Cosmic-ray transport in magnetized turbulence

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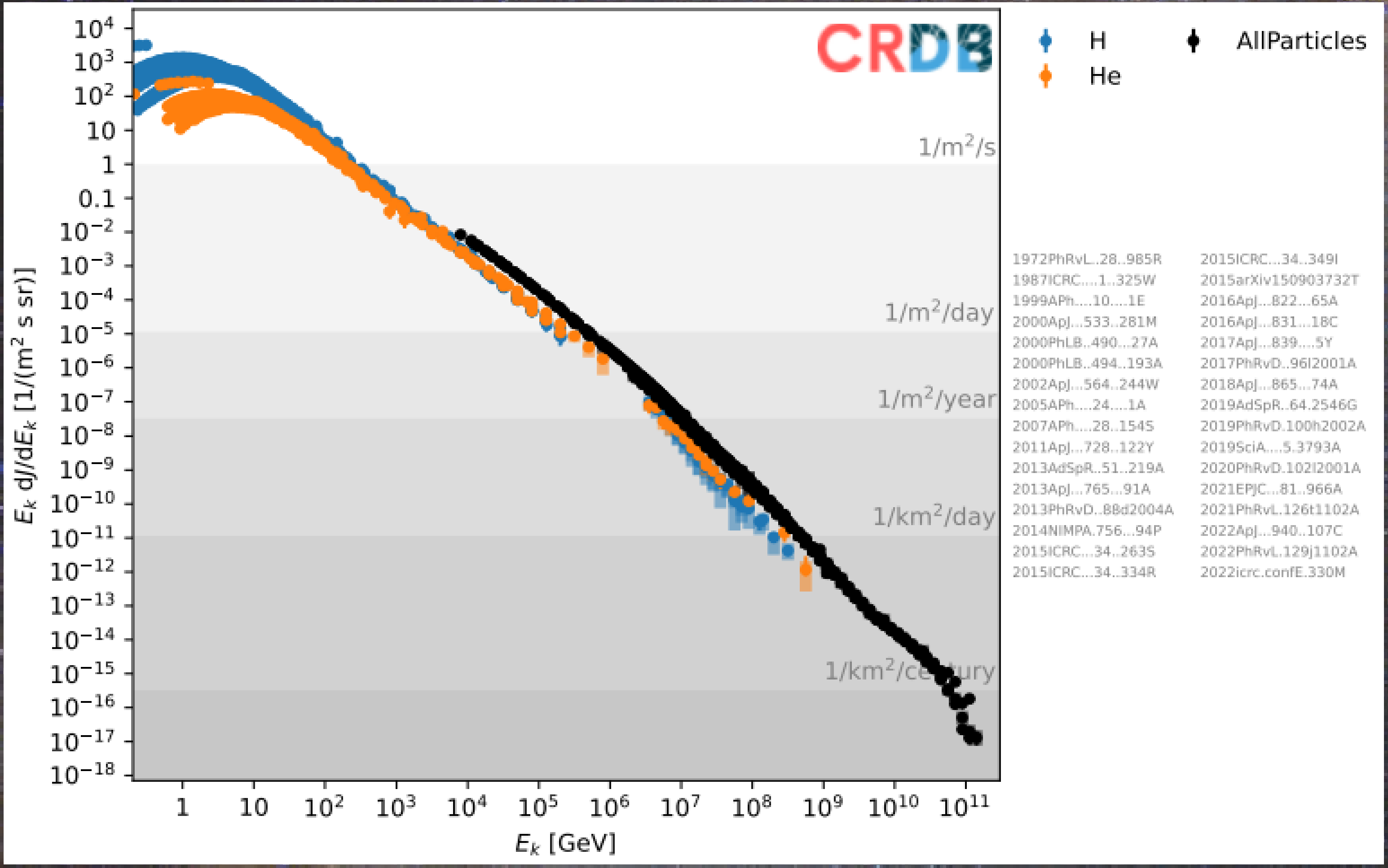
cnrs

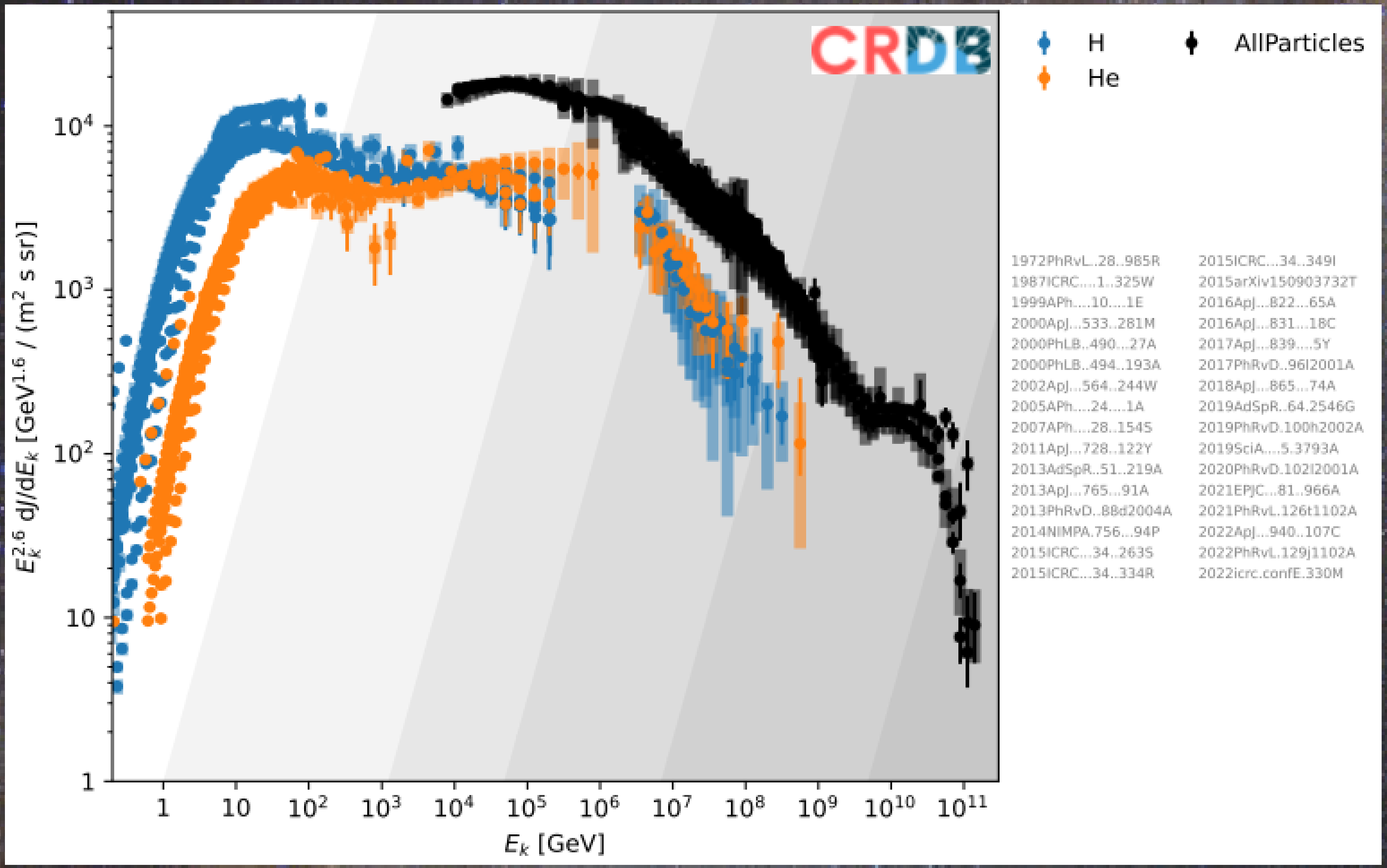
LAPTh

Kinetic Physics of Astrophysical Plasma – June 20225



https://github.com/crdb-project/tutorial/blob/main/gallery.ipynb

Introduction : the precision era



Introduction : the precision era

Some pending questions of galactic CRs :

Sources

What are the sources of GCRs/acceleration mec.?
Is CR acceleration universal?
What is their respective contribution to the flux?
What is the maximum energy of GCRs?
Does the escape impact the injected flux?
What is their distribution in the galaxy?
Are there exotic (!=astrophysical) sources?
...

Transport

What are the dominating transport mech.?
Is the transport universal?
How does the transport depend on the ISM?
What is the origin of the diffusive halo?
Is the transport homogeneous in the galaxy?
...

Local environement

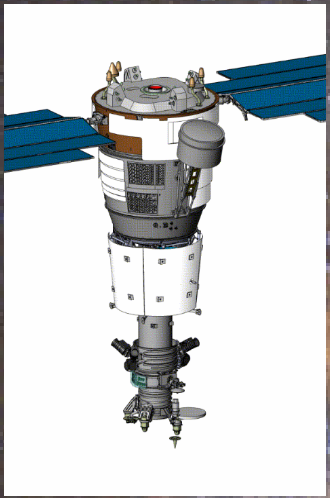
What is the effect of the solar wind on local fluxes?
Is the local flux close to the averaged galactic one?
What is the contribution of local sources?
What is the origin of the anisotropies?
Is the local underdensity affecting local fluxes?
...

See also the recent review: Gabici+ (2019)

Introduction : the precision era

Game changer: high-quality data!

→ In this talk focus on direct detection experiments

	AMS02	NUCLEON	CALET	DAMPE	ISS-CREAM
					
Op. since:	14yrs	10.5yrs	10yrs	9.5yrs	5.5yrs
Published E-range	1 GV – 1.9 TV	1 TeV – 500 TeV	10 GeV – 100TeV	10 GeV – 100 TeV	1TeV-500TeV

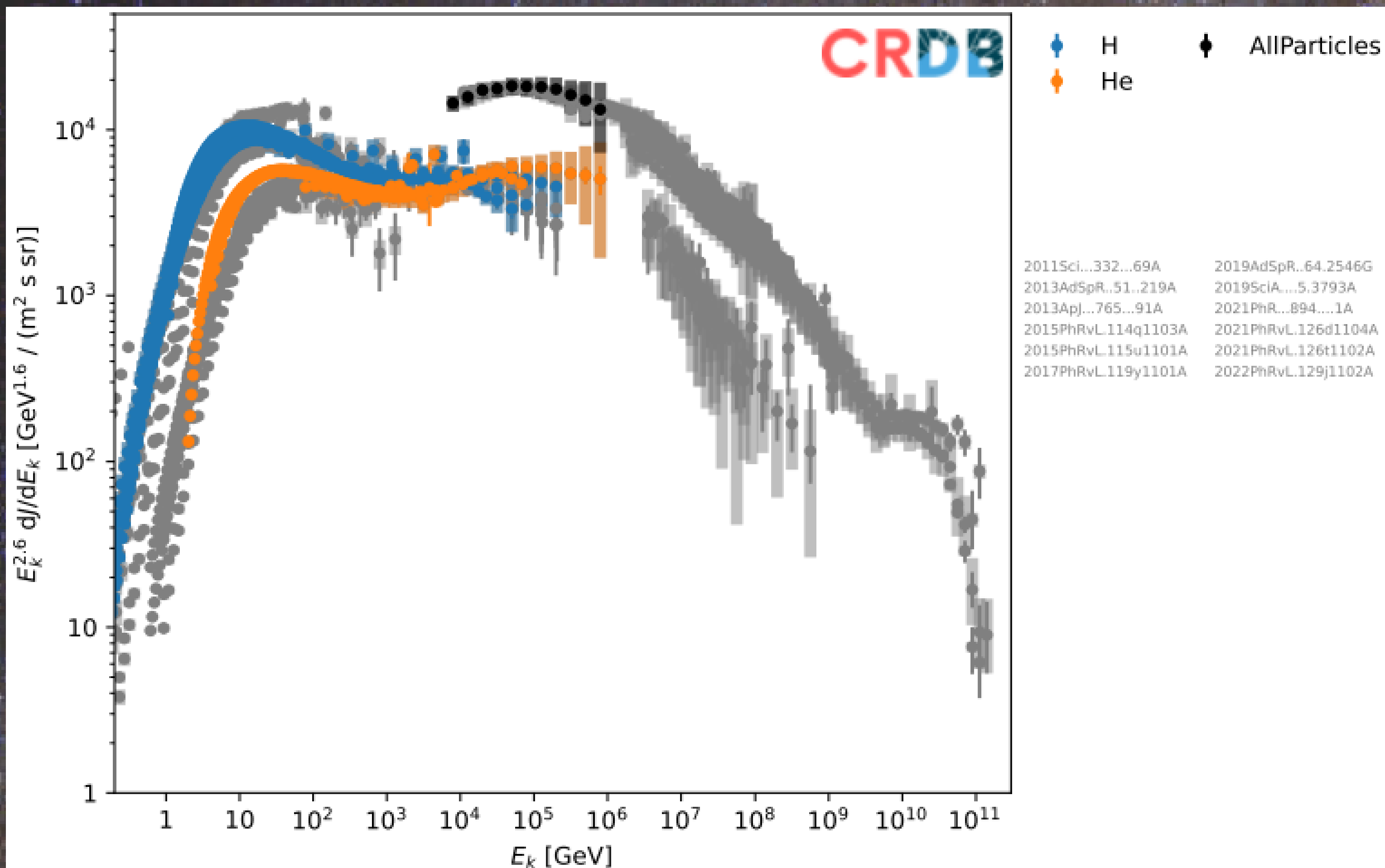
Spectrometer

→ Precision level % from GV to TV
→ Spectrometer : able to measure isotopes

Calorimeters

→ High-statistics up to 100TeV
→ Bridging the gap with air-shower experiments

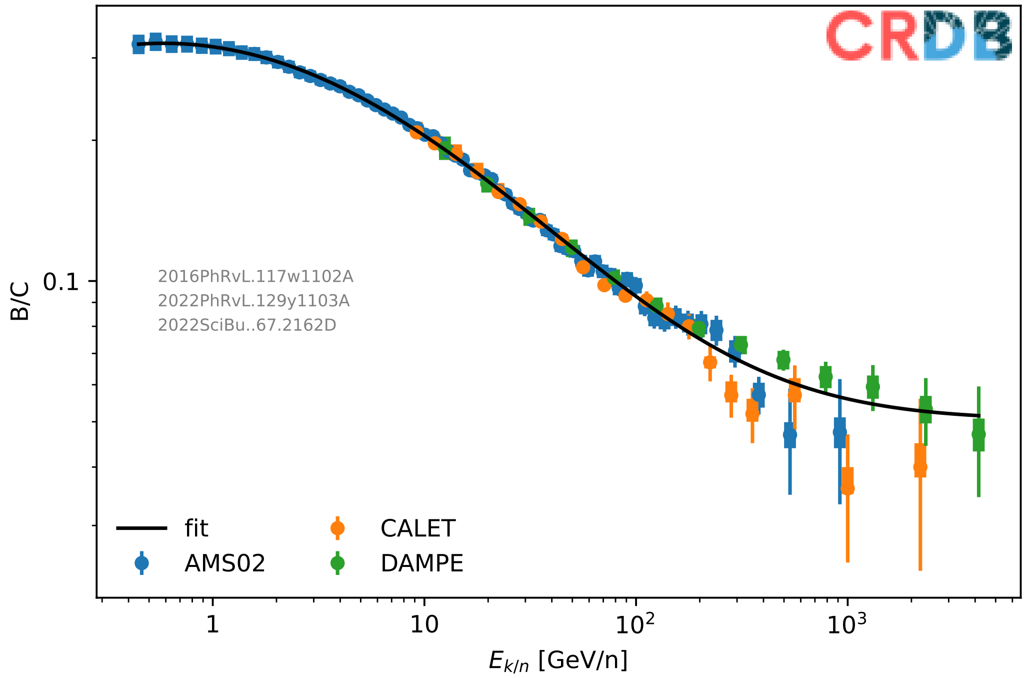
Game changer: high-quality data!



→ Data need carefull treatment of the errors (systematic VS statistical uncertainties)

Game changer: high-quality data!

Examples : AMS02 / DAMPE / CALET : B/C



Introduction : the precision era

Galactic cosmic-ray transport

Microphysics of cosmic-ray transport
→ Some motivations
→ Synthetic turbulence
→ MHD

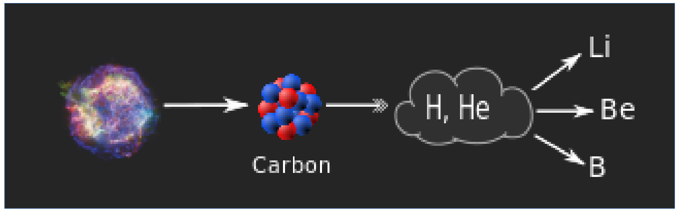
Conclusion

Resolution of **CR transport equation** in steady state:

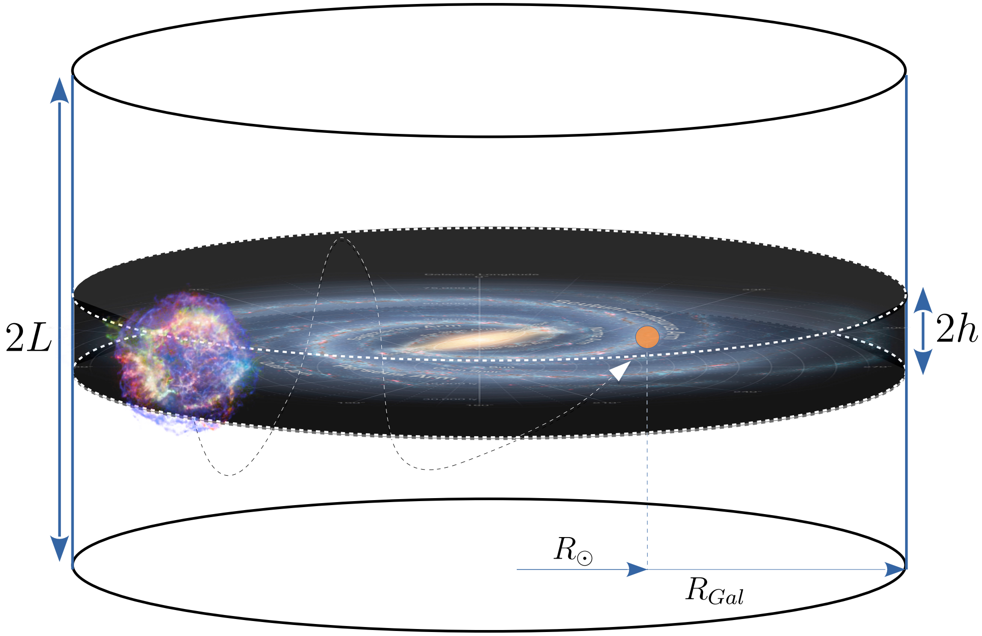
$$\frac{\partial \psi_\alpha}{\partial t} - \vec{\nabla}_{\mathbf{x}} \left\{ K(E)\, \vec{\nabla}_{\mathbf{x}} \psi_\alpha - \vec{V}_c \psi_\alpha \right\} + \frac{\partial}{\partial E} \left\{ b_{\rm tot}(E)\, \psi_\alpha - \beta^2 K_{pp} \frac{\partial \psi_\alpha}{\partial E} \right\}$$

$$+\sigma_\alpha\, v_\alpha\, n_{\rm ism}\, \psi_\alpha + \Gamma_\alpha\, \psi_\alpha = q_\alpha + \sum_\beta \left\{ \sigma_{\beta\to\alpha} v_\beta n_{\rm ism} + \Gamma_{\beta\to\alpha} \right\} \psi_\beta \,.$$

Ginzburg&Syrovatskii (1964)



in a **cylindrical geometry**.



Remarks on the **CR transport equation**

→ Diffusion, convection, E-losses, reacceleration, spallation

→ Ingredients introduced ~60 yrs ago still satisfying

→ Non exhaustive list of fitted parameters :

$K = K_0 \beta R^{\delta}$ / V_c / V_A / L / ...

→ **Effective** transport param. = average over kpc scales

- pros : learn generic properties of transport/sources
- cons : several processes intricated

→ Precise determination of transport param.

- link **µ-physics**
- **prediction secondaries (antipart.)**

Resolution of **CR transport equation** in steady state:

$$\cancel{\frac{\partial \psi_\alpha}{\partial t}} - \vec{\nabla}_{\mathbf{x}} \left\{ K(E)\, \vec{\nabla}_{\mathbf{x}} \psi_\alpha - \vec{V}_c \psi_\alpha \right\} + \frac{\partial}{\partial E} \left\{ b_{\rm tot}(E)\, \psi_\alpha - \beta^2 K_{pp} \frac{\partial \psi_\alpha}{\partial E} \right\}$$

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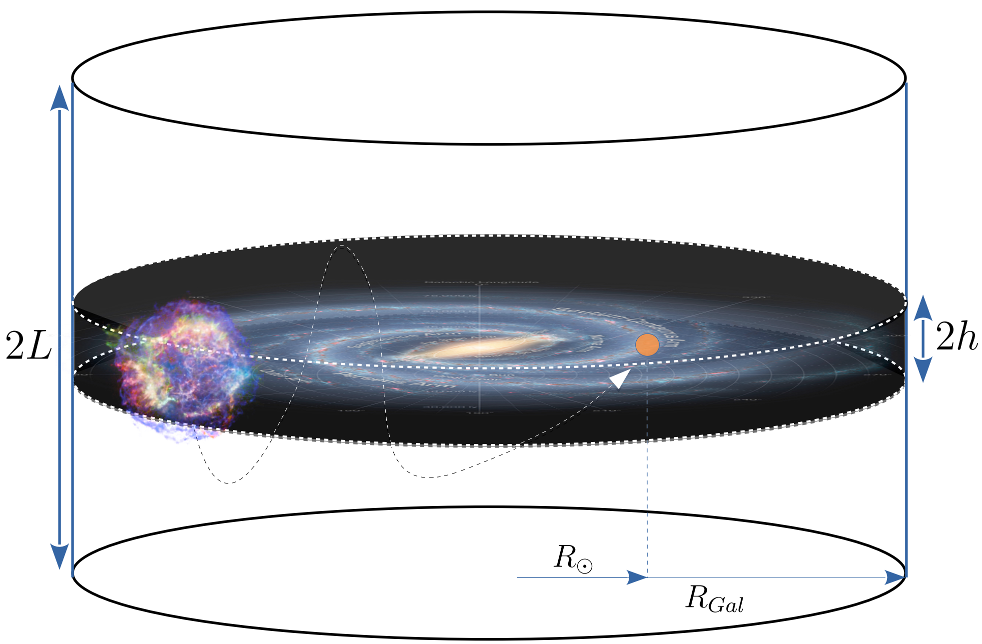
Ginzburg&Syrovatskii (1964)

in a **cylindrical geometry**.

Challenged by % precise data! → **Usual assumptions** of the resolution



- → Steady state is reached
- → Sources are distributed homogeneously in the galaxy
- → Injection scaling : single powerlaw q = C x R $^\alpha$
- → Diffusion is homogeneous and isotropic
- → Diffusion scaling : single powerlaw K = K_0 **β**R $^\delta$
- → Injection and diffusion are universal (i.e. among species)
- → Spallation cross sections are well-known
- → Energy losses are well-known
- → Local ISM has no impact on local fluxes
- → ...

Resolution of **CR transport equation** in steady state:

$$\cancel{\frac{\partial \psi_\alpha}{\partial t}} - \vec{\nabla}_{\mathbf{x}} \left\{ K(E)\, \vec{\nabla}_{\mathbf{x}} \psi_\alpha - \vec{V}_c \psi_\alpha \right\} + \frac{\partial}{\partial E} \left\{ b_{\rm tot}(E)\, \psi_\alpha - \beta^2 K_{pp} \frac{\partial \psi_\alpha}{\partial E} \right\}$$

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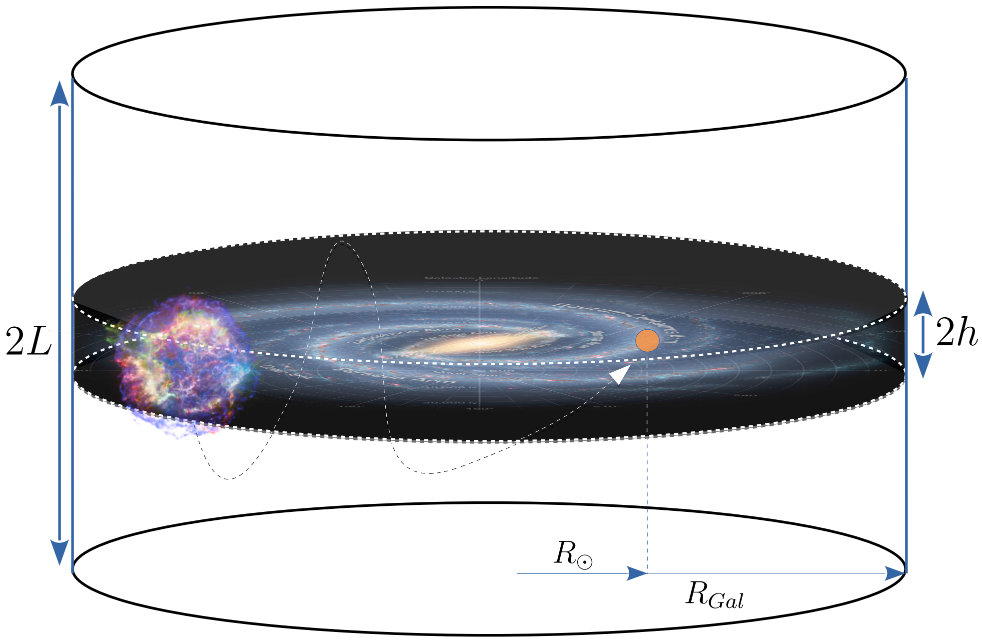


Challenged by % precise data! → **Usual assumptions** of the resolution

Rules to challenge hypothesis

→ Chose a minimal setup based on usual assumptions

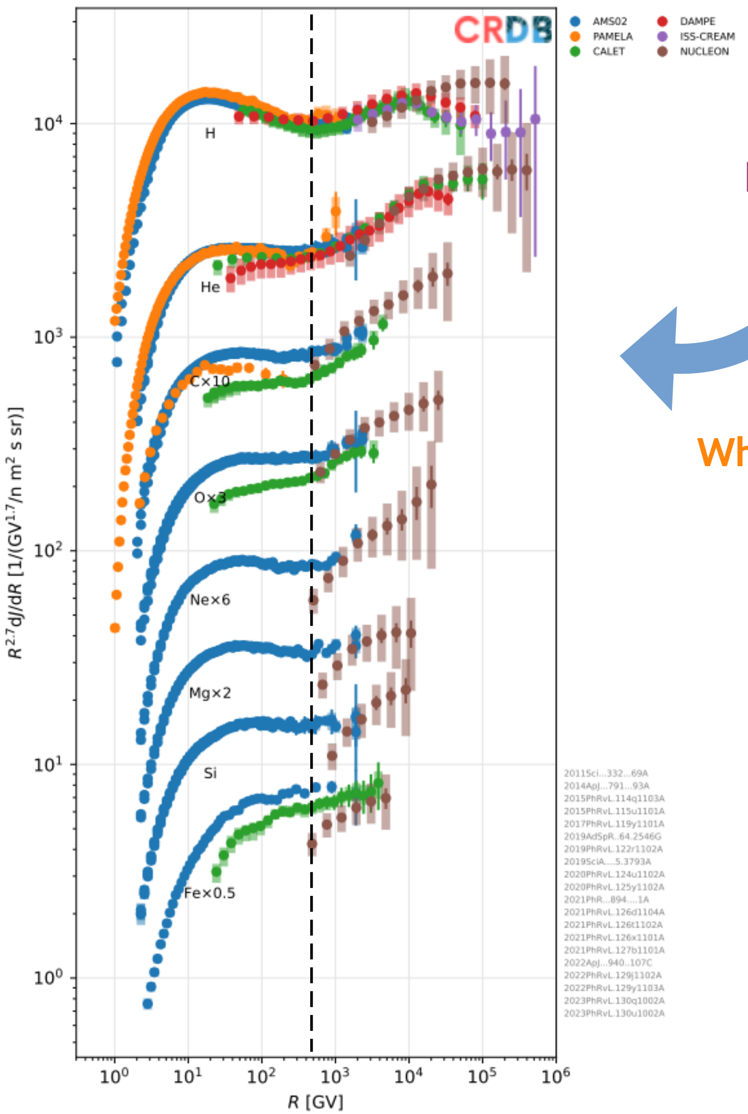
→ Add a novel ingredient

→ Check the preference of the data on a statistical basis

→ Covariance matrix required

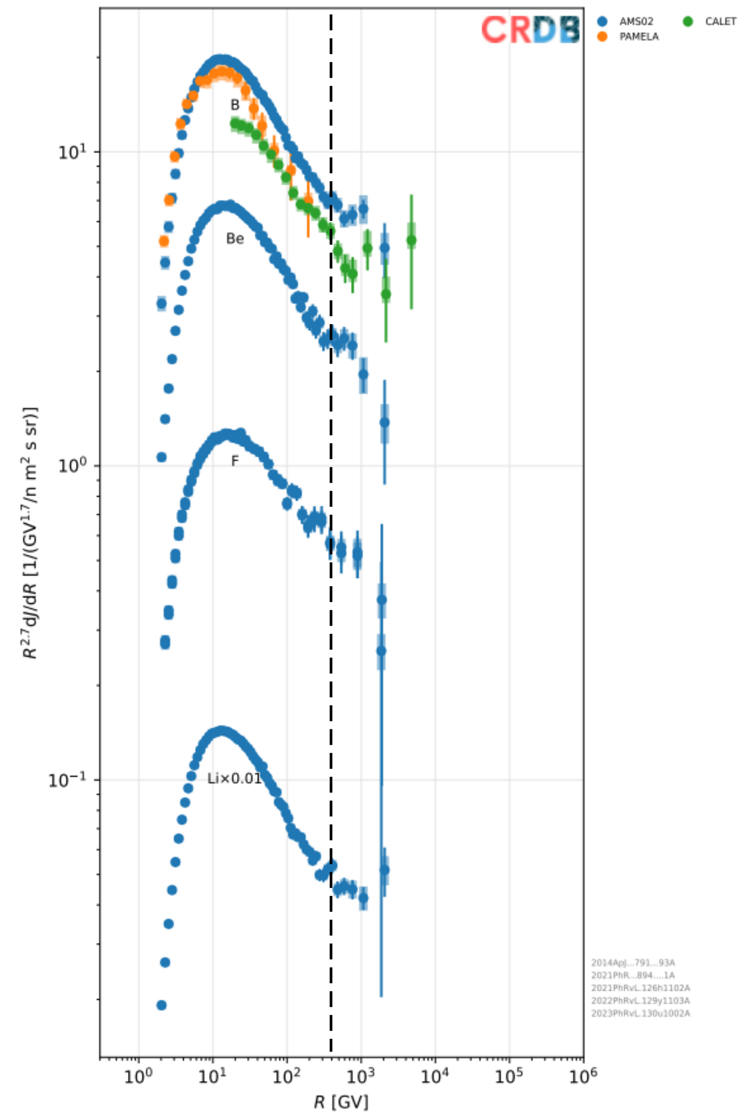
Cosmic-ray transport → Breaks?

Universal break in the spectra around 300 GV!



primaries and secondaries

What is the origin of the break?



Cosmic-ray transport → Breaks?

Universal break(s) in the spectra!

$$\frac{\partial \psi_\alpha}{\partial t} - \vec{\nabla}_{\mathbf{x}} \left\{ K(E)\, \vec{\nabla}_{\mathbf{x}} \psi_\alpha - \vec{V}_c \psi_\alpha \right\} + \frac{\partial}{\partial E} \left\{ b_{\mathrm{tot}}(E)\, \psi_\alpha - \beta^2 K_p \frac{\partial \psi_\alpha}{\partial E} \right\}$$
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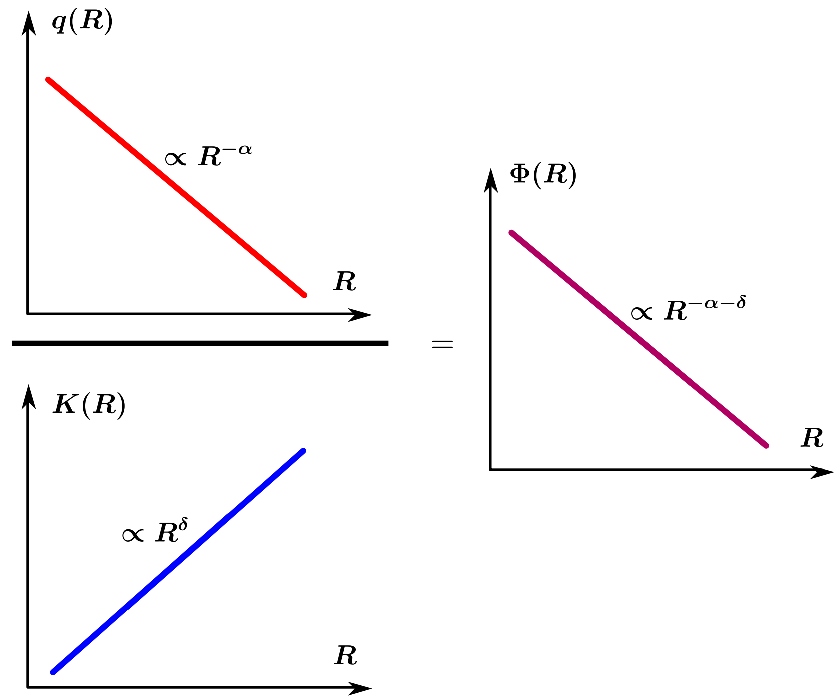
Solution of CR transport equation : pure diffusive case

What is their origins?

Injection? **Local source?** **Diffusion?**

For pure primary species: $\Phi(R) \propto \frac{q}{K} =$



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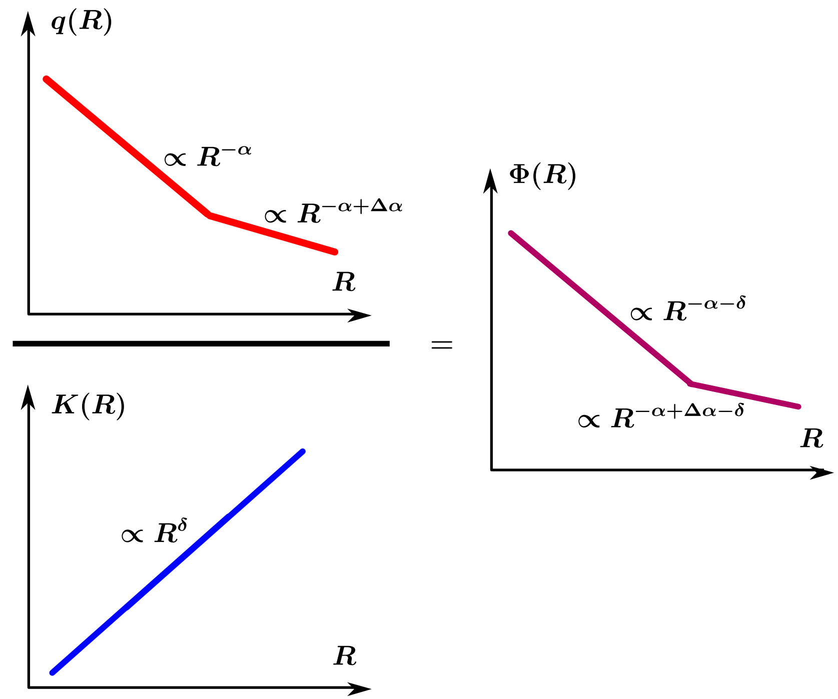

Primaries e.g. Vladimirov+ (2012), Niu+ (2018, 2019, 2020); Tomassetti+ (2015)
Secondaries e.g. Tomassetti+ (2012);Y.G.+ (2014);Tomassetti+ (2017); Zhang+ (2023)
Reacceleration e.g. Tomassetti+ (2012); Yuan+ (2020)

Cosmic-ray transport → Breaks?

Universal break(s) in the spectra!

$$\frac{\partial \psi_\alpha}{\partial t} - \vec{\nabla}_x \left\{ K(E)\, \vec{\nabla}_x \psi_\alpha - \vec{V}_c \psi_\alpha \right\} + \frac{\partial}{\partial E} \left\{ b_{tot}(E)\, \psi_\alpha - \beta^2 K_p \frac{\partial \psi_\alpha}{\partial E} \right\}$$
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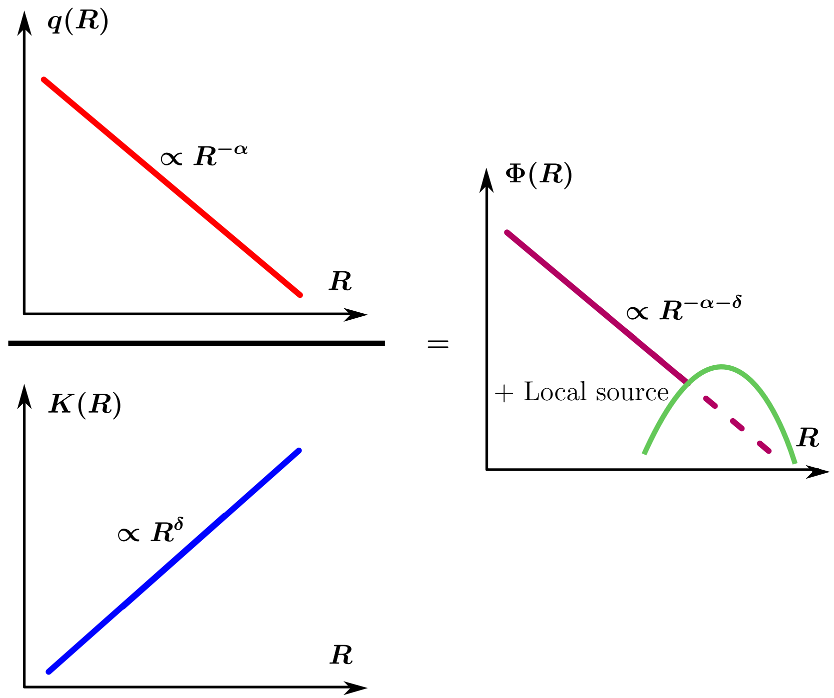
Solution of CR transport equation : pure diffusive case

What is their origins?

Injection? | **Local source?** | **Diffusion?**

For pure **primary** species: $\Phi(R) \propto \frac{q}{K} =$



Local source fits:
Bernard+ (2012)
Thoudam+ (2012)
Wei+ (2014)
Mertsch+ (2014/2021)
Savchenko+ (2015)
Bouyahiaoui+ (2018)
Lagutin+ (2019)
Yue+ (2020)
Tang+ (2022)
Anisotropies
Ahlers+ (2016)

Statistical approach:
Hadrons
Y.G+ (2017)
Evoli+ (2022)
Leptons
Mertsch (2011,2018,2018)

Stochasticity kicks in when the effective number of sources (Ns) is close to 1 →

Ns ~ 1 at 10 TV for Leptons
Ns ~ 1 at 10 PV for Hadrons

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Universal break(s) in the spectra!

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Solution of CR transport equation : pure diffusive case

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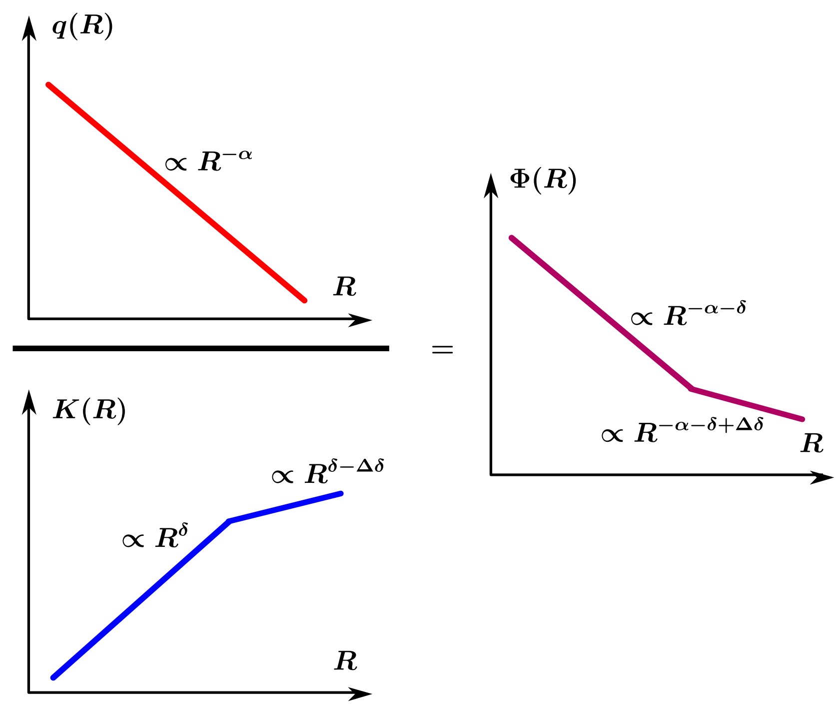

Pheno : Vladimirov+ (2012); Y.G+ (2017); Niu+ (2020);
Explanation : Tomassetti (2012);Amato+ (2012);Evoli +(2019);

Cosmic-ray transport → Breaks?

Universal break(s) in the spectra!

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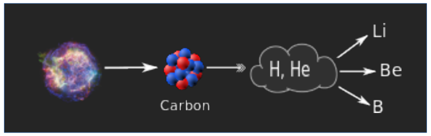
Solution of CR transport equation : pure diffusive case

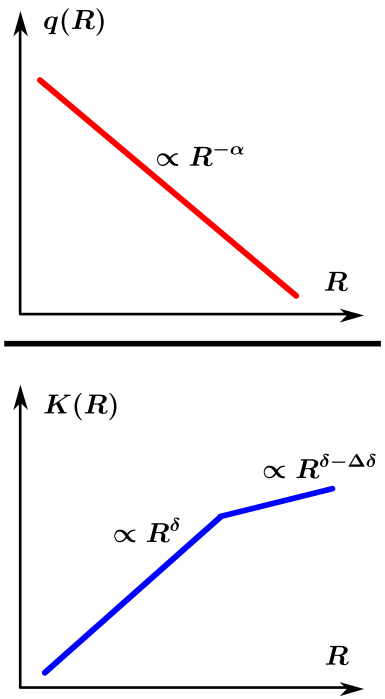
For pure secondary species:



$\Phi^{\rm sec}(R)$

||

For pure primary species: $\Phi(R) \propto \frac{q}{K} =$



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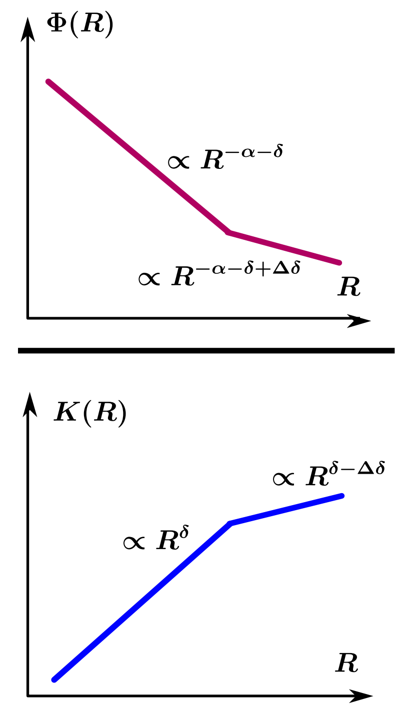

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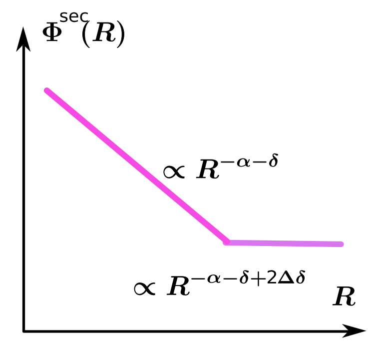

Pheno : Vladimirov+ (2012); Y.G+ (2017); Niu+ (2020);
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Cosmic-ray transport → Breaks?

Universal break(s) in the spectra!

What is their origins?

Injection? **Local source?** **Diffusion?**

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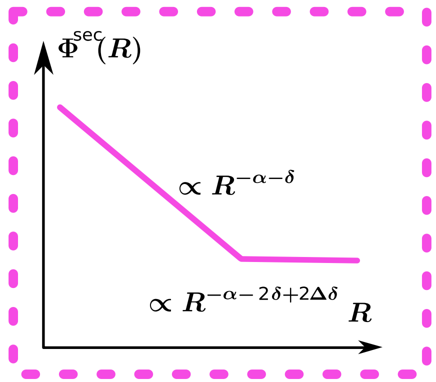
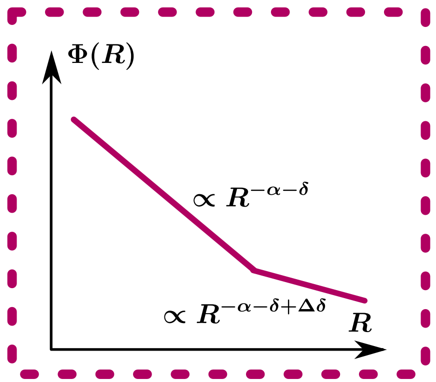
Solution of CR transport equation : pure diffusive case

For **secondary**/**primary** ratio: $\dfrac{\text{Secondary flux}}{\text{Primary flux}}$ =





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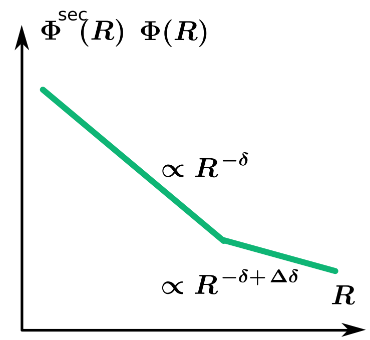
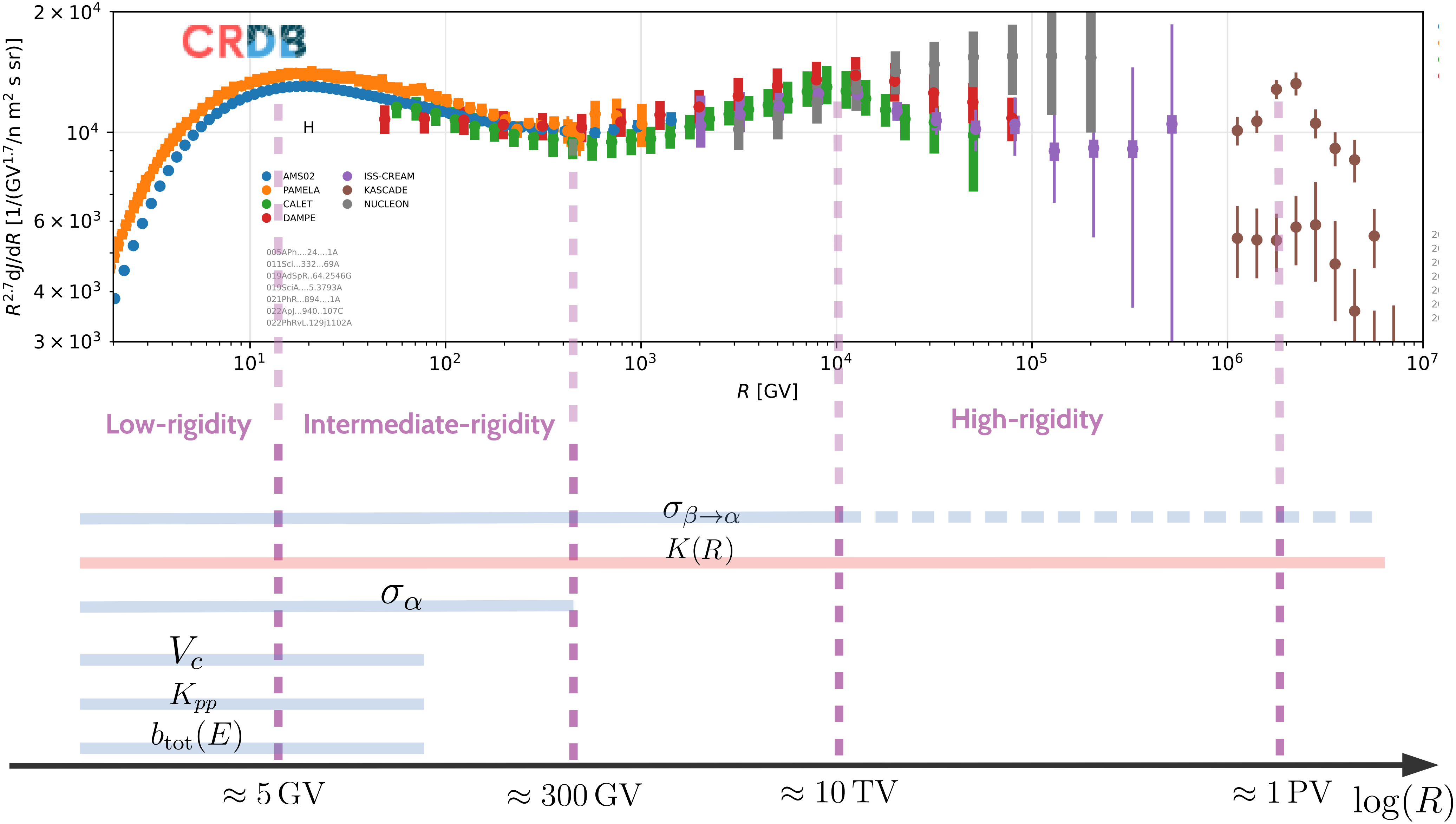


→ e.g. Li/C, Be/C, B/C, B/O, ...

Some words of caution:
Vecchi et al. (2022)

Cosmic-ray transport → Explaining the spectra

Universal break(s) in the spectra!



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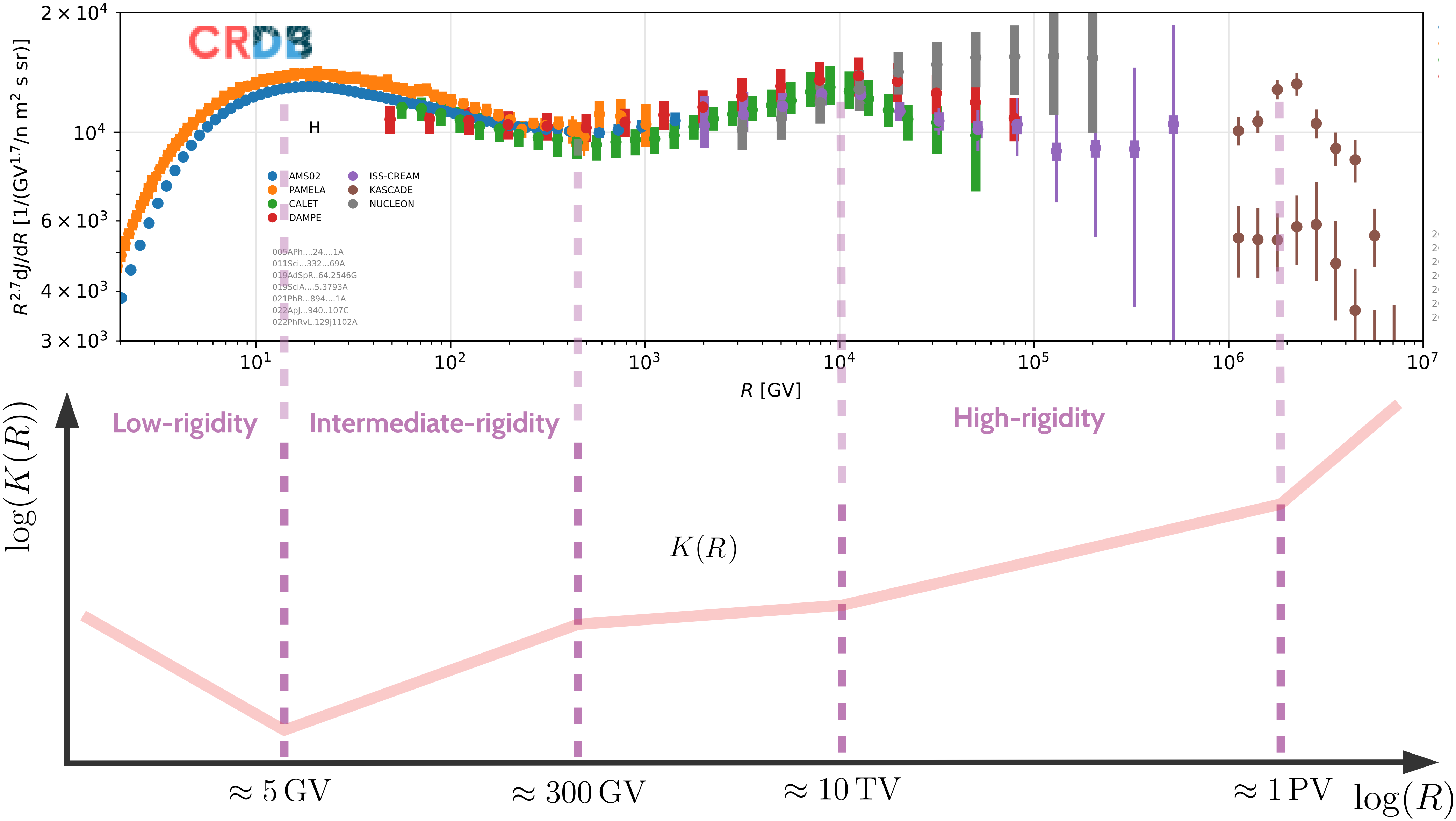

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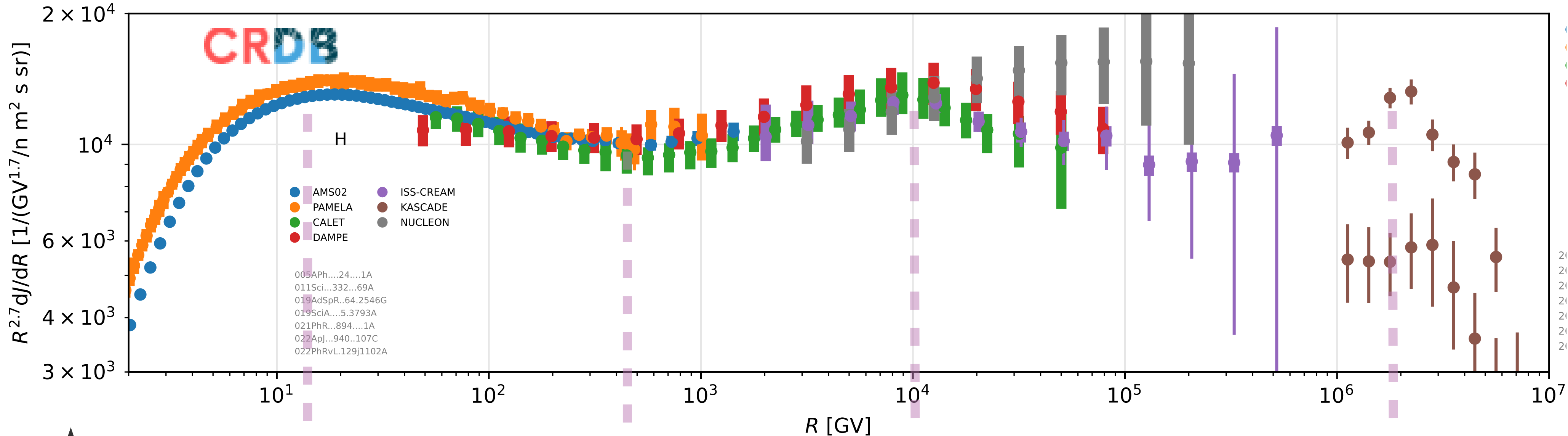
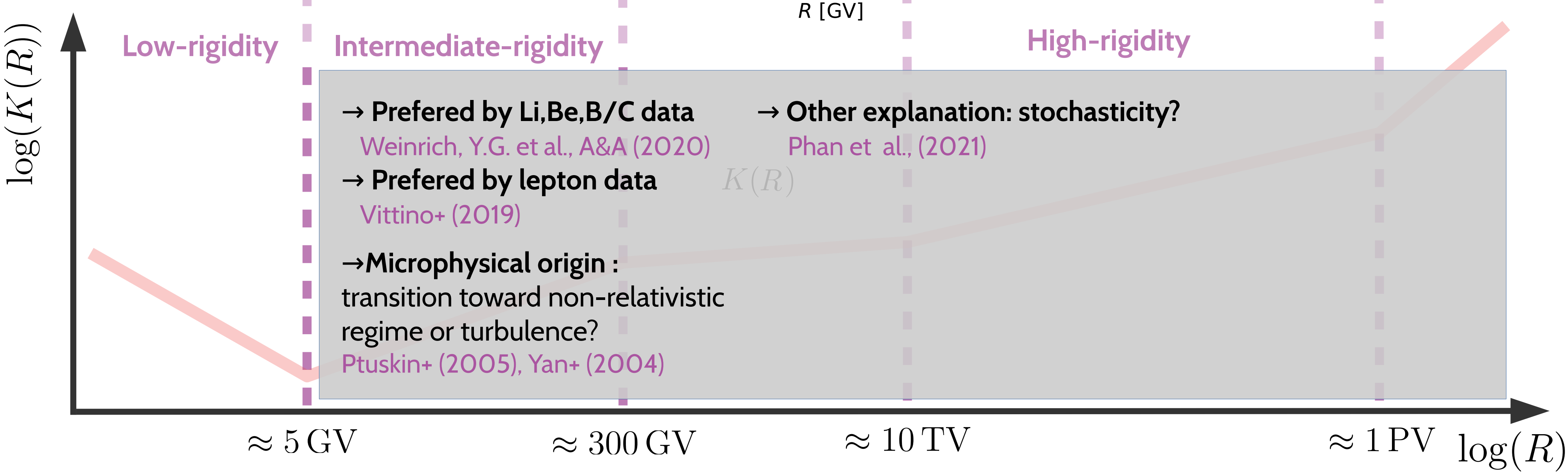




→ Prefered by Li,Be,B/C data
Weinrich, Y.G. et al., A&A (2020)

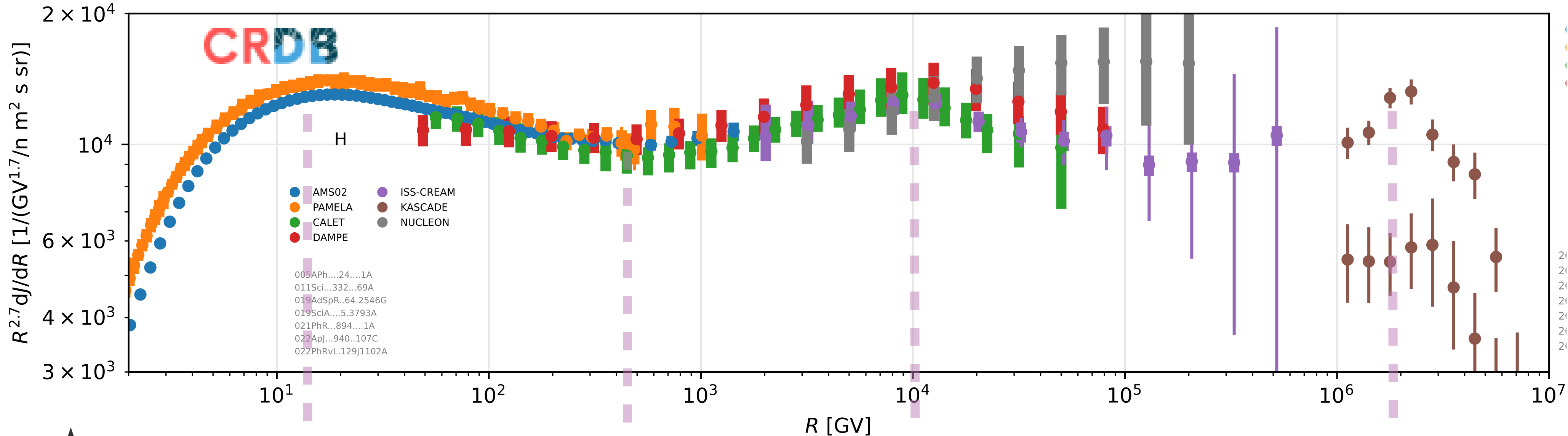
→ Prefered by lepton data
Vittino+ (2019)

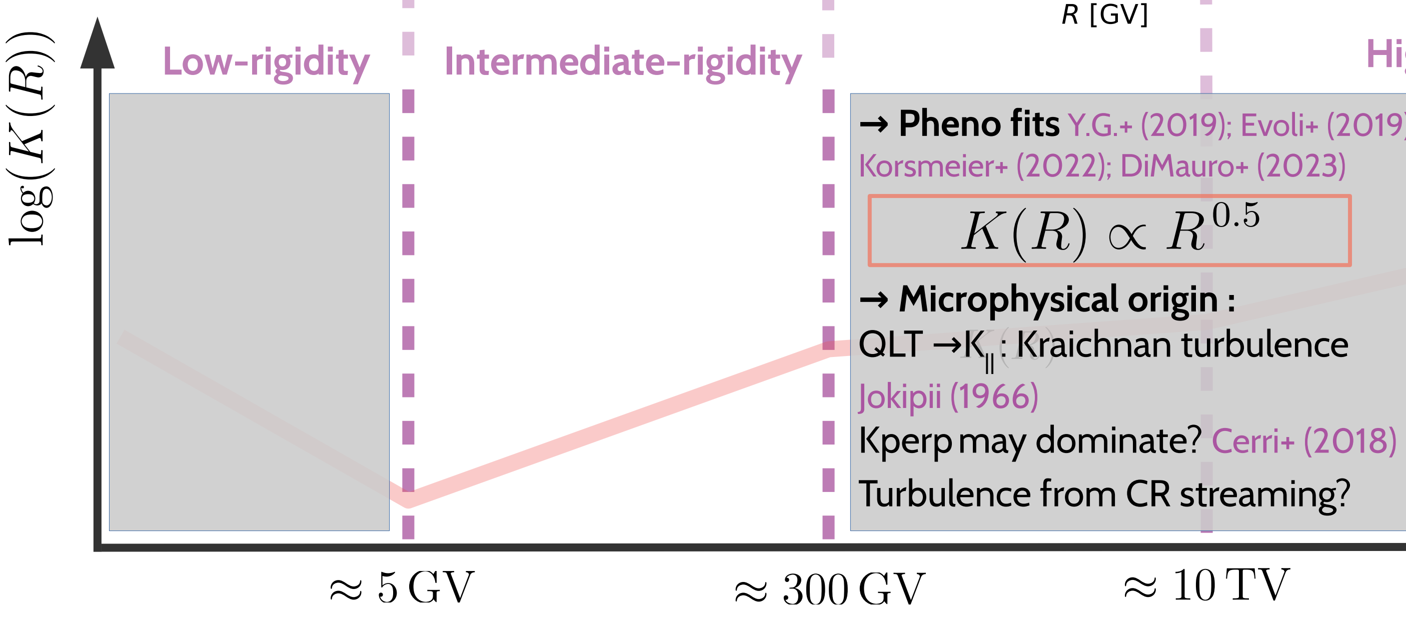
→Microphysical origin :
transition toward non-relativistic regime or turbulence?
Ptuskin+ (2005), Yan+ (2004)

→ Other explanation: stochasticity?
Phan et al., (2021)

Cosmic-ray transport → Explaining the spectra

Universal break(s) in the spectra!





→ Pheno fits Y.G.+ (2019); Evoli+ (2019) Korsmeier+ (2022); DiMauro+ (2023)

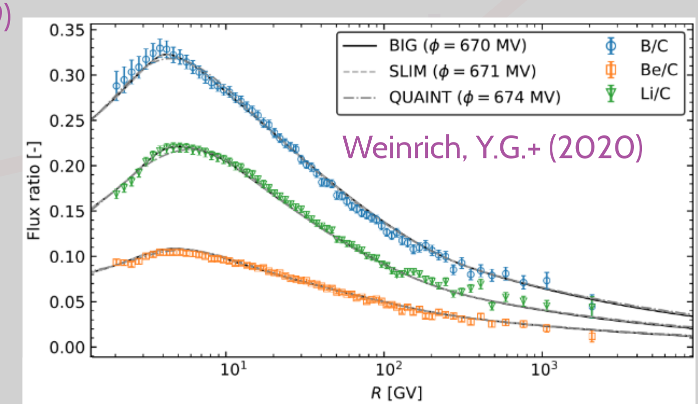
$$K(R) \propto R^{0.5}$$

→ Microphysical origin :

QLT →$K_{||}$: Kraichnan turbulence Jokipii (1966)

Kperp may dominate? Cerri+ (2018)

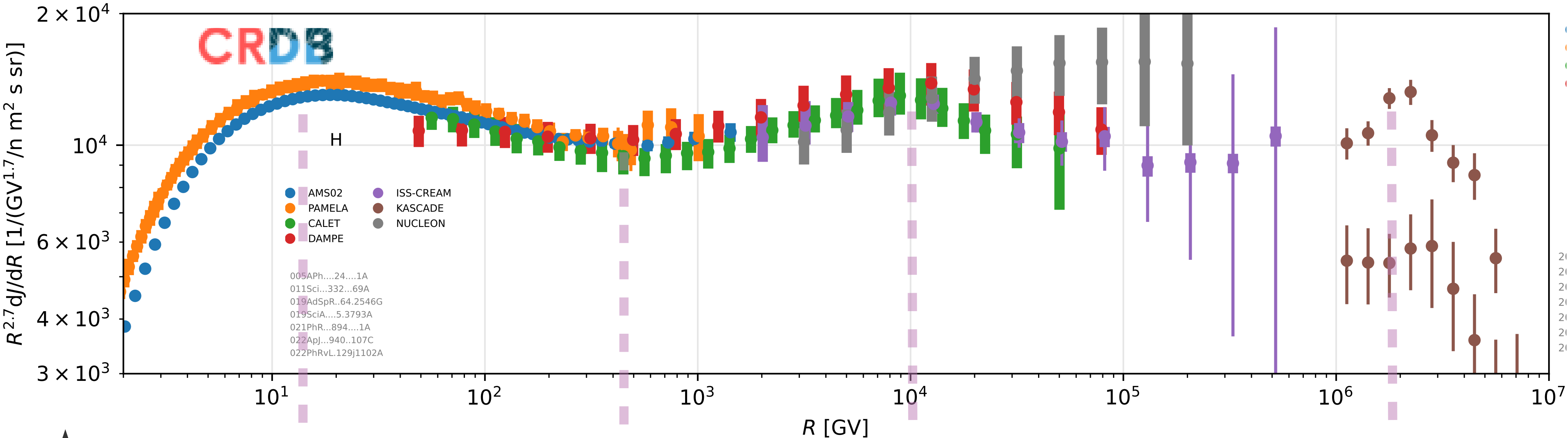
Turbulence from CR streaming?

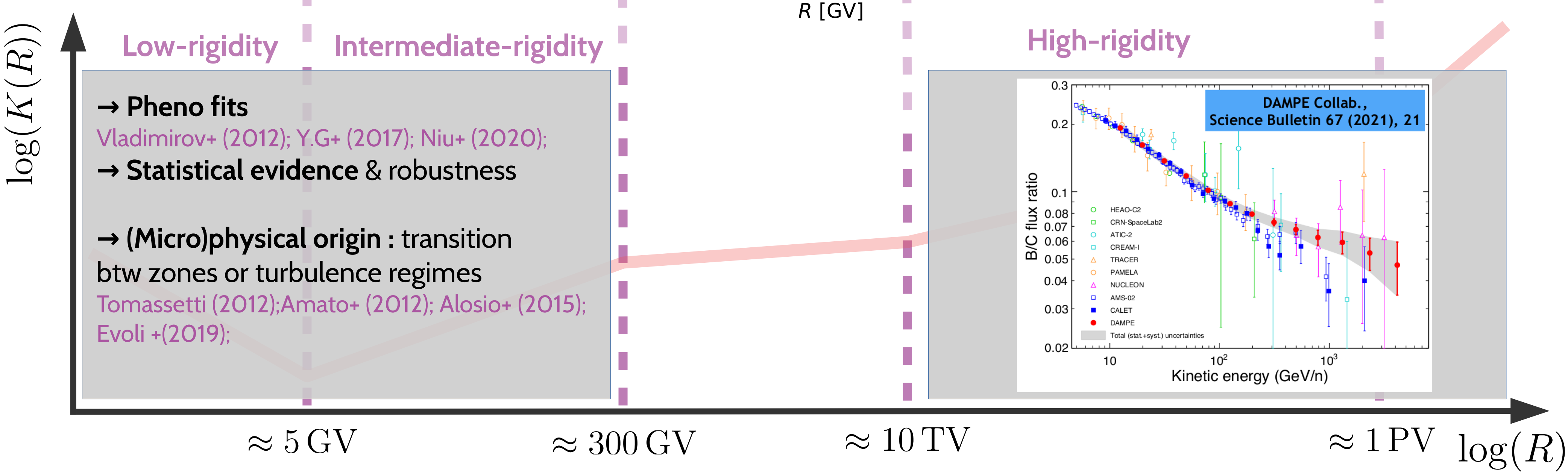


Weinrich, Y.G.+ (2020)

Cosmic-ray transport → Explaining the spectra

Universal break(s) in the spectra!



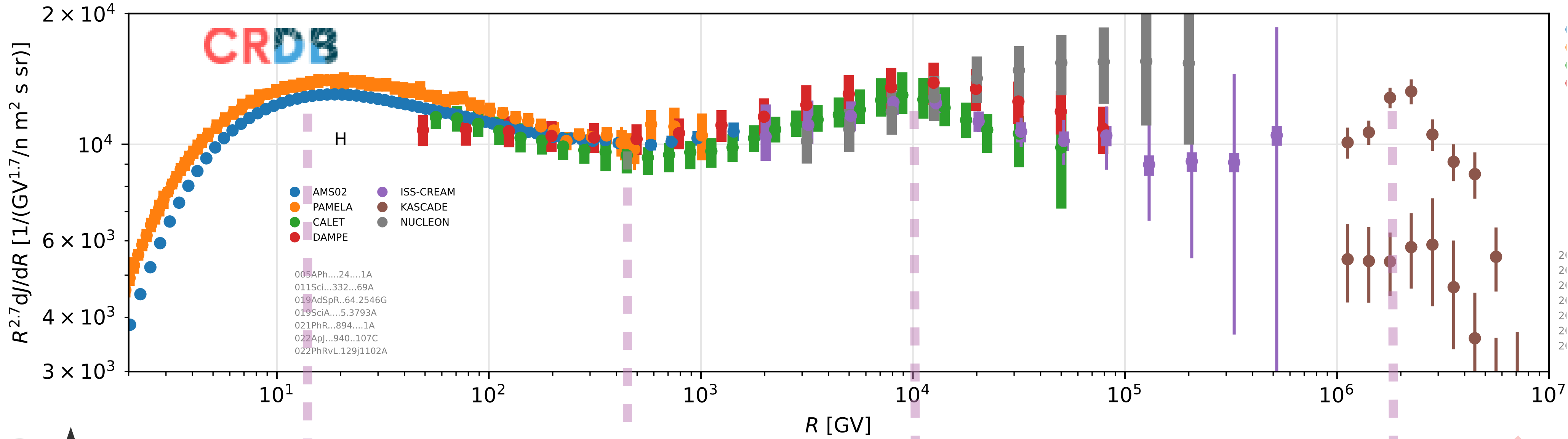


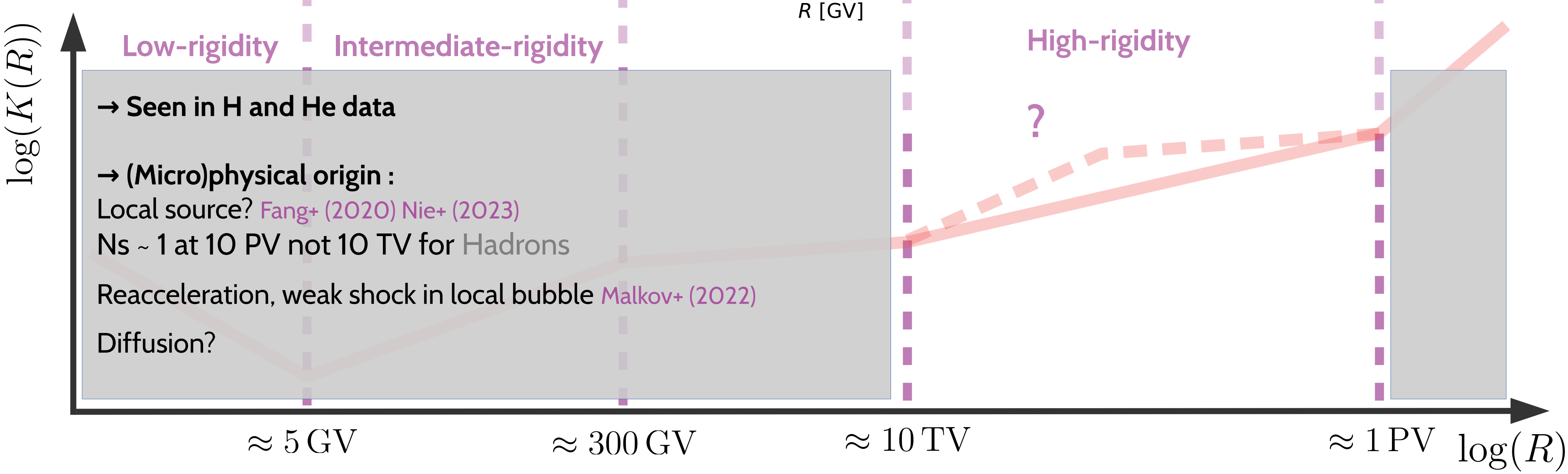
→ Pheno fits
Vladimirov+ (2012); Y.G+ (2017); Niu+ (2020);
→ Statistical evidence & robustness

→ (Micro)physical origin : transition btw zones or turbulence regimes
Tomassetti (2012);Amato+ (2012); Alosio+ (2015); Evoli +(2019);

Cosmic-ray transport → Explaining the spectra

Universal break(s) in the spectra!





→ Seen in H and He data

→ (Micro)physical origin :

Local source? Fang+ (2020) Nie+ (2023)

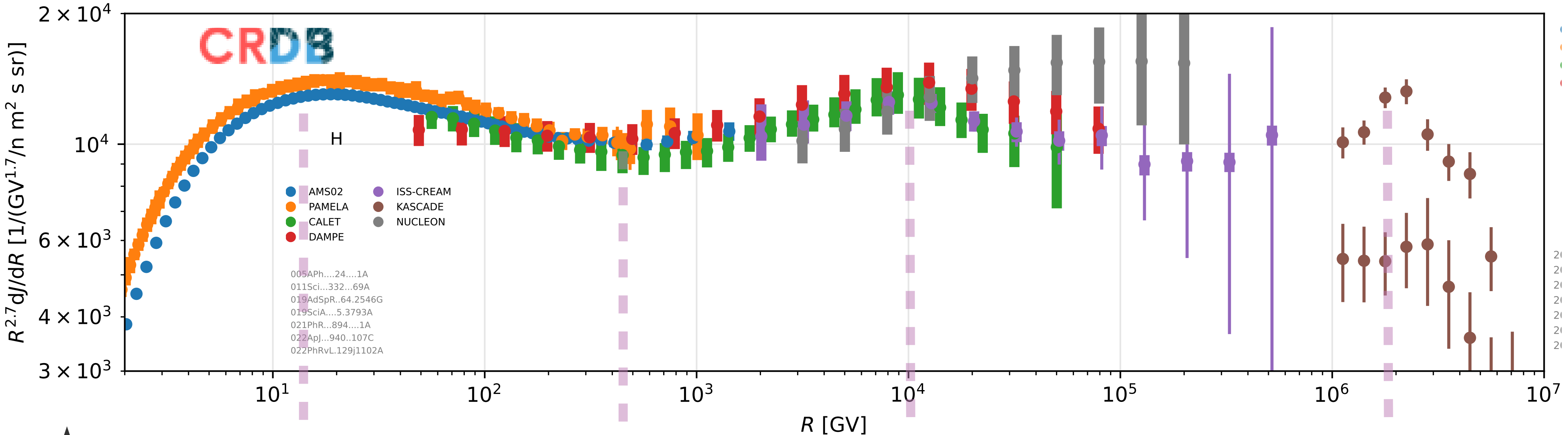
Ns ~ 1 at 10 PV not 10 TV for Hadrons

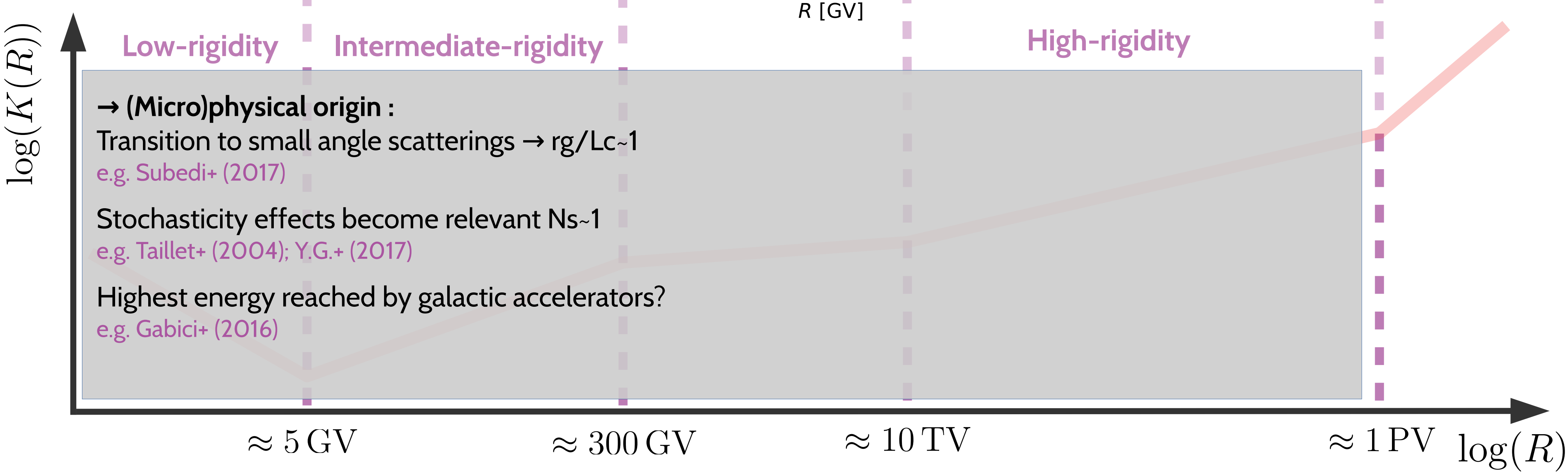
Reacceleration, weak shock in local bubble Malkov+ (2022)

Diffusion?

Cosmic-ray transport → Explaining the spectra

Universal break(s) in the spectra!





→ (Micro)physical origin :

Transition to small angle scatterings → rg/Lc~1
e.g. Subedi+ (2017)

Stochasticity effects become relevant Ns~1
e.g. Taillet+ (2004); Y.G.+ (2017)

Highest energy reached by galactic accelerators?
e.g. Gabici+ (2016)

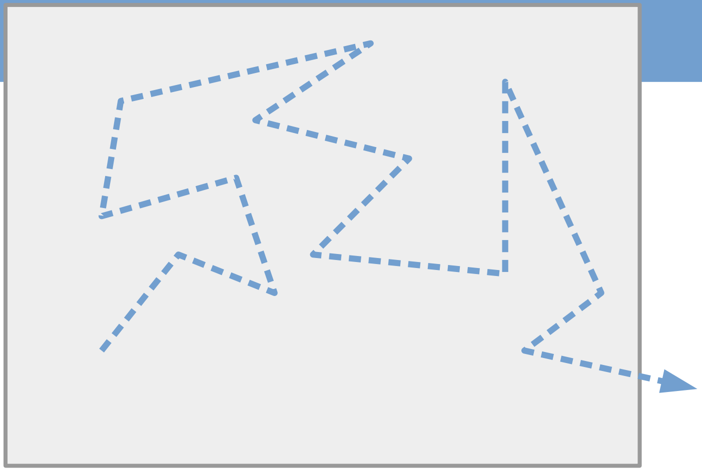
Introduction : the precision era

Galactic cosmic-ray transport

Microphysics of cosmic-ray transport
→ Some motivations
→ Synthetic turbulence
→ MHD

Conclusion

Some motivations



→ Spherical cow galactic diffusion model :

- cylindrical diffusion box
- homogeneous & isotropic diffusion tensor

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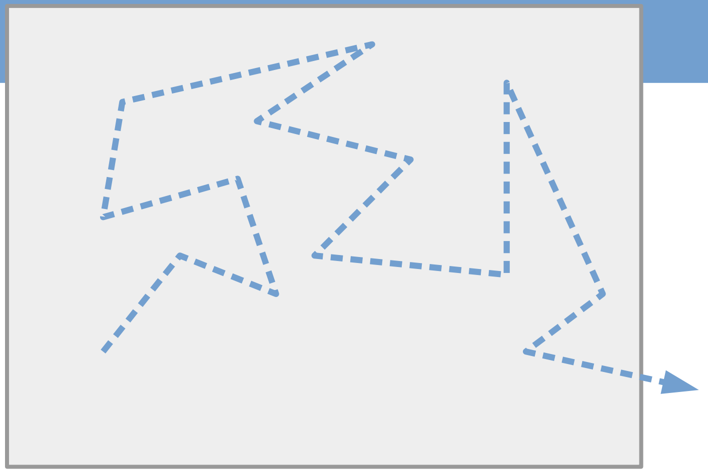
- pheno model reproducing the data but not explaining :
  K dependence wrt R / CR small scales anisotropies / CR spectral hardening toward GC / ..

Some motivations



→ Spherical cow galactic diffusion model :
- cylindrical diffusion box
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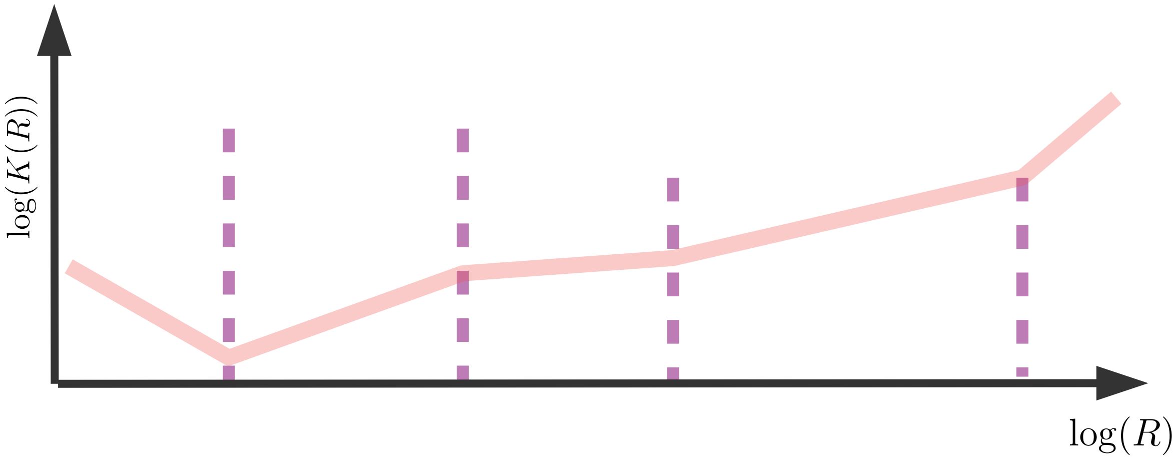

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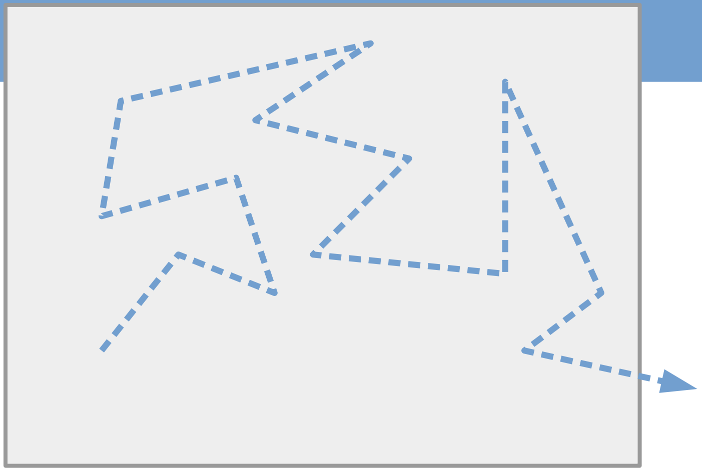

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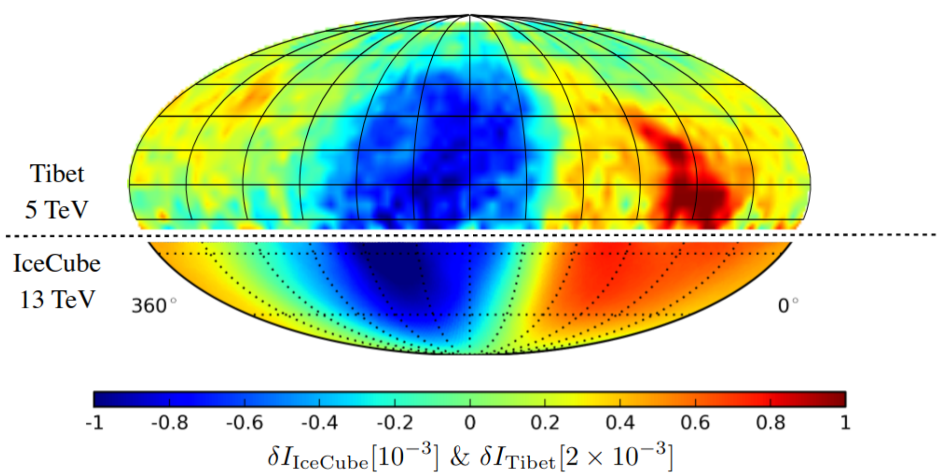
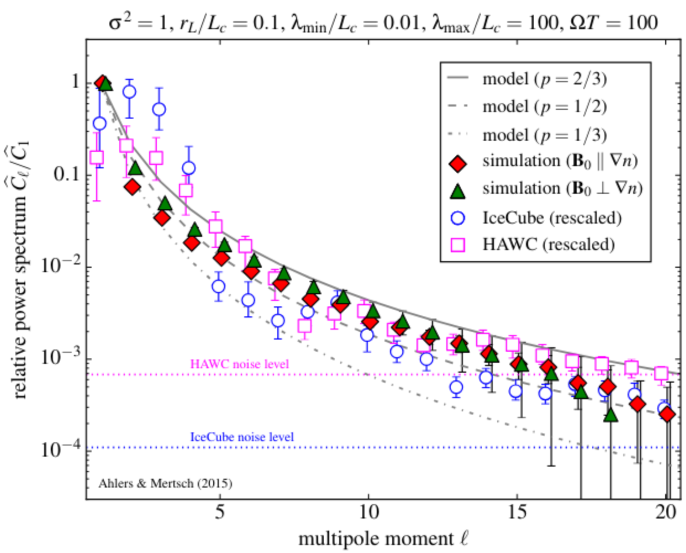


Ahlers & Mertsch (2016)



Ahlers & Mertsch (2015)

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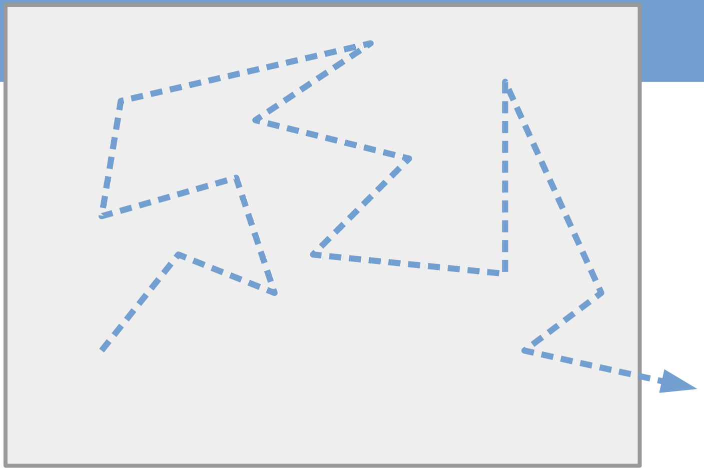

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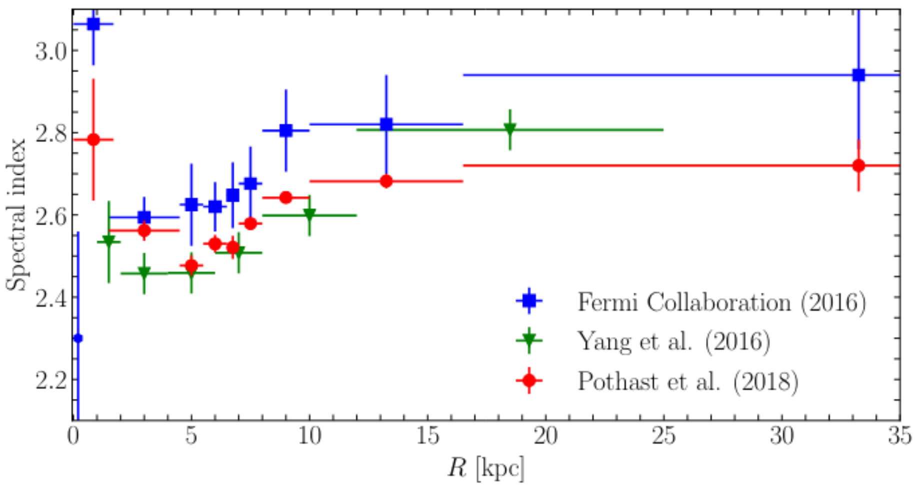
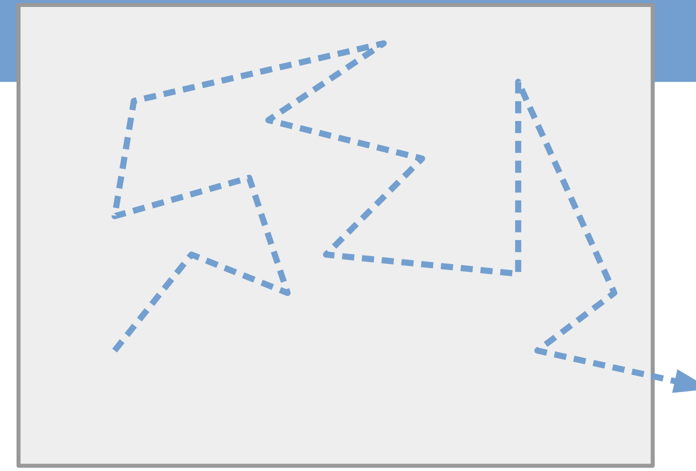


Gabici+ (2019)

Some motivations



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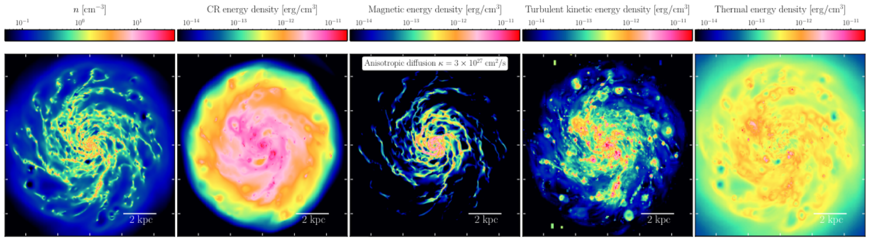
- pheno model reproducing the data but not explaining :
  K dependence wrt R / CR small scales anisotropies / CR spectral hardening toward GC / ..

Going for more realistic!

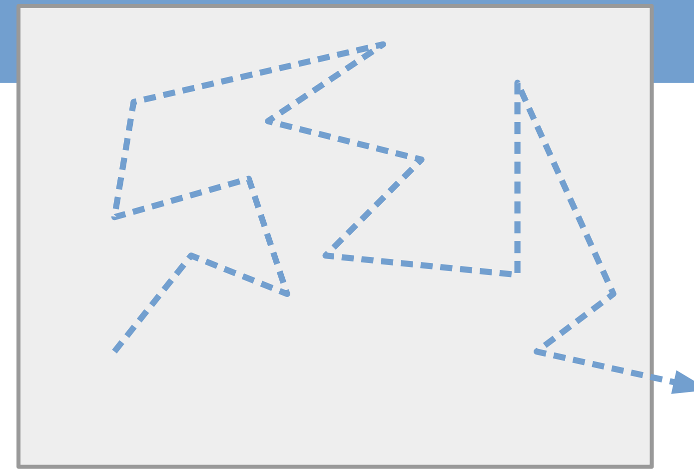
→ Design more complex model : MHD turbulence driven by Galactic dynamics + Cosmic ray feed backs + Kinetic module
Derive the CR and galactic observable data ... Hopkins+ (2022)
[CR a game changer in the galactic dynamic e.g. Hopkins+ (2020)]



Núñez-Castiñeyra+ (2022)

→ Not yet there!
- Some physics still unknown
- Numerically challenging (read impossible)
  Wide range of scales to be covered : L_{out}= 10 pc scale >> r_L(1 TeV)= 10^{-4} L_{out} – r_L(1GeV)= 10^{-7} L_{out}

Some motivations



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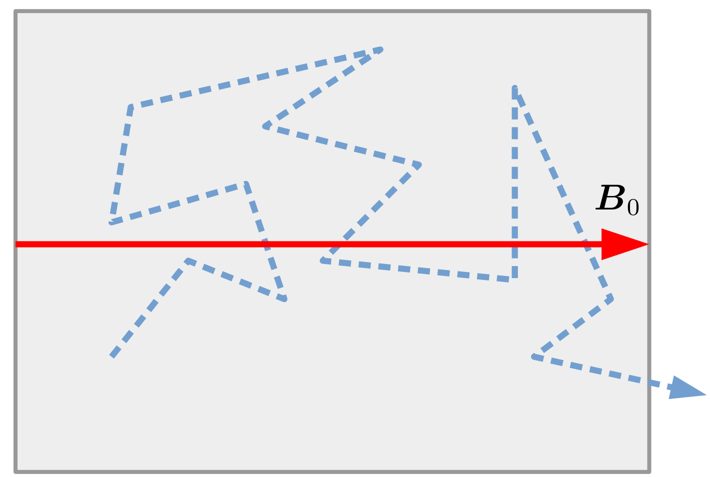
Going for more realistic!

$$\boldsymbol{K} = \begin{pmatrix} K_\perp & 0 & 0 \\ 0 & K_\perp & 0 \\ 0 & 0 & K_\parallel \end{pmatrix}$$

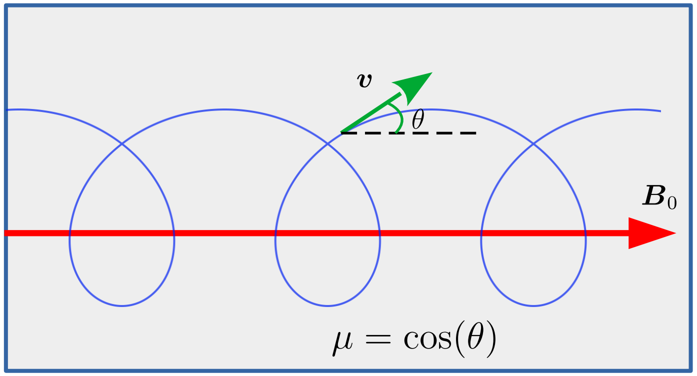
- Parallel transport to the bckg fields B_0 driven by pitch angle scattering

- Perpendicular transport driven by
  - field line random walk
  - parallel transport
  - transverse complexity

See review by Shalchi 2020



How well do we know CR scattering in a static turbulent B field?



→ **Quasi linear theory in a nutshell:** Jokipii (1966)

- Vlasov equation:

$$\frac{\partial f}{\partial t} + \frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t} \cdot \boldsymbol{\nabla}_{\boldsymbol{r}} f + e\left(\cancel{\boldsymbol{E}} + \frac{\boldsymbol{v} \times \boldsymbol{B}}{c}\right) \cdot \boldsymbol{\nabla}_{\boldsymbol{p}} f = 0$$

- Perturbative expansion $\boldsymbol{B} = \boldsymbol{B}_0 + \delta \boldsymbol{B}$ & $f = \langle f \rangle + \delta f$

$$\frac{\partial \langle f \rangle}{\partial t} + \frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t} \cdot \boldsymbol{\nabla}_{\boldsymbol{r}} \langle f \rangle + e\frac{\boldsymbol{v} \times \boldsymbol{B}_0}{c} \cdot \boldsymbol{\nabla}_{\boldsymbol{p}} \langle f \rangle \simeq \int_{t_0}^{t} \mathrm{d}t' \left\langle e\frac{\boldsymbol{v} \times \delta\boldsymbol{B}}{c} \cdot \nabla_{\boldsymbol{p}} \left[e\frac{\boldsymbol{v} \times \delta\boldsymbol{B}}{c} \cdot \boldsymbol{\nabla}_{\boldsymbol{p}} \langle f \rangle \right]_{P(t')} \right\rangle$$

Unperturbed orbits!

- Further hypotheses ..

($\delta B \ll B_0$, gyrotropy, adiabatic evolution, finite correlation times, homogeneous & stationary turbulence)

Pitch-angle diffusion equation:

Resonance function

$$\frac{\partial \langle f \rangle}{\partial t} + v\mu \frac{\partial \langle f \rangle}{\partial z} = \frac{\partial}{\partial \mu}\left(D_{\mu\mu} \frac{\partial \langle f \rangle}{\partial \mu} \right) \longrightarrow \text{with } D_{\mu\mu} = \frac{2\pi v^2 (1-\mu^2)}{r_L^2 B_0^2} \int \mathrm{d}k_\parallel \, P(k_\parallel) \boxed{\delta(k_\parallel v \mu - \Omega)} \sim \left(\frac{\delta B}{B_0}\right)^2 \left(\frac{r_L}{L_c}\right)^{\alpha - 2}$$

- Taking the isotropic part of the phase space distribution $\langle f \rangle(\boldsymbol{x}, p, \mu, t) = f_0(\boldsymbol{x}, p, t) + f_1(\boldsymbol{x}, p, \mu, t)$

  Spatial diffusion equation:

$$\frac{\partial f_0}{\partial t} - \frac{\partial}{\partial z}\left(K_\parallel \frac{\partial f_0}{\partial z} \right) = 0 \longrightarrow \text{with } K_\parallel = \frac{v^2}{8} \int_{-1}^{1} \mathrm{d}\mu \frac{(1-\mu^2)^2}{D_{\mu\mu}} \sim \left(\frac{B_0}{\delta B}\right)^2 \left(\frac{r_L}{L_c}\right)^{2-\alpha}$$

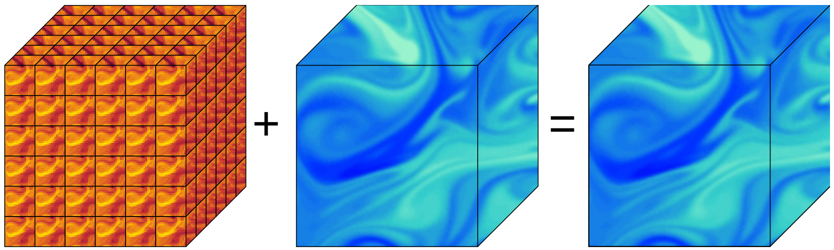
How well do we know CR scattering in a static turbulent B field?

Test-particle simulations in **synthetic turbulence**

Generating synthetic turbulence?

[see Mertsch (2020)]

Method	Store in memory	Main pros	Main cons
Harmonic method Giacalone&Jokipii (1994): $\boldsymbol{\delta B}(\boldsymbol{r}) = \sum_{n=0}^{N-1} A_n \hat{\boldsymbol{\xi}}_n \cos\left[k_n \hat{\boldsymbol{k}}_n \cdot \boldsymbol{r} + \beta_n\right]$	$\{A_n, \hat{\boldsymbol{\xi}}_n, \hat{\boldsymbol{k}}_n, \beta_n\}$	Dynamical range	Computing time
Grid method e.g Qin+ (2002): $\delta B_j(\boldsymbol{r}_{n_1,n_2,n_3}) = \frac{1}{N^3} \sum_{l_1,l_2,l_3=0}^{N-1} e^{-2\pi i(\frac{l_1 n_1}{N} + \frac{l_2 n_2}{N} + \frac{l_3 n_3}{N})} \delta\tilde{B}_j^{l_1,l_2,l_3}$	$\delta B_j(\boldsymbol{r}_{n_1,n_2,n_3})$	Computing time	Dynamical range
Nested grid Giacinti+ (2012):	$\delta B_j(\boldsymbol{r}_{n_1,n_2,n_3})$	Computing time	Memory storage



Mertsch (2020)

→ Propagate particles in these turbulences!

- Going more realistic see e.g. Lübke+ (2024) Durrive+ (2020,2022) Maci+ (2024)

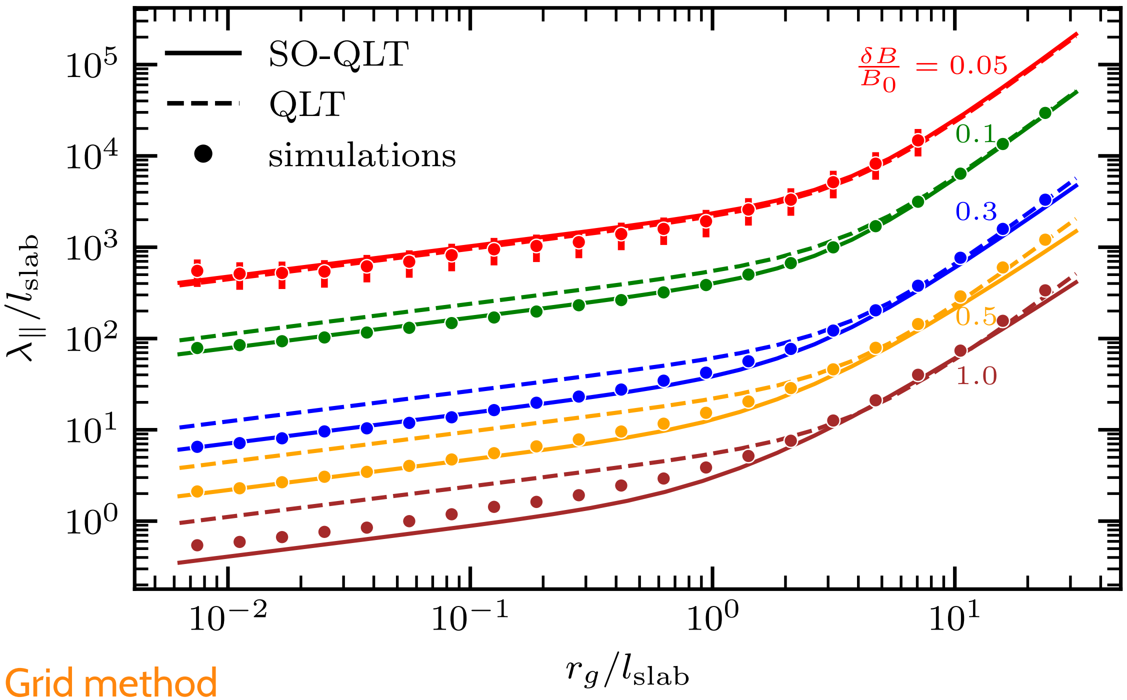
How well do we know CR scattering in a static turbulent B field?

Test-particle simulations in **synthetic turbulence**

Probing parallel transport → $K_{//}$

Slab turbulence : [Dundovic+ (2020)]

$$S^{\rm slab}(k_\parallel, k_\perp) = E^{\rm slab}(k_\parallel)\frac{\delta(k_\perp)}{2\pi k_\perp} = 2\frac{C^{\rm slab}(s)\delta B^2_{\rm slab} l_{\rm slab}}{\left[1+(k_\parallel l_{\rm slab})^2\right]^{s/2}}\frac{\delta(k_\perp)}{2\pi k_\perp} \rightarrow D_{\mu\mu} = \frac{\pi C^{\rm slab}(s) v\, \delta\tilde{B}^2_{\rm slab}}{l_{\rm slab}}\frac{(1-\mu^2)\,\mu^{s-1}}{(1+\mu^2\tilde{r}_g^2)^{s/2}}\tilde{r}_g^{s-2}$$



Grid method

→ Good agreement for small delta B/B

→ Transition resonant/small scattering regimes (only with this spectrum)

Going beyond QLT: (see Tautz+ 2006)

→ 90° problem $D_{\mu\mu}$ → 0

→ Evaluate perturbation over the perturbed orbit

→ Broaden the resonance function

→ Example: Shalchi+ (2004) Weakly Non Linear Theory or SO-QLT

$$\delta(k_\parallel v\mu - n\Omega) \rightarrow \frac{K_\perp k_\perp^2 + \omega}{(K_\perp k_\perp^2 + \omega)^2 + (k_\parallel v\mu - n\Omega)^2}$$

Needs a closure relation / iterative solver

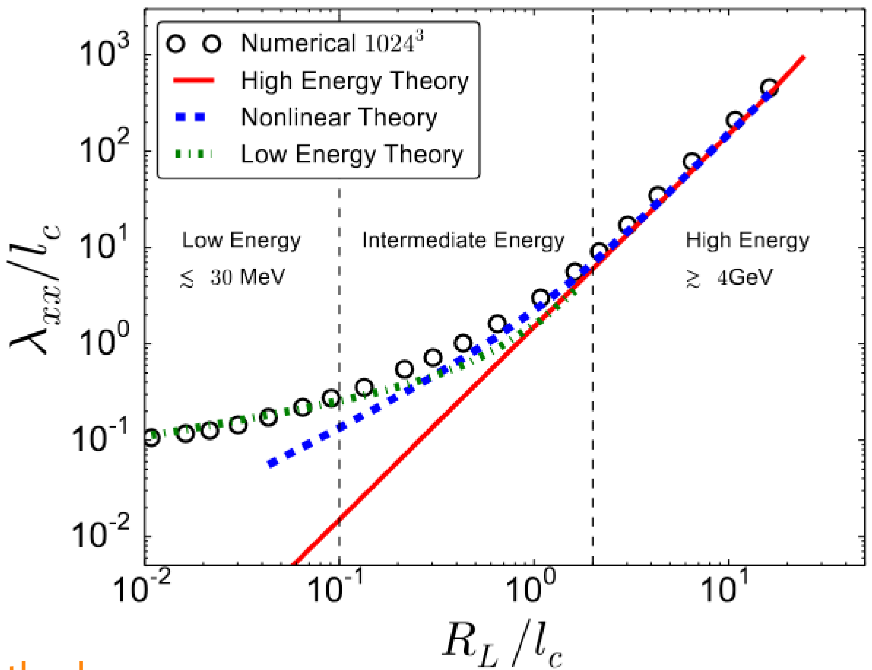
How well do we know CR scattering in a static turbulent B field?

Test-particle simulations in **synthetic turbulence**

Probing isotropic transport → $K_{//}(B_0 \to 0)$

3D isotropic turbulence : $S^{\rm iso}_{\ell m}(\mathbf{k}) = \frac{S^{\rm iso}(k)}{2}\left(\delta_{\ell m} - \frac{k_\ell k_m}{k^2}\right)$

Subedi+ (2017)



Grid method

→ Good agreement for small R_L/l_c values

Going beyond QLT:

(see Tautz+ 2006)

→ 90° problem

→ Example: Subedi+ (2017)

Low Energy theory

Non Linear theory

→ Difficulties to predict $K_\perp$

How well do we know CR scattering in a static turbulent B field?

Test-particle simulations in **synthetic turbulence**

The physics of perpendicular transport

$$\boldsymbol{K} = \begin{pmatrix} K_\perp & 0 & 0 \\ 0 & K_\perp & 0 \\ 0 & 0 & K_\parallel \end{pmatrix}$$

- Parallel transport to the bckg fields B_0 driven by pitch angle scattering

- Perpendicular transport driven by
  - field line random walk
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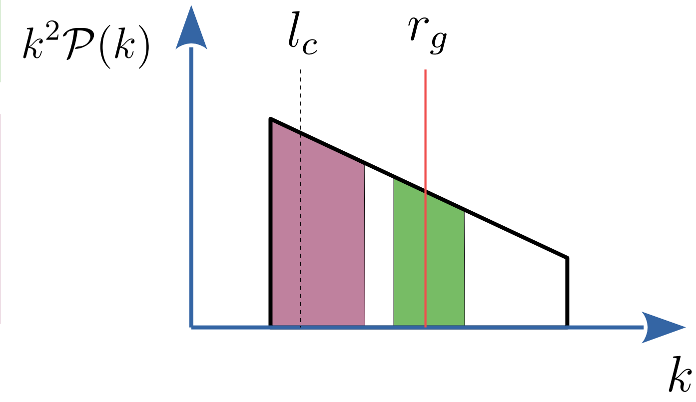
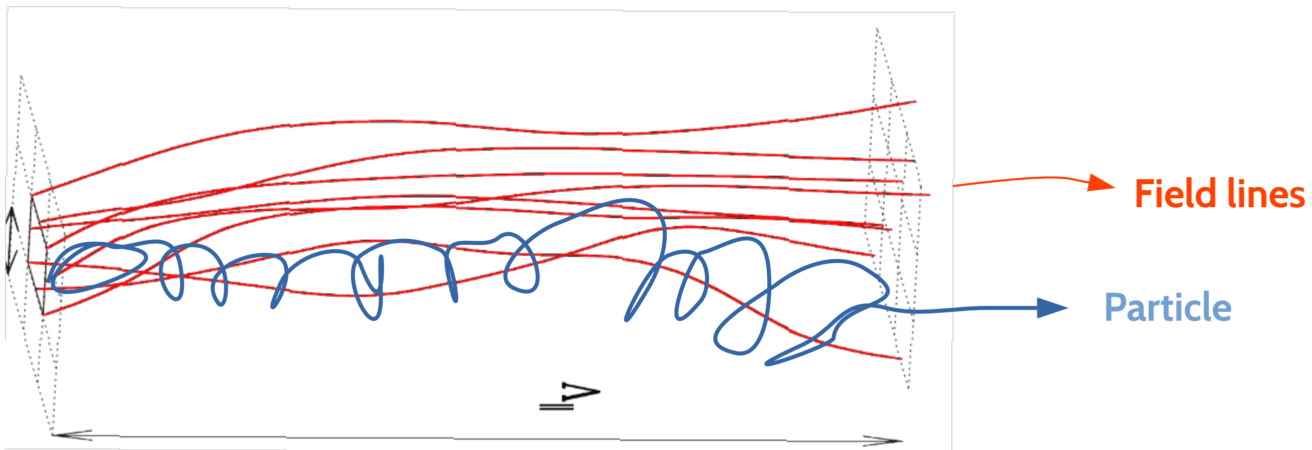
  - transverse complexity





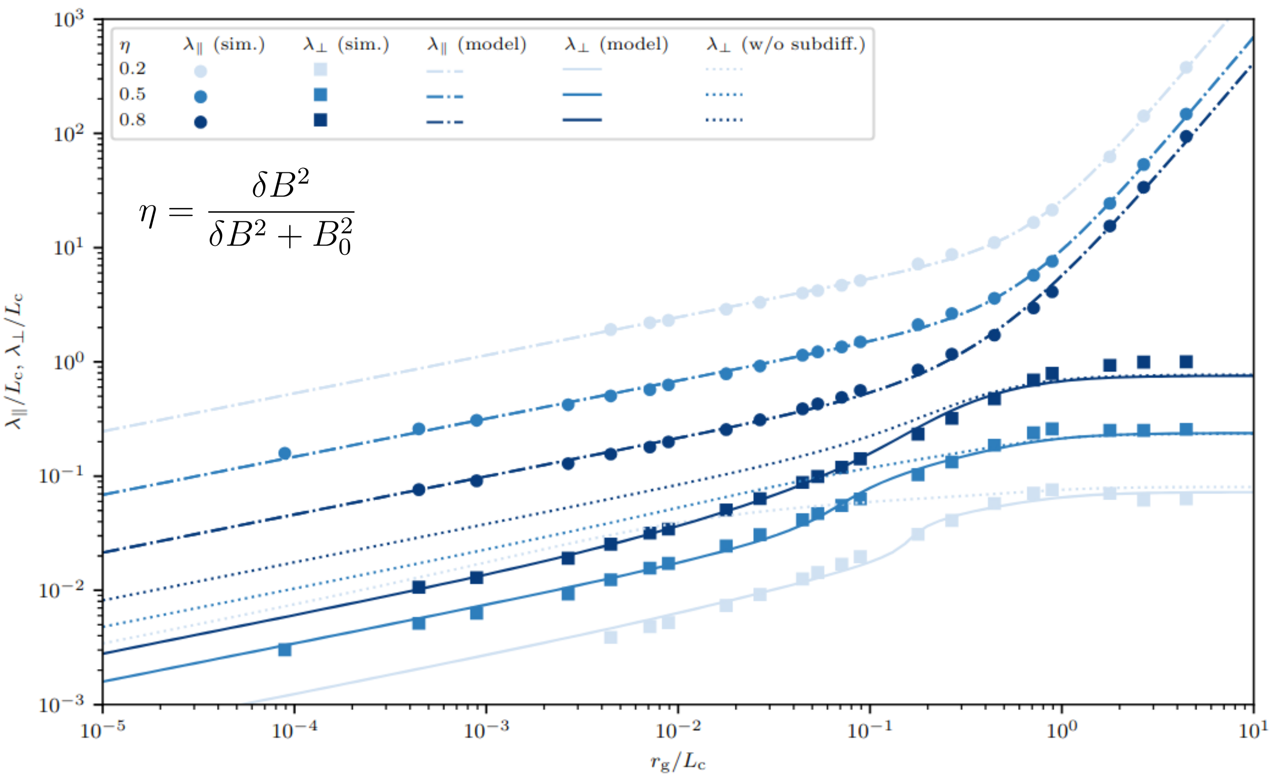
Lithwick & Goldreich (2001)

How well do we know CR scattering in a static turbulent B field?

Test-particle simulations in **synthetic turbulence**

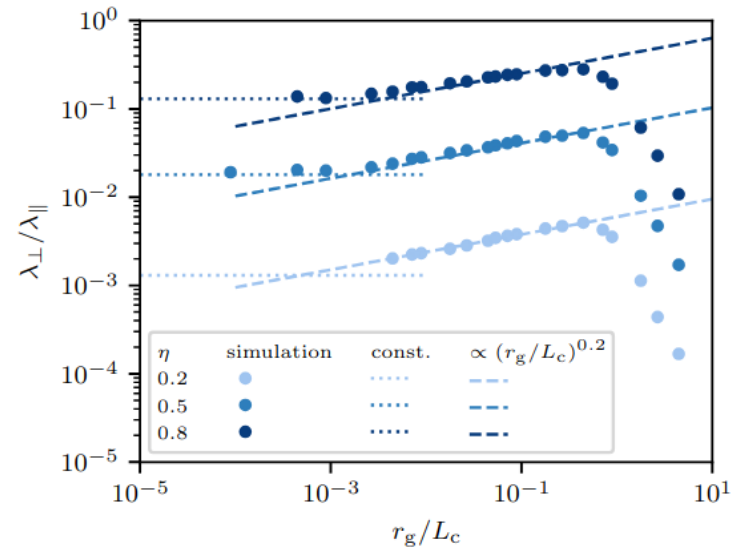
Probing parallel & perpendicular transport → $K_{//}$, $K_{\perp}$

Kuhlen+ (2023) [See also DeMarco et al. (2007), Snodin et al. (2016), Giacinti et al. (2018)]



Nested-grid method

Kuhlen+ (2023)



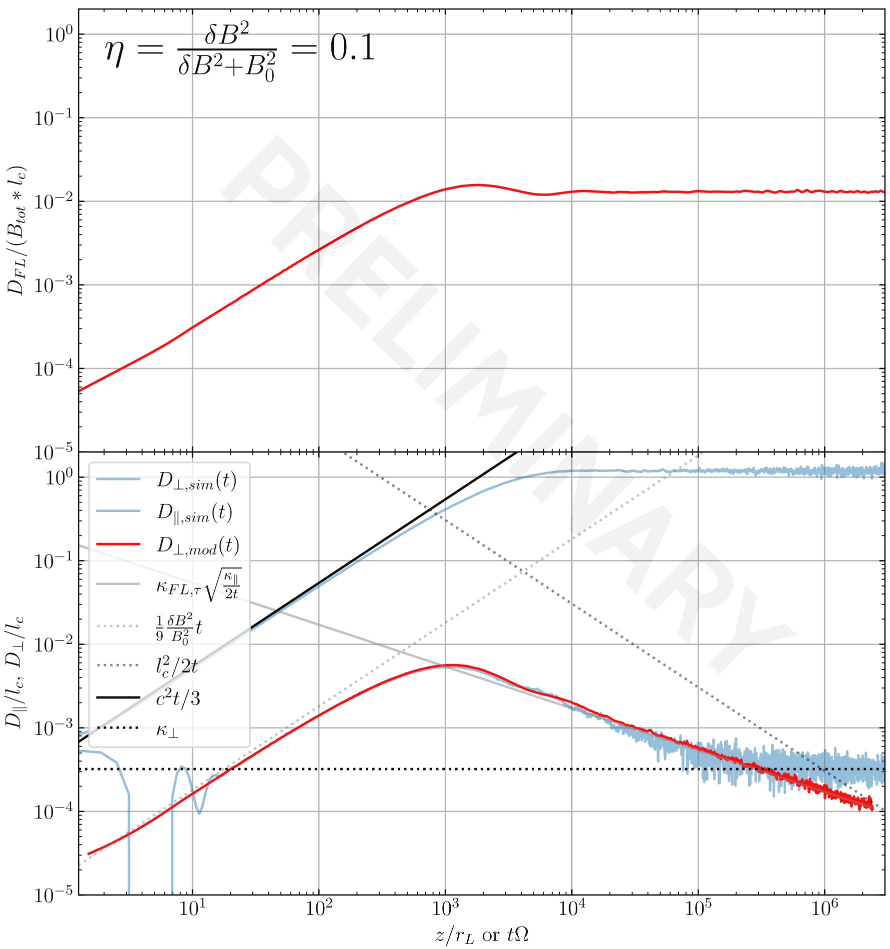
$K_{//}$ & $K_{\perp}$ scale differently at medium rigidites but they scale the same at low rigidities

How well do we know CR scattering in a static turbulent B field?

Test-particle simulations in **synthetic turbulence**

Probing parallel & perpendicular transport → $K_{//}$, $K_{\perp}$

Bouchet+ in prep.!



→ Running diffusion coefficient of FL and particles

→ FL: Ballistic then diffusive
→ Particles // : Ballistic then diffusive
→ Particles ⊥ : Ballistic then subdiffusive then diffusive

→ Following the heuristic description by Shalchi (2019)

$$D_\perp(t) = \frac{D_{FL}(\sqrt{\langle z^2(t)\rangle})}{\sqrt{\langle z^2(t)\rangle}} D_\parallel(t)$$

→ Subdiffusive regime explained by:

$$D_\perp(t) = \frac{K_{FL}}{\sqrt{\langle z^2(t)\rangle}} K_\parallel \propto t^{-1/2}$$

→ Particles have moved a distance ~ l_c in the ⊥ direction at :

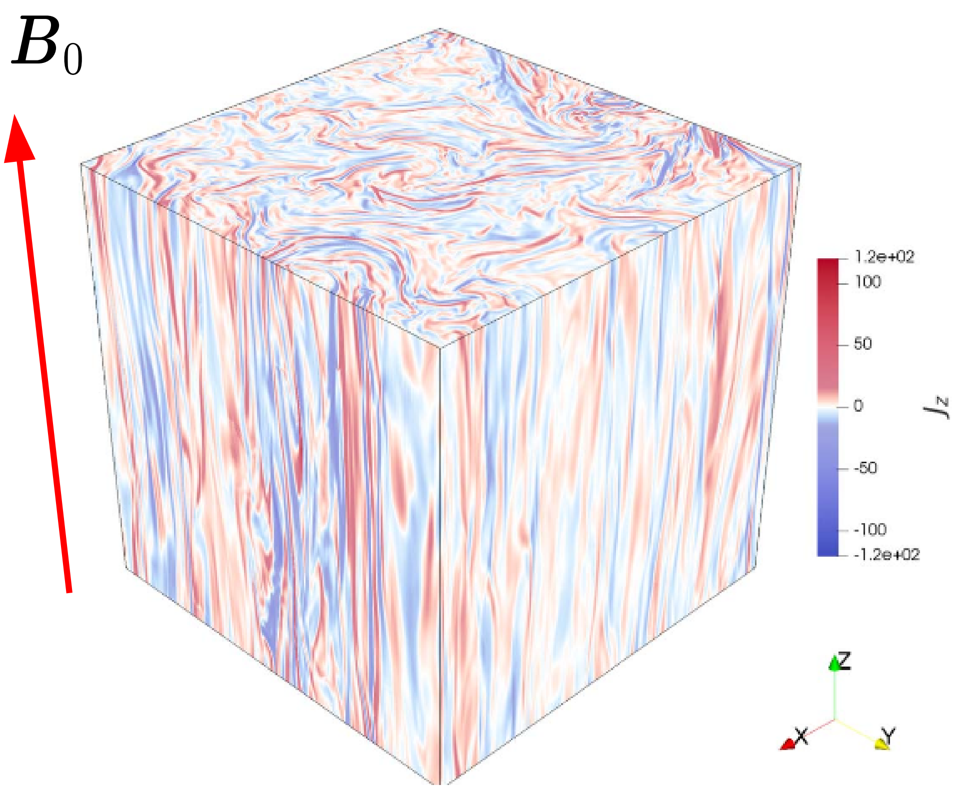
$$D_\perp \approx \frac{l_c^2}{2\tau_c}$$

("transverse complexity") → Under scrutiny now!
→ Toward a refined theory

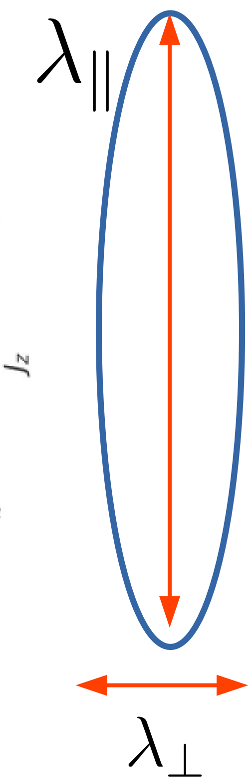
How well do we know CR scattering in a static turbulent B field?

Test-particle simulations in **MHD turbulence**

Which waves are driving the CR scattering in MHD ?



Pugliese & Dmitruk (2022)



→ MHD turbulence is anisotropic: critical balance

$$\tau_{\perp} \sim \frac{\lambda_{\perp}}{\delta v(\lambda_{\perp})} \sim \frac{\lambda_{\|}}{v_A} \Rightarrow \frac{\lambda_{\|}}{\lambda_{\perp}} \sim \frac{v_A}{\delta v(\lambda_{\perp})} \gg 1$$

→ Suppress turbulent power in the // direction

Inefficient pitch angle scattering!

Chandran (2000)

→ Cascade of fast modes (magnetosonic) is isotropic & weak and could provide an alternative

Yan&Lazarian (2004) / Zakharov&Sagdev (1970)

→ Beyond ideal MHD : fast modes significantly damped below some scale.

→ Maybe not an issue? CRSI instability can source the turbulence below this scale?

→ Maybe we need another scattering mech.?

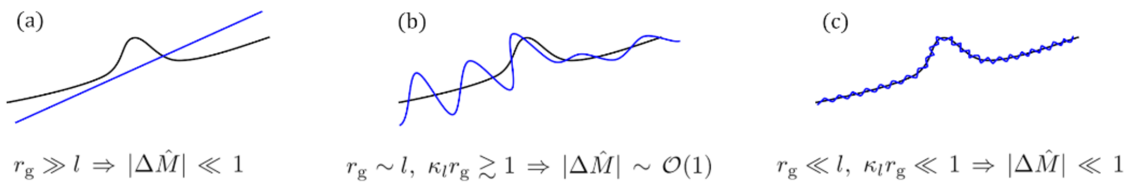
How well do we know CR scattering in a static turbulent B field?

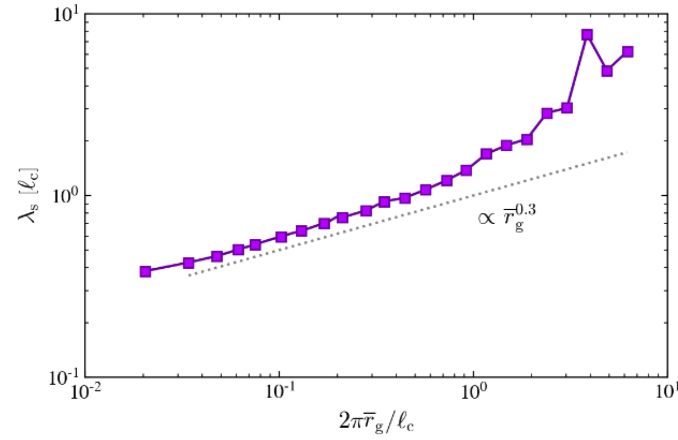
Test-particle simulations in **MHD turbulence**

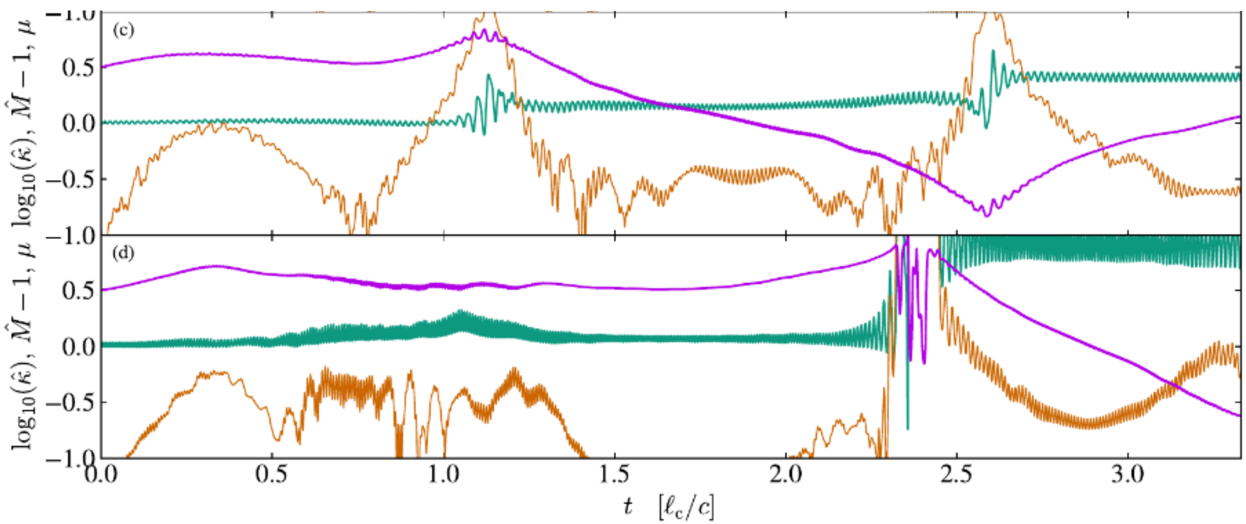
Transport driven by structures in the turbulence Lemoine (2023)

→ MHD turbulence contains magnetic mirrors but also magnetic bends: $\boldsymbol{B} = m\boldsymbol{B} + \kappa B\boldsymbol{n}$

→ Particle magnetic moments invariant except when interacting with a sharp magnetic field bend of size $r_L \kappa \sim 1$





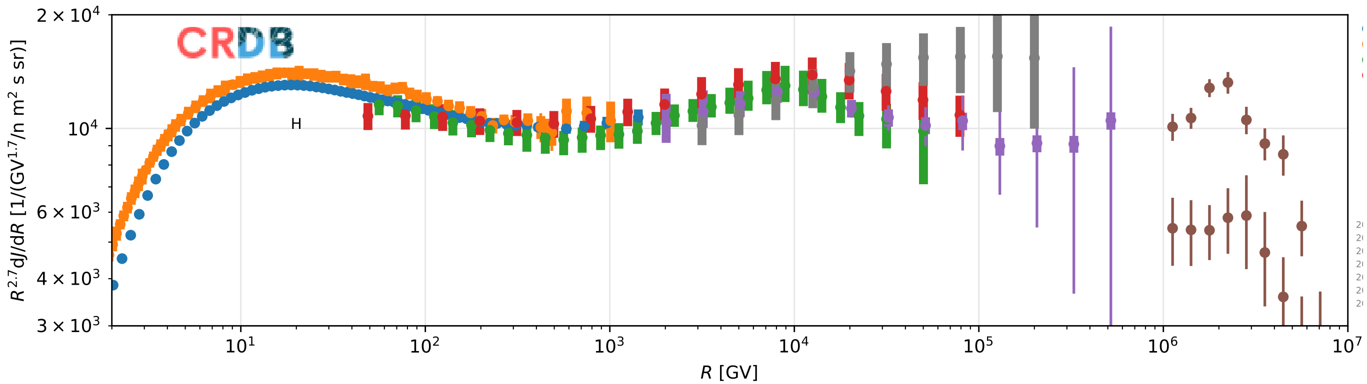


→ Particle scattering controlled by interactions with the bending distribution

→ Same arguments presented in Kempski+ (2023)

Magnetic moment

FL curvature



→ Context : cosmic-ray data (direct and indirect) with high precision
Phenomenology → Microphysics

→ Understanding/quantifing CR scattering in a (simple) synthetic turbulence still ongoing

→ Reality ~ MHD turbulence. What is the origin of CR scattering?

→ CR feedback?

Requires more theory more simulations .. more people power !

Thank you !

New measurements are required!

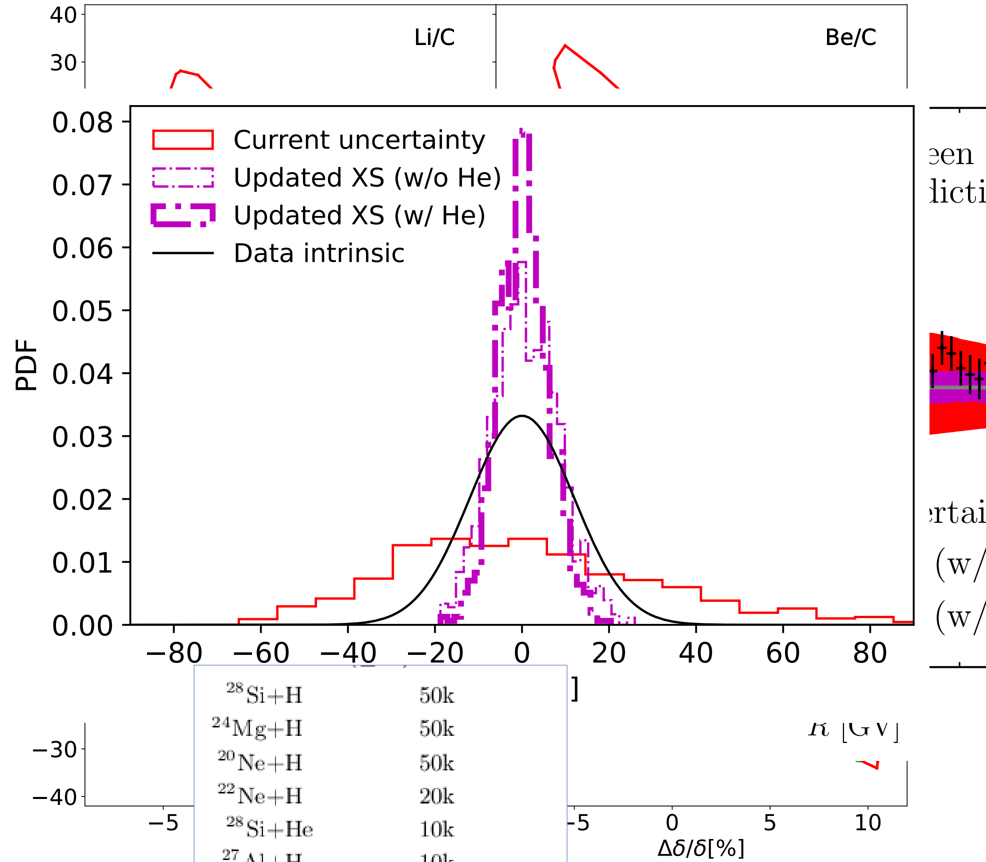
Dificulty : more than
1000 reactions are involved ..

reaction	N_{int}
^{16}O+H	60k
^{12}C+H	50k
^{16}O+He	20k
^{11}B+H	10k
^{15}N+H	10k
^{14}N+H	10k
^{12}C+He	10k
^{10}B+H	5k
^{13}C+H	5k
^{7}Li+H	5k
	$N(\leq \mathrm{O}) = 1.9 \times 10^5$

→ Selection rules Y.G. + (2018)

→ Proposition of new measurements
beam + target experiment (e.g. : NA61)
Y.G., Maurin, Moskalenko, Unger (2023)

→ Quantifying the **improvments**



^{28}Si+H	50k
^{24}Mg+H	50k
^{20}Ne+H	50k
^{22}Ne+H	20k
^{28}Si+He	10k
^{27}Al+H	10k
^{26}Mg+H	10k
^{24}Mg+He	10k
^{23}Na+H	10k
^{25}Mg+H	10k
^{21}Ne+H	10k
^{20}Ne+He	10k
^{32}S+H	5k
^{29}Si+H	5k
^{22}Ne+He	5k
	$N(\leq \mathrm{Si}) = 3.8 \times 10^5$
^{56}Fe+H	30k
^{56}Fe+He	10k
	$N(\leq \mathrm{Fe}) = 4.2 \times 10^5$

kalenko, Unger (2023)

→ **Pre... transport parameters**

→ **Cha...**

→ **Scr... ondaries**