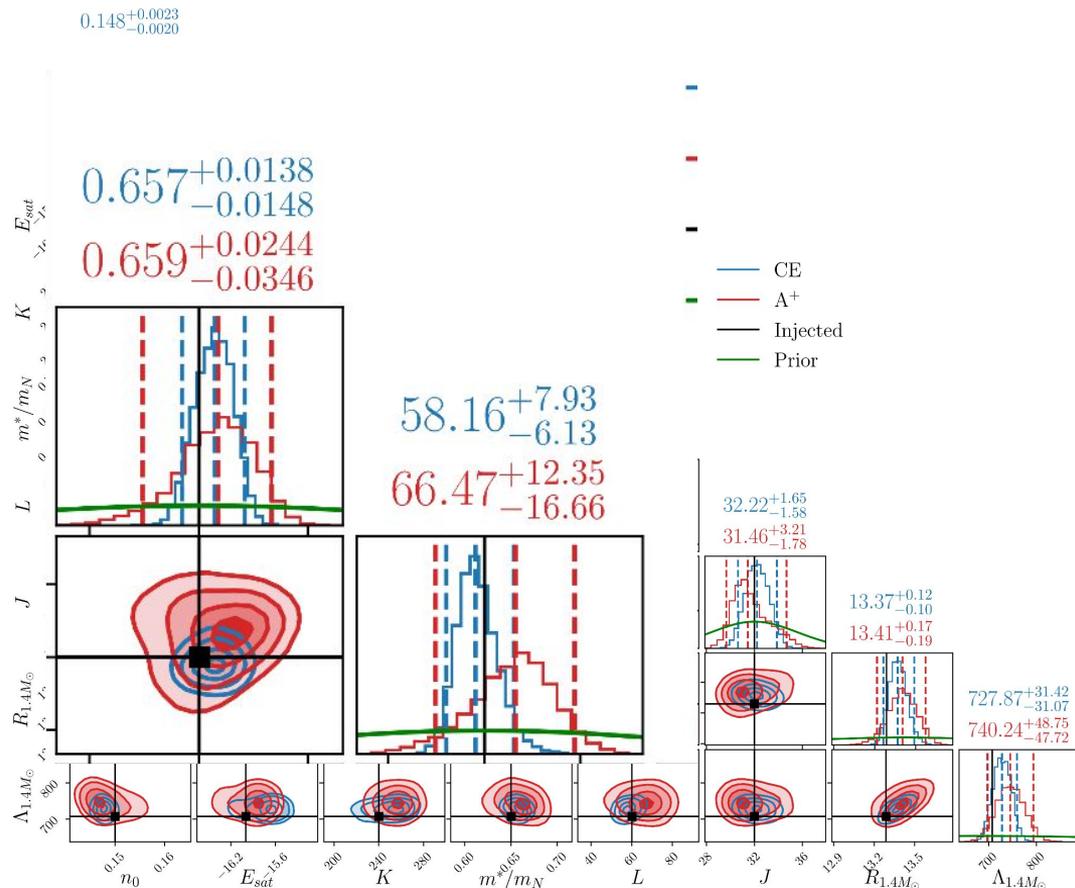
Methodology: For Future Events



Constraints on Nuclear parameters with GWs from BNSs

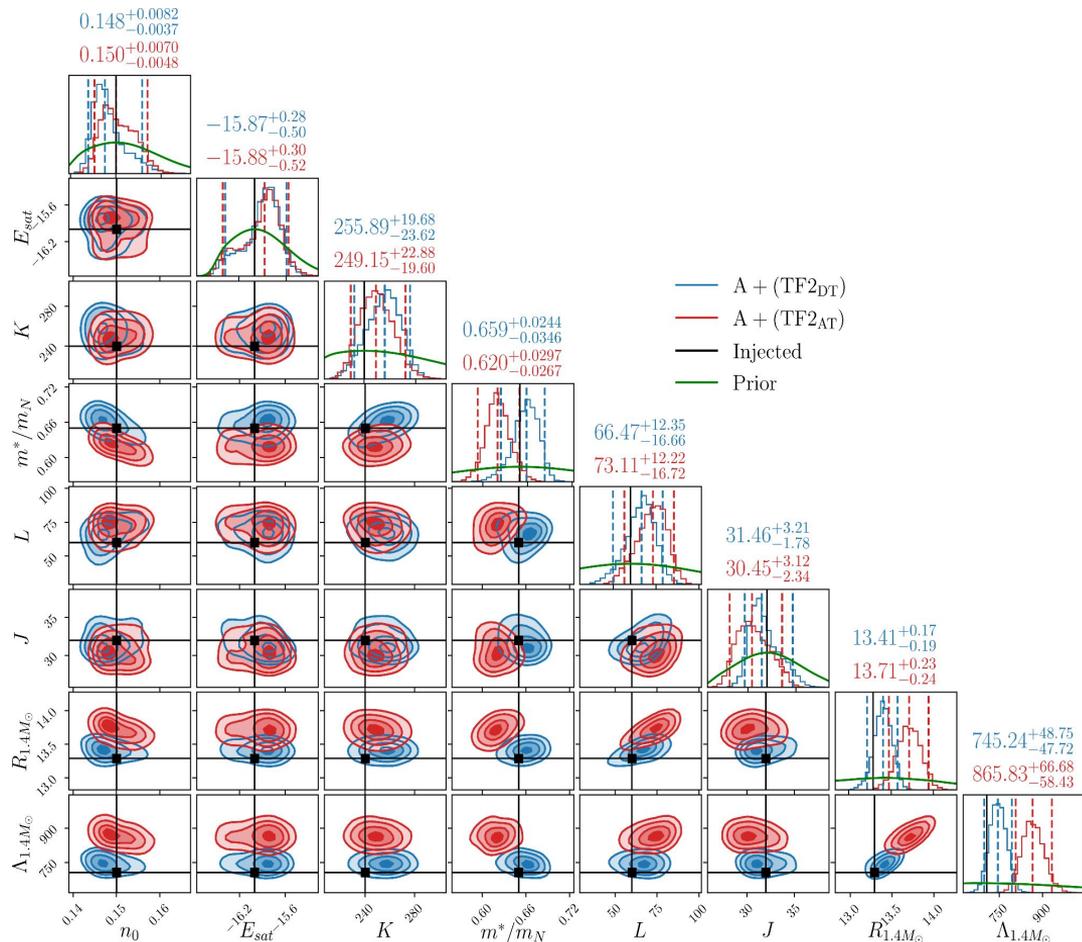
13 events with
 A⁺ : HLV with O5 sensitivity
 CE : Cosmic Explorer

- m^* (at 90% CI) \sim 5% (A⁺) and 3% (CE).
- $L \sim$ 20% and 15% with A⁺ and CE, respectively.
- $R_{1.4M_\odot}$ can be constrained to \sim 2% (in A⁺) and 1% in (CE).
- $\Lambda_{1.4M_\odot}$ can be constrained to \sim 10% (in A⁺) and 5% in (CE).



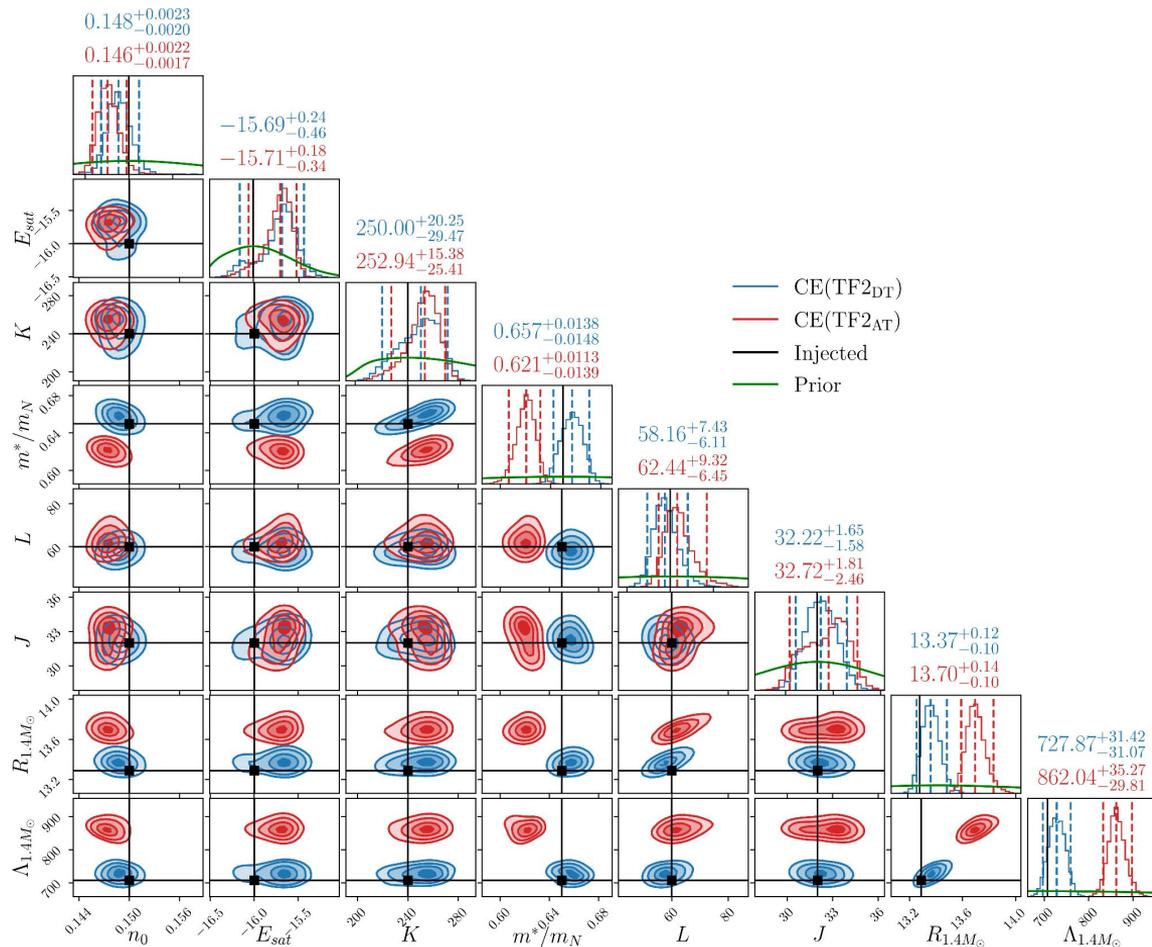
Results: Injection Studies

- ~6% bias on median of m^* due to ignorance of dynamical tide.
- m^* (at 90% CI) ~ 5%

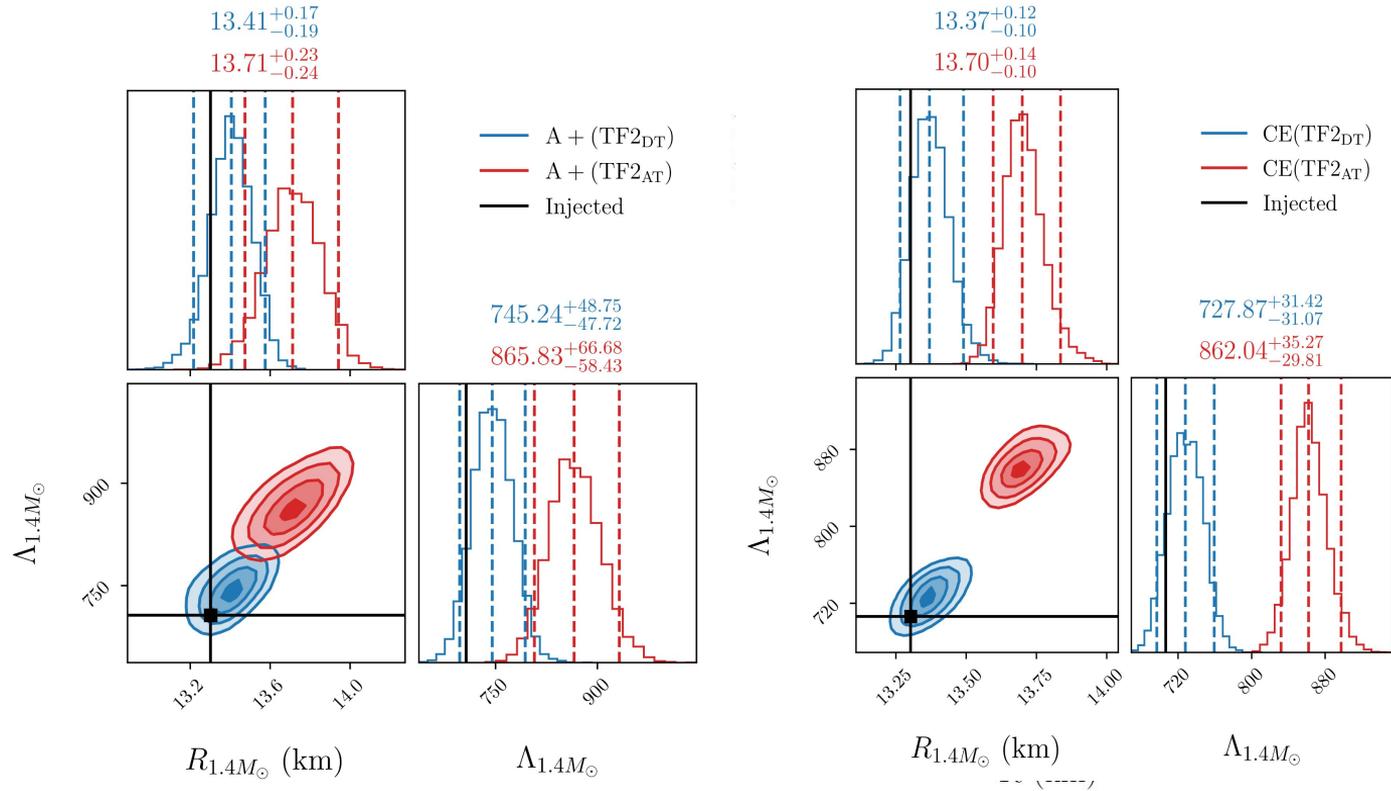


Results: Injection Studies

- ~6% bias on median of m^* due to ignorance of dynamical tide.
- 15% bias in $\Lambda_{1.4M_\odot}$.
- m^* (at 90% CI) ~ 3%



Results: Injection Studies



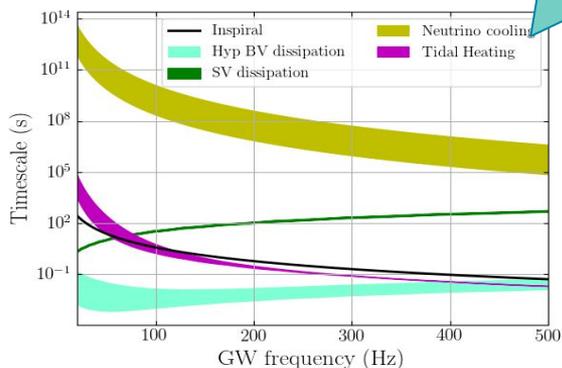
- Biases are consistent with [G. Pratten+ PRL 129, 081102 \(2022\)](#). Where polytropic model was considered.

F-mode excitation in binary inspiral and Tidal Heating in Presence of Hyperons

➤ S. Ghosh, B. K. Pradhan, D. Chatterjee, *PRD* 109, 10, 103036, [arXiv:2306.14737](https://arxiv.org/abs/2306.14737).

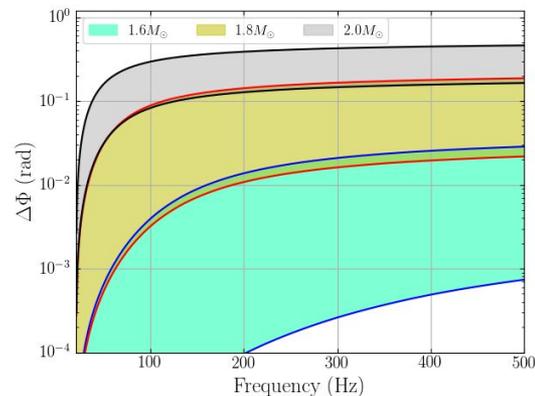
- During the binary inspiral, viscous processes in NS matter can damp out the tidal energy induced by the companion and convert this to thermal energy to heat up the star.
- Damping is small for normal neutron matter viscosity to have any significant effect. Bildsten & Cutler, *ApJ* 400(1992), D. Lai *MNRAS* 270(1994).
- Dominant non-leptonic channel $n + p \longleftrightarrow p + \Lambda$

Dissipation and heating happens well within the timescale of inspiral !!



$$\Delta \mathcal{N} = - \int_{f_a}^{f_b} t_D \left(\frac{\dot{E}_{visc}}{\dot{E}_{gw}} \right) df$$

$$\Delta \mathcal{N} = \frac{15\pi}{384} \left(\frac{c^2 R}{GM} \right)^6 \omega_0^{-4} Q_0^2 \frac{1}{q(1+q)} \left(\frac{R}{c} \right)^2 \int_{f_a}^{f_b} \gamma_{bulk} df.$$



- Hyperon BV in the core is high enough to heat the star up to 0.1-1 MeV during the inspiral, but not high enough to require inclusion of thermal corrections to the EoS.
- The dissipated energy can induce a net phase difference $\sim 10^{-3}$ - 0.5 rad depending on component masses.

Summary

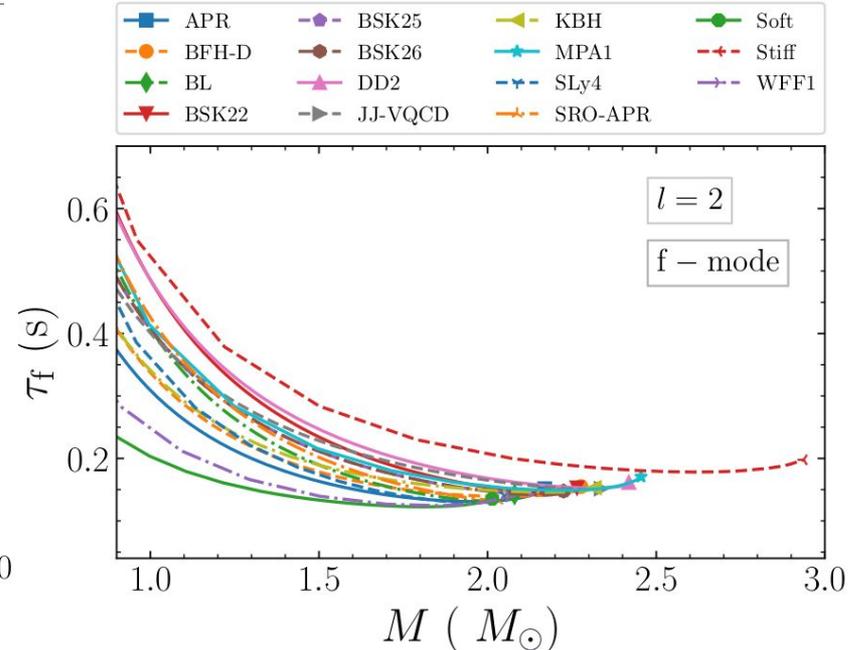
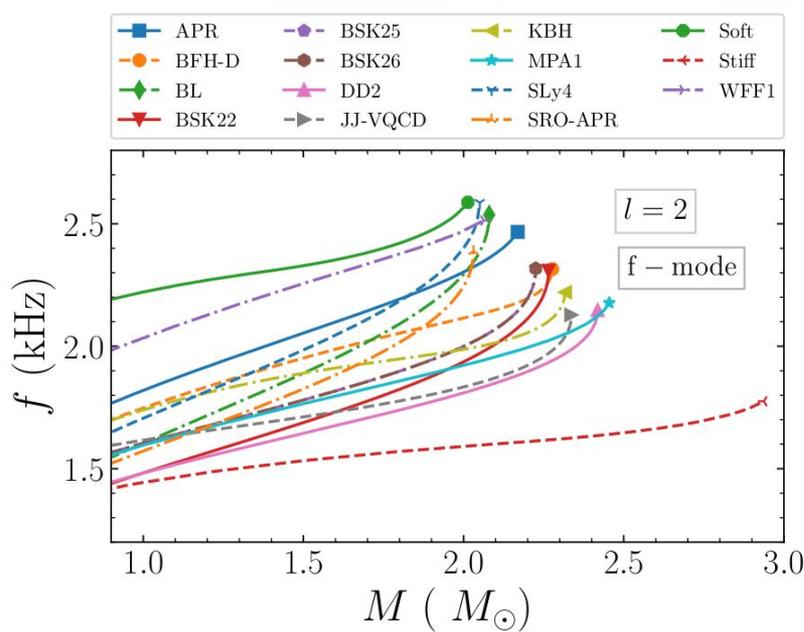
Discussed the Constraints in Nuclear Parameters and NS properties

- For the RMF model considered, m^* and L get well constrained.
- $R_{1.4M_{\odot}}$ can be constrained to $\sim 2\%$ (in A+) and 1% in (CE).
- $\Lambda_{1.4M_{\odot}}$ can be constrained to $\sim 10\%$ (in A+) and 5% in (CE).

f-mode Dynamical tidal effect

- For GW170817, dynamical tide has no significant impact on the inferred nuclear parameter.
- Important for future observations.
- Presence of hyperons in NS may lead to a detectable tidal-heating in BNS.
- F-mode dissipation in quark star binaries are discussed in our new work :<https://arxiv.org/abs/2504.07659>.

AsteroSeismology: With Isolated NSs



- Non-radial oscillations of NS are source of GWs.
- GWs depends upon the mode properties.
- Observing NS oscillation properties, NS interior composition can be inferred.

f-mode GW and glitching Pulsars

- The waveform is modelled as an exponentially damped oscillation.

$$h(t) = h_0 \exp(-t/\tau_f) \sin(2\pi\nu_f t), \quad t > 0$$

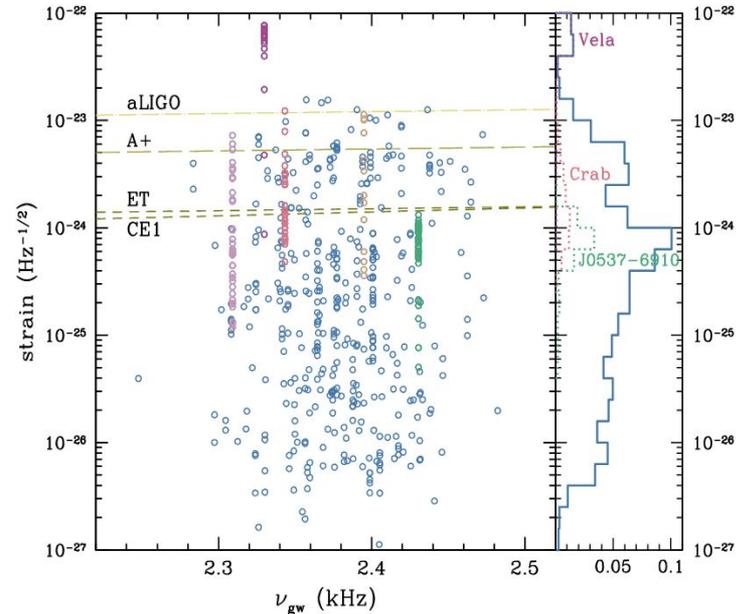
(B.J. Owen, 2010)

$$h_0 = 4.85 \times 10^{-17} \sqrt{\frac{E_{\text{gw}}}{M_{\odot} c^2}} \sqrt{\frac{0.1 \text{ sec} \cdot 1 \text{ kpc}}{\tau_f d}} \left(\frac{1 \text{ kHz}}{\nu_f} \right)$$

$$E_{\text{gw}} = E_{\text{glitch}} = 4\pi^2 I \nu^2 \left(\frac{\Delta\nu}{\nu} \right)$$

F-mode GW

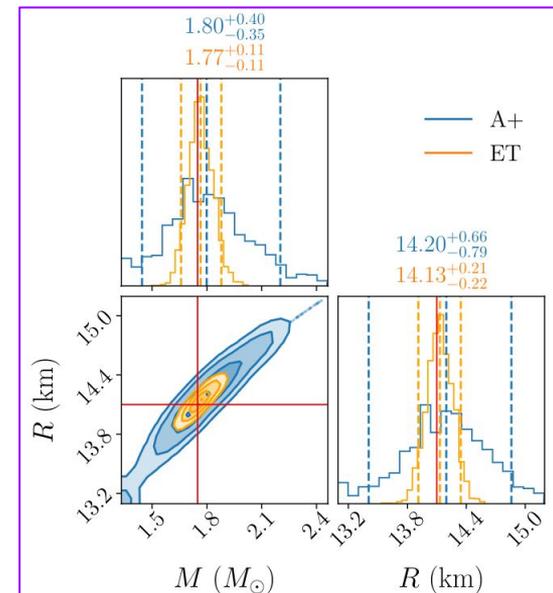
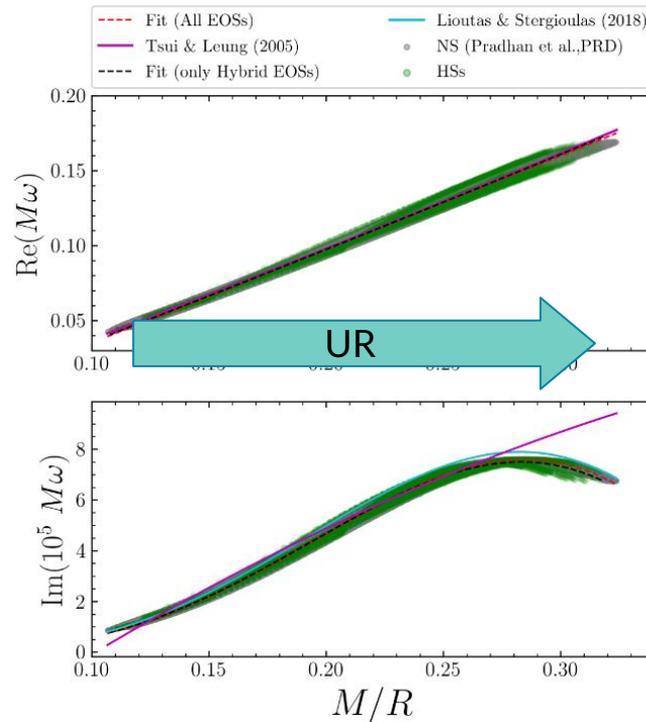
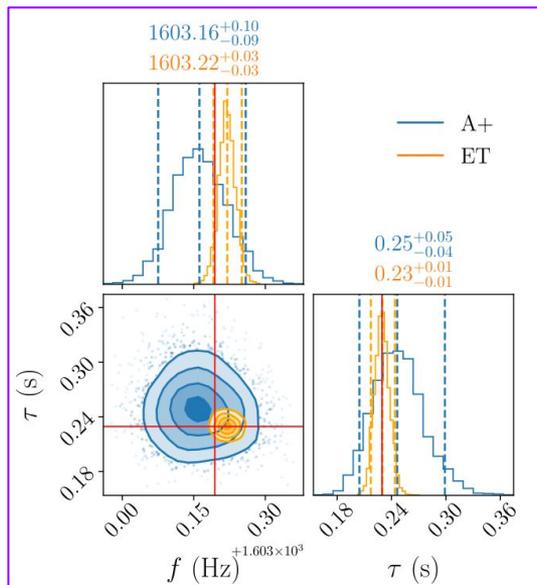
- B. Abbott et al., LVC, [ApJ 874 163, 2019](#);
- R. Abbott et al., LVK, [PhRvD, 104, 122004, 2021](#).
- R. Abbott, et al., LVK, [arXiv:2210.10931, 2022](#).
- R. Abbott, et al., LVK, [arXiv:2203.12038, 2022](#).
- D. Lopez et al., [PhRvD, 106, 103037, 2022](#)



W.C. G. Ho, D. I. Jones, N. Andersson, and C. M. Espinoza, [PRD 2010, 103009](#)

URs in Asteroseismology

➤ [N. Andersson and K. D. Kokkotas, PRL 77, 4134, \(1996\) and MNRAS 299, 1059–1068, \(1998\).](#)



➔ [BKP, D. Chatterjee, D. E. Alvarez-Castillo, MNRAS, 4640–4655, 2023](#)

Constraining Nuclear Parameters Using GWs from f -mode Oscillations in Neutron Stars

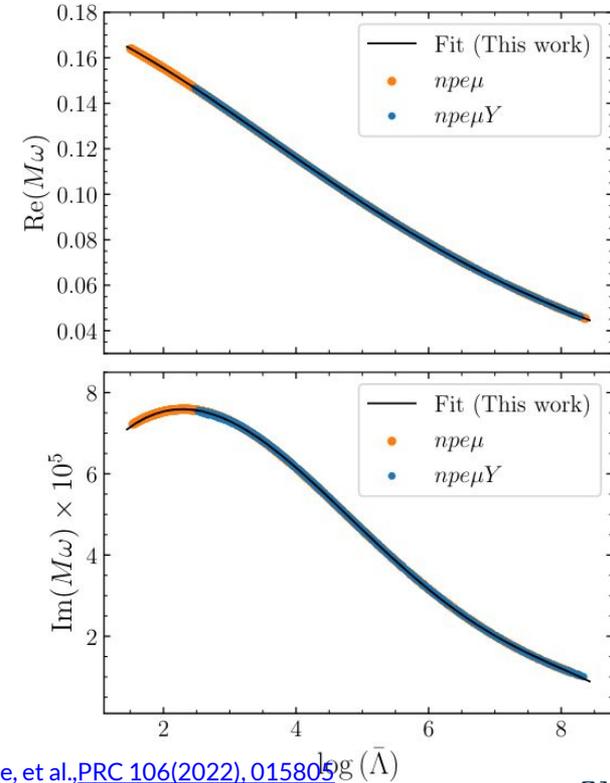
➤ **B. K. Pradhan, D. Pathak, and D. Chatterjee, [APJ 956 \(2023\) 1, 38](#)**

$$P(\theta|D) = \frac{P(D|\theta)\pi(\theta)}{P(D)}$$

$$\theta = \{n_0, E_{sat}, K, m^*, J, L\}$$

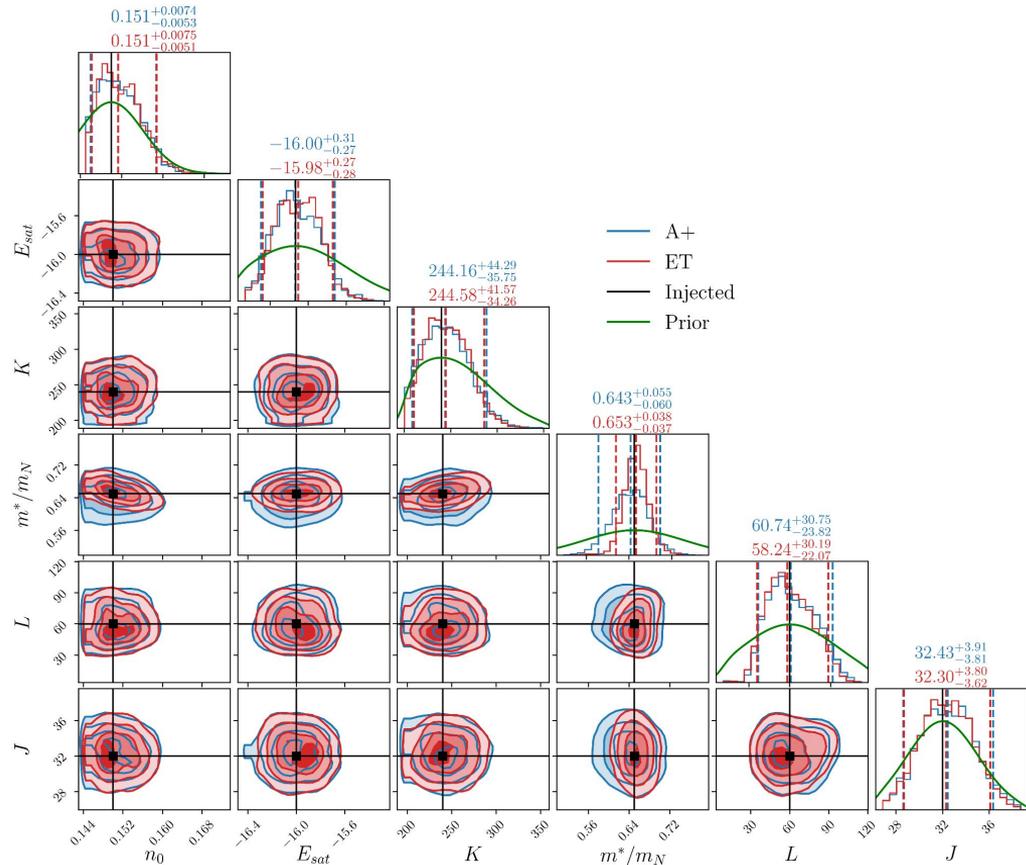
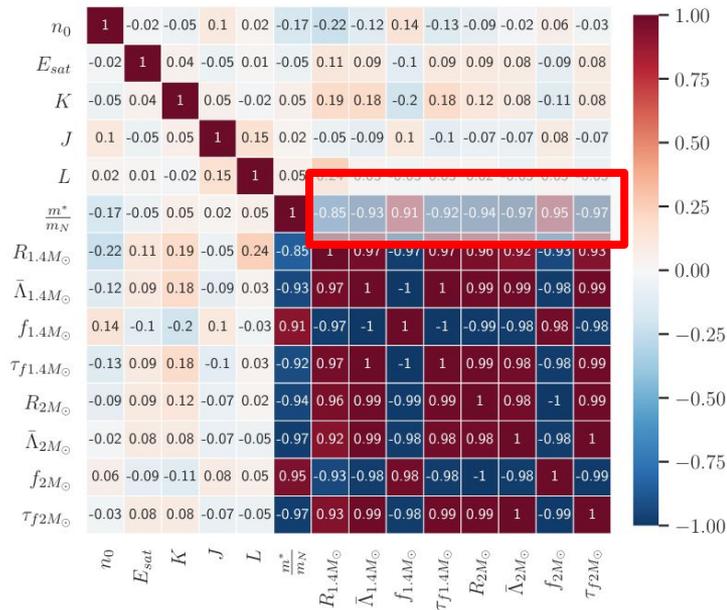
$$P(D|\theta) \propto \int^{M_{max}(\theta)} dm P(D|f(m), \tau(m)) p(m|\theta)$$

- ❖ Solve TOV for M, R and use f, τ -C(= M/R) URs.
- ❖ Solve for $M-\Lambda$, use f -Love and τ -Love URs.

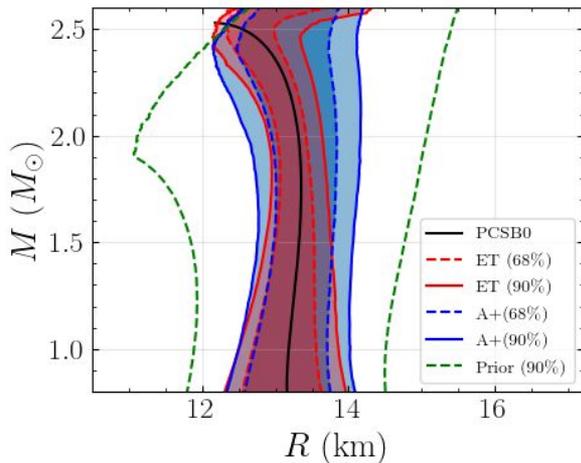
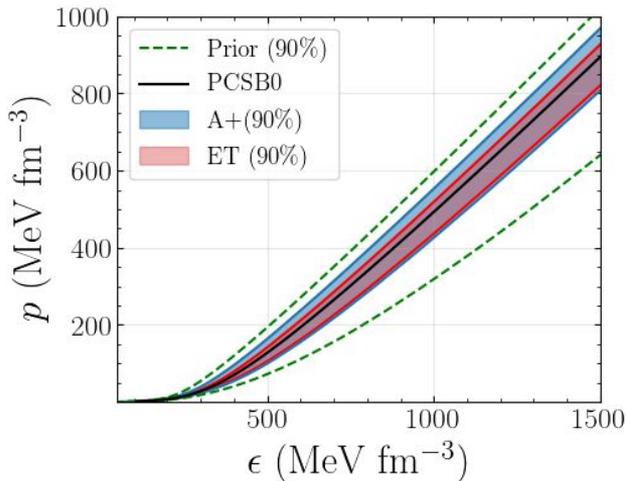


Results : From a single event from Vela Pulsar

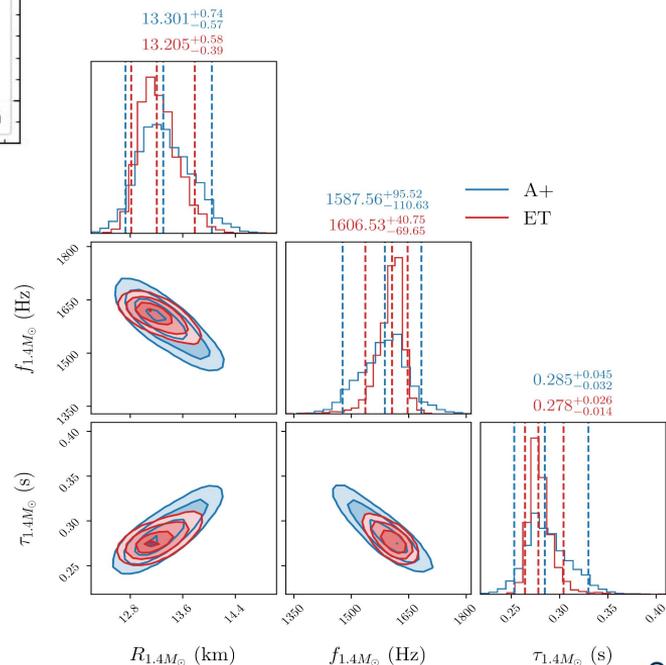
- Within 90% CI, $m^* \sim 10\%$ (in A+) and $\sim 5\%$ in ET.



Results : From a single event from Vela Pulsar



- $f_{1.4M_{\odot}}$ can be estimated (within 90% CI) upto ~ 100 Hz (in A+) and ~ 70 Hz (in ET).
- For multiple events $f_{1.4M_{\odot}}$ (at 90% CI) ~ 20 Hz (in ET).
- Measurement of EOS and Nuclear parameters improve with Multiple events.



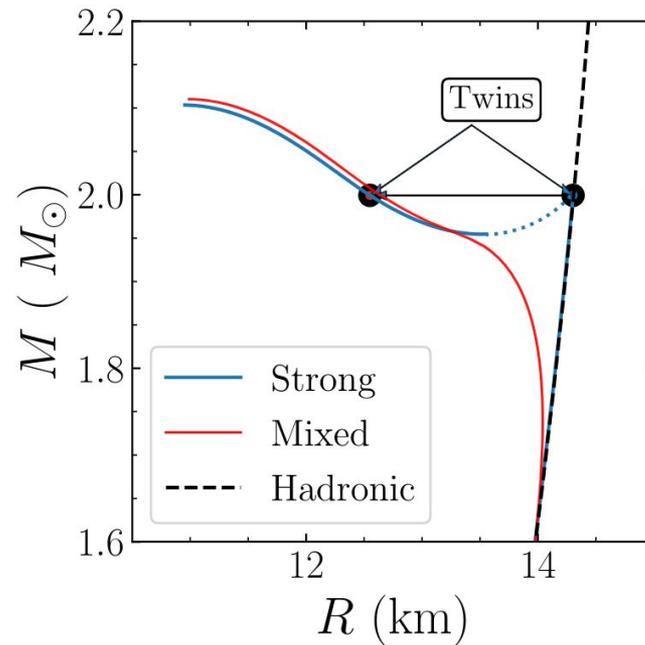
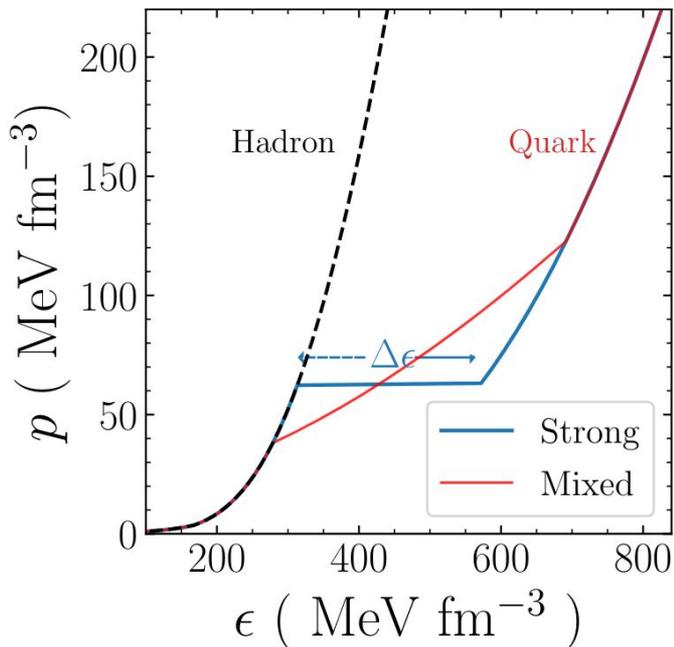
Compact Star EOS and Twin Stars

1. Probing hadron-quark phase transition in twin stars using f -modes

→ BKP, D. Chatterjee, and D. E. Alvarez-Castillo, [*MNRAS*, vol. 531, pp. 4640–4655, 06 2024](#)

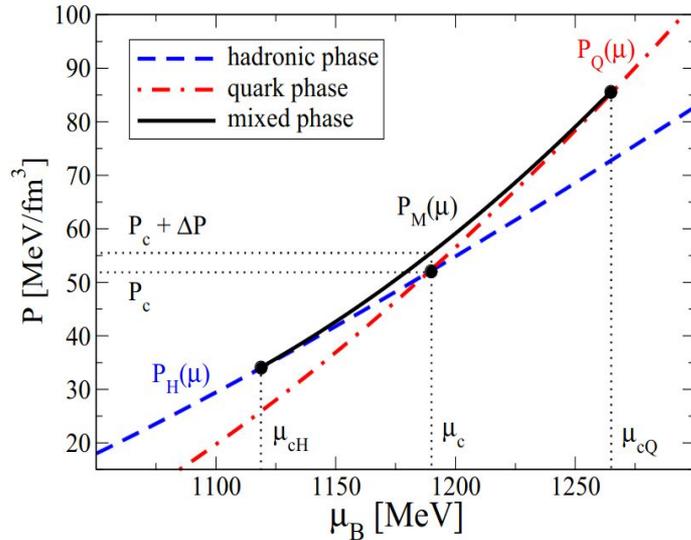
❖ Hadron-Quark phase transition inside the NS core

➤ The puzzle: Strong or Crossover ?



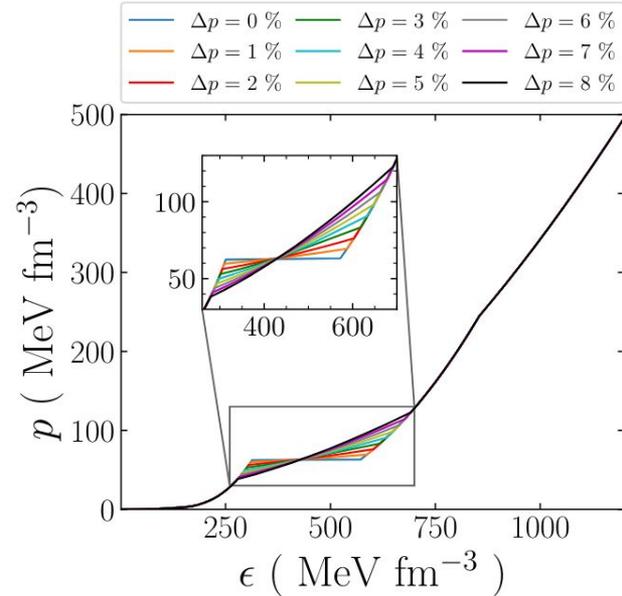
The EOS Model : Phenomenological Description

- ★ Surface tension effect leads to existence of pasta phases.
- ★ A parabolic interpolation method used to construct the mix phase.



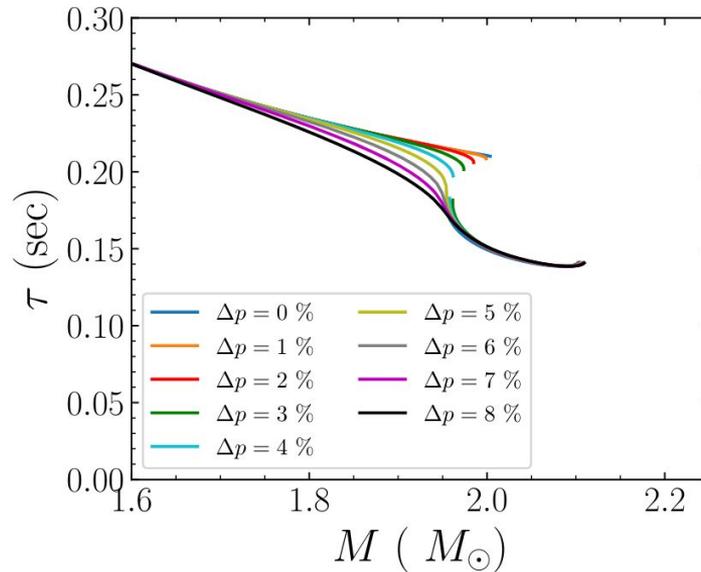
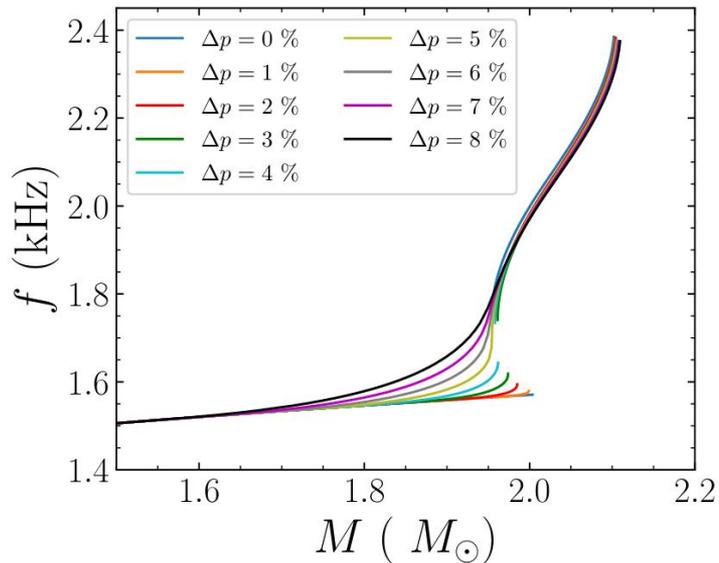
➤ [V. Abgaryan et al, Universe, 4, 94 \(2018\).](#)

- ★ Mixed Phase is parametrized by $\Delta p = \Delta P/P_c$.
- ★ $\Delta p = 0$: Maxwell Construction.



$$P(n) = \kappa_i \left(\frac{n}{n_0} \right)^{\Gamma_i}, \quad n_i < n < n_{i+1}, \quad i = 1, \dots, 4$$

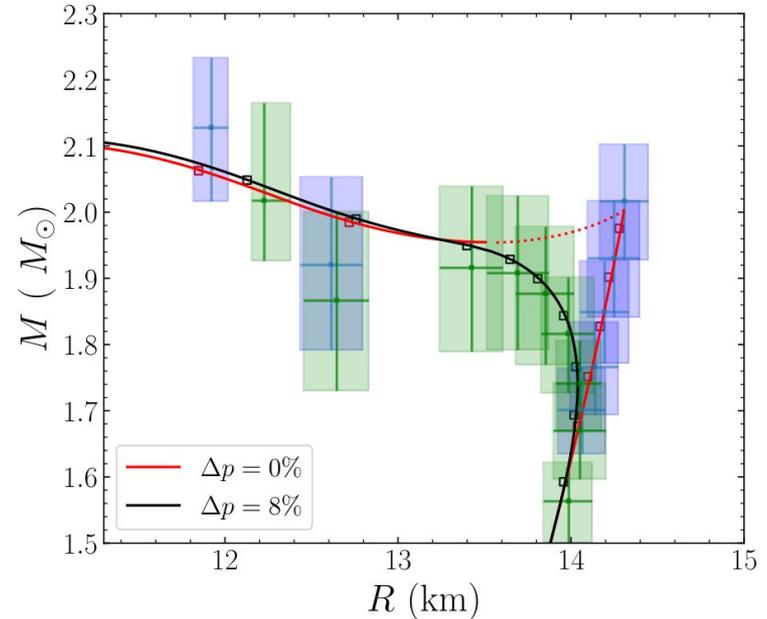
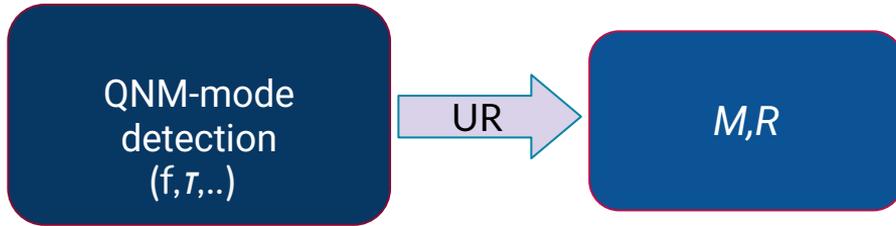
f-mode characteristics



➤ F-mode characteristics are obtained within **General relativistic** formalism.

f-mode and twins

- Glitching PSR data : [Jodrell Bank Glitch catalogue](#).
- Consider few random mass configurations with an assumed EOS model .



- The measurement of R from f-mode observation may confirm the presence of twins.
- More challenging for low mass twins.
- Differentiating the nature of Δp is more challenging.

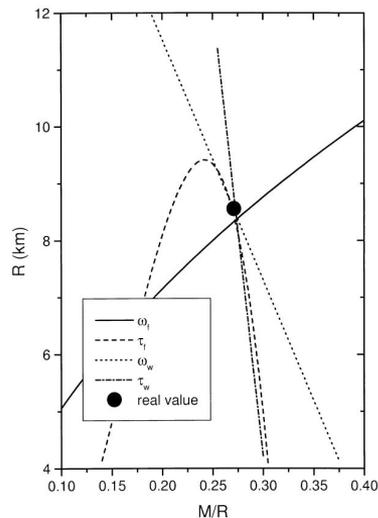
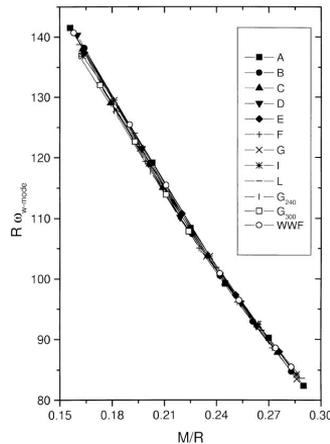
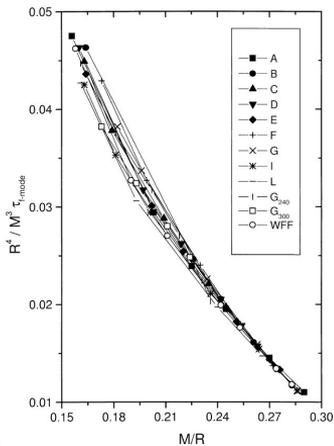
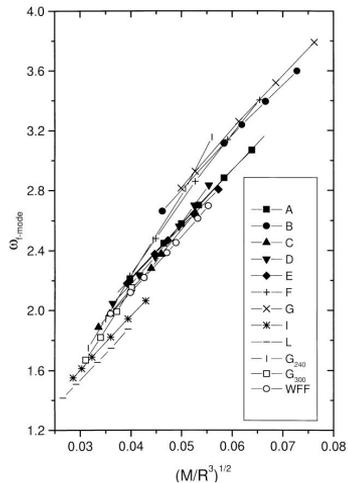
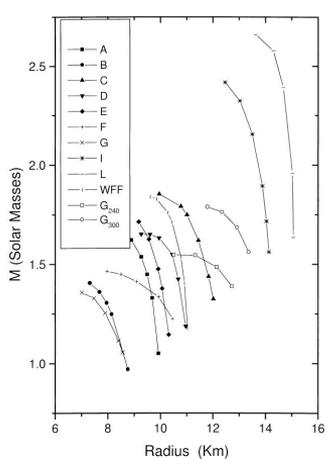
Summary

- Detection of f-mode can constrain the EOS/nuclear parameter.
- The measurement of f-mode observation may confirm the presence of twins.

Thank You

Neutron star Asteroseismology and Universal Relations (UR)

➤ Nils Andersson and Kostas D. Kokkotas, Phys. Rev. Lett. 77, 4134, (1996) and Mon. Not. R. Astron. Soc. 299, 1059–1068, (1998).



$$\bar{M} = \frac{M}{1.4 M_{\odot}} \quad \text{and} \quad \bar{R} = \frac{R}{10 \text{ km}}$$

$$\omega_f (\text{kHz}) \approx 0.78 + 1.635 \left(\frac{\bar{M}}{\bar{R}^3} \right)^{1/2}$$

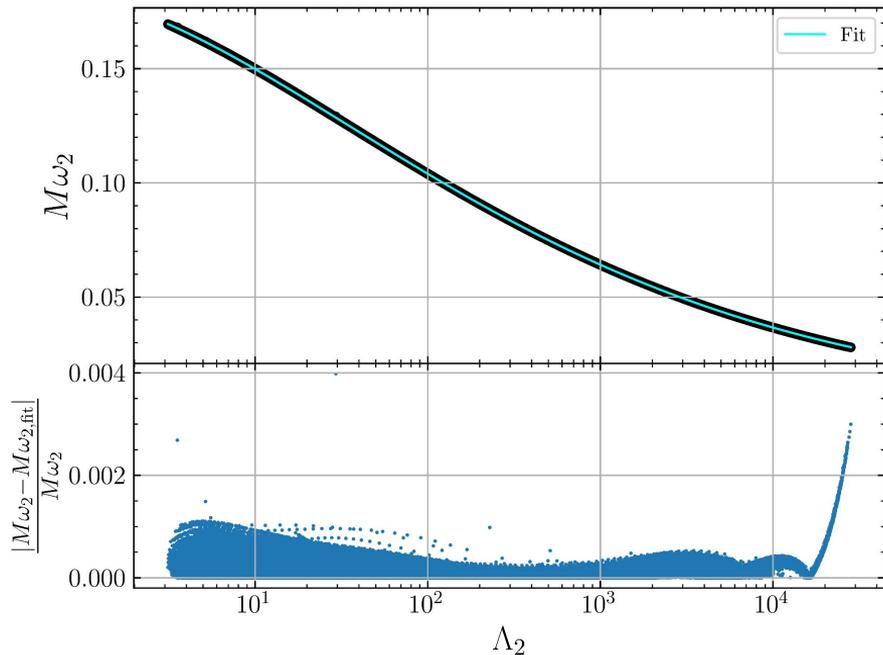
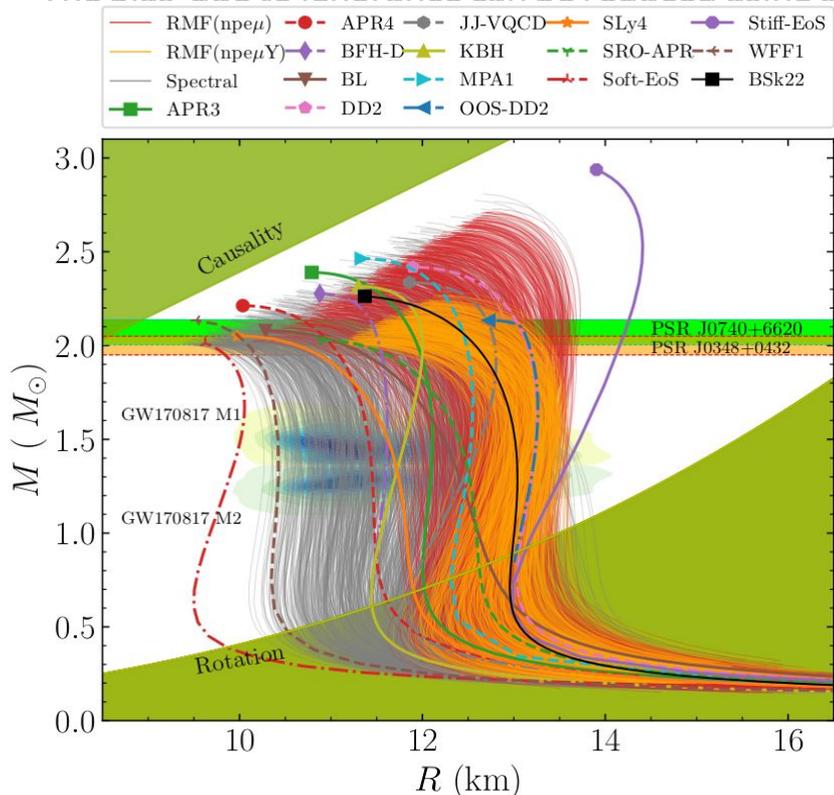
$$\frac{1}{\tau_f (\text{s})} \approx \frac{\bar{M}^3}{\bar{R}^4} \left[22.85 - 14.65 \left(\frac{\bar{M}}{\bar{R}} \right) \right]$$

$$\omega_w (\text{kHz}) \approx \frac{1}{\bar{R}} \left[20.92 - 9.14 \left(\frac{\bar{M}}{\bar{R}} \right) \right]$$

$$\frac{1}{\tau_w (\text{ms})} \approx \frac{1}{\bar{M}} \left[5.74 + 103 \left(\frac{\bar{M}}{\bar{R}} \right) - 67.45 \left(\frac{\bar{M}}{\bar{R}} \right)^2 \right]$$

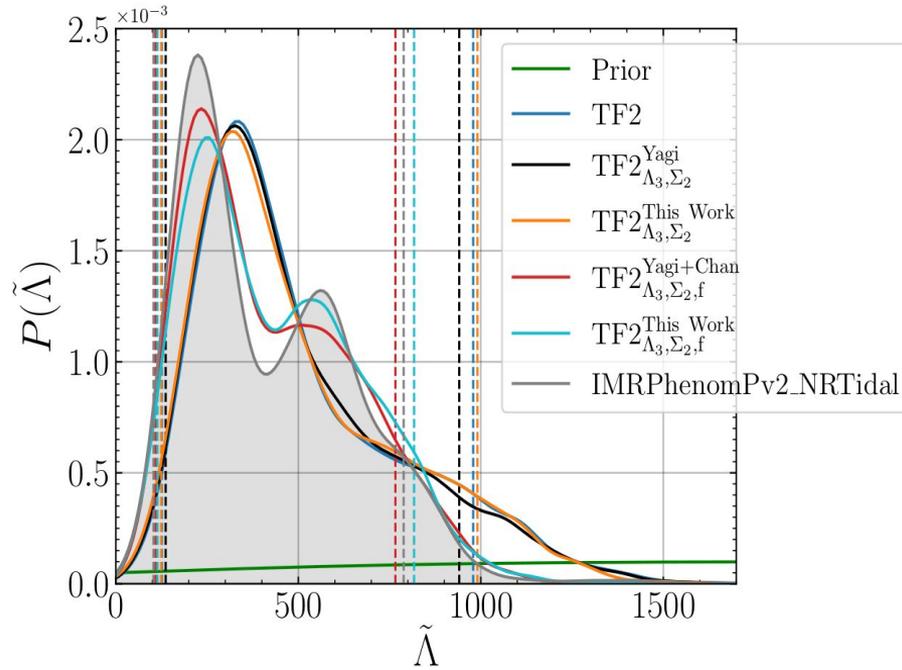
Multipolar Love and f-Love relations

- EoS is highly uncertain.
- Higher order correction with Λ_3, Λ_4 or Σ_2 terms are often ignored.
- Measuring the $\Lambda_3, \Lambda_4, \Sigma_2$ and ω are difficult.
- The bias due to ignorance can be reduced using universal relations (URs).



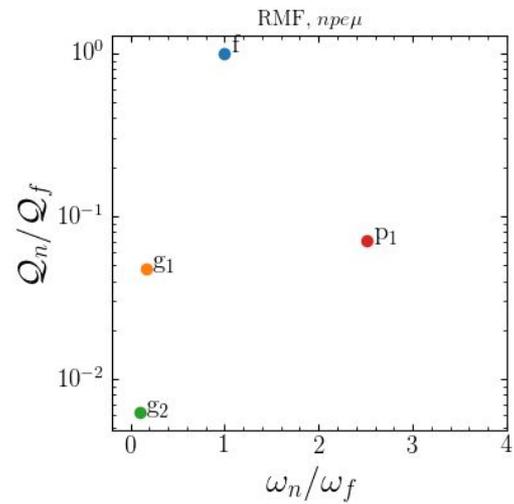
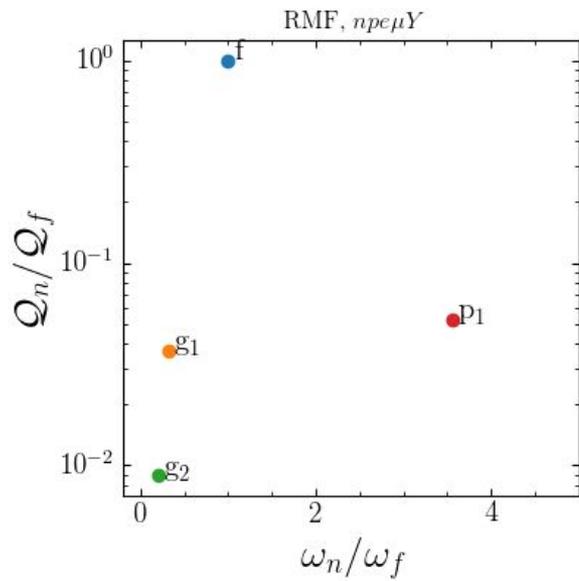
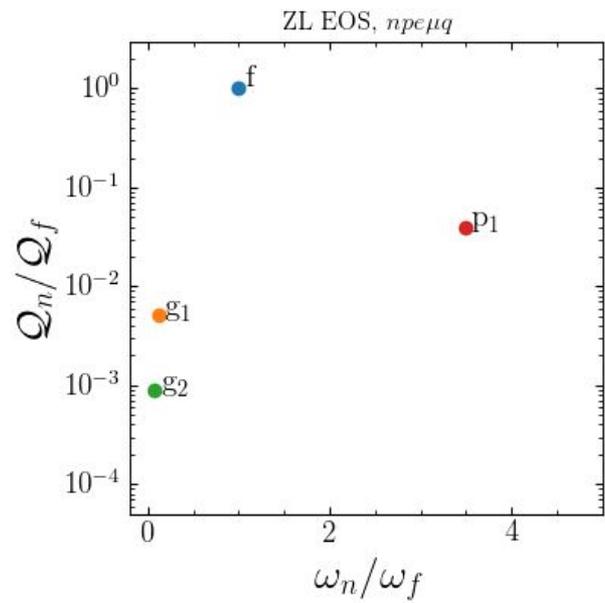
B. K. P, A. Vijaykumar, and D. Chatterjee [PRD 107 \(2023\) 2, 023010](#)

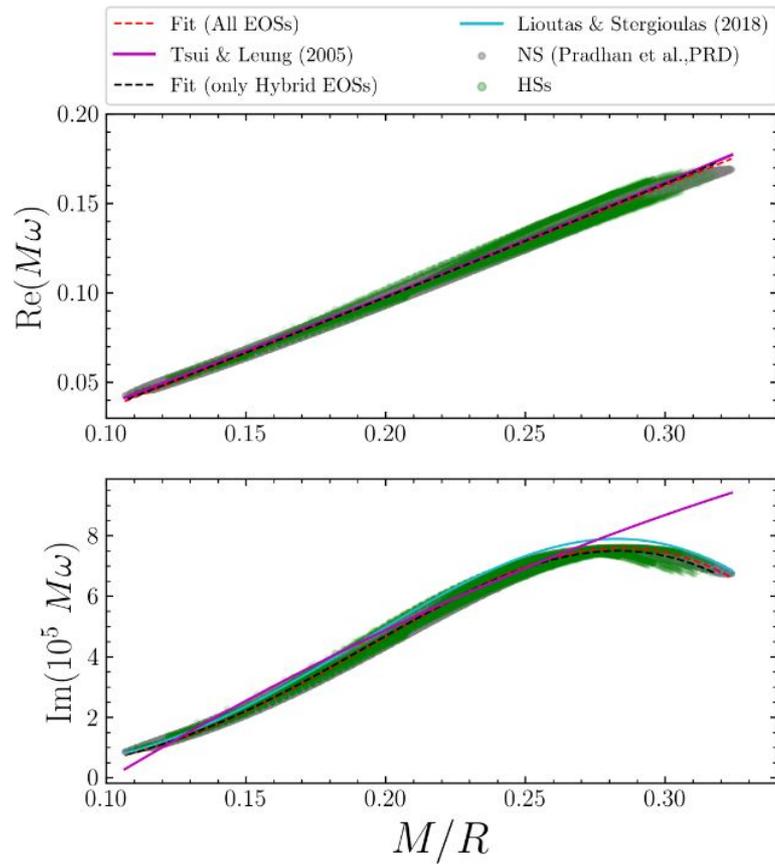
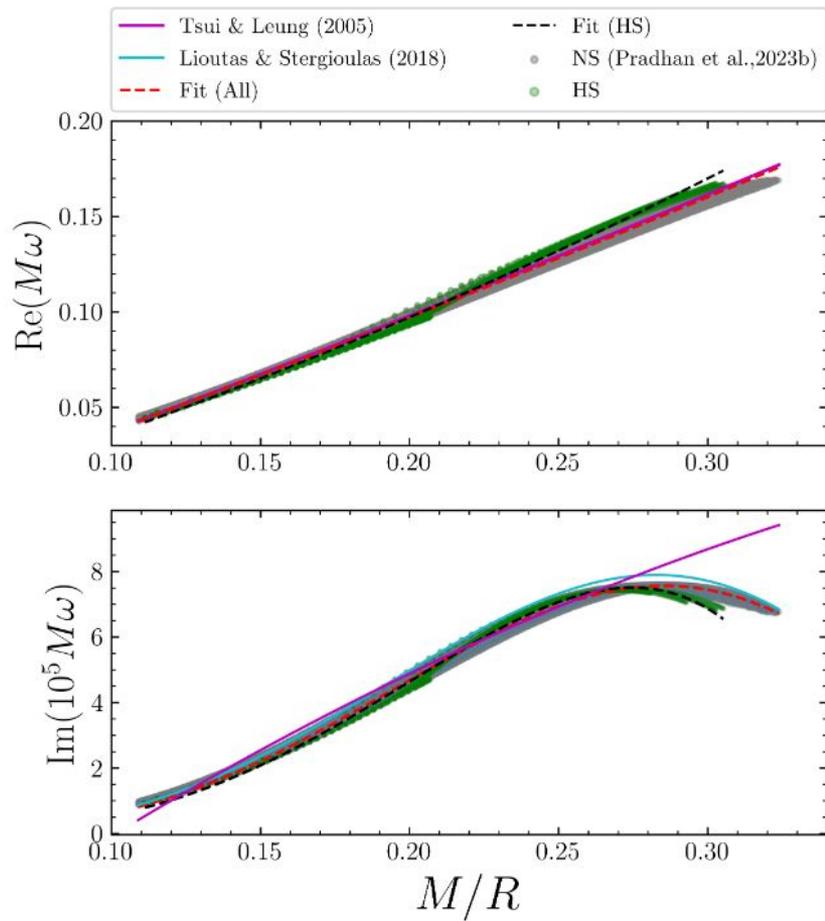
Results: GW170817



$$\tilde{\Lambda} = \frac{8}{13} \left[(1 + 7\eta - 31\eta^2)(\Lambda_{2,1} + \Lambda_{2,2}) + \sqrt{1 - 4\eta}(1 + 9\eta - 11\eta^2)(\Lambda_{2,1} - \Lambda_{2,2}) \right]$$

- This work : URs from BKP et al., [*PRD 107 \(2023\) 2, 023010*](#)
- Inclusion of f-mode dynamical phase lowers the 90% upper bound of $\tilde{\Lambda}$ by ~15%.
- Λ_3 and Σ_2 or the choice of multipole Love relation has no significant effect .





The EOS Model : Phenomenological Description

- [A. Ayriyan, H. Grigorian, EPJ Web of Conferences, p. 03003, \(2018\).](#)
- [A. Ayriyan, N. Bastian, D. Blaschke, H. Grigorian, K. Maslov, D. N. Voskresensky, PRC 97, 045802 \(2018\).](#)
- [V. Abugaryan, D. Alvarez-Castillo, A. Ayriyan, D. Blaschke, H. Grigorian, Universe, 4, 94 \(2018\).](#)

- ★ Surface tension effect leads to existence of pasta phases.
- ★ A parabolic interpolation method used to construct the mix phase.

$$p(\mu) = \begin{cases} p^H(\mu), & \mu \leq \mu_{cH}, \\ P^M(\mu) = \alpha_2(\mu - \mu_c)^2 + \alpha_1(\mu - \mu_c) + P_c + \Delta P, & \mu_{cH} \leq \mu \leq \mu_{cQ}, \\ p^Q(\mu), & \mu \geq \mu_{cQ} \end{cases}$$

$\alpha_1, \alpha_2, \mu_{cH}, \mu_{cQ}$

Determined from the continuity of pressure and its derivative.

- ★ Mixed Phase is parametrized by $\Delta p = \Delta P/P_c$.
- ★ $\Delta p = 0$: Maxwell Construction.

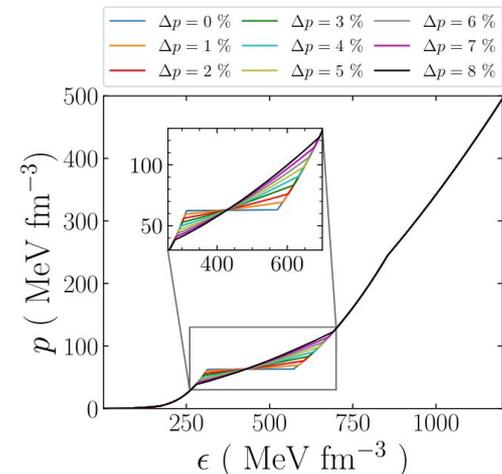
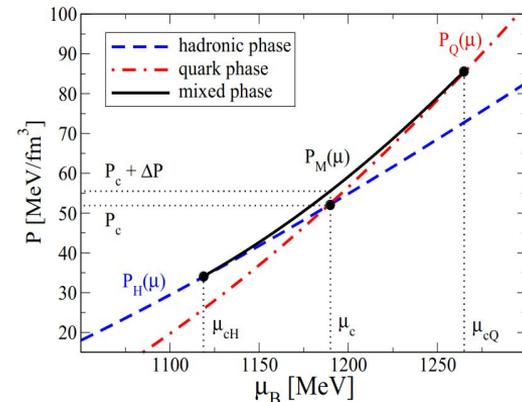
ACB4 Parametrization:

- [D. E. Alvarez-Castillo, D. Blaschke PRC. 96, 045809, \(2017\).](#)
- [V. Paschalidis, K. Yagi, D. Alvarez-Castillo, D. Blaschke, A Sedrakian, PRD. 97, 084038, \(2018\).](#)

$$P(n) = \kappa_i \left(\frac{n}{n_0} \right)^{\Gamma_i}, \quad n_i < n < n_{i+1}, \quad i = 1, \dots, 4$$

$$P(\mu) = \kappa_i \left[(\mu - m_{0,i}) \frac{\Gamma_i - 1}{\kappa_i \Gamma_i} \right]^{\frac{\Gamma_i}{(\Gamma_i - 1)}}$$

i	Γ_i	κ_i [MeV fm ⁻³]	n_i [fm ⁻³]	$m_{0,i}$ [MeV]
1	4.921	2.1680	0.1650	939.56
2	0.0	63.178	0.3174	939.56
3	4.00	0.5075	0.5344	1031.2
4	2.80	3.2401	0.7500	958.55



Inclusion of Observational Uncertainties

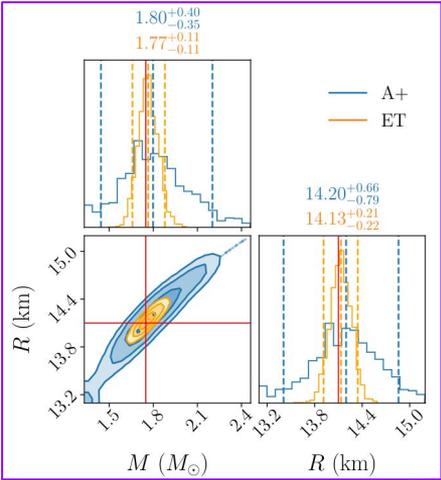
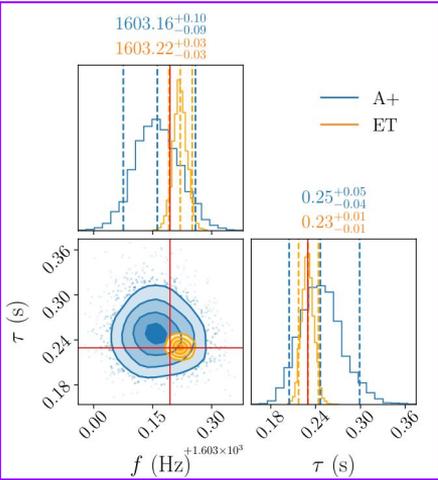
- Parameter Estimation for GW signal parameters are carried out using **Bilby**.
- Priors are kept,
 - logUniform in E_{gw} .
 - $f \in \text{U}[800,3500]$ Hz.
 - $\tau \in \text{U}[0.05,0.7]$ s.
 - We fix the distance.

$$h(t) = h_0 \exp(-t/\tau_f) \sin(2\pi\nu_f t), \quad t > 0$$

$$h_0 = 4.85 \times 10^{-17} \sqrt{\frac{E_{\text{gw}}}{M_{\odot} c^2}} \sqrt{\frac{0.1 \text{sec} \text{ kpc}}{\tau_f d}} \left(\frac{1 \text{kHz}}{\nu_f} \right)$$

- Frequency can be measured accurately in A+ and ET.
- Damping time can have error ~20-50% in A+ and ~5-15% in ET.
- With a 90% CI, M and R can be measured to ~6% and ~2% in ET.
- With a 90% CI, R can be measured to ~2% in ET.
- Spin correction is ignored
- Fastest spinning pulsars are also ignored.

$$\frac{\sigma_c^s}{\sigma_0} = 1.0 - 0.27 \left(\frac{\Omega}{\Omega_K} \right) - 0.34 \left(\frac{\Omega}{\Omega_K} \right)^2$$

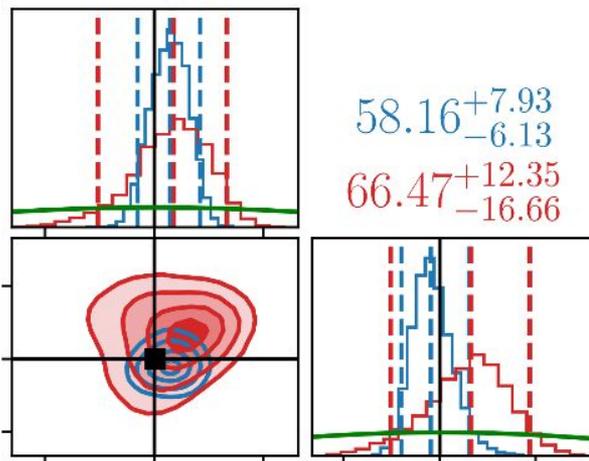


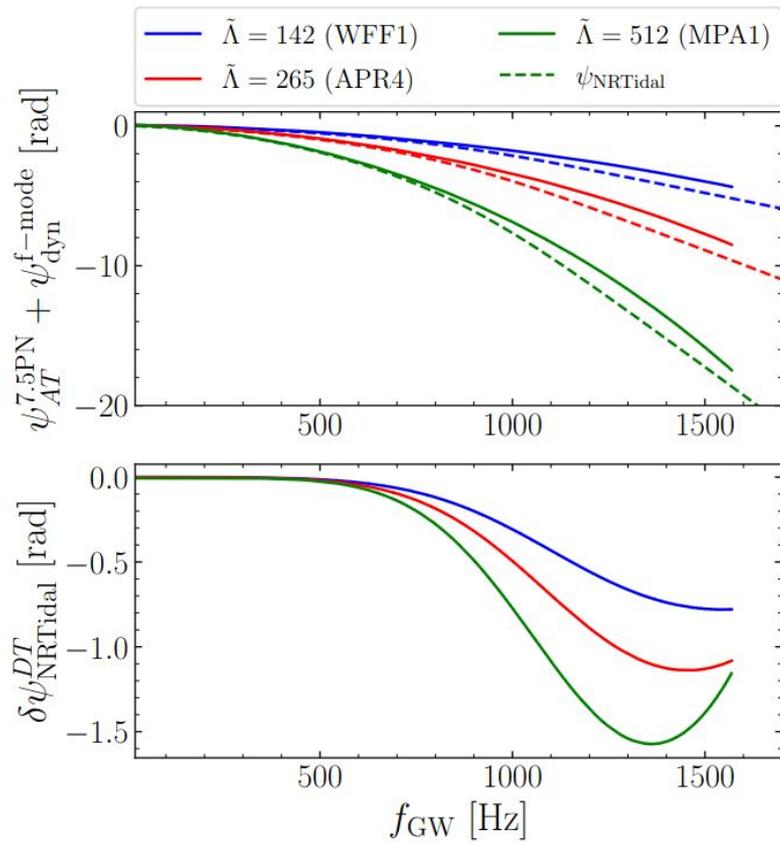
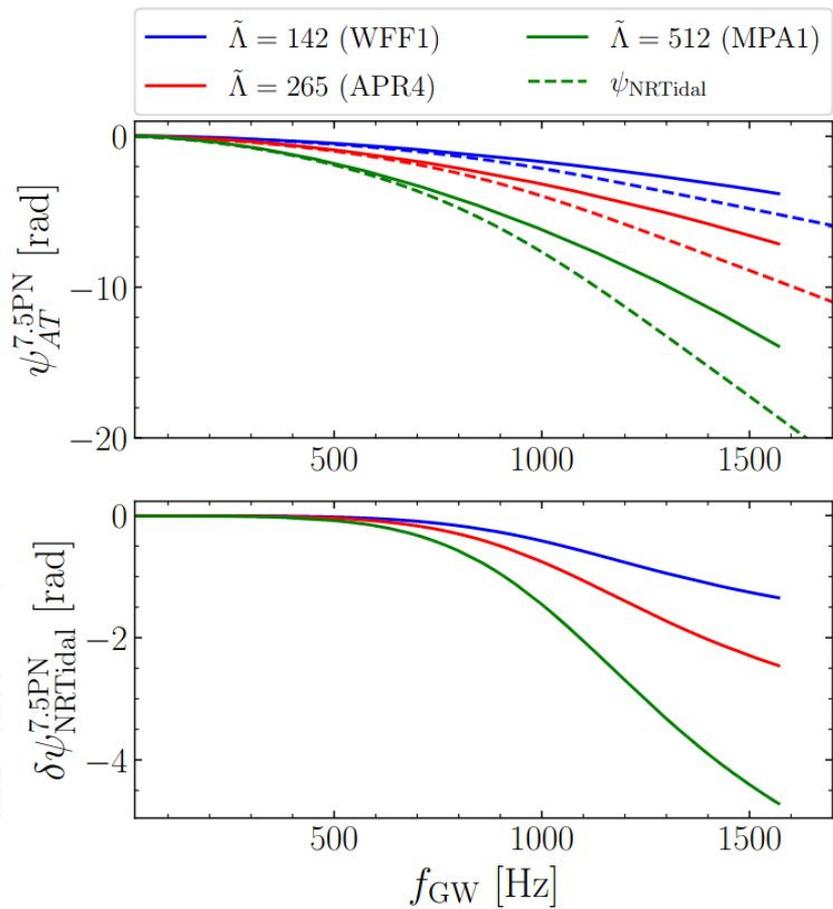
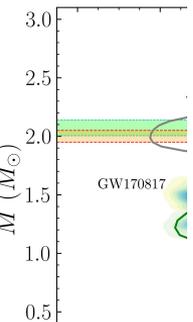
$0.657^{+0.0138}_{-0.0148}$

$0.659^{+0.0244}_{-0.0346}$

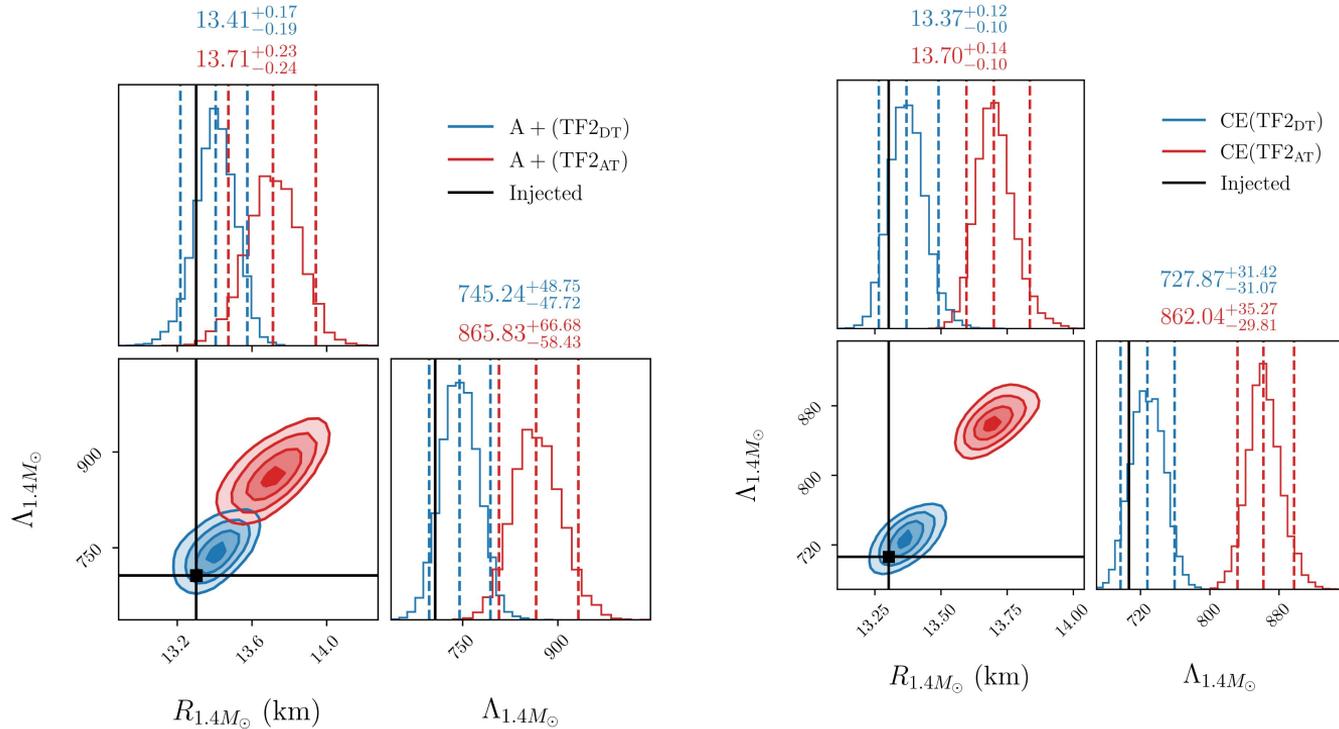
$58.16^{+7.93}_{-6.13}$

$66.47^{+12.35}_{-16.66}$



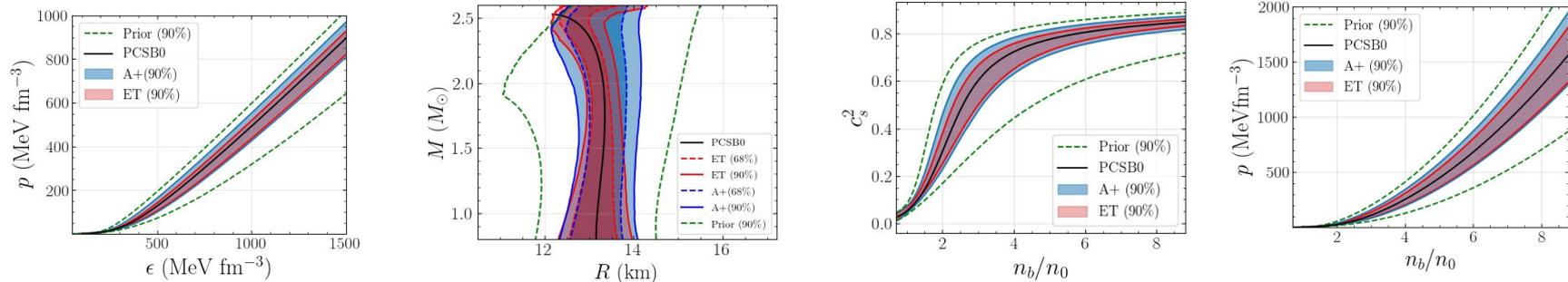


Results: Injection Studies



- Ignorance of dynamical tide can lead a bias of 15% in $\Lambda_{1.4M_\odot}$.
- Ignorance of dynamical tide can lead a bias of 3% in $R_{1.4M_\odot}$.

Results : From a single event from Vela Pulsar



- $f_{1.4M_\odot}$ can be estimated (within 90% CI) upto ~ 100 Hz (in A+) and ~ 70 Hz (in ET).
- In A+, within 90% SCI the $R_{1.4M_\odot}$, and $\tau_{1.4M_\odot}$ can be measured with in 6%, and 18%, respectively.
- With ET, the 90% SCI of $R_{1.4M_\odot}$, $f_{1.4M_\odot}$, and $\tau_{1.4M_\odot}$ can be measured within 4%, 4%, and 9%, respectively.

