Relativistic Wave-Particle Interactions in Strong Magnetic Fields

Takayoshi Sano
(Institute of Laser Engineering, Osaka University)
Collaborators:
S. Isayama, S. Matsukiyo (Kyushu U), M. Hata (QST),
K. Sugimoto (Kyoto U), Y. Sentoku (Osaka U)

Workshop: Kinetic physics of astrophysical plasmas
2025/6/18-20 (Sorbonne U, Paris)

Laser Fusion
Laser Astrophysics

GEKKO(激光) & LFEX

1 kJ

100 um

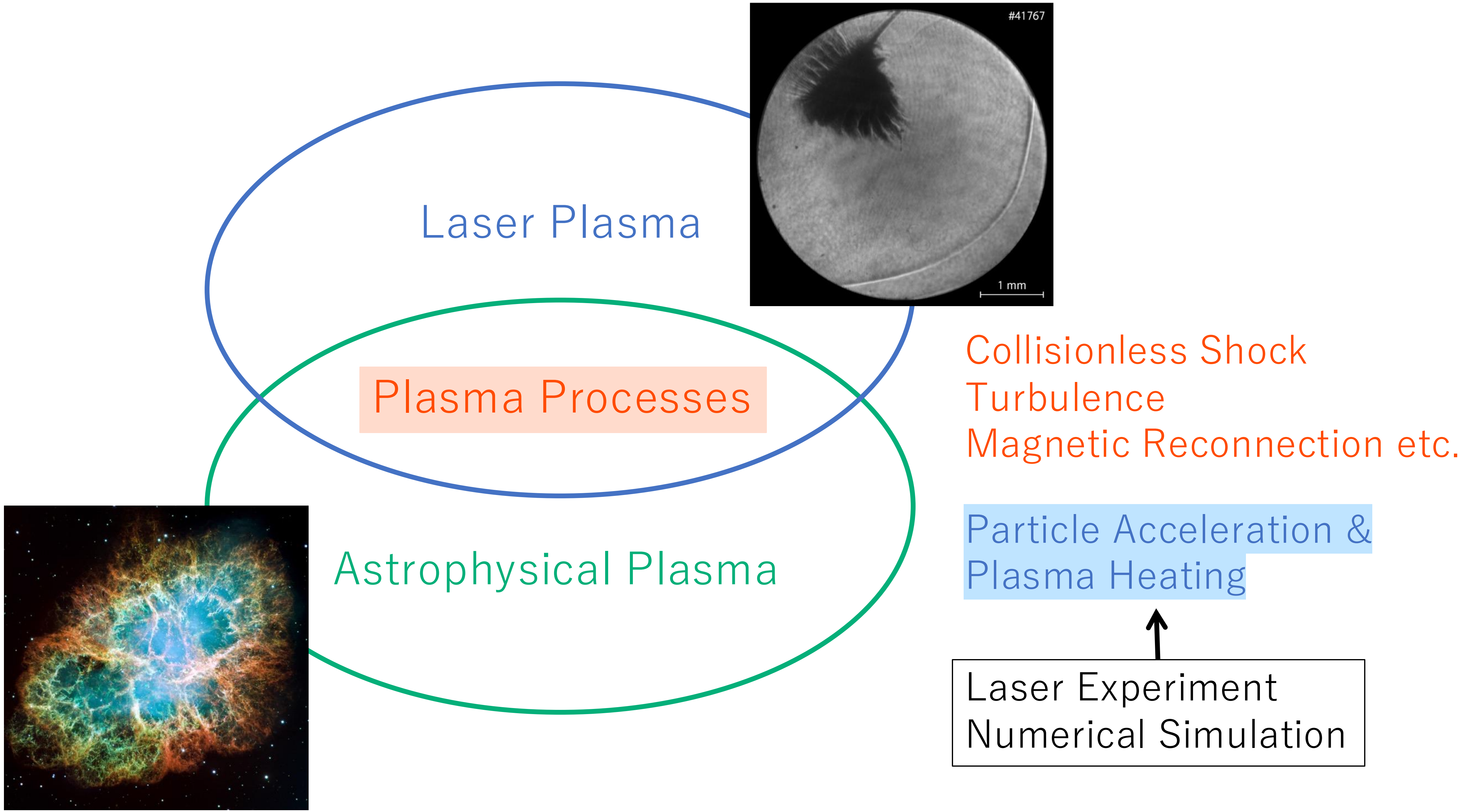
ns laser ↔ ps laser

10 GPa ↔ 10 PPa

High Energy Density (High Pressure) State

$$I_L = \frac{E_L}{\tau_L A} \sim 10^{20}\ [\mathrm{W/cm^2}]$$

$$P = \frac{I_L}{c} \sim 10\ [\mathrm{PPa}]$$


#41767
1 mm
Laser Plasma
Plasma Processes
Astrophysical Plasma
Collisionless Shock
Turbulence
Magnetic Reconnection etc.
Particle Acceleration &
Plasma Heating
Laser Experiment
Numerical Simulation

High Intensity Laser (Relativisitic Intensity Laser)

- Equation of Motion for Electrons in Electromagntic Fields

$$\frac{d\boldsymbol{p}}{dt} = -e\boldsymbol{E} \quad \xrightarrow{\text{Normalized}} \quad \frac{d\widetilde{\boldsymbol{p}}}{d\widetilde{t}} = -\boldsymbol{a}_0$$

$$\widetilde{\boldsymbol{p}} = \frac{\boldsymbol{p}}{m_e c} \qquad \widetilde{t} = \omega_0 t \qquad \boldsymbol{a}_0 = \frac{e\boldsymbol{E}}{m_e c \omega_0}$$

<u>Typical Energy</u>

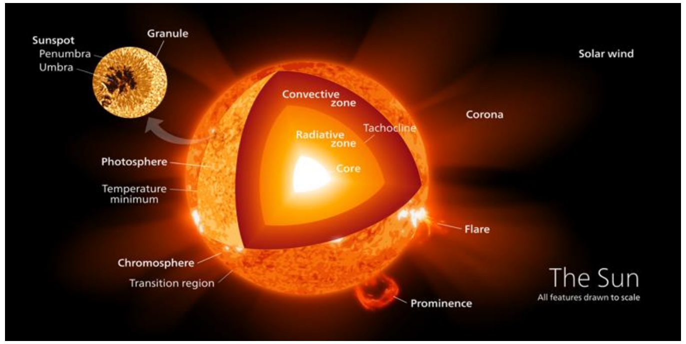
$$\widetilde{p} \sim a_0$$

$$\gamma - 1 \sim \sqrt{1+\widetilde{p}^2} - 1 \sim a_0$$

- Relativistic Intensity: $a_0 > 1$
- For a Typical Laser Case (Wavelength = 1 micron)

$$I > 10^{18}\ [\mathrm{W/cm^2}]$$

$$P \sim 10\left(\frac{I}{10^{20}}\right)\ [\mathrm{PPa}]$$



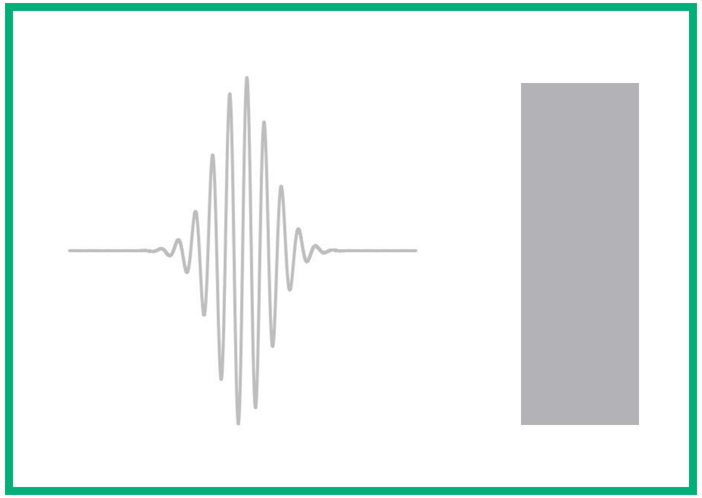
Laser Plasma Parameters

- Laser Parameter (Characteristic)

$$\lambda_0 = 1\ \mu\mathrm{m} \qquad \omega_0 = 2\pi c/\lambda_0 \qquad \tau_0 = 3\ \mathrm{fs}$$

Laser Amplitude

$$a_0 = \frac{eE_0}{mc\omega_0}$$



Plasma Density

$$\widetilde{n}_e = \frac{n_e}{n_c} = \frac{\omega_{pe}^2}{\omega_0^2}$$

$$\frac{d\boldsymbol{p}}{dt} = -e\boldsymbol{E}$$

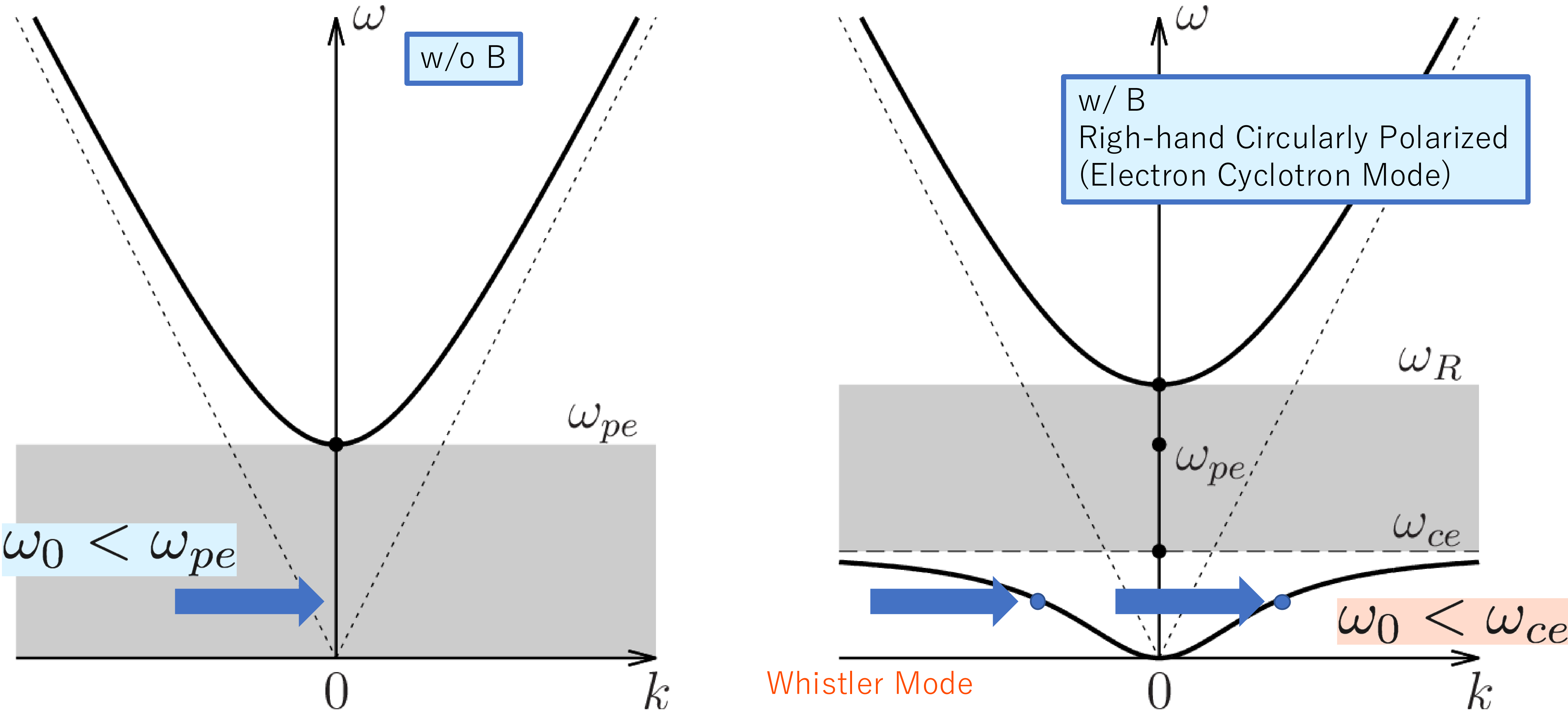
Normalized

$$\frac{d\widetilde{\boldsymbol{p}}}{d\widetilde{t}} = -\boldsymbol{a}_0$$

$$\widetilde{\boldsymbol{p}} = \frac{\boldsymbol{p}}{m_e c} \qquad \widetilde{t} = \omega_0 t$$

$$n_c \sim 10^{21} \left(\frac{\lambda_0}{1\ \mu\mathrm{m}}\right)^{-1} \quad [\mathrm{cm}^{-3}]$$

Electromagntic Wave along a Magnetic Field



Why Whistler Wave?

Great Advantages for Plasma Heating

- No Cut-off Density: Direct Interaction with Dense Plasma
- Right-hand Circlarly Polarized: Cyclotroron Resonance with Electrons

→ Energy Conversion from EM Waves to Ions and Electrons

Strong External B Field	$\widetilde{B} = \frac{B_{\mathrm{ext}}}{B_c} = \frac{\omega_{ce}}{\omega_0} \gg 1$
Large Amplitude EM Wave	$a_0 = \frac{eE_0}{m_e c \omega_0} \gg 1$

Plasma Conditions: "Strong" Magnetic Field and "Relativistic-amplitude" Laser (Electromagnetic Wave)

- External Magnetic Field

Strong Field

$$\widetilde{B} = \frac{B_{\mathrm{ext}}}{B_c} = \frac{\omega_{ce}}{\omega_0}$$

$$B_c \equiv \frac{m_e \omega_0}{e}$$

$$B_c \sim 10 \left(\frac{\lambda_0}{1\ \mu\mathrm{m}} \right)^{-1} [\mathrm{kT}]$$

100 MG

- Laser Amplitude

Relativistic Intensity

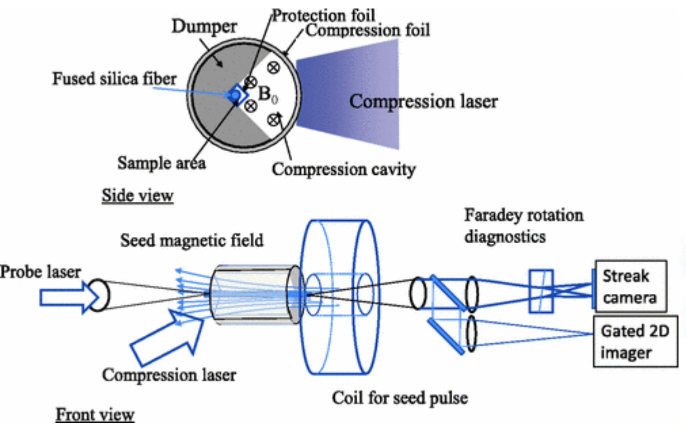
$$a_0 = \frac{eE_0}{m_e c \omega_0}$$

Generation of kilo-Tesla magnetic fields has been achieved by high-power lasers.

- Strong B Field Available in Laser Exp.
- Method (Using GEKKO Laser in Osaka)
  - Coil + Compression
  - Capacitor Coil

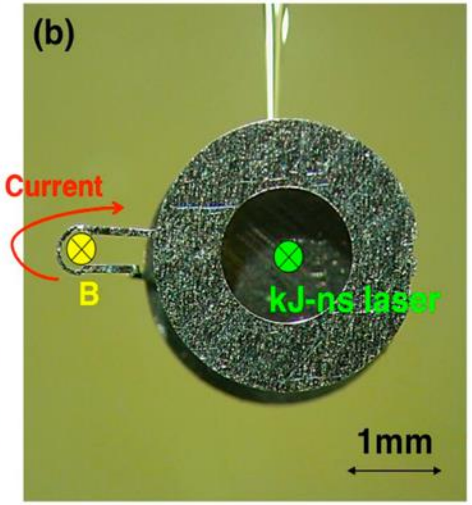
Motivation：
To Control Electron Dynamics by Strong Magnetic Fields

cf.) 1 kT = 10 MG, Permanent Magnet ～1 T

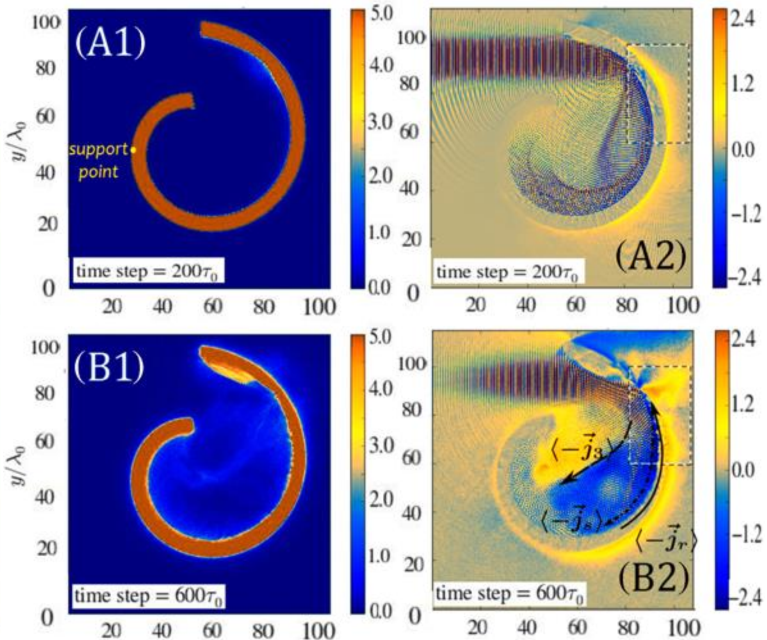


Yoneda+ 2012

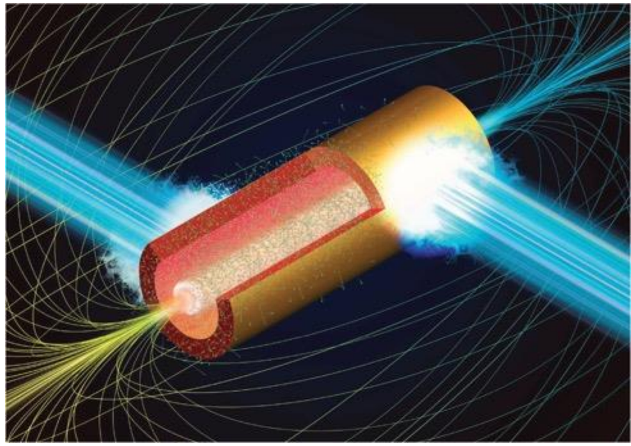
Fujioka+ 2013



Korneev+ 2015



Mega Tesla?
(10^{10} G)



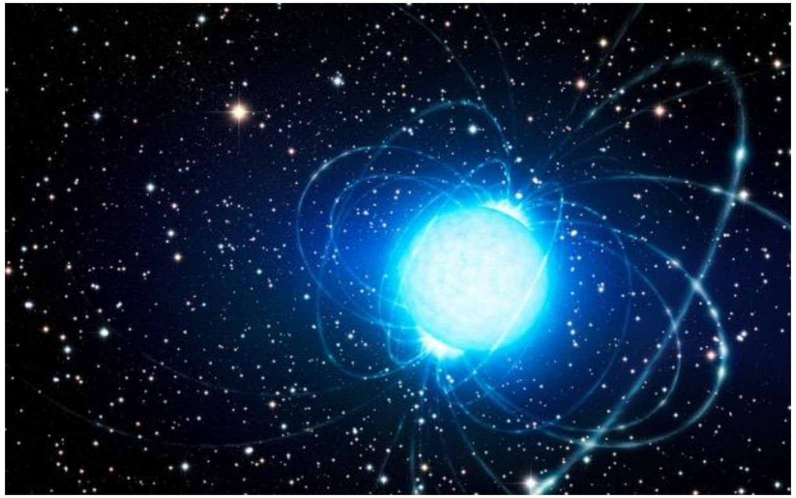
Shokov+ 2022

Magnetosphere of neutron star has similar plasma parameters to laser experiments.

- Fast Radio Burst (FRB)
- Emission mechanism is still unclear.
- At least one FRB is associated with a magnetar.
- Key Question: Can a strong radio wave escape the magnetosphere of magnetar?

Beloborodov (2021; 2022)

Qu+ (2022)

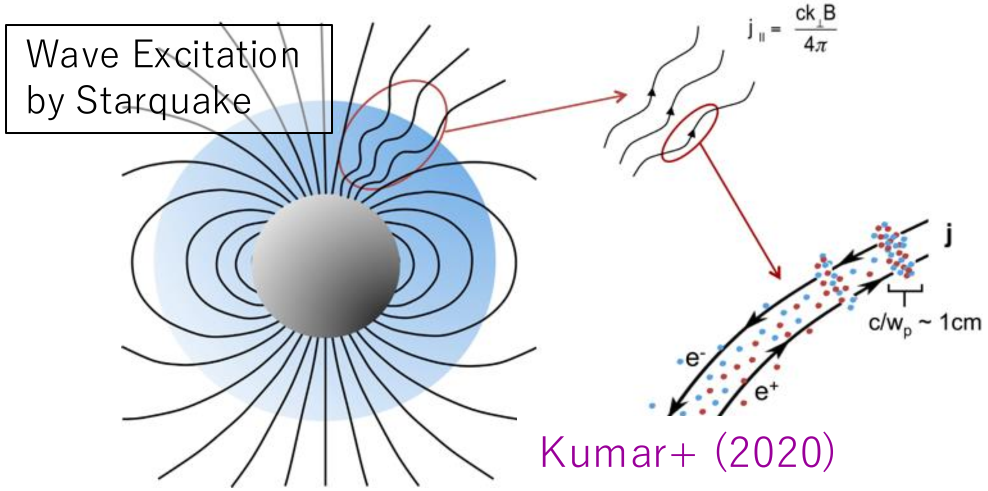


- Alfven Wave in Magnetosphere of Magnetar

$\widetilde{n} \sim 10^3$ High Density

$\widetilde{B} \sim 10^4$ Strong Field

$a_0 \sim 10^4$ Relativistic Amplitude

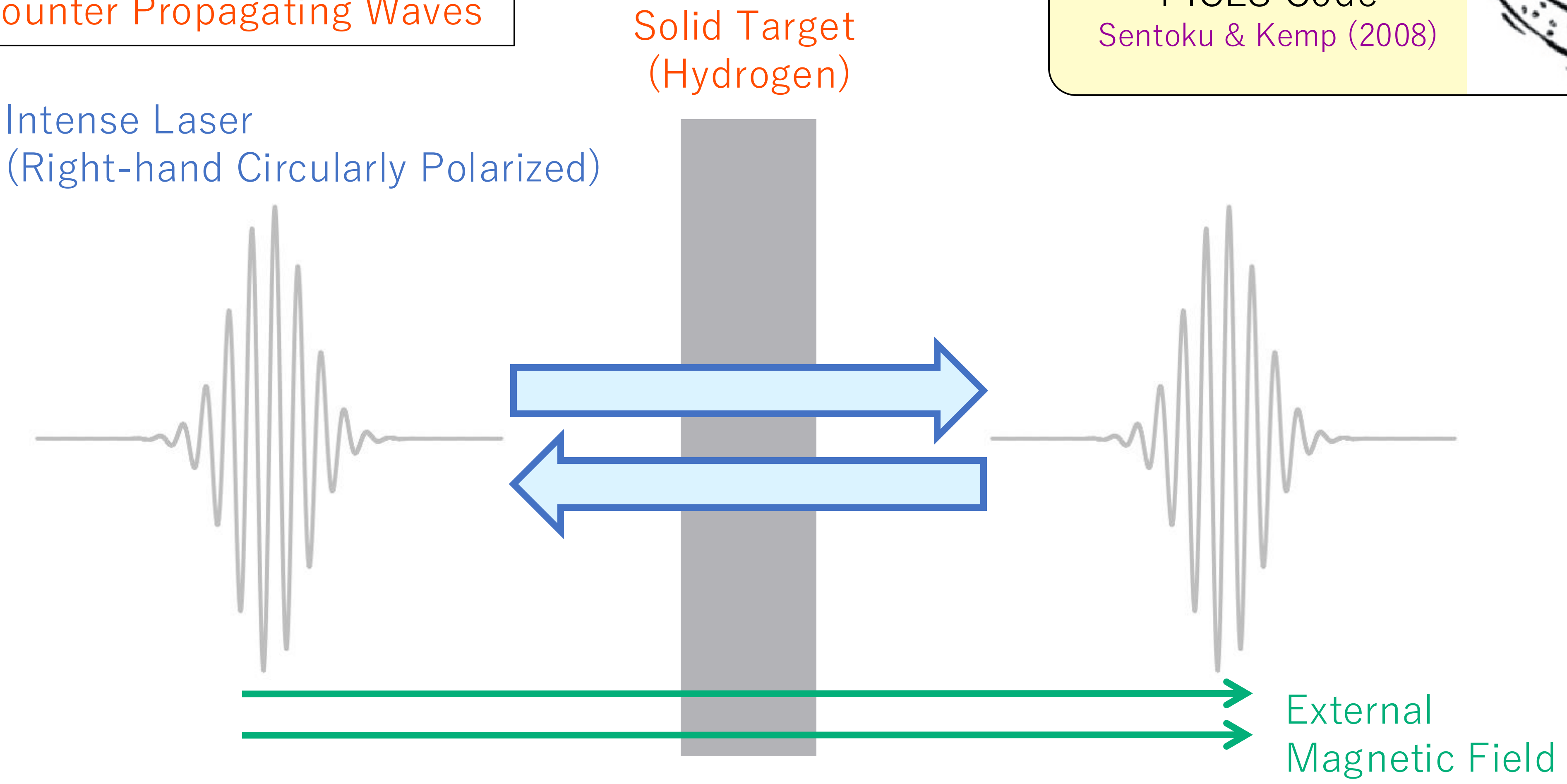


Kumar+ (2020)

Numerical Setup: Electron acceleration in standing whistler wave is examined by1D PIC simulation

Standing Wave Formation
by Counter Propagating Waves

PICLS Code
Sentoku & Kemp (2008)

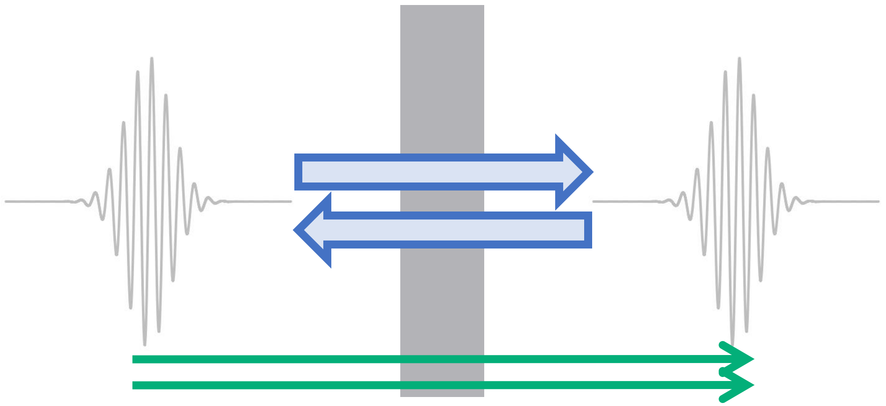


Energy convergence rate is depending on the external magnetic field strength.

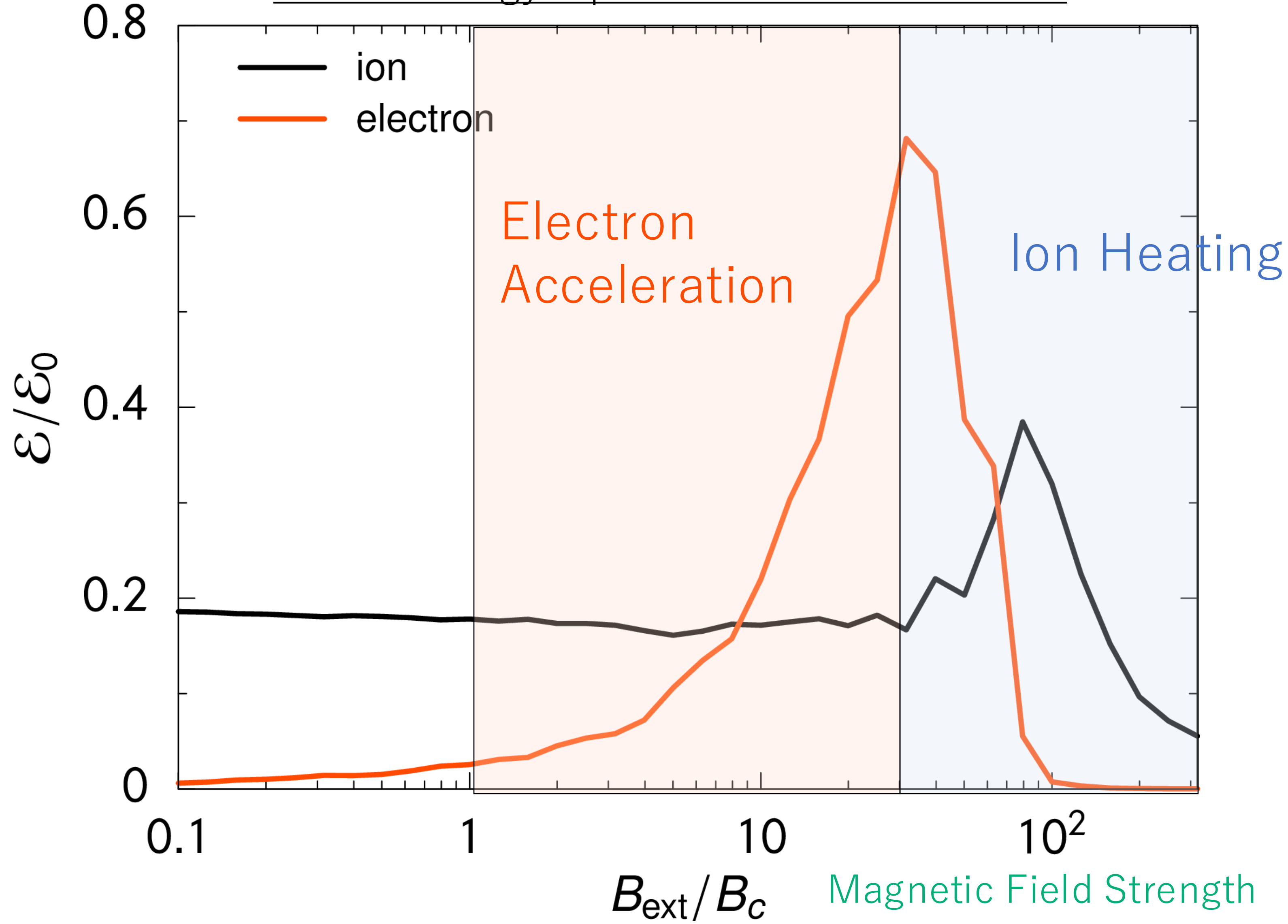
$$\frac{n}{n_c} = 100$$

$$a_0 = 30$$

$$\frac{\tau_0}{t_0} = 10$$



Plasma Energy Aquired from Counter Waves



Short Abstract of My Research

- Standing whistler waves can accelerate either electrons or ions depending on the external magnetic field strength.

$$\widetilde{B} = \frac{B_{\mathrm{ext}}}{B_c} = \frac{\omega_{ce}}{\omega_0}$$

- If magnetic fields in excess of 10 kT become available, this could be the subject of new laser astrophysics experiments.

Magnetic Field Strength

$\widetilde{B} \sim 1$

Electron Acceleration

$$1 \lesssim \frac{\omega_{ce}}{\omega_0} \lesssim a_0$$

$\widetilde{B} \sim a_0$

$$\frac{\omega_{ce}}{\omega_0} \gtrsim a_0$$

Ion Heating

Electron Acceleration in Standing Whistler Waves

Sano et al. PRE (2017)
Isayama et al. ApJ (2023)
Sano et al. PRE (2024)

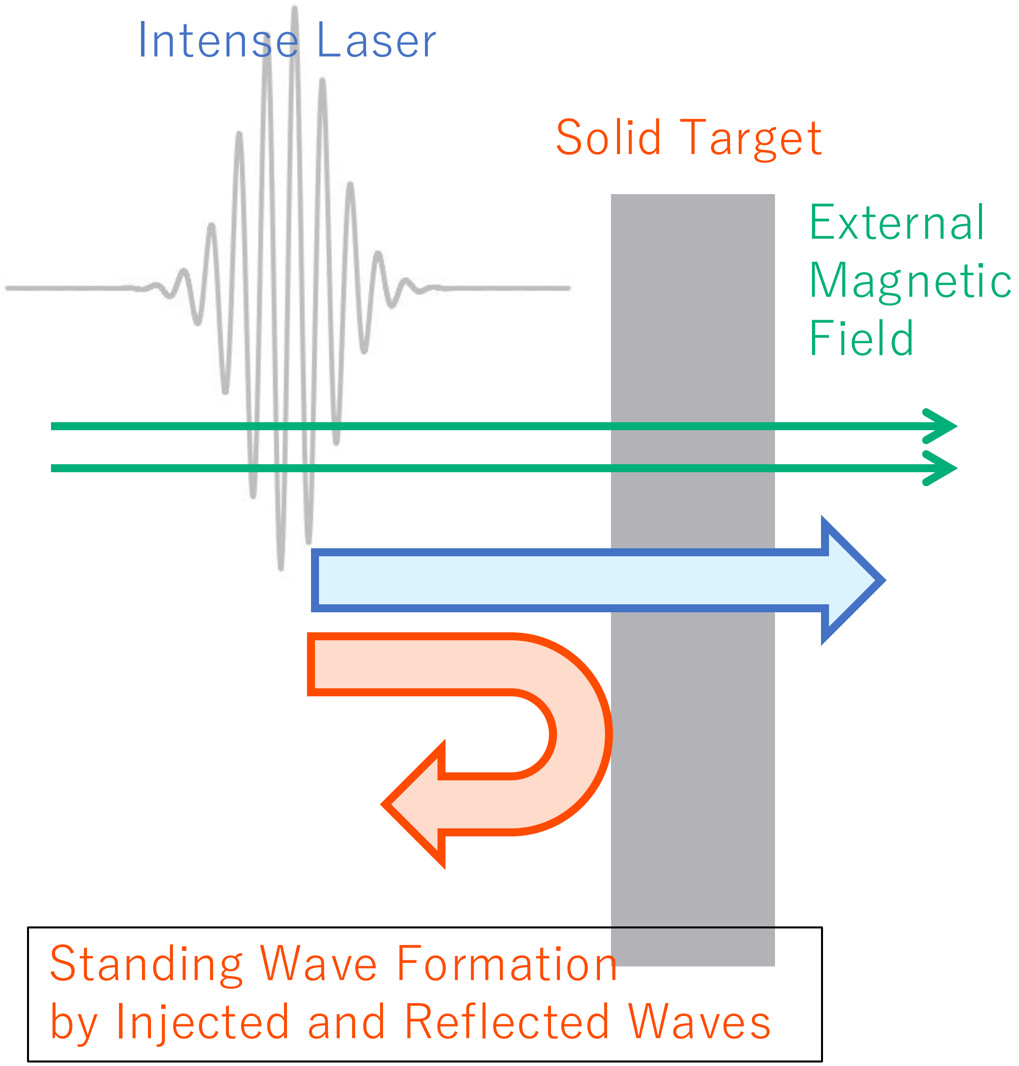
Electron acceleration in standing whistler wave is examined by1D PIC simulation

- Target:
  - Thin Carbon Foil (Diamond)
  - Thickness = 1 um + Preplasma (Scale Length = 1 um)
- External Magnetic Field:
  - Parallel to Laser Injection
- Laser:
  - Right-hand Circularly Polarized
  - Wavelength = 1 um
  - Pulse Duration = 30 fs

$$\tilde{n} = 600$$

$$\tilde{B} = 30$$

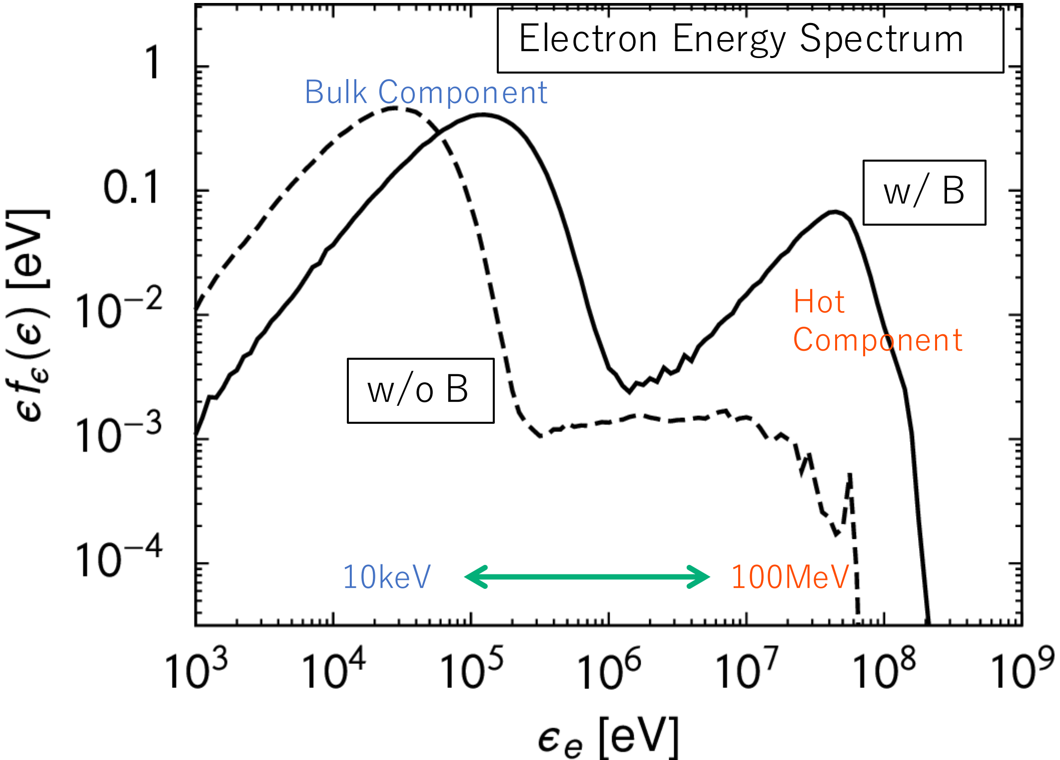
$$a_0 = 30$$

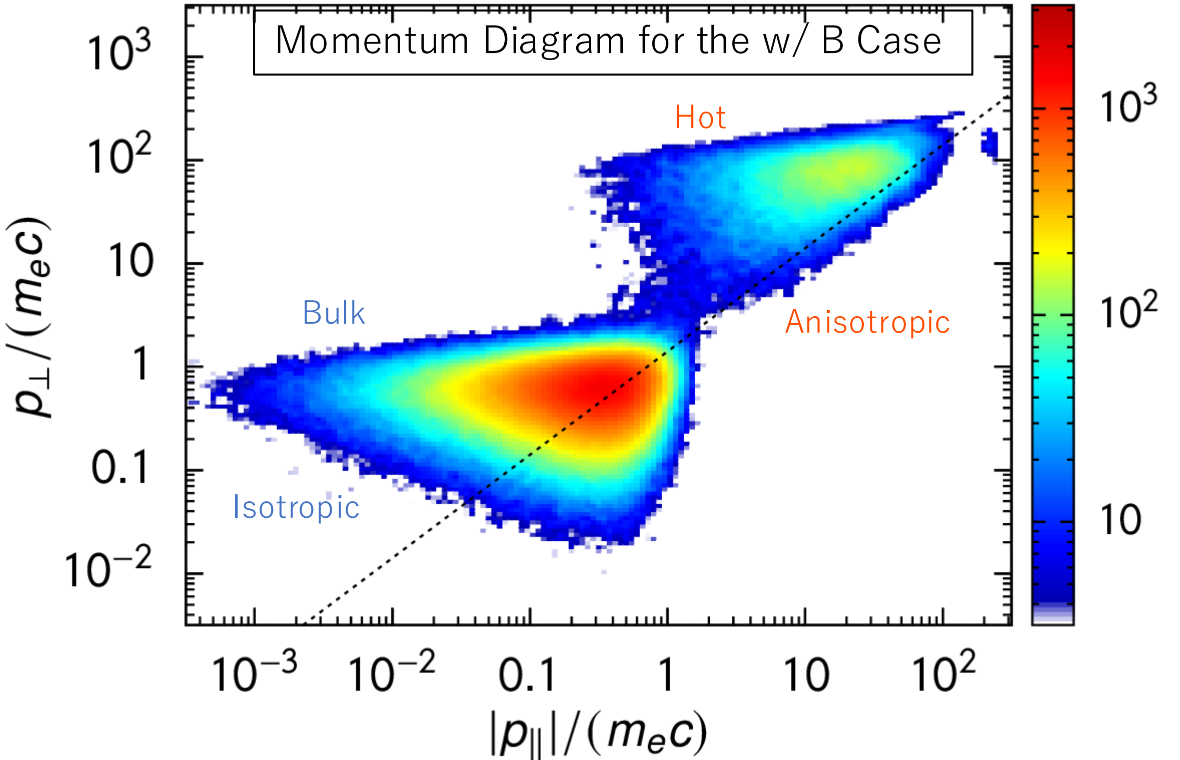


Applying magnetic field enhances the energy and number fraction of relativistic electron.

- Enhancement of Hot Electron Energy
- Clear Dichotomization with Bulk Component (Double Peak in the Spectrum)

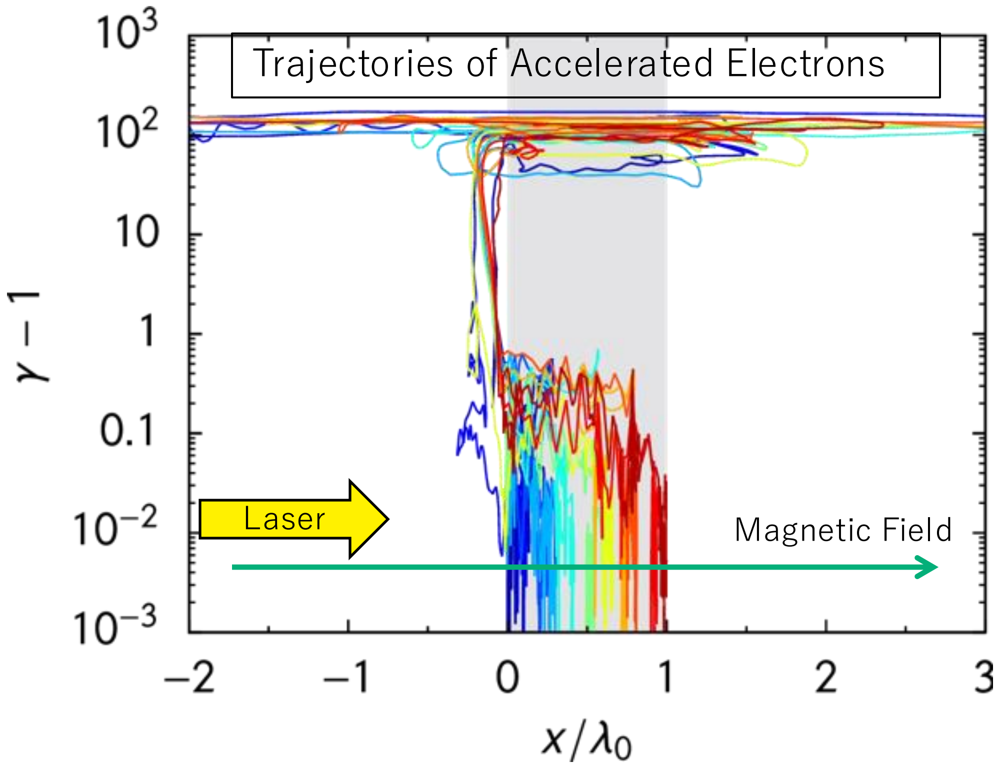
$\tilde{n} = 600$ $\tilde{B} = 30$ $a_0 = 30$

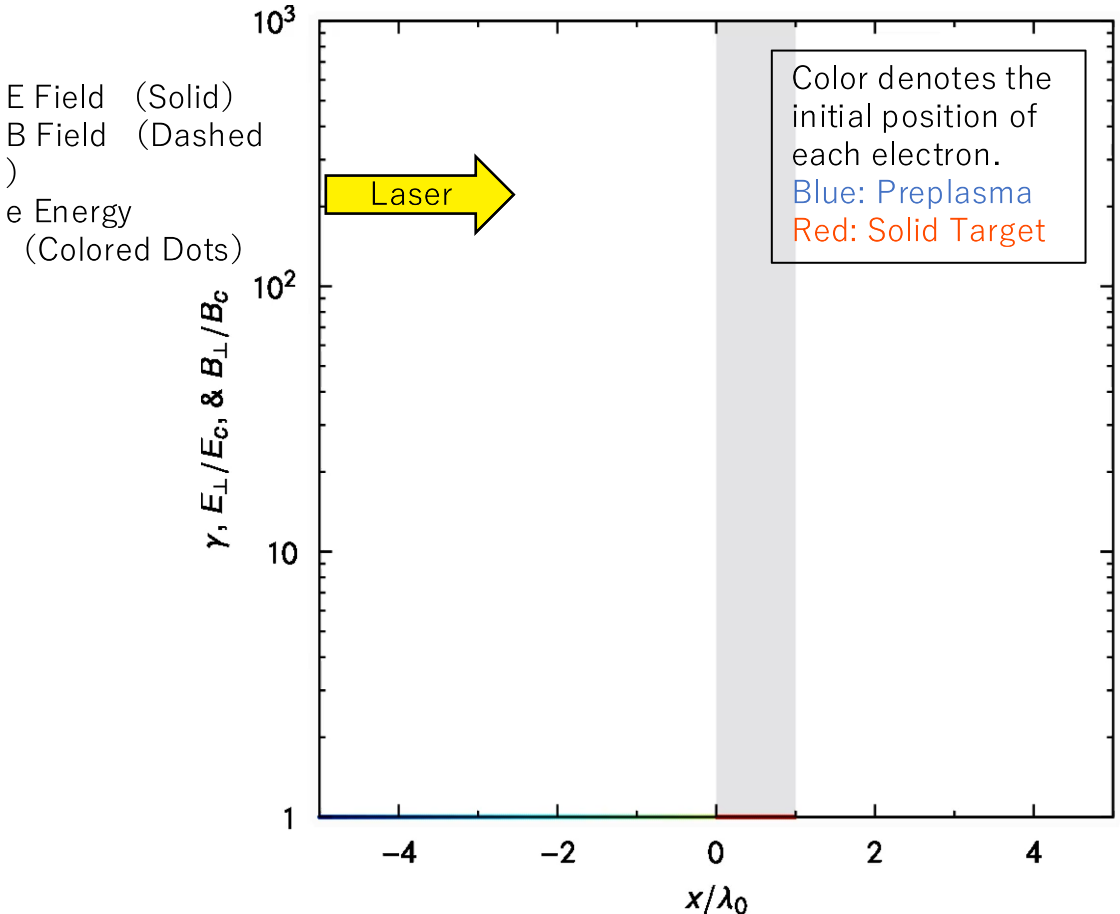




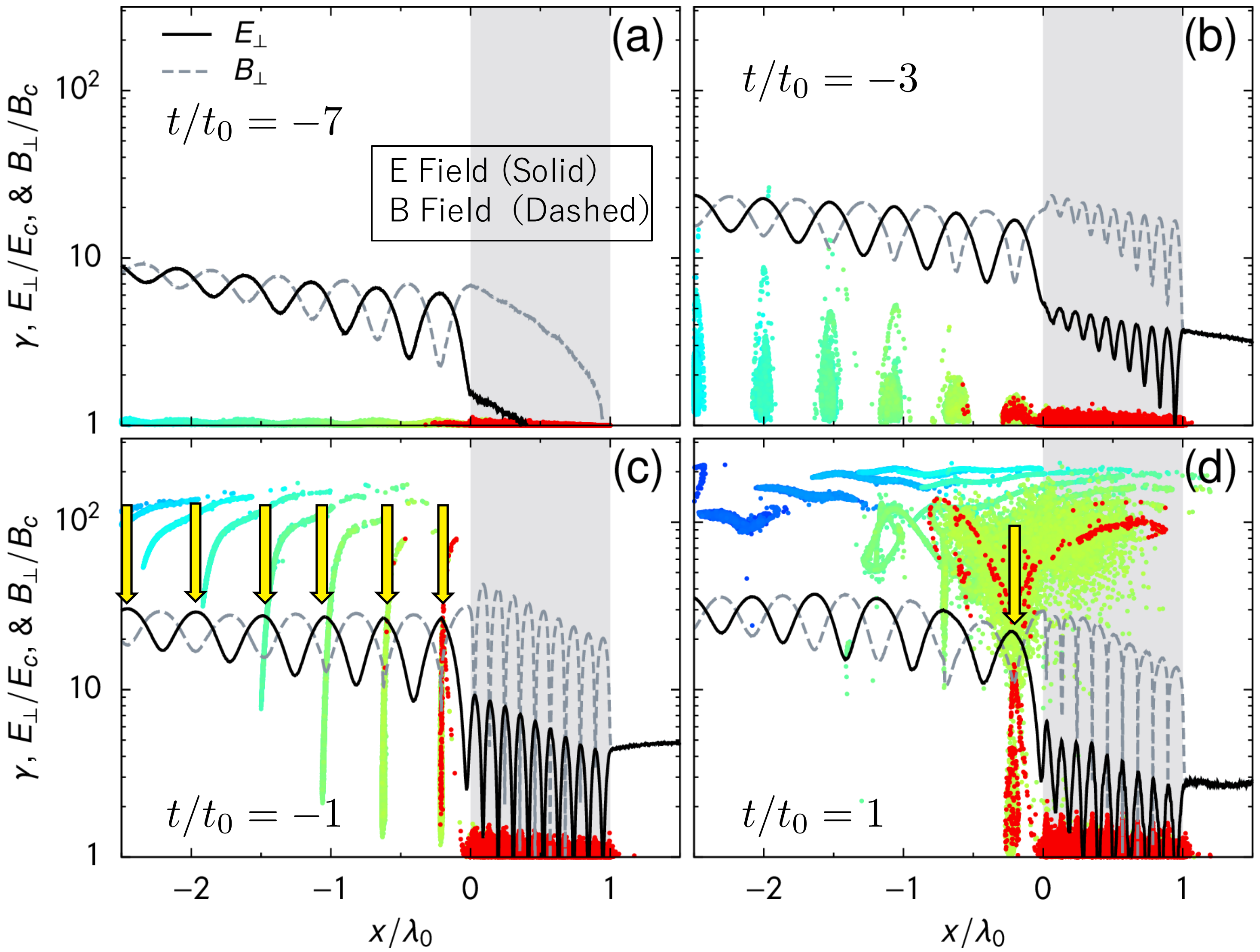
Acceleration occurs in standing whistler wave at the target surface without exception.

- Acceleration point is just outside of the front surface.
- Acceleration takes place at the same location from non-relativistic velocity to relativistic at once.
- Standing wave is essential.





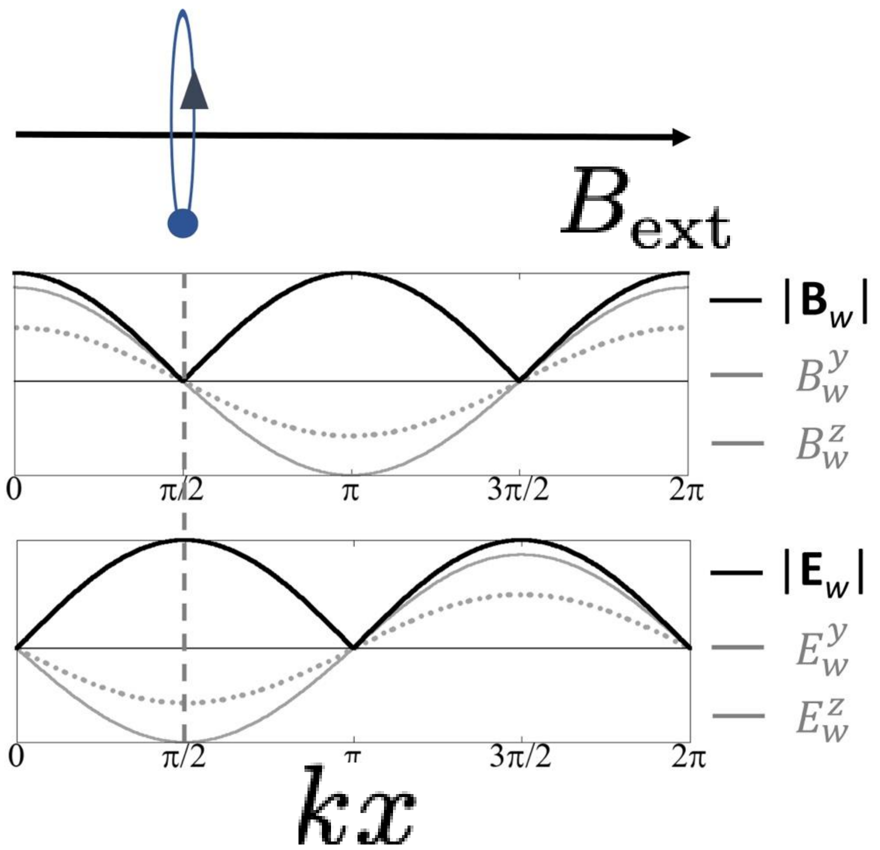
Acceleration always takes place at the trough of magnetic field in standing wave.



Sano et al. PRE (2024)

Relativistic resonant acceleration in counterpropagating Alfvén (Whistler) waves

Matsukiyo & Hada (2009), S. Isayama+ (2023), Sano+ (2024)



at $kx = \pi/2$

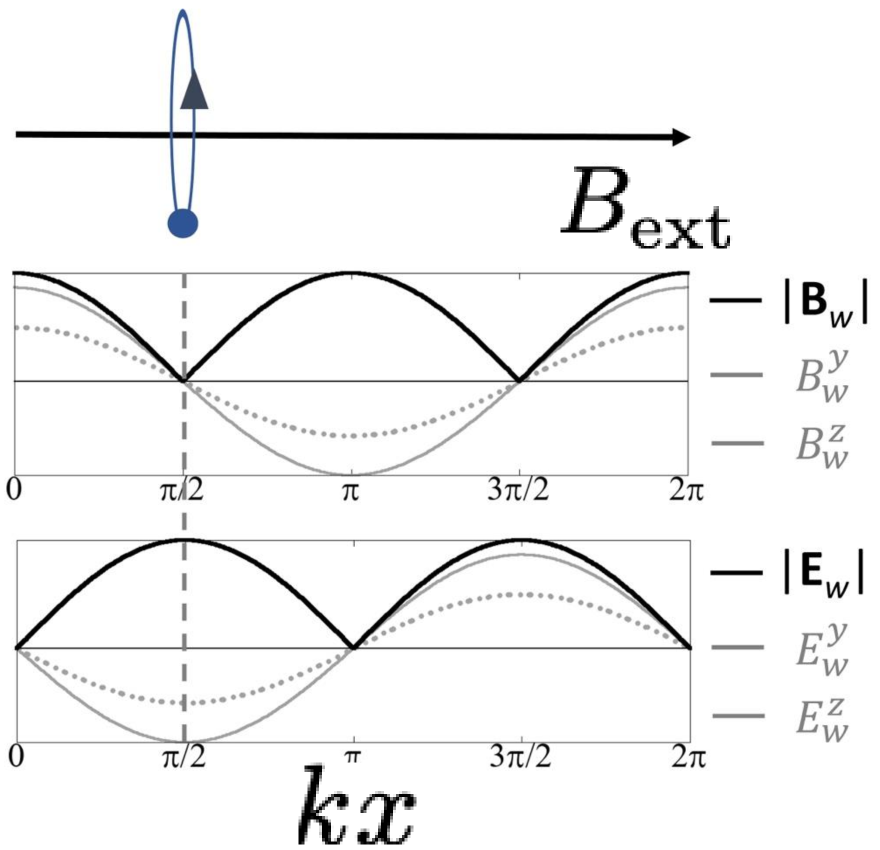
$$\boldsymbol{B} = \begin{pmatrix} B_{\text{ext}} \\ 0 \\ 0 \end{pmatrix}$$

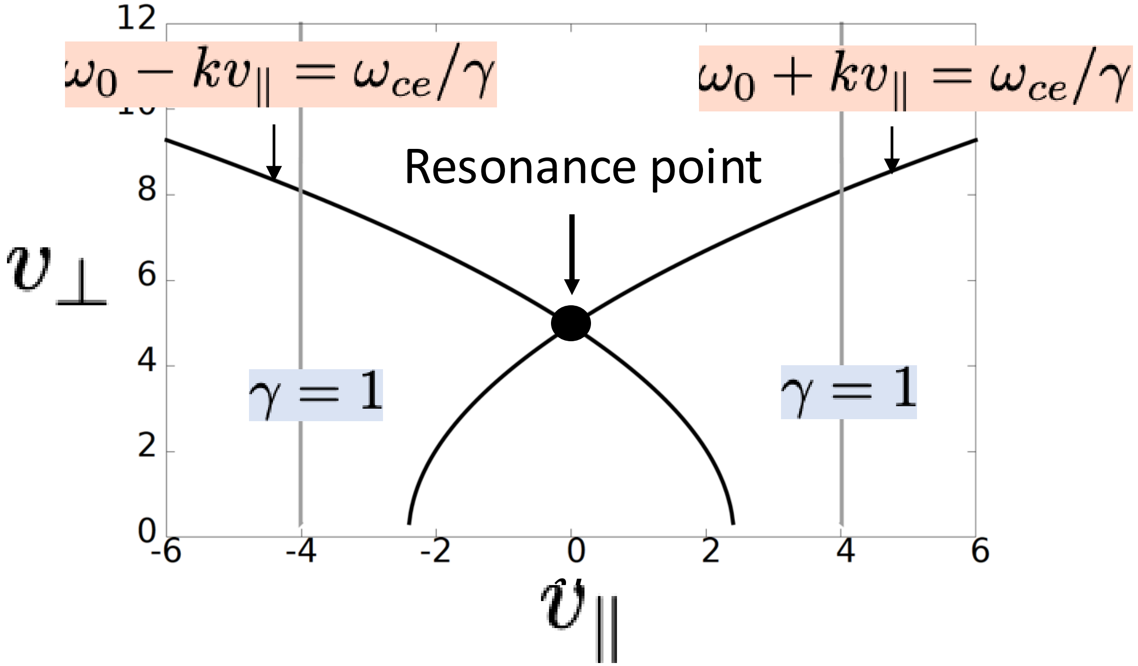
$$\boldsymbol{E} = -2E_0 \begin{pmatrix} 0 \\ \cos\omega_0 t \\ \sin\omega_0 t \end{pmatrix}$$

Simultaneous resonant acceleration occurs in wave envelope.

Relativistic resonant acceleration in counterpropagating Alfvén (Whistler) waves

Matsukiyo & Hada (2009), S. Isayama+ (2023), Sano+ (2024)





Simultaneous resonant acceleration occurs in wave envelope.

Electron trajectory at the acceleration point

Electron equation of motion

$$\frac{d\boldsymbol{p}}{dt} = -e\left(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}\right)$$

assuming

$$\boldsymbol{p} = \begin{pmatrix} p_{\parallel} \\ p_{\perp}\cos\phi \\ p_{\perp}\sin\phi \end{pmatrix} \qquad \psi = \phi - \omega_0 t$$

$$\begin{cases} \dfrac{d\widetilde{p}_{\parallel}}{d\widetilde{t}} = 0 \\ \dfrac{d\widetilde{p}_{\perp}}{d\widetilde{t}} = 2a_0\cos\psi \\ \dfrac{d\psi}{d\widetilde{t}} = -2a_0\dfrac{1}{\widetilde{p}_{\perp}}\sin\psi + \dfrac{\widetilde{B}_{\text{ext}}}{\gamma} - 1 \end{cases}$$

Hamiltonian for electron orbits

using $\chi \equiv \widetilde{p}_{\perp}^2$

$$H(\chi,\psi) = 4a_0\sqrt{\chi}\sin\chi - 2\widetilde{B}_{\text{ext}}\sqrt{\chi+1} + \chi$$

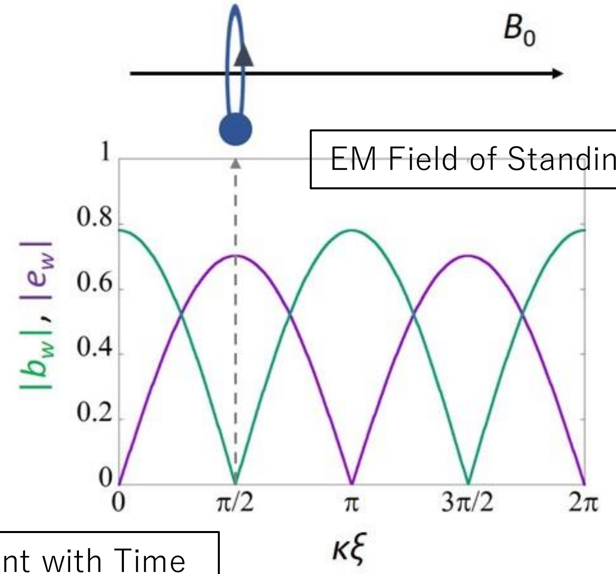
Phase Transition in Electron Trajectory: Free from the "Injection Problem"

- Momentum Equation at the Acceleration Point

Matsukiyo & Hada (2009)

$$\frac{d\widetilde{p}_\perp}{d\widetilde{t}} = 2a_0 \cos\psi$$

$$\frac{d\psi}{d\widetilde{t}} = -\frac{2a_0}{\widetilde{p}_\perp}\sin\psi + \frac{\widetilde{B}_{\mathrm{ext}}}{\gamma} - 1$$

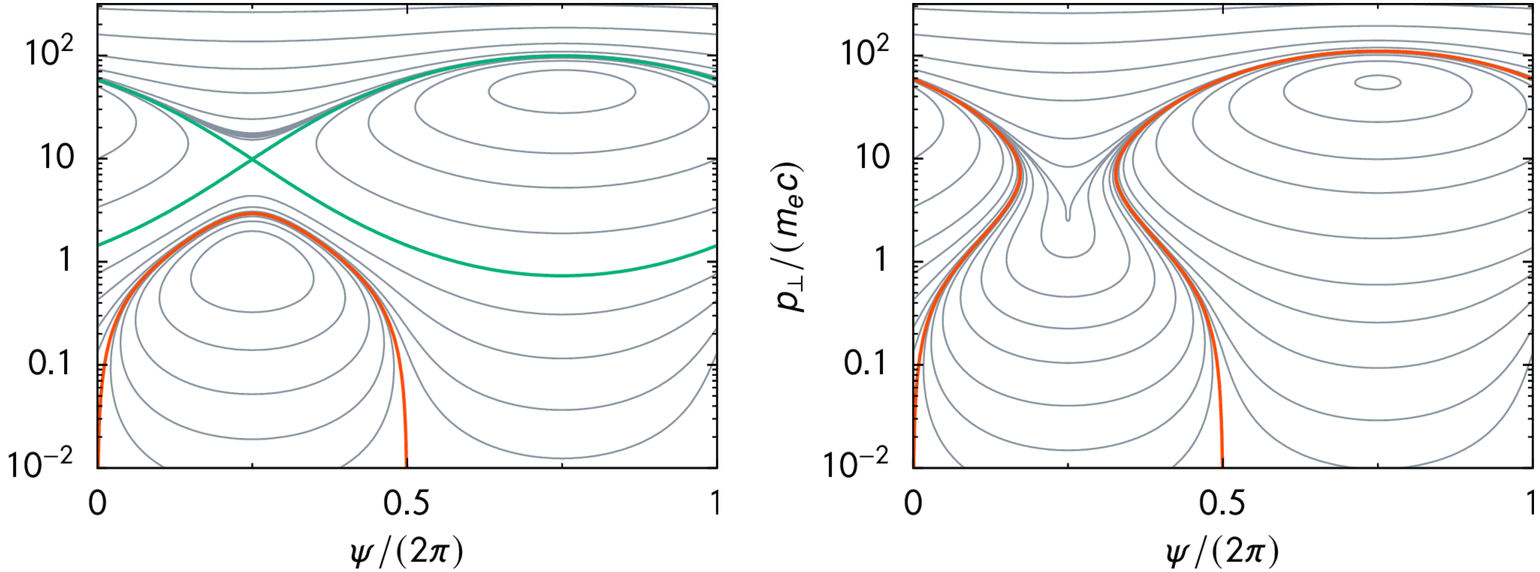


EM Field of Standing Wave

Constant with Time

Electron Trajectory in Momentum-Phase Diagram

Obtained as countour line of Hamiltonian



Small Amplitude Wave

- Non-relativistic and relativistic orbits are separated.

Isayama et al. (2023)

Large Amplitude Wave

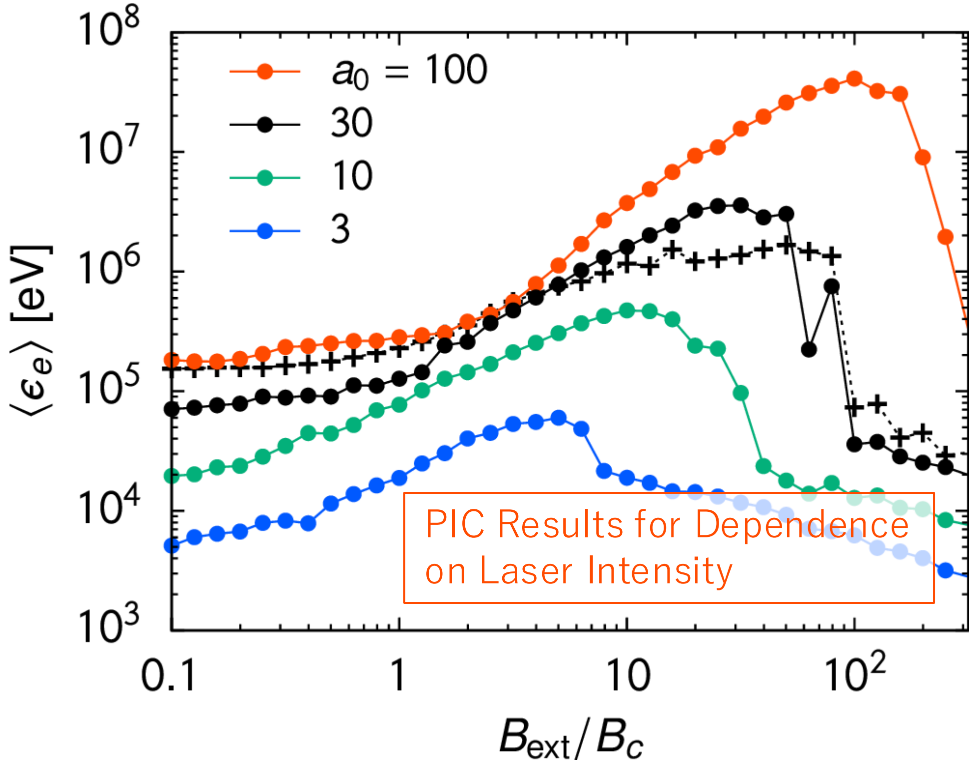
- All electrons can gain relativistic velocities
- Two-wave resonance

Requirement for phase transition is that wave amplitude is larger than the external field.

- Condition for Phase Transition

$$2a_0 \gtrsim (\widetilde{B}^{2/3} - 1)^{3/2}$$

$$1 \lesssim \frac{B_{\rm ext}}{B_c} \lesssim a_0$$

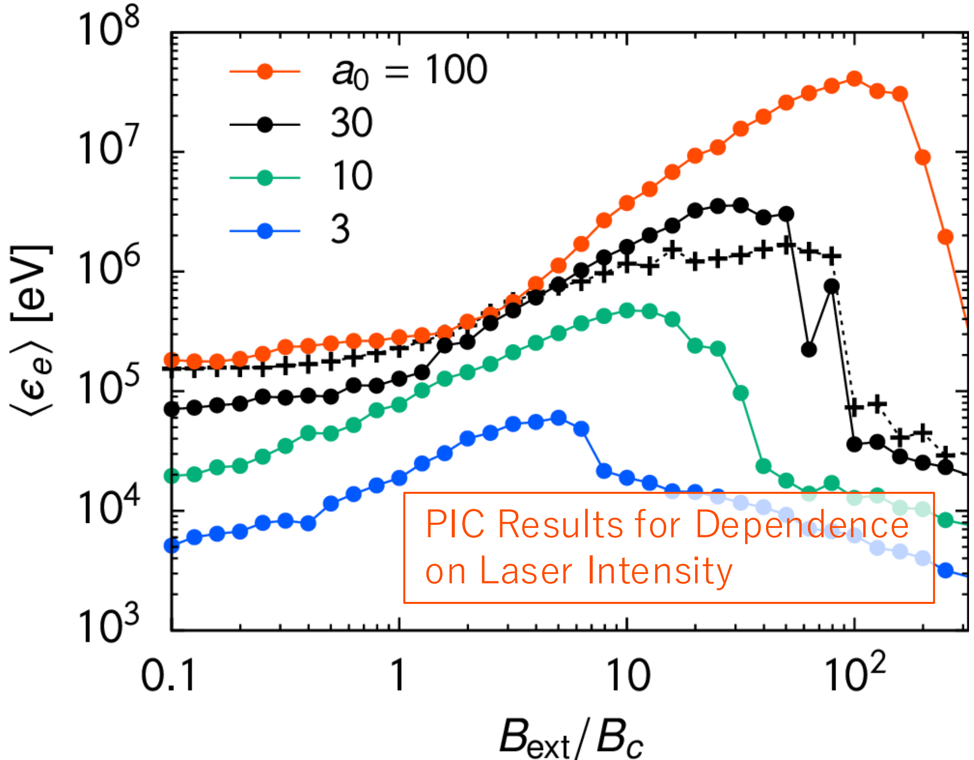


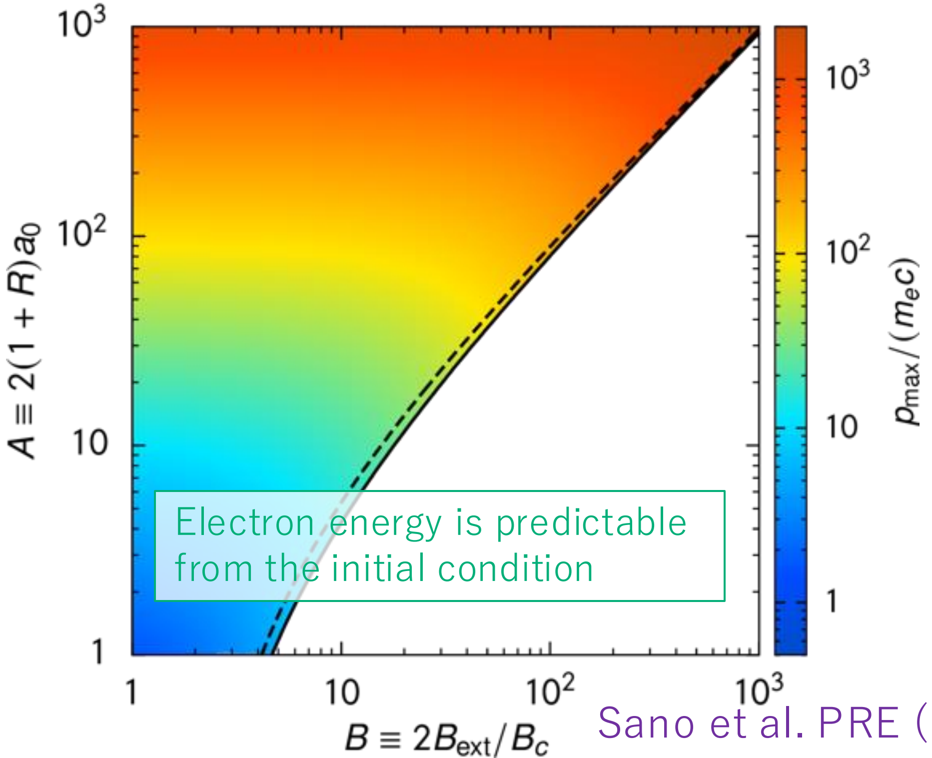
Sano et al. PRE (2024)

Requirement for phase transition is that wave amplitude is larger than the external field.

- Condition for Phase Transition $2a_0 \gtrsim (\widetilde{B}^{2/3} - 1)^{3/2}$
- The maximum energy can also be derived analytically.

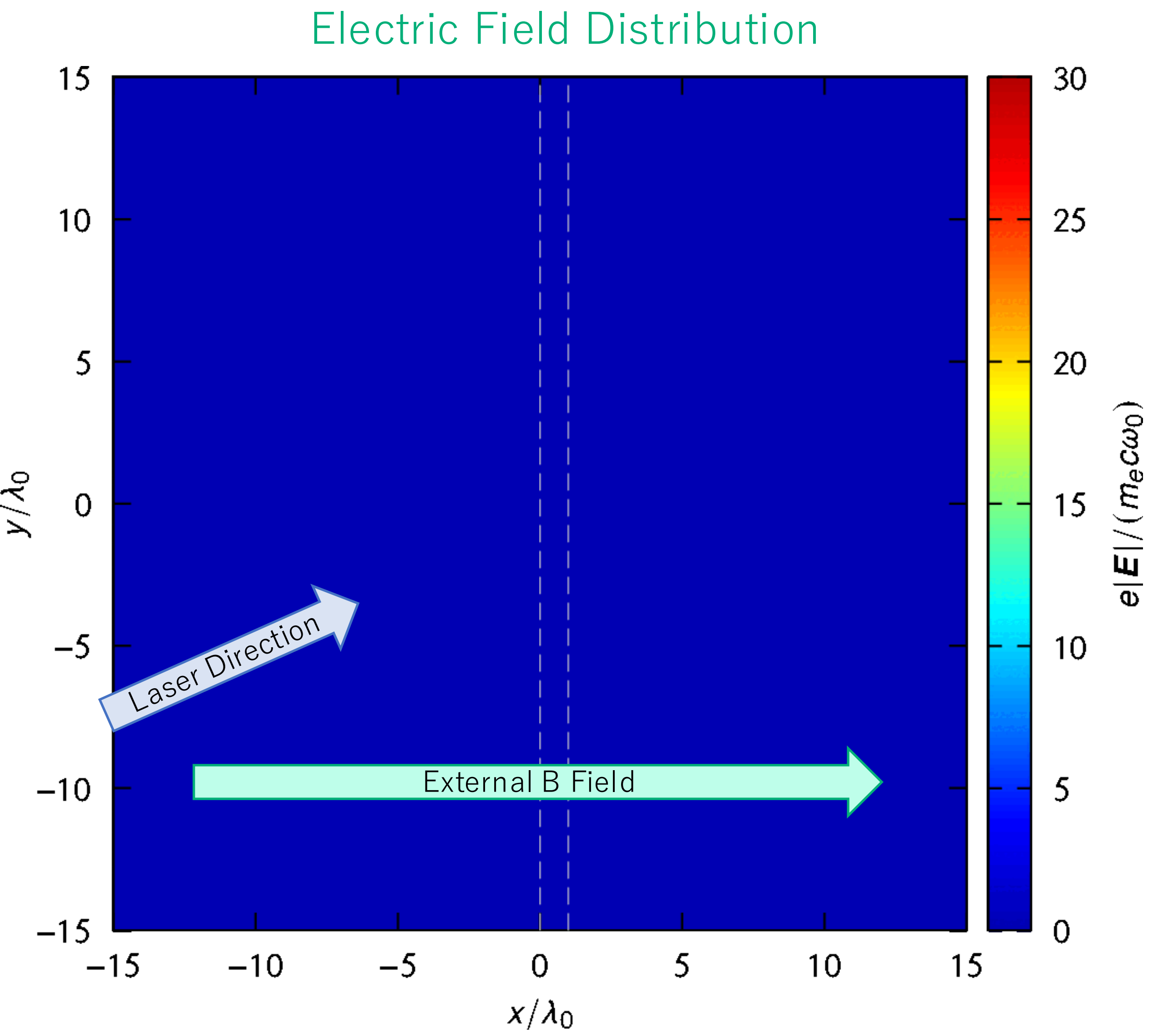
$$\widetilde{p}_{\max} \approx 4a_0 + 2[\widetilde{B}(\widetilde{B} - 1)]^{1/2}$$

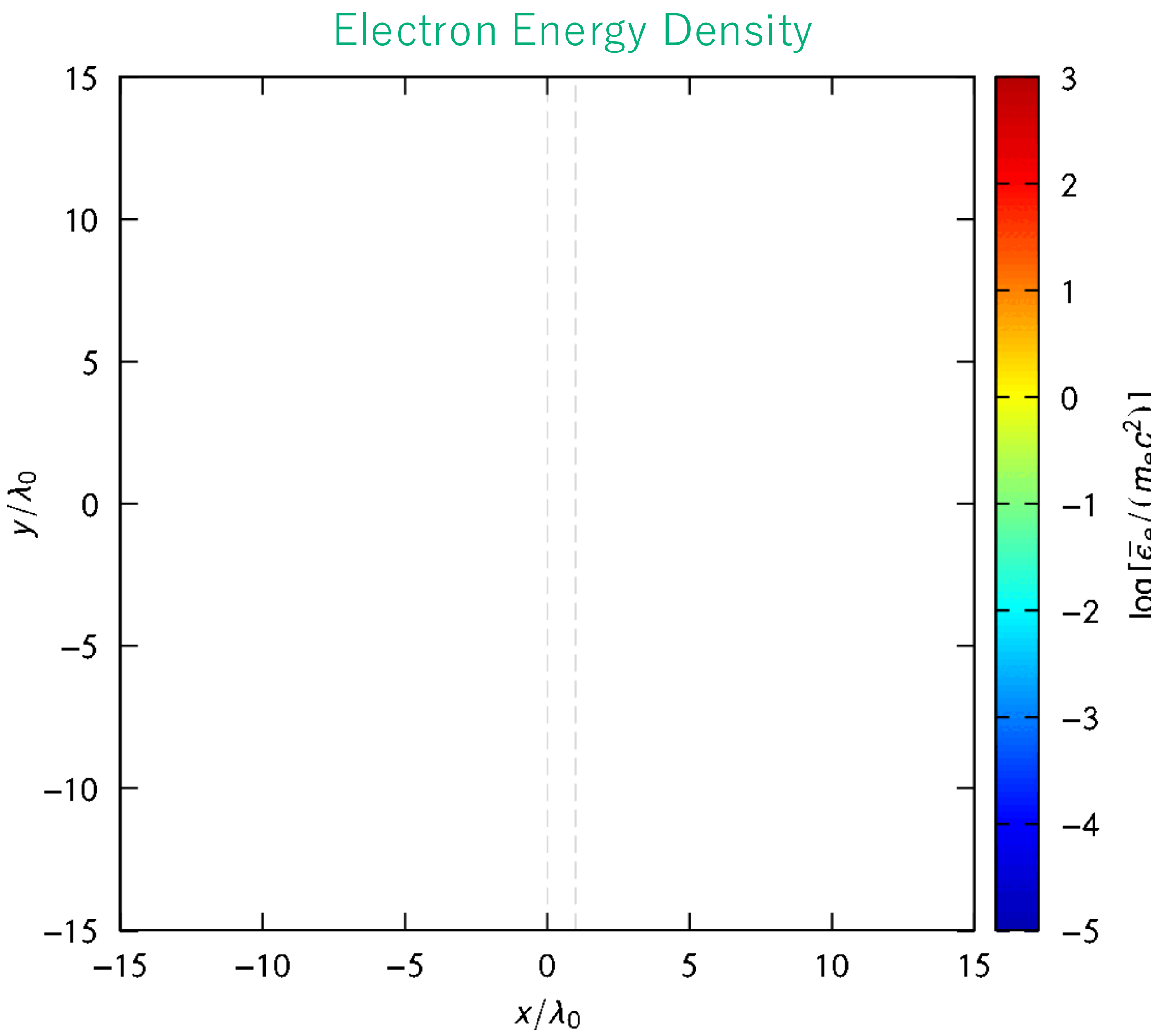




Sano et al. PRE (2024)

Hot electrons are generated by the same mechanism evne in 2D PIC simulations.





Summary

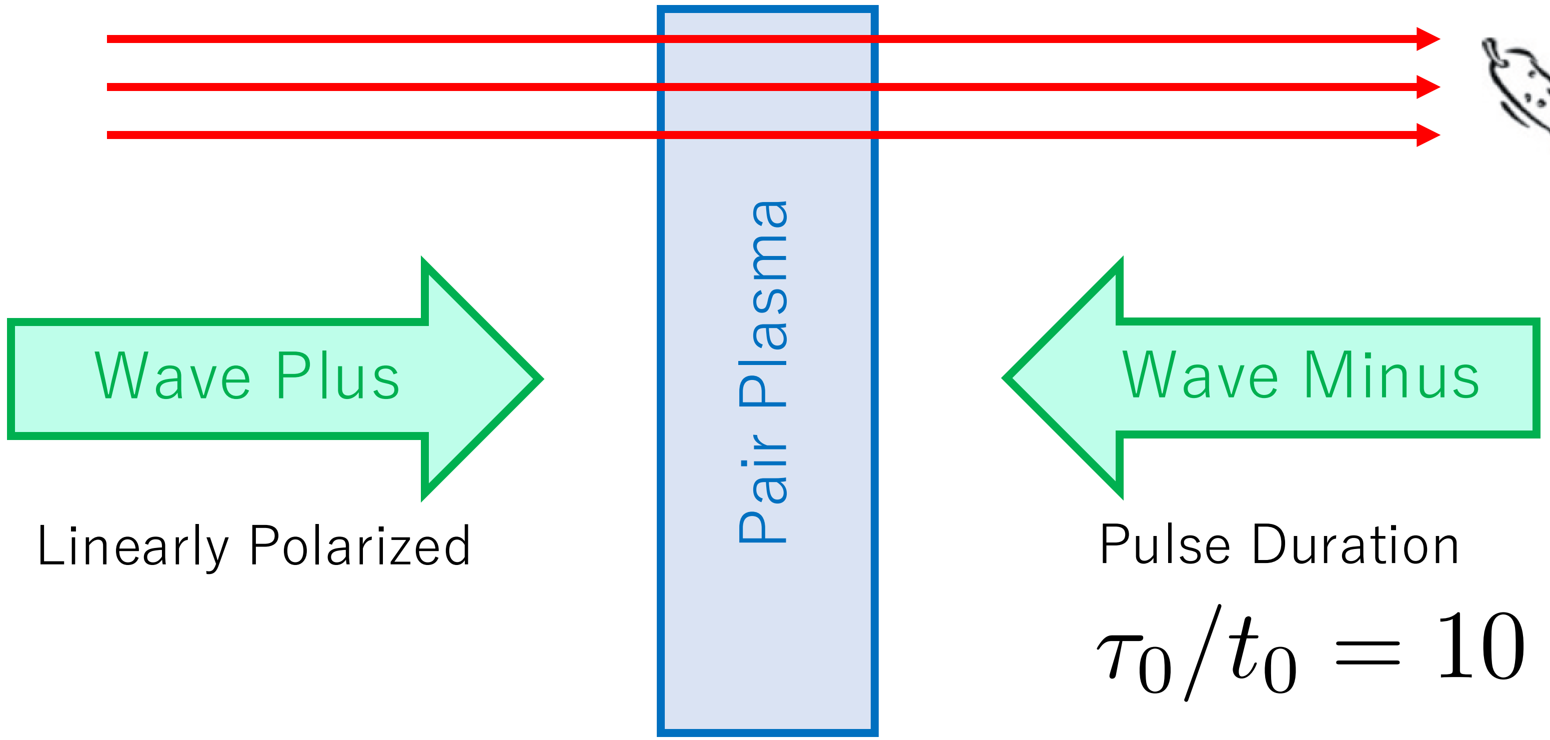
- Laser-plasma interaction in a strong magnetic field is an important process not only in laser plasmas but also in astrophysical phenomena.
- Efficient plasma heating occurs in standing waves created by opposing whistler (Alfven) waves.
- Depending on the strength of the magnetic field, the laser energy is transported to electrons or ions.
- If magnetic fields in excess of 10 kT become available, this could be the subject of new laser astrophysics experiments.

$$\widetilde{n} = \frac{n_e}{n_c} = \frac{\omega_{pe}^2}{\omega_0^2} \qquad \widetilde{B} = \frac{B_{\mathrm{ext}}}{B_c} = \frac{\omega_{ce}}{\omega_0} \qquad a_0 = \frac{eE_0}{m_e c \omega_0}$$

Key parameters are "plasma density", "B field strength", "wave amplitude".

Pair plasma target is irradiated by linearly-polarized two waves to generate a standing wave in the target.

- Initial Setup for 1D PIC Simulation including Radiation Damping



PICLS

Linearly Polarized

Pulse Duration

$$\tau_0/t_0 = 10$$

Plasma Thickness

$$L_x/\lambda_0 = 30$$

Key Parameters

$$\widetilde{n}_{e0} = 100$$

$$\widetilde{B}_{\mathrm{ext}} = 300$$

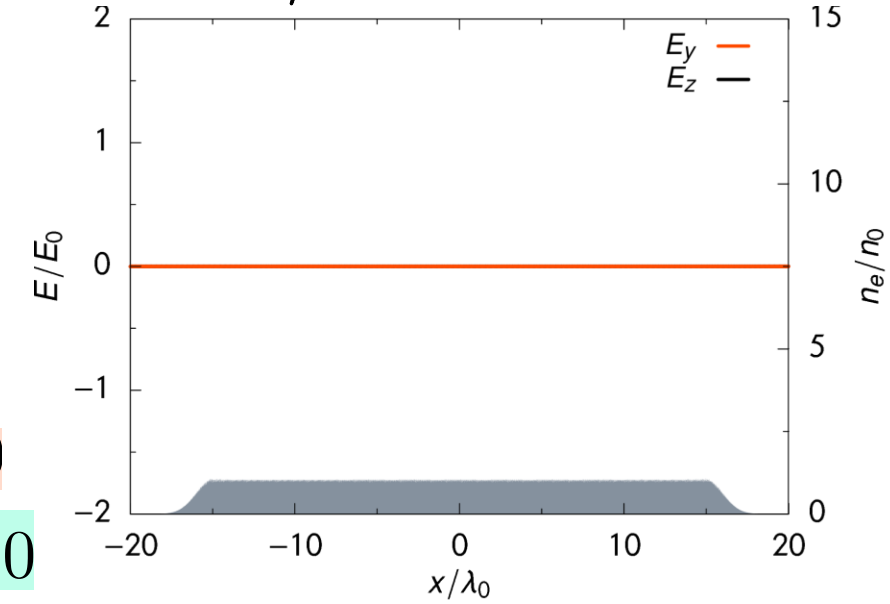
$$a_0 = 300$$

$$\widetilde{n}_e \equiv \frac{n_e}{n_c}$$

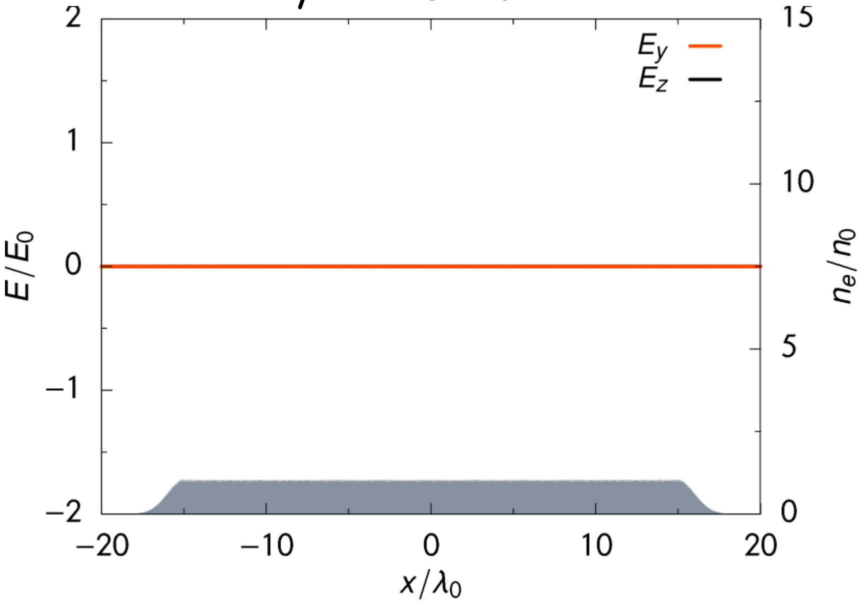
$$\widetilde{B}_{\mathrm{ext}} \equiv \frac{B_{\mathrm{ext}}}{B_c}$$

A linearly polarized electromagneticwave can propagate stably in overdense pair plasma.

$$a_0 = 30$$
$$\delta B / B_{\rm ext} = 0.1$$



$$a_0 = 300$$
$$\delta B / B_{\rm ext} = 1$$



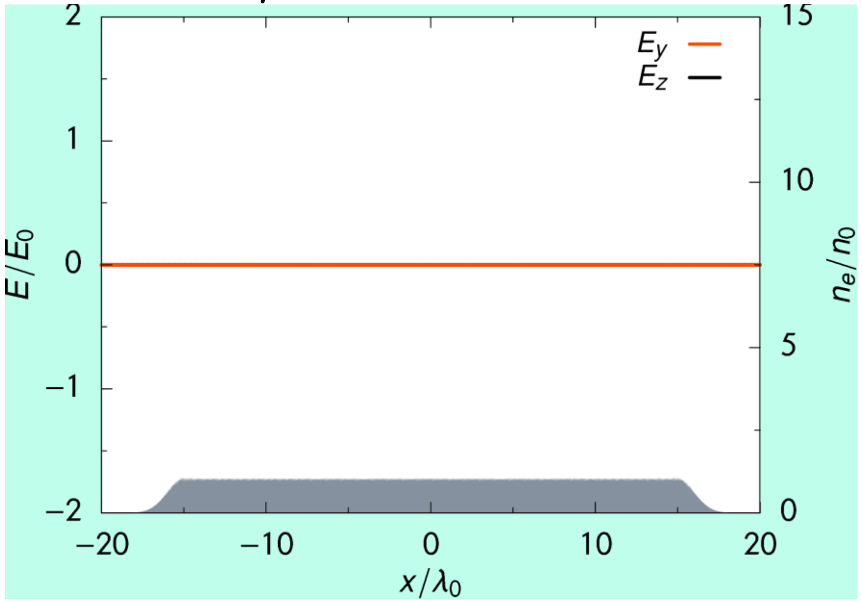
$\widetilde{n}_{e0} = 100$

$\widetilde{B}_{\rm ext} = 300$

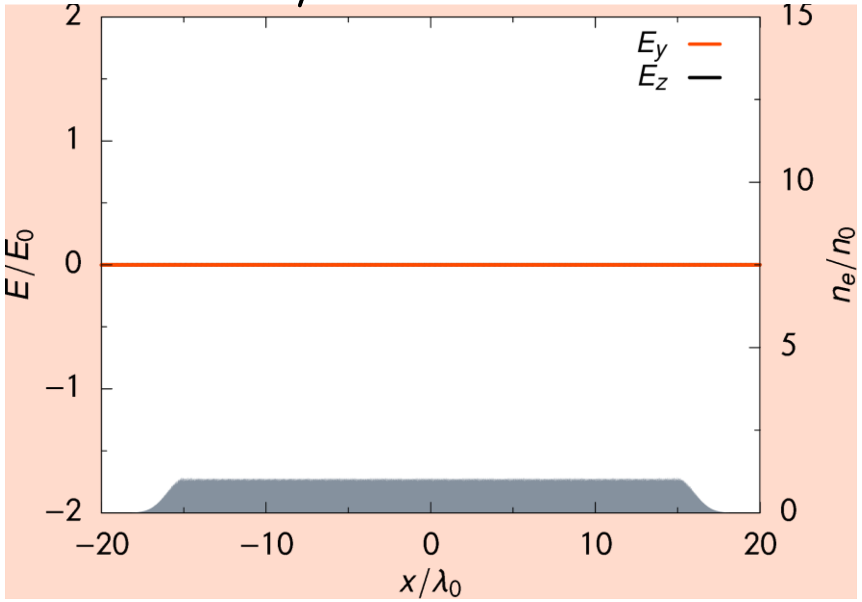
Characteristics of counter wave propagation depends strongly on the wave amplitude.

- When the magnetic field amplitude of a wave is less than the external field, counter waves pass through.
- However, efficient energy conversion from the wave to electrons occurs if the wave amplitude becomes comparable to the external field.

$$\delta B/B_{\mathrm{ext}} = 0.1$$

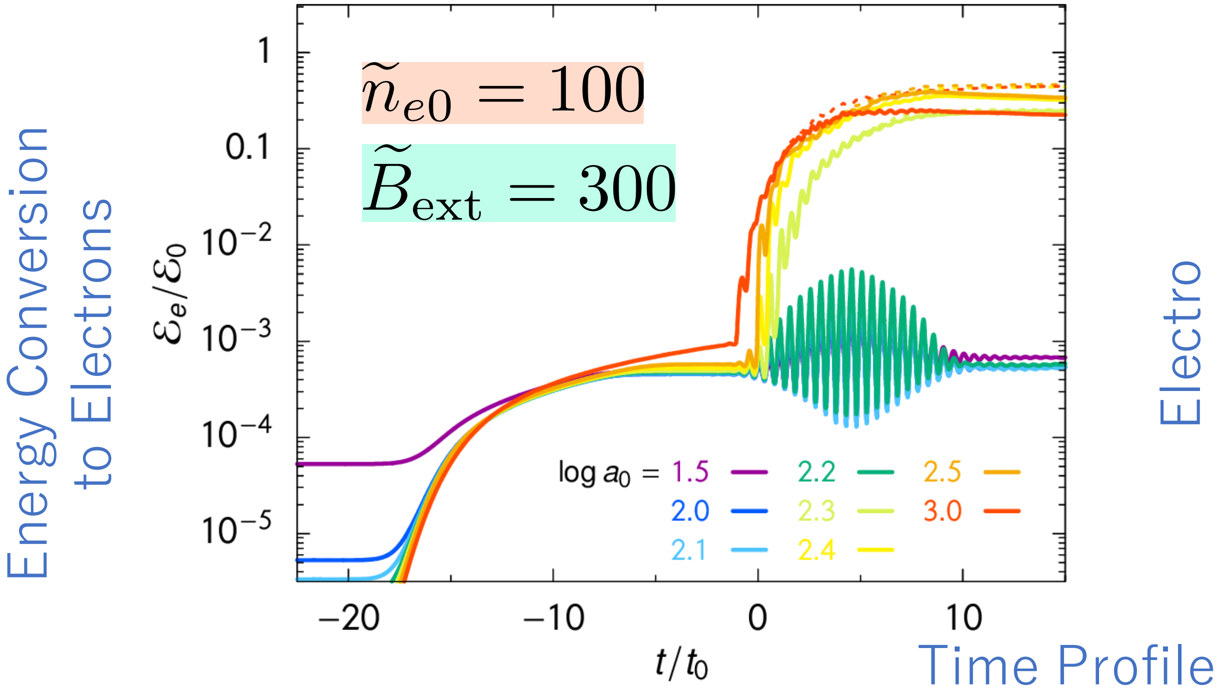


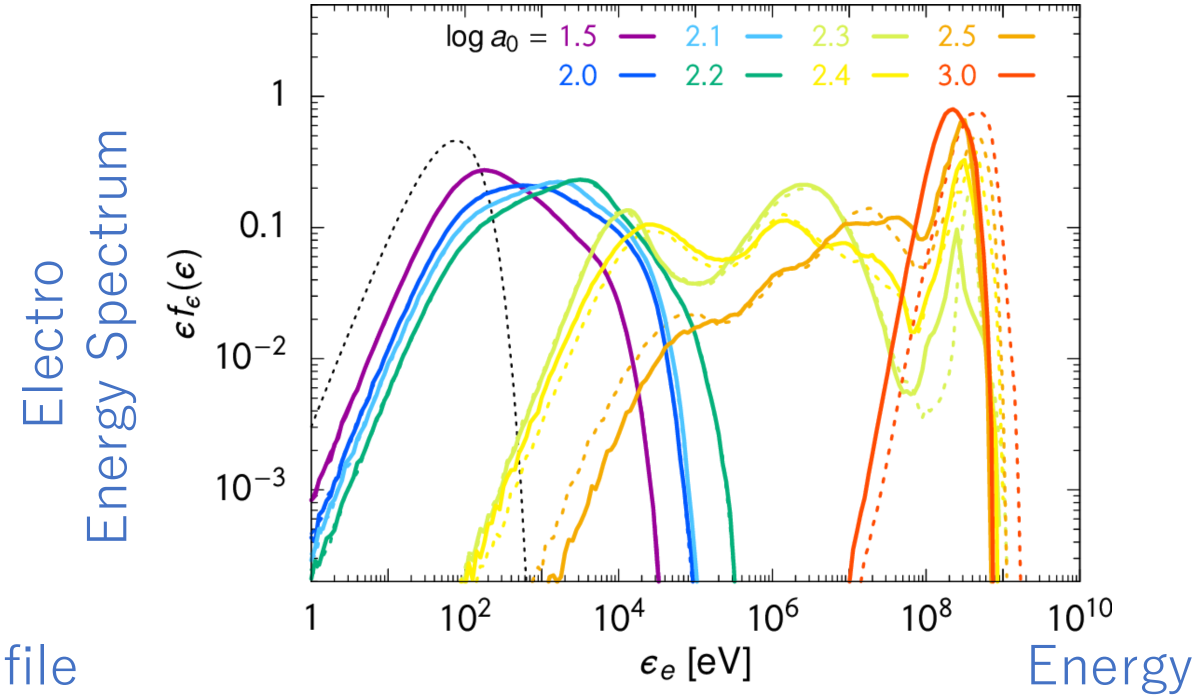
$$\delta B/B_{\mathrm{ext}} = 1$$



The conditions for efficient electron acceleration is derived by two-wave resonance in a standing wave.

- There is a rapid transition to electron acceleration, depending on the amplitude of the wave.
- Entire electrons in standing wave can be accelerated to relativistic velocity in a short timescale.





Radiation damping effect is significant when the wave amplitude becoms highly relativistic.

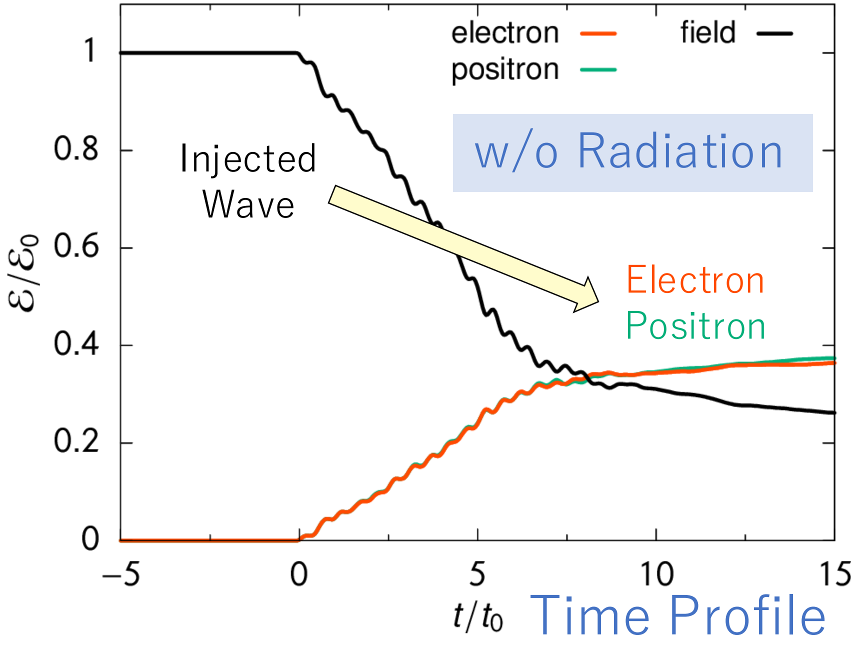
- High-energy electrons are generated in a standing whistler wave, which can emit gamma-rays.
- Injected laser energy is converted to the radiation energy when the wave amplitude is very high.

$$a_0 \gtrsim 1000$$

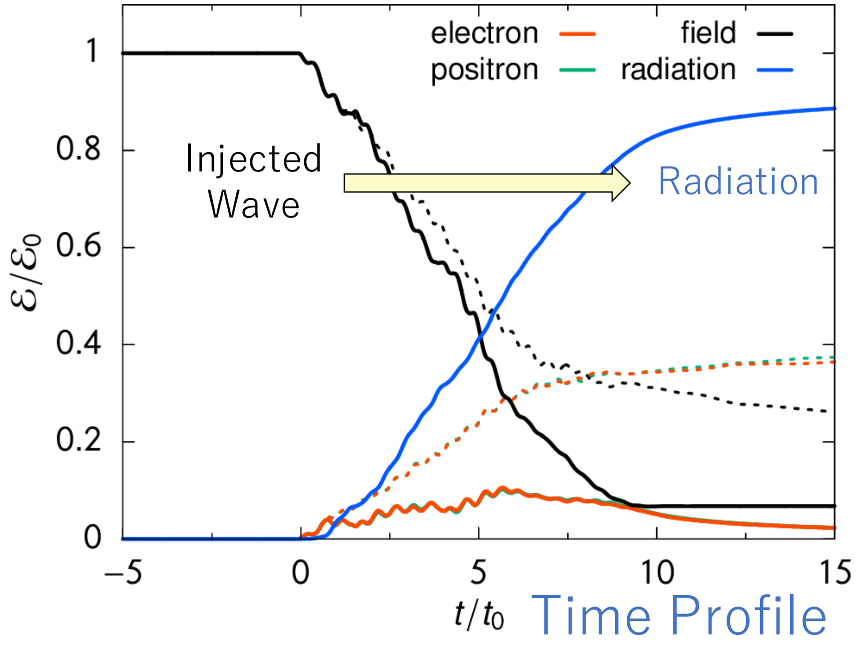
$$\tilde{n}_{e0} = 100$$

$$\tilde{B}_{\rm ext} = 1000$$

$$a_0 = 1000$$



w/ Radiation



Summary

- Standing whistler waves can accelerate either electrons or ions depending on the external magnetic field strength.

$$\widetilde{B} = \frac{B_{\mathrm{ext}}}{B_c} = \frac{\omega_{ce}}{\omega_0}$$

- If magnetic fields in excess of 10 kT become available, this could be the subject of new laser astrophysics experiments.

Magnetic Field Strength

$\widetilde{B} \sim 1$

Electron Acceleration

$$1 \lesssim \frac{\omega_{ce}}{\omega_0} \lesssim a_0$$

$\widetilde{B} \sim a_0$

$$\frac{\omega_{ce}}{\omega_0} \gtrsim a_0$$

Ion Heating