MHD turbulence: a short review

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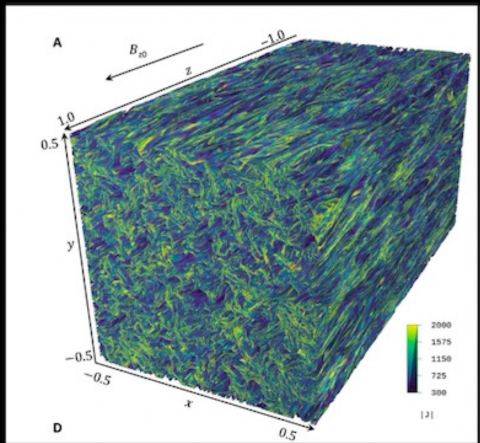
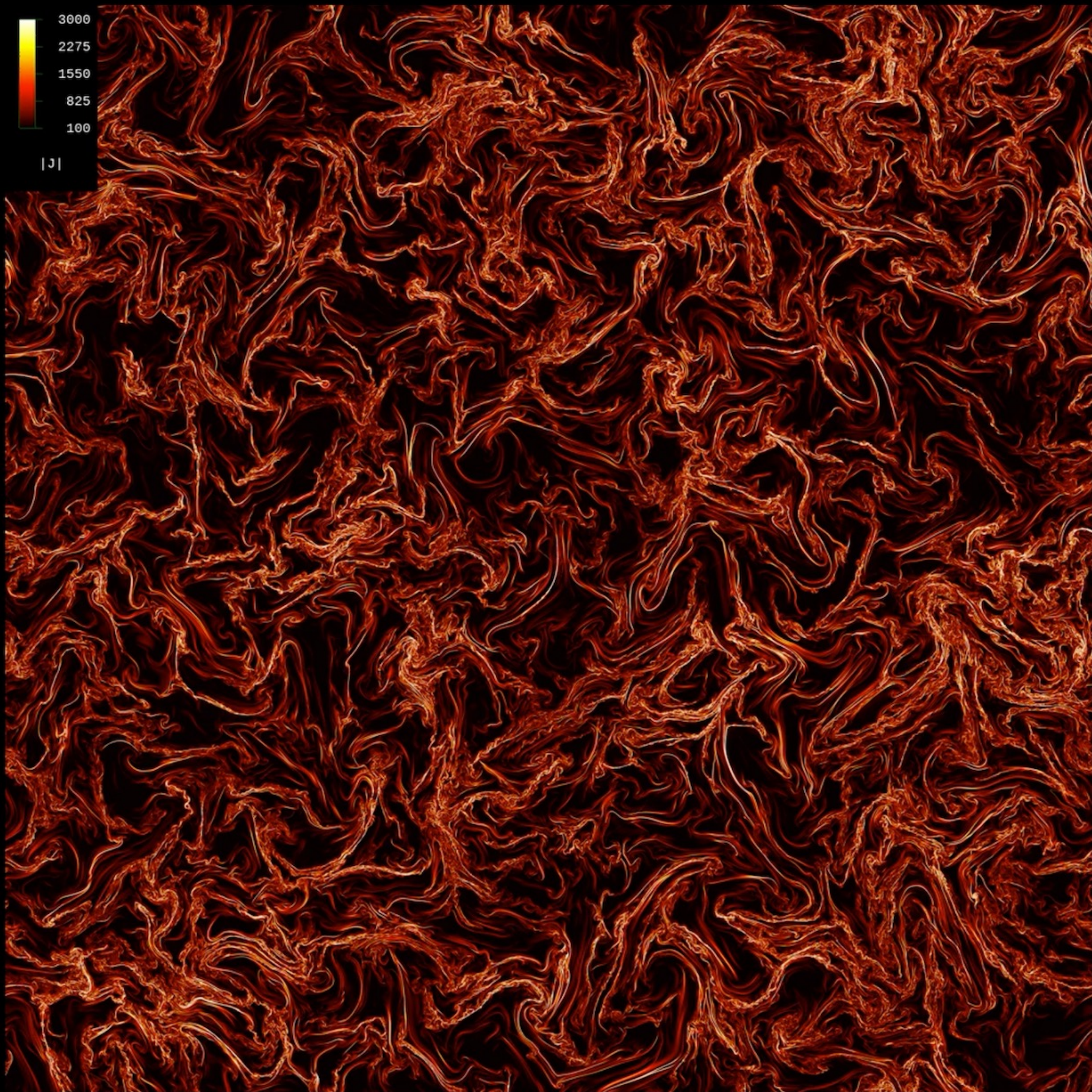
10^4 x 10^4 x 5000 points
compressible MHD
200 10^6 CPU hours



[Dong+, Sci. Adv., 2022]



Magnetohydrodynamics (MHD)

- Generalized (non-relativistic) MHD equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0\,, \qquad \mathrm{P} = \mathrm{K}n^{\gamma}$$

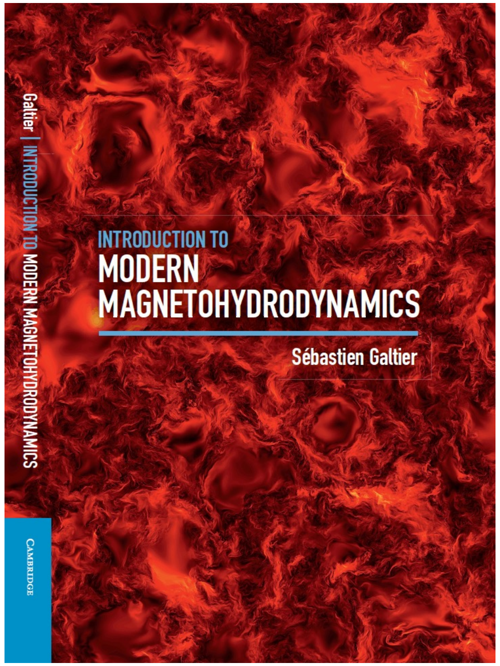
$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla P + \frac{1}{\mu_0}(\nabla\times\mathbf{B})\times\mathbf{B} + \tilde{\nu}\Delta\mathbf{u} + \frac{\tilde{\nu}}{3}\nabla(\nabla\cdot\mathbf{u})\,,$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{u}\times\mathbf{B}) - \nabla\times\left(\frac{(\nabla\times\mathbf{B})\times\mathbf{B}}{\mu_0 n e}\right) + \eta\Delta\mathbf{B}\,,$$

$$\nabla\cdot\mathbf{B} = 0\,.$$

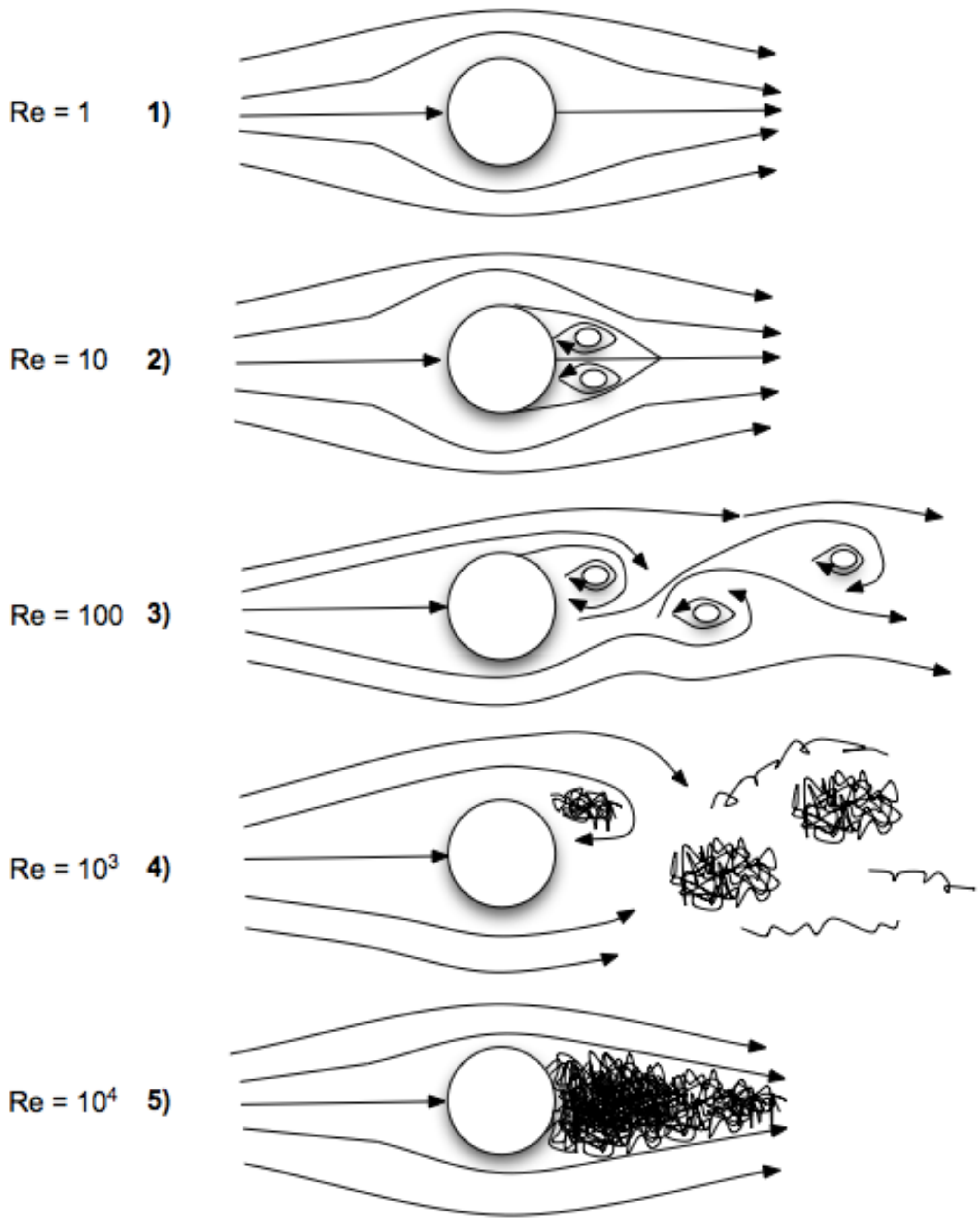
$\tilde{\nu}$ is the dynamic viscosity

η is the **magnetic diffusivity**



What is turbulence ?

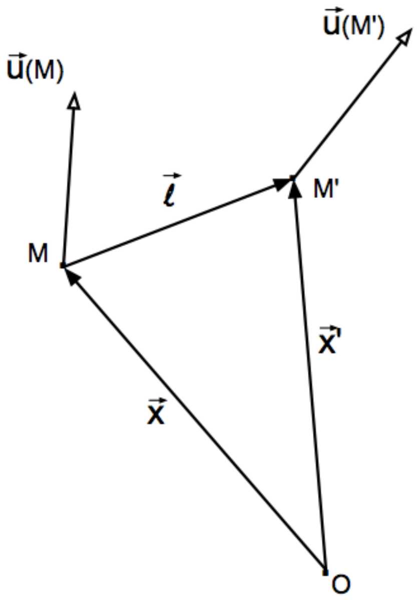
$$R_e = \frac{UL}{\nu}$$

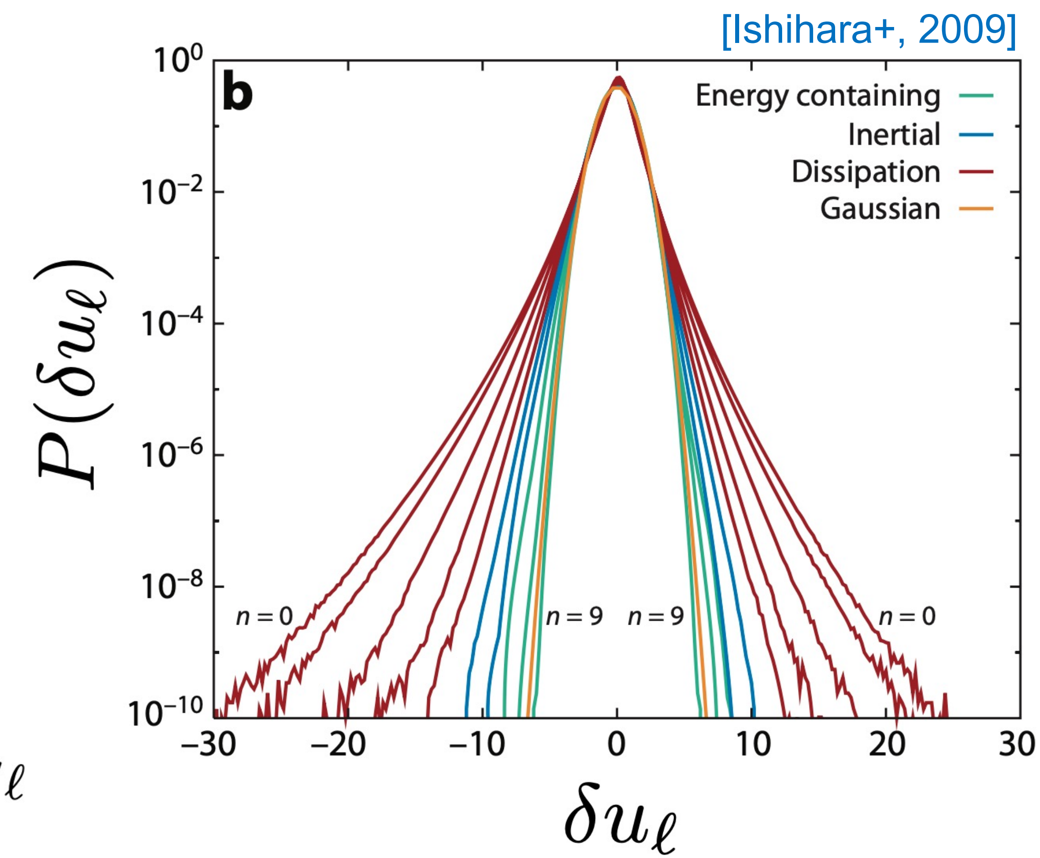


Fully developed turbulence

What is turbulence ?

Distribution of increments are not Gaussian and asymmetric





Increments:

$$\delta u_\ell = (\mathbf{u}(\mathbf{M}') - \mathbf{u}(\mathbf{M})) \cdot \boldsymbol{\ell}/\ell$$

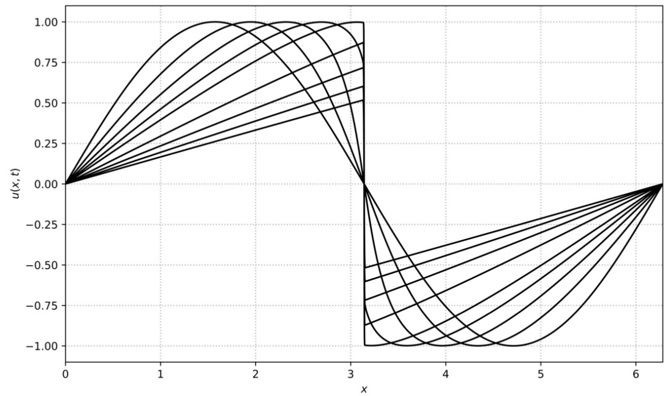
Navier-Stokes equations (DNS); 4096^3

The Burgers equation

$$\boxed{\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \nu \frac{\partial^2 u}{\partial x^2}}$$

$\mathrm{Re} \approx 10^4$

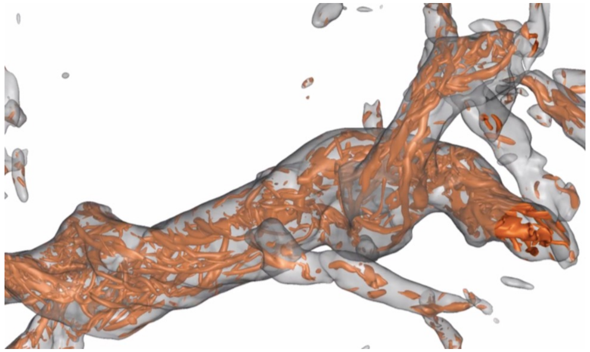
$$u(x,t) = \frac{1}{t}\left[x - L\tanh\left(\frac{xL}{2\nu t}\right)\right] \quad \xrightarrow[\nu \to 0^+]{x>0} \frac{1}{t}(x-L), \qquad \xrightarrow[\nu \to 0^+]{x<0} \frac{1}{t}(x+L).$$



Mean rate of energy dissipation $\varepsilon_\nu = -\nu \left\langle u \frac{\partial^2 u}{\partial x^2} \right\rangle \xrightarrow[\nu \to 0]{} \frac{L^2}{3t^3}$ non-zero value !

Anomalous dissipation / *spontaneous symmetry breaking*

Anomalous dissipation



From Navier-Stokes equations $(\rho_0 = 1)$

$$\frac{dE}{dt} = \frac{d\langle \mathbf{u}^2/2\rangle}{dt} = -\nu\langle \mathbf{w}^2\rangle$$

with $\mathbf{w} \equiv \nabla \times \mathbf{u}$ the **vorticity**

Zeroth law of turbulence:

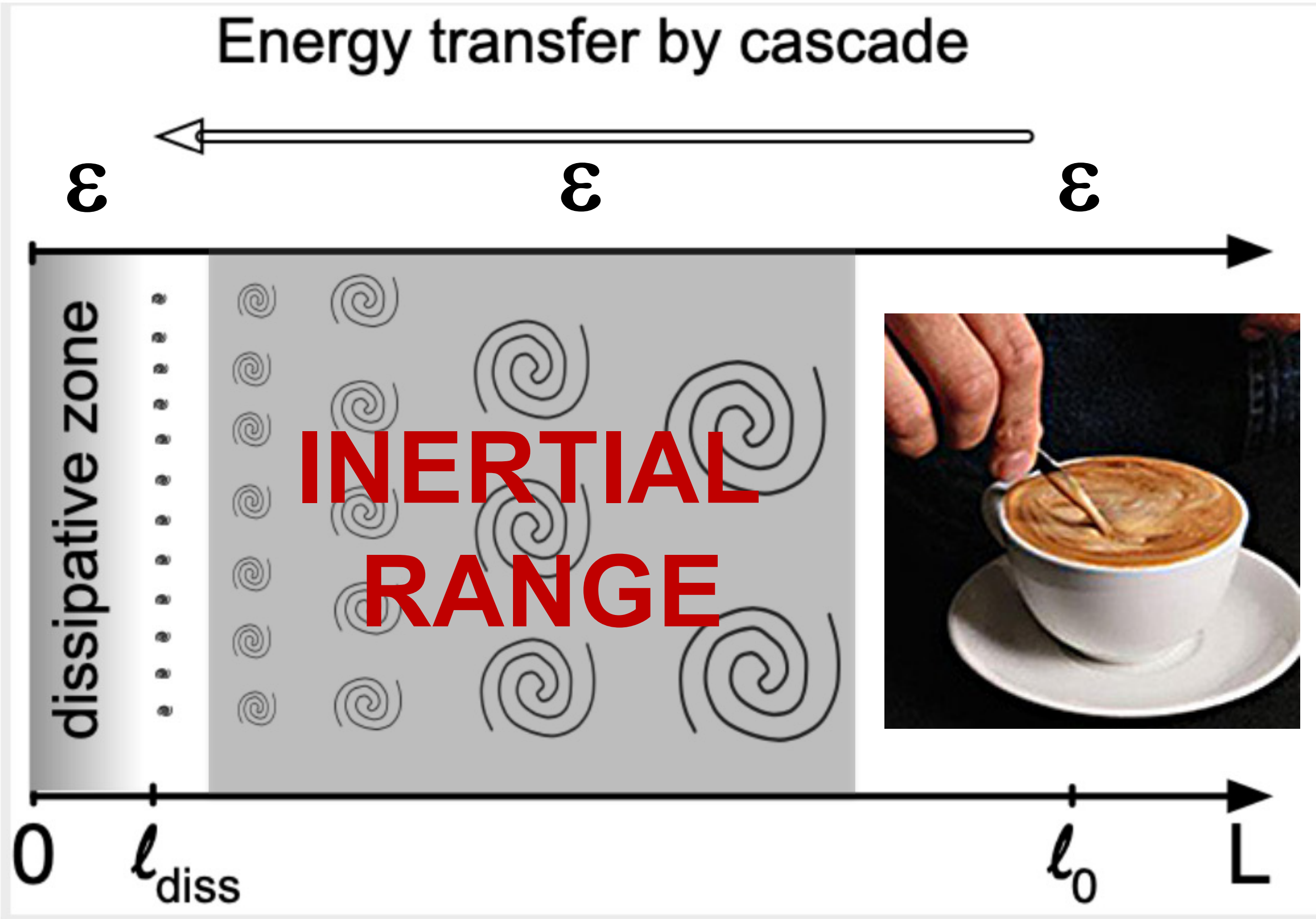
The mean rate of energy dissipation (per unit mass) ε reaches a finite limit (when $\nu \to 0$) independent of the viscosity ν

$$\lim_{\substack{\nu\to 0\\ \eta}} \frac{dE}{dt} = -\varepsilon \neq \frac{dE}{dt}\Big|_{\substack{\nu=0\\ \eta}} ,$$

HD/Exp. [Ravelet+, JFM, 2008; Saint-Michel+, PoF, 2014]

MHD/DNS [Mininni & Pouquet, PRE, 2009; Bandyopadhyay+, PRX, 2018]

Stationary turbulence

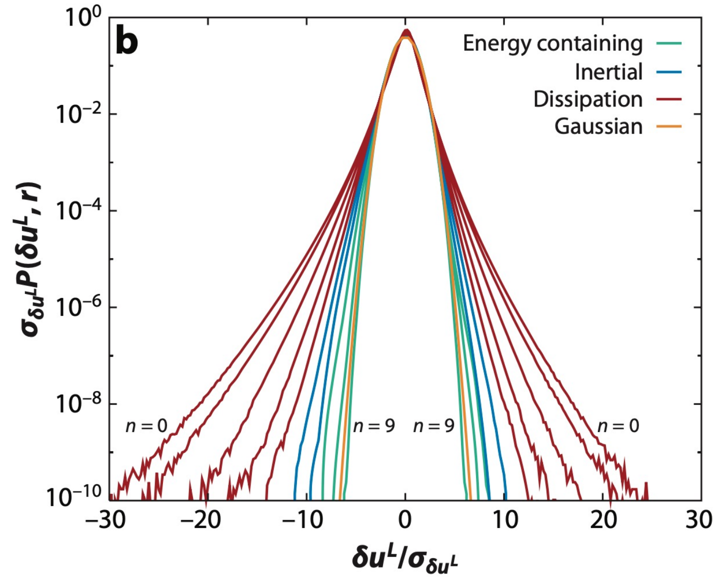


Turbulent cascade: smaller vortices are generated by a scale-by-scale energy transfer. The range of scales ℓ between the energy injection ℓ_0 and dissipation ℓ_{diss} (in practice $\ell_{diss} \ll \ell \ll \ell_0$) forms the inertial zone.

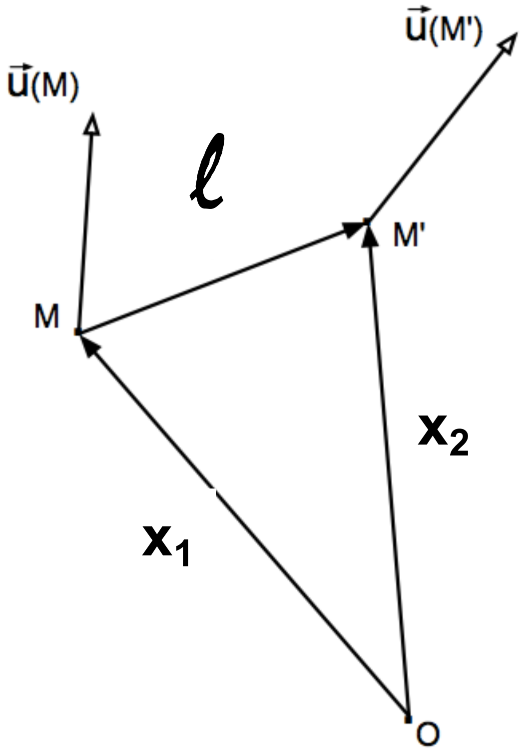
Strong turbulence

beyond phenomenology

Exact law



Increments: $\delta u_\ell = (\mathbf{u}(\mathbf{M}') - \mathbf{u}(\mathbf{M})) \cdot \boldsymbol{\ell}/\ell$



3D incompressible HD

Homogeneous & isotropic turbulence
R_e >>1; stationary; direct cascade

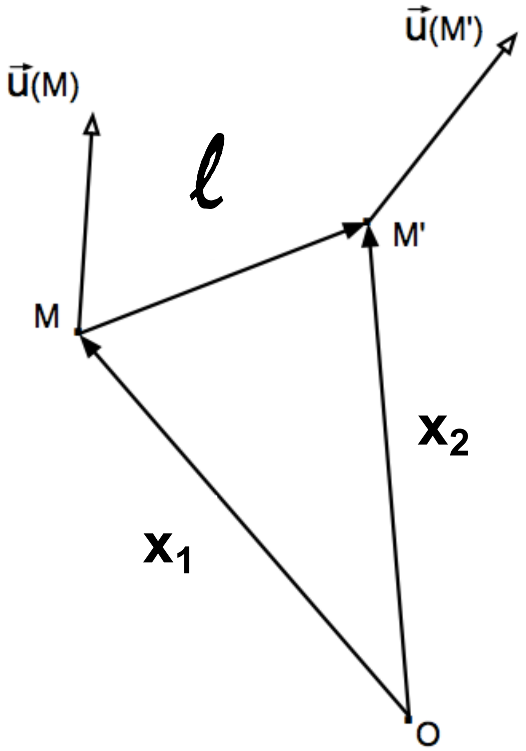
$$\boxed{-\frac{4}{5}\varepsilon\ell = \langle(\delta u_\ell)^3\rangle\,,}$$

Kolmogorov's law
[Kolmogorov, 1941]

Valid in the inertial range (universality)

Exact law

Increments: $\delta u_\ell = (\mathbf{u}(\mathbf{M}') - \mathbf{u}(\mathbf{M})) \cdot \boldsymbol{\ell}/\ell$
$\delta b_\ell = (\mathbf{b}(\mathbf{M}') - \mathbf{b}(\mathbf{M})) \cdot \boldsymbol{\ell}/\ell$



3D incompressible MHD

Homogeneous & isotropic turbulence
R_e, R_{em} >>1; stationary; direct cascade

[Politano & Pouquet, PRE, 1998]

$$\boxed{-\frac{4}{3}\varepsilon^T \ell = \langle (\delta \mathbf{u} \cdot \delta \mathbf{u} + \delta \mathbf{b} \cdot \delta \mathbf{b}) \delta u_\ell \rangle - 2 \langle (\delta \mathbf{u} \cdot \delta \mathbf{b}) \delta b_\ell \rangle \,,}$$

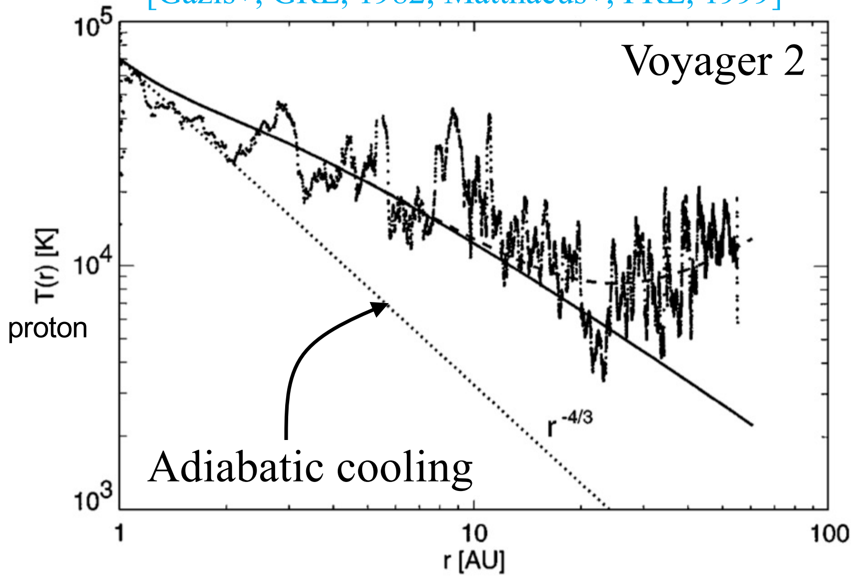
$$\mathbf{z}^\pm \equiv \mathbf{u} \pm \mathbf{b} \quad \boxed{-\frac{4}{3}\varepsilon^\pm \ell = \langle (\delta \mathbf{z}^\pm \cdot \delta \mathbf{z}^\pm) \delta z_\ell^\mp \rangle \,,}$$

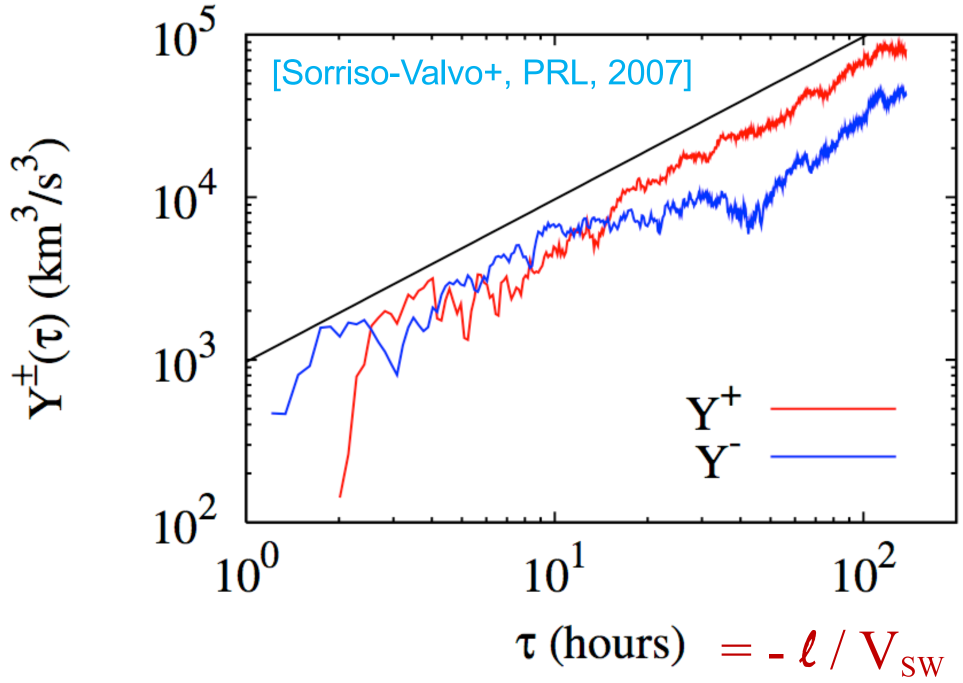
ε^T can be interpreted as a heating rate for the plasma

Application: solar wind heating

Turbulence is a **source** of **local** heating

[Gazis+, GRL, 1982; Matthaeus+, PRL, 1999]





$$\mathbf{z}^{\pm} \equiv \mathbf{u} \pm \mathbf{b}$$

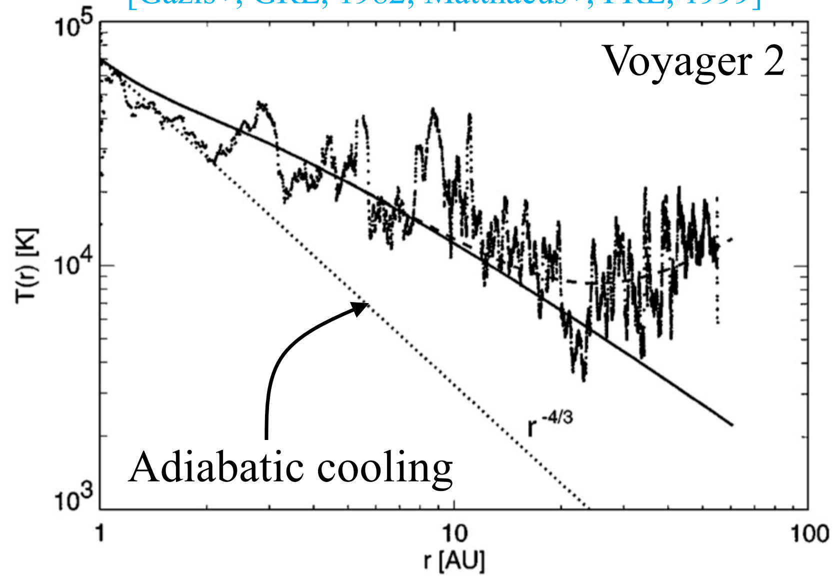
$$\boxed{-\frac{4}{3}\varepsilon^{\pm}\ell = \overbrace{\langle(\delta\mathbf{z}^{\pm}\cdot\delta\mathbf{z}^{\pm})\delta z_{\ell}^{\mp}\rangle}^{Y^{\pm}},} \Longrightarrow \varepsilon^{\pm} \sim 10^{2}\ \mathrm{J\,kg^{-1}\,s^{-1}}$$

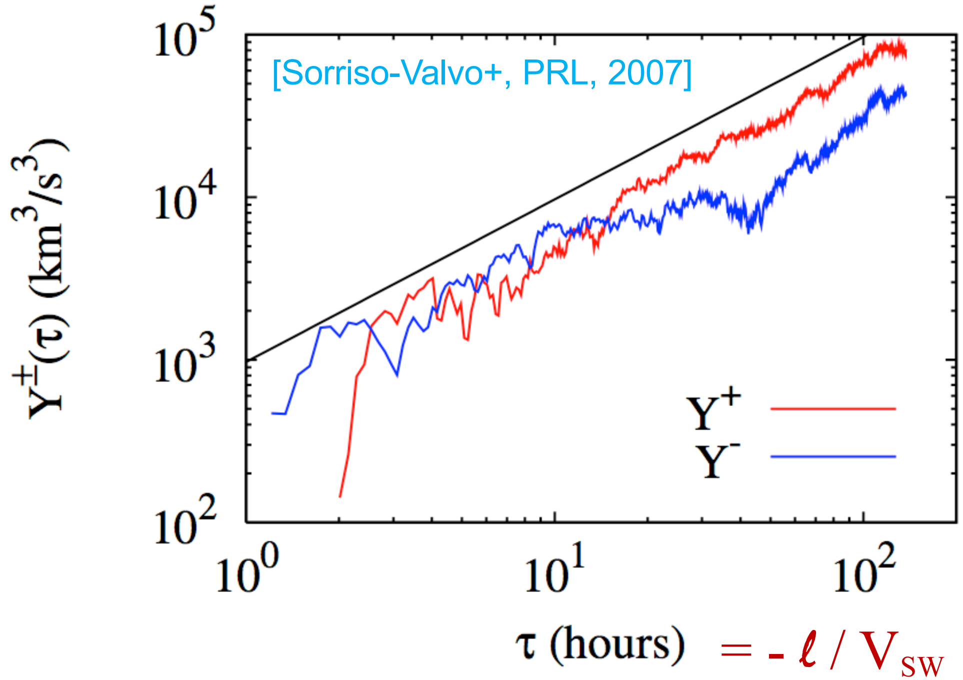
Extension to incompressible Hall MHD is possible [SG, PRE, 2007]

Application: solar wind heating

Turbulence is a **source** of **local** heating

[Gazis+, GRL, 1982; Matthaeus+, PRL, 1999]





$$\mathbf{z}^{\pm} \equiv \mathbf{u} \pm \mathbf{b}$$

$$\boxed{-\frac{4}{3}\varepsilon^{\pm}\ell = \overbrace{\langle(\delta\mathbf{z}^{\pm}\cdot\delta\mathbf{z}^{\pm})\delta z_{\ell}^{\mp}\rangle}^{Y^{\pm}},}$$

$$\Longrightarrow \quad \varepsilon^{\pm} \sim 10^2 \ \mathrm{J\,kg^{-1}\,s^{-1}}$$

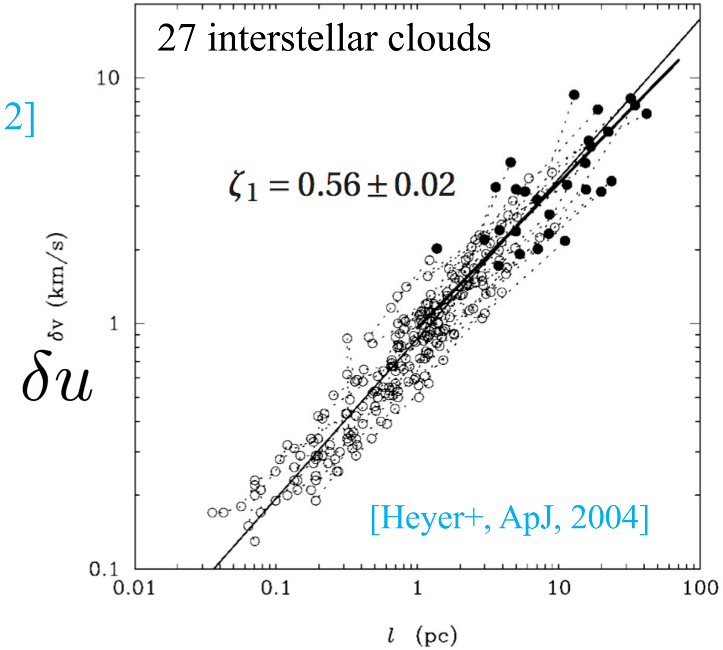
Compressible MHD [Banerjee & SG, PRE, 2013-2016; Andrés+, PRE, 2016-2018; Ferrand+, JPP, 2021]

Solar wind data [Banerjee+, ApJL, 2016; Hadid+, PRL, 2018; Andrés+, PRL, 2019; Simon+, PRE, 2022]

Application: interstellar medium

- ISM is highly turbulent [Hennebelle & Falgarone, AAR, 2012]
- ISM turbulence is supersonic with $\mathrm{Mach}_{\mathrm{turb}} > 10$
- Low star formation rate; why?

[Arzoumanian+, A&A, 2011; Federrath+, MNRAS, 2016]



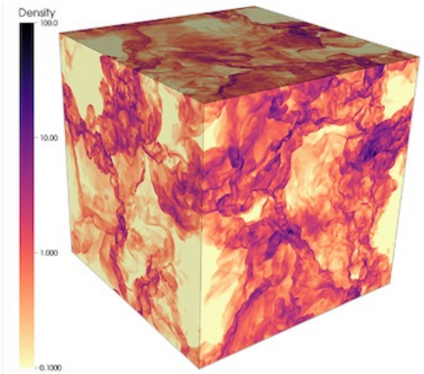
Supersonic MHD turbulence simulations are necessary
but isothermal HD is often used

[Kritsuk+, ApJ, 2007, JFM, 2013; Schmidt+, PRL, 2008; Federrath+, A&A, 2010, MNRAS, 2016]

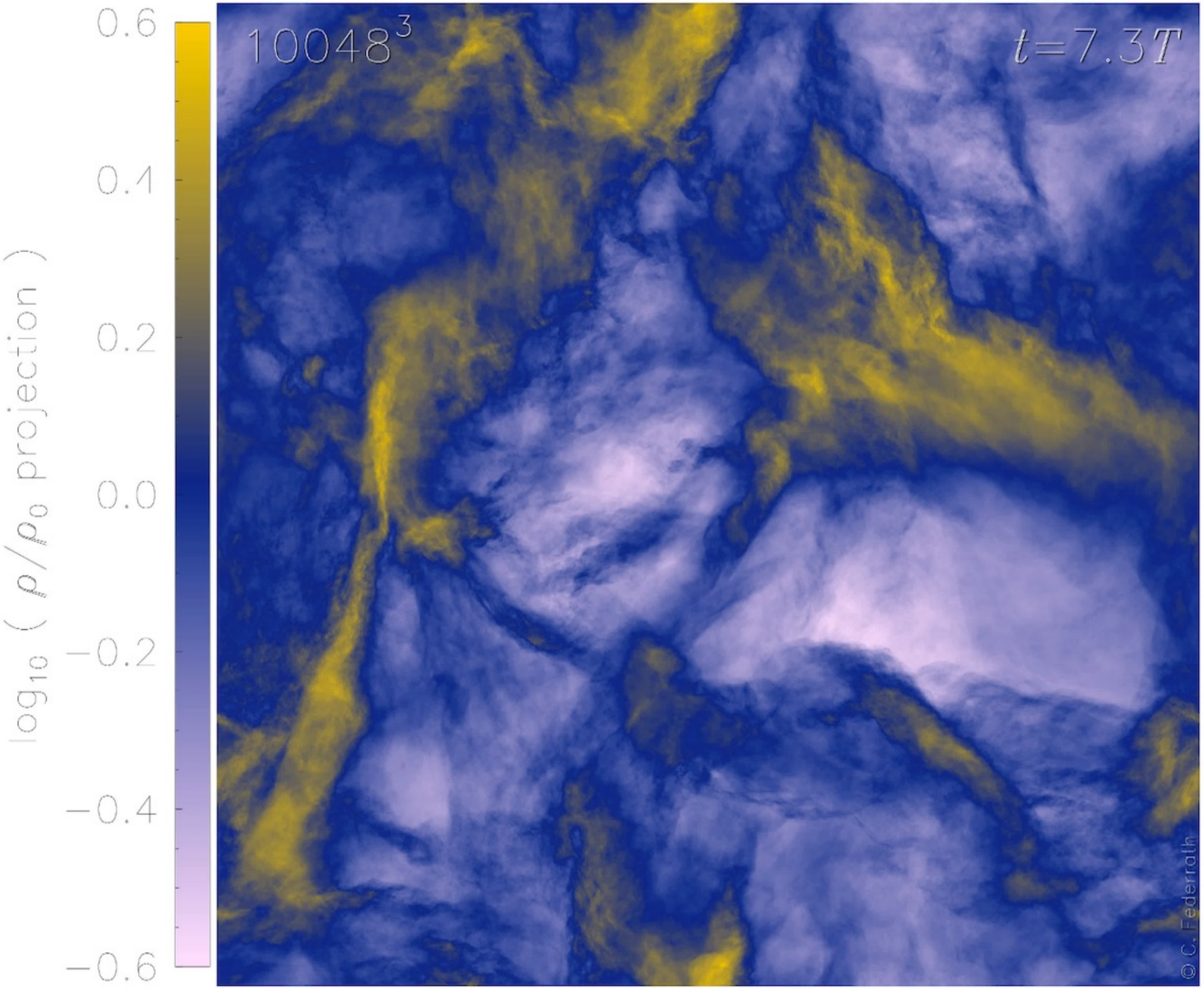
$$\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0,$$
$$\partial_t(\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u}\mathbf{u}) = -\nabla P + \mu \Delta \mathbf{u} + \frac{\mu}{3} \nabla(\nabla \cdot \mathbf{u}) + \mathbf{f},$$
$$P = C_s^2 \rho$$

[Federrath et al., 2020]

$\mathrm{Mach}_{\mathrm{turb}} = 6$



$N^3 = 10\,048^3$
52×10^6 h CPU time
65 536 processors



Exact law for isothermal HD turbulence

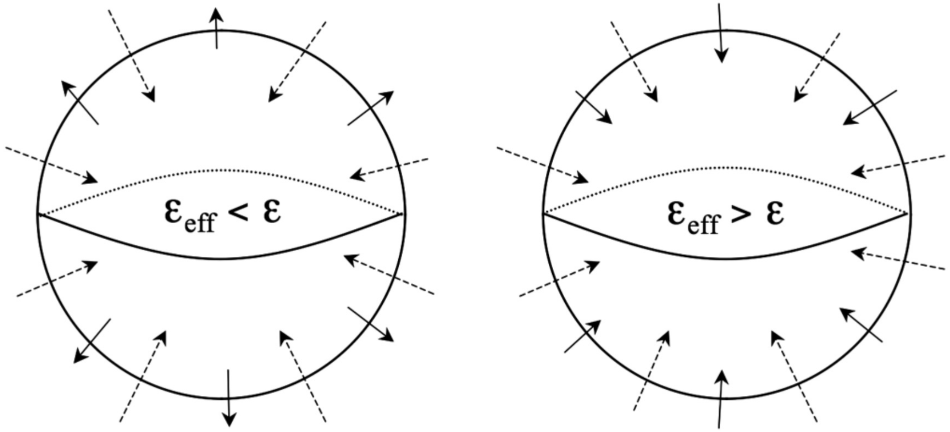
$$\boxed{-4\varepsilon = \nabla_\ell \cdot \underbrace{\langle \bar{\delta}\rho(\delta \boldsymbol{u})^2 \delta \boldsymbol{u} \rangle}_{\text{Flux}} \underbrace{- \frac{1}{2}\langle (\rho\theta' + \rho'\theta)(\delta \boldsymbol{u})^2 \rangle}_{\text{Source (S)}}}$$

$$\bar{\delta}\xi = (\xi + \xi')/2 \qquad \delta\xi = \xi' - \xi \qquad \theta \equiv \nabla \cdot \boldsymbol{u}$$

[SG & Banerjee, PRL, 2011]

Physical interpretation:

$$\varepsilon_{\rm eff} \equiv \varepsilon + S/4$$



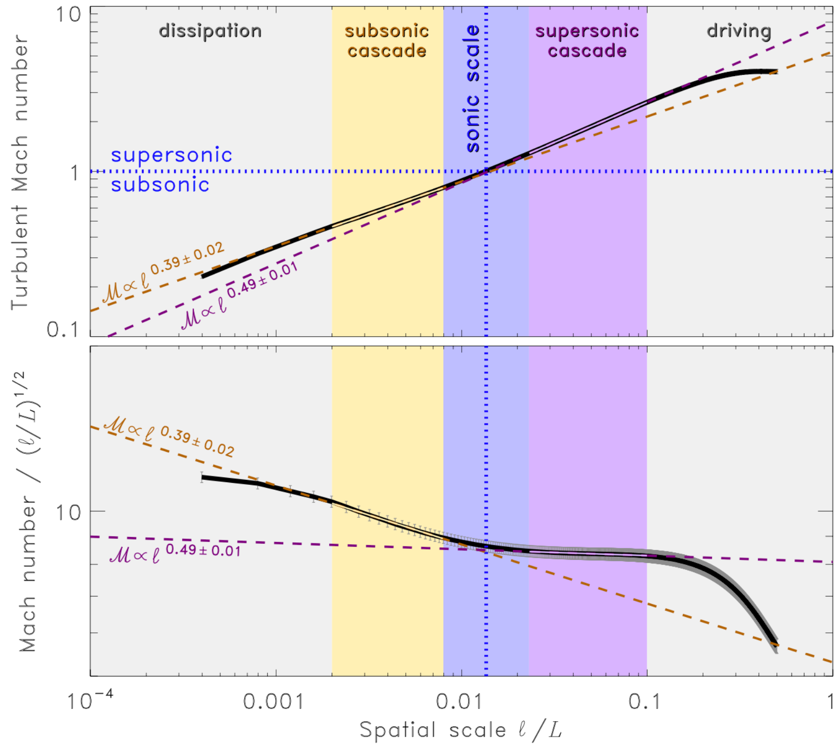
Compression/dilatation increases/decreases the effective mean rate of energy transfer ($\varepsilon_{\rm eff} > \varepsilon$; $\varepsilon_{\rm eff} < \varepsilon$)

$\rightarrow$ global (non-local) effect

Exact law for isothermal HD turbulence

[Federrath+, Nat. Astro., 2021]

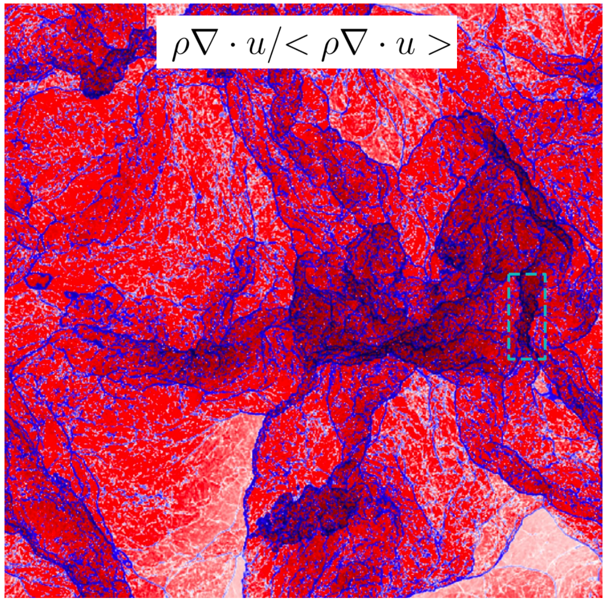
[see also Kritsuk+, JFM, 2013]



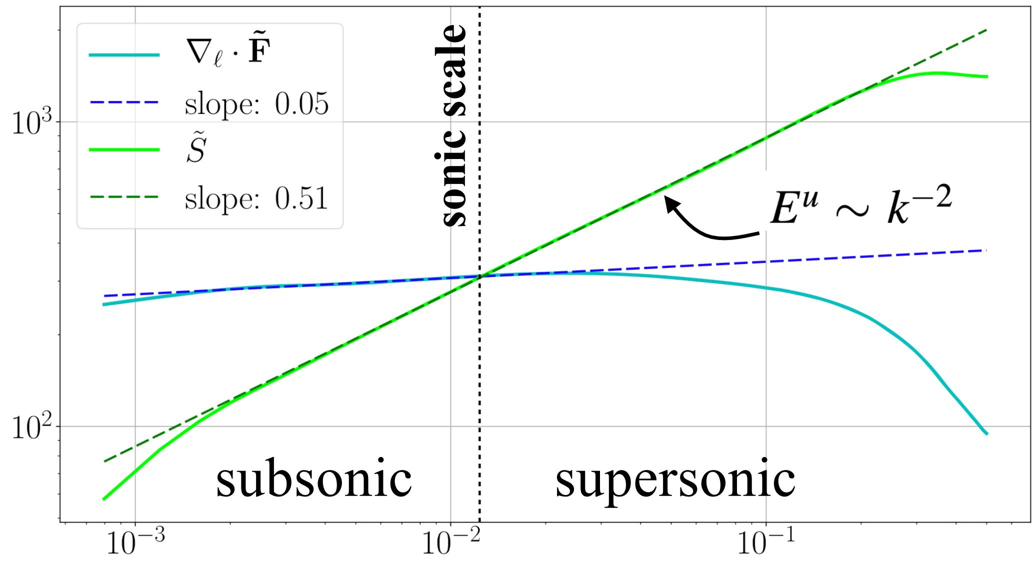
$$\boxed{-4\varepsilon = \nabla_{\boldsymbol{\ell}} \cdot \mathbf{F} + S}$$

$\mathrm{Mach_{turb}} = 4$

$10\,048^3$ points



[Ferrand+, ApJ, 2020]

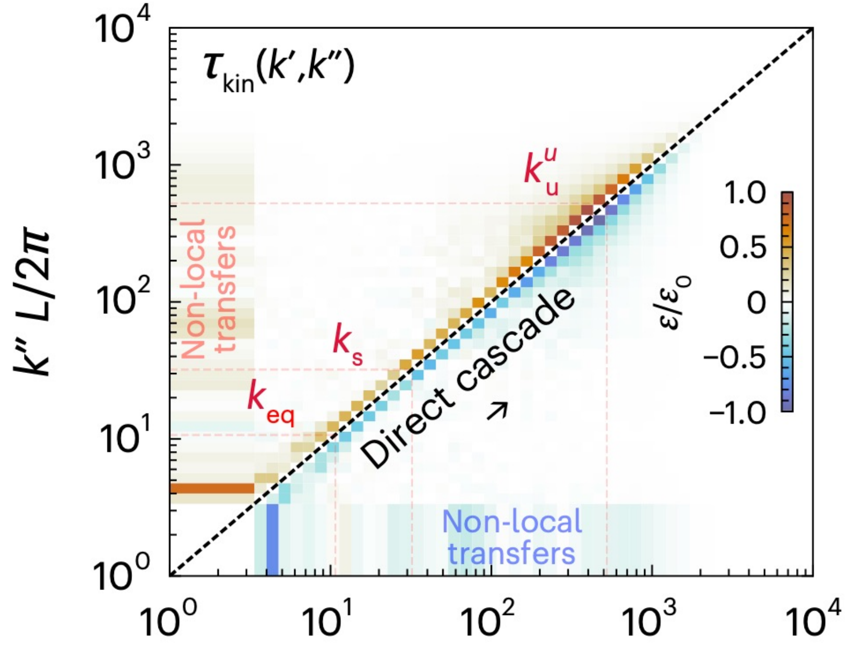


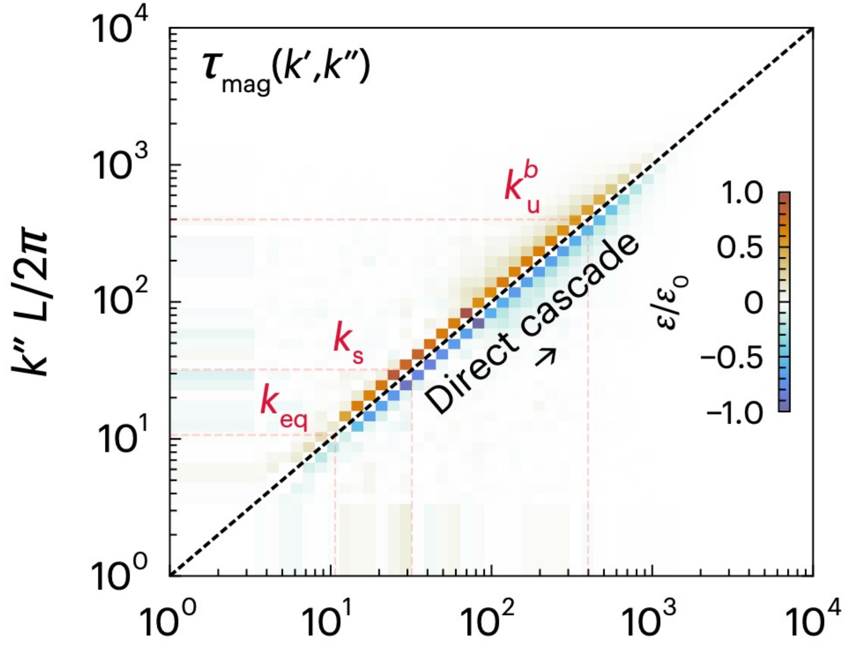
Exact law for isothermal MHD turbulence

$$\boxed{-4\varepsilon = \nabla_{\boldsymbol{\ell}} \cdot \mathbf{F} + S}$$

[Beattie+, Nat. Astro., 2025]

10 080^3 points





$\mathrm{Mach_{turb}} = 4$

Compressible ~~Hall~~ MHD

Isothermal closure

$$-4\bar{\varepsilon} = \nabla_{\boldsymbol{\ell}} \cdot \left\langle \frac{\bar{\delta}\rho}{\rho_0}|\delta \mathbf{u}|^2\delta\mathbf{u} + |\delta\mathbf{b}|^2\delta\mathbf{u} - 2(\delta\mathbf{u}\cdot\delta\mathbf{b})\delta\mathbf{b} \right\rangle$$

$$- \left\langle \frac{(\rho\theta' + \rho'\theta)}{2\rho_0}|\delta\mathbf{u}|^2 \right\rangle - 2\left\langle \frac{\delta\rho}{\rho_0}\,\delta\mathbf{u}\cdot\bar{\delta}\left(\mathbf{j_c}\times\mathbf{b}\right) \right\rangle$$

$$\cancel{+d_i \nabla_{\boldsymbol{\ell}} \cdot \langle 2(\delta\mathbf{b}\cdot\delta\mathbf{j_c})\delta\mathbf{b} - |\delta\mathbf{b}|^2\delta\mathbf{j_c}\rangle - 2d_i\langle\delta(\mathbf{j}\times\mathbf{b})\cdot\delta\mathbf{j_c}\rangle\,,}$$

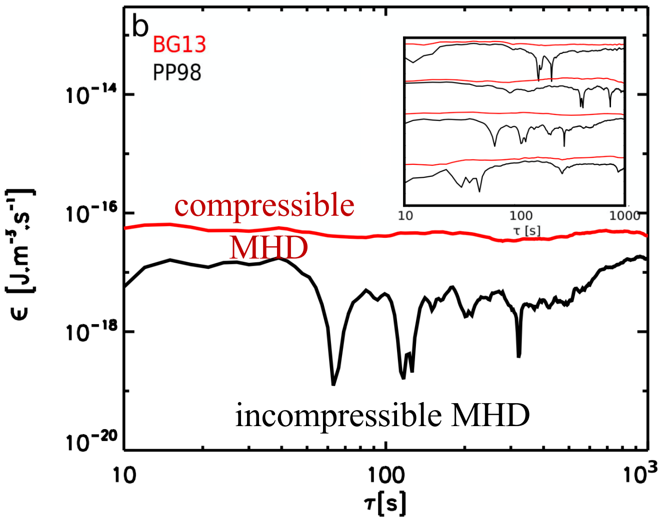
$$\bar{\varepsilon} \equiv \varepsilon/\rho_0$$
$$\bar{\delta}X \equiv (X + X')/2$$
$$\theta = \nabla\cdot\boldsymbol{u}$$
$$\rho_0 \equiv \langle\rho\rangle$$
$$(\rho/\rho_0)\mathbf{j_c} \equiv \mathbf{j} = \nabla\times\mathbf{b}$$
$$\mathbf{b} \equiv \mathbf{B}/\sqrt{\mu_0\rho_0}$$

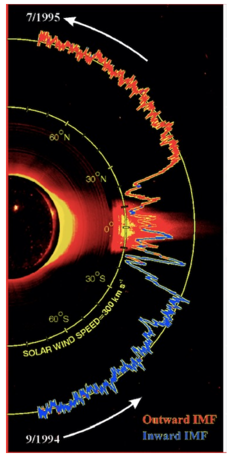
DNS shows that source terms are negligible at subsonic scales but dominant at supersonic scales

[Andrés+, JPP, 2018; Ferrand+, ApJ, 2020; SG, CUP, 2022]

Solar wind:

[Hadid+, ApJ, 2017]





Local exact law

- A weak formulation [Duchon & Robert, Nonlin, 2000; SG, JPAMT, 2018]
- Energy dissipation without viscosity/resistivity [Onsager, 1949]

$$\partial_t E^\sigma(\boldsymbol{x}) + \nabla \cdot \boldsymbol{\Pi}^\sigma(\boldsymbol{x}) = -\,\mathcal{D}_{\mathrm{I}}^\sigma(\boldsymbol{x}) \qquad \nu = \eta = 0$$

$$\boxed{\mathcal{D}_{\mathrm{I}}^\sigma(\boldsymbol{x}) = \frac{1}{4}\int_{\mathbb{R}^3} \nabla\varphi^\sigma(\boldsymbol{\xi}) \cdot \boldsymbol{Y}(\boldsymbol{x},\boldsymbol{\xi})\mathrm{d}\boldsymbol{\xi},} \qquad \boldsymbol{Y}(\boldsymbol{x},\boldsymbol{\xi}) = (|\delta\boldsymbol{u}|^2 + |\delta\boldsymbol{b}|^2)\delta\boldsymbol{u} - 2(\delta\boldsymbol{u}\cdot\delta\boldsymbol{b})\delta\boldsymbol{b},$$

inertial dissipation

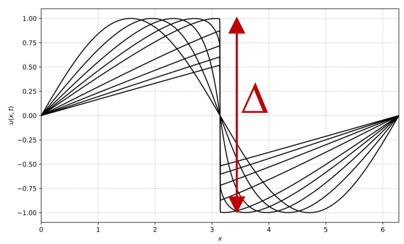
test function (Gaussian) of thickness σ

$$\mathcal{D}_{\mathrm{I}}(\boldsymbol{x}) \equiv \lim_{\sigma \to 0} \mathcal{D}_{\mathrm{I}}^\sigma(\boldsymbol{x})$$ is a generalized function

---

Burgers equation: $\mathcal{D}_I = \dfrac{\Delta^3}{12}\delta(x) \qquad \Delta = 2L/t$

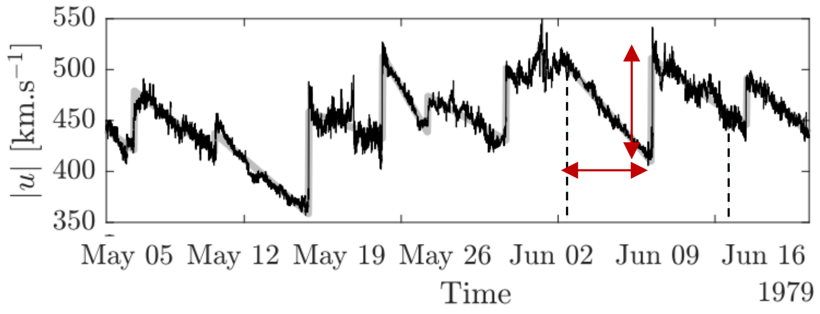
[Dubrulle, JFM, 2019] $\rightarrow$ the zeroth law is proved !



Application to interplanetary shocks [David & SG, PRE, 2021]

Local exact law

Voyager 2 (5.1 AU)



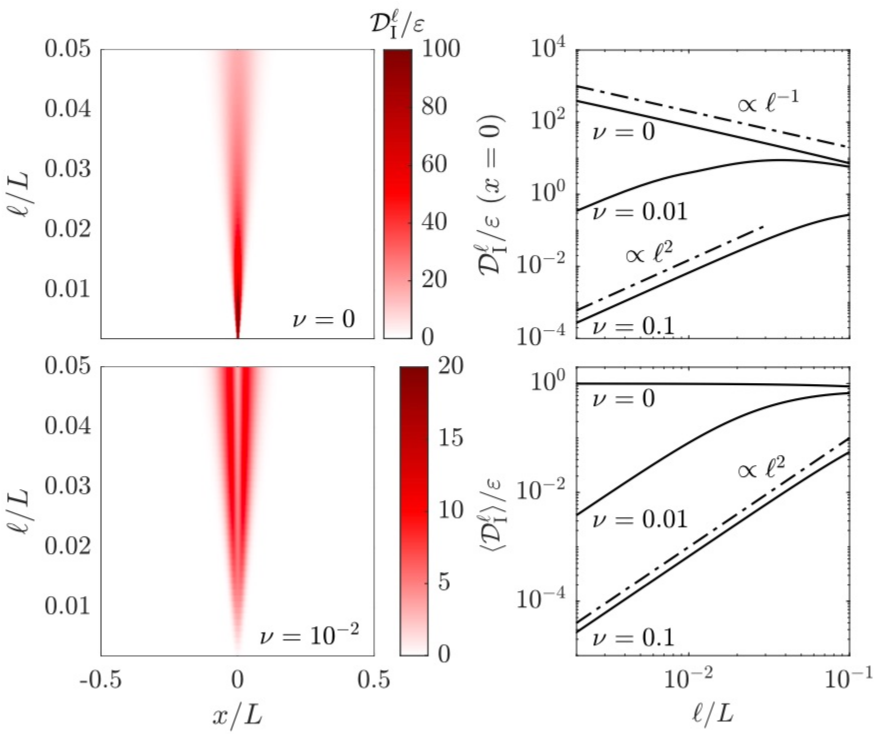
$$\partial_t u + u\partial_x u + b_y\partial_x b_y + b_z\partial_x b_z = \cancel{\nu\partial_{xx} u},$$

$$\partial_t b_y + u\partial_x b_y + b_y\partial_x u = \cancel{\eta\partial_{xx} b_y},$$

$$\partial_t b_z + u\partial_x b_z + b_z\partial_x u = \cancel{\eta\partial_{xx} b_z},$$

Shock heating

$$\boxed{\rho\langle\mathcal{D}_I\rangle = \frac{1}{6}\frac{\rho\Delta^3}{L} \sim 10^{-18}\,\mathrm{J\,m^{-3}\,s^{-1}}.}$$



[David & SG, PRE, 2021]

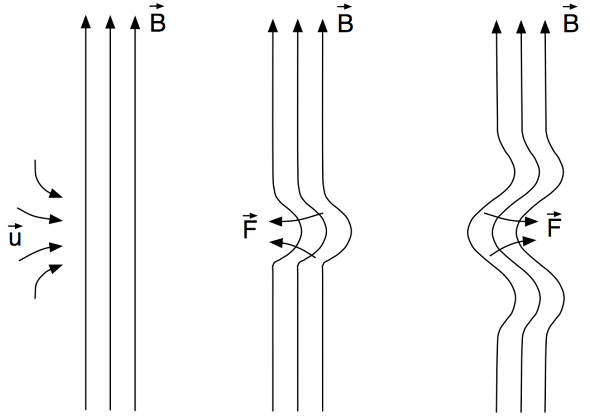
Wave turbulence

beyond phenomenology

Alfvén wave turbulence

$\boxed{\mathbf{z}^{\pm} \equiv \mathbf{u} \pm \mathbf{b}}$ Elsässer fields

Alfvén waves



$$\frac{\partial \mathbf{z}^{\pm}}{\partial t} \mp \mathbf{b}_0 \cdot \nabla \mathbf{z}^{\pm} + \underbrace{\mathbf{z}^{\mp} \cdot \nabla \mathbf{z}^{\pm}}_{\text{collision}} = -\nabla P_* \quad \text{(no dissipation)}$$

$$\tau_\omega \sim 1/\omega \sim \ell / b_0$$

Linear solution: $\boxed{\omega_k^2 = (\mathbf{k} \cdot \mathbf{b}_0)^2}$ $\quad \mathbf{b} = \mathbf{B}/\sqrt{\mu_0 \rho_0}$

Alfvén wave turbulence is the result of collisions between counterpropagating waves

Iroshnikov-Kraichnan spectrum

Phenomenology of Alfvén wave turbulence

$z^+ \sim z^- \sim z,$ balanced turbulence

$$\tau_\omega \sim 1/\omega$$
$$\tau_{nl} \sim \ell/z_\ell$$

$$\tau_{cascade} \sim \tau_\omega/\epsilon^2$$ (3-wave interactions)

$$\epsilon \sim \frac{\tau_\omega}{\tau_{nl}} \ll 1$$ stochastic collisions of wave packets of weak amplitude

[Iroshnikov, SA, 1964; Kraichnan, PoF, 1965]

$$\varepsilon \sim \frac{z_\ell^2}{\tau_{cascade}}$$

$$\boxed{E(k) \sim \sqrt{\varepsilon b_0}\, k^{-3/2}}$$

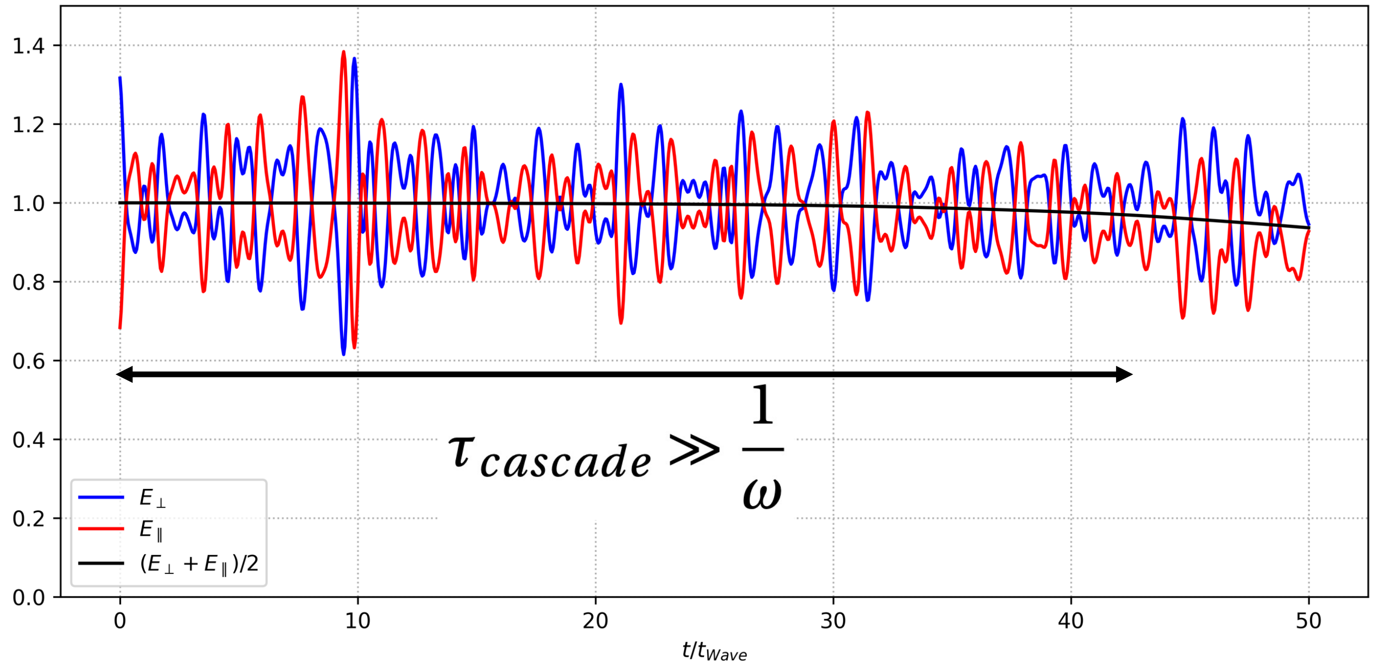
However, Alfvén wave turbulence is anisotropic !

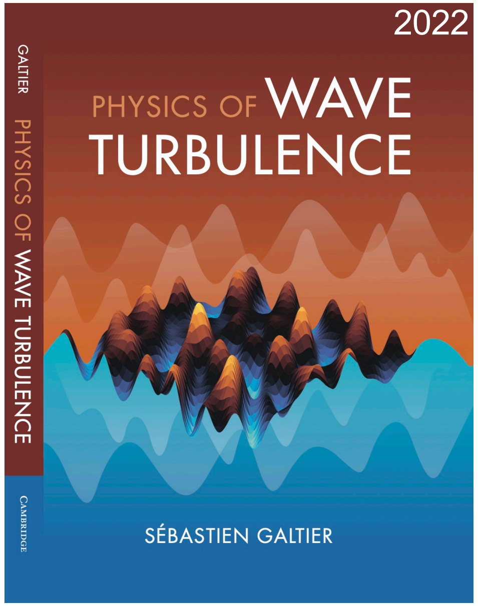
What is wave turbulence?

Weak wave turbulence theory is analytical: there is a natural asymptotic closure [Benney & Saffman, 1966; Benney & Newell, 1967; Deng & Hani, 2023] [Zakharov & Filonenko, JAMTP, 1967]

WWT is a theorem

It is a multiple time scale problem





Resonance condition

$$\frac{\partial \mathbf{z^s}}{\partial t} - s\mathbf{b_0}\cdot\nabla\mathbf{z^s} = -\mathbf{z^{-s}}\cdot\nabla\mathbf{z^s} - \nabla P_*\,,$$

$$\nabla\cdot\mathbf{z^s} = 0\,. \qquad s = \pm$$

$$z_j^s(\mathbf{x},t) \equiv \int_{\mathbb{R}^3} A_j^s(\mathbf{k},t)\, e^{i\mathbf{k}\cdot\mathbf{x}}\, d\mathbf{k}\,,$$

$$A_j^s(\mathbf{k},t) \equiv \epsilon a_j^s(\mathbf{k},t) e^{-is\omega_k t}\,,$$

$$\frac{\partial a_j^s(\mathbf{k})}{\partial t} = -i\epsilon k_m P_{jn} \int_{\mathbb{R}^6} a_m^{-s}(\mathbf{q})\, a_n^s(\mathbf{p})\, e^{is(\omega_k-\omega_p+\omega_q)t} \underbrace{\delta(\mathbf{k}-\mathbf{p}-\mathbf{q})}_{\text{Triadic interactions}}\, d\mathbf{p}d\mathbf{q}$$

where $P_{jn}(k) \equiv \delta_{jn} - k_j k_n/k^2$ $\qquad \epsilon \ll 1$

Resonance condition: $\qquad \omega_k \equiv k_{\|} b_0$

$$\begin{cases} \omega_k & = \omega_p - \omega_q\,, \\ \mathbf{k} & = \mathbf{p} + \mathbf{q}\,, \end{cases} \Longrightarrow$$

Alfvén wave turbulence

$q_{\|} = 0 \quad \text{and} \quad \mathrm{k}_{\|} = \mathrm{p}_{\|}$

no cascade along $\mathbf{b_0}$

[Montgomery & Turner, PoF, 1981; Shebalin+, JPP, 1983]

Gives an explanation to observations of anisotropy (in B-confinement)

[Robinson & Rusbridge, PoF, 1971; Zweben+, PRL, 1979]

Kinetic equation

$$\frac{\partial N(\mathbf{k})}{\partial t} = \epsilon^2 \int_{\mathbb{R}^{2d}} S(\mathbf{k},\mathbf{p},\mathbf{q})(N_p N_q - N N_p - N N_q)\underbrace{\delta(\omega-\omega_p-\omega_q)\delta(\mathbf{k}-\mathbf{p}-\mathbf{q})}_{\text{resonance condition}} d\mathbf{p}d\mathbf{q}$$

$\epsilon \ll 1$ slow dynamics

[SG, Nazarenko, Newell & Pouquet, JPP, 2000]

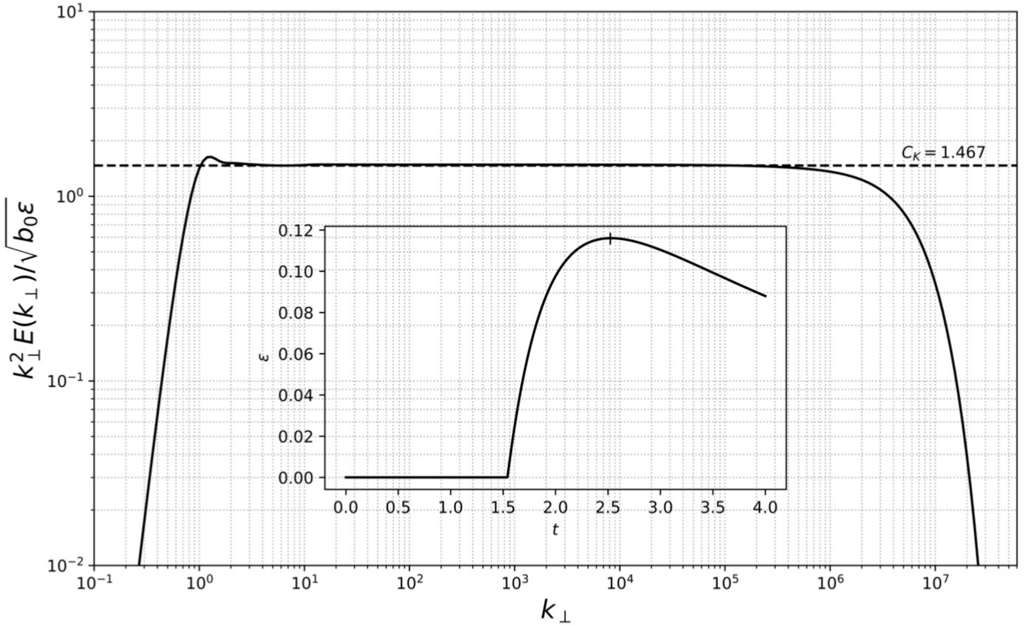
Balanced Alfvén wave turbulence:

(Kolmogorov-Zakharov spectrum)

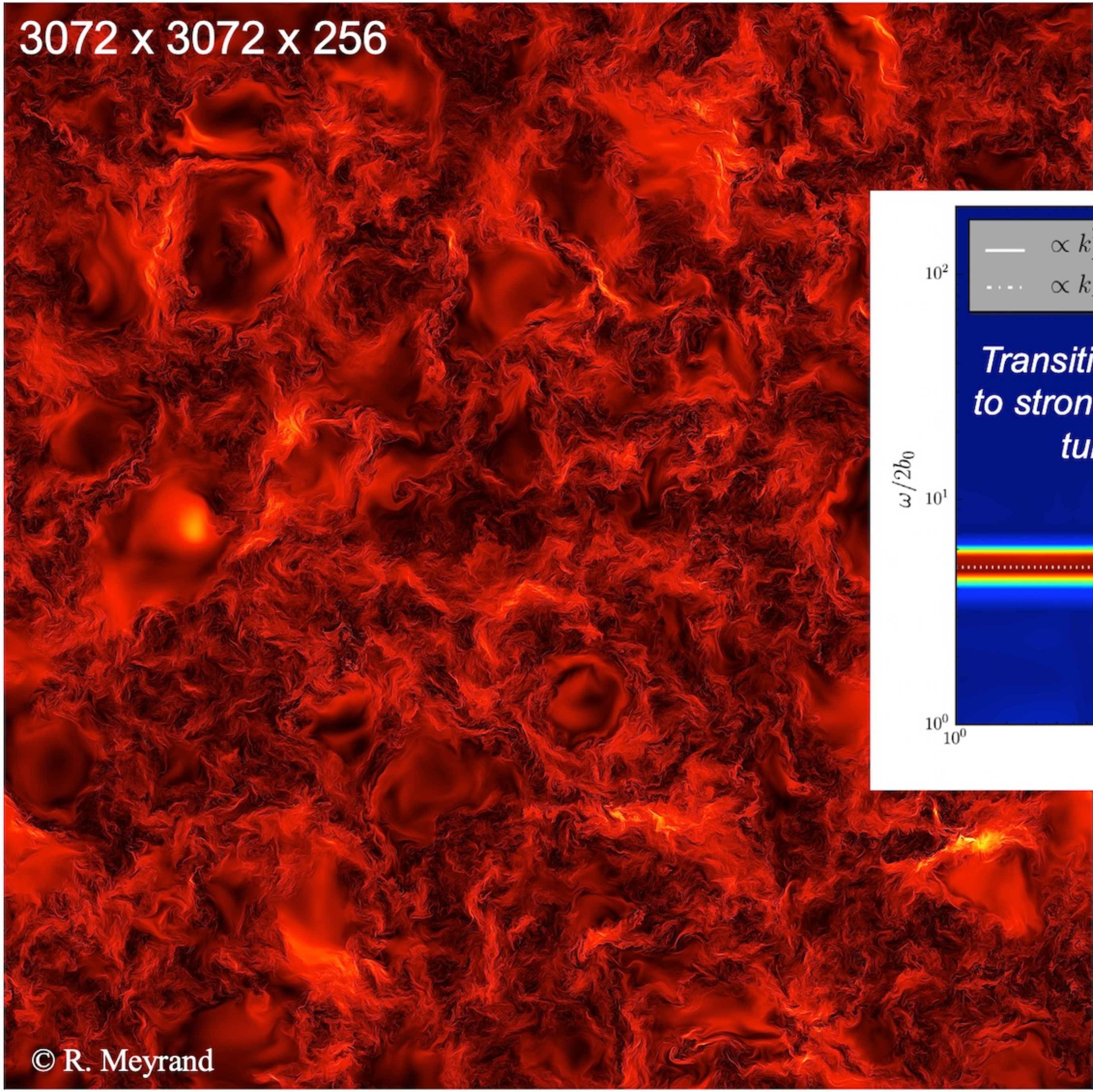
$$\boxed{E(k_\perp) = C_K \sqrt{b_0 \varepsilon} k_\perp^{-2}}\,.$$

$C_K \simeq 1.467$ (exact solution)

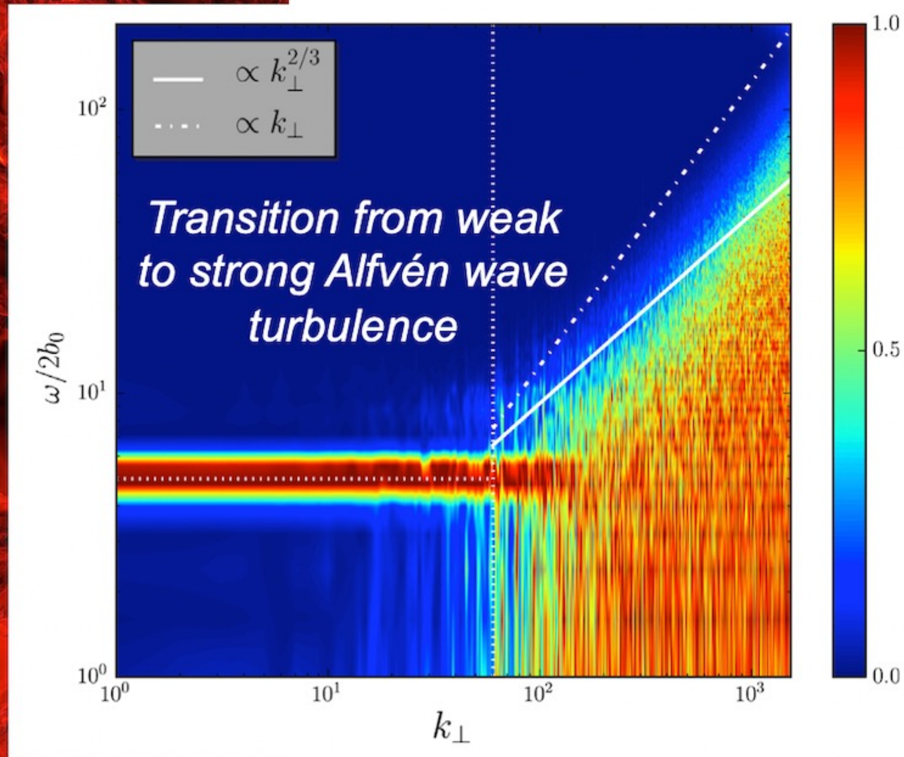
Direct cascade is proved ($\varepsilon > 0$)



DNS of Alfvén wave turbulence



ω – k energy spectrum



[Meyrand+, PRL, 2016]

Observation of Alfvén wave turbulence

nature astronomy

Article

https://doi.org/10.1038/s41550-024-02249-0

Identification of the weak-to-strong transition in Alfvénic turbulence from space plasma

Cluster data
(Earth's magnetosheath)

Received: 17 January 2023

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A&A 386, 699–708 (2002)
DOI: 10.1051/0004-6361:20020305

Astronomy & Astrophysics

Evidence for weak MHD turbulence in the middle magnetosphere of Jupiter

J. Saur[1,⋆], H. Politano[1], A. Pouquet[2], and W. H. Matthaeus[3]

Goldreich-Sridhar spectrum

Phenomenology for strong wave turbulence

$$\boxed{\text{Scale-by-scale critical balance: } \tau_\omega \sim \tau_{nl}}$$

[Phillips, JFM, 1958; Higdon, ApJ, 1984; Goldreich & Sridhar, ApJ, 1995; SG, Mangeney & Pouquet, PoP, 2005]

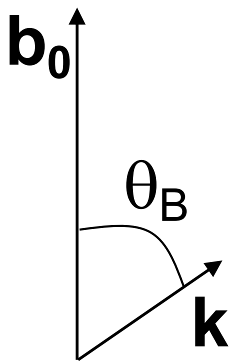
$$\Longrightarrow \quad \boxed{E(k_\perp) \sim k_\perp^{-5/3}}$$

$$E(k_\parallel) \sim k_\parallel^{-2}$$

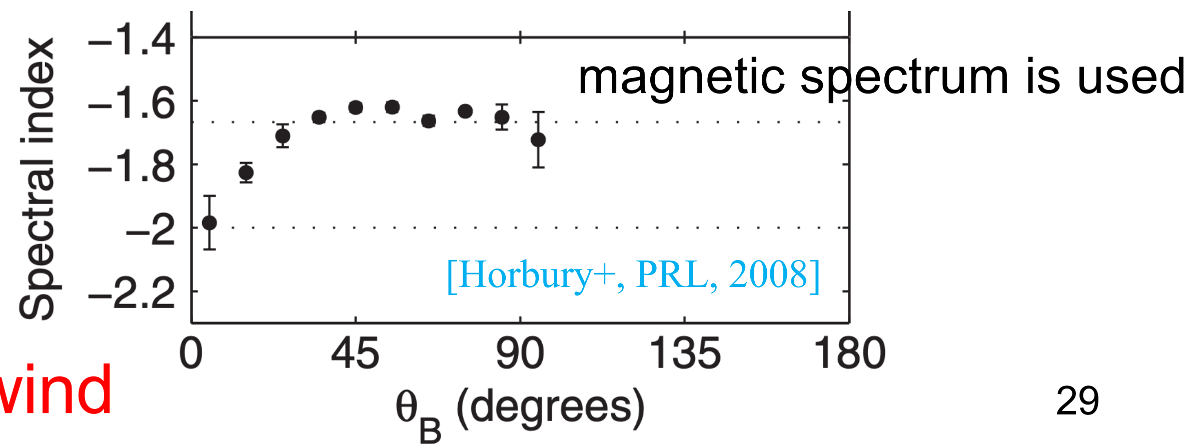
$$\boxed{k_\parallel \sim k_\perp^{2/3}}$$

Anisotropy

[Cho & Vishniac, ApJ, 2000]

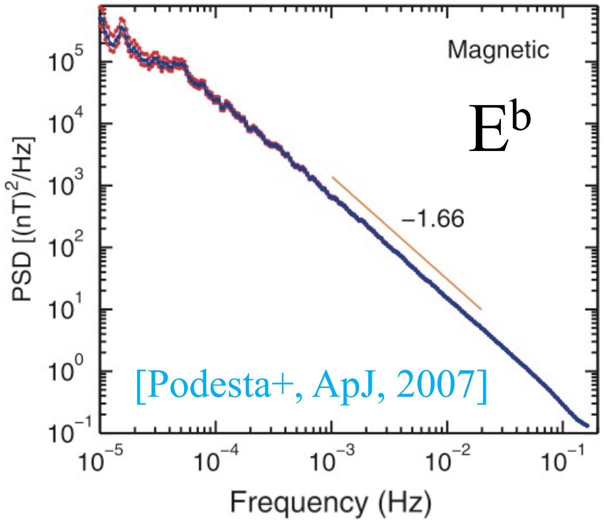


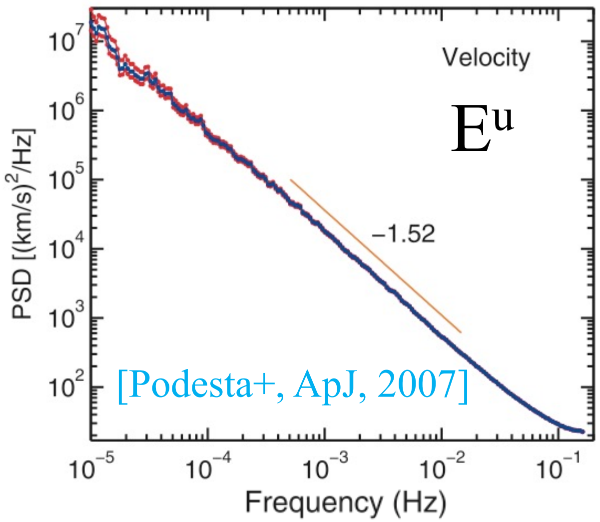
Solar wind

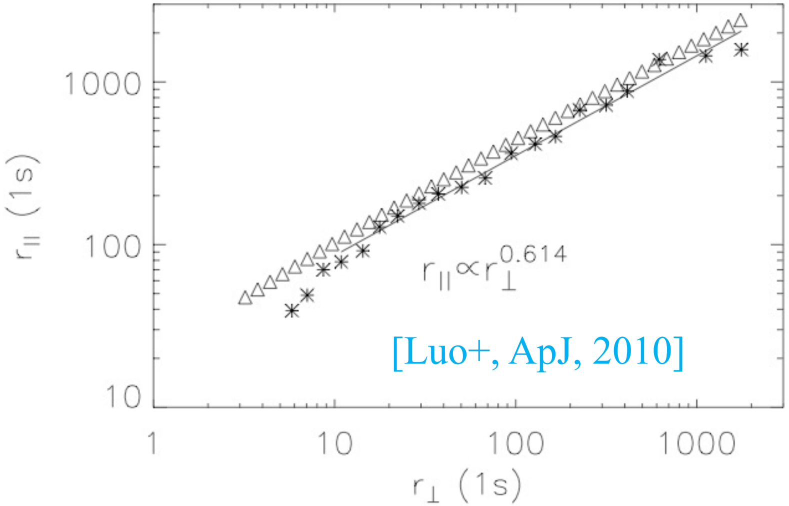


Comparison with data

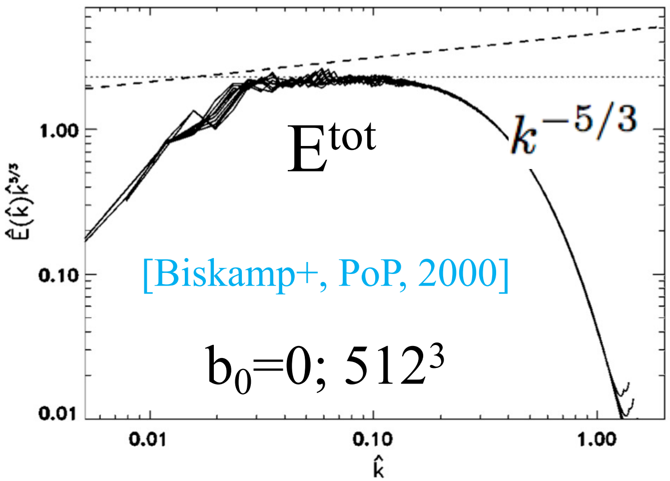
Solar wind data

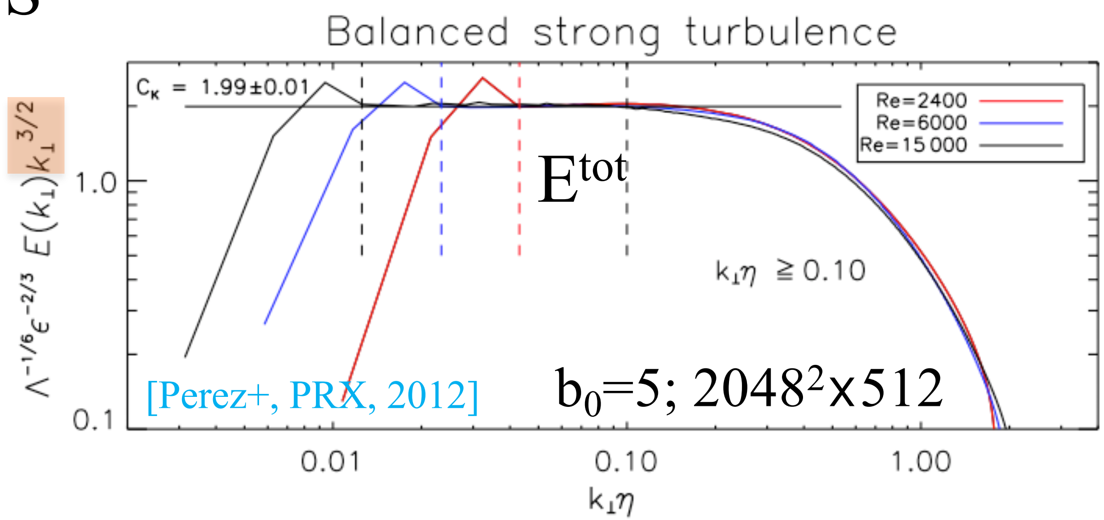






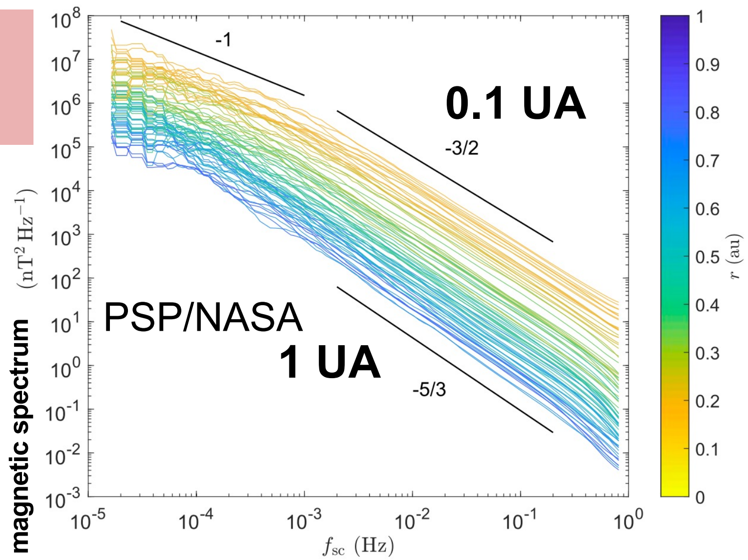
DNS





Fast magneto-acoustic wave turbulence

Solar wind



[Chen+, ApJ, 2020]

Phenomenology

$$E^f(k) \sim k^{-3/2}.$$

DNS [Cho & Lazarian, ApJ, 2002]

Solar wind [Zhao +, ApJ, 2022]

Wave turbulence theory

$$\frac{\partial E_k^s}{\partial t} = \frac{\pi \epsilon^2 K_{\theta,\phi}}{32 b_0} \int_{\Delta_\perp} \sum_{s_p s_q} \delta(sk + s_p p + s_q q) \frac{s}{pq} \left[s k^3 E_p^{s_p} E_q^{s_q} + s_p p^3 E_k^s E_q^{s_q} + s_q q^3 E_k^s E_p^{s_p} \right] \mathrm{d}p\, \mathrm{d}q,$$

[SG, JPP, 2023]

$$E_k = \sqrt{\frac{b_0 \varepsilon}{K_{\theta,\phi}}} C_K k^{-3/2} \qquad C_K = \sqrt{\frac{6}{\pi(\pi - 1 + 4\ln(2))}} \simeq 0.623$$

Kolmogorov-Zakharov isotropic specrum; direct cascade of energy

Observation of fast-MHD wave turbulence

The Astrophysical Journal, 937:102 (14pp), 2022 October 1 https://doi.org/10.3847/1538-4357/ac822e

Multispacecraft Analysis of the Properties of Magnetohydrodynamic Fluctuations in Sub-Alfvénic Solar Wind Turbulence at 1 au

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Abstract

We present observations of three-dimensional magnetic power spectra in wavevector space to investigate the anisotropy and scalings of sub-Alfvénic solar wind turbulence at magnetohydrodynamic (MHD) scale using the Magnetospheric Multiscale spacecraft. The magnetic power distributions are organized in a new coordinate determined by wavevectors ($\hat{\boldsymbol{\kappa}}$) and background magnetic field ($\hat{\boldsymbol{b}}_0$) in Fourier space. This study utilizes two approaches to determine wavevectors: the singular value decomposition method and multispacecraft timing analysis. The combination of the two methods allows an examination of the properties of magnetic field fluctuations in terms of mode compositions without any spatiotemporal hypothesis. Observations show that fluctuations ($\delta B_{\perp 1}$) in the direction perpendicular to $\hat{\boldsymbol{\kappa}}$ and $\hat{\boldsymbol{b}}_0$ prominently cascade perpendicular to $\hat{\boldsymbol{b}}_0$, and such anisotropy increases with wavenumbers. The reduced power spectra of $\delta B_{\perp 1}$ follow Goldreich–Sridhar scalings: $\hat{P}(k_\perp) \propto k_\perp^{-5/3}$ and $\hat{P}(k_\parallel) \propto k_\parallel^{-2}$. In contrast, fluctuations within the $\hat{k}\hat{b}_0$ plane show isotropic behaviors: perpendicular power distributions are approximately the same as parallel distributions. The reduced power spectra of fluctuations within the $\hat{k}\hat{b}_0$ plane follow the scalings $\hat{P}(k_\perp) \propto k_\perp^{-3/2}$ and $\hat{P}(k_\parallel) \propto k_\parallel^{-3/2}$. Comparing frequency–wavevector spectra with theoretical dispersion relations of MHD modes, we find that $\delta B_{\perp 1}$ are probably associated with Alfvén modes. On the other hand, magnetic field fluctuations within the $\hat{k}\hat{b}_0$ plane more likely originate from fast modes based on their isotropic behaviors. The observations of anisotropy and scalings of different magnetic field components are consistent with the predictions of current compressible MHD theory. Moreover, for the Alfvénic component, the ratio of cascading time to the wave period is found to be a factor of a few, consistent with critical balance in the strong turbulence regime. These results are valuable for further studies of energy compositions of plasma turbulence and their effects on energetic particle transport.