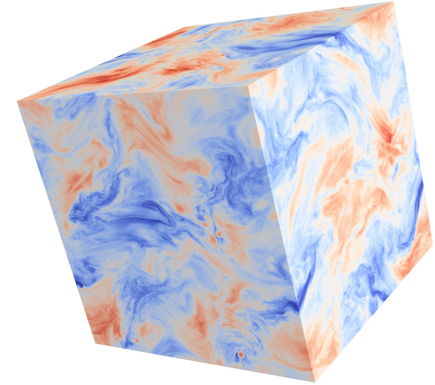
$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial \mu}\left(D_{\mu\mu}\frac{\partial f}{\partial \mu}\right) + \frac{\partial}{\partial \mu}\left(D_{\mu p}\frac{\partial f}{\partial p}\right) + \frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2\left(D_{p\mu}\frac{\partial f}{\partial \mu} + D_{pp}\frac{\partial f}{\partial p}\right)\right]$$



Kinetic Turbulence in Relativistic Astrophysical Plasmas

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(recently moved from Univ Colorado)

Thanks to my Univ Colorado group:

Vladimir Zhdankin, Kai Wong, Lia Hankla, Greg Werner, Mitch Begelman

Support: NASA, NSF

Kinetic Physics of Astrophysical Plasmas Workshop, Paris, June 20, 2025

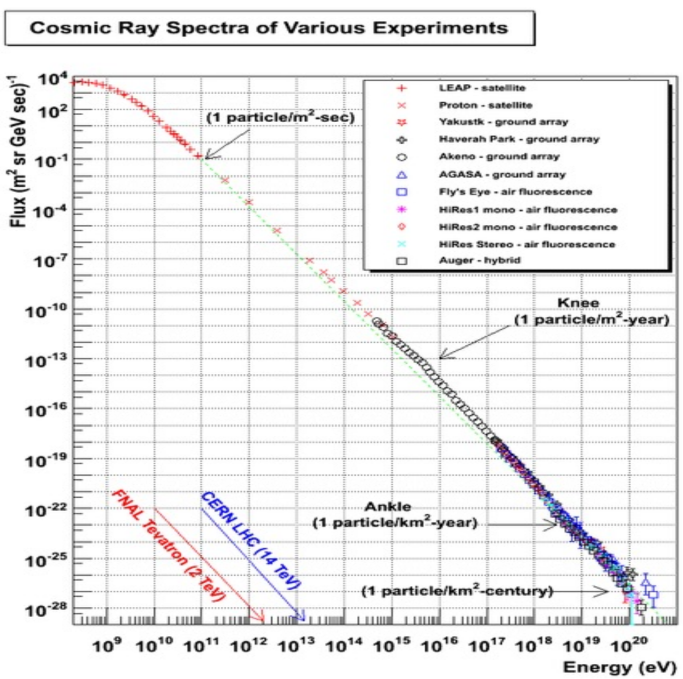
OUTLINE

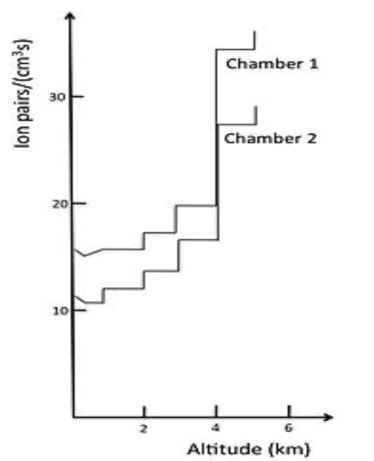
- Relativistic Particle Acceleration in Astrophysics:
  - Cosmic Rays (CRs)
  - Supernova Remnants (SNR) and Pulsar Wind Nebulae (PWN)
  - Relativistic Jets from Active Galactic Nuclei (AGN)
- Theory of Turbulent Particle Acceleration -- a History of Ideas:
  - Diffusive Fermi Acceleration
  - Quasi-Linear Theory (QLT) and the Fokker-Planck Equation
  - Beyond QLT
- Numerical Simulation Studies:
  - First-principles Particle-in-Cell (PIC) simulations of kinetic turbulence
  - Direct PIC-based numerical tests of the Fokker-Planck framework
- Conclusions
- Current Challenges and New Theoretical Directions

Nonthermal Radiation and Relativistic Particle Spectra are Ubiquitous in the Universe

I. Cosmic Rays

- Discovered by Victor Hess (1911-12) with hot-air balloon measurements
- Relativistic particles (mostly protons) with (broken) power-law energy spectrum from $m_p c^2$ = 1 GeV up to 10^{20} eV (>10 decades!)
- The original **Multi-Messenger Astrophysics** (non-EM)!
- Main driver of theoretical research of nonthermal particle acceleration



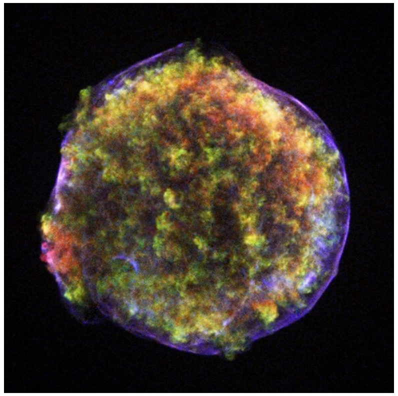


Astro Motivation:

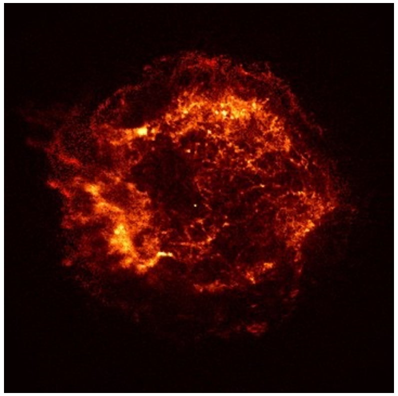
Nonthermal Radiation and Relativistic Particle Spectra are Ubiquitous in the Universe

II. EM Radiation from Supernovae & Pulsar Wind Nebulae

Supernova Remnants

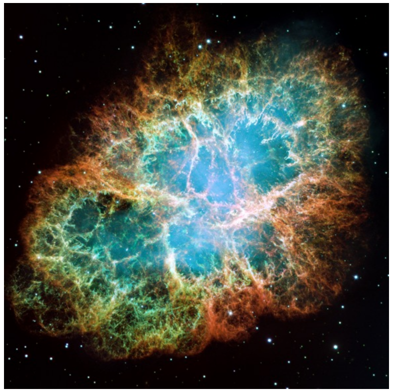


Tycho SNR 1572



Cas A SNR (~1690) X-rays

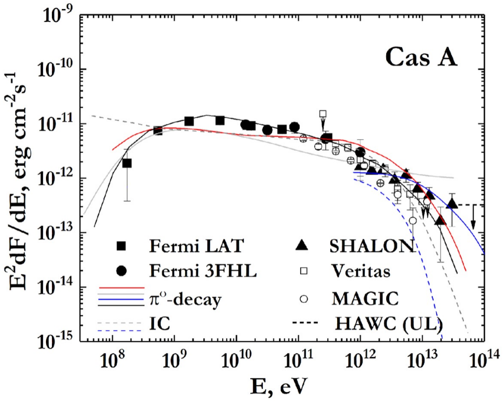
Pulsar Wind Nebulae

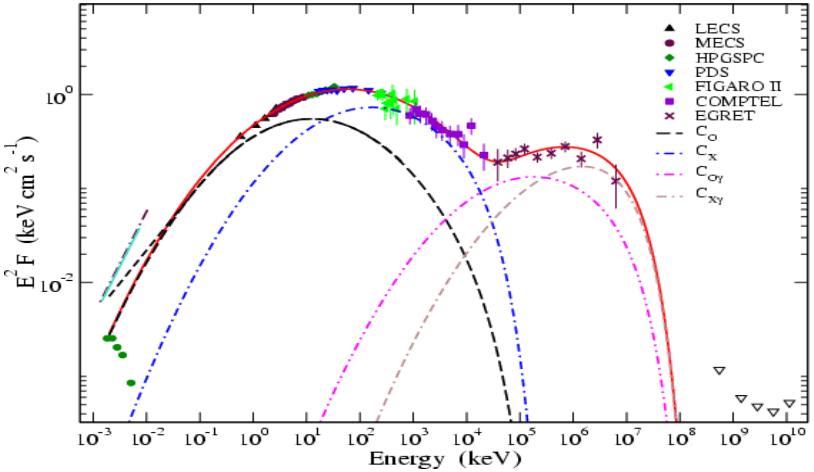


Crab Nebula (M1); HST

Crab Nebula; HST+Chandra

Supernova shocks
- main CR accelerators up to PeV.

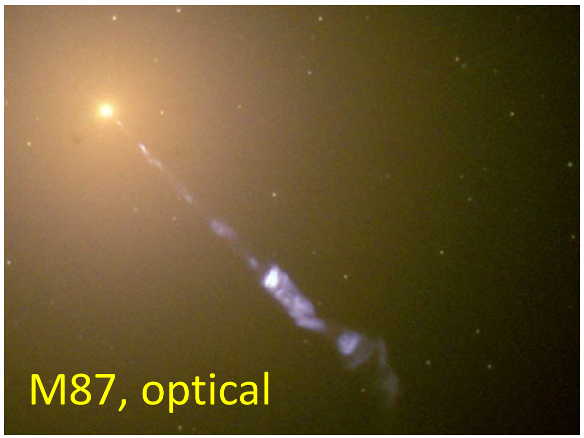


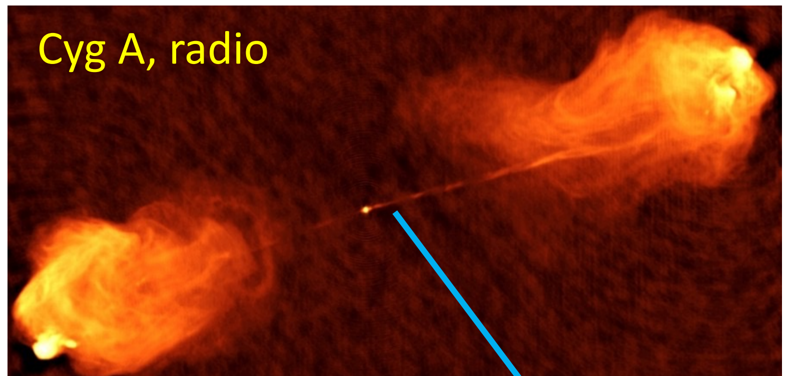


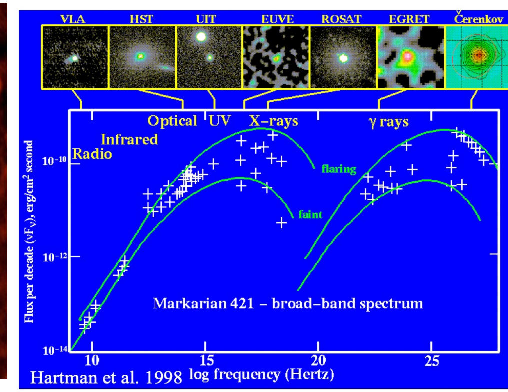
Astro Motivation:

Nonthermal Radiation and Relativistic Particle Spectra are Ubiquitous in the Universe

III. Relativistic Jets driven by Supermassive Black Holes (SMBH) in Active Galactic Nuclei (AGN), incl. blazars

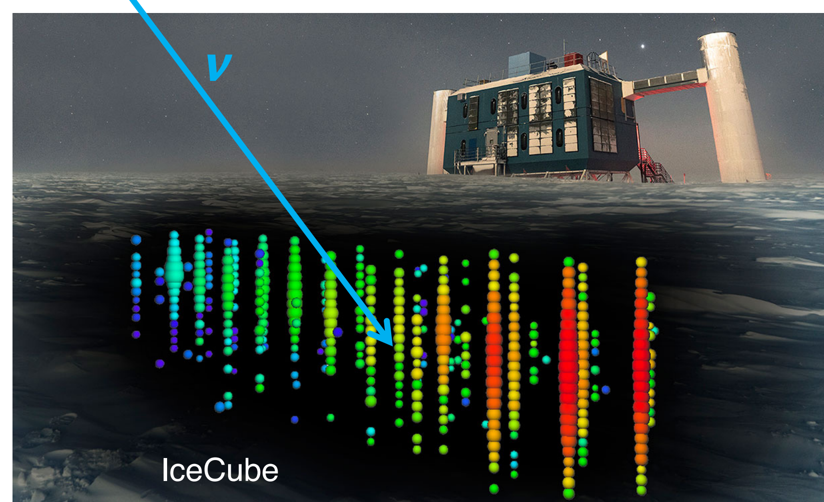






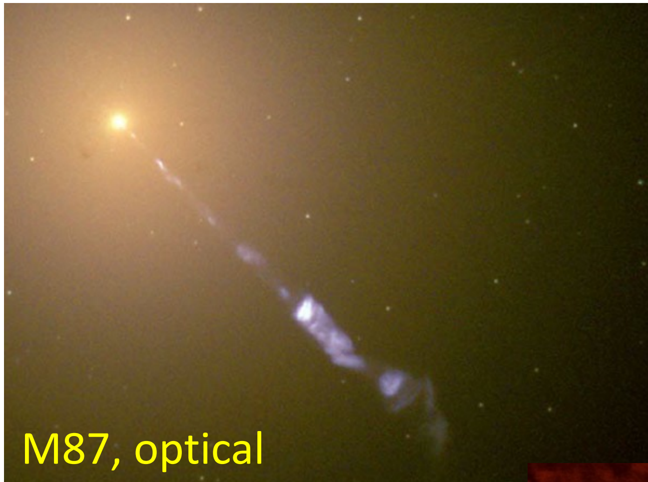
AGN jets are multi-messenger sources:

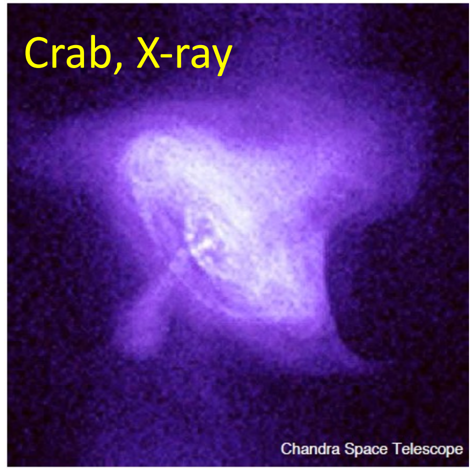
- main CR accelerators beyond PeV, including UHECRs
- Likely sources of PeV (IceCube) neutrinos

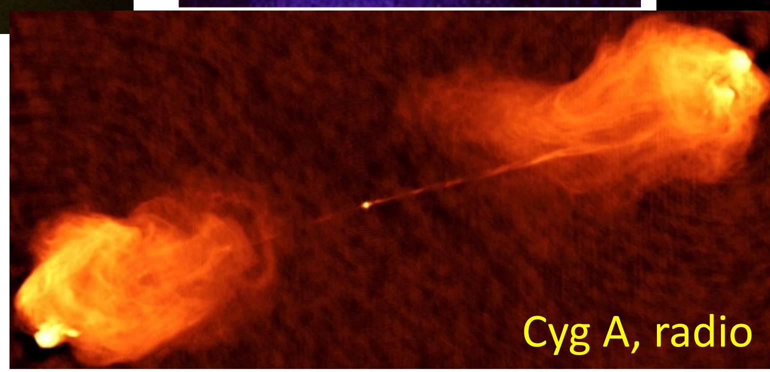


Astro Motivation:

High-Energy Particle Acceleration and Emission in Astrophysical Plasmas







- Astrophysical plasmas ***shine***.
- Radiative cooling time << travel time from source → *in-situ particle acceleration.*

Dissipation and Particle Acceleration Mechanisms

In Astrophysics, particles are accelerated by nonlinear ***collective plasma processes***.

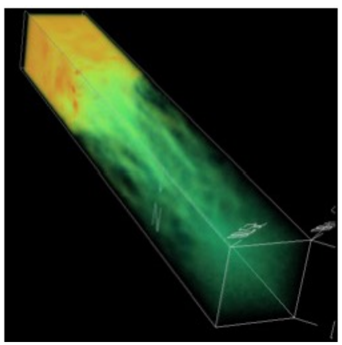
AVAILABLE FREE ENERGY

Bulk Kinetic

Magnetic (Poynting flux)

- longitudinal:

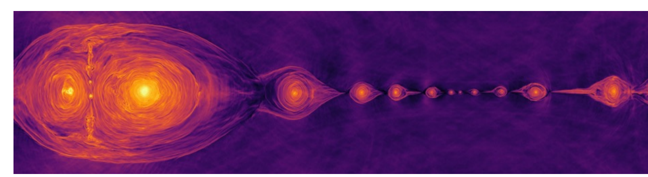
shocks



SINGULAR STRUCTURE

current sheets:

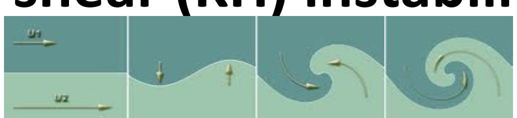
Magnetic reconnection

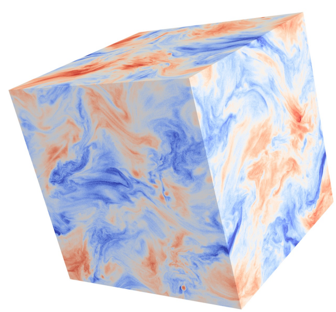


TURBULENCE

- transverse:

shear (KH) instability



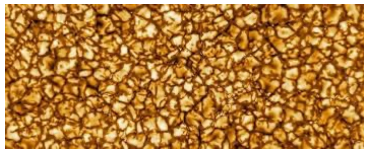


Astro Motivation:

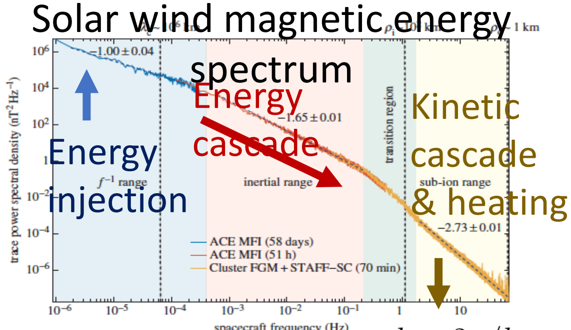
Turbulence is Ubiquitous in the Universe

Astrophysical plasmas are generally manifestly turbulent

- Solar/Stellar Convection Zone

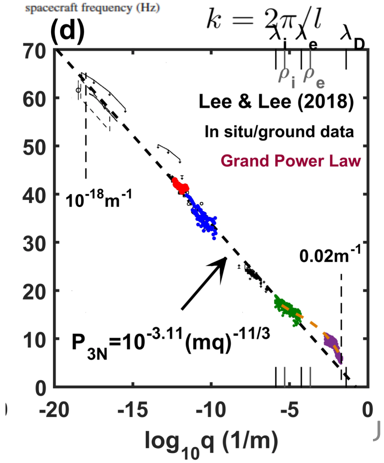


- **Solar Wind:**

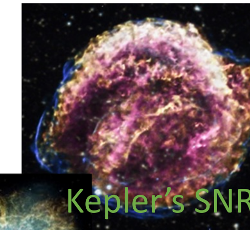


- **ISM:**

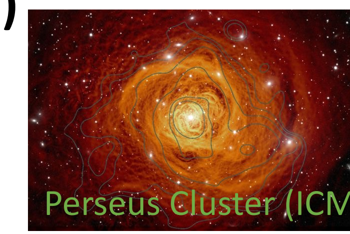
Big Power Law In the Sky



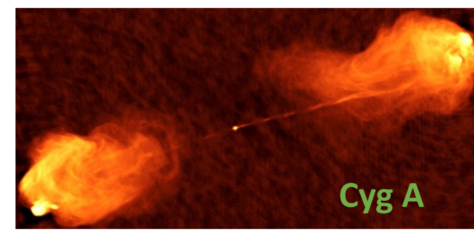
- **SN remnants, PWN**



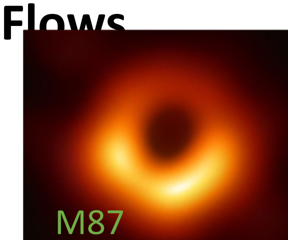
- **Galaxy Clusters (ICM)**



- **AGN jets**

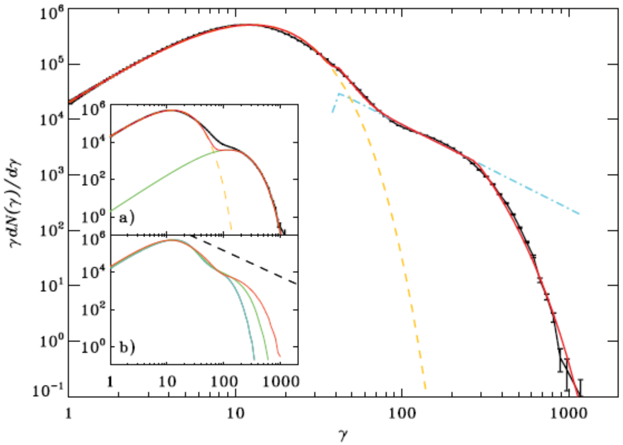


- **Black-Hole Accretion Disks, Advection-Dominated Flows**

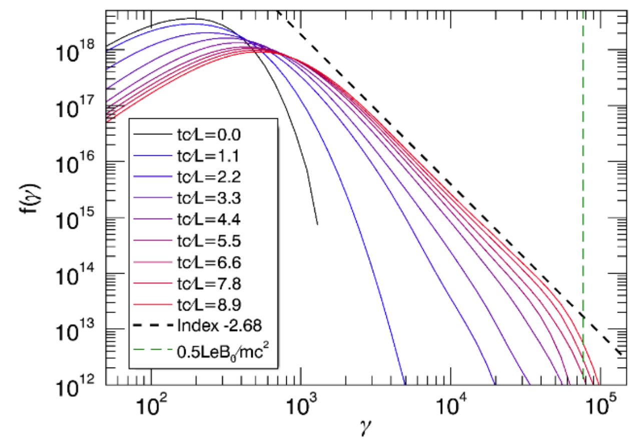


Nonthermal Particle Acceleration (NTPA) in PIC simulations

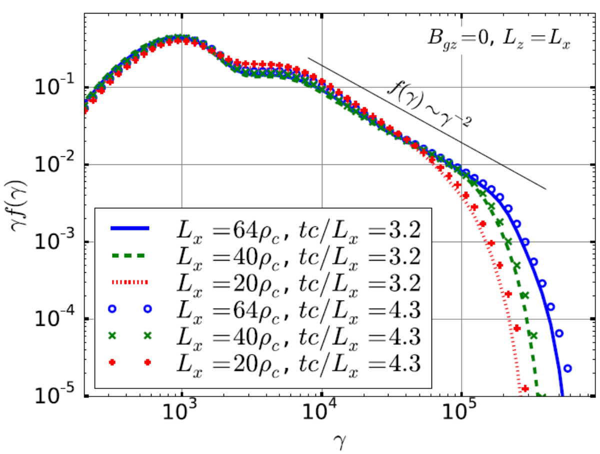
Collisionless shocks (e.g. Spitkovsky+ 2008)



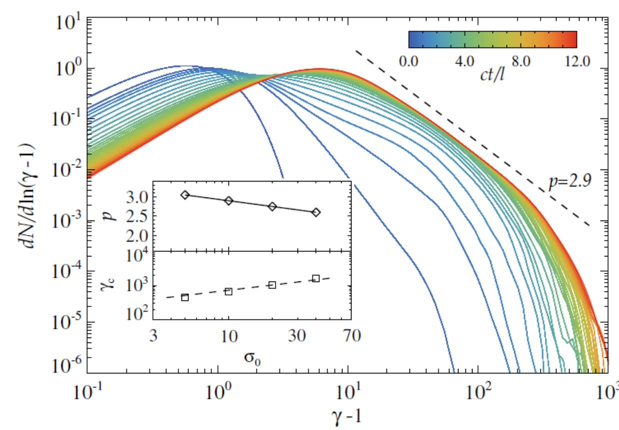
- Driven turbulence (e.g., Zhdankin+ 2017)



Reconnection (e.g. Werner+ 2016)



- Decaying turbulence (e.g. Comisso+ 2018)



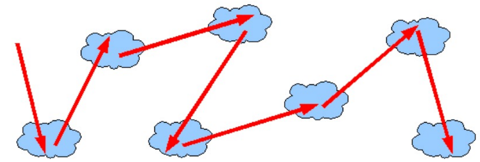
Theory:

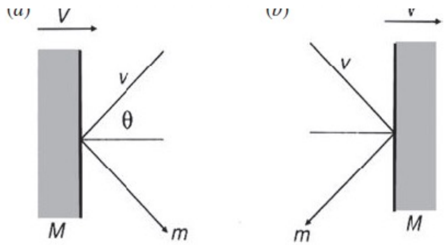
Theory of Turbulent Relativistic Nonthermal Particle Acceleration (NTPA)

I. Fermi Acceleration:

Diffusive, 2nd-order Fermi ('49, '54) Acceleration of CRs:

- Particles scatter off randomly moving magnetic clouds
- Head-on collisions (gaining energy) more probable than tail collisions (losing energy)
- Particle momentum performs biased random walk
- Average energy gain ∝ cloud velocity squared, $\left(\frac{v}{c}\right)^2$
- Generalised to any scatterer type: plasma waves, turbulence, …
- In combination with particle escape, particle spectrum develops steady-state power law: $f(\gamma) \sim \gamma^{-\alpha}$, with slope $\alpha = 1 + \tau_{acc} / \tau_{esc}$





$\epsilon = \gamma\, m\, c^2$

Fermi's ideas have guided turbulent particle acceleration research ever since.
Astrophys. applications of turb NTPA are diverse, but most theory work done for CRs.

But what are these scattering centers, how rapidly do they move, how do they scatter particles?

Theory:

Theory of Turbulent Relativistic Nonthermal Particle Acceleration (NTPA)

What are these scattering centers, how rapidly do they move, how do they scatter particles?

II. Quasi-Linear Theory (QLT) of Wave-Particle Interaction:

The Language of the (analytical) Realm!

Basic idea:

- Turbulence is ensemble of EM perturbations, (e.g., Alfven, Fast, Slow MHD waves)
- Wave properties [dispersion relation ω(k), polarization] described by linear theory
- Particle momentum distribution is split into the slowly evolving spatially and gyro-average part and rapidly fluctuating part: $f(t,\mathbf{x},\mathbf{p}) = f_0(t,p,\mu) + f_1(t,\mathbf{x},\mathbf{p})$ ($\mu = \cos\theta$)
- $f_1(t,\mathbf{x},\mathbf{p})$ governed by linearizing relativistic Vlasov equation: $\frac{\partial f_e}{\partial t} + \mathbf{v}_e \cdot \nabla f_e - e\left(\mathbf{E} + \frac{\mathbf{v_e}}{c} \times \mathbf{B}\right) \cdot \frac{\partial f_e}{\partial \mathbf{p}} = 0$

evolves due to product of fluctuating EM fields and *momentum derivative of f_0*

- $f_0(t,\mathbf{p})$ evolves slowly (2nd order), due to correlation between fluctuating EM fields and momentum derivative of *momentum derivative of f_1*
- Generic Result: QLT evolution equation for $f_0(t,\mathbf{p})$ is a Fokker-Planck ***diffusion equation*** in momentum space.

Theory:

Theory of Turbulent Relativistic Nonthermal Particle Acceleration (NTPA)

QLT motivates **Fokker-Planck** approach to stochastic particle acceleration, yielding practical, convenient, widely used prescriptions for NTPA (esp. for CRs)

Fokker-Planck equation:

- Derived from Vlasov equation using QLT for given spectrum of EM
- Describes slow evolution of the space- and gyro-averaged momentum distribution $f_0(t,p,\mu)$, as a function of particle momentum p and pitch angle θ due to wave-particle interactions.
- General form: diffusion equation in momentum space:

$\mu = \cos\theta$

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial \mu}\left(D_{\mu\mu}\frac{\partial f}{\partial \mu}\right) + \frac{\partial}{\partial \mu}\left(D_{\mu p}\frac{\partial f}{\partial p}\right) + \frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2\left(D_{p\mu}\frac{\partial f}{\partial \mu} + D_{pp}\frac{\partial f}{\partial p}\right)\right]$$

- Simplified form: pitch-angle-averaged; diffusion in energy space only:

$$\partial_t f = \partial_\gamma\left(D\partial_\gamma f\right) - \partial_\gamma(Af)$$

- $f(\gamma,t)$: particle energy distribution
- $D(\gamma,t)$: energy diffusion coefficient – second-order acceleration
- $A(\gamma,t)$: energy advection coefficient – first-order acceleration

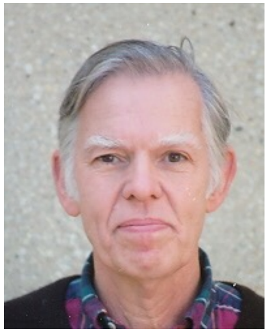
$\gamma = \epsilon/mc^2 = p/mc$

Theory:

Theory of Turbulent Relativistic Nonthermal Particle Acceleration (NTPA)

Analytical QLT of Turbulent NTPA: Historical Development (1960-90s)

- Kennel and Engelmann (1966); Lerche (1968): "*Quasilinear Theory of Resonant Diffusion in a Magneto-Active, Relativistic Plasma*"



- Kulsrud & Pearce (1969), Kulsrud & Ferrari (1971): relativistic QLT for Alfven-wave MHD turbulence starting with relativistic Vlasov equation.

(Interlude: important parallel developments in 1st-order Fermi shock acceleration 1977-1978: Krymskii '77, Axford etal '77, Bell '78, Blandford & Ostriker '78; see Blandford & Eichler 1987 review)

- Schlickeiser (1989-1993): QLT for spatial and momentum diffusion for $f_0(p,\mu)$ for Alfven waves (parallel or antiparallel to background $\boldsymbol{B_0}$, $k_\perp = 0$) with a power-law $k_\parallel$-spectrum.

What properties of MHD turbulence determine QL diffusion coefficient?

Subsequent development of QLT of turbulent particle acceleration was influenced by advances in theory of magnetized plasma turbulence itself.

Theory:

Theory of Turbulent Relativistic Nonthermal Particle Acceleration (NTPA)

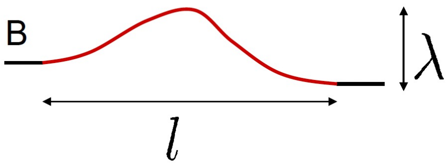
Important development in incompressible MHD turbulence theory: Goldreich-Sridhar 1995, 1997

(cf. Shebalin et al 1983; Higdon 1984)

- Strong Alfvenic MHD Turbulence is **anisotropic**: $\boldsymbol{k_\perp \gg k_\parallel}$
- Critical Balance: $\tau_{lin} = \omega_A = k_\parallel V_A \sim \tau_{nl} = 1/(k_\perp u_\perp)$

  Here, $\boldsymbol{l} = k_\parallel{}^{-1}$ and $\lambda = k_\perp{}^{-1}$
- Result: GS95 spectral scalings:

$$E(k_\perp) \sim \varepsilon^{2/3} k_\perp^{-5/3}$$
$$k_\parallel \sim \varepsilon^{1/3} v_A^{-1} k_\perp^{2/3}$$
$$E(k_\parallel) \sim \varepsilon v_A^{-1} k_\parallel^{-2}$$



Chandran (2000):

updated Alfvenic-turbulence QLT accounting for GS95 theory.

Key findings: strong $\boldsymbol{k_\perp \gg k_\parallel}$ anisotropy suppresses strong pitch-angle scattering and NTPA.

But is MHD Turbulence truly Alfvenic?

Theory:

Theory of Turbulent Relativistic Nonthermal Particle Acceleration (NTPA)

But is MHD turbulence truly Alfvenic?

- MHD turbulence is not limited to incompressible Alfven waves; also includes compressible Fast and Slow MS waves.
- Standard view: (nonrel.) MHD turbulence divides into two decoupled cascades:
  - Solenoidal (Alfvenic, Goldreich & Sridhar 1995)
  - Compressive (Fast Mode, Cho & Lazarian 2002+)
- QLT analyses of CR transport/acceleration by compressible FMS/SMS turbulence: transit-time damping (n=0) + gyroresonant (n≠0) wave-particle interactions
  - Fast modes are more efficient particle accelerators
  - Schlickeiser & Miller (1998): isotropic FMS turbulence with Kolmogorov spectrum
  - Yan & Lazarian (2002), Cho & Lazarian (2003, 2006): strong & weak FMS+SMS turbulence.
  - Chandran (2003), Chandran & Maron (2004): SMS turbulence
- Comprehensive analytical relativistic QLT for stochastic NTPA in general (fast, slow, Alfvenic) strong relativistic MHD turbulence, including finite-lifetime resonance broadening and GS95 anisotropy (Demidem, Lemoine, Casse 2020), finding:

$$D^{A}_{pp} \sim \beta_A{}^3 p^2$$

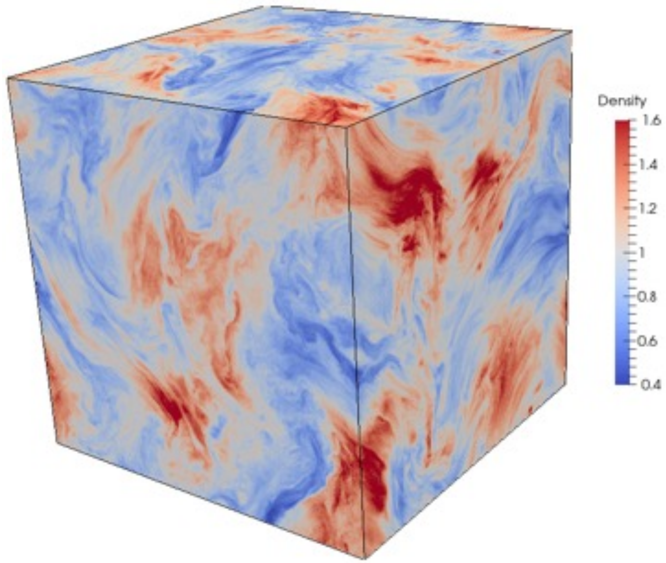
Theory:

Theory of Turbulent Relativistic Nonthermal Particle Acceleration (NTPA)

Beyond Quasilinear Theory: New Innovative Theoretical Ideas

- Role of strong intermittent, coherent dissipative structures, e.g. intense current sheets, sharp magnetic bends:
  - Sophisticated analytical theory by Lemoine 2021-2025
  - also work Vlahos et al. 2004, Isliker et al. 2017, Zhdankin et al. 2013, Comisso & Sironi 2018-2019, Davis et al. 2024
- Generalized entropies, Casimir Invariants (Zhdankin 2021, 2022)
- Interplay between pitch-angle scattering and trapping in accelerating magnetic structures (Vega, Boldyrev, Roytershteyn, Medvedev 2022+)
- Lynden-Bell's stat. mech. framework (Ewart et al. 2023)

Numerical Studies of Relativistic Turbulent NTPA



Three numerical approaches:

- Test particles in synthetic turbulent EM fields
- Test particles in MHD simulations
- **Particle-in-Cell (PIC) kinetic simulations**

Types of Turbulence:

- **Driven (e.g., Zhdankin et al)**
- Decaying (e.g., Comisso)

Kinetic turbulence in the relativistic regime

High magnetization: $\sigma \equiv \dfrac{\overline{B^2}}{4\pi h} \gtrsim 1$ $\quad (h \sim 4 n_0 \bar{\gamma} m c^2 / 3)$

Relativistically hot: $\theta_s \equiv T_s / m_s c^2 \gtrsim 1 \quad (\bar{\gamma}_s \equiv \bar{E}_s / m_s c^2 \sim 3\theta_s \gg 1)$

Relativistic turbulence: $\delta v / c \sim 1 \quad v_A / c = \sqrt{\sigma / (\sigma + 1)}$

Major questions involve not turbulence *per se*, but particle energization:

- Is (relativistic) turbulence a viable and efficient astrophysical particle accelerator?
- What are mechanisms of heating and/or acceleration?
- What is a minimal reduced model for describing nonthermal distributions?
- How is dissipated energy partitioned between electrons and ions?
- What are observable radiative signatures? (spectra, variability)

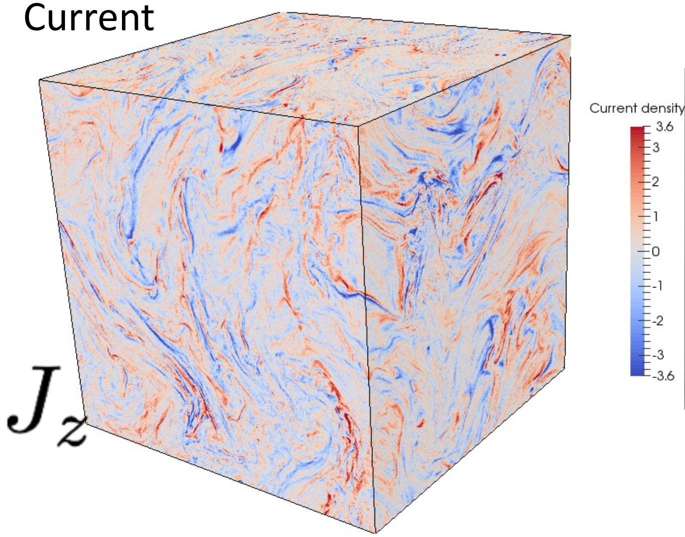
Based on a slide by V. Zhdankin

Simulation:

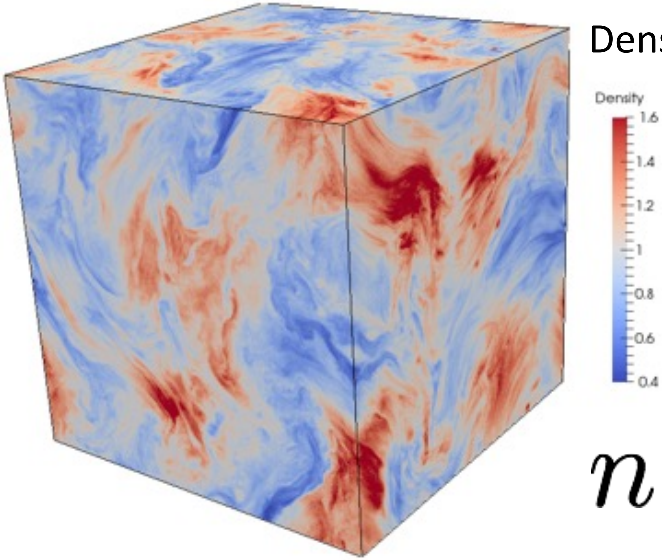
3D Kinetic (PIC) Simulations of Driven Turbulence in Relativistic Collisionless Plasmas

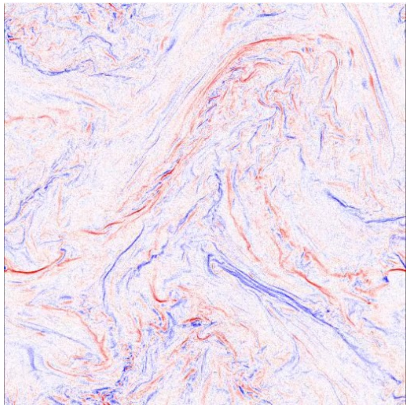
***V. Zhdankin**, K. Wong, A. Hankla, G. Werner, D. Uzdensky, M. Begelman (2017-2025)*

Current

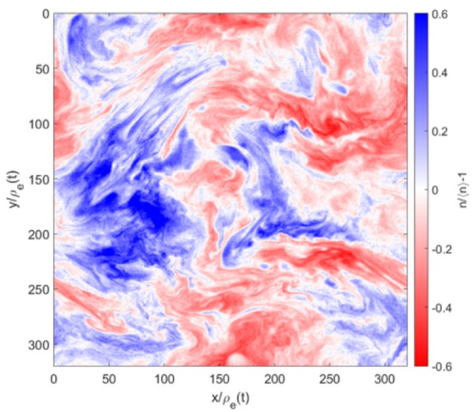


Density





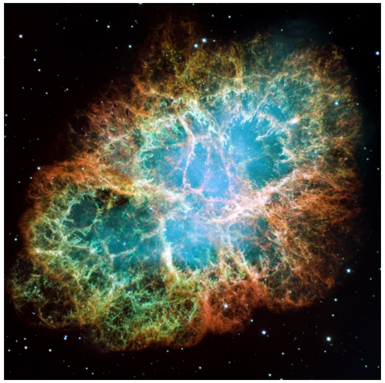
- 3D periodic box; Zeltron PIC code
- Relativistically hot pair plasma
- Driven magnetized plasma turbulence
- Uniform guide field: $B_0 \sim \delta B_{\rm rms}$
- System size: $L/2\pi\rho_e \to 163$
- Hundreds of billions macroparticles



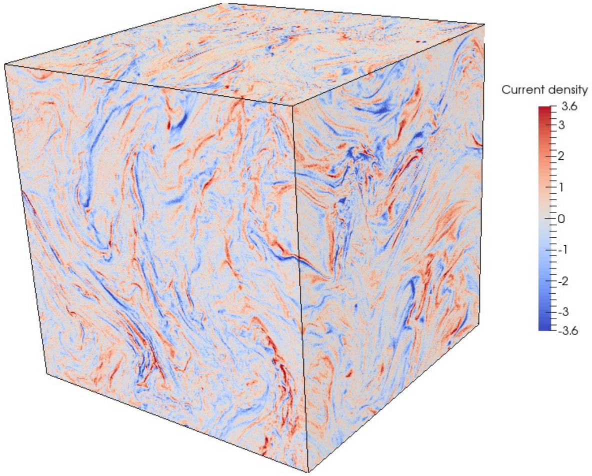
Simulation:

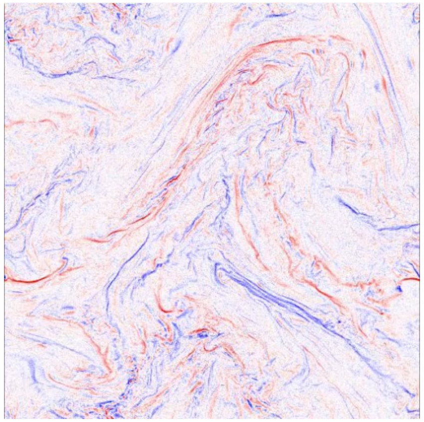
3D PIC Simulations of Nonradiative Driven Relativistic Collisionless Turbulence in Pair Plasmas

(Zhdankin, Werner, Uzdensky, Begelman 2017-2018)



Crab Nebula (M1)





Key Questions:

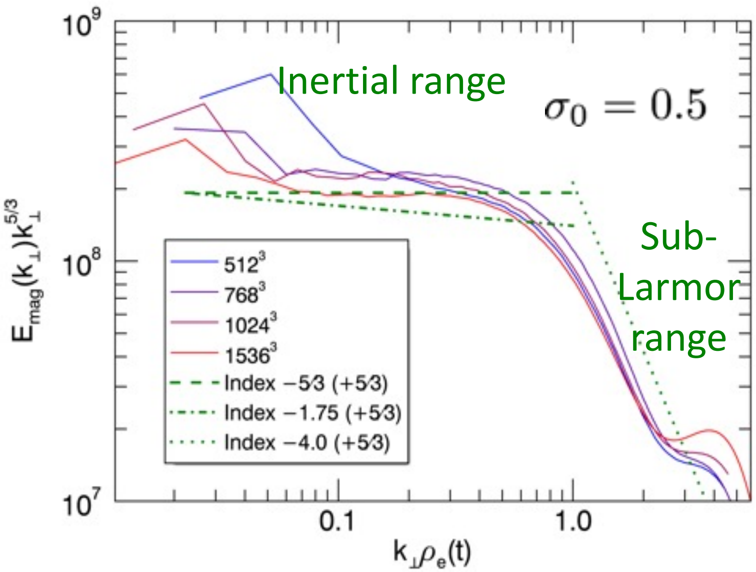
- What are the statistical properties of turbulence: **power spectra** in inertial (MHD) range ($l > \rho$) and sub-Larmor (kinetic) range ($l < \rho$)?
- **Is turbulence a viable mechanism for nonthermal particle acceleration?**

Simulation:

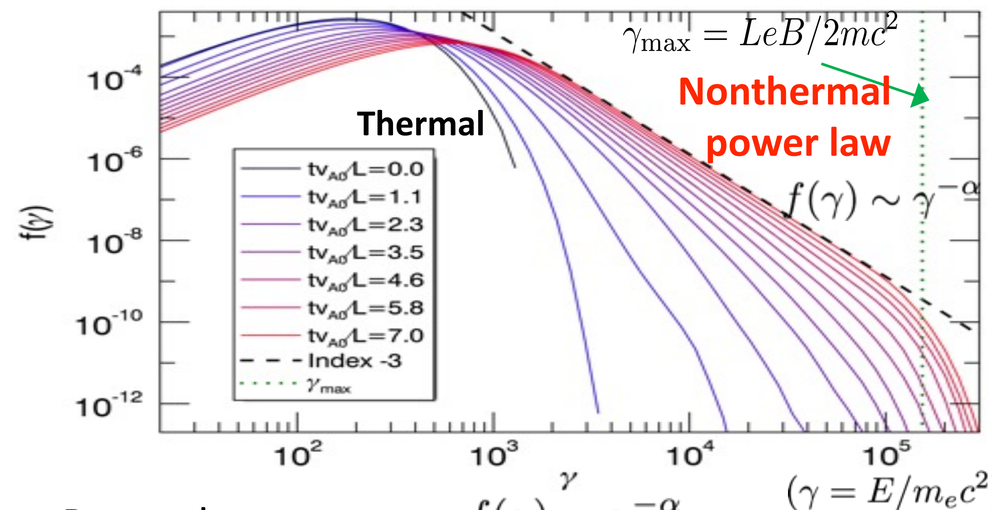
3D PIC Simulations of Nonradiative Driven Relativistic Collisionless Turbulence in Pair Plasmas

(Zhdankin, Werner, Uzdensky, Begelman 2017-2018)

Turbulent magnetic energy spectrum



Nonthermal Particle Acceleration (NTPA)



Power-law segment: $f(\gamma) \sim \gamma^{-\alpha}$

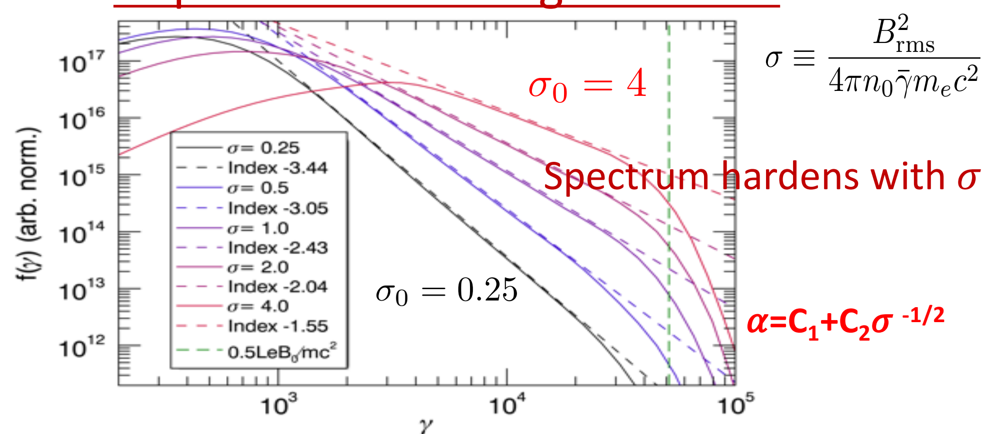
reaches system-size-limit: $\gamma_{\max} = LeB/2mc^2$ $(\rho_e \sim L/2)$

Turbulent magnetic fluctuations spectrum:

- **E(k)~k $^{-5/3}$** in inertial range ($k_\perp < \rho_e^{-1}$)
- **E(k)~k $^{-4}$** in kinetic range ($k_\perp > \rho_e^{-1}$)

Dependence on Magnetization

$$\sigma \equiv \frac{B_{\rm rms}^2}{4\pi n_0 \bar{\gamma} m_e c^2}$$

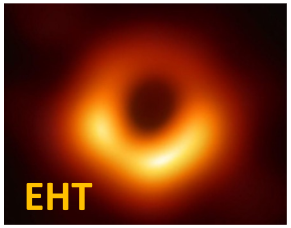


Two dimensionless parameters:

- Length-scale separation: L/ρ_0
- Initial Magnetization: $\sigma_0 = B_{rms}^2/(4\pi\, n\, \gamma m c^2)$

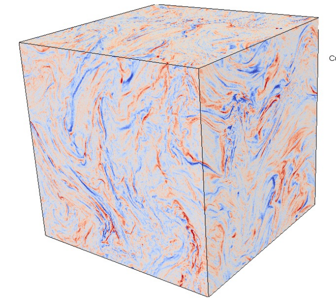
Simulation:

Driven kinetic turbulence in nonradiative collisionless semirelativistic electron-ion plasma



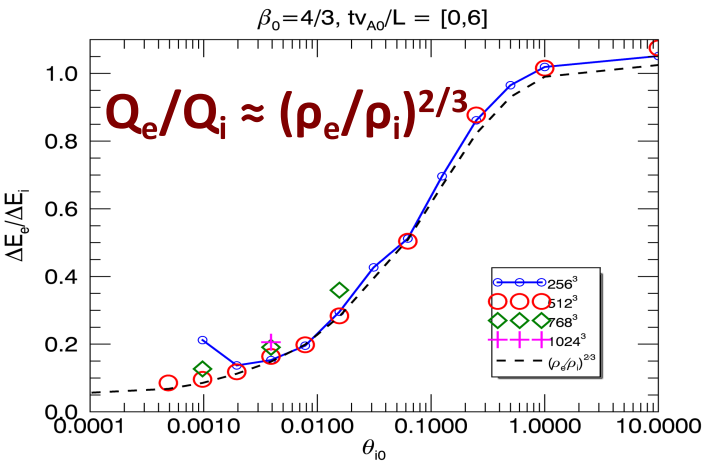
(Zhdankin, Werner, Uzdensky, Begelman 2019)

(e.g., hot BH accretion flows in M87 and Sgr A*)

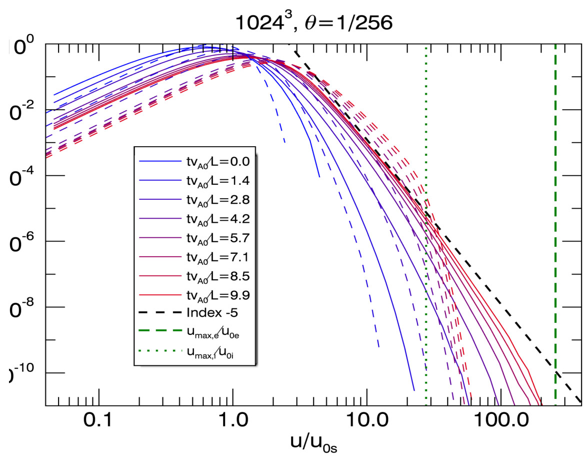


Focus on semi-relativistic regime: ultra-rel. electrons, sub-rel. ions

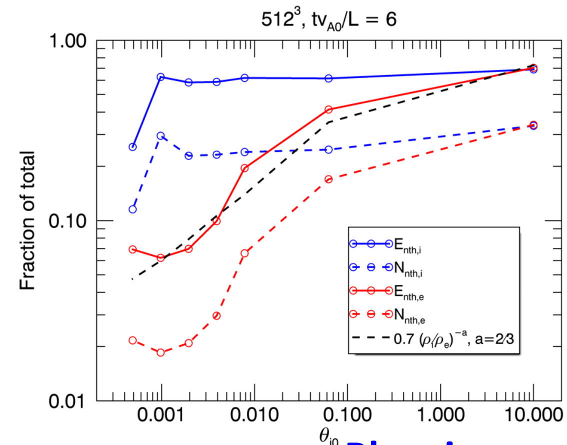
Electron-Ion Energy Partition



Particle Energy Spectra



Nonthermal Fraction



[Also applies to radiative *e/i* plasma] *(Zhdankin, Kunz, Uzdensky 2021)*

Robust nonthermal acceleration of **ions**
Modest acceleration of **electrons**

Blue: ions
Red: electrons

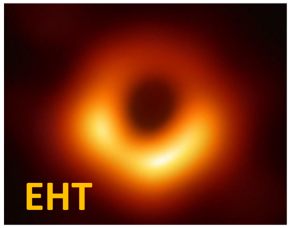
Caveat: these simulations are

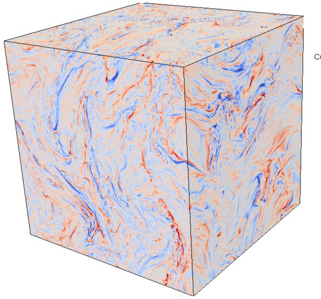
- Driven by EM antenna
- With strong fluctuations: $\delta B_{rms} = B_0$

Simulation:

Driven kinetic turbulence in nonradiative collisionless semirelativistic electron-ion plasma

(Zhdankin 2021)

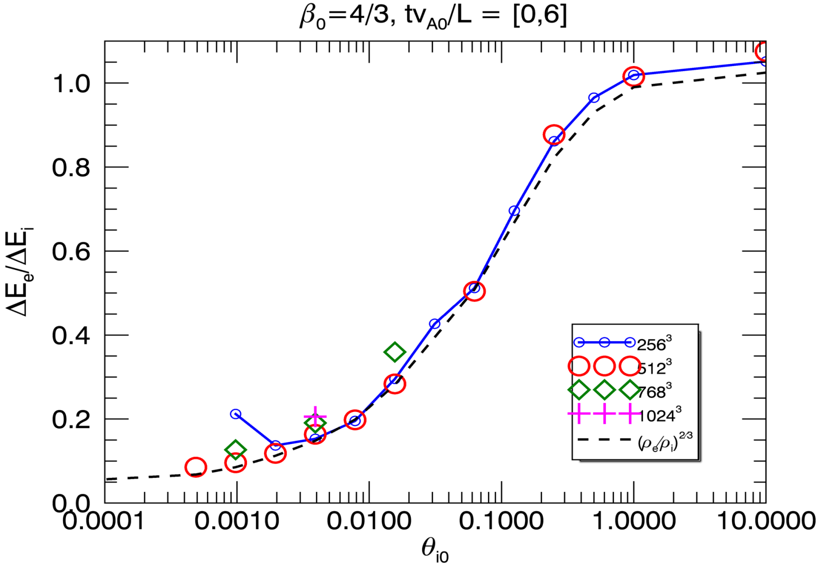




$Q_e/Q_i \approx (\rho_e/\rho_i)^{\alpha}$

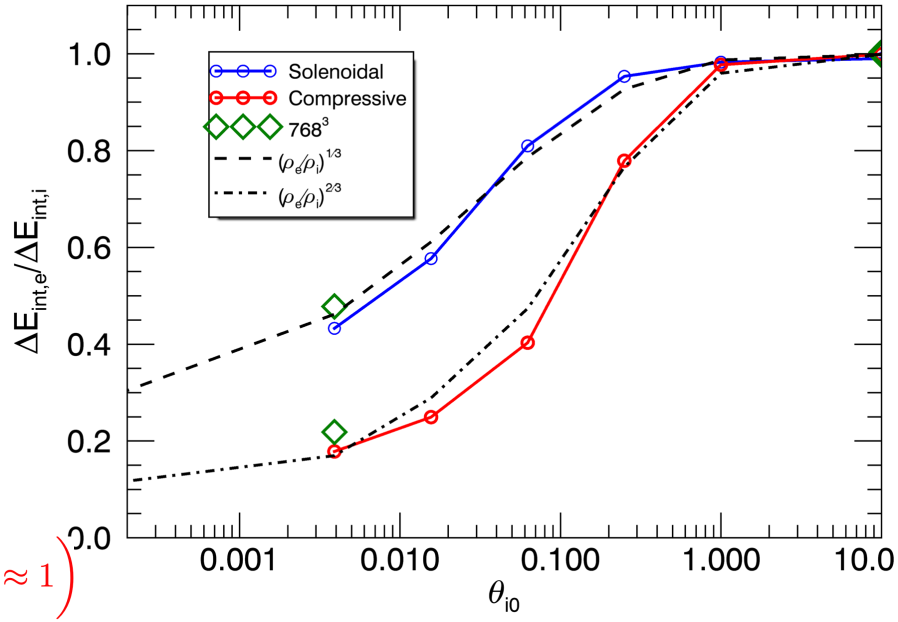
Electron-ion energy partitioning depends on **type of turbulent driving** (Zhdankin 2021):

Electromagnetic driving (Langevin Antenna) $\alpha \approx 2/3$



External Force on Particles: $\boldsymbol{F}_{\rm ext}$

- Compressive $\nabla_\perp \times \boldsymbol{F}_{\rm ext} = 0$ $\alpha \approx 2/3$
- Solenoidal: $\nabla \cdot \boldsymbol{F}_{\rm ext} = 0$ $\alpha \approx 1/3$



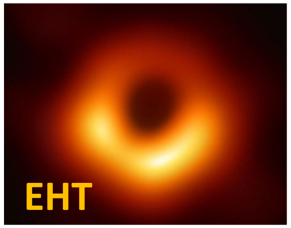
$$\rho_s = \frac{\gamma_s m_s v_s c}{eB_{\rm rms}} \qquad \theta_i \equiv T_i/m_i c^2 \qquad \left(\beta \approx 1,\ \frac{\delta B_{\rm rms}}{B_0} \approx 1\right)$$

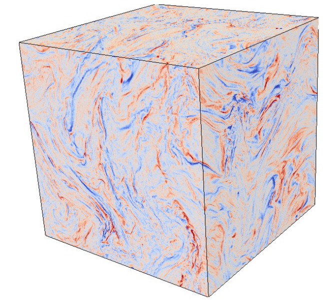
See also 2D PIC Meringolo+ 2023

Simulation:

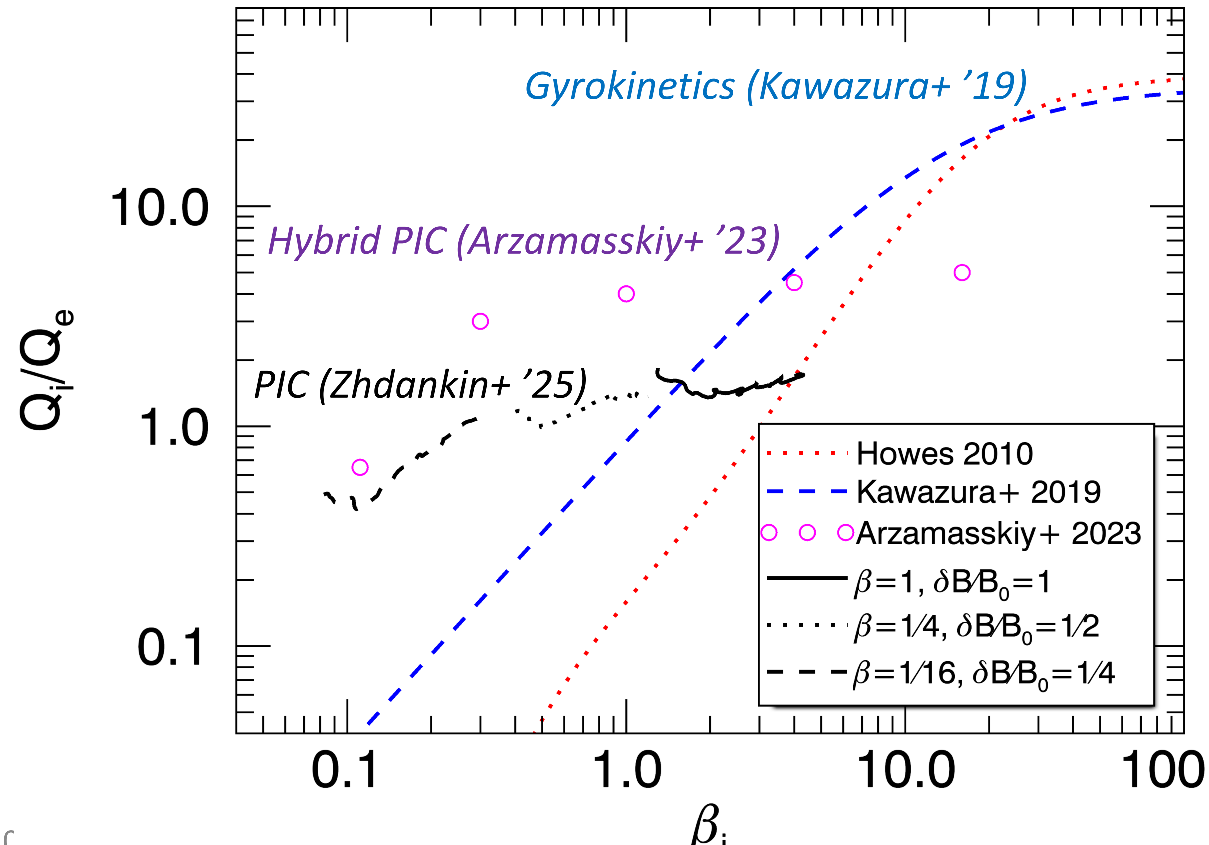
Driven kinetic turbulence in nonradiative collisionless semirelativistic electron-ion plasma

(Zhdankin et al 2025)





Electron-ion energy partitioning depends on **guide field** (Zhdankin et al 2025):
In strong guide field, heating ratio close to 1, independent of plasma beta



$$\rho_s = \frac{\gamma_s m_s v_s c}{e B_{\rm rms}}$$

$$\theta_i \equiv T_i/m_i c^2$$

Simulation:

Direct Numerical (PIC) Test of Fokker-Planck Model for Turbulent Particle Acceleration

Kai Wong, V. Zhdankin, D. Uzdensky, G. Werner, M. Begelman 2020, 2025

(also Comisso & Sironi 2019)

Questions:

- Is turbulent particle acceleration indeed diffusive?
- Do particle energies undergo random walks?
- Is Fokker-Planck (FP) framework an appropriate description of turbulent NTPA?
- How do the Fokker-Planck coefficients $D(\gamma)$, $A(\gamma)$ depend on σ (magnetization), L/ρ_e (system size)?

Simplest Fokker-Planck formulation:

advection-diffusion equation in energy:

$$\partial_t f = \partial_\gamma \left(D \partial_\gamma f \right) - \partial_\gamma (A f)$$

- $f(\gamma, t)$: particle energy distribution
- $D(\gamma, t)$: energy diffusion coefficient
- $A(\gamma, t)$: energy advection coefficient

$(\gamma = E/m_e c^2)$

Method:

- PIC simulation of driven 3D relativistic pair-plasma turbulence
- Large statistical ensembles of tracked particles
- Directly measure stat. properties of NTPA, FP coefficients $D(\gamma, t)$, $A(\gamma, t)$ and their dependence on $\sigma, L/\rho_e$

System parameters:

Relativistic pair plasma:

$$\langle \gamma \rangle_{\rm init} = 300$$

Initial plasma magnetization:

$$\sigma_0 = B_0^2 / 16\pi n_0 T_0 = 3/8$$

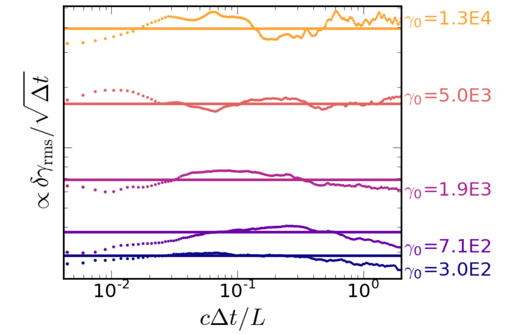
Simulation:

Direct Numerical (PIC) Test of Fokker-Planck Model for Turbulent Particle Acceleration

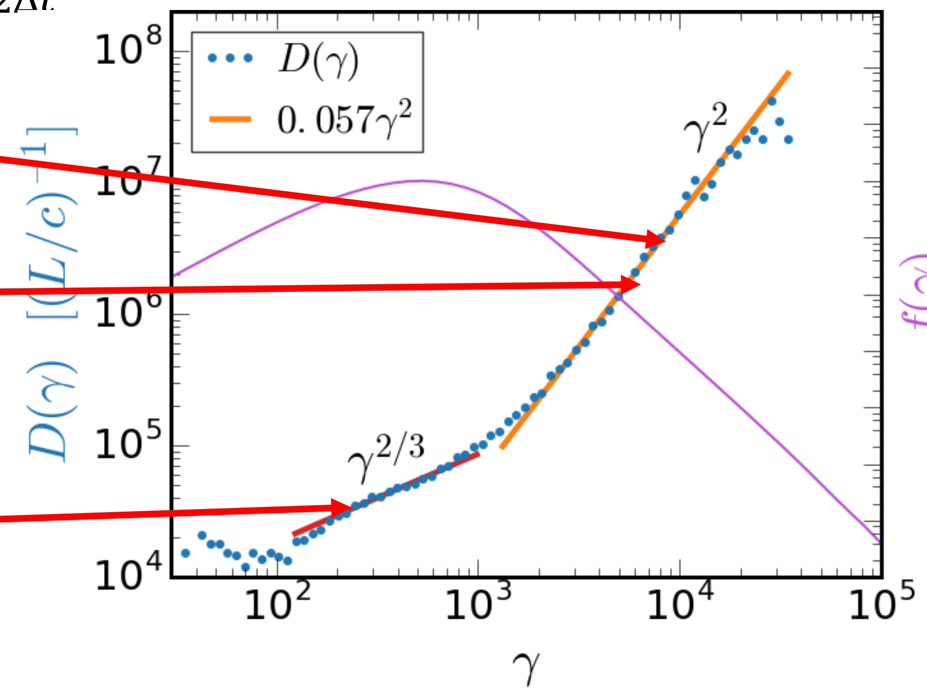
Wong et al. 2020

Results:

Random walk in energy and energy-diffusion coeff.



- Consistent with random walk:
  $\delta\gamma_{rms} \sim \sqrt{\Delta t}$
- Extract Diffusion Coefficient: $D = \frac{\delta\gamma^2_{rms}}{2\Delta t}$
- $D(\gamma) \sim \gamma^2$ in the nonthermal tail, corresponding to inertial range of turbulence.
- 2^{nd}-order Fermi Accel: $D(\gamma) = D_0\,\gamma^2$
- Much shallower $D(\gamma)$ scaling at low energies…



Simulation:

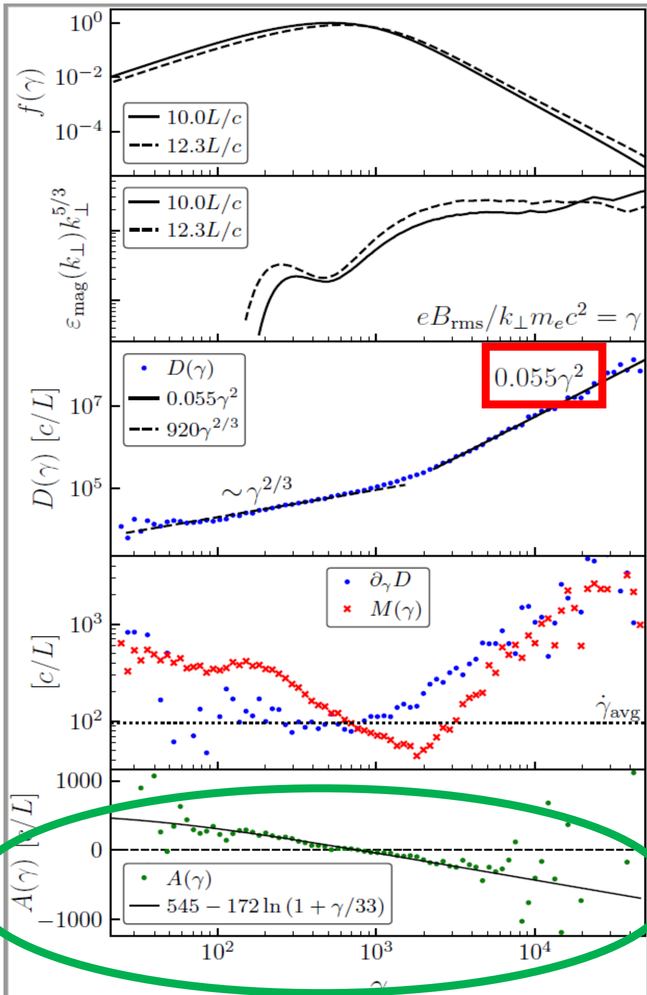
Direct Numerical (PIC) Test of Fokker-Planck Model for Turbulent Particle Acceleration

Results:

Wong et al. 2020

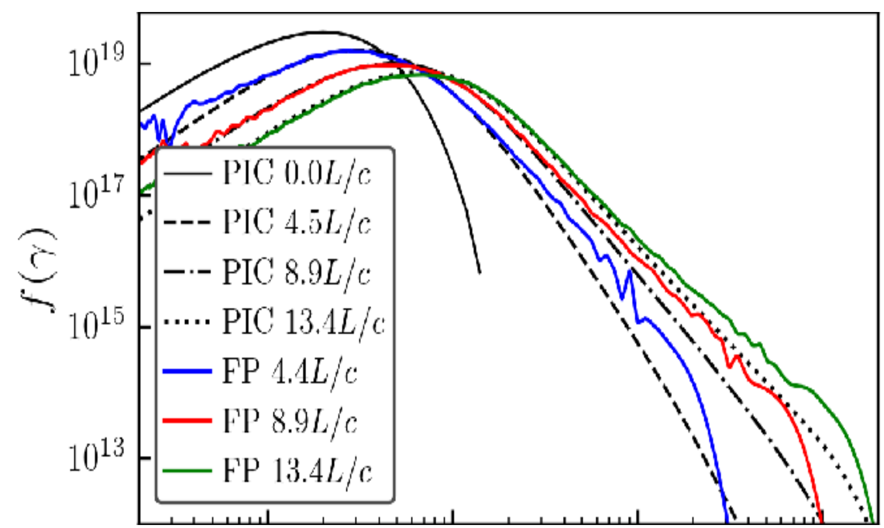
$$\partial_t f = \partial_\gamma (D \partial_\gamma f) - \partial_\gamma (A f)$$

Advection term is essential!



Fokker-Planck Framework works well!

Fokker-Planck Evolution vs. actual PIC

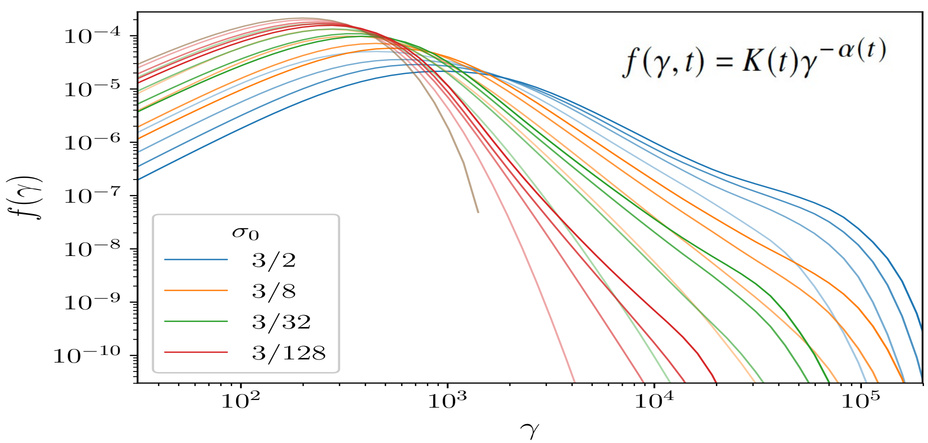


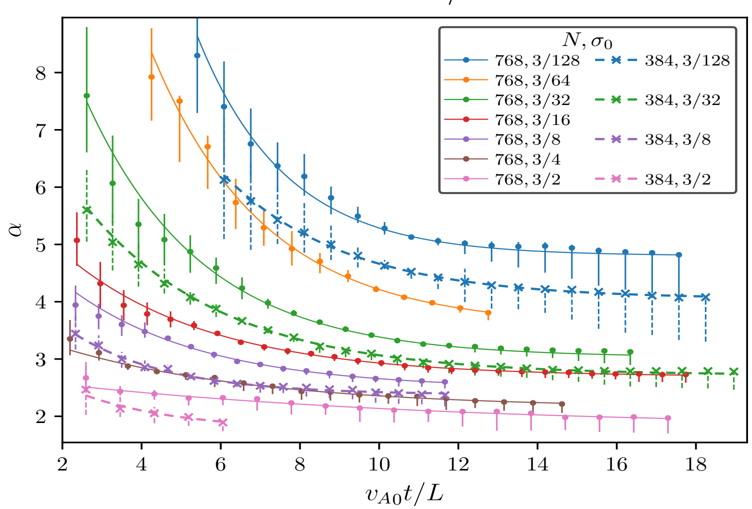
Simulation:

Energy Diffusion and Advection Coefficients in Kinetic Simulations of Relativistic Plasma Turbulence

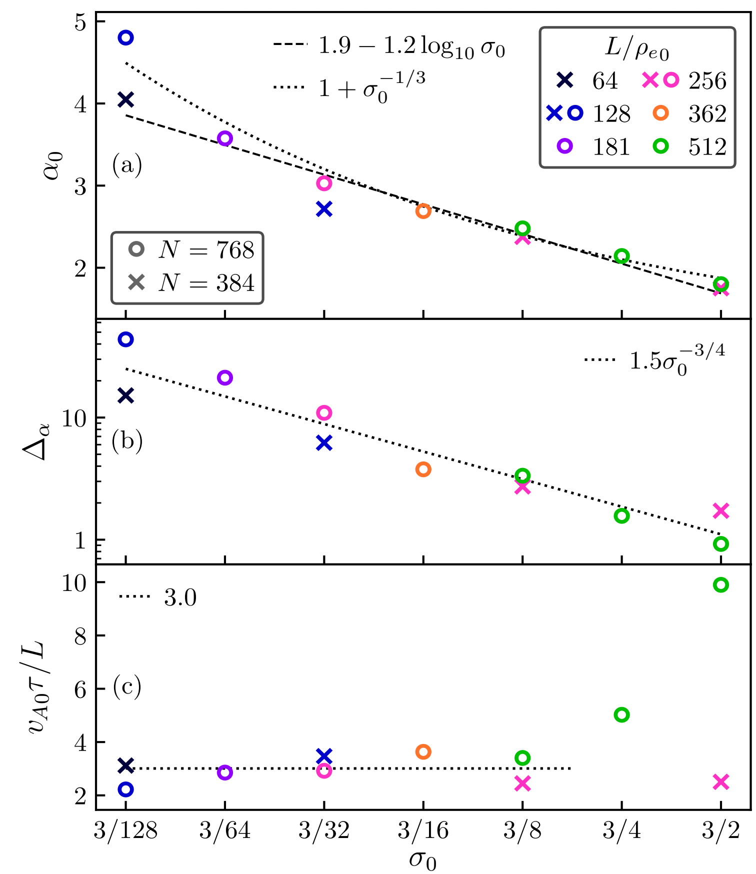
Wong et al. 2025

How are energy-space FP coefficients $D(\gamma)$ and $A(\gamma)$ related to the spectral evolution, e.g., power-law index $\alpha(t)$?





$$\alpha(t) = \alpha_\infty + \Delta_\alpha \exp(-t/\tau)$$



Simulation:

Energy Diffusion and Advection Coefficients in Kinetic Simulations of Relativistic Plasma Turbulence

Wong et al. 2025

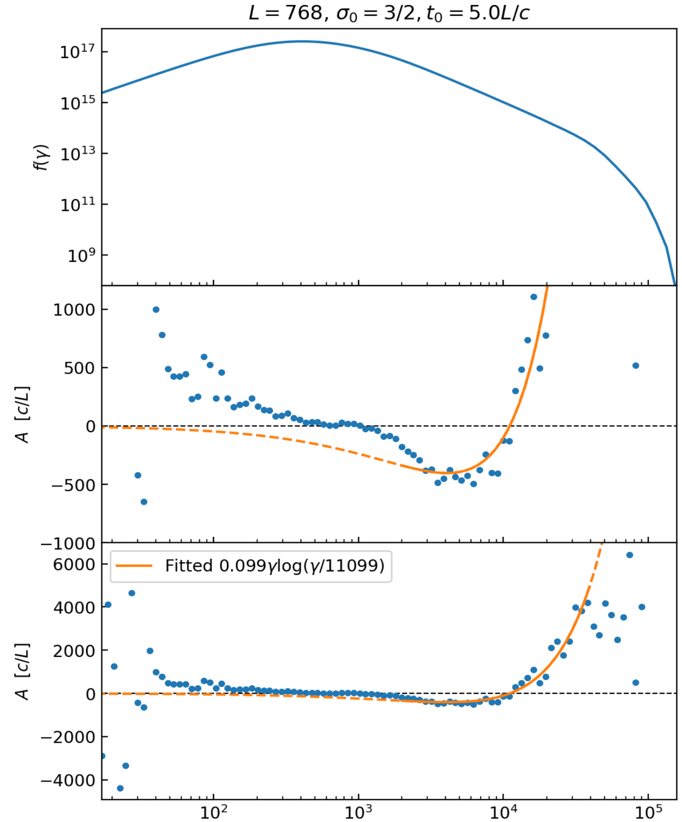
Can the evolution of the spectral index $\alpha(t)$ be reproduced with a FP model?

- Energy **advection-diffusion** FP equation:

$$\partial_t f = \partial_\gamma\left(D\partial_\gamma f\right) - \partial_\gamma(Af)$$

  - $f(\gamma, t)$: particle energy distribution
  - $D(\gamma, t)$: energy diffusion coefficient
  - $A(\gamma, t)$: energy advection coefficient
- Nonthermal power law: $f(\gamma, t) = K(t)\gamma^{-\alpha(t)}$
- Diffusion Coefficient: $D(\gamma, t) = D_0(t)\gamma^2$
- What should $A(\gamma, t)$ be?
- Prediction for γ-functional form of A:

$$A = A_0\,\gamma\,\log(\gamma/\gamma_A^*)$$



$$A_0(t) = \frac{\dot{\alpha}}{1-\alpha} = -\frac{d\log(\alpha-1)}{dt}, \quad \gamma_A^*(t) = \exp\left[\frac{\dot{K}/K + D_0(\alpha-\alpha^2)}{\dot{\alpha}} + \frac{1}{1-\alpha}\right]$$

Simulation:

Energy Diffusion and Advection Coefficients in Kinetic Simulations of Relativistic Plasma Turbulence

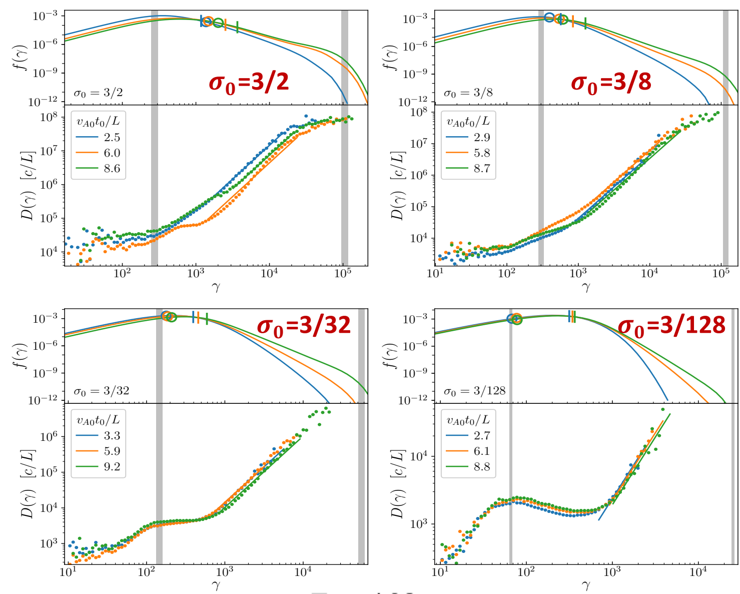
Wong et al. 2025

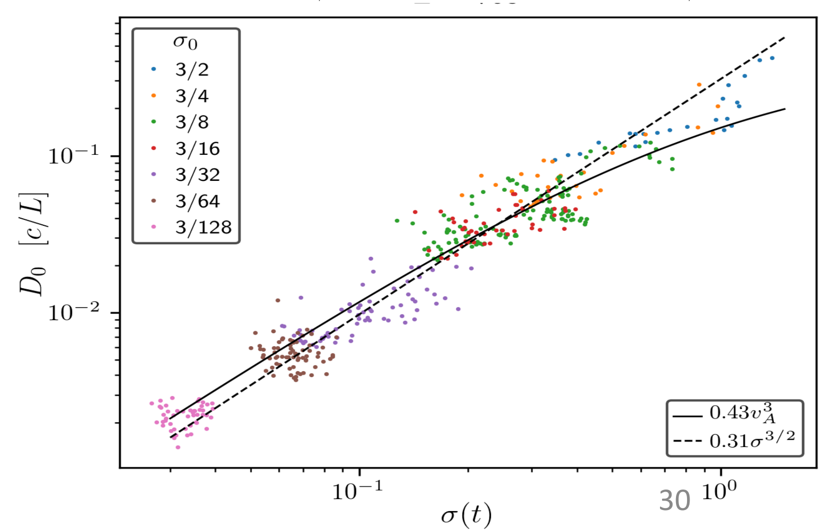
$$\sigma_0 = B_0^2/16\pi n_0 T_0$$

Dependence of $D(\gamma, t)$ on magnetization $\sigma(t)$

- Multiple runs with varying initial magnetization σ_0.
- Universal $D(\gamma) \sim \gamma^2$ in high-γ nonthermal tail $\gamma \gg \gamma_{\text{peak}}$, but non-universal σ_0-dependent slope at lower γ.
- Measure D_0 at many time instances.
- Magnetization decreases over time as plasma heats up.
- D_0 depends mainly on instantaneous $\sigma(t)$, not initial σ_0.
- D_0 scales as $\sigma^{3/2}$ or V_A^3 instead of σ or V_A^2.
- Agrees with *Demidem et al 2020*:

  Extra power of V_A from resonance broadening
- Knowing $D_0(\sigma)$ helps us:
  - discriminate among NTPA theories
  - predict $f(\gamma, t)$ given $\sigma(t)$





Other recent PIC work on kinetic relativistic plasma turbulence:

- Turbulence with self-consistent radiation reaction (extreme plasma astrophysics):
  - opt thin inverse-Compton: Zhdankin 2021, 2023
  - Opt thin synchrotron:
  - Finite optical-depth cooling Grosel
  - QED radiative regime: Klein-Nishina inverse-Compton cooling and pair production

- Diverse Driving methods:
  - Decaying turbulence
  - MRI-driven turbulence
  - Imbalanced turbulence

- Turbulence with high-energy particle escape

SUMMARY

- Turbulent Relativistic NTPA is ubiquitous in the Universe
- Quasi-Linear Theory (QLT) has been a leading theoretical approach, informed by advances in theory of MHD turbulence
- PIC simulation is the main working horse of modern numerical studies
- Relativistic magnetized plasma turbulence is a robust particle accelerator, producing extended (to Hillas limit) power-law spectra.
- Validation of energy-space Fokker-Planck (FP) model of turbulent relativistic particle acceleration using direct PIC measurements:
  - Demonstrated that relativistic NTPA is diffusive.
  - FP agrees well with PIC evolution of $f(\gamma, \mathrm{t})$ in fully-developed turbulence.
- Measured $D(\gamma)$, $A(\gamma)$:
  - $D(\gamma) \sim \gamma^2$ in the nonthermal tail, much shallower scaling at low energies.
  - Dependence on magnetization is $D_0 \sim \sigma^{\frac{3}{2}}$
  - Energy Advection Coefficient $A(\gamma)$ is essential

THANK YOU!