The Electromagnetic Spectrometers

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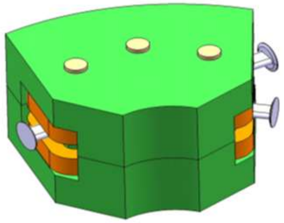
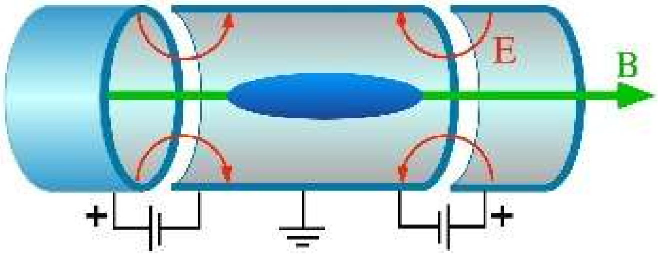
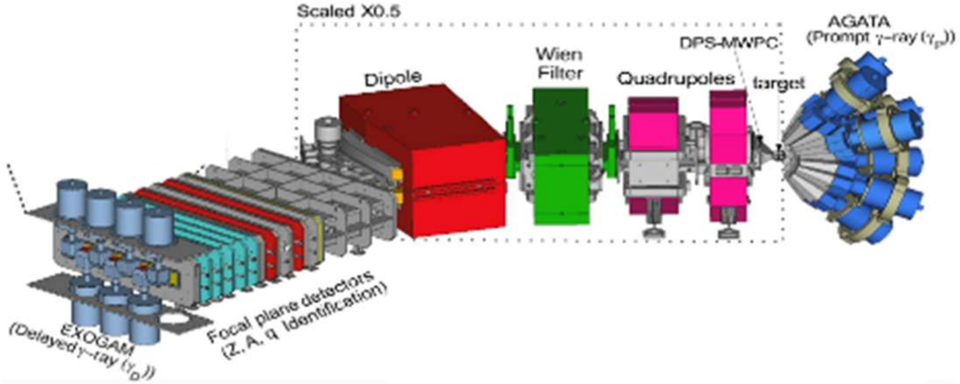
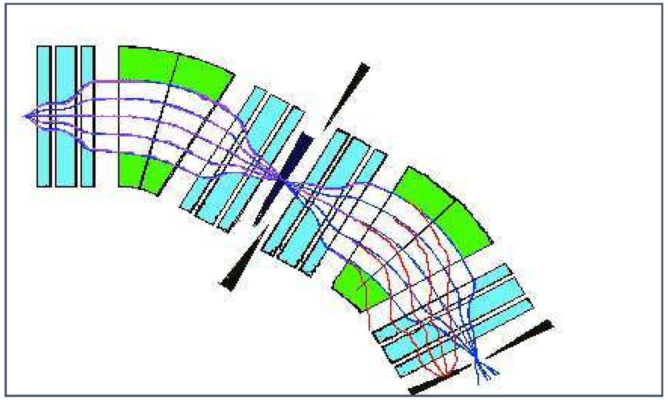








Du détecteur à la mesure : Roscoff 2025

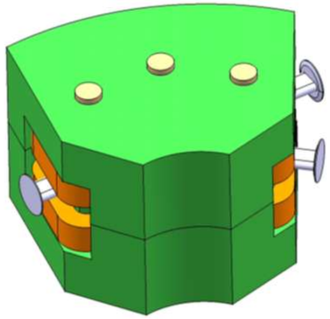
Summary

- ❑ **The different technics : basic concepts and history**

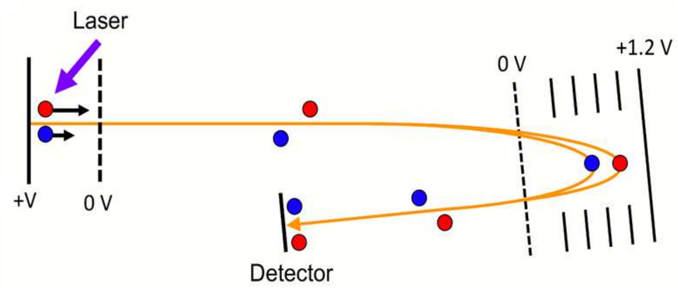
- ❑ **Magnetic spectrometers with accelerator beams**
  - **Part 1**
  - **Part 2**

- ❑ **Spectrometers without particle accelerators**

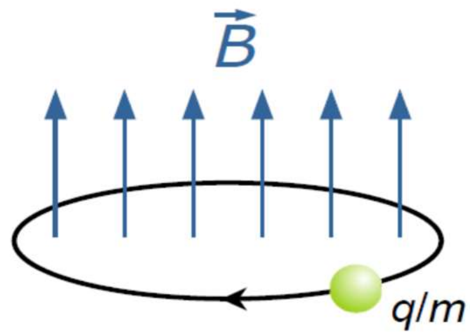
Basic concepts in the EM Spectrometers



- **Magnetic deflection** $B\rho$
- **Electrostatic deflection**
- **Magnetic+ Electrostatic deflection**



- **Time of flight (TOF)**
- **Cyclotron motion** (cyclo frequency ωc)

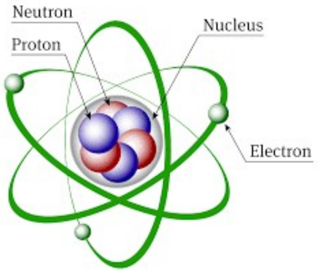


Electromagnetic forces (B,E) : $F=q\,(\vec{E} + \vec{v} \times \vec{B})$

Convenient to manipulate charged particles

- Many elementary particles are charged (p,e-,e^+, π^+, π-, $\mu-$,….)

Atoms can be ionized



Action at a certain distance , change the trajectories as a function of charge, masses, velocities (q,M,v)

Electromagnetic spectrometers : measuring particle motion (detectors) under magnetic or electric field

Motion in a transverse magnetic field

Lorentz force :

$$\frac{d\mathbf{p}}{dt} = \mathbf{F} = q\,(\mathbf{v} \times \mathbf{B}\;)$$

$(\mathbf{v} \perp \mathbf{F})$ so $\mathbf{v}.\, d\mathbf{p}/dt \sim d\mathrm{v}^2/dt \;\; = \mathbf{0}$

hence the modulus $|\mathbf{v}| = \text{constant}$

and $\gamma = \text{constant}$

The motion is circular and uniform

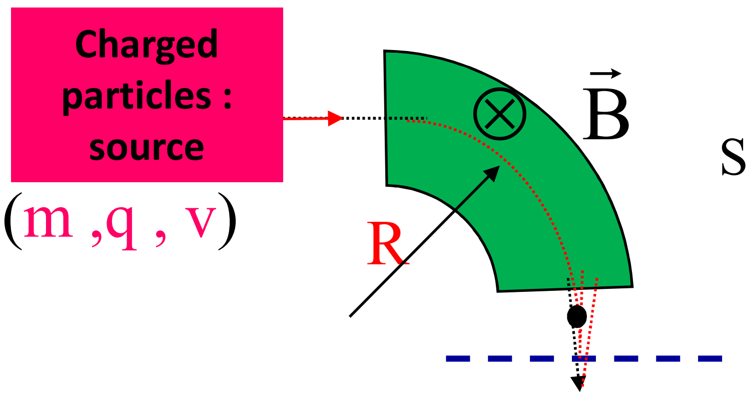
$$\frac{d\mathbf{v}}{dt} = \frac{|\mathbf{v}|^2}{R} e_r$$

trajectory Radius R $\quad R = \gamma \dfrac{m\mathrm{v}}{\mathrm{q\,B}}$

$$\gamma\, m\, \mathrm{v}^2 / R = q\, |\mathbf{v}|\, |\mathbf{B}|$$

We define the magnetic rigidity : Bρ [Tesla.m]

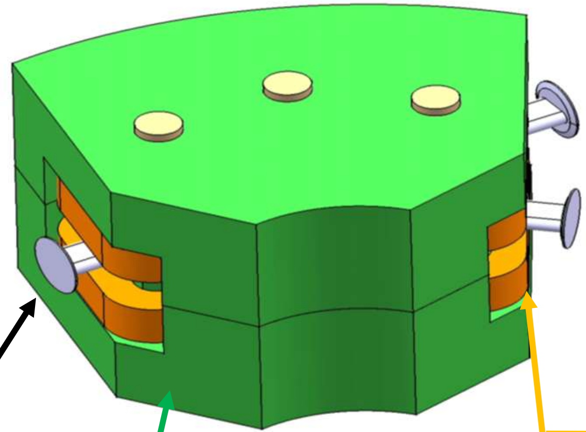
$$B\rho \overset{def}{=} \gamma \frac{m\mathrm{v}}{\mathrm{q}}$$



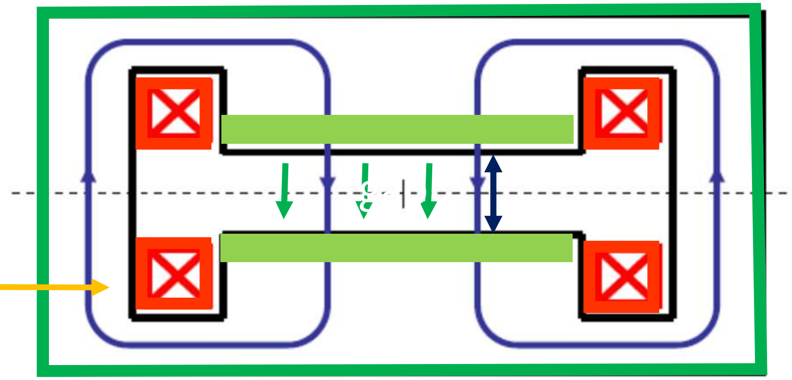
So the trajectory radius given By R= Bρ /B

$$R = \frac{B\rho}{B} = \gamma \frac{m\mathrm{v}}{\mathrm{q\,B}}$$

Magnetic dipole : technical details



H type magnetic dipole



Beam pipe (vacuum chamber)

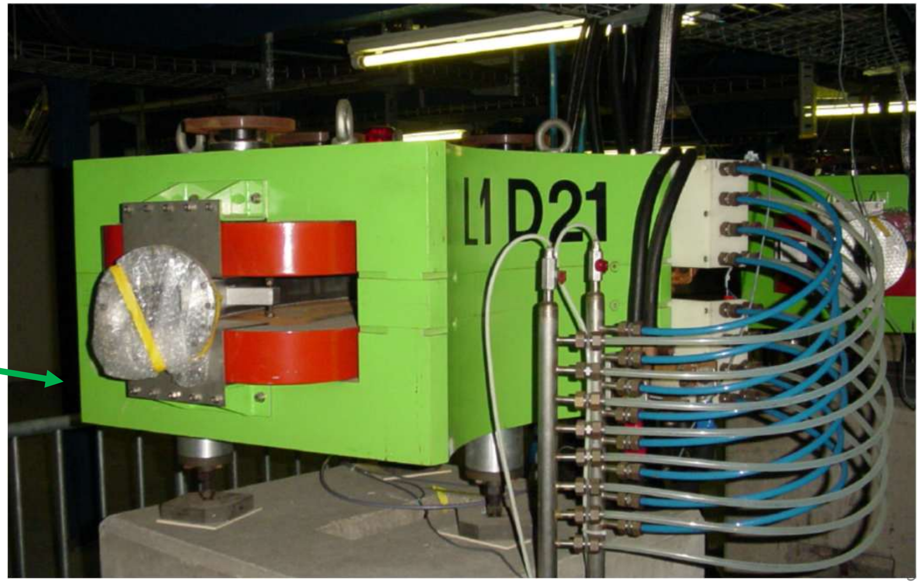
Coils (copper)

Yoke & Poles (N & S)

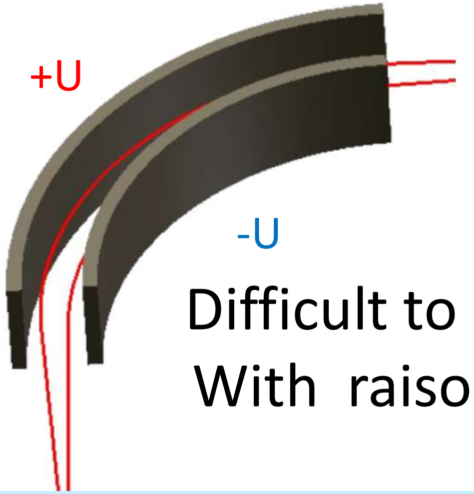
Poles saturates at 1.6-2.0 Tesla

B ~ I (power supply)

Power supply (100-1000A)

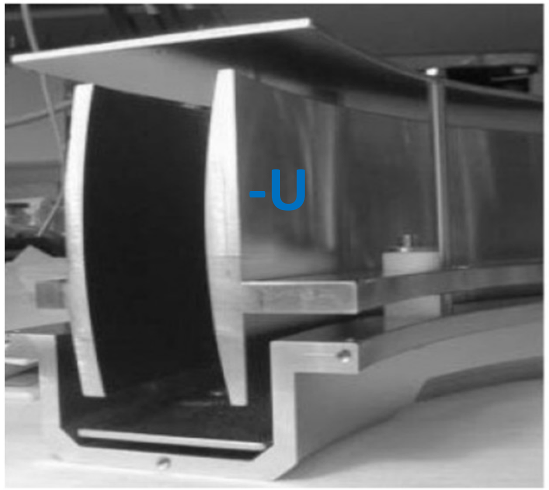


Electrostatic deflection



$$F = q\boldsymbol{E}$$

Difficult to bend energetic particles
With raisonnable E.field : sparking



Limitations :

transverse electric forces are not very efficient

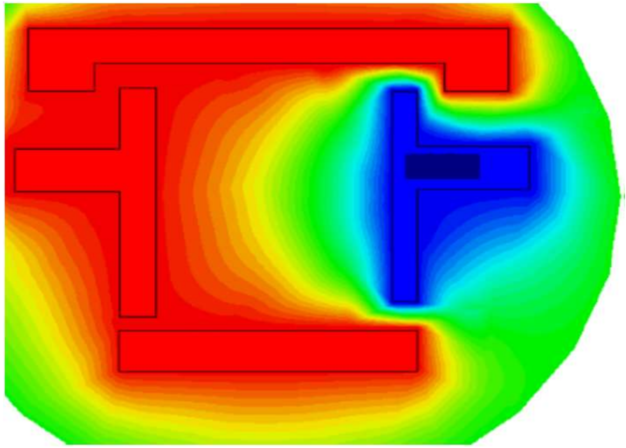
Not strong : Electric force $\mathbf{F_E} \sim q\ E$

Magnetic force $\mathbf{F_B} \sim q\ V \times B$: proportional to **v,**
adapted to high energy particle

Not selectiv in mass with electrostatic acceleration

Radius= $\gamma\ m\ V^2 /\ qE\ =\ 2\ U_{source}\ /\ Ed$

R independant of m, q



*E not very uniform with end fields

Electrostatic selection : many limitations but …

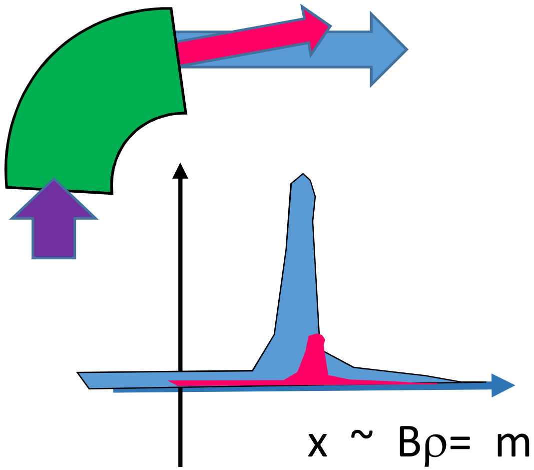
- Electric *deflector* can complement a magnet

(velocity compensation possible $R_E \sim \gamma\, mV^2/\, qE$ $R_B \sim \gamma\, mV/\, q$

Aston : Nobel prize in 1929 with a « mass spectrograph » = Electric+ Magnetic field

identification of ^{20}**Ne** ^{22}**Ne**

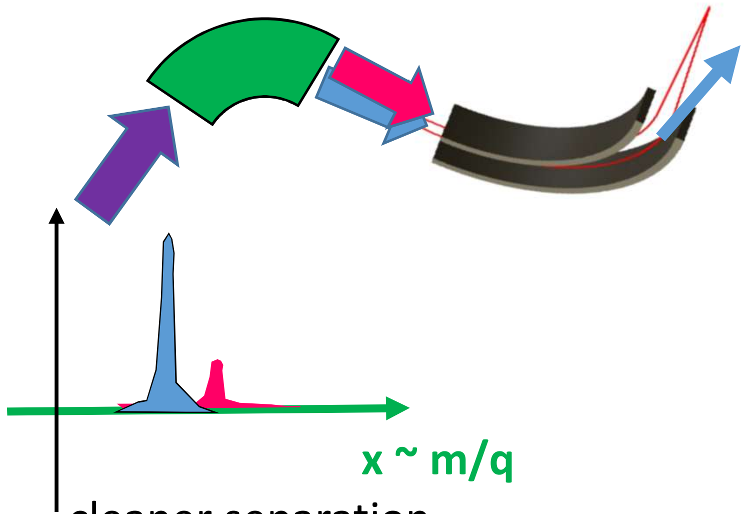
1) B field only



x ~ Bρ= m v/q

velocity distribution tails

2) Aston device « B + **E** » field



x ~ m/q

cleaner separation

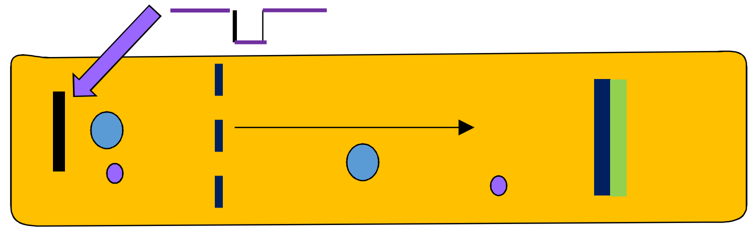
position x independant of v

TOF spectrometer for mass analysis

Time Of Flight : ionisation (pulsed laser) + acceleration

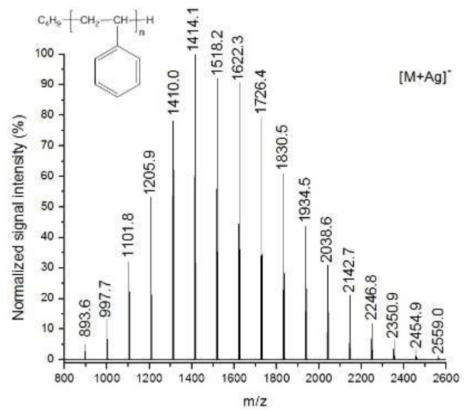
laser

$$+ q\,U = mV^2/2 = m\,(L/\,Tof)^2$$



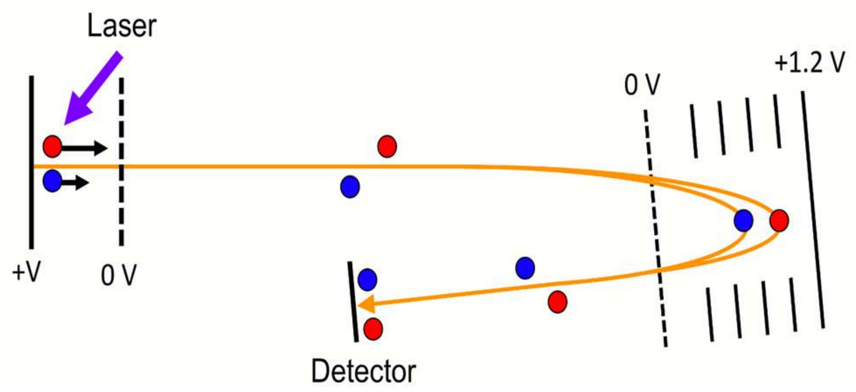
$$T.o.F = L\,/\,V = L\,.(\,m/qU\,)^{1/2}$$

+U 0Volt Time detector



Reflectron (ToF ~ m/q)

isochronism (TOF no dependance with U)



$$Tof = L\,(m/q)^{1/2}$$

$$L = F(U)$$

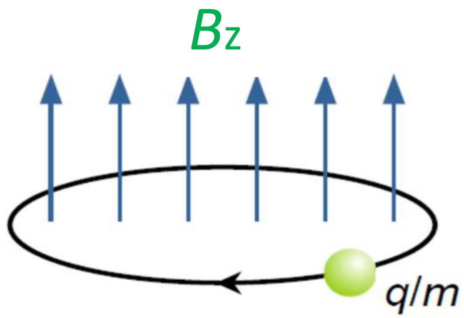
for $U = U0$ $\quad$ $L = L0$

for $U = U0 + \Delta U$ $\quad$ $L = L0 + \Delta L$

Cyclotron motion :
the secret of the Penning Trap

In a homogenous transverse field, B_z

The motion is circular and uniform



$$R = \gamma \frac{m\mathrm{v}}{\mathrm{q\,B}}$$

$F_{revolution}$ (s^{-1}) = v / 2π R

R = γ mv /q B

. $\omega_c = 2\pi$ **$F_{revolution}$ = q B / γ m**

Measuring the ion frequency ω_c at low energy (γ=1) gives **m/q**

(if B_z is well known)

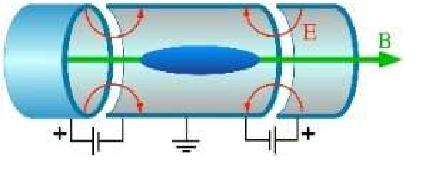
History / evolution of the electromagnetic spectrometers

- Thomson (1897) : cathode rays measurement
  Noble prize in 1906 for discovery of the electron
  : measurement of (m_e / q_e) ratio with

- Aston (1929) : Magnetic selection+ Electrostatic
identification of isotopes Ne,Cl & mass measurement

- 1942 : Manhattan project U235/U238 enrichment

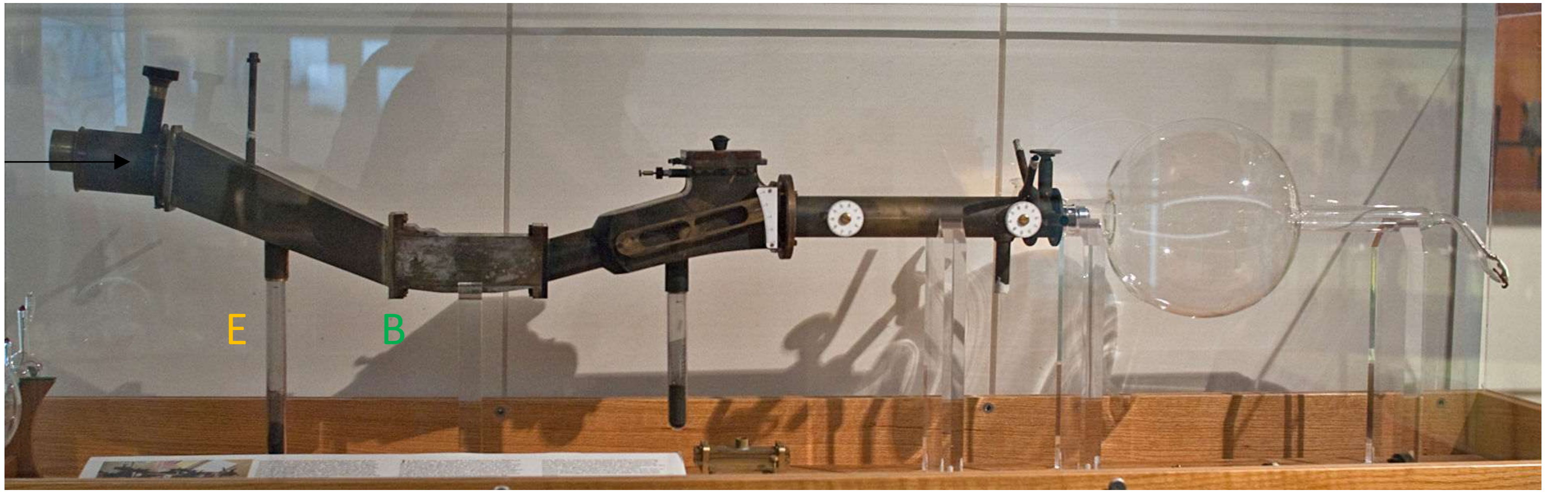
- H. Dehmelt (1955) : Penning Traps

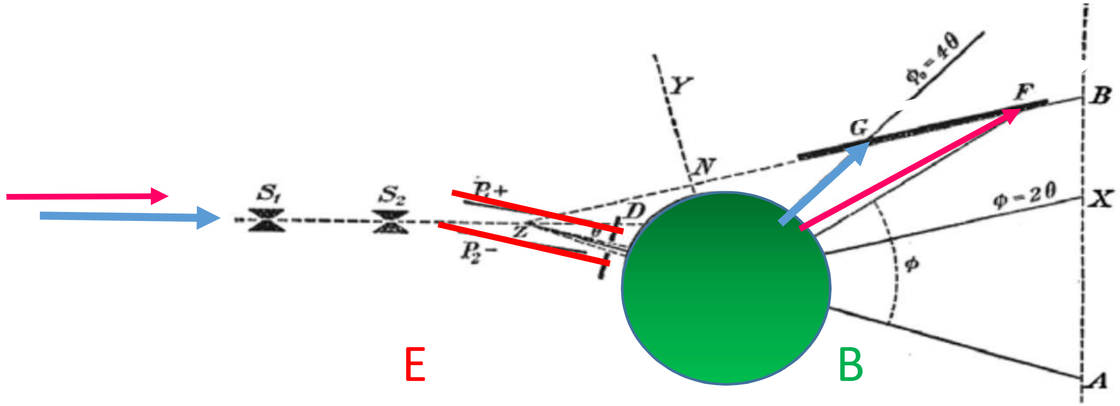


- « Commercial » mass analyzer : (TOF , MALDI, ESI,...)
Small size tools commercially available for many applications

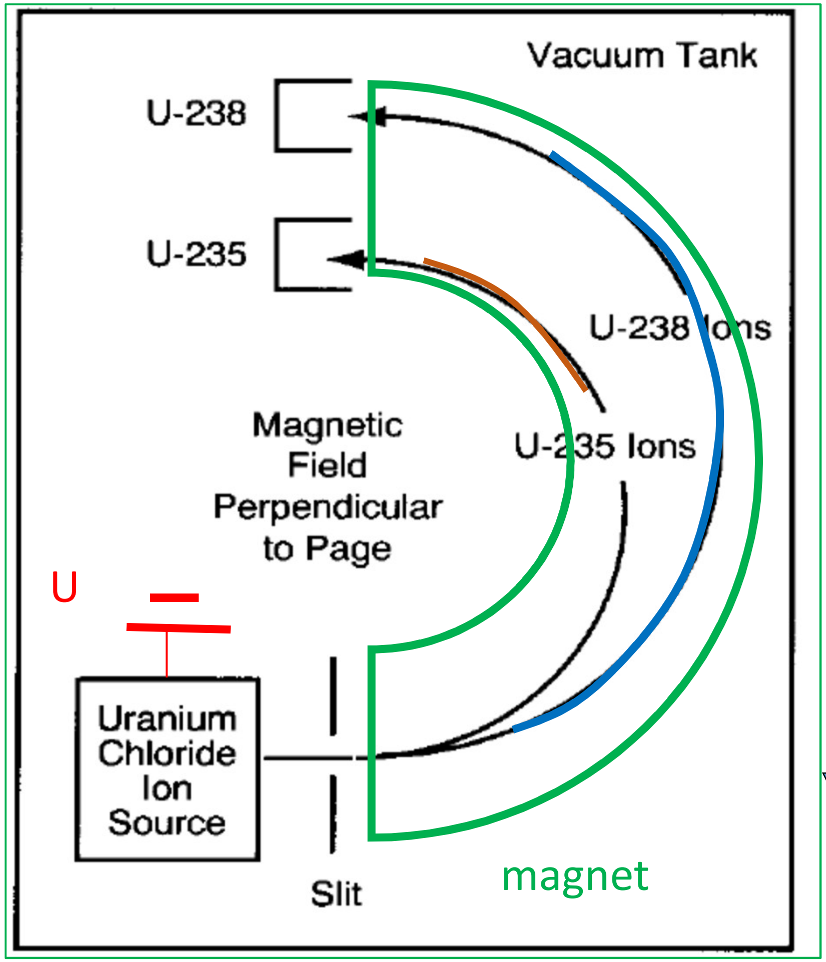
....

F.W Aston, Nobel Prize (1929):
Stable isotopes discovery : $^{20-22}Ne$; $^{35-37}Cl$





Isotope separator for atomic bomb (1942)

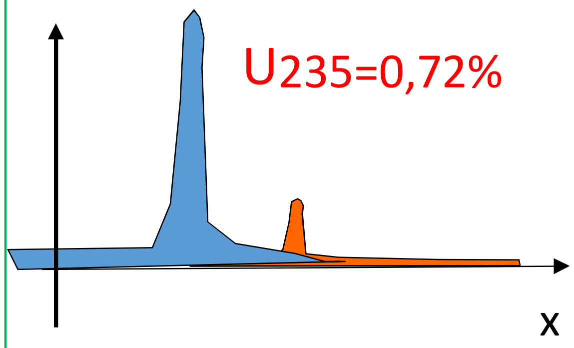


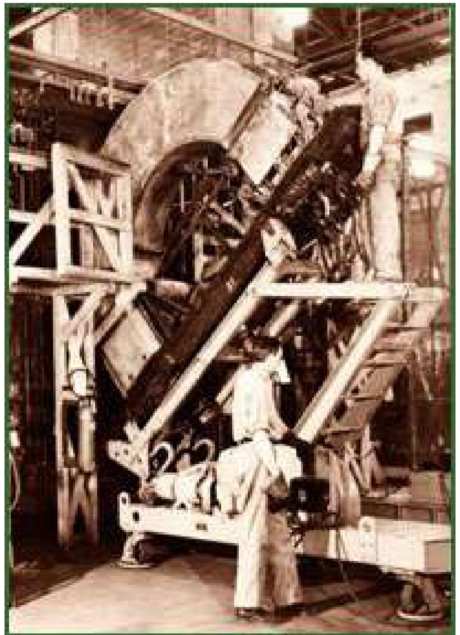
Radius= m v / (q B)

$V = (2\ q\ U\ /m)^{1/2}$

$R_{235}/R_{238} \sim (235/238)^{1/2}$

U238 : 99,27%





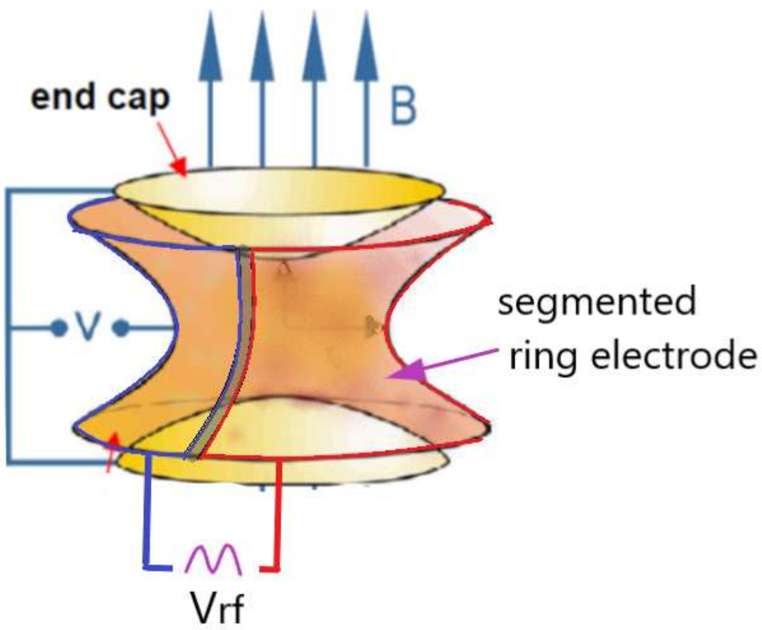
A "C-shaped" alpha calutron tank, Y-12, Oak Ridge.

1152 isotope magnetic separators working in parallel

at Oak Ridge (USA) in the 1940's for ^{235}U bomb

Penning Traps : high resolution mass spectroscopy
Rm ~ 10^{-5}-10^{-9}

Developped in the 50's : H. Dehlmet (Nobel in 1989) , F.M. Penning
Used in research but the applications are expending



3 tricks to mesure ω_c

1) Confinement (B_z+E_z)
static EM fields

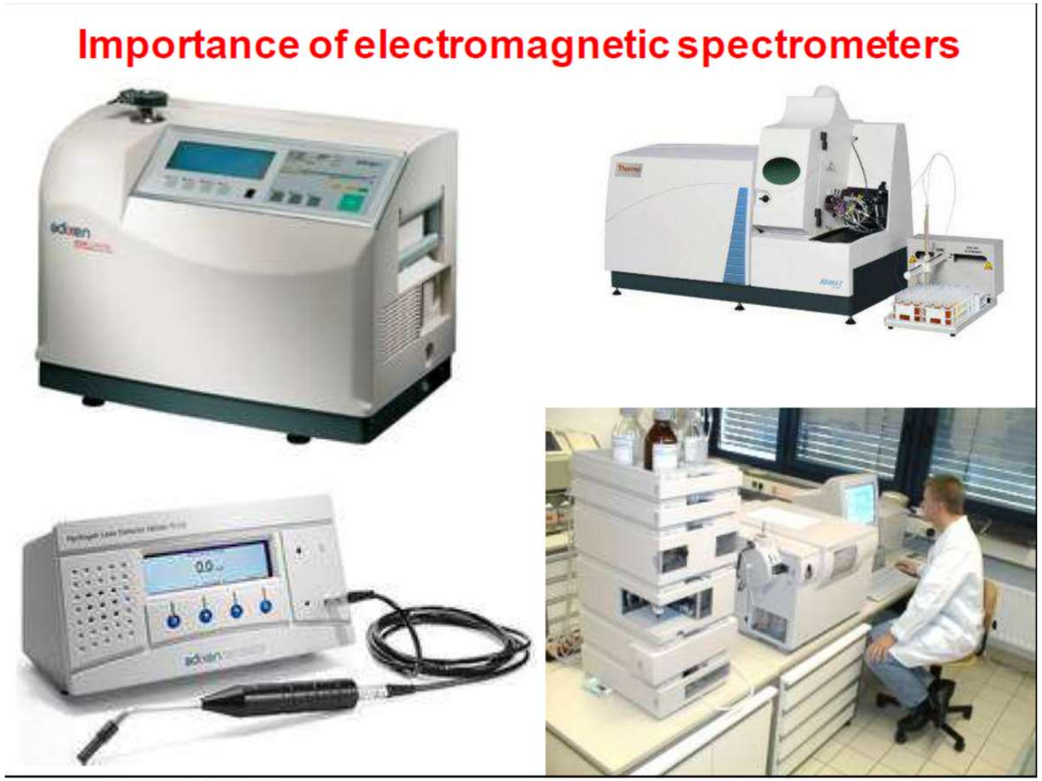
2) Excitation (RF field)

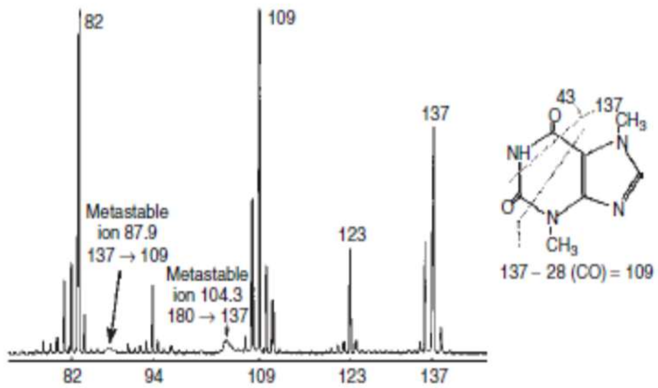
3) Extraction
Time of flight measurement

Commercial Mass analysers

Time Of Flight or magnetic

Chemistry, Pharmacy, industry
environment





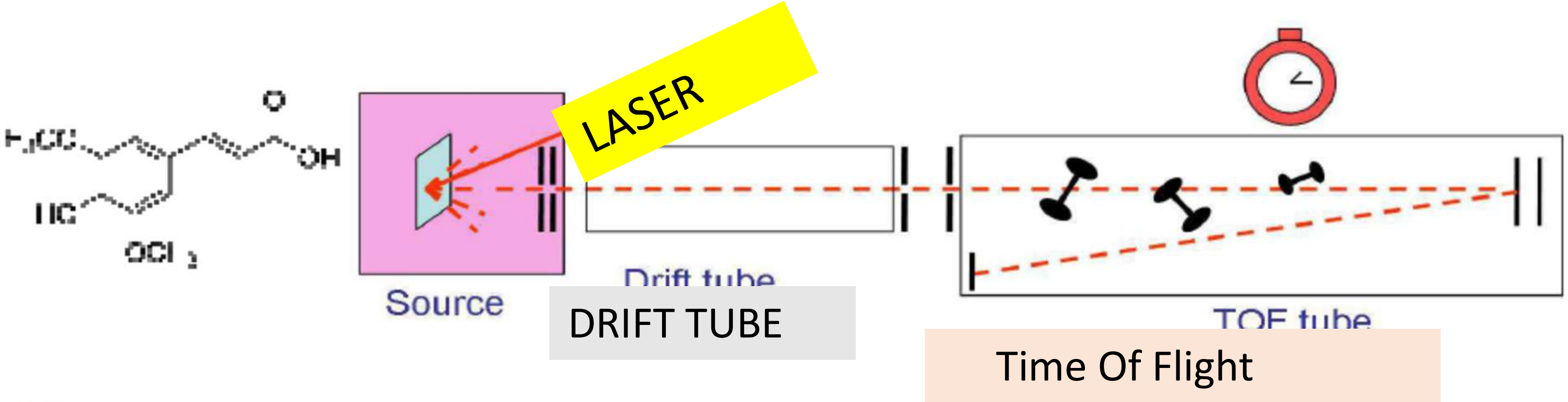
Molecule identification
Accurate mass measurements

Quantitation
Isotope ratio measurements

Commercial tools for Mass spectroscopy

MALDI = Matrix Assited LASER desorption /Ionization

MALDI-TOF
matrix assisted laser desorption/ionization TOF

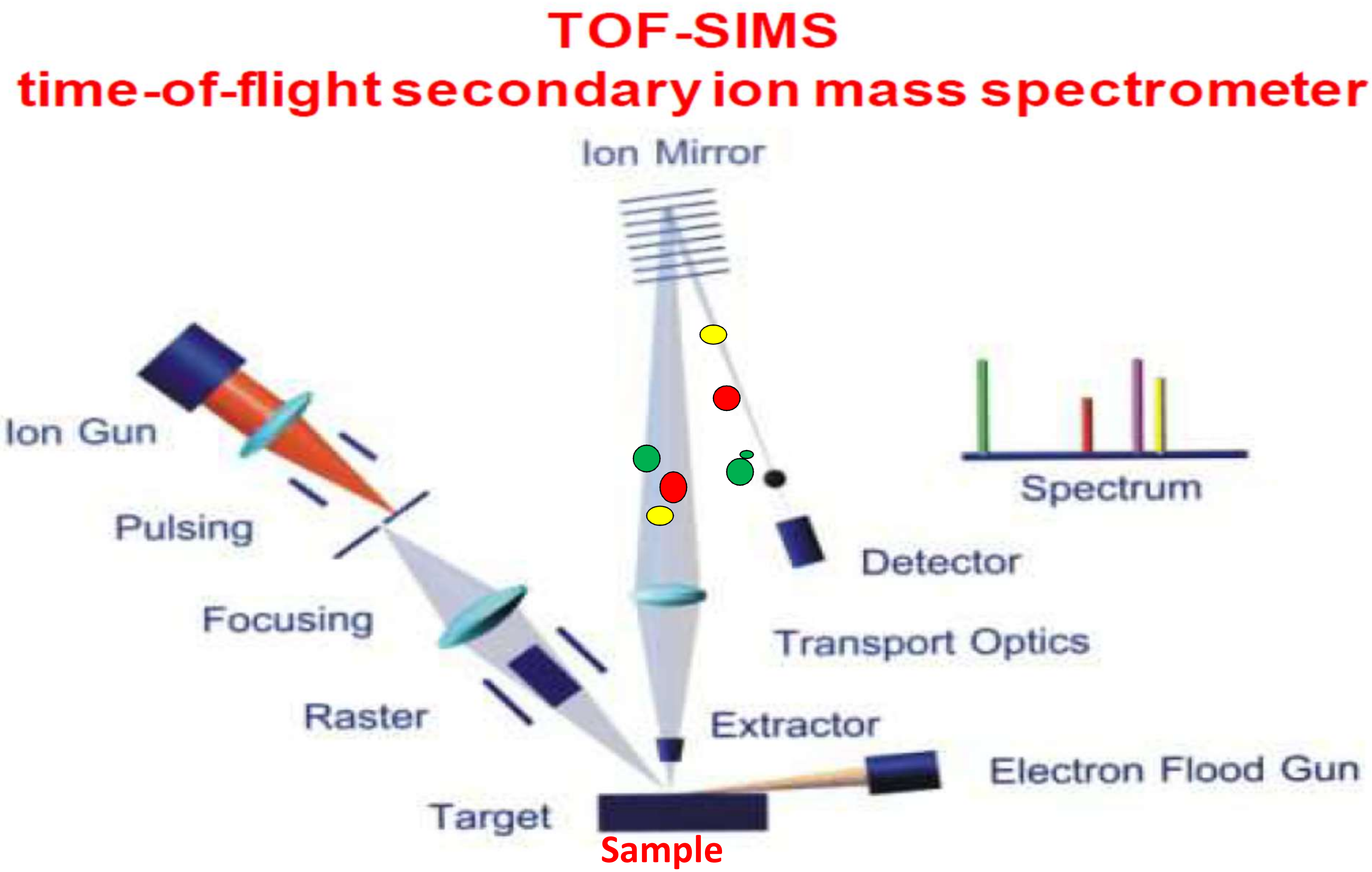


Matrix:

- low vapor pressure for operation at low pressure
- polar groups for use in aqueous solutions
- strong absorption in UV or IR for efficient evaporation by laser
- low molecular weight for easy evaporation
- acidic: provides easily protons for ionization of analyte

Commercial tools for Mass spectroscopy

SIMS = ionization of a sample with an ion gun

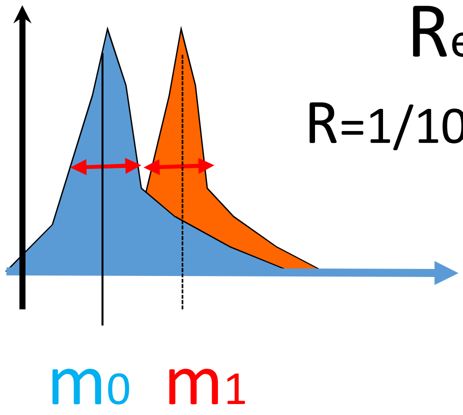


Spectrometer measurement : What are the performances ?

- Resolution, experimental definition : closest peak separation

$$\text{Resolution} = \delta m / m_0 = [m_0 - m_1]/m_0$$

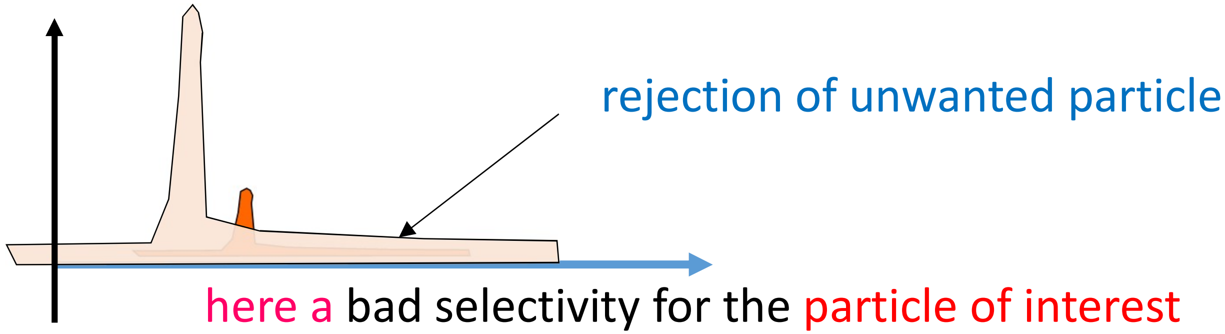
R=1/1000 means separation of mass with 1 per mille difference



x= position or Time Of Flight

- Selectivity : easy to select rare events (clean or not clean) ?

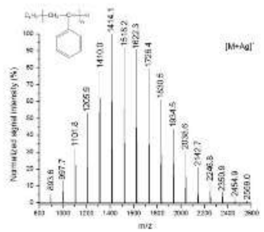
noise, background, distribution tails..

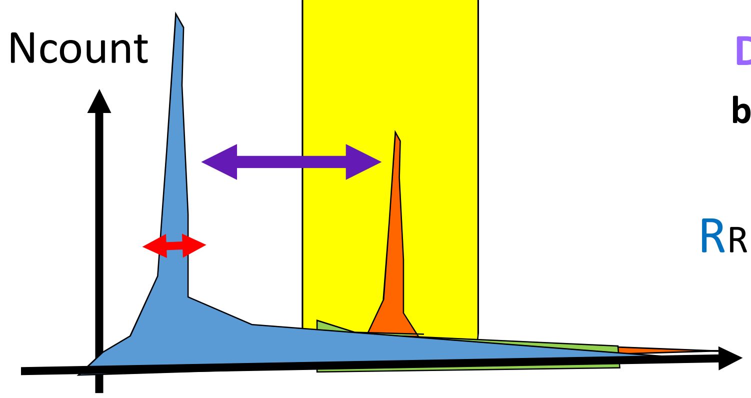


here a bad selectivity for the particle of interest

Spectrometer measurements :
performances depend on the context

- **Resolution** $R_{RMS} = \sigma_x$ / Dispersion





Dispersion = separation distance (meter)
bewteen two peaks with 100% difference (Bρ, Mass)

$R_{RMS} = 1/100$: what does it means ?
separation of two peaks with 1% mass difference at 1sigma

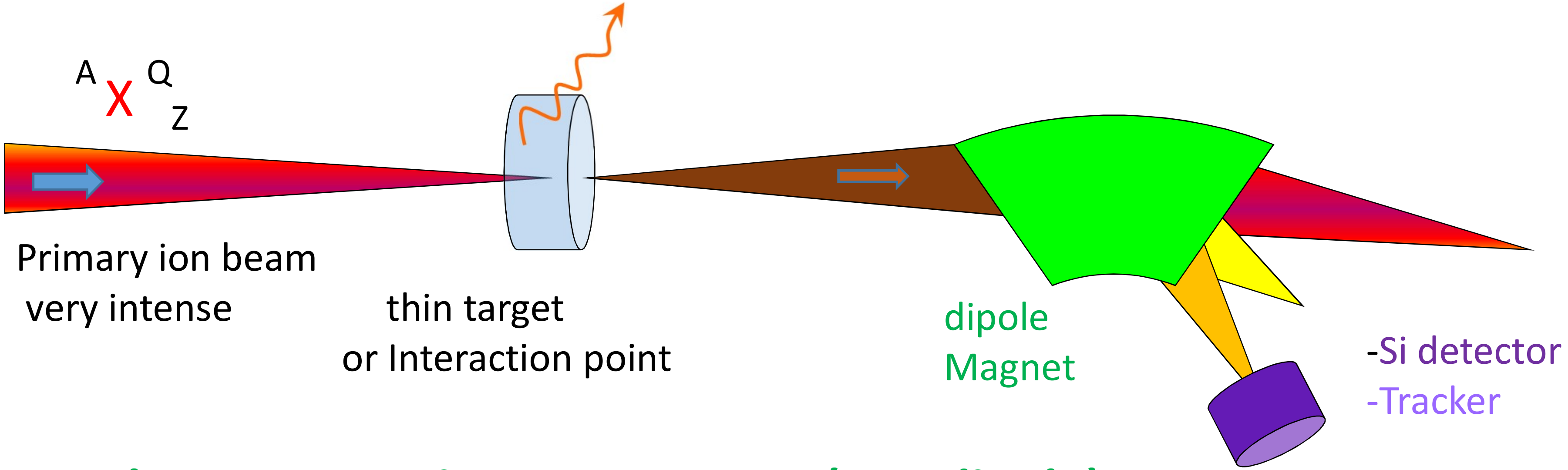
? many definitions ? $R_{fwhm} = \Delta x\_fwhm$ / Dispersion

$R_{5\sigma} = 5\ \sigma_x$ / Dispersion

- **Acceptance** : - Mass range
  (do you want to measure many different masses at the same time ?)
  - Angular acceptance (solid angle: steradian => efficency)

- **Rejection** = Nb of particles of interest (final)/ Nb of unwanted particles (initial)
  Rejection : distribution tails (context ,tools, noise, detector)

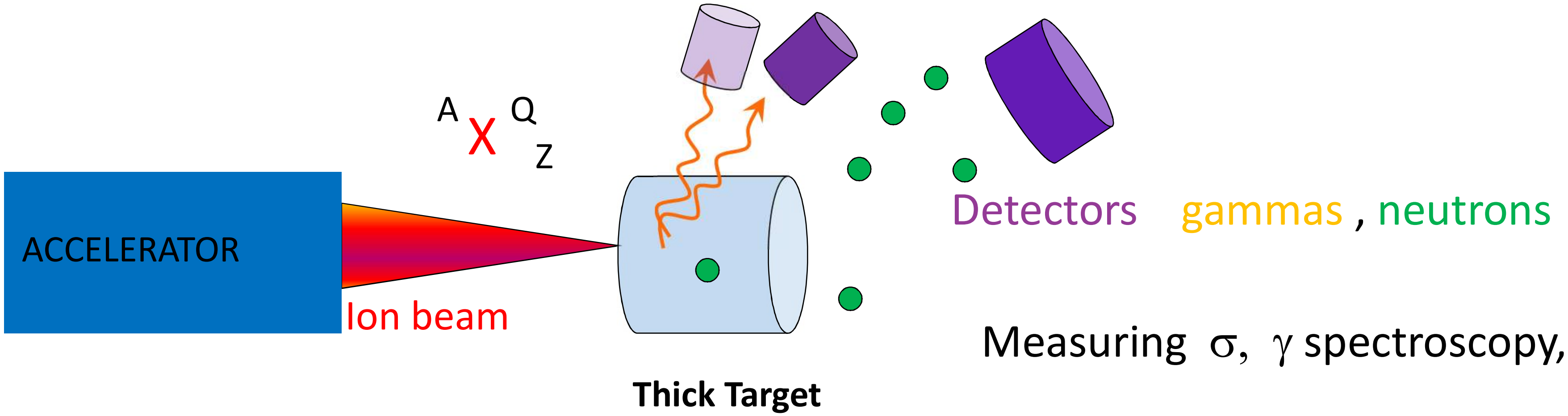
An other experiment
with a thin target & and a spectrometer



Electromagnetic spectrometer (one dipole)

- **Eliminate primary beam** (unreacted beam)
  (~ 10^{11-14} particles per second) = Separator
- **Measure Momentum** with very good resolution
  (final position gives R , P/Q =<B><R>)
- Help to **identify the reaction products**
- Select **very rare events** (selectivity)

An experiment in nuclear physics without spectrometer : not very selectiv



Measuring σ, γ spectroscopy,...

$$^{A}X_{Z} + \text{Target} \rightarrow n + {}^{A-1}X_{Z}^{*} \rightarrow \gamma + {}^{A-1}X_{Z}$$

Reaction of interest, but :

$^{A}X_{Z}$ + Target -> 2n + $^{A-2}X_{Z}^{*}$

$^{A}X_{Z}$ + Target -> 3n + 1p+ $^{A-3}X_{Z-1}$

Many other reaction channels

Reaction products not identified , ion energy (or P) not measured

Why an electromagnetic spectrometer ?

- ❑ **In application, where the velocity distribution is sharp**

  $B\rho$, TOF => perfect identification

A Spectrometer is always good

- ❑ **In nuclear/particle physics : reaction**

- velocity distribution is wide
- Accelerator beam, unwanted particles to be rejected (to observe rare events)

Whith a spectrometer Better selectivity / lower acceptance

permit to reconstruct momentun P /Q (with a good resolution)

P is not sufficient P/Q for identification (M,q, Z,…)

limitation of the acceptance (Magnet)

Whithout spectrometer : better acceptance, lower resolution

acceptance : size of the dectector (less constraint)

momentum resolution : given only by dectectors (E , TOF …), generally lower selectivity should be obtained only by a given radiation (gamma,…)

What is the best compromise ACCEPTANCE // SELECTIVITY and RESOLUTION ?

No general answer

Summary

- The basic concepts : Magnetic, TOF , Bρ

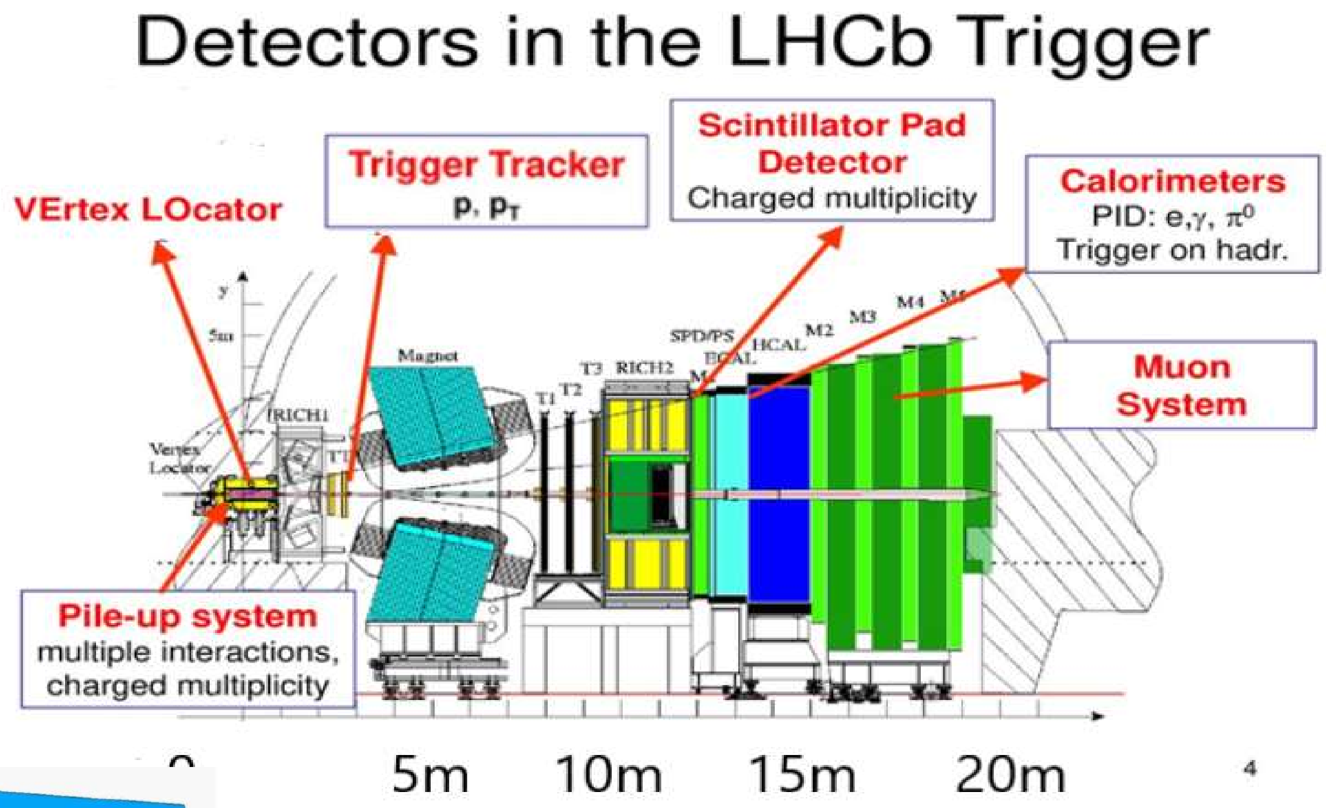
- **Magnetics spectrometer with accelerator's beams**
  - **Exemples (LHCb@Cern , Enge Split pole , VAMOS@ganil)**
  - **Technical devices : quadrupole, dipoles**
  - **Beam optics concept**

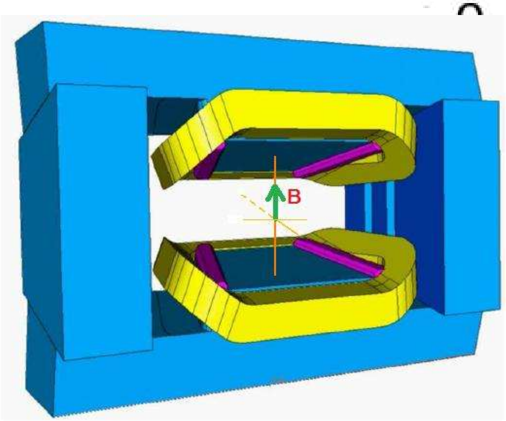
Part 2

- Beam optics concept
- Fragment separator

- **Spectrometers without accelerator**

A spectrometer in particle physics : LHCb





LHCb magnet $\int B_y \, dl = 4T.meter$

Angular acceptance ± 300mrad

Magnetic deflection in vertical field Bz of very high rigidity particles

$d\mathbf{p}/dt = q.\, v_s.\, B_y$

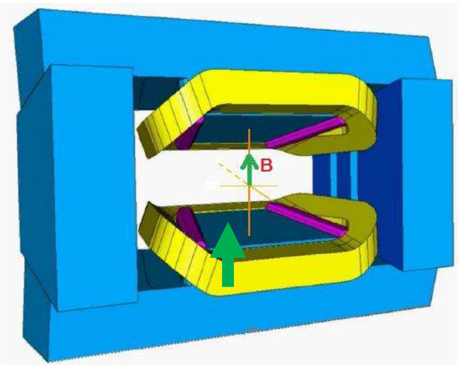
$\Delta \mathbf{p} = q\, v_s\, B_y\ \Delta t = q\ v_s\ B_y\ \Delta t = q\, B_y\ (v_s\, \Delta t) = q\, B_y\ (ds/dt).\, \Delta t$

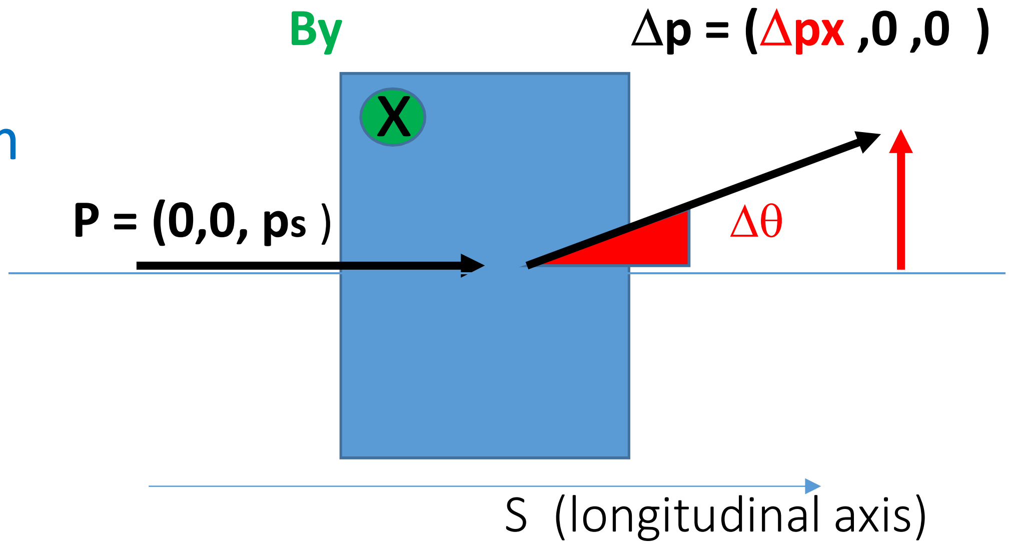
$\Delta p_x = q \int B_y\, ds$

$\Delta p_x / p_s = \tan \Delta\theta \sim \Delta\theta$ besides $B\rho = p_s / q$

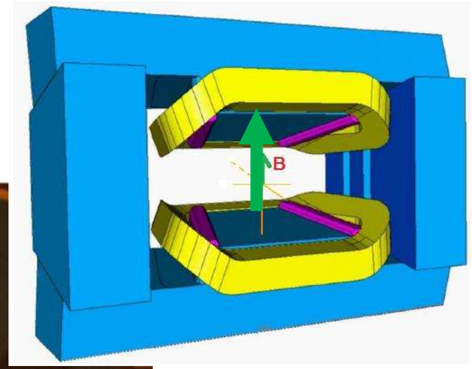
$\Delta\theta \sim \int B_y\ ds\ / B\rho$

LHCb **dipole** $\int B\, ds = 4$ T.m





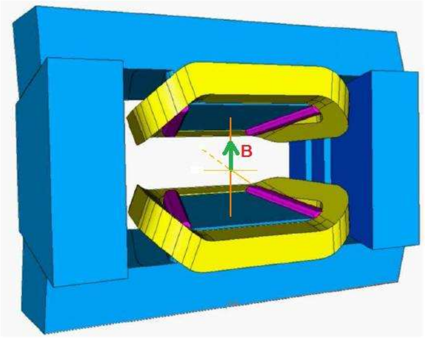
LHCb Magnet Pe=4.2 MWatt



$\Delta\theta \sim \int By \; ds \; / B\rho = 4.T.m \; / B\rho$

strong deviation $\Delta\theta$ = good $B\rho$ resolution

A spectrometer in particle physics :LHCb
momentum reconstruction ($B\rho=P/Q$)



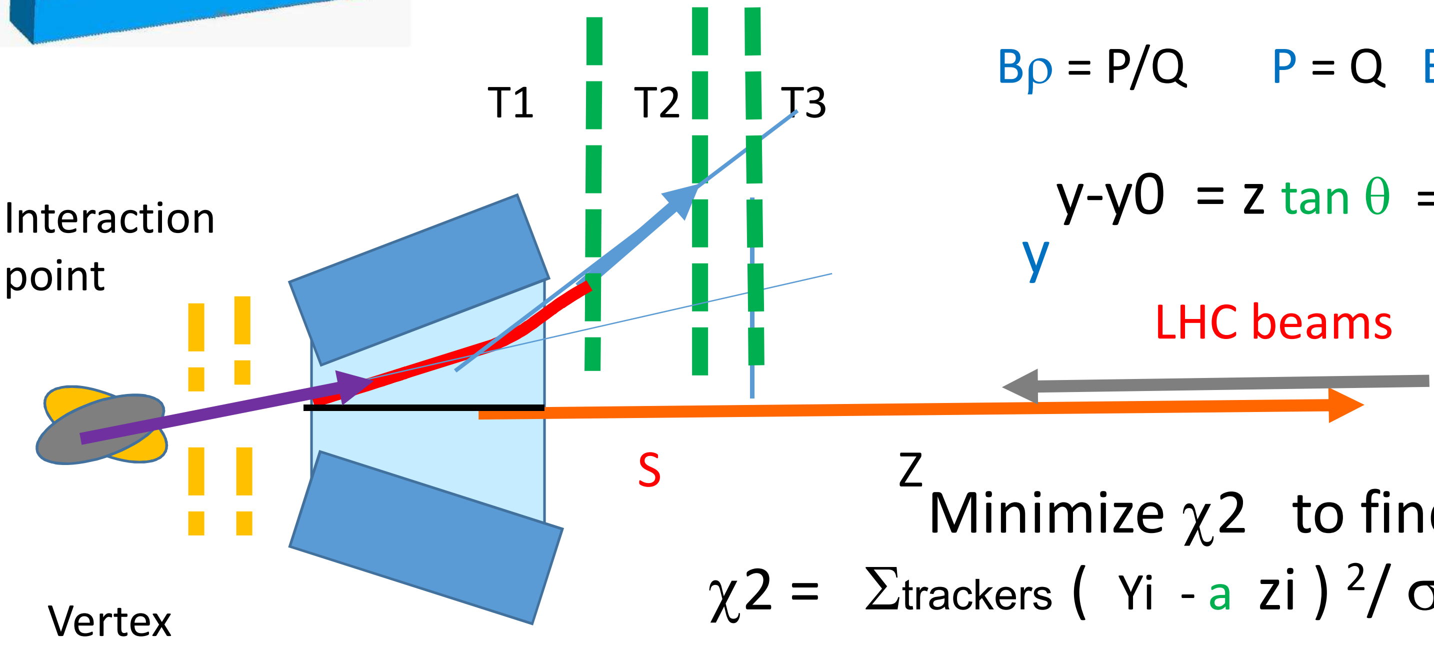
In homegonous field , the arc length S

$R\,\Delta\theta = S$

R is the radius of curvature

$B\rho = P/Q \qquad P = Q\ B\ R = Q\ B\ (S / \Delta\theta)$

$y\text{-}y0 = z \tan\theta = z\ dy/dz$



Minimize $\chi 2$ to find $\Delta\theta$

$\chi 2 = \Sigma_{\text{trackers}} \left(Y_i - a\ z_i \right)^2 / \sigma y^2$

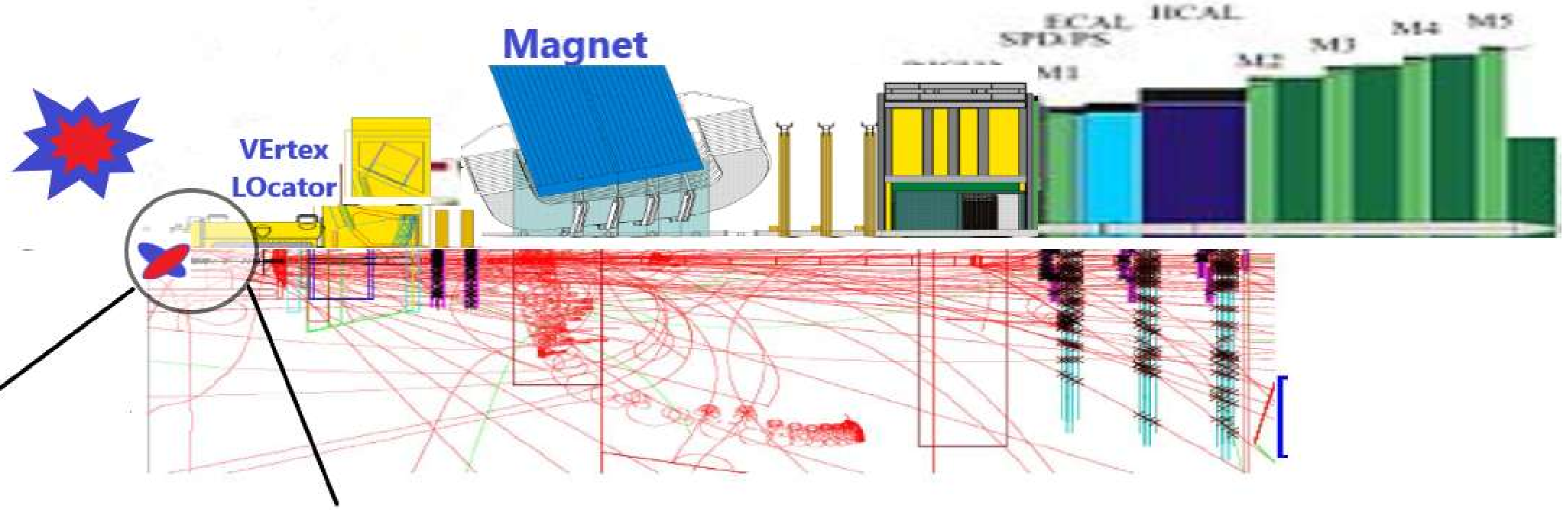
find a : $\tan\theta = a$

$P = Q\ BR \sim Q\,B\,(s/ \theta) \sim Q\ B\ (S / \text{atan}\ a)$

Typical Tracks in LHCb

Production and decay of a B^0_s meson (T ~ 10^{-12} s)

B^0_s : 2 quarks $(\underline{b}s)$



p+p ► B_s^0 ► D^+ ► π^- K^+ K^- ► μ^+ μ^- e^+ e^-

π^- ► γ γ ► e^+ e^-

Vertex S_0 — S_0 + 7mm — S_0 + 5m

bunch crossing region

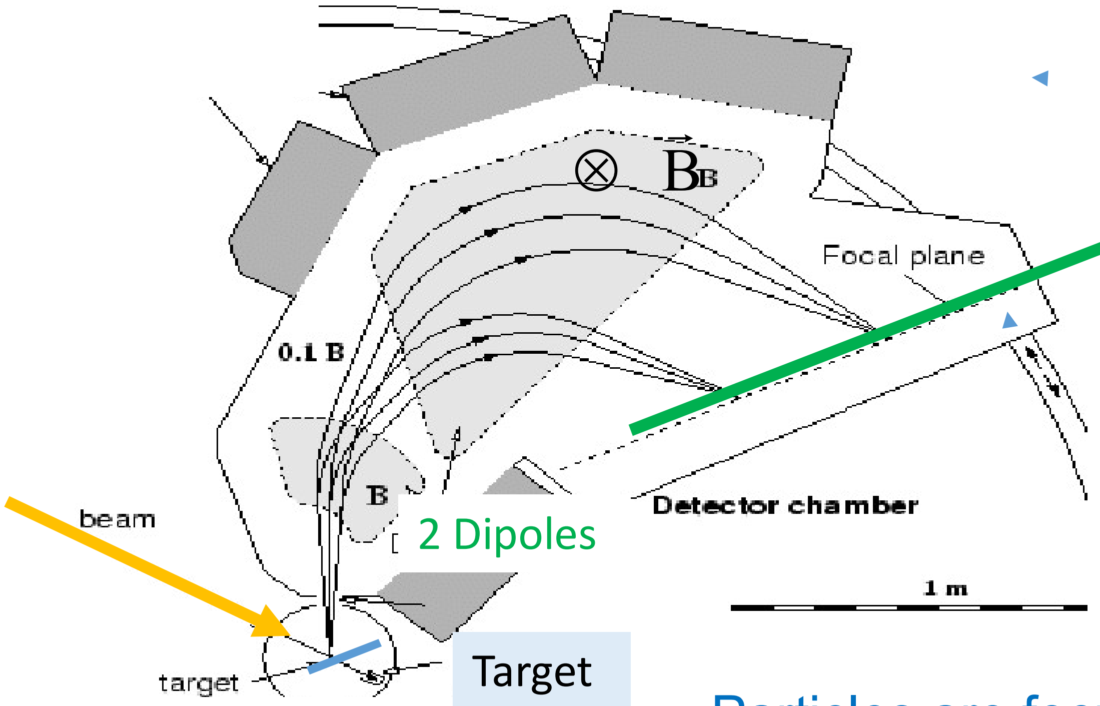
-

Jeroen van Tilburg, Track reconstruction,
https://fr.wikipedia.org/wiki/LHCb

Spectrometer exemple in nuclear physics

Split pole spectrometer : 2 magnetic dipoles

IJCLab (Fr) , TUNL (USA) , Florida SU, Notre Dame U (USA), …

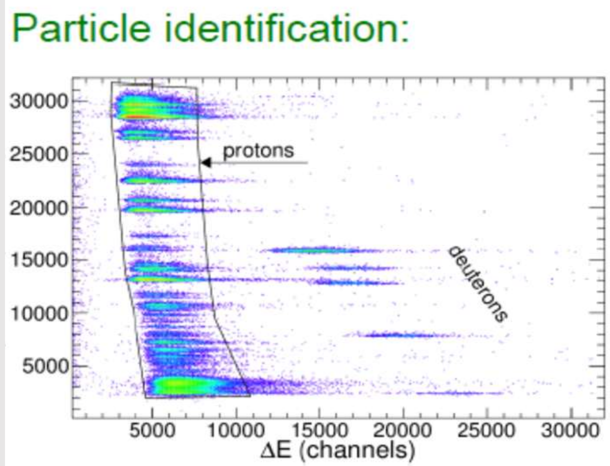


2 Dipoles

Target

Accelerator beams
Heavy ions

Position focal plane



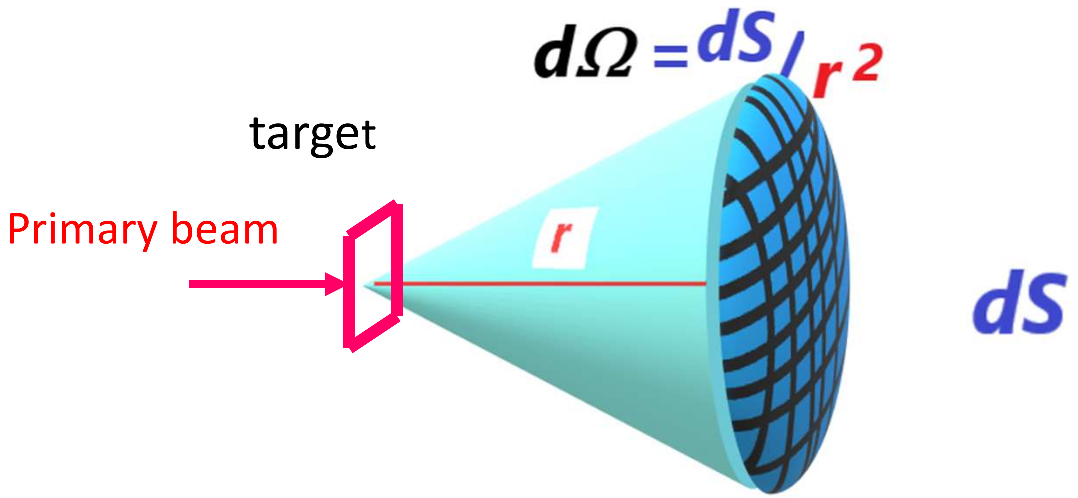
Particles are focused on a plane (the focal plane)
1 position gives the $B\rho$
Resolution $\Delta B\rho / B\rho = \Delta p/p = 1/1000$

Low angular acceptance $\Delta\Omega$= 1.7mstrd
~ ± 1.5° in Horizontal& Vertical plane

Spectrometer properties : the angular acceptance

The reaction products exit from the target with an angular dispersion

Vacuum chamber limitation / magnet aperture /detector Area



$$d\Omega(strd) = \frac{dS}{r^2}$$

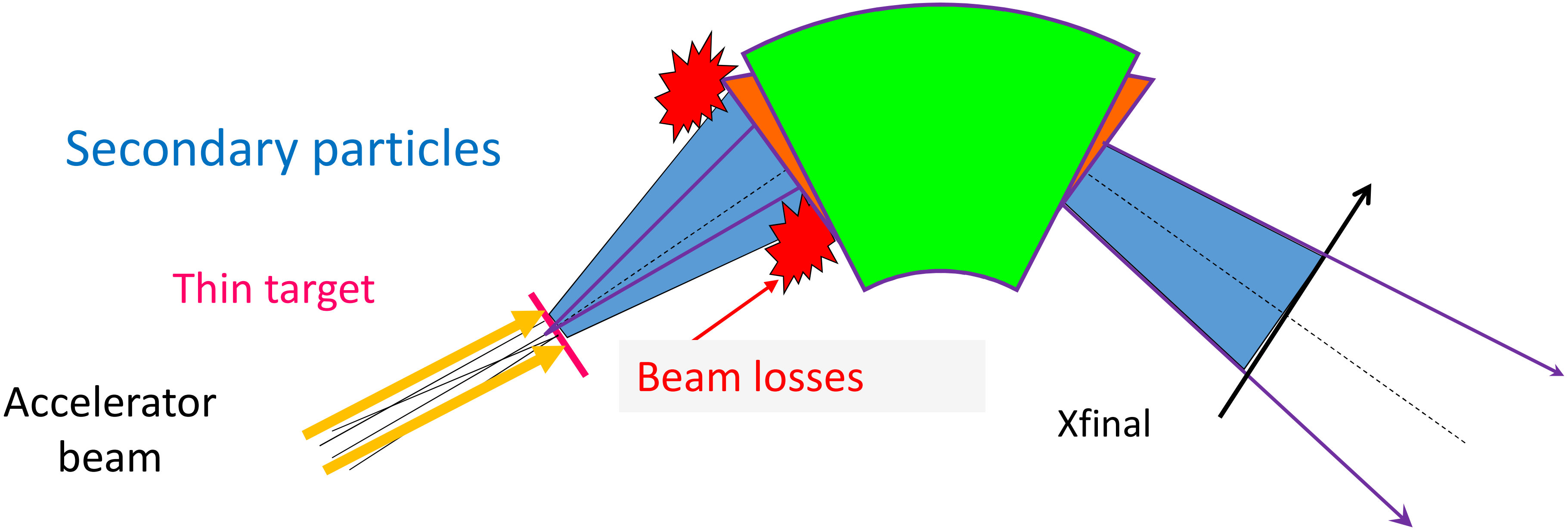
Small acceptance = less transmission, less efficiency

Example: If the particles inside $\pm$50mrad (Horizontal & vertical) are transmitted

at r=1m , **dS=** π 0.050m*0.050m=0.0078 m^2

Acceptance is $d\Omega = 0.0078$ strd= 7.8 mstrd

General problems with 1 dipole magnet : limited acceptance & identification



- Many particles can be lost in the magnet : limited in acceptance

- Trajectories are complex : how to avoid many trackers

$$X_{final} = \text{Function}(B\rho,\ \theta_i,\ \phi_i,\ X_0,\ Y_0)$$

Reconstruction of $B\rho = P/Q$

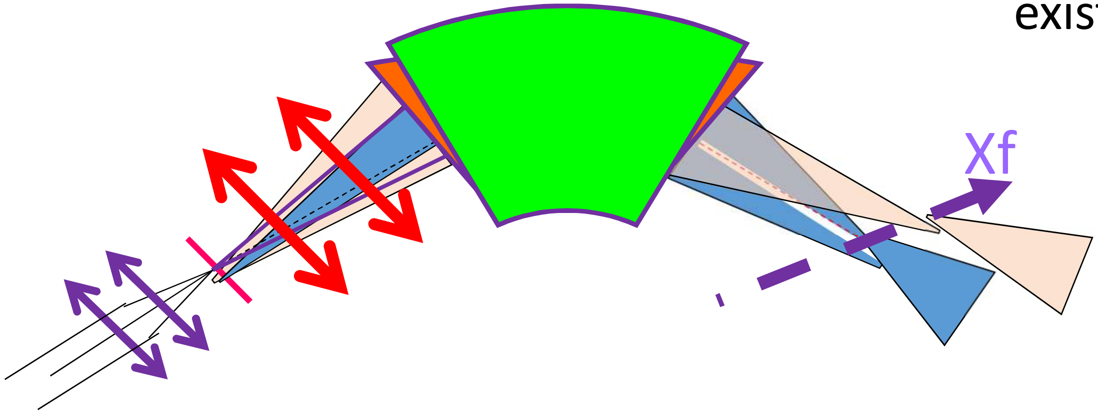
- Measurement of two initial positions X_0, Y_0

  Measurement of two initial angles $(\theta_i,\ \phi_i)$

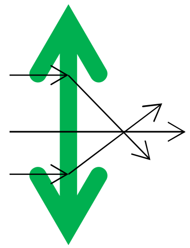
  Measurement of final position X_{final}

Measuring angles are difficult with low energy particles (energy loss in tracker)

2 problems solved with focusing lenses



Imagine that a focusing lenses exist like in light optics



With Focusing lenses Xfinal = F(Bρ, θi, ϕi , X0=0 ,Y0= 0)

- less unknowns (focusing on target)
- Less beam losses in dipole (refocusing downstreal target)

Select the detector position : choose the focal plane, if possible

At one location s (the detector location, called focal plan)

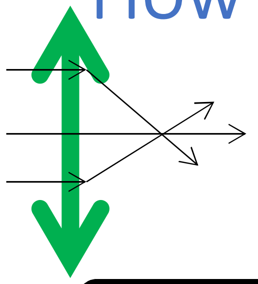
The trajectoires are independant of the initial angles θi, ϕi

and x0=0, y0=0

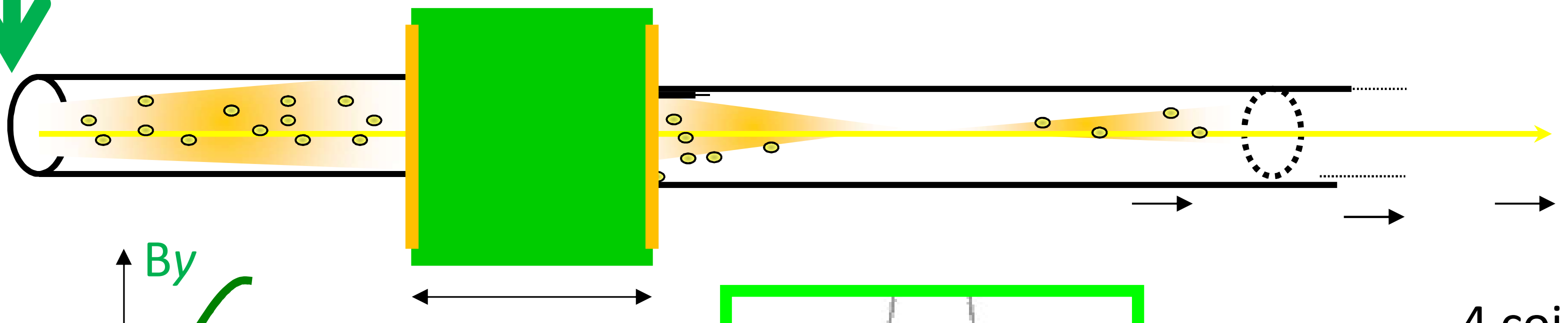
Xfocal =F(Bρ) reconstruction $B\rho = F^{-1}(Xfocal)$

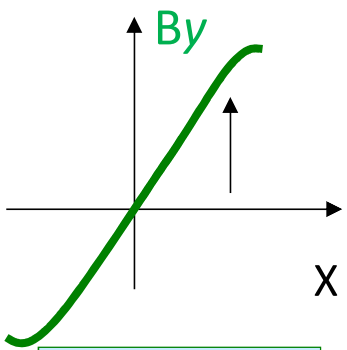
How to construct a focusing lens for charged particles

Magnet with 4 poles (+,-,+,-)

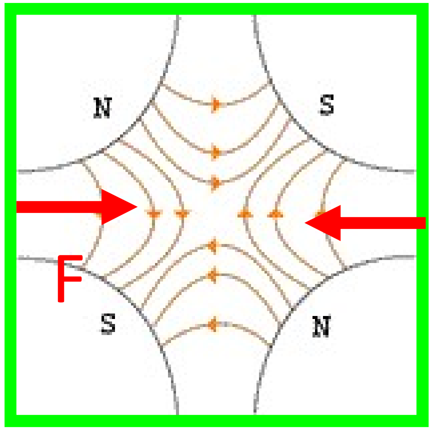


$F=q\ (v \times B)$





G=dBy/dx

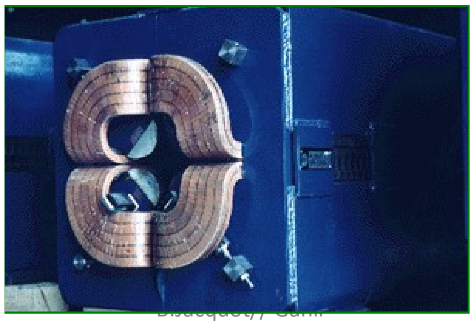


4 coils

+ 4 hyperbolic poles

By=G.X

G is called GRADIENT
[Tesla/m]



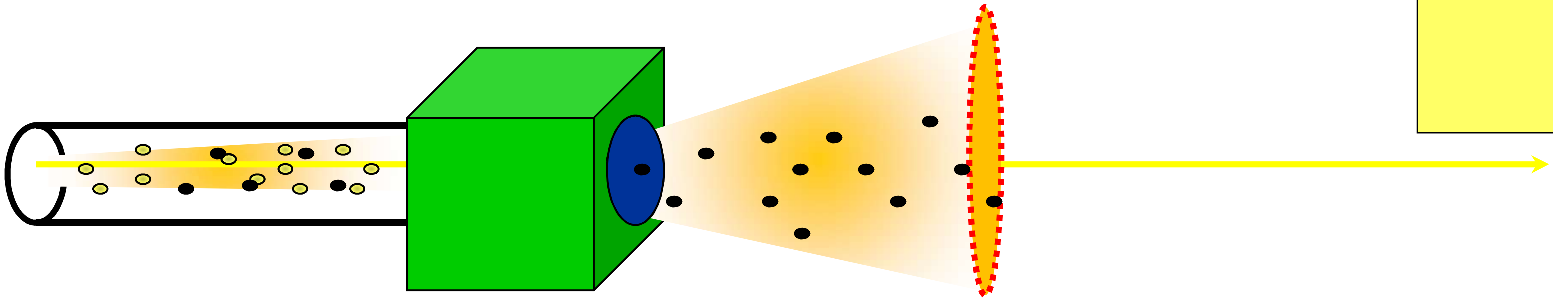
This quadrupole magnet is focusing in horizontal plane

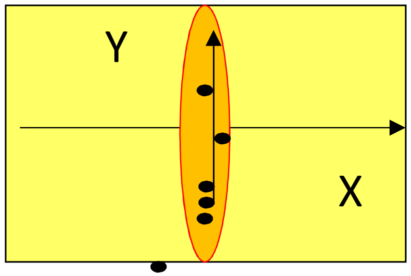
Nota: In the center, the force is zero

What happen in vertical plane ?

1 quadrupole magnet

Quadrupole = Focusing lens in horizontal
But defocusing in vertical





The beam becomes narow in X and large in Y

B_x= G.Y

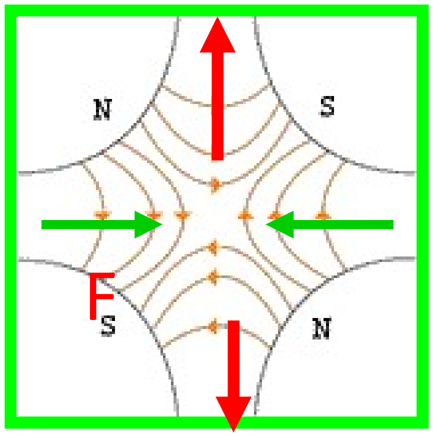
B_y= G.X

B_s= 0

Focusing   in X   (G>0)

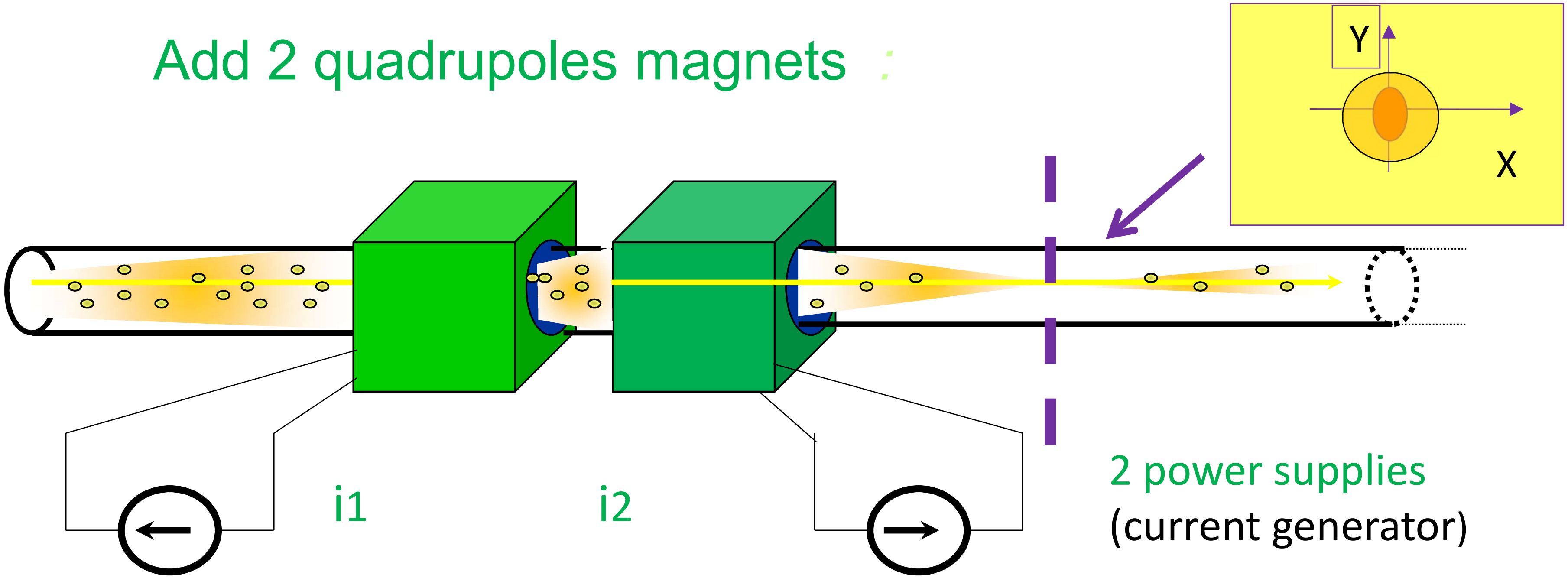
Defocusing in Y

Since the Force is defocusing in vertical



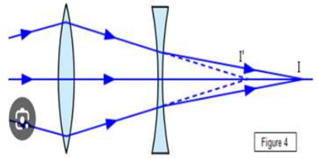
How to construct a focusing lens system In horizontal AND vertical plane

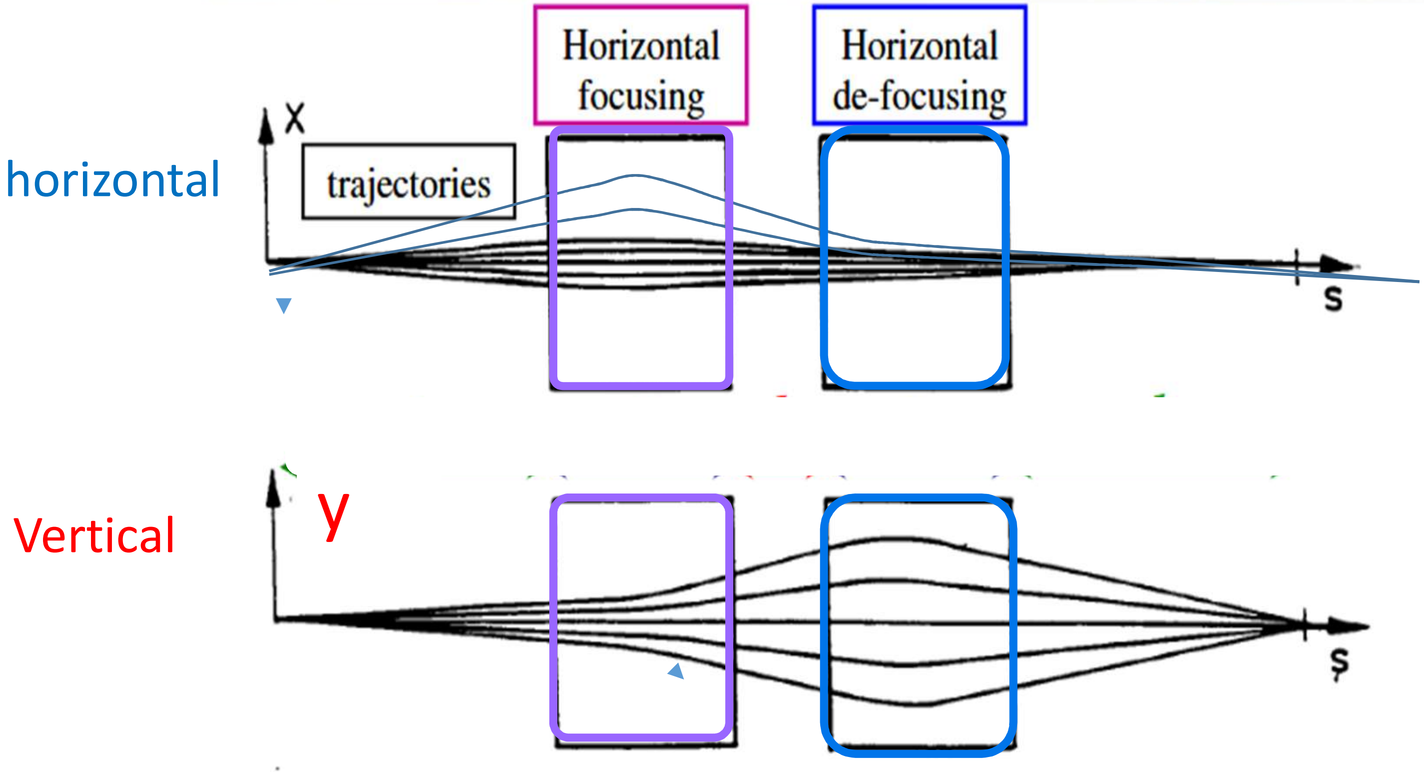
Add 2 quadrupoles magnets :



If you tune i1 and i2 with opposite polarities , the beam can be focused in X and Y

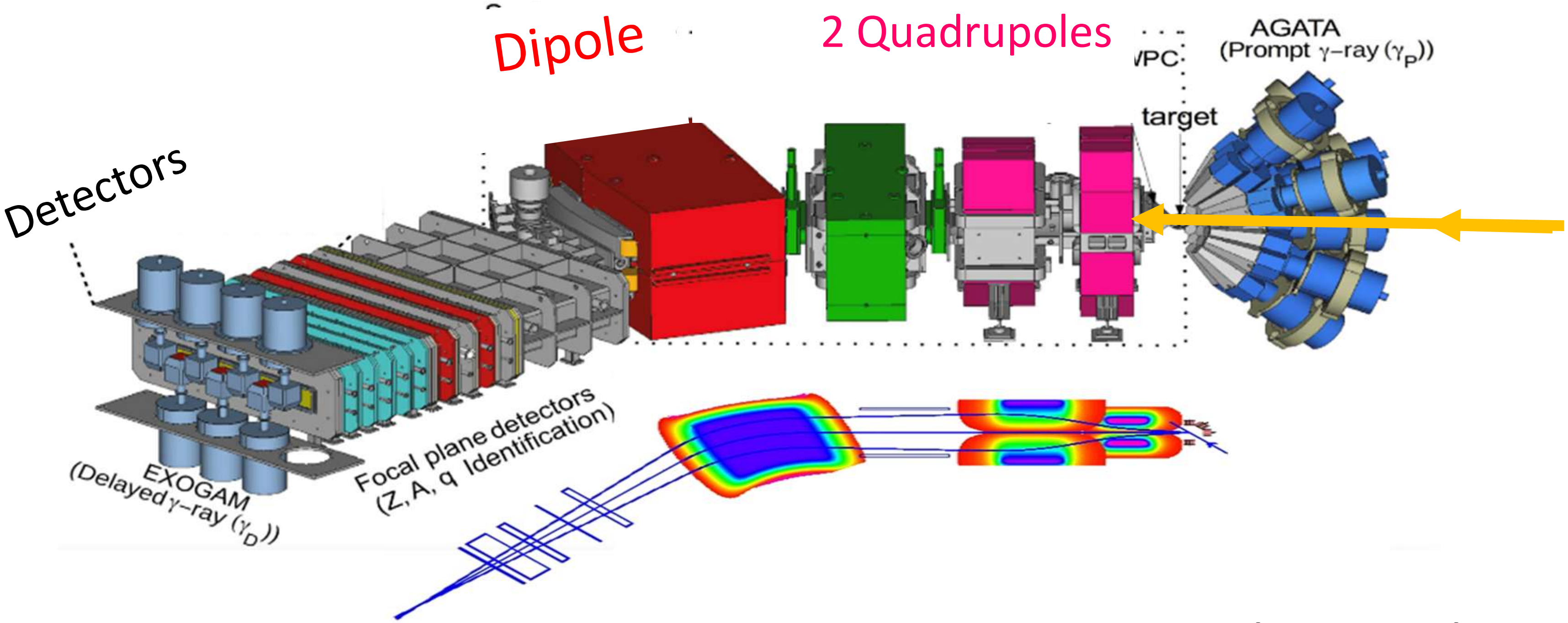
Homework : Demonstrate that a quadrupole doublet with opposite polarities is focusing in horizontal plane (x) and vertical plane (y)…





Bx= +G.Y	horizontal field	**Fx**	~	q Vz . By	~ X
By = -G.X	vertical field	**Fy**	~	q Vz. Bx	~ Y
Bz= 0					

A spectrometer in nuclear physics at GANIL (France)



VAMOS spectrometer at GANIL (France)

Identiy different atomic nuclei produced in a reaction

- 8 meter long, 2 quads + 1 dipole
- Acceptance 100 mstrd (±7° Horizontal& Vertical)
- 2 Trackers + TOF + Energy loss + Residual Energy

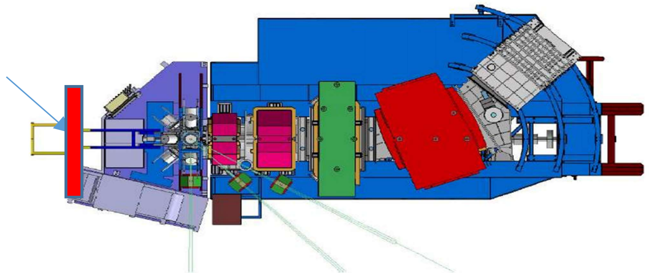
A spectrometer in nuclear physics VAMOS@GANIL

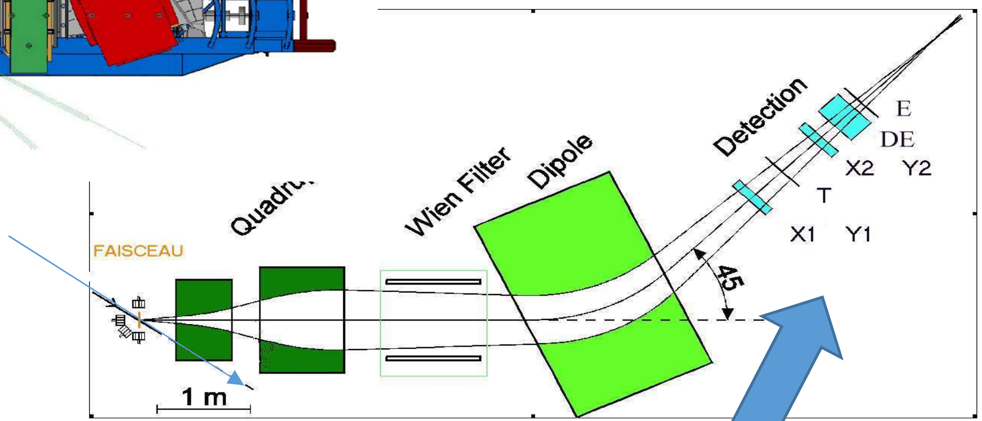
Identifification :

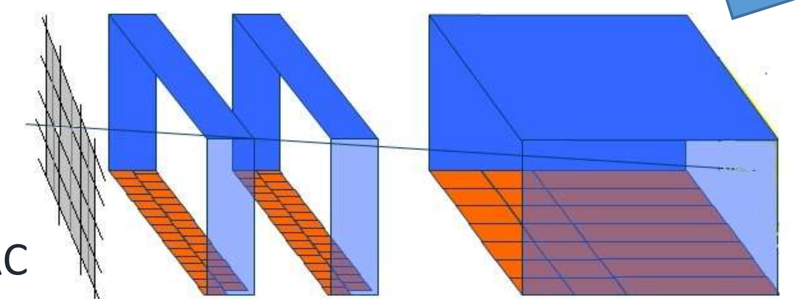
Measure ion after reaction

Masse (N neutron , Z proton)

~ A nucleons



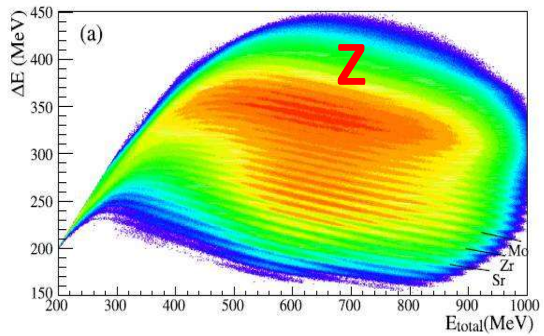




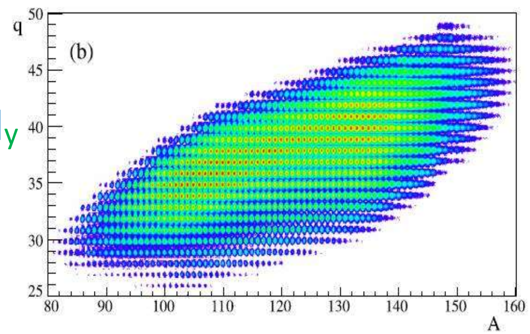
MWPPAC
(TimeoF =v)

2 Positions
($B\rho = Mv/q$)

Ionisation. Chamber
(ΔE,E) =Z (atomic number

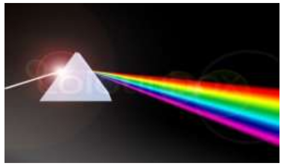


q_y



M

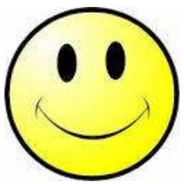
Beam optics



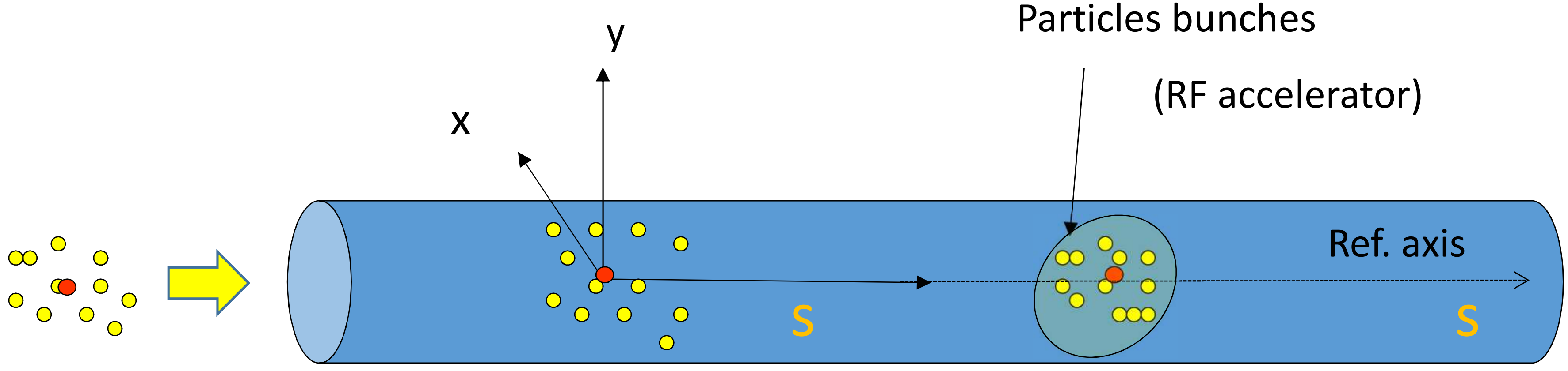
- Focalisation with quadrupoles DONE
- Dispersion with dipole DONE
- Magnetic rigidity : $B\rho = \gamma\, Mv/Q = P/Q$ DONE

- Particles coordinates
- Equations in field B & E
- 1rst order approximation : Optical Matrices

The solution to simulate the transport of charged particles in many magnets (up to 1000 !) with few mutliplications



Beam Optics convention : Particle coordinates



Particle coordinates ? (energy, velocity, angle, Bρ, ?)

6 coordinates needed (x,px), (y,py), (s, ps)

Define a reference particle (x_0,y_0 , Bρ_o ,t_o)

At a given **S**, a particle is described

with 6 coordinates

2 positions ($X-X_0$) ,($Y-Y_0$)

2 angles θ, ϕ

magnetic rigidity $B\rho = B\rho_o (1+\delta)$

time advance $(t - t_0)$

Optical conventions :

Angle in Horizontal plane noted as

$X' = dx/ds = \tan\theta \sim \theta$

Angle in Vertical plane

$Y' = dy/ds = \tan\phi \sim \phi$

Time coordinate expressed in meter

$L = v_0 (t - t_0)$

Beam optics convention

The reference particle : $B\rho_0 = P_0/Q_0$

it is traveling in the Center of the beam lines : $X_0=0$, $Y_0=0$

reference « angles » : $X'_0 = 0$, $Y'_0=0$

At the location s_i , a particle is represented by a vector ***Z***(s_i)

$$\vec{Z}(s_i) = \begin{pmatrix} x \\ x' \\ y \\ y' \\ \ell \\ \delta \end{pmatrix}_{s_i}$$

x = horizontal position with respect to beam pipe center

$x' = dx/ds = \tan\theta$

y = vertical position

$y' = dx/ds = \tan\phi$

$\ell = V_0 (t - t_0)$

$\delta = (B\rho - B\rho_0) / B\rho_0$

$\vec{Z}(s) = (x, x', y, y', l, \delta)$

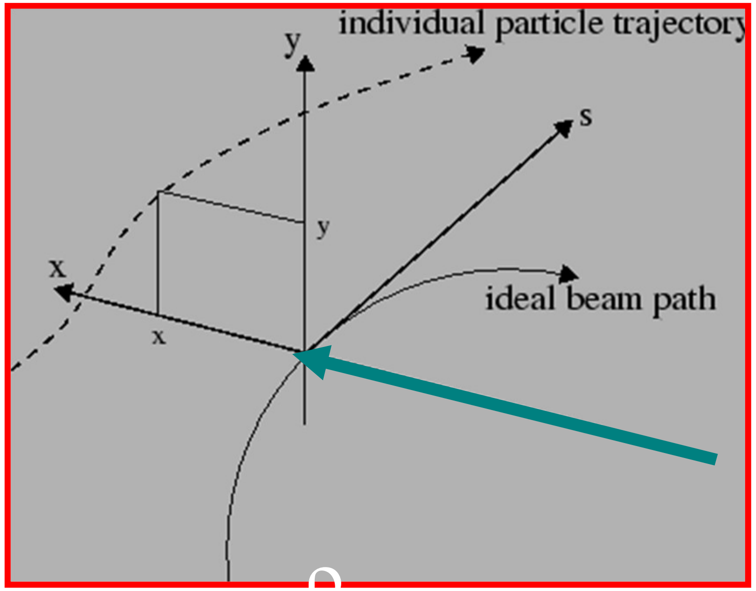
1 particle at location s = 6 Dimensional vector ***Z***

Trajectory equations for 1 particle

How to compute x(s), y(s) ?

We use a curvilinear Reference Frame

which follows the reference particle



$$\frac{d}{dt}\left[m\gamma\ \mathbf{v}\right] = q \cdot (\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$\frac{d}{dt} = \dot{s}\,\frac{d}{ds}$$

Coordinate change t $\Rightarrow$ s

$$\frac{d}{ds}\left[m\gamma\ \mathbf{v}\right] = \ldots$$

x(t),y(t) → x(s),y(s)

We want to compute x,y at a detector location

s=s0

Trajectories : exact equations

$$\frac{d}{ds}\left[m\gamma\,\dot{x}\right] = m\gamma\,\dot{s}(1+\frac{x}{\rho}) + q(t'Ex + y'B_s - \dot{s}(1+\frac{x}{\rho})\cdot B_y)$$

$$\frac{d}{ds}\left[m\gamma\,\dot{y}\right] = q(t'E_y + (1+\frac{x}{\rho})\cdot B_x - x'\cdot B_s)$$

$$\frac{d}{ds}\left[m\gamma\,\dot{s}(1+\frac{x}{\rho})\right] = -\frac{m\gamma\,\dot{x}}{\rho} + q(t'E_s + x'\cdot B_y - y'\cdot B_x)$$

Trajectory simulation (x(s) , y(s))

1) Knowing B(x,y,s) and E(x,y,s,t) : field map 3D

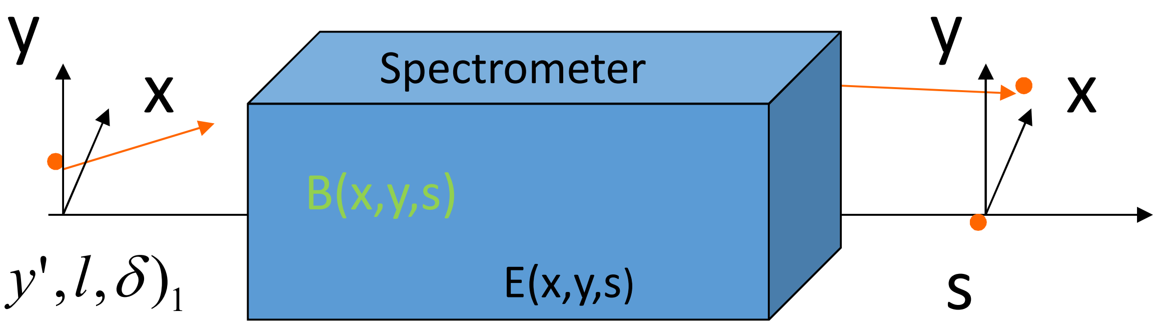
2) Integrate the equations for all the particles

(computer+ Numerical method: **Runge-kutta**)

Generally we can do <u>simpler</u>

Matrix approach (1rst order approximation)

Beam optics with Matrices: dx/ds <<1



$$Z_1 = (x, x', y, y', l, \delta)_1$$

$$\begin{aligned} \mathbf{Z}_2 &= f_{1\to 2} \quad (\mathbf{Z}_1, B, E, l......) \\ &= R_{1\to 2} \quad .\mathbf{Z}_1 + 0(\mathbf{Z}_1.^2) + ... \\ &\approx R_{1\to 2} \quad .\mathbf{Z}_1 \end{aligned}$$

Exact Dynamic (non linear)

Taylor expansion : X and Y Are small,

: angle dx/ds are small

Taylor 1rst order = Linear dynamics

$$\begin{bmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{bmatrix}_2 = \begin{bmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} & R_{16} \\ R_{21} & R_{22} & R_{23} & R_{24} & R_{25} & R_{26} \\ R_{31} & R_{32} & R_{33} & R_{31} & R_{31} & R_{36} \\ R_{41} & R_{42} & R_{43} & R_{44} & R_{45} & R_{46} \\ R_{51} & R_{52} & R_{53} & R_{54} & R_{55} & R_{56} \\ R_{61} & R_{62} & R_{63} & R_{64} & R_{66} & R_{66} \end{bmatrix} . \begin{bmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{bmatrix}_1$$

$$l = v_0(t - t_0)$$

$$\delta = \frac{B\rho - B\rho_0}{B\rho_0}$$

The simplest transport matrix:
Rmatrix for a straight section L (drift)

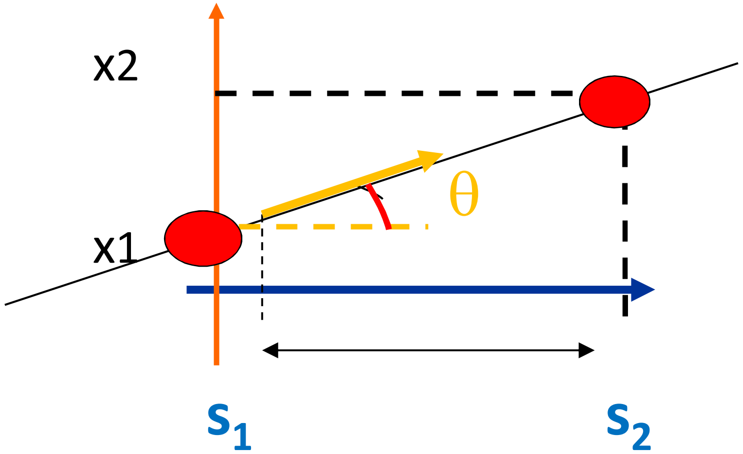
Particle evolution in « drift length » between s1 & s2 : x=x(s) ?

$$X_2 = X_1 + \tan(\theta 1).\ (s_2 - s_1)$$

$$\theta 1 = \theta 2 \quad \text{(no field)}$$

nota: $\tan(\theta 1) = \Delta x_1 / \Delta s \;= x_1'$

and $\Delta s = (s_2 - s_1) = L$

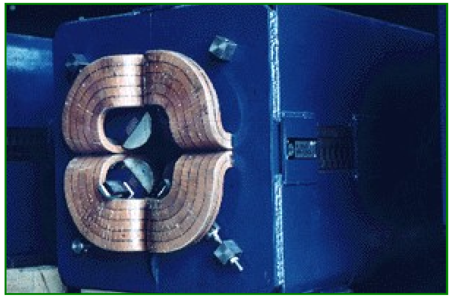


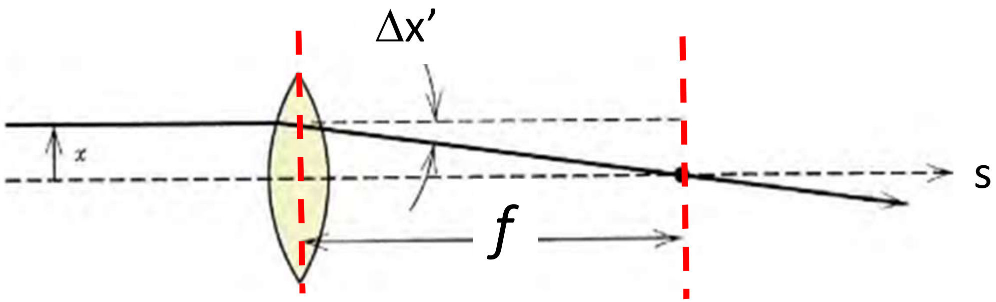
$$\begin{pmatrix} x_2 \\ x2' \\ y_2 \\ y2' \\ \dots \end{pmatrix} = \begin{pmatrix} 1 & L & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_1' \\ y_1 \\ y_1' \end{pmatrix}$$

$$R_{d1} = \begin{bmatrix} 1 & L & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & L & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & L/\gamma^2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The magnetic quadrupole

like a concave lens= focusing



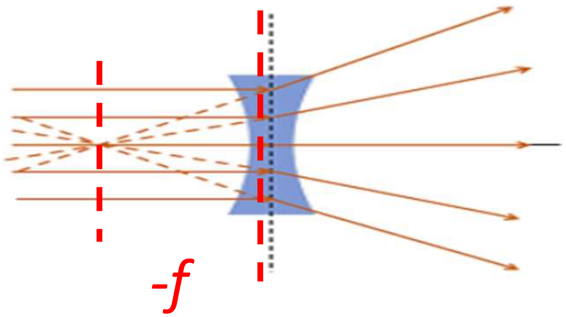


$$\begin{pmatrix} x \\ x' \end{pmatrix}_{out} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_{in}$$

Matrix 2x2 : in horizontal plane : focusing

$x' \sim \theta$ deviation in horizontal plane

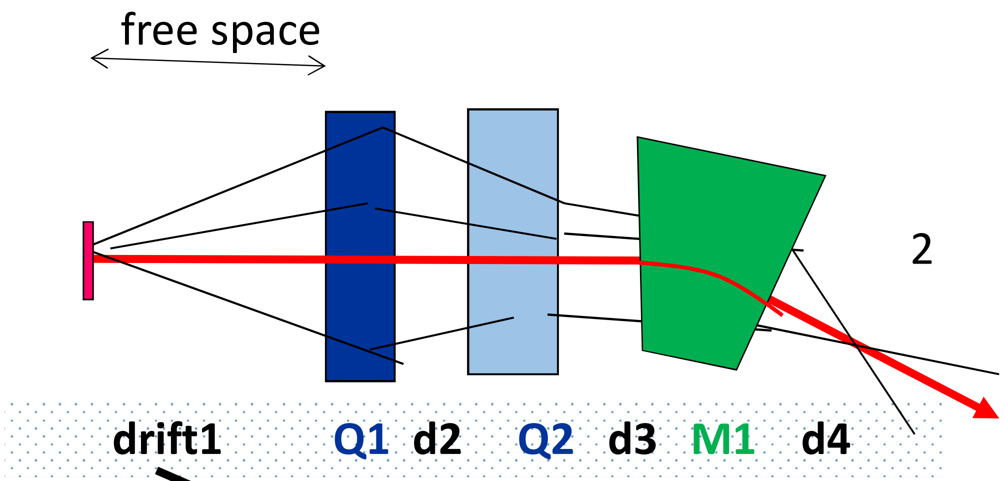
In y ,y' : in Vertical plane : Defocusing (like a convex lens)



$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}_{out} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1/f & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/f & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}_{in}$$

More on « transport matrix »:
how to compute the Rmatrix of a spectrometer ?

The total transport matrix R is the product of the matrices representing each elements (drift , **quadrupole** dipole magnet)



Quad matrix $k_x = GL/B\rho_0$

$$R_{M1} = \begin{bmatrix} \cos k_x L & \frac{\sin k_x L}{k_x} & 0 & 0 & 0 & M_{16} \\ -k_x \sin k_x L & \cos k_x L & 0 & 0 & 0 & M_{26} \\ 0 & 0 & \cos k_y L & \frac{\sin k_x L}{k_x} & 0 & 0 \\ 0 & 0 & -ky \sin k_y L & \cos k_y L & 0 & 0 \\ M_{26} & M_{16} & 0 & 0 & 1 & M56 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Drift Matrix (Free space length L)

$$R_{d1} = \begin{bmatrix} 1 & L & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & L & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & L/\gamma^2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Matrix product

$$R = R_{drift4} \cdot R_{M1} . R_{d3} . R_{Q2} . R_{d2} . R_{Q1} . R_{drift1}$$

The transport Matrix R allows the computation of the coordinates of a particle at the end of a spectrometer

Zin= (x,x’,y,y’,l,δ)$_0$ coordinate at the entrance

Zout= (x,x’,y,y’,l,δ)$_1$ coordinate at the exit

$$\vec{Zout} = R . \vec{Zin}$$

$$\begin{bmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{bmatrix}_1 = \begin{bmatrix} R_{11} & R_{12} & 0 & 0 & 0 & R_{16} \\ R_{21} & R_{22} & 0 & 0 & 0 & R_{26} \\ 0 & 0 & R_{33} & R_{31} & 0 & 0 \\ 0 & 0 & R_{43} & R_{44} & 0 & 0 \\ R_{51} & R_{52} & 0 & 0 & R_{55} & R_{56} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{bmatrix}_0$$

$$l = \mathrm{v}0(t - t_0)$$

$$\delta = \frac{p - p_0}{p_0}$$

Interpretation of R

$$Rij = \left(\frac{\partial Zi\ out}{\partial Zj\ in}\right)$$

$ex:$

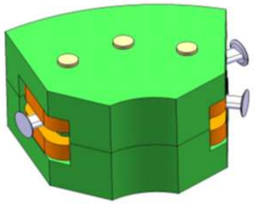
$$R_{11} = \left(\frac{\partial Z_1}{\partial Z_1}\right) = \left(\frac{\partial x\ out}{\partial x\ in}\right) \qquad R_{12} = \left(\frac{\partial Z_1}{\partial Z_2}\right) = \left(\frac{\partial x\ out}{\partial x'\ in}\right)$$

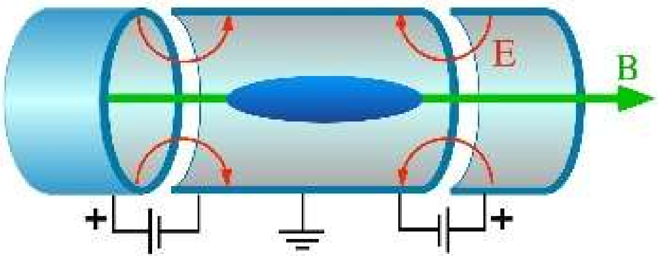
$$R_{16} = \left(\frac{\partial Z_1}{\partial Z_6}\right) = \left(\frac{\partial x\ out}{\partial \delta\ in}\right)$$

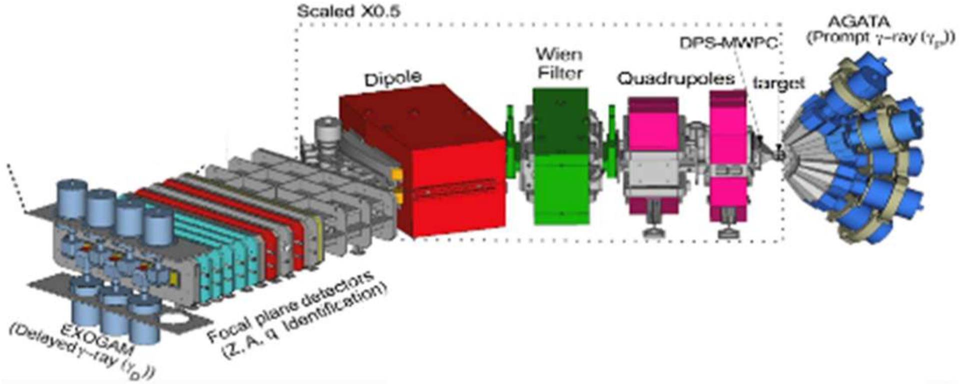
The transport Matrix **R**=R*ij* depends on the

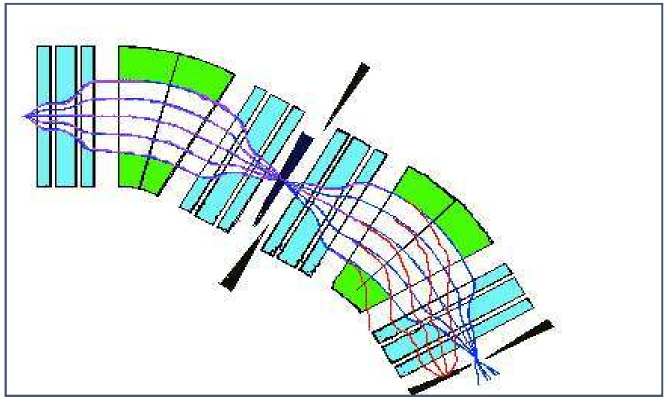
- spectrometer/accelerator geometry
- tuning of the quadrupoles

Electromagnetic Spectrometers

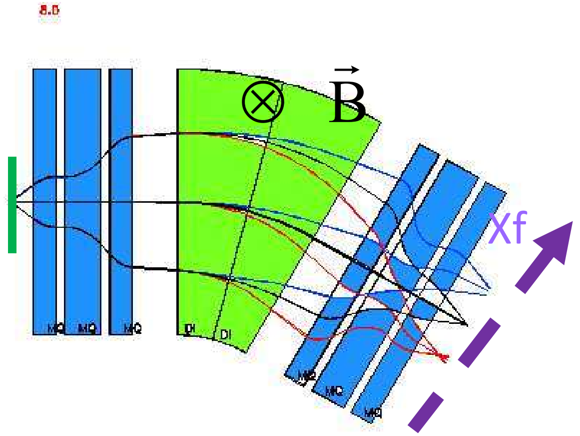








Spectrometer matrix
allows the simulation of 1 trajectory



$$\begin{bmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{bmatrix}_{FINAL} = \begin{bmatrix} R_{11} & 0 & 0 & 0 & 0 & R_{16} \\ R_{21} & R_{22} & 0 & 0 & 0 & R_{26} \\ 0 & 0 & R_{33} & 0 & 0 & 0 \\ 0 & 0 & R_{43} & R_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & L/\gamma^2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{bmatrix}_{TARGET}$$

The Typical spectrometer Matrix
is simple

$$\boxed{X_{Final} = R_{11}\ \overset{\sim 0}{X_{target}} + \overset{=0}{R_{12}}\ x' + R_{16}\ \delta}$$

$$\approx \quad R_{16}\ \delta$$

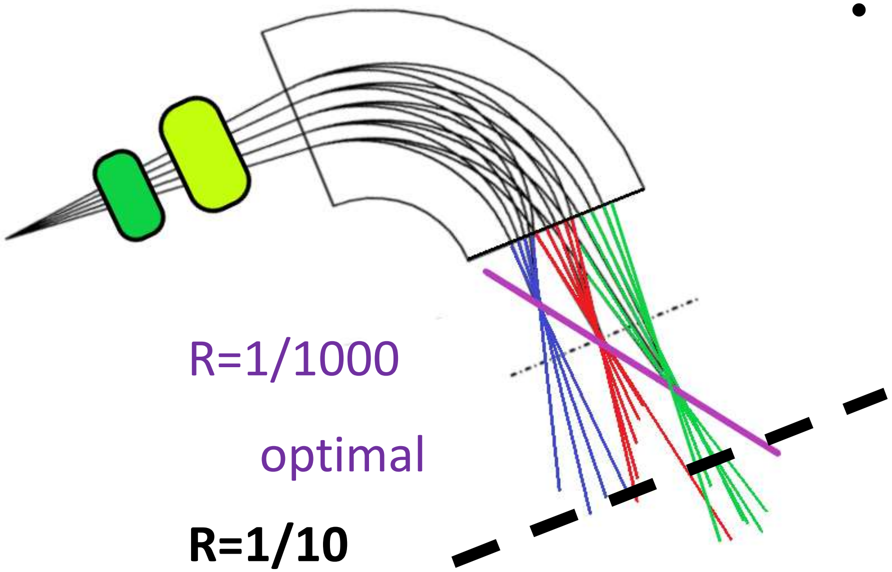
$$\delta = (B\rho - B\rho_o) / B\rho_o$$

$$R_{16} = \left(\frac{\partial x_F}{\partial \delta}\right)_{Target}$$

Dispersion

$$B\rho_o = B_{dipole} \cdot R_{dipole}$$

The resolution is optimal at the focal plane



R=1/1000

optimal

R=1/10

Not optimal

- The 3 beams with ≠ magnetic rigidities

$B\rho_{ref} = B\rho_0 = B \times R_{dipole}$

$B\rho = B\rho_0(1-\delta)$

$B\rho = B\rho_0(1-\delta)$

- The 3 beams are well separated « **at the focal plan** »

When σx is small and R_{16} (dispersion) is large

Resolution RMS= $\sigma x / R_{16}$ Dispersion $R_{16}= dxf/d\delta$

HOMEWORK

- **Exercise 1**: Imagine a spectrometer with a dispersion R_{16}=2 m (=2cm/ %) and

beam width σx =2 mm on the focal plan detector,

What is the resolution R in Bρ

Resolution RMS = σx /R_{16} =0.001

- **Exercise 2 :**

A spectrometer (R_{16}=1.5 cm/ % =1.5m) is tuned for $B\rho_0$=2.0 T.m

A particle arrives on the focal plane at Xf=3cm,

What is the particle rigidity?

$x_f = R_{16}\ (B\rho - B\rho_0)\ /\ B\rho_0$

- **Exercise 3 :**

_How to measure the dispersion (R_{16}) in a spectrometer ?

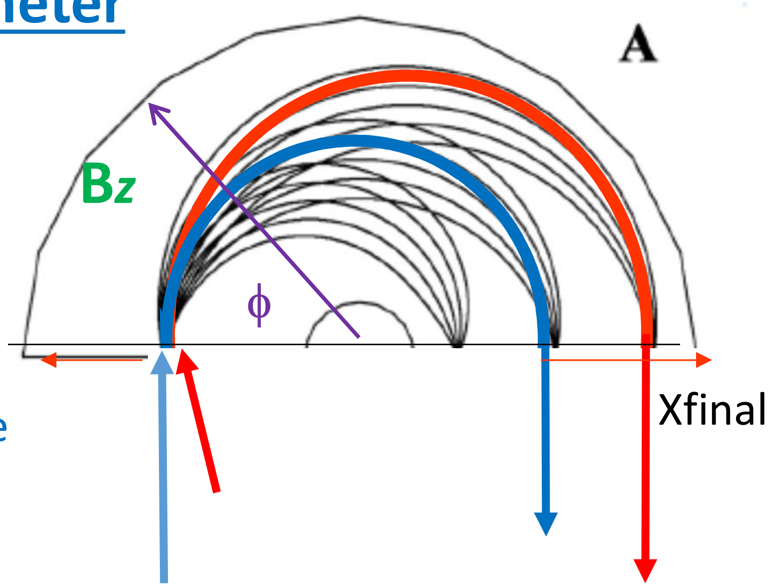
HOMEWORK : How to compute a particle trajectory
Compare to an ideal particle?

- ## Exercise : the 180° spectrometer

Deviation : ϕ=180°

$$\boxed{B\rho \overset{def}{=} \gamma \frac{m\mathrm{v}}{\mathrm{q}}}$$

Magnetic field is tuned for the reference particle : $B_z R_{ref} = B\rho_{ref}$



Blue trajectory : the reference particle

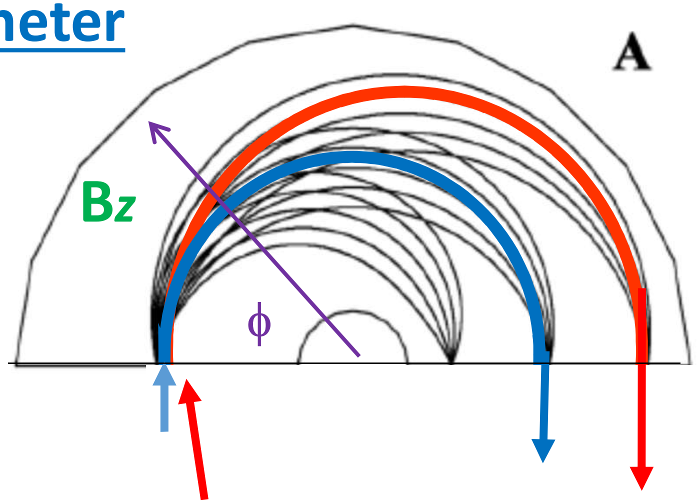
$$X_{final}\,(\phi=180°) = -x_0 + 2\,(R - R_{ref}) = -x_0 + 2\,(R_{ref}\,.\,\delta\,)$$

$(R - R_{ref}) = (BR - BR_{ref})\,/B = R_{ref}\,[B\rho - B\rho_{ref}\,]/\,B\rho_{ref}$

x_0 = The initial position compared to reference particle
X'_0 = The initial angle
$\delta = [B\rho - B\rho_{ref}\,]/\,B\rho_{ref}$ = The relative rigidity deviation

HOMEWORK

- **Exercise : the 180° spectrometer**



$$X_{final}(\phi) = x_0 \cos(\phi) + x'_0 R_{ref} \sin(\phi) + \delta R_{ref}(1-\cos(\phi))$$

x_0 = The initial position
X'_0 = The initial angle
δ = $B\rho - B\rho_{ref} / B\rho_{ref}$ = The relative rigidity deviation

Xfinal =correspond to optical matrix

$$M_{dipole} = \begin{pmatrix} \cos\varphi & R\sin\varphi & 0 & 0 & 0 & R(1-\cos\varphi) \\ -1/R\cdot\sin\varphi & \cos\varphi & 0 & 0 & 0 & \sin\varphi \\ 0 & 0 & 1 & R\varphi & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -\sin\varphi & -R(1-\cos\varphi) & 0 & 0 & 1 & R\varphi/\gamma^2 - R(\varphi-\sin\varphi) \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

The R matrix of a spectrometer :

1^{rst} order approximation

A spectrometer generally

A) Starts with a focus (on target)

B) End up with a focus ($R_{12}=R_{34}=0$)

C) The spectrometer is chromatic ($R_{16} \neq 0$)

typical matrix (8 coefficients)

R_{16} is called dispersion

$$\begin{bmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{bmatrix}_1 = \begin{bmatrix} R_{11} & 0 & 0 & 0 & 0 & R_{16} \\ R_{21} & R_{22} & 0 & 0 & 0 & R_{26} \\ 0 & 0 & R_{33} & 0 & 0 & 0 \\ 0 & 0 & R_{43} & R_{44} & 0 & 0 \\ - & - & 0 & 0 & 1 & L/\gamma^2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{bmatrix}_0$$

$$l = \mathrm{v}_0(t-t_0)$$

$$\delta = \frac{B\rho - B\rho_0}{B\rho_0}$$

R_{11} $_1$ is called MAGNIFICATION

$R_{11} = \Delta x_F / \Delta x_{Target}$

Coordinates

At focal (detectors)

Coordinates

on target

$$x^F \approx \sum_{j=1\ldots6} R_{1j} Z_i^0 = R_{11}.x_0 + R_{12}.x'_0 + R_{13}.y_0 + R_{14}.y_0' + R_{15}.l^0 + R_{16}.\delta^0$$

Non linear effects in optical system
Some effects can not be represented in Matrix form

1rst order = Matrix ...

$$\vec{Z_2} = R\,\vec{Z_1} + \varepsilon....$$

for large angle, large Bρ deviation 2nd order, third order is required.

$$Z2_i = \sum_{j=1}^{6} R_{ij}\ .Z1_j + \sum_{k=1}^{6}\ \sum_{j=1}^{6} T_{ijk}\ \ Z1_j . Z1_k + ...$$

1rst order 2nd order + ...

Linear Approximation holds for small angle, small Bρ deviation... (#30mrad, δ<2%)

$$Z1 = (x, x', y, y', l, \delta)_1$$

Effects of second order :

-Inclination of focal plane

-the Focusing strenght of quads is bρ dependant

-Large angle particles are not well focused

Non linearities (ABERRATIONS) come with

- large acceptance (large angle x' , y' and large br deviation $\delta = B\rho - B\rho 0 / B\rho 0$

- field defects in quads and dipoles

Summary

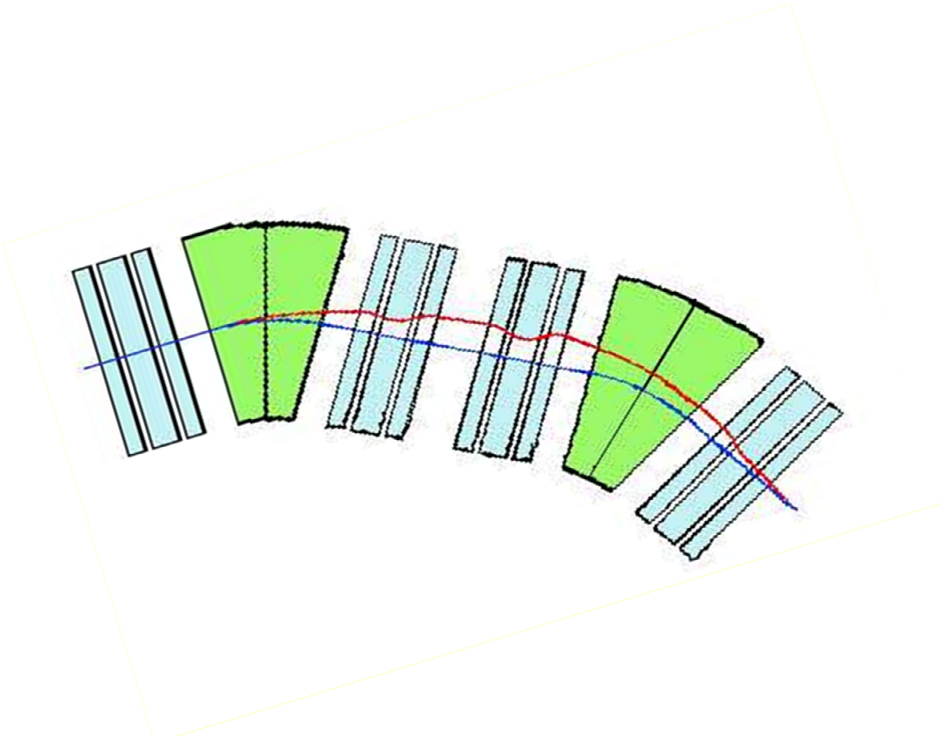
- ❑ The different technics : basic concepts and history

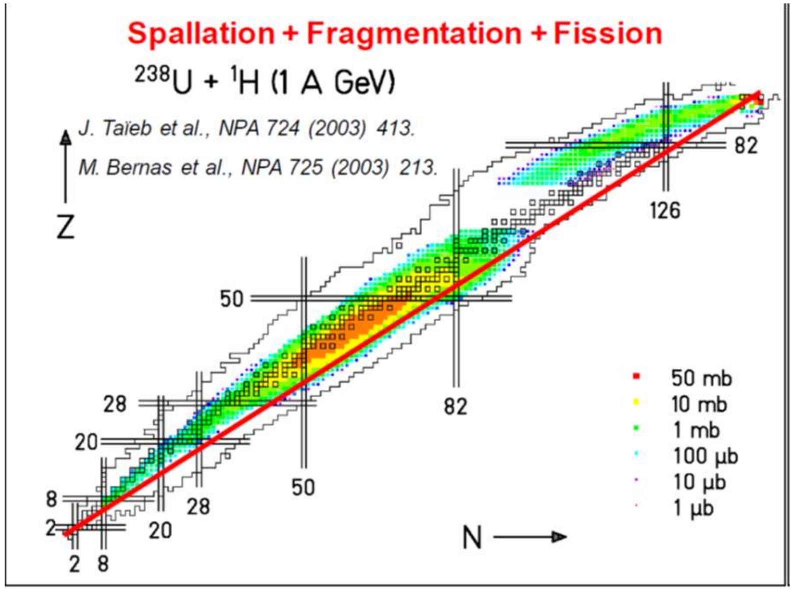
- ❑ **Magnetic spectrometer with accelerator beams**
  - **Part 1**
  - **Part 2 : Exemple of a large fragment separator**

- ❑ Spectrometers without particle accelerator

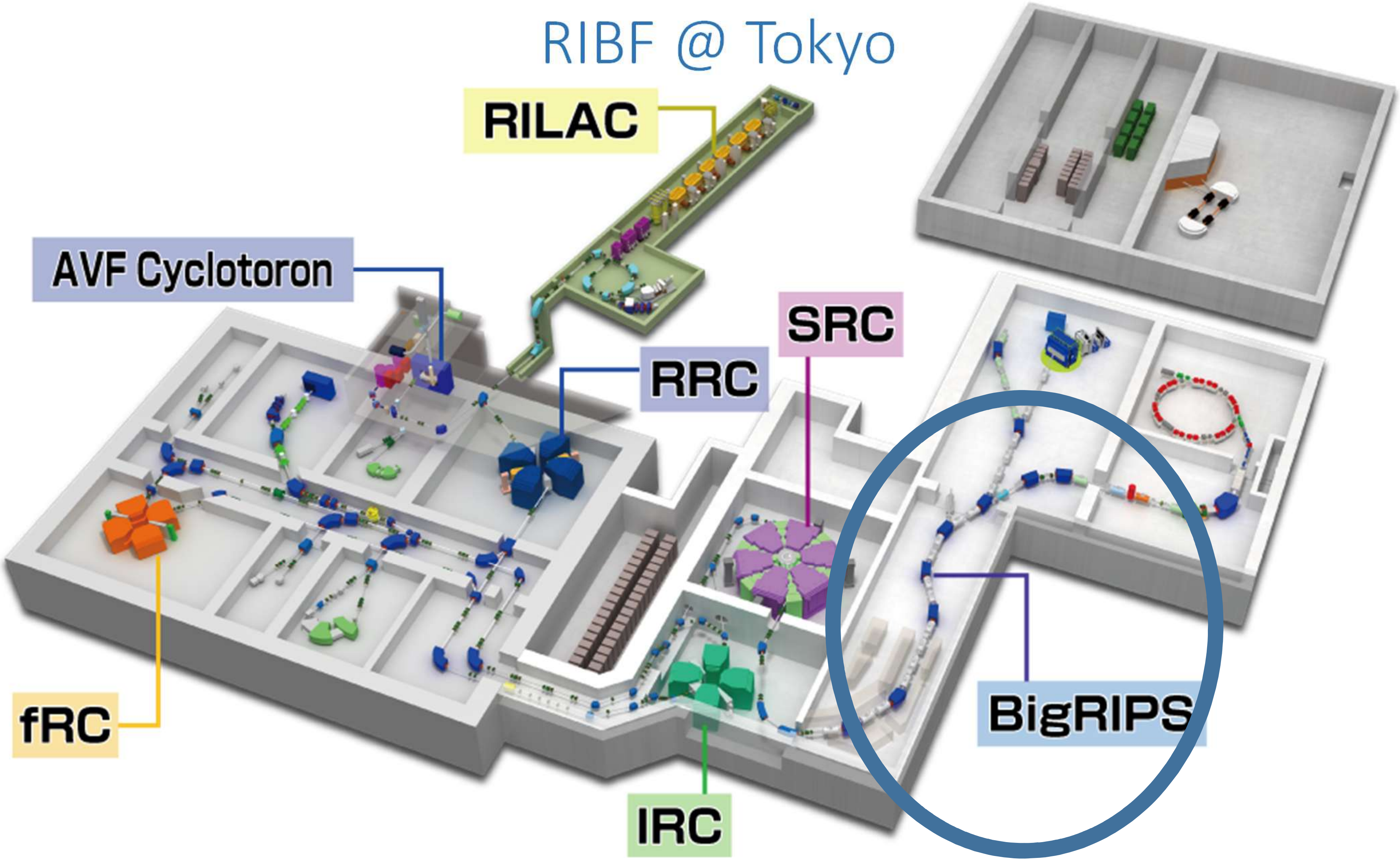
Spectrometers
with accelerator beams

part n°2
A very specific spectrometer in nuclear physics
A fragment separator in Japan (Tokyo, RIBF)





RIBF @ Tokyo



5 cyclotrons, **The biggest superconducting cyclotron (SRC)**

1 Linac

Heavy ions up to Uranium beam

Fragment Separator BIG RIPS
study of rare radioactive isotopes ($T_{1/2}$ ~ 1ms-1s)

Reaction : U fission

accelerator beams Uranium beam at 300 MeV/A

^{238}U beam on Be Target@300MeV/A

Production of many istopes

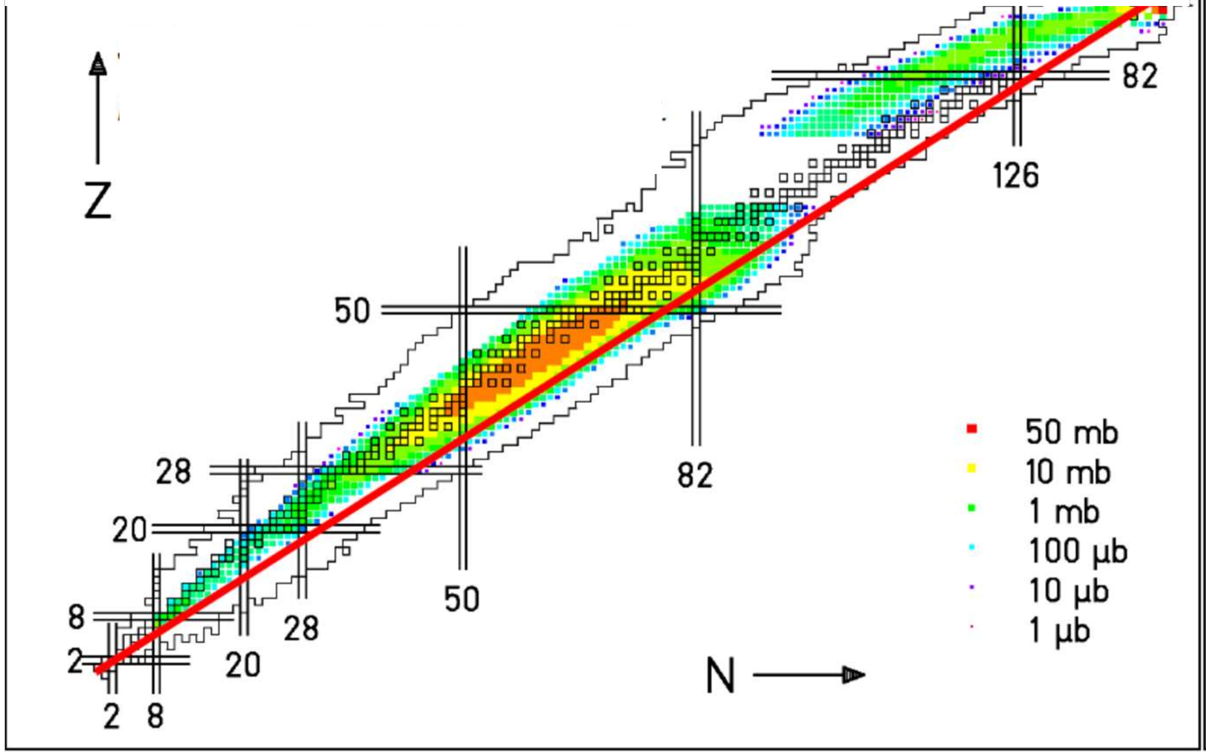
GOALs

-Production

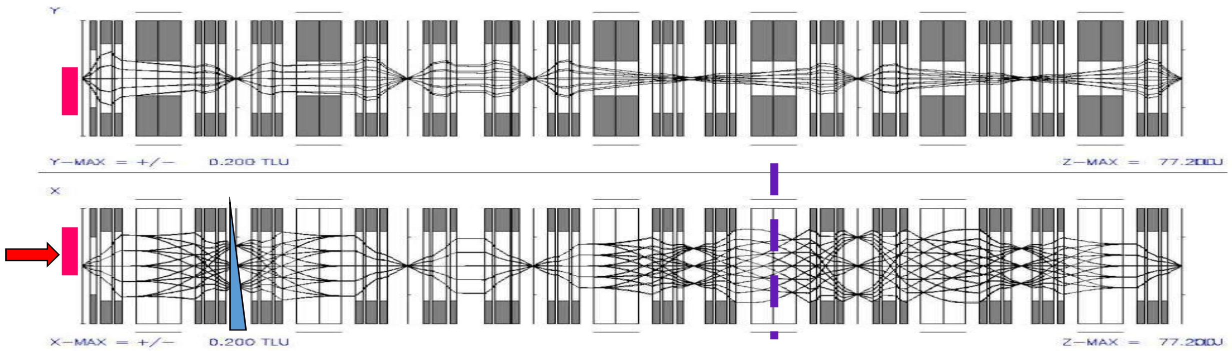
-Separation

-Study of exotic nuclei

beyond the region of nuclei known today



Fragment separator : BIG RIPS (Tokyo)



Specifications

L=77m

$B\rho_{max}$ = 7 Tm

$\Delta p/p$ = ± 3%

$\Delta\theta = \Delta x' = \pm$ 50mrad

$\Delta\varphi = \Delta y' = \pm$ 60mrad

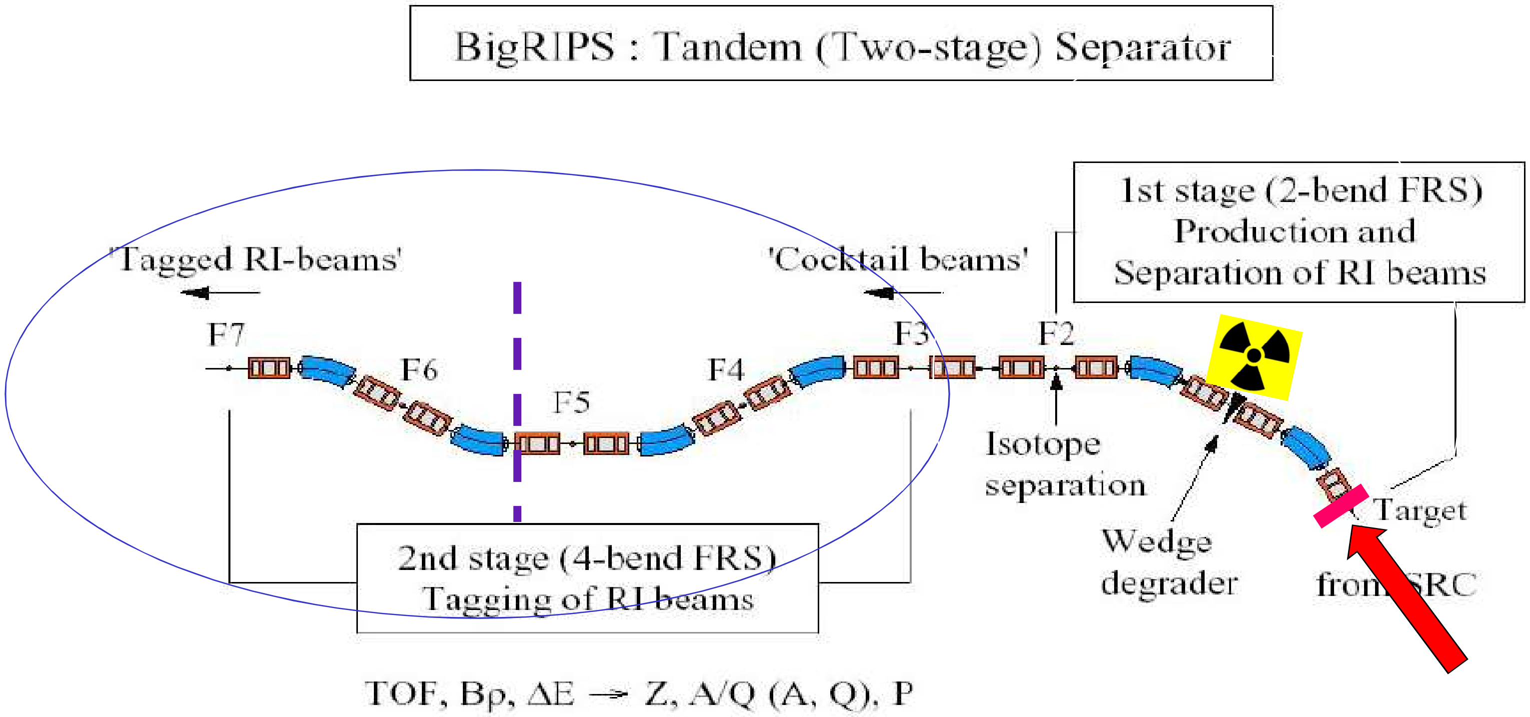


Fig. 2. A schematic diagram of the RI-beam tagging in the BigRIPS separator.

Fragments separator @RIBF (Japan)
E#300-500 MeV/A   Length=77m
6 dipoles magnets, 42 quadrupole magnets

Suppression of the primary beam

(many dipoles, degrador selection)

Help the selection of very rare nuclei

Selection of 4-5 nuclei

Identification (DE-TOF)

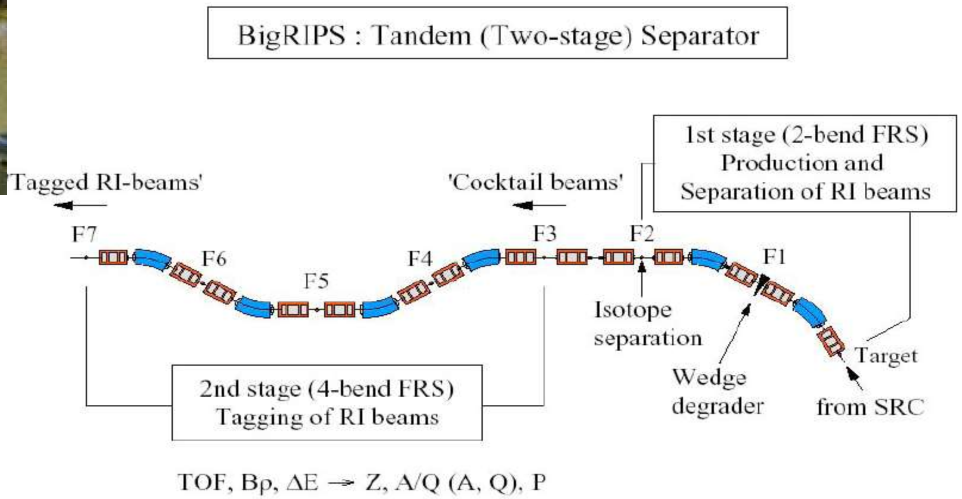
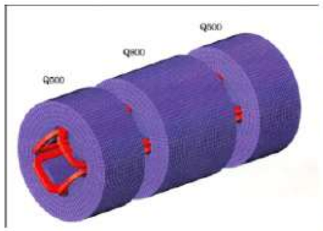


Fig. 2. A schematic diagram of the RI-beam tagging in the BigRIPS separator.



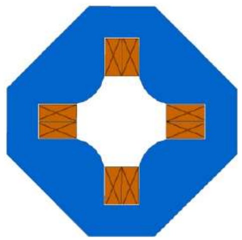
Superferric quads

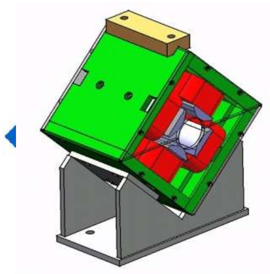
Quadrupole technology

1 :Normal conducting quadrupole

hyperbolic pole (Fe)
coils (Cu)

G~ 10 Tesla/m





Larger Aperture

or/and

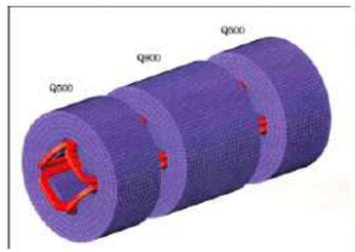
Higher strength

2 : Superferric quad

hyperbolic pole (Fe)
coils (NbTi)

Higher Gradient , larger aperture possible (A1900, BigRips, Synchro.)

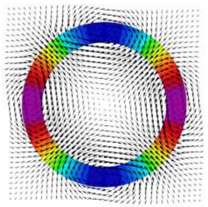
G~ 20-30 Tesla/m

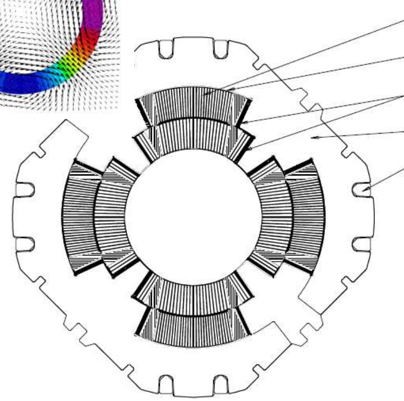


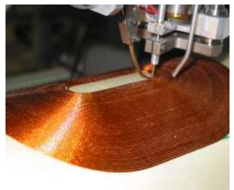
3 : Superconducting quadrupoles

No pole !!!!!!
cos(2θ) coils (NbTi)

G~ 40-200 Tesla/m (Cern LHC…)







BIG RIPS quads

Beam very rigid : $B\rho = \gamma\, mv/Q$ =7 T.m (Beam at Ek=300MeV/A)

with high velocity !

Super-ferric quadrupole triplet :

Very strong focusing : supraconducting coils (NbTi), with poles (Fe)

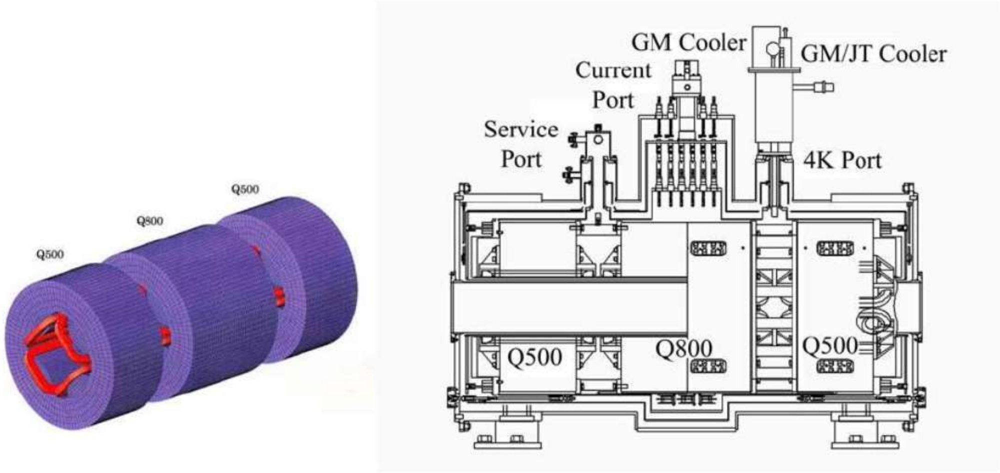


Figure 22: Schematic view of the RIKEN prototype quadrupole triplet (left side) and its installation into the cryostat (right side) [24].

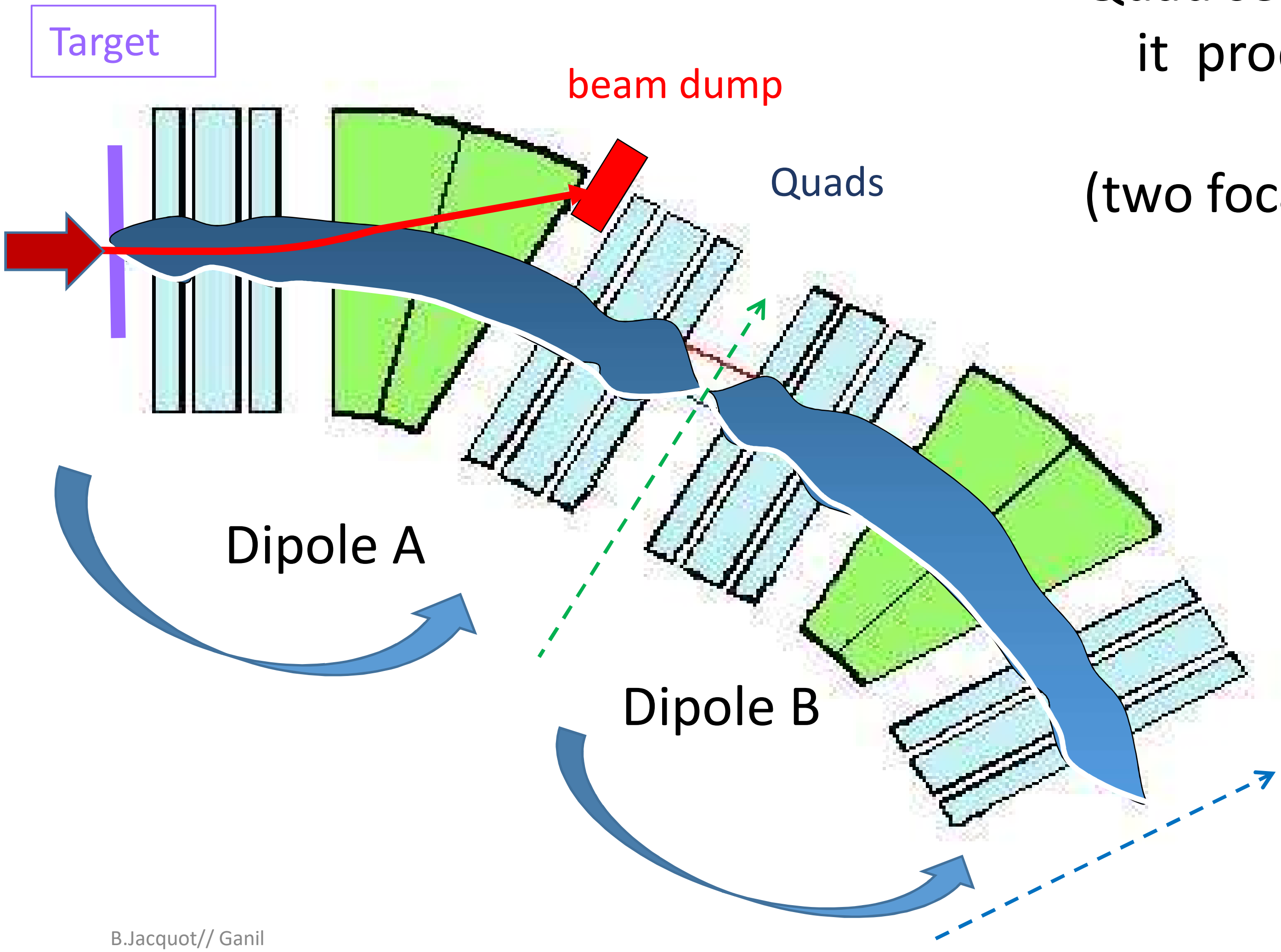
- Supra-conducting coils (i very large, B close to saturation)

-Raperture very large =0.1m ; Bpole-max# 2 Teslas

- GradientMax=2T/0.1m= 20.T/m

Fragment separator : the principle

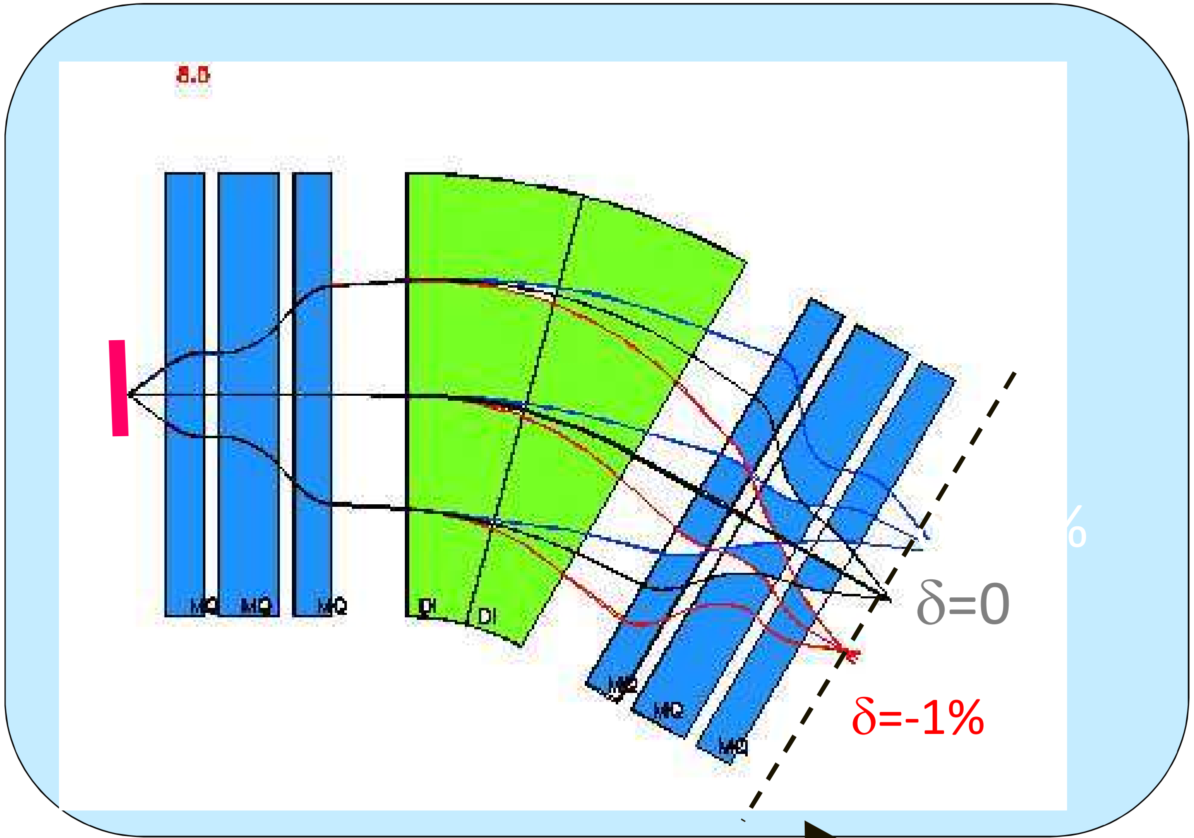
2 symetric sections : A & B



Quad setting :
it produces two focii

(two focal planes)

Fragment separators : dispersiv section optics



Dispersiv focal plane

$$Rmatrix(A) = \begin{bmatrix} R_{11} & 0 & 0 & 0 & 0 & R_{16} \\ R_{21} & R_{22} & 0 & 0 & 0 & R_{26} \\ 0 & 0 & R_{33} & 0 & 0 & 0 \\ 0 & 0 & R_{43} & R_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & L/\gamma^2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Section A :

Focusing

$R_{12}=0$ (Horizontal)

$R_{34}=0$ (Vertical)

dispersion

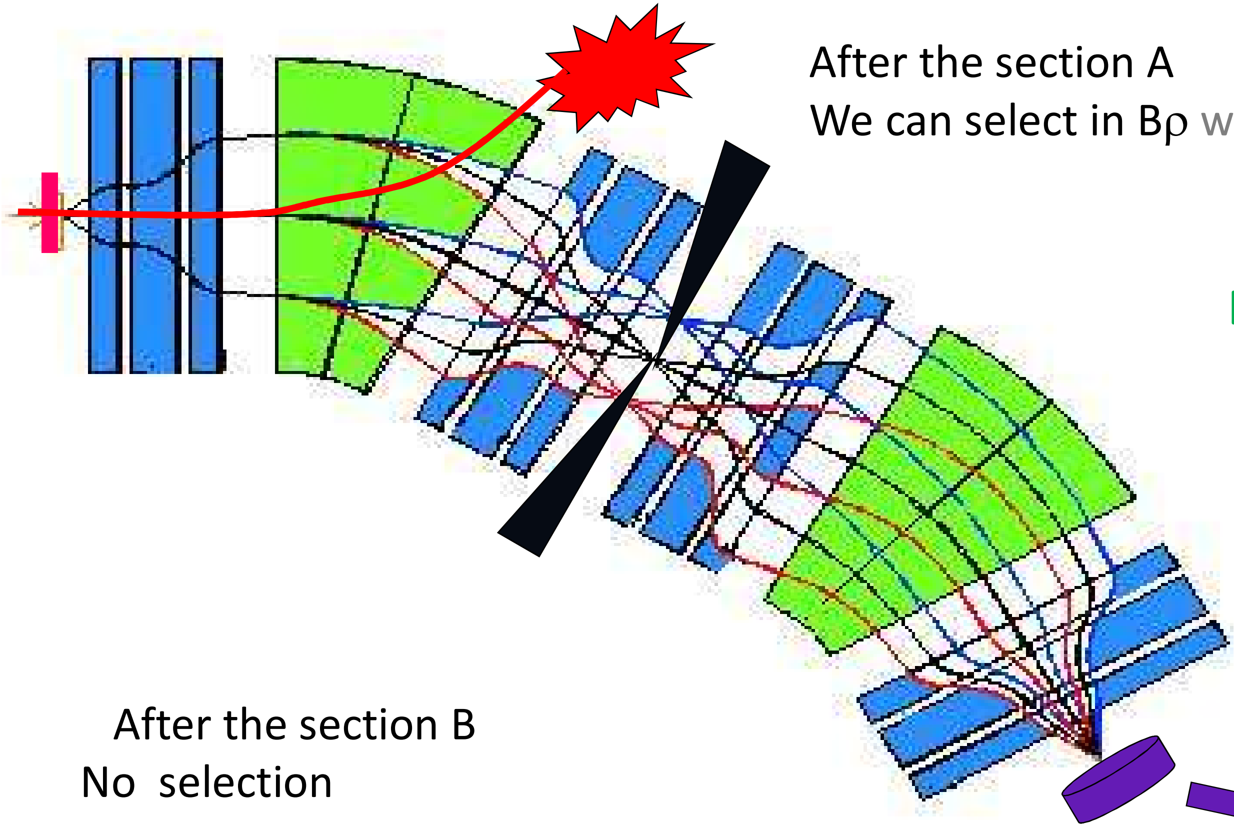
$R_{16}(A) \neq 0$

$\delta = (B\rho - B\rho_o) / B\rho_o$

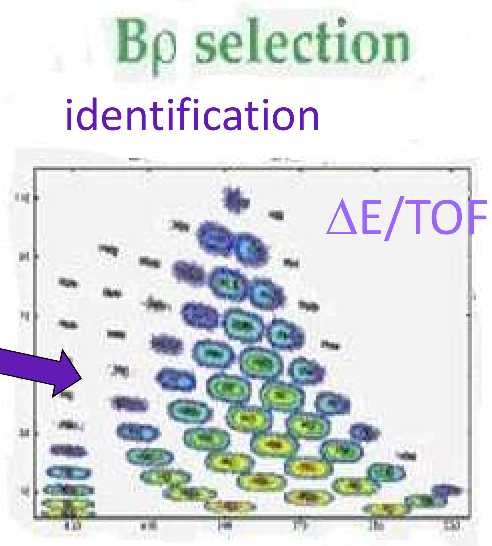
$B\rho_o$ = Bdipole. Rdipole

1 Bρ Selection in Fragments separators is not sufficient

After the section A
We can select in Bρ with movable slit



Bρ Selection
Is not good enough

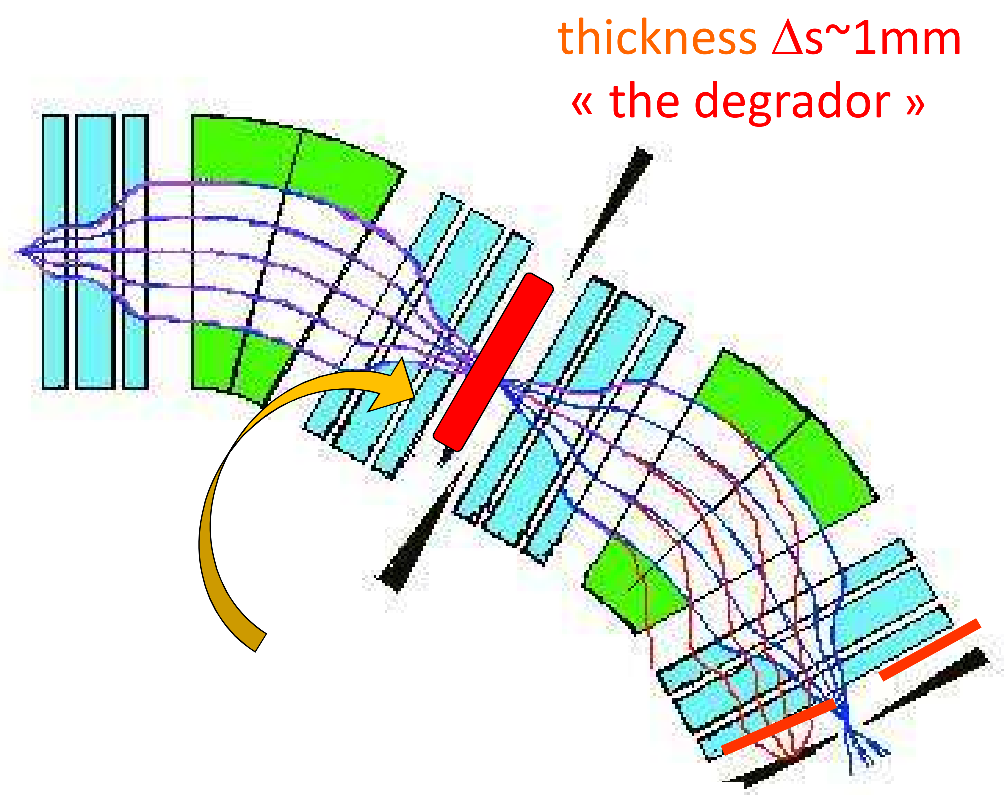


After the section B
No selection

Primary beam is eliminated
, but too many isotopes (≠Z,A)
are produced and transported up to the end

Magnetic separator with a degrador increases the purification (Z dependance)

We consider 2 isobars (A=34, Z=14) ; (A=34, Z=15) with same Bρ



Selection 2: Energy loss

Bρ selection is independant

from Atomic number (Z)

Bρ= γ M v /Q

in the degrador Energy loss

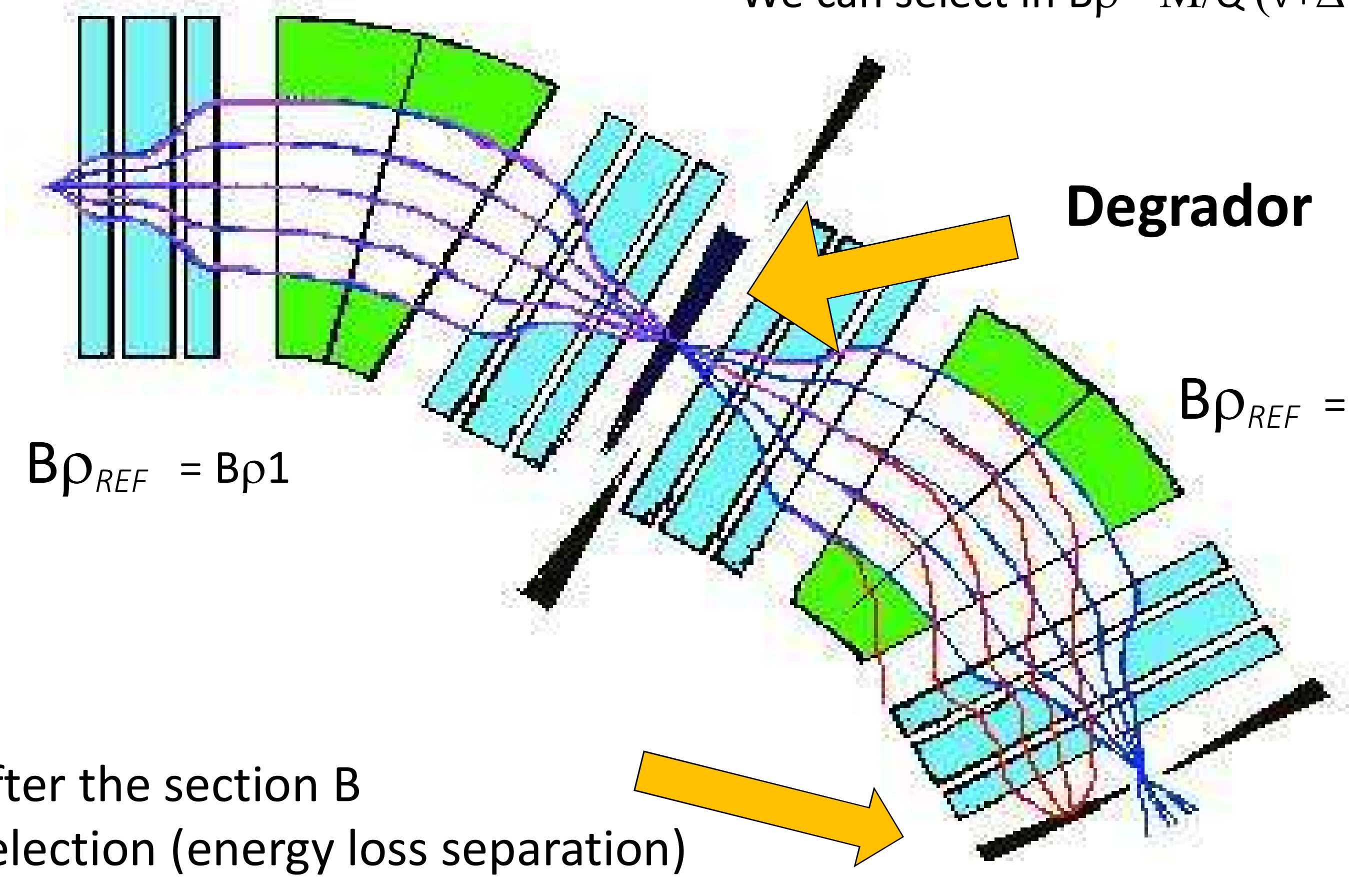
is « Z dependant »

~Bethe-Bloch formula :

$$\Delta E = k\, Z^2/A * \Delta s$$

2 Selections in Fragments separators
Bρ + Energy loss (degrador)

1) After the section A
We can select in $B\rho \sim M/Q\ (v+\Delta v)$



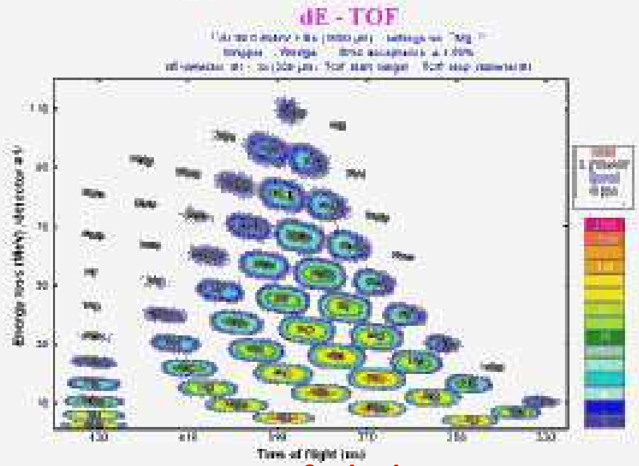
2) After the section B
Z selection (energy loss separation)

Selection in Fragments separators & identification

Bρ selection

ΔE

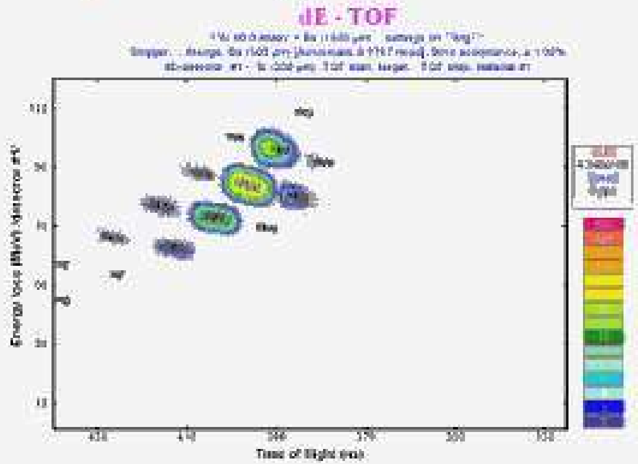
~Z



Time of Flight~M/Q

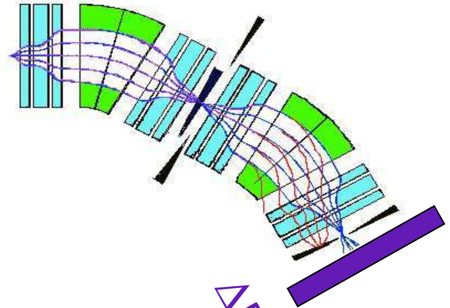
Bρ + degrador selection

ΔE



Time of Flight

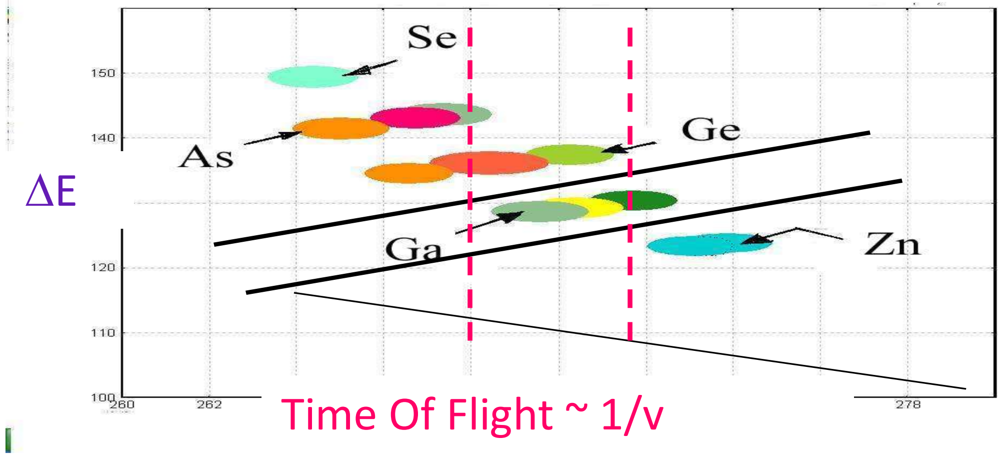
Detector :
Thin Silicone



2 selections
Is much better for
purety

Often, Isotopes are not well identified (ΔE,TOF)

detector Plot (TOF, ΔE)



Isotopes are mixed in TOF

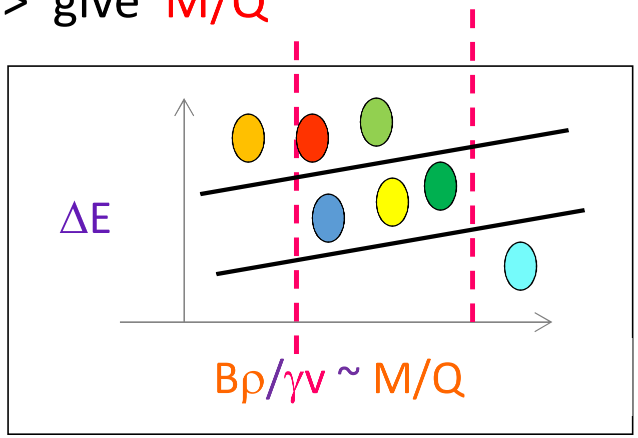
Large velocity distribution Δv

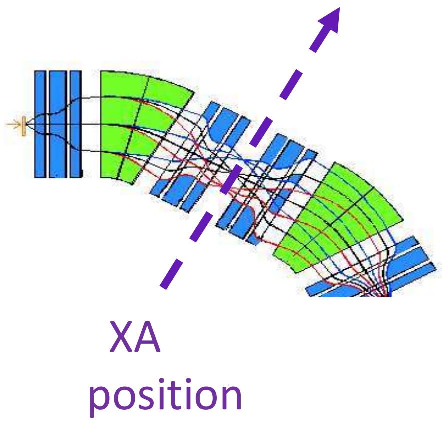
Solution : Measure X_A for each ion : $B\rho = B\rho_0 \,(1 + X_A / R_{16})$

The two measurements (TOF,Bρ) => give M/Q

$v = TOF/L$

$M/Q = B\rho \;/ (v\,.\gamma)$





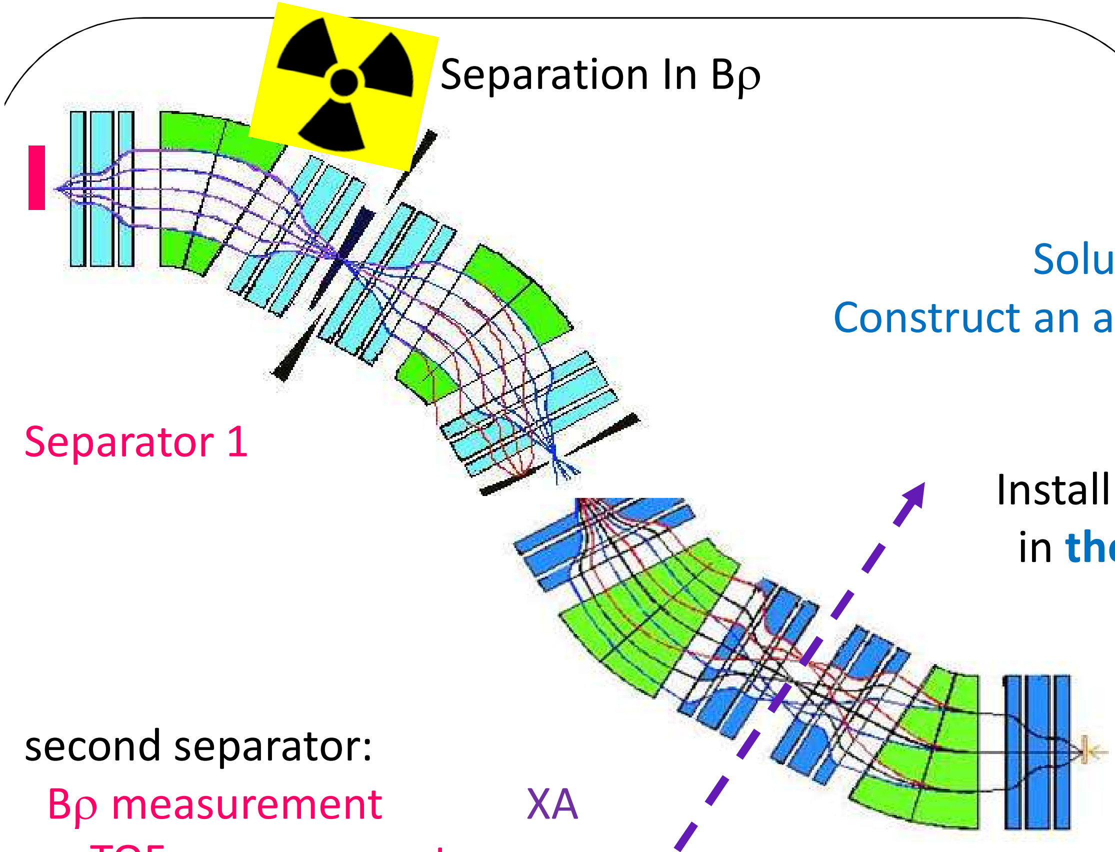
Isotopes are not well identified with (ΔE,TOF)

Install a Detector position for XA : $B\rho = B\rho 0\,(1+XA/R16)$

<u>NOT POSSIBLE</u>

(too much intensity before Z selection)

Separation In $B\rho$



Separator 1

Solution choosen in BigRIPS :

Construct an additiv spectrometer to measure $B\rho$

(€ !!)

Install the position Detector

in **the second separator**

second separator:

$B\rho$ measurement

TOF measurement

Summary

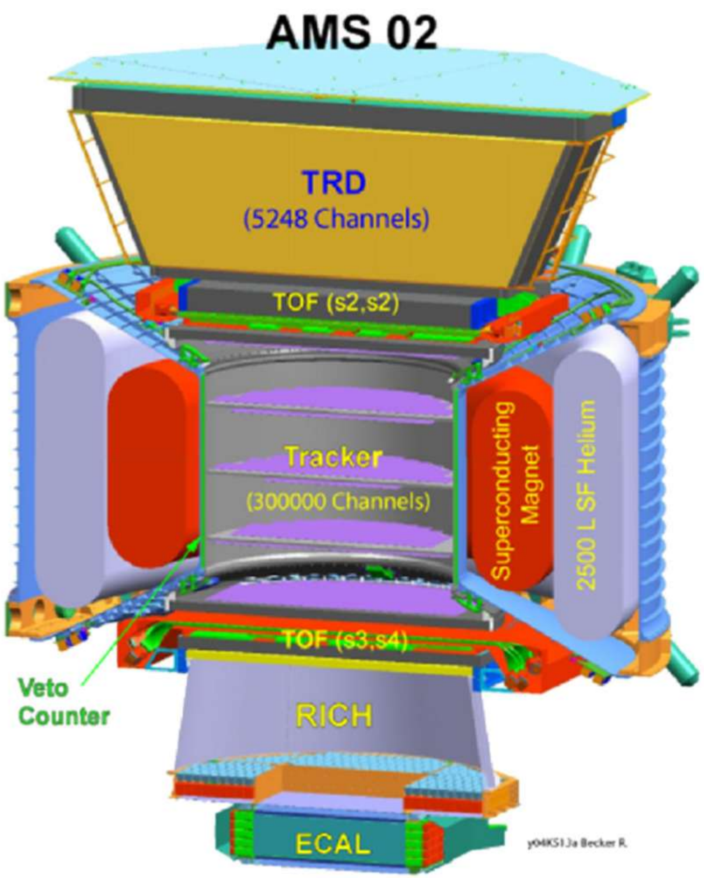
- ❑ Basic concepts and history

- ❑ Magnetics spectrometer with accelerator beams

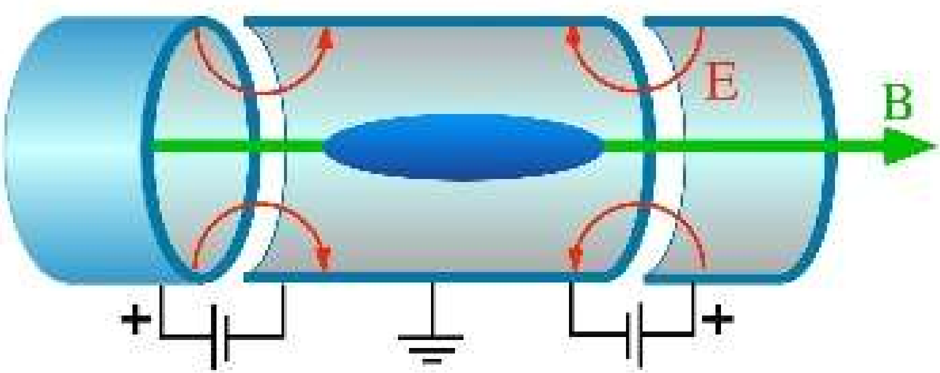
  - Part 1 : Exemples at Cern, Orsay, optical matrix
  - Part 2 : Fragment separator at Tokyo

- ❑ **Spectrometers without particle accelerators**

Spectrometer experiments without accelerators



AMS at ISS (10-300GeV e-)



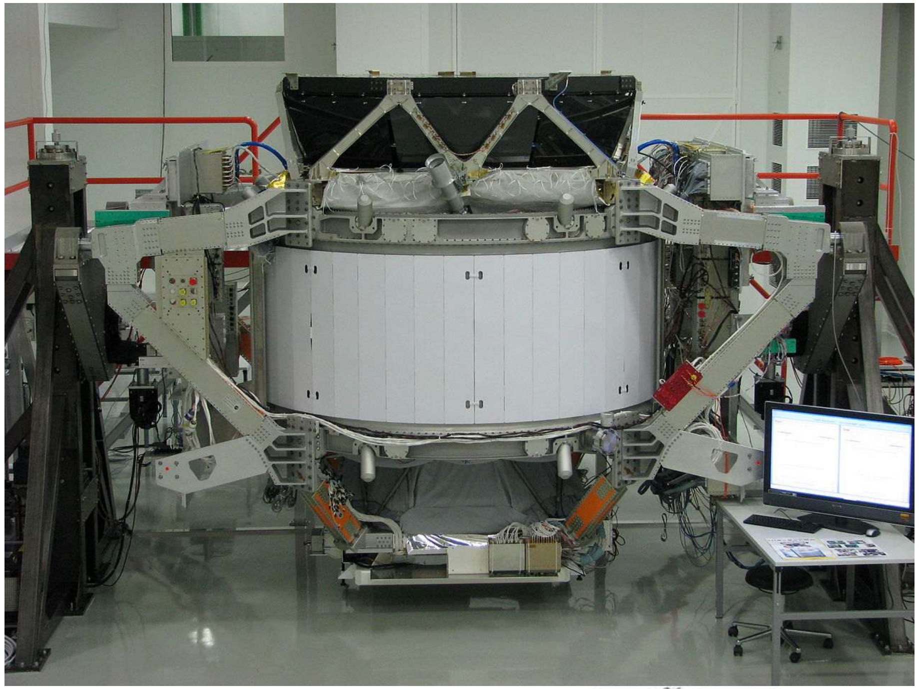
Penning Traps

1eV ions

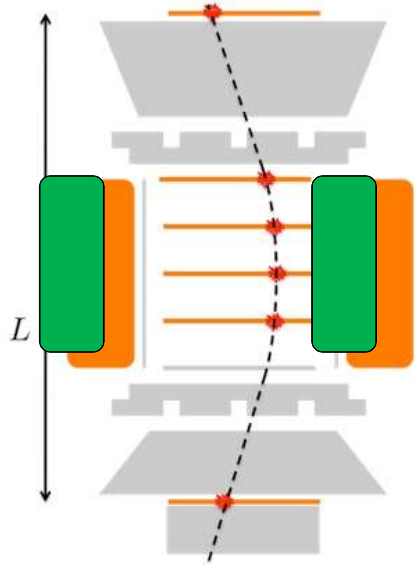
AMS-02 : a spectrometer in space
Alpha Magnetic Spectrometer (in ISS since 2011)

7.5 tons : 1 dipole magnet + trackers + 1Calorimeter…

P=2,3 kW S_{active}=0,8 m2



$\overline{R} \propto \overline{B_{scm} l^2}$



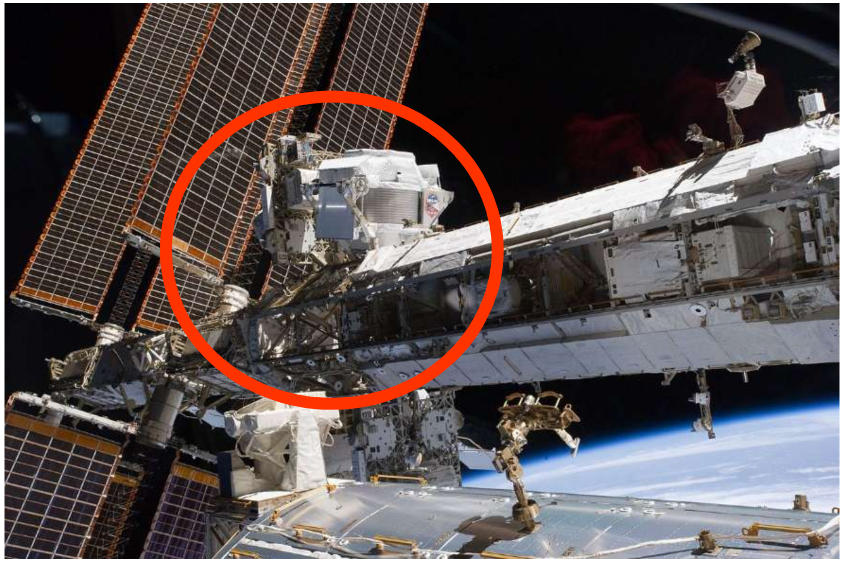
Permanent Magnet Scenario

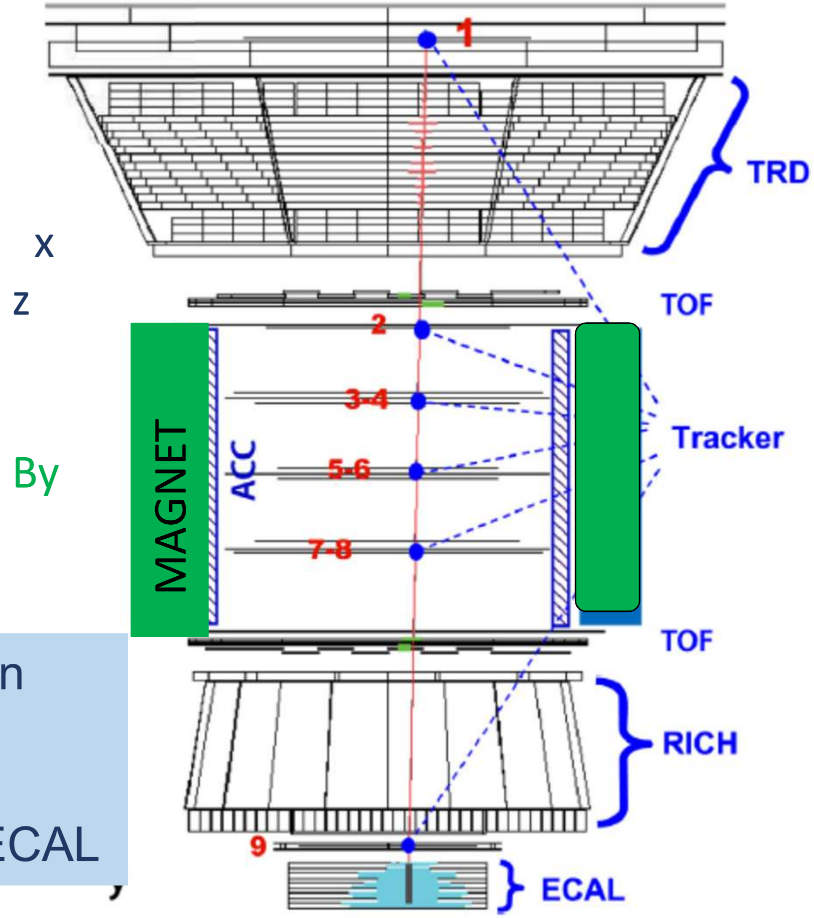
$$\frac{\Delta R}{R} \propto \frac{1}{B_{pm} l L}$$

Cosmic Ray Measurement + quantification of Matter/ Antimatter

e- //e+ ; p // pbar ; heavy ions He; Li ;….

AMS-02 at International Space Station (ISS)





The reconstruction of a 300 GeV electron measured by AMS2

with the signals in TOF, tracker, RICH and ECAL

- Spectrometer + detector that measures antimatter

in cosmic rays : e- // e+ [10-300 GeV]

p // p bar

AMS2 with « a dipolar field » B=Bx

higher field=higher dispersion= better Resolution

Electromagnet excluded : too heavy // high power

So 2 magnet options : sufficient field ? & uniformity ?

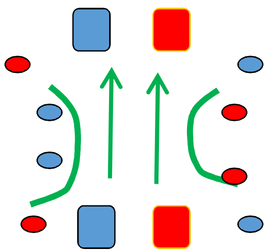
Superconducting coils :

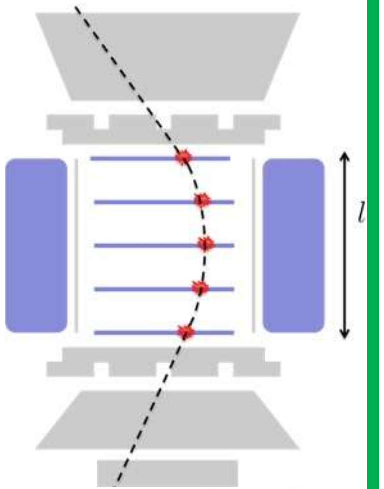
Bz ~ 0,8 T

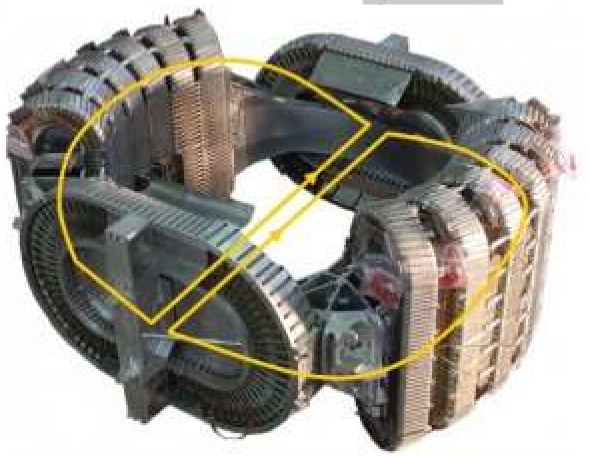
Bx Produces

by courant distribution

I(θ) # Io cos(θ)

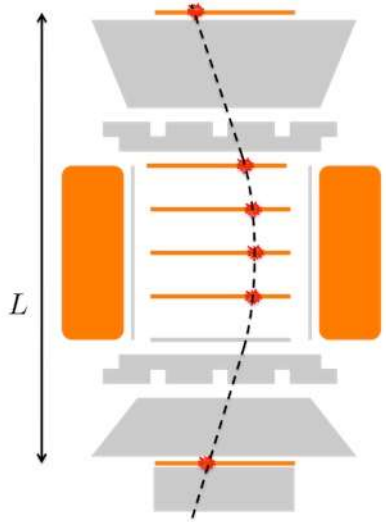


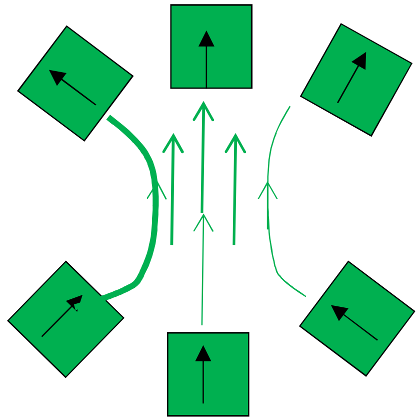




Permanent magnets : more reliable

Bz ~ 0,15 T



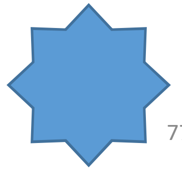


Permanent Magnet Scenario

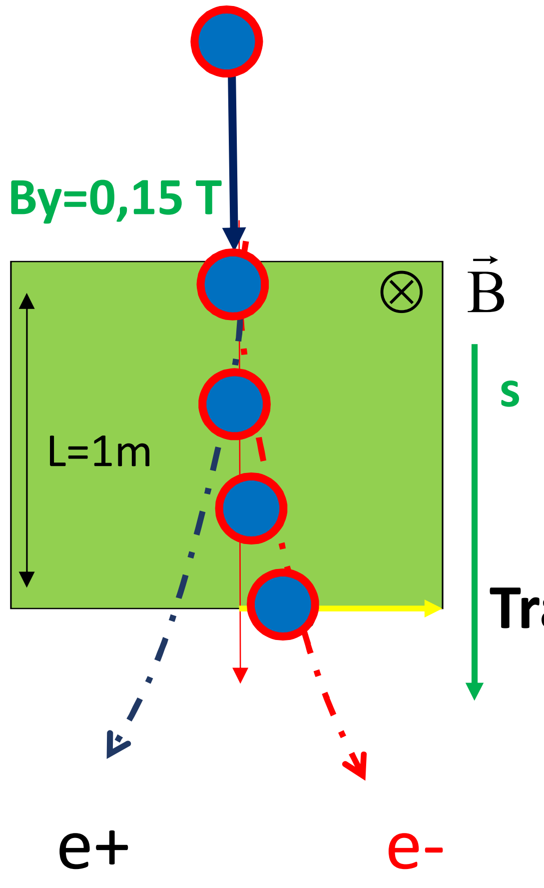
$$\frac{\Delta R}{R} \propto \frac{1}{B_{\mathrm{pm}} l L}$$

Bx produces by magnetization distibution

M(θ) =Mo cos(θ)



AMS2 :Traking in a dipole magnet



Particle electron (<50GeV) **$B\rho_0$ < 167 Tm**

Radius of trajectory $R_0 = B\rho_0 / By$ = 167T.m / 0,15T =

R_0 =1100m

deviation angle x' ~ ∫B ds / $B\rho_0$ ~ 1 mrad

Trajectory in magnetic field =

Circle of Radius R ~ 100-1100m

Four points (trackers) on a circle with a huge radius

in fact the trajectories are close to a parabola

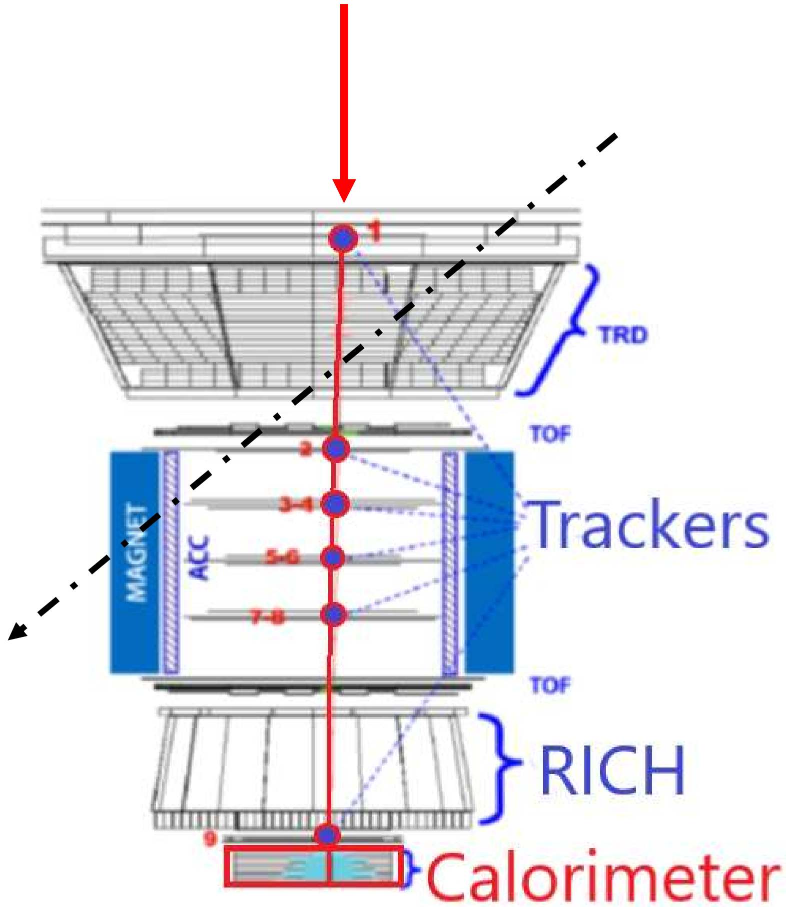
$$X(s) = A + B\,s + C\,S^2 + \ldots$$

The curvature of the parabola is C = $R_0/2$

Fit X(s) with N trackers in Magnet in field ($\chi 2$ minimization)

and get (A,B,C) => the magnetic rigidity is obtained $B\rho$= **By** . R_0 = **By .2C**

AMS2 :Tracking in a dipole magnet and full identification



2 T.o.F : veto : if **2 signals event is ok**

Trackers : Fit X(s) on the trackers

find R and compute $B\rho = B.R$

Calorimeter = gives E

RICH gives β

$B\rho = P/Q$ (Tracker)

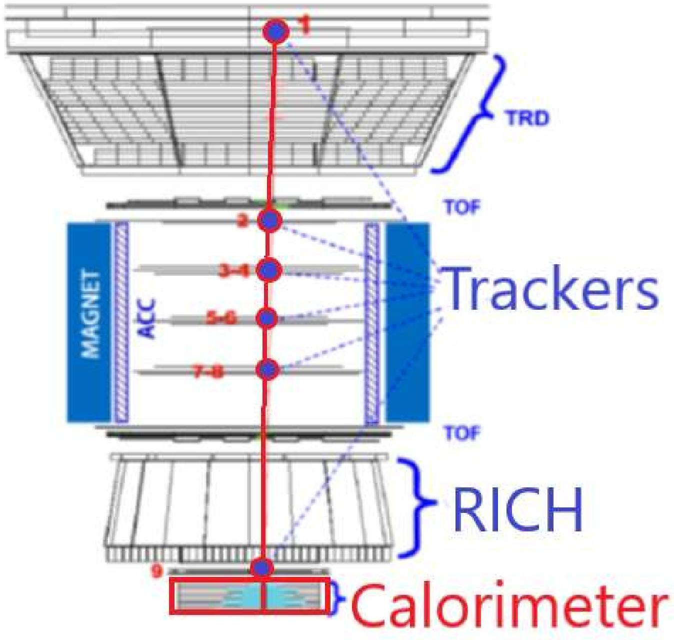
$E=(\gamma-1)\,mc^2$ (calorimeter)

Z & $\beta = v/c$ (RIng CHerenkov Detector)

Identification = (m, Z, E, β)

AMS-02 detectors and results

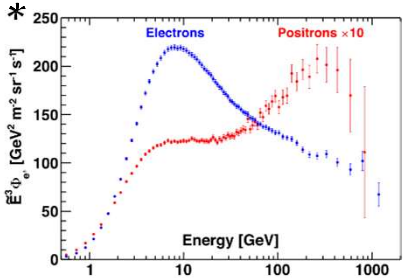
-**TRD** :A transition Radiation detector (gas)
Allows the separation of e //p
-Ring Cerenkov detector :$\Delta\beta/\beta$ ~ 0,1 / Z %
z measurement (ions)
-2 TOF detectors : are a fast trigger (coinc.)
- Trackers : 10 micron resolution for e/p
e: R=2,5% in Brho up to 100GeV



[2011-2025]

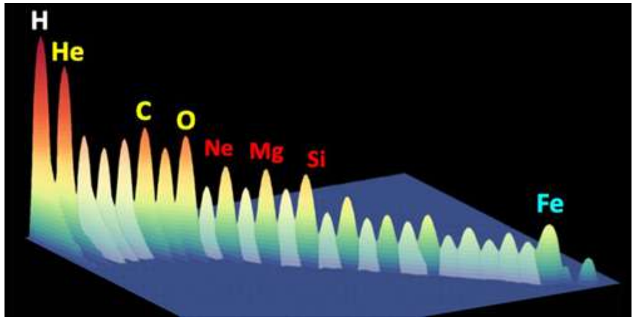
Energy Distribution electron and positron different

*

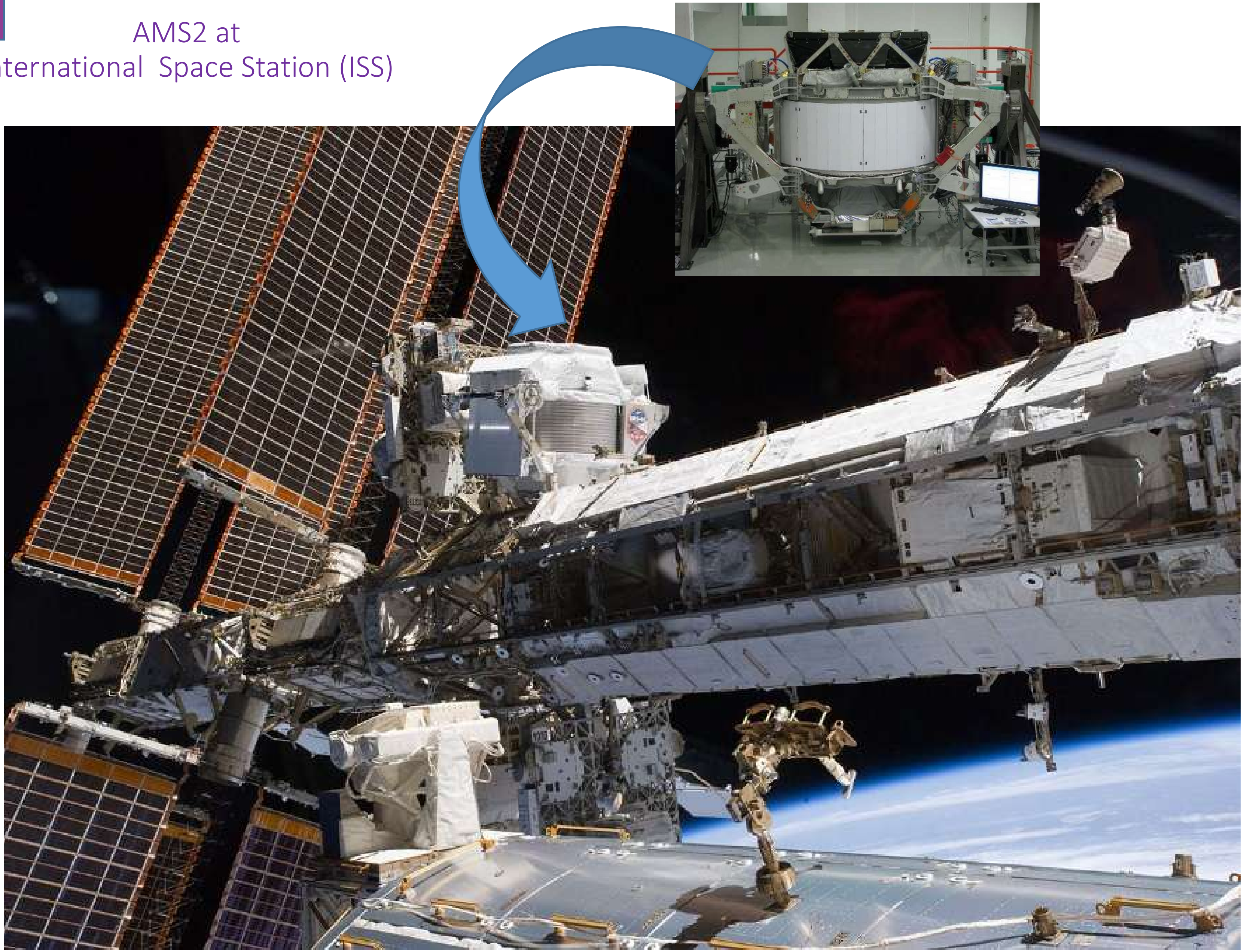


Large difference
Source of emission different ?

Data has been collected for radioprotection issues *
(long flight toward mars ?)
ions distribution P, He, Li,...Ni



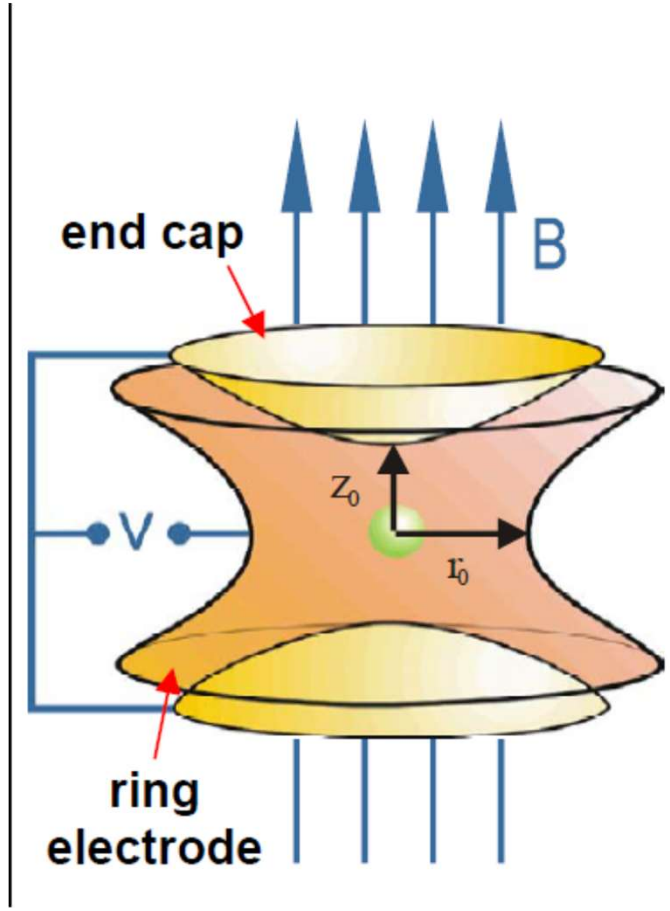
AMS2 at
International Space Station (ISS)



Penning Traps : high resolution mass spectroscopy

Used in research but applications expending

The ultimate mass resolution device

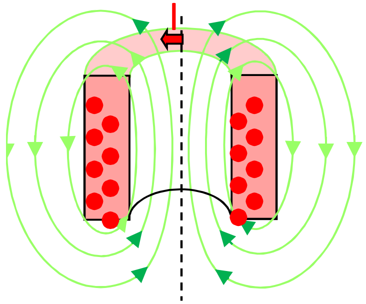


3 tricks

1) Confinement (Bz+Ez) static EMfieds

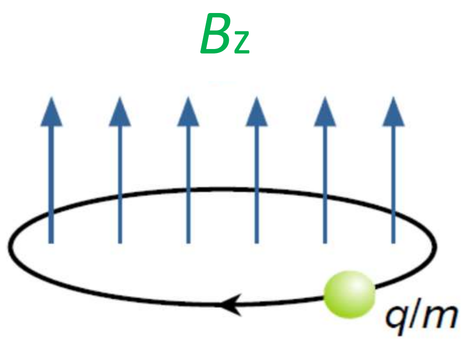
2) Excitation (RF field)

3) Extraction (Tof measurement)



Penning Traps (1) : confinement

Solenoid coil : field (Bz) is axial

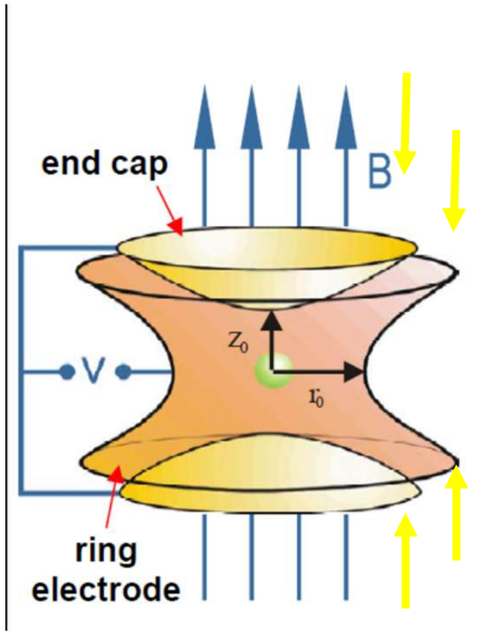


3D trapping technics (Radial+longitudinal)

1a) Radial (x,y) confinement with Bz

-natural cyclotron frequency

ω_c= frevolution x 2π = q**B**/m

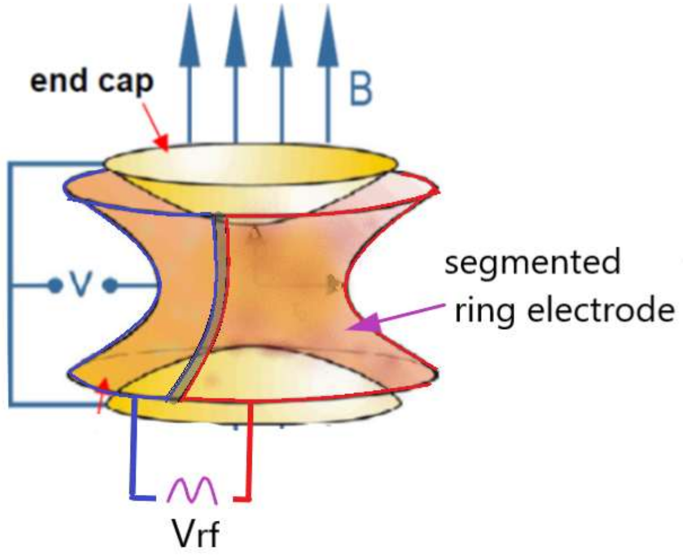


1b) Longitudinal (z) confinement with Electric field

$V = U_0\,(2z^2 - x^2 - y^2)$ $F_z = -4\,z\,U_0$

Hyperbolic Electric field : End caps + Ring electrode

Penning Traps (2) : RF excitation at resonance



Excitation RF : segmented Ring electrode

natural cyclotron frequency

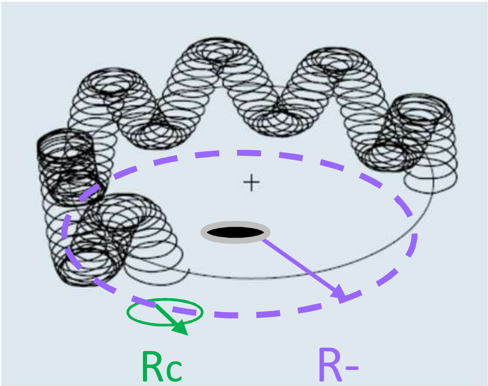
$\omega_c = f_{revolution}/2\pi = qB/m$

Excitation with RF electic field

$V(t) = U \cos(\omega_{rf}\, t)$

if $\omega_{rf} = \omega_c$

Radial motion increases : resonance



in reality, the motion is complex :

Coupling between radial (Bz) and axial motion (Fz)

$\omega_c = \omega^+ + \omega^-$ $\quad\quad \omega_z^{\,2} = 2\,\omega^+ \cdot \omega^-$

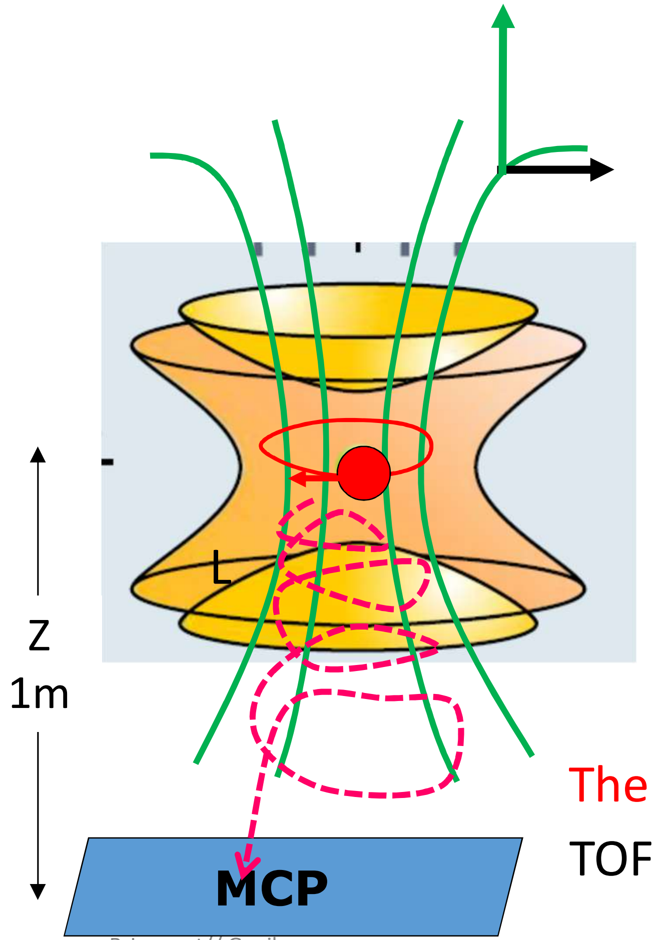
$\omega_c^{\,2} = \omega^{+\,2} + \omega^{-\,2} + \omega_z^{\,2}$

Penning Traps (3) : extraction

Extraction with a time of flight measurement

- axial field Bz = radial Force(radial) = q [$v \times Bz$]
  : confinement

$\mathbf{B} = Bz + B_y + B_x$



Extraction in solenoid's end fields

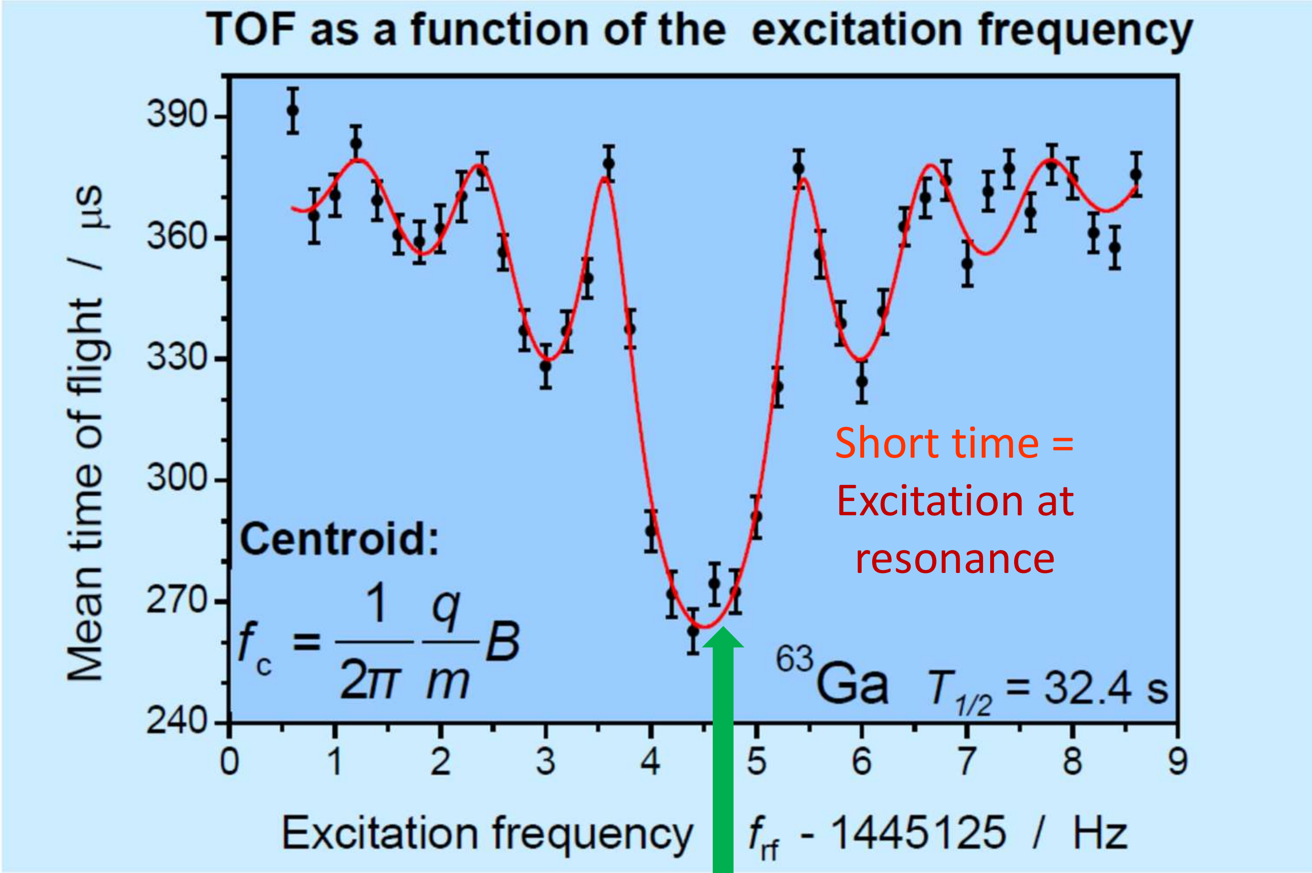
Force (z) = q [$v_x \; B_y \; - v_y \; B_x$]

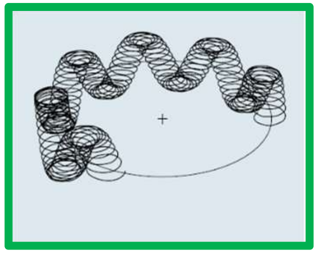
(By+Bx) act on the excited ions with large Vradial

The most Excited ions reach the detector (MCP) quickly

TOF Measurement TOF= f(Vradial) =f (resonance)

Penning Trap : mass measurement



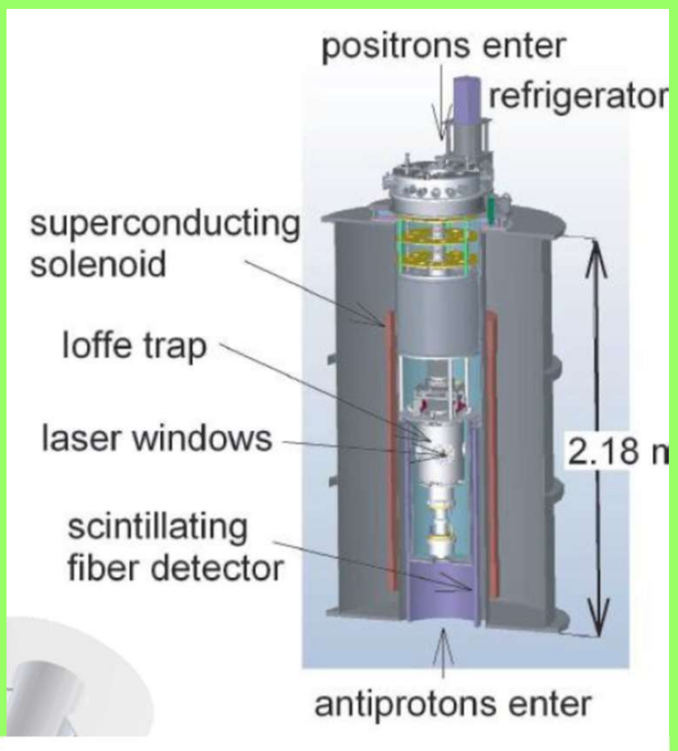


When the ions are perfectly excited ions by the good RF frequency

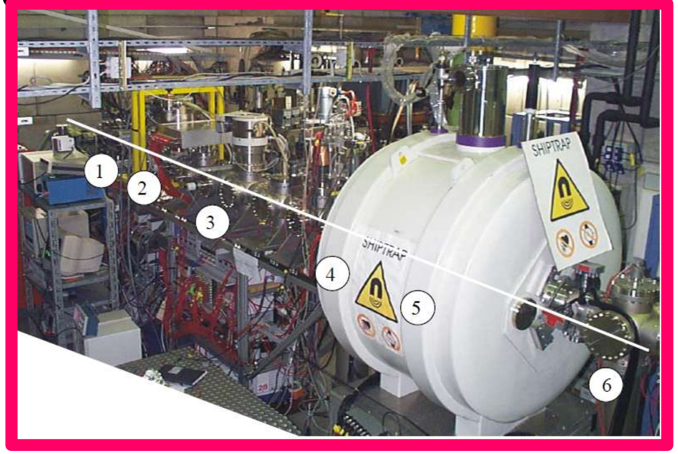
reach the detector (MCP) quickly : f_{rf}= Frevolution $=2\pi \ qB/m$

Penning Traps : Many experiments operated in the world

Magnet = superconducting solenoid (Bz # 3-6 Tesla)



$$\overline{\mathrm{p}} + \mathrm{e}^{+} + \mathrm{e}^{+} \rightarrow \overline{\mathrm{H}} + \mathrm{e}^{+}$$



Particle Physics at Cern

M/q of P and Pbar ($R=10^{-10}$)

-ATrap - ATrap2, Athena …

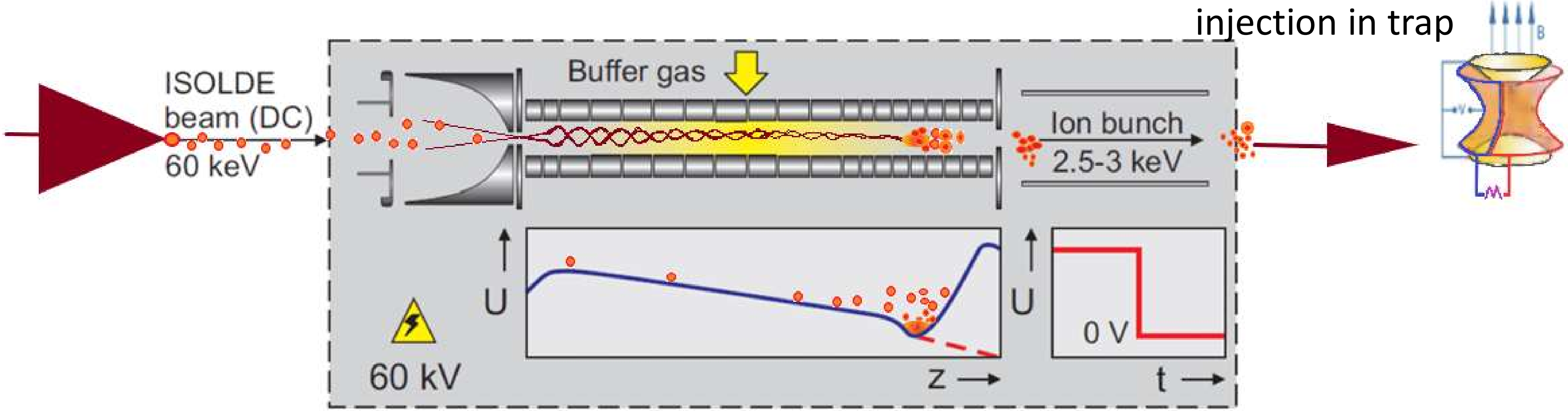
- **Anti Hydrogen** Production & cooling..
- Decay measurement
- Atomic spectroscopy

Nuclear Physics

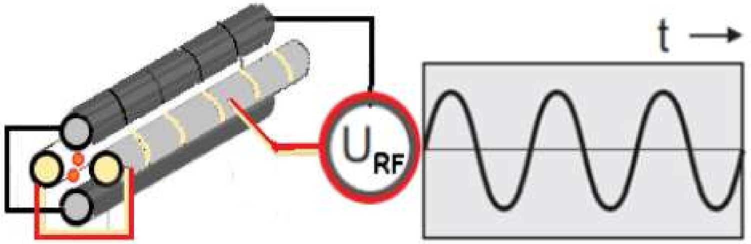
- Ship Trap (GSI)
Z>92 mass measurement
- Isolde Trap (CERN)
- JYFL Trap (finland)
- Piperade (LP2IB Bordeaux => GANIL)
- MLLTrap (IJClab=> Ganil)

Complex beam preparation before a trap measurement
Isoldetrap exemple at CERN

Low emitance
ready for
injection in trap



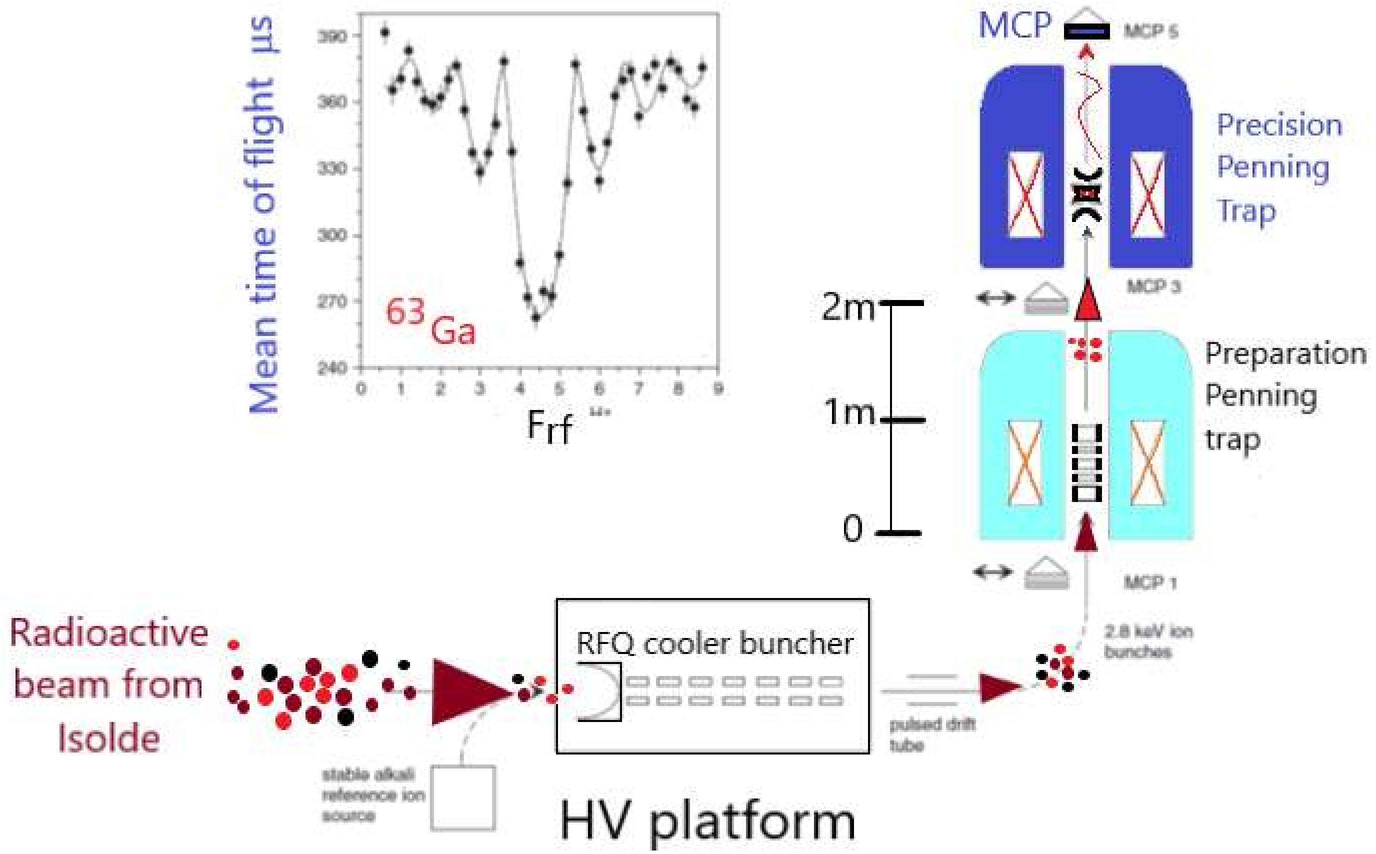
Radio frequency
quadrupole
Cooler



r_0	6 mm
L_{rods}	880 mm
U_{RF}	150 V
f	1 MHz
P_{He}	10^{-2} mbar

Cooling in the RFQ cooler : **improves beam quality** (emittance)
beam slowing down
bunching for injection in trap (cycle of injection, excitation, extraction)

Mass measurement of radioactive ions in CERN Isolde



END

- **The different technics : basic concepts and history**
  magnetic rigidity $B\rho = \gamma M v / q$

- **Magnetic spectrometer with accelerator beams**
- **Beam optics, full dynamics, matrix, quadrupoles**
- **Exemples : LHCb, Split pole (Alto_IJCLab), VAMOS (Ganil)**
- **Fragment separator Big RIPS facility (RIBF_Tokyo)**

- **Spectrometers without accelerator**
  Penning Traps, AMS at ISS

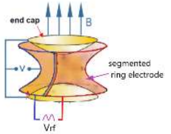
summary

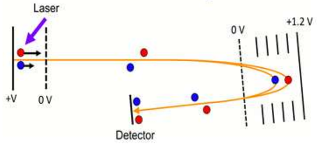
❑ **Most of the magnetic spectrometers select charged particle with their magnetic rigidities $B\rho = \gamma m v / q$**

Identification (m) requires often additive measurements (Brho+ TOF,…)

❑ **Some specific devices give directly the m/q ratio**

(Penning trap, Mass spectrometer with E+B field, Reflectron

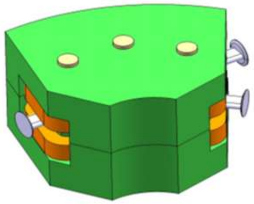


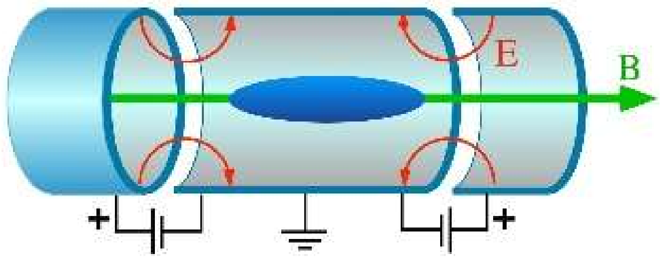


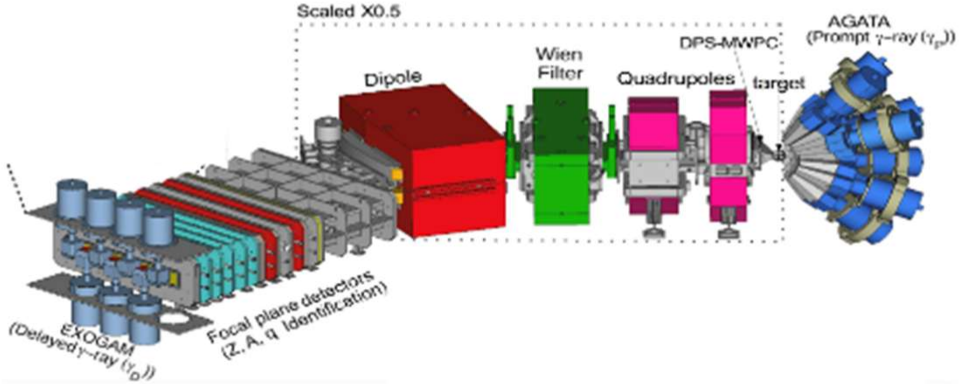
❑ **The properties of spectrometers are optimised with many tricks**

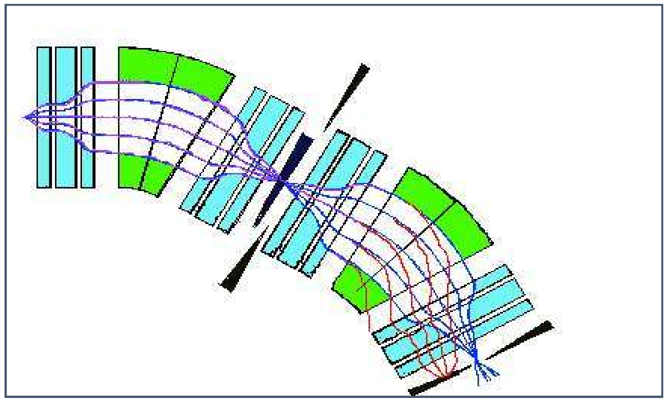
- **Resolution (large dispersion and good detector resolution)**
- **Acceptance (large dipole or detector, focusing quadrupole,..)**
- **selectivity (elimination of distribution tails, several selection stages)**

Electromagnetic Spectrometers









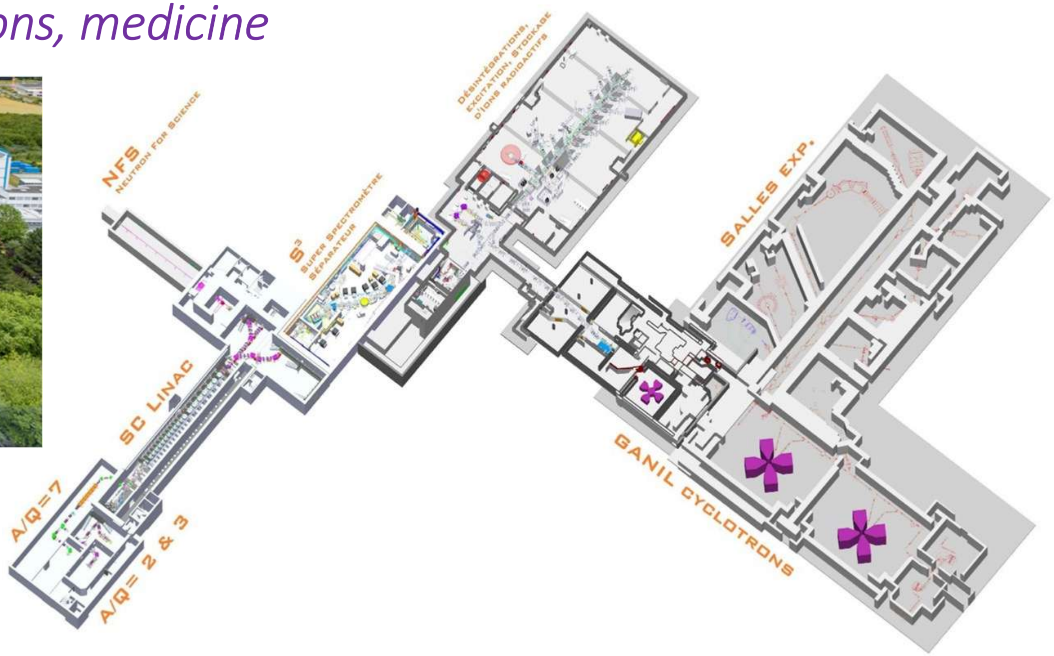
Useful relations :

$$E_0 = mc^2 \; ; \; E = E_0\gamma = mc^2\gamma \; ; \; p = mc\beta\gamma \; ; \; cp = mc^2\beta\gamma = E_0\beta\gamma \; ; \; E^2 = E_0^2 + p^2c^2$$

$$\beta\gamma = \frac{cp}{E_0} \; ; \; \gamma = (1-\beta^2)^{-\frac{1}{2}} \; ; \; \beta^2\gamma^2 = \gamma^2 - 1 \; ; \; W = E - E_0 \; ; \; \frac{mc\beta\gamma}{q} = B\rho \; .$$

	β	γ	W	cp
β	β	$\frac{\sqrt{\gamma^2-1}}{\gamma}$	$\frac{\sqrt{(1+W/E_0)^2-1}}{1+W/E_0}$	$\frac{cp/(mc^2)}{\sqrt{1+[cp/(mc^2)]^2}}$
γ	$\frac{1}{\sqrt{1-\beta^2}}$	γ	$1+W/E_0$	$\sqrt{1+\left(\frac{cp}{mc^2}\right)^2}$
W	$\left(\frac{1}{\sqrt{1-\beta^2}}-1\right)E_0$	$E_0\,(\gamma - 1)$	W	$mc^2\left[\sqrt{1+\left(\frac{cp}{mc^2}\right)^2}-1\right]$
cp	$mc^2\frac{\beta}{\sqrt{1-\beta^2}}$	$E_0(\gamma^2 - 1)^{1/2}$	$[W(2E_0+W)]^{1/2}$	cp

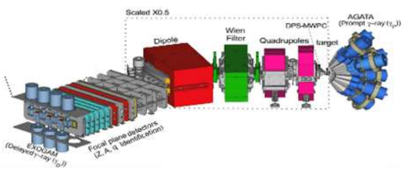
GANIL : *Nuclear physics, atomic physics , solid state physics, industrial applications, medicine*

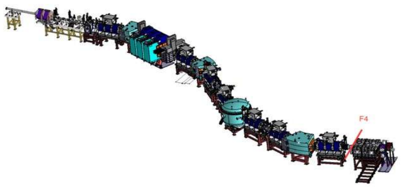


- ***1 superconducting linac***

- *5 cyclotrons*

- **Many electromagnetic spectro.(VAMOS, LISE, S^3, HRS DESIR , Trap...)**

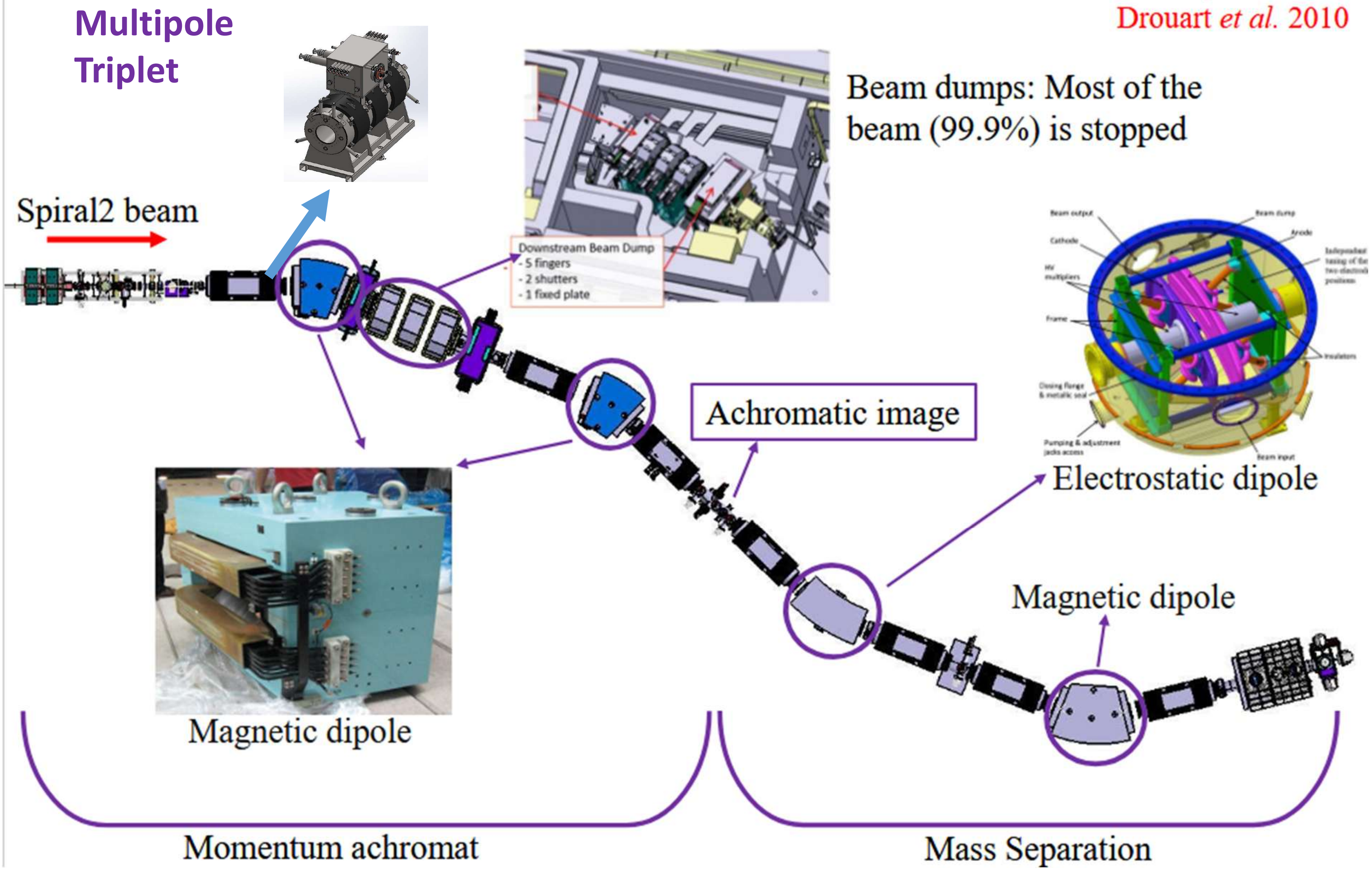




...

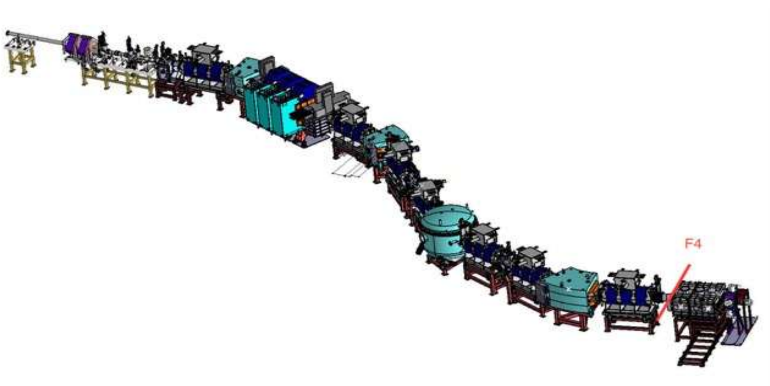
The Super Separator Spectrometer (S^3)

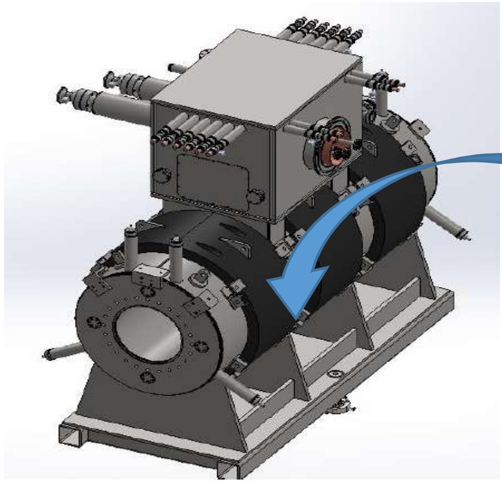
Dechery *et al.* 2016
Drouart *et al.* 2010

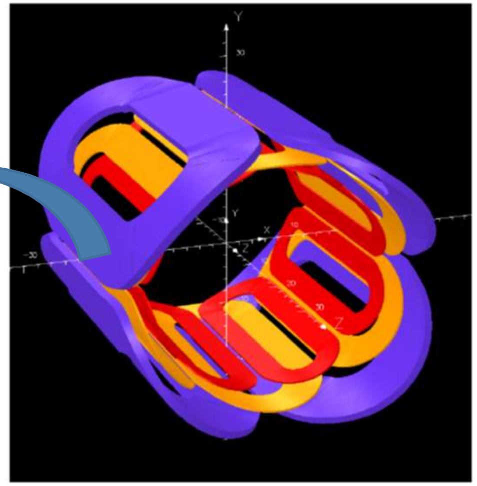


S^3 : The challenge

7 Superconducting Multipole Triplets







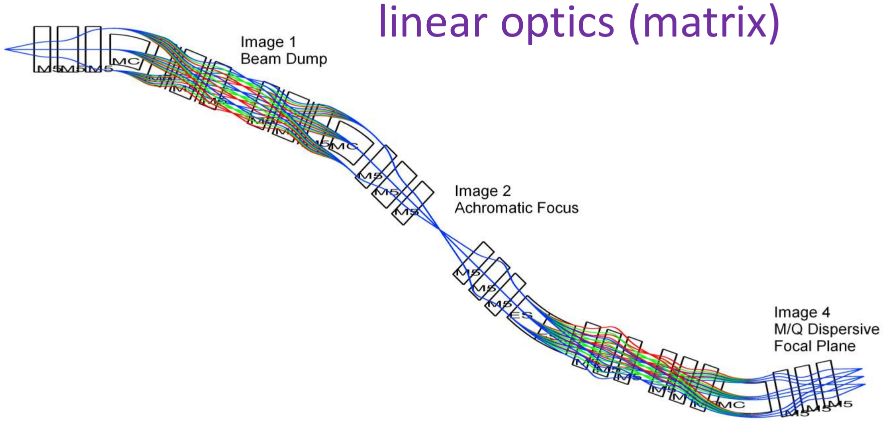
Quadrupoles+hexapoles+ octupoles

Beam optics : controlling the ion optical aberations up the third order

$$Z_i^F = \sum_l R_{ij} Z_j^0 \quad + \quad \sum_j \sum_k T_{ijk} Z_j^0 Z_l^0 \quad + \quad \sum_j \sum_k \sum_l U_{ijkl} Z_j^0 Z_l^0 Z_k^0 + \ldots$$

linear optics (matrix)

Aberrations= non linearities



Improving resolution with many corrections in the magnets