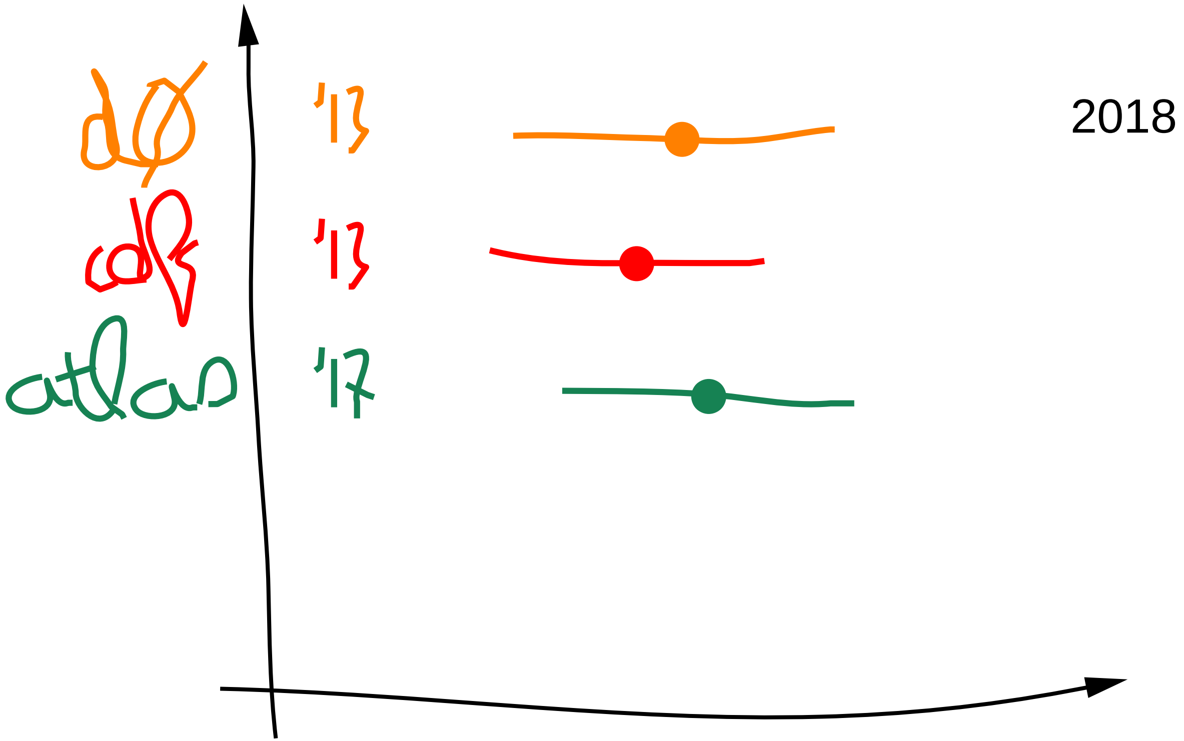
W mass combinations

- State of the art
- Comparisons
- Combinations
- What to expect?



M. Boonekamp

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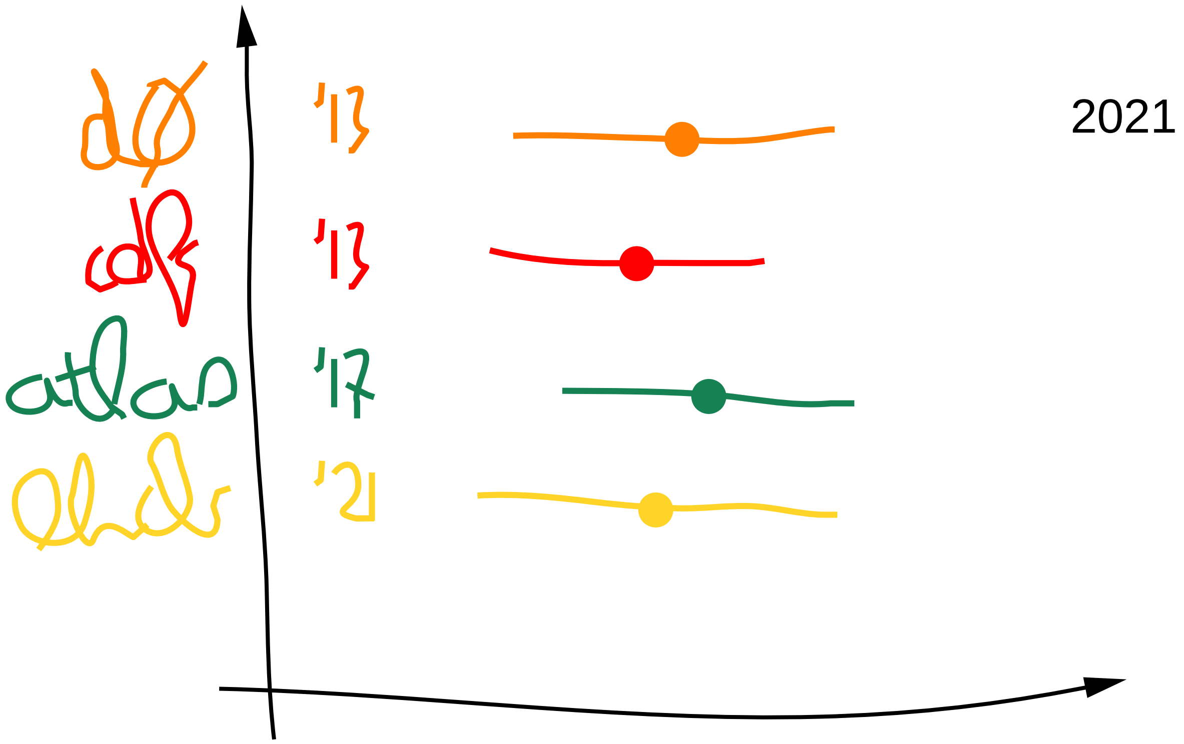

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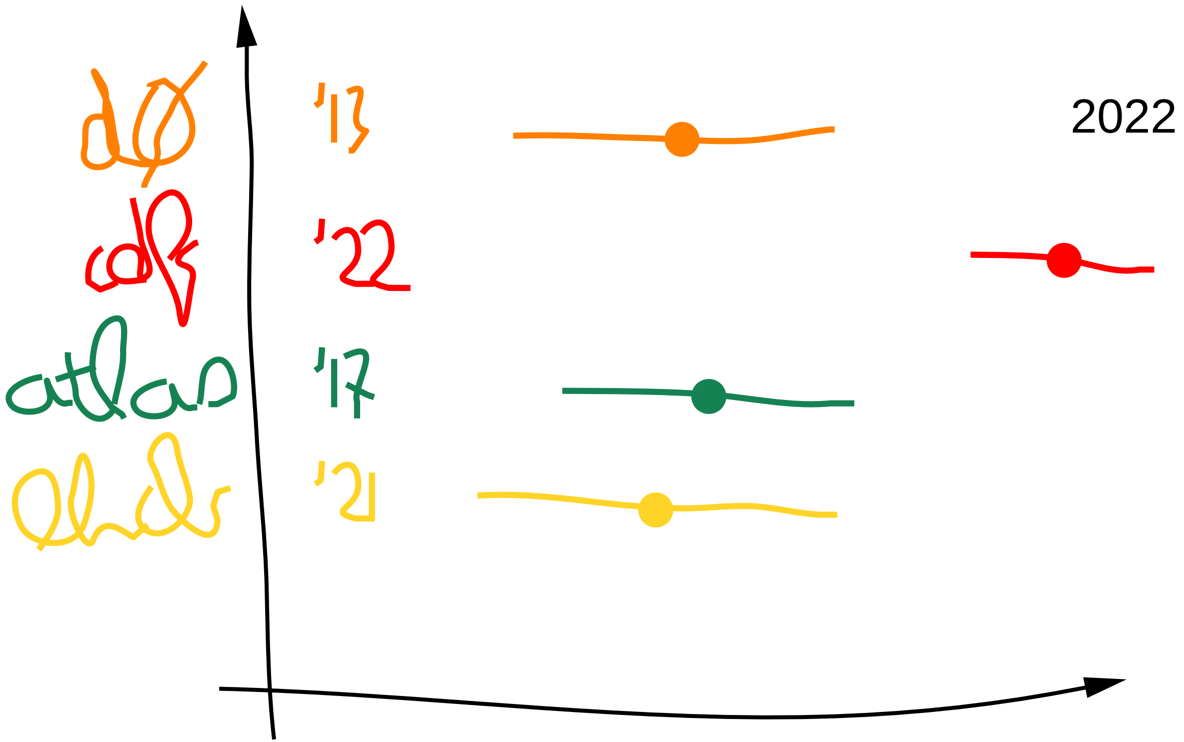

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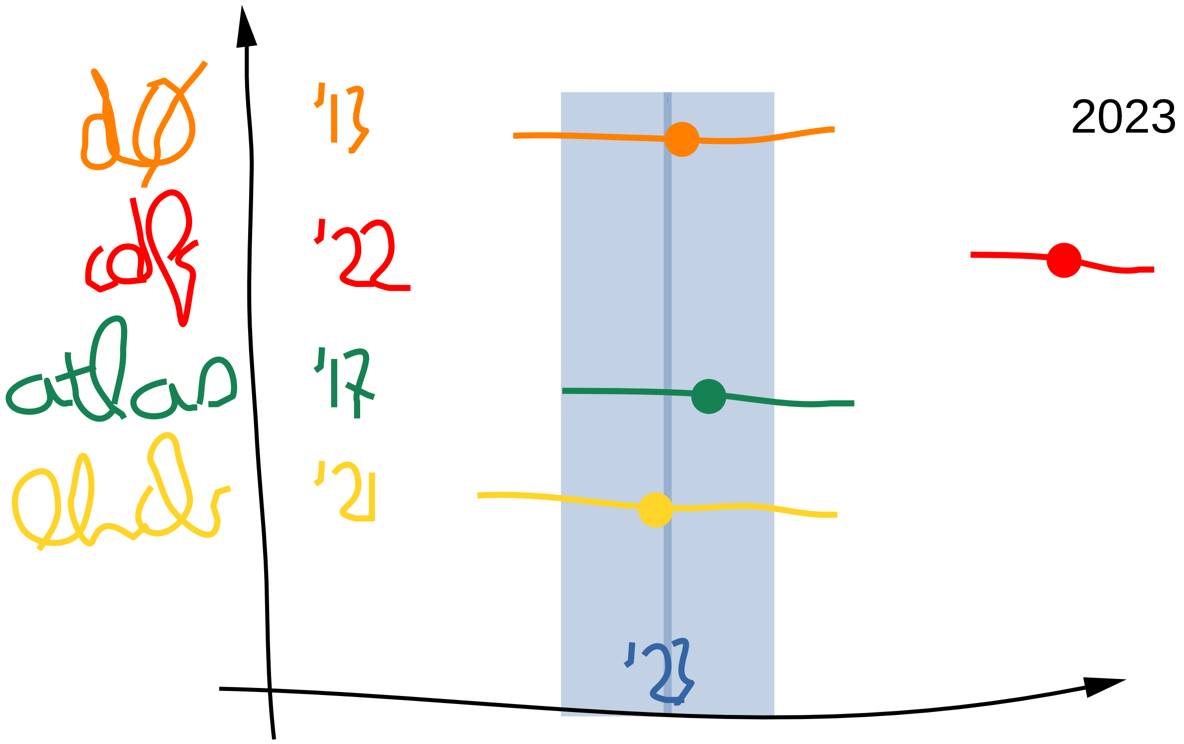

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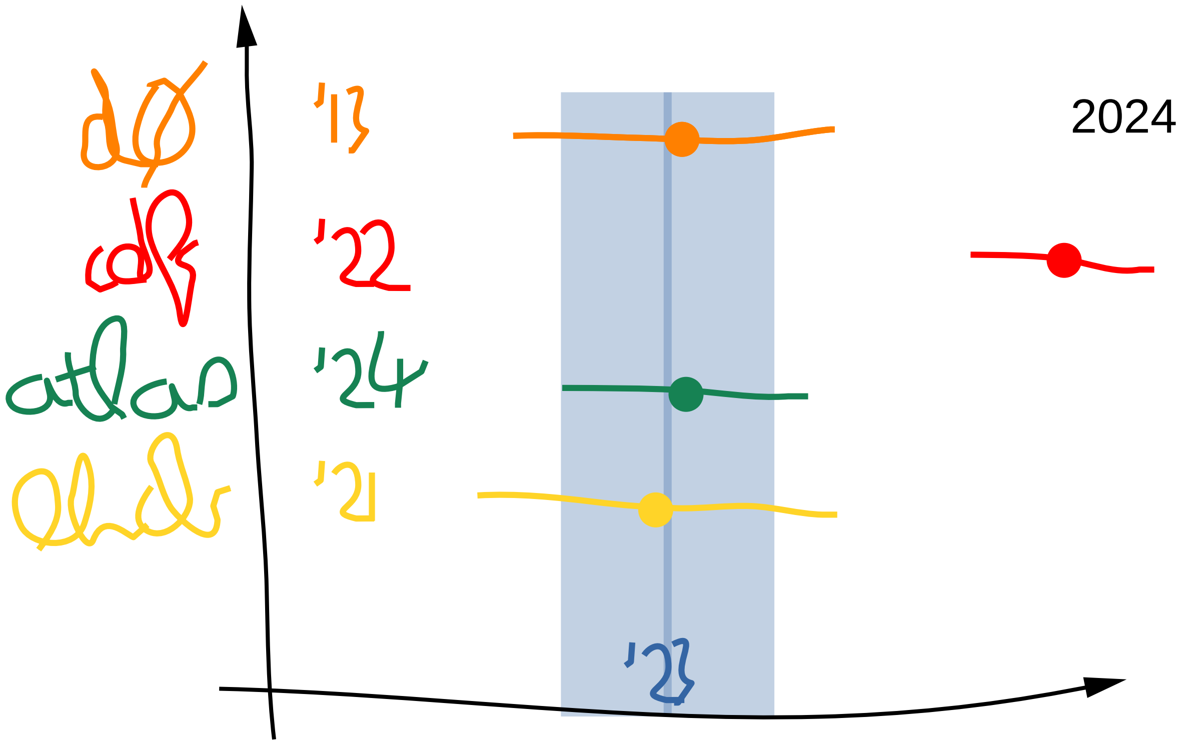

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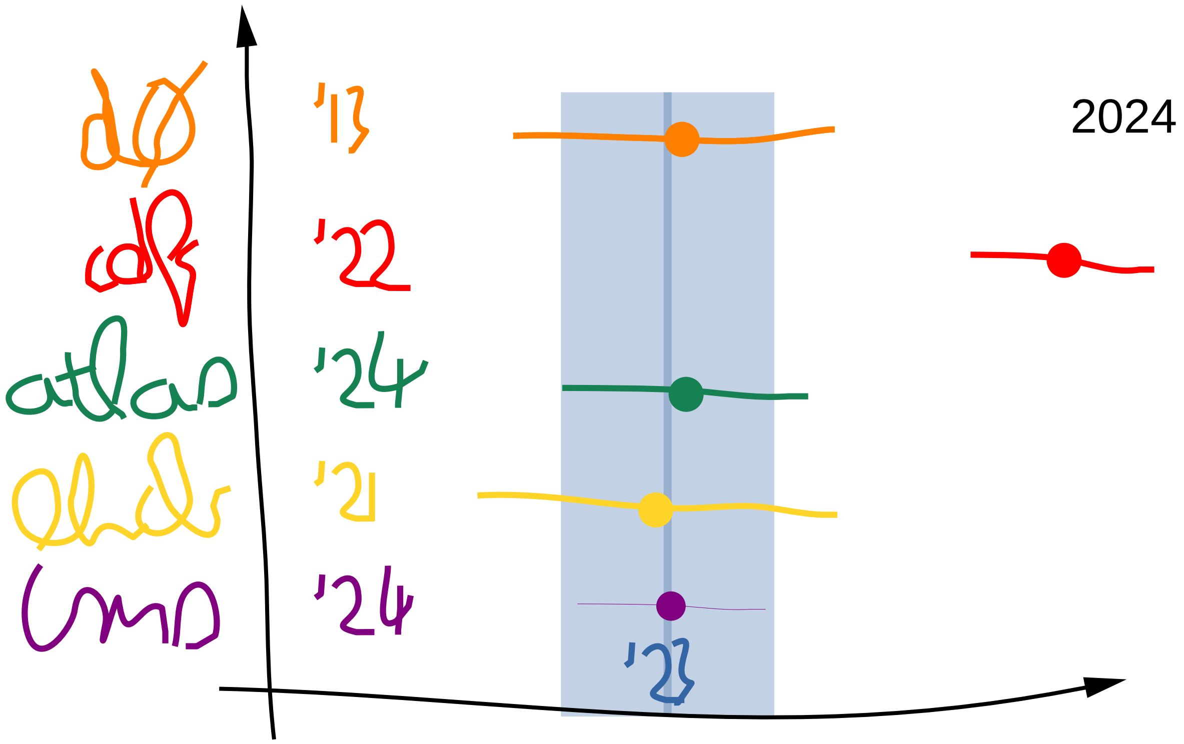
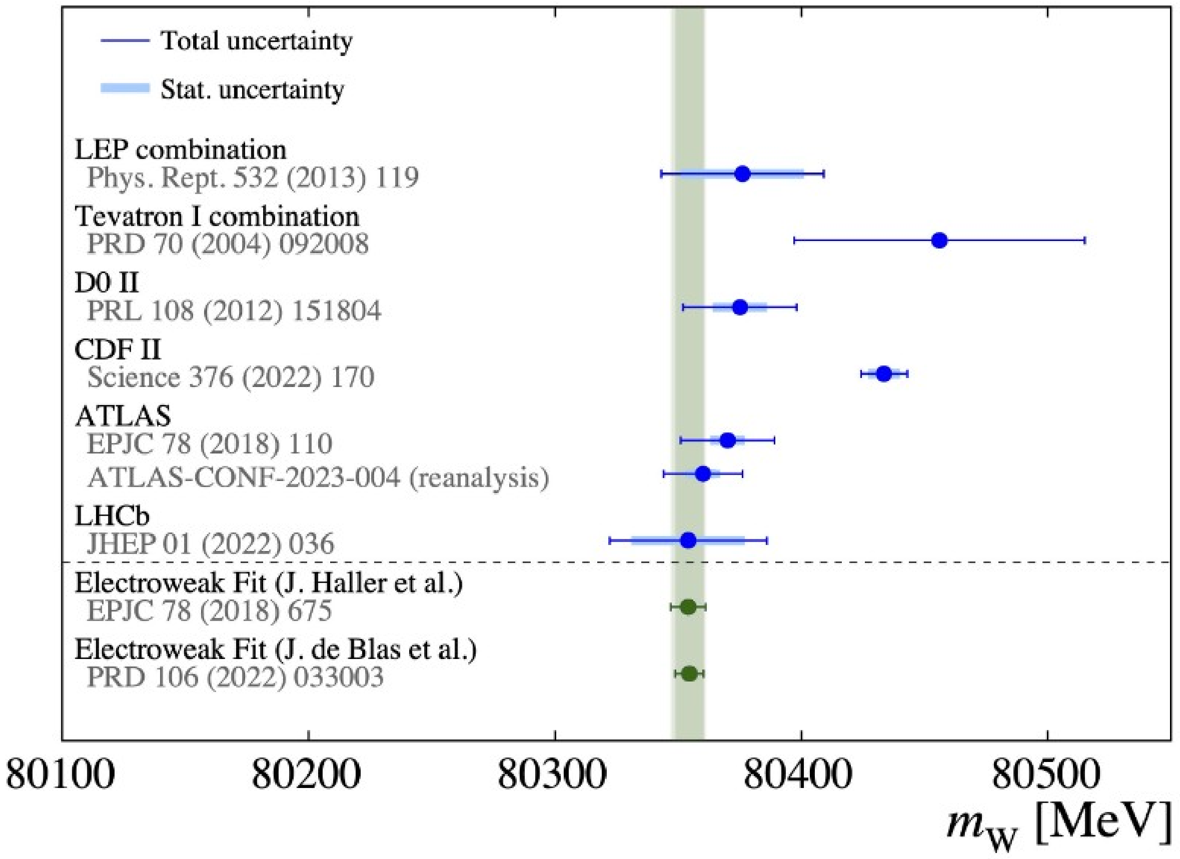


Summary of 2023 combination



Input measurements

- CDF – $p\bar{p}$ collisions @ $\sqrt{s}$ = 1.96 TeV; fit variables are p_T^l , p_T^ν and m_T.
- D0 – two separate measurements using $p\bar{p}$ collisions @ $\sqrt{s}$ = 1.96 TeV; fit variables are p_T^e, m_T and p_T^ν.
- ATLAS – pp collisions @ $\sqrt{s}$ = 7 TeV; central region at LHC; fit variables are p_T^l and m_T. *[Original analysis used following agreement to use published results]*
- LHCb – pp collisions @ $\sqrt{s}$ = 13 TeV; forward region at LHC; fit variable is q/p_T^μ.

- LEP – legacy combination from LEP experiments.

Experiment	Event requirements	Fit ranges
CDF	$30 < p_T^\ell < 55$ GeV $\lvert\eta_\ell\rvert < 1$ $30 < E_T^{miss} < 55$ GeV $65 < m_T < 90$ GeV $u_T < 15$ GeV	$32 < p_T^\ell < 48$ GeV $32 < E_T^{miss} < 48$ GeV $60 < m_T < 100$ GeV
D0	$p_T^e > 25$ GeV $\lvert\eta_\ell\rvert < 1.05$ $E_T^{miss} > 25$ GeV $m_T > 50$ GeV $u_T < 15$ GeV	$32 < p_T^e < 48$ GeV $65 < m_T < 90$ GeV
ATLAS	$p_T^\ell > 30$ GeV $\lvert\eta_\ell\rvert < 2.4$ $E_T^{miss} > 30$ GeV $m_T > 60$ GeV $u_T < 30$ GeV	$32 < p_T^\ell < 45$ GeV $66 < m_T < 99$ GeV
LHCb	$p_T^\mu > 24$ GeV $2.2 < \eta_\mu < 4.4$	$28 < p_T^\mu < 52$ GeV

QCD aspects

- Measurements made over a number of years
- QCD understanding has significantly improved over this time (modelling of PDFs, Matrix Element, Soft Radiation).
- Newer measurements typically use an improved QCD model than older measurements.

- Seek to 'update' measurements to a common QCD framework before compatibility assessed and combination.

Update to common PDF

Additional (small) updates

$$m_W^{update} = \underbrace{m_W^{ref}}_{\text{Published value}} + \delta m_W^{PDF} + \underbrace{\delta m_W^{pol}}_{\text{Common W boson polarization update}} + \delta m_W^{other}$$

Published value

Common W boson polarization update

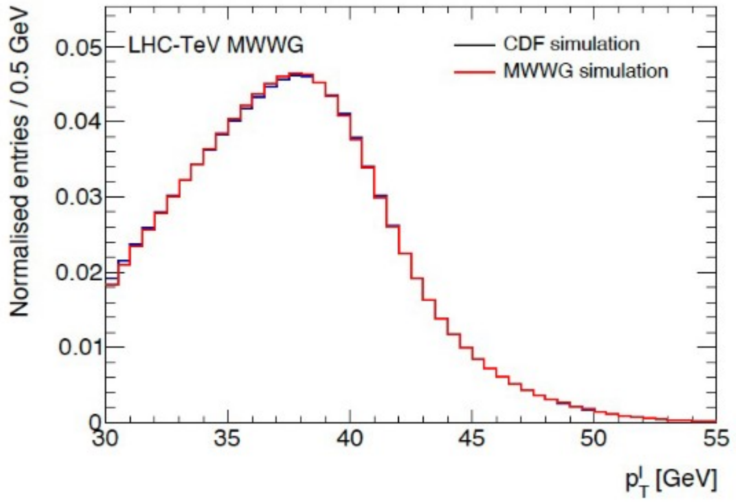
QCD aspects

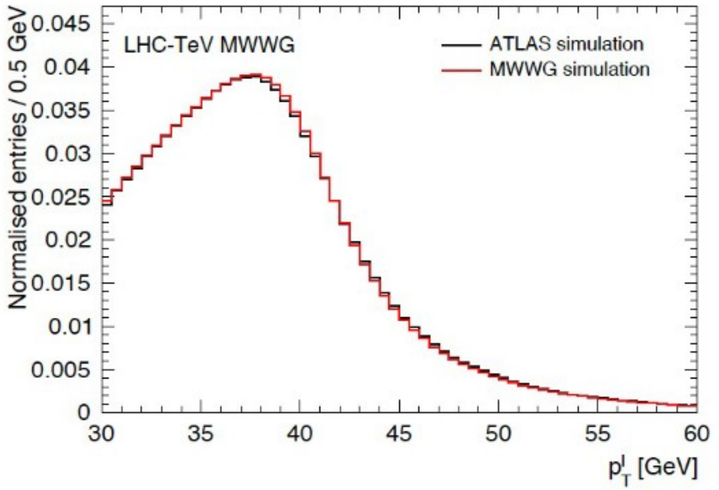
- Starting point of fits to data therefore crucial.
  - D0: RESBOS CP (N2LO, N2LL) with CTEQ66 PDFs (NLO)
  - CDF: RESBOS C (NLO, N2LL) with CTEQ6M PDFs (NLO) [CDF publication applied a correction to reproduce Resbos2 + NNPDF3.1]
  - ATLAS: POWHEG + Pythia8 (NLO+PS) with DYTurbo for Angular Distribution (N2LO) with CT10 PDFs (NNLO)
  - LHCb: POWHEG + Pythia8 (NLO+PS) with DYTurbo for Angular Distribution (N2LO) with averaged result from MSHT20, NNPDF31 and CT18 PDFs (NLO)

- Approach taken:
  - LHCb measurement "repeated" using same code framework but with PDF updates.
  - Effect of updates on other measurements using simulated samples from two models.

Detector Emulation

- ATLAS, CDF and D0 detectors emulated.
  - η- and p_T-dependent smearing of leptons.
  - Recoil modelling includes lepton removal and and event activity effects.
  - Agreement typically at the percent level between full simulation and LHC-TeV MWWG emulation.
  - Small imperfections in emulation lead to MeV-level uncertainties on δm_W.





Analysis code rerun

PDF effects

Using the POWHEG MiNNLO generator to study impact of changes in the relevant distribution.

δm_W^{PDF}

PDF set	D0 p_T^ℓ	D0 E_T^{miss}	CDF p_T^ℓ	CDF E_T^{miss}	ATLAS W^+	ATLAS W^-	LHCb
CTEQ6	-17.0	-17.7	0.0	0.0	–	–	–
CTEQ6.6	0.0	0.0	15.0	17.0	–	–	–
CT10	0.4	-1.3	16.0	16.3	0.0	0.0	–
CT14	-9.7	-10.6	5.8	6.8	-1.2	-5.8	-1.1
CT18	-8.2	-9.3	7.2	7.7	12.1	-2.3	6.0
ABMP16	-19.6	-21.5	-1.4	-2.4	-22.5	-3.1	7.6
MMHT2014	-10.4	-12.7	6.1	5.5	-2.6	9.9	11.8
MSHT20	-13.7	-15.4	3.6	4.1	-20.9	4.5	2.0
NNPDF3.1	-1.0	-1.2	14.0	15.1	-14.1	1.8	-6.0
NNPDF4.0	6.7	8.1	20.8	24.1	-22.4	6.9	-8.3

$\sigma(m_W^{PDF})$

PDF set	D0	CDF	ATLAS	LHCb
CTEQ6	–	14.1	–	–
CTEQ66	15.1	–	–	–
CT10	–	–	9.2	–
CT14	13.8	12.4	11.4	10.8
CT18	14.9	13.4	10.0	12.2
ABMP16	4.5	3.9	4.0	3.0
MMHT2014	8.8	7.7	8.8	8.0
MSHT20	9.4	8.5	7.8	6.8
NNPDF3.1	7.7	6.6	7.4	7.0
NNPDF4.0	8.6	7.7	5.3	4.1

Note Tevatron combination did not consider δm_W^{PDF}(CTEQ6, CTEQ6.6)~17 MeV.

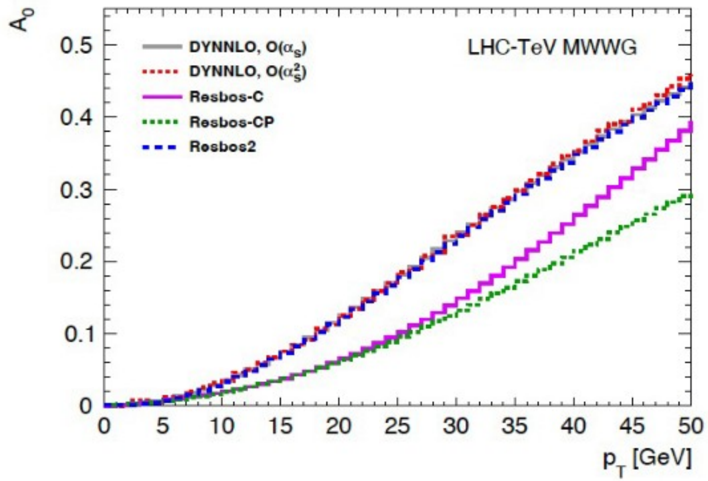
Uncertainties larger here than in some original publications, e.g. NNPDF3.1 ~6.6 MeV (CDF) here v 3.9 MeV in original publication (which used principal component analysis).

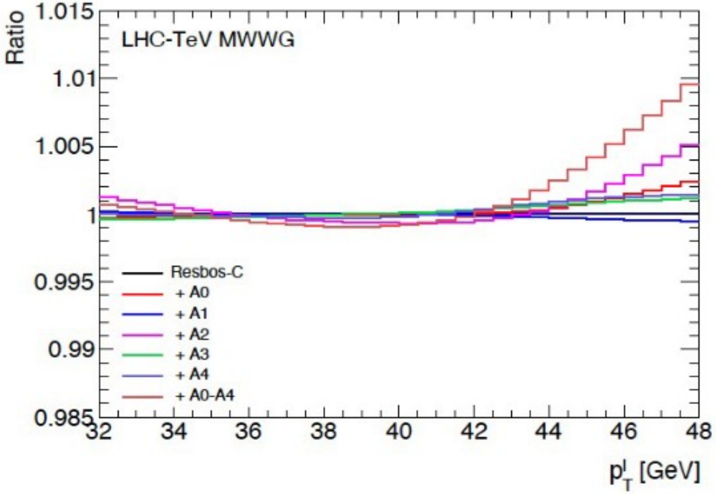
S. Carraza *et al.*, EPJC 75 (2015) 369

W boson polarisation

$$\frac{d\sigma}{dp_T^W\,dy\,dm\,d\Omega} = \frac{d\sigma}{dp_T^W\,dy\,dm}[(1+\cos^2\theta) + \frac{1}{2}A_0(1-3\cos^2\theta) + A_1\sin 2\theta\cos\phi + \frac{1}{2}A_2\sin^2\theta\cos 2\phi + A_3\sin\theta\cos\phi + A_4\cos\theta + A_5\sin^2\theta\sin 2\phi + A_6\sin 2\theta\sin\phi + A_7\sin\theta\sin\phi]$$

- ATLAS and LHCb use DYTurbo treatment of polarisation – no update made.
- Fits to data using RESBOS-C (CDF) and RESBOS-CP (D0) ported so that $A_0 - A_4$ coefficients match O(α_s) predictions using RESBOS2.

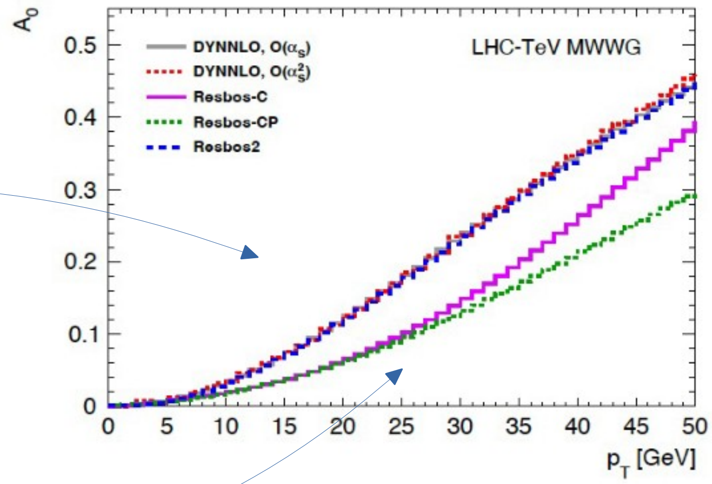


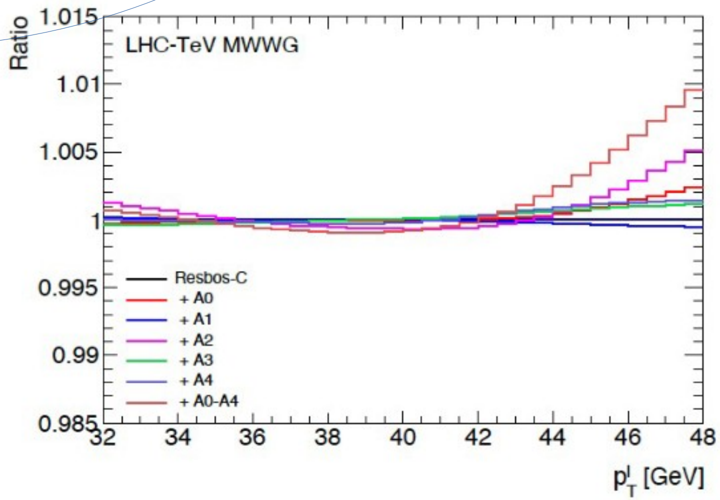


W boson polarisation

e.g. $$A_0 = \frac{\sigma_0}{\sigma_{unpol}} = \frac{\sigma_0^{FO}}{\sigma_{unpol}^{FO}}$$ (most codes, data)

$$= \frac{\sigma_0^{FO}}{\sigma_{unpol}^{Resum.}}$$ (Resbos1)





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W boson polarisation

$$
\begin{aligned}
\frac{d\sigma}{dp_T^W\,dy\,dm\,d\Omega} = \frac{d\sigma}{dp_T^W\,dy\,dm}[(1+\cos^2\theta) \\
&+ \frac{1}{2}A_0(1-3\cos^2\theta) + A_1\sin 2\theta\cos\phi \\
&+ \frac{1}{2}A_2\sin^2\theta\cos 2\phi + A_3\sin\theta\cos\phi \\
&+ A_4\cos\theta + A_5\sin^2\theta\sin 2\phi \\
&+ A_6\sin 2\theta\sin\phi + A_7\sin\theta\sin\phi]
\end{aligned}
$$

- ATLAS and LHCb use DYTurbo treatment of polarisation – no update made.
- Fits to data using RESBOS-C (CDF) and RESBOS-CP (D0) ported so that $A_0 - A_4$ coefficients match O(α_s) predictions using RESBOS2.

CDF δm_W^{pol}

Coefficient	m_T	p_T^ℓ	p_T^ν
A_0	-6.3	-2.6	-9.1
A_1	1.1	1.3	0.3
A_2	-0.7	0.4	-3.2
A_3	-2.1	-4.2	1.0
A_4	-1.4	-3.3	-1.6
$A_0 - A_4$	-9.5	-8.4	-12.5
ResBos2	-10.2±1.1	-7.6±1.2	-11.8±1.4
Difference	-0.7±1.1	0.8±1.2	0.7±1.4

D0 δm_W^{pol}

Coefficient	m_T	p_T^ℓ	p_T^ν
A_0	-9.8	-7.3	-15.6
A_1	1.9	2.4	1.8
A_2	3.0	3.3	-2.7
A_3	-1.6	-2.9	0.4
A_4	0.2	-2.3	0.5
$A_0 - A_4$	-6.4	-6.9	-15.8
ResBos2	-7.8±1.0	-6.6±1.1	-16.5±1.2
Difference	-1.4±1.0	0.3±1.1	-0.7±1.2

pT(W) modelling

- Keep each experiment's approach to pT(W) modelling – unified approach beyond scope of this study.
- Treat pT(W) uncertainties as uncorrelated between experiments since the different experiments use different generators to evaluate these uncertainties.

Experiment	**Uncertainty**
CDF	2.2 MeV
D0	2.4 MeV
ATLAS	6 MeV
LHCb	11 MeV

Other ‘updates’ and modelling effects

- Treatment of W boson width
  - Experiments use slightly different values for W boson width in fits.
  - Default value of 2089.5 MeV used.

- W boson resonance
  - Update to RESBOS 2 invariant mass modelling causes a 2 MeV shift in W boson mass for CDF.

- Higher-order electroweak effects
  - ATLAS and D0 use same shower models to study these effects, so uncertainties considered fully correlated in combination. CDF and LHCb are considered uncorrelated.

Combination

- Results combined using BLUE.

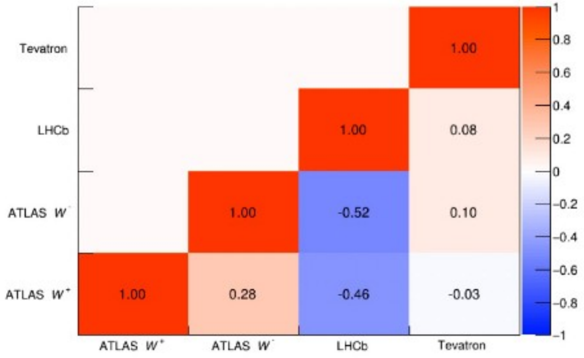
- Approach validated by reproducing internal experimental combinations.

- CDF measurement itself contains $\sim$3MeV δm_W effect (CTEQ6M $\rightarrow$ NNPDF31; mass modelling; polarisation) – this is removed as a first step before combination performed.

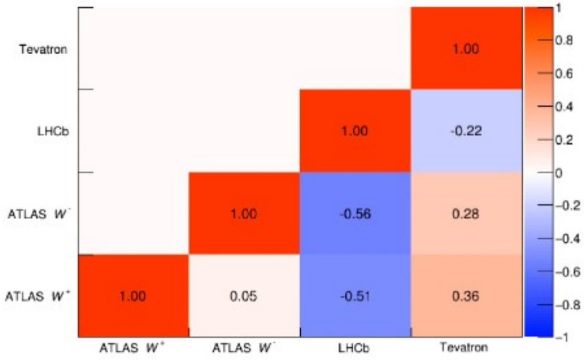
Combination: PDF Correlations

- PDF correlations determined in order to provide combination.
- Significantly different correlations reported by different PDF sets.
- Presence of anti-correlation provides stable results with reduced PDF dependence.

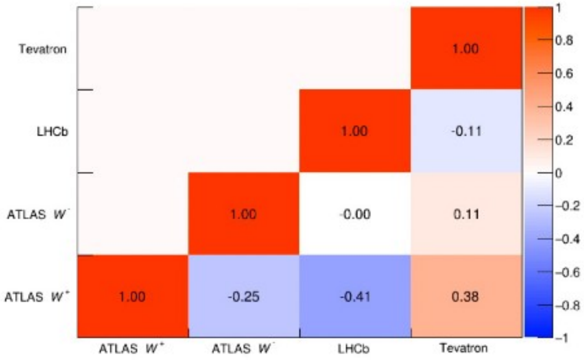
CT18



MSHT 20

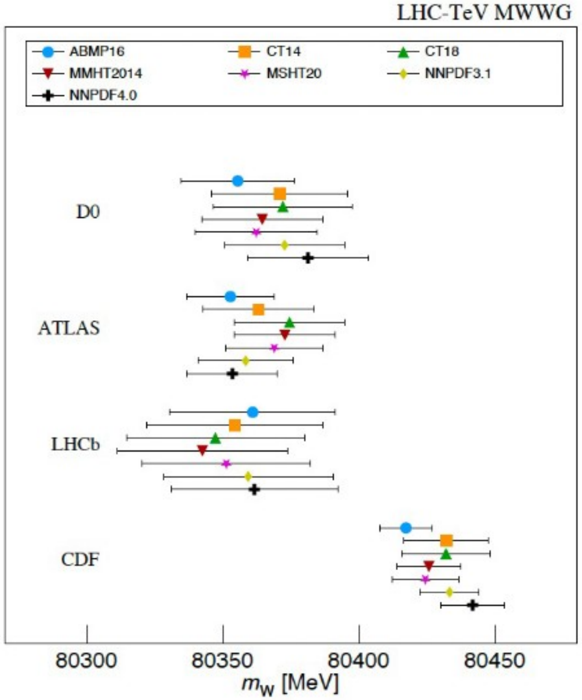


NNPF40



Combination

Input measurements with updates applied

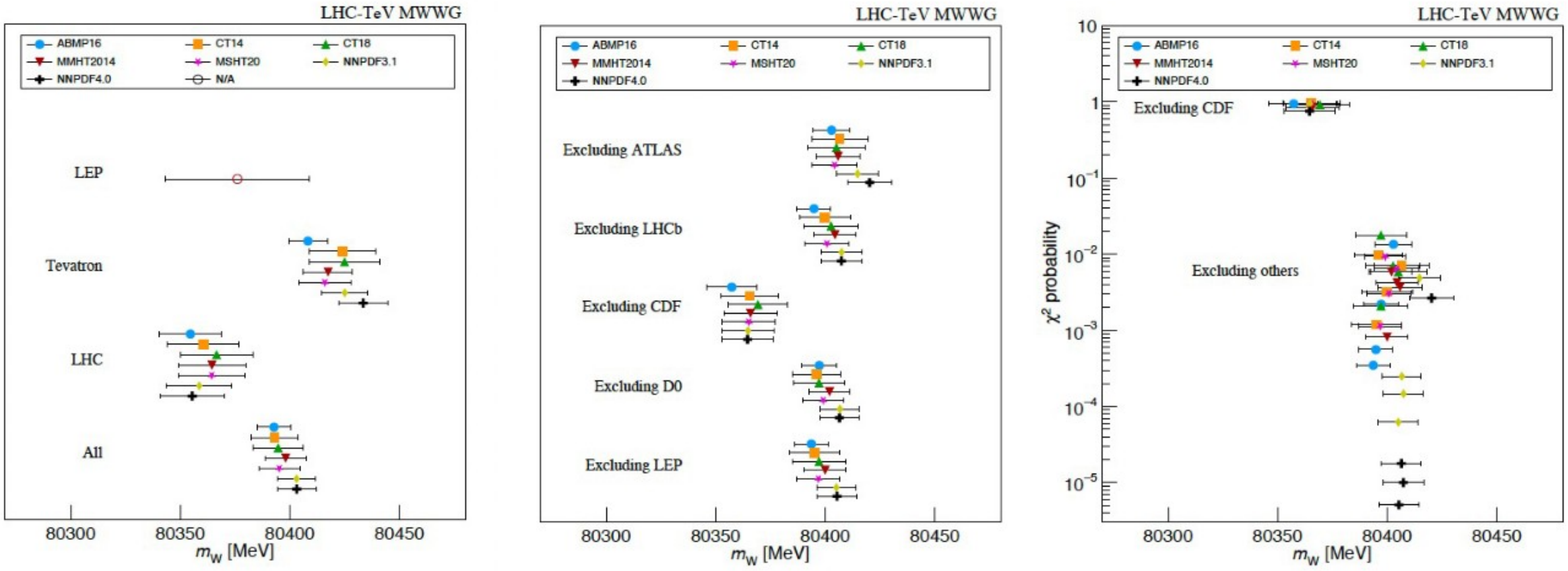


Combinations for different PDF sets

All experiments				
PDF set	m_W	σ_{PDF}	χ^2	$p(\chi^2)$
ABMP16	80392.7 ± 7.5	3.2	29	0.0008%
CT14	80393.0 ± 10.9	7.1	16	0.3%
CT18	80394.6 ± 11.5	7.7	15	0.5%
MMHT2014	80398.0 ± 9.2	5.8	17	0.2%
MSHT20	80395.1 ± 9.3	5.8	16	0.3%
NNPDF3.1	80403.0 ± 8.7	5.3	23	0.1%
NNPDF4.0	80403.1 ± 8.9	5.3	28	0.001%

Note: no combination of all measurements provides a good χ^2 probability, so the full combination is disfavoured.

Sub-combinations



PDF spread reduced compared to input measurements to roughly 5 MeV (without ABMP), 10 MeV (with ABMP)

Sub-combinations

- Combinations with CDF excluded have good compatibility.
  - **mW = 80369.2$\pm$13.3 MeV** (CT18)

    CDF, Science 376 (2022) 170

    [CDF measurement value (mW = 80433.5$\pm$9.4 MeV)].
  - Combination without CDF has a χ^2 probability of 91%.
  - Relative weights: 42% (ATLAS); 23% (D0); 18% (LHCb); 16% (LEP).

- Difference between "All-CDF" combination and the updated CDF value here is 3.6σ when the CT18 PDF set is used.

Towards the next iterations

- New measurements since : ATLAS’24, and most importantly CMS’24
- “Historical” complications are now treated and shouldn’t be of relevance in the future
  - Measurements will come natively assuming the relevant PDF sets of their time
  - Polarisation questions solved
  - CDF measurement very precise, and for the moment not considered
- New challenges
  - Statistical :
    - how to combine several profile-likelihood measurements, and/or with offset measurements?
    - Profiling of PDF uncertainties
  - Modelling : measurement correlations beyond PDFs

Statistical aspects : combine PL and offset measurements

- Option 1 : maintain mW-only combinations using BLUE
  - Requires to decompose uncertainties of the PL fit results, such that the uncertainty contributions have the usual interpretation in terms of error propagation.
    Motivated 2307.04007

NP-NP cov.

$$
\begin{array}{ccccc|c}
C_{11} & \cdots & C_{1t} & \cdots & C_{1n} & \epsilon_1 \\
\vdots & & \vdots & & \vdots & \vdots \\
C_{t1} & \cdots & C_{tt} & \cdots & C_{tn} & \epsilon_t \\
\vdots & & \vdots & & \vdots & \vdots \\
C_{n1} & \cdots & C_{nt} & \cdots & C_{nn} & \epsilon_n \\
\hline
\epsilon_1 & \cdots & \epsilon_t & \cdots & \epsilon_n & \sigma_\theta^2
\end{array}
$$

NP-POI cov. POI-POI cov.

Parameter	α_1	$\cdots$	α_r	$\cdots$	α_n	θ
Syst 1	C_{11}	$\cdots$	C_{1r}	$\cdots$	C_{1n}	ϵ_1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Syst t	C_{t1}	$\cdots$	C_{tr}	$\cdots$	C_{tn}	ϵ_t
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Syst n	C_{n1}	$\cdots$	C_{nr}	$\cdots$	C_{nn}	ϵ_n
Stat	$\sigma_{\alpha_1}^{\text{stat}}$	$\cdots$	$\sigma_{\alpha_r}^{\text{stat}}$	$\cdots$	$\sigma_{\alpha_n}^{\text{stat}}$	$\sigma_{\alpha_\theta}^{\text{stat}}$
Total	$\sqrt{C_{11}}$	$\cdots$	$\sqrt{C_{rr}}$	$\cdots$	$\sqrt{C_{nn}}$	σ_θ

BLUE

Statistical aspects : combine PL and offset measurements

- Option 2 : combine complete information
  - PL - PL

$$\mathrm{cov}(\mathcal{L}_M)|_{M=A,B} = \begin{pmatrix} C_M & \vec{\epsilon}_M \\ \vec{\epsilon}_M^T & \sigma^2_{\theta_M} \end{pmatrix}$$

The general covariance matrix can be constructed as,

$$\mathrm{cov}(\mathcal{L}_{\mathrm{cmb}}) = \begin{pmatrix} \mathrm{cov}(\mathcal{L}_A) & \mathrm{cov}(\mathcal{L}_{AB}) \\ \mathrm{cov}(\mathcal{L}_{AB})^T & \mathrm{cov}(\mathcal{L}_B) \end{pmatrix}$$

where,

$$\mathrm{cov}(\mathcal{L}_{AB}) = \begin{pmatrix} C_A \cdot C_B & C_A \cdot \vec{\epsilon}_B \\ \vec{\epsilon}_A^T \cdot C_B & \vec{\epsilon}_A^T \cdot \vec{\epsilon}_B \end{pmatrix}$$

Andres Pinto

Statistical aspects : combine PL and offset measurements

- Option 2 : combine complete information
  - PL - NP

$$\mathrm{cov}(\mathcal{L}_{\mathrm{cmb}}) = \left(\begin{array}{cc|c} C_A & \vec{\epsilon}_A & C_A \cdot \vec{\epsilon}_B \\ \vec{\epsilon}_A^{\,T} & \sigma^2_{\theta_A} & \vec{\epsilon}_A^{\,T} \cdot \vec{\epsilon}_B \\ \hline \vec{\epsilon}_B^{\,T} \cdot C_A & \vec{\epsilon}_B^{\,T} \cdot \vec{\epsilon}_A & \sigma^2_{\theta_B} \end{array}\right)$$

BLUE colored terms recover the BLUE combination matrix between A and B, i.e. if we have no information of C_A. This matrix considers that the NPs from the B measurement are Gaussian constrained, i.e. 0 ± 1.
The off-diagonal blocks of the matrix propagate the NPs information through B, i.e. $C_A \cdot \vec{\epsilon}_B$.

Andres Pinto

Profiling of PDFs

- In principle a question to be addressed upstream, by the measurements

- But not only : combinations also profile PDFs, even if not explicitly

  BLUE doesn’t make PDF NPs explicit, but they are still there, and unavoidably constrained in the process

Profiling of PDFs

- In principle a question to be addressed upstream, by the measurements
  - Claim : tolerance criteria used in PDF fits should be treated consistently in downstream fits
  - Proposal 1 (CTEQ)

1. Start by defining the correspondence between $\Delta\chi^2$ and cumulative probability level: 68% c.l. $\Leftrightarrow$ $\Delta\chi^2 = T^2$.
2. Write the **augmented** likelihood density for this definition:

$$P(D_i|T_i) \propto e^{-\chi^2/(2T^2)}$$

3. When profiling 1 new experiment with the prior imposed on PDF nuisance parameters $\lambda_{\alpha,th}$:

$$\chi^2(\vec{\lambda}_{\text{exp}}, \vec{\lambda}_{\text{th}}) = \sum_{i=1}^{N_{pt}} \frac{\left[D_i + \sum_\alpha \beta_{i,\alpha}^{\text{exp}} \lambda_{\alpha,\text{exp}} - T_i - \sum_\alpha \beta_{i,\alpha}^{\text{th}} \lambda_{\alpha,\text{th}}\right]^2}{s_i^2} + \sum_\alpha \lambda_{\alpha,\text{exp}}^2 + \sum_\alpha T^2 \lambda_{\alpha,\text{th}}^2. \qquad \beta_{i,\alpha}^{\text{th}} = \frac{T_i(f_\alpha^+) - T_i(f_\alpha^-)}{2},$$

new experiment

priors on expt. systematics and PDF params

4. Alternatively, we can reparametrize $\chi^{2\prime} \equiv \chi^2/T^2$, so that 68% c.l. $\Leftrightarrow$ $\Delta\chi^{2\prime} = 1$. We have

$$P(D_i|T_i) \propto e^{-\chi^{2\prime}/2}$$

consistent redefinition

$$\chi^{2\prime}(\vec{\lambda}_{\text{exp}}, \vec{\lambda}_{\text{th}}) = \sum_{i=1}^{N_{pt}} \frac{\left[D_i + \sum_\alpha \beta_{i,\alpha}^{\text{exp}} \lambda_{\alpha,\text{exp}} - T_i - \sum_\alpha \beta_{i,\alpha}^{\text{th}} \lambda_{\alpha,\text{th}}\right]^2}{s_i^2\, T^2} + \sum_\alpha \frac{\lambda_{\alpha,\text{exp}}^2}{T^2} + \sum_\alpha \lambda_{\alpha,\text{th}}^2.$$

Kills any prospect...

Pavel Nadolsky

Profiling of PDFs

- In principle a question to be addressed upstream, by the measurements
  - Claim : tolerance criteria used in PDF fits should be treated consistently in downstream fits
  - Improved Proposal 2 (MSHT)

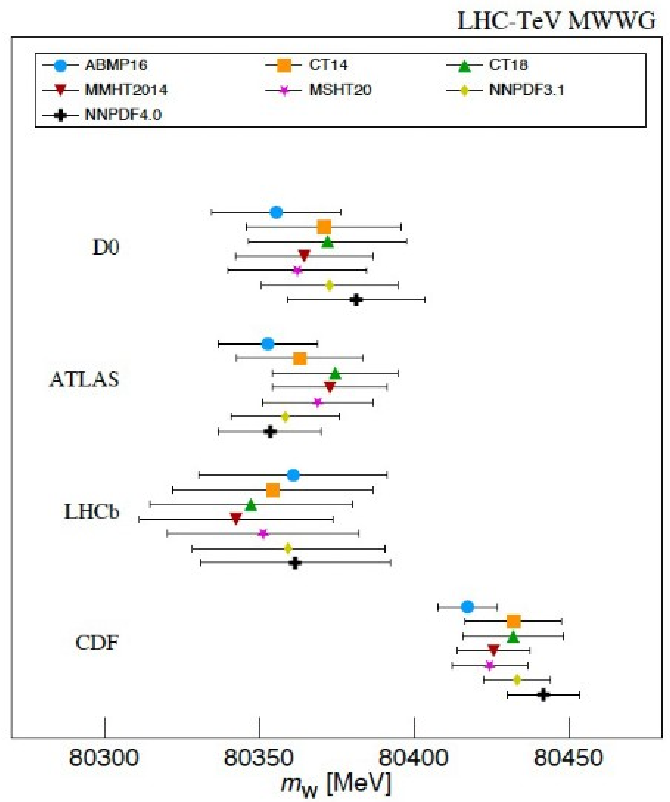
> - Include tolerance by two-step profiling:
>   1. First profile PDFs including factor of T^2 to correctly account for existing PDF uncertainties $\Rightarrow$ obtain consistently profiled PDFs.
>   2. Then perform fit with $\Delta\chi^2 = 1$ and obtain PDF uncertainty by scanning profiled PDFs as new input set.
>
> *Tom Cridge*

has the nice property that it reproduces offset error propagation exactly, for large T

  - NB : regardless, such treatments will enhance differences between “aggressive” PDF sets defined without tolerances, and “conservative” sets defined with tolerances

Profiling of PDFs

- In principle a question to be addressed upstream, by the measurements
  - Personal perspective :
    - PDF model dependence is too large already for offset measurements, and error inflation is in order. Experimenters should be given tools to understand the reasons for the differences (cf. E.Nocera, PDF4LHC21 – generalize!)
    - PDF profiling is not so much about reducing uncertainties, but about reducing model dependence. Propagating tolerances downstream makes PDFs stiffer and doesn't go into that direction



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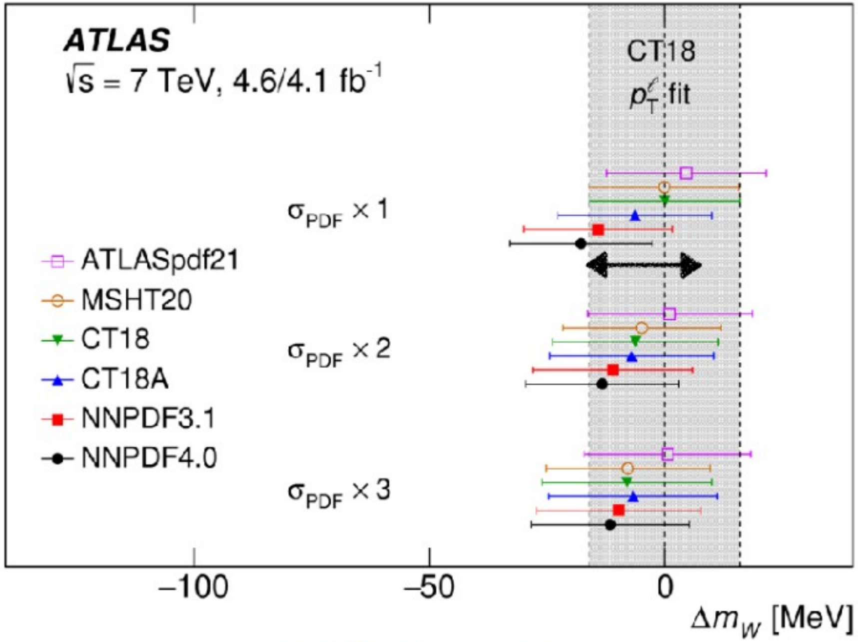
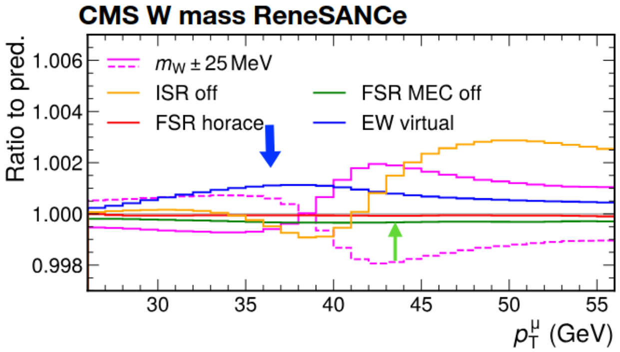
  - Personal perspective :
    - PDF model dependence is too large already for offset measurements, and error inflation is in order. Experimenters should be given tools to understand the reasons for the differences (cf. E.Nocera, PDF4LHC21 – generalize!)
    - PDF profiling is not so much about reducing uncertainties, but about reducing model dependence. Propagating tolerances downstream makes PDFs stiffer and doesn't go into that direction



STDM-2019-24

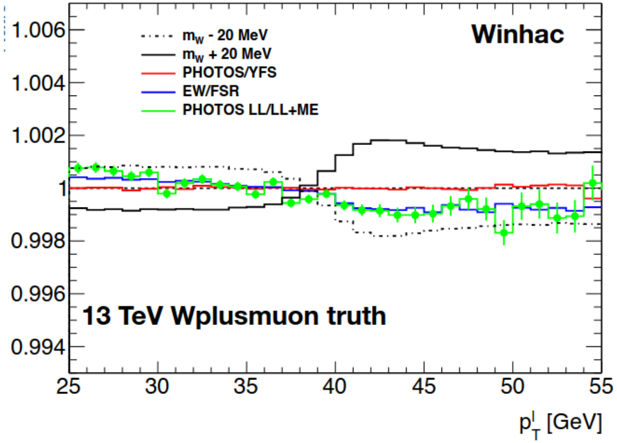
Correlations beyond QCD

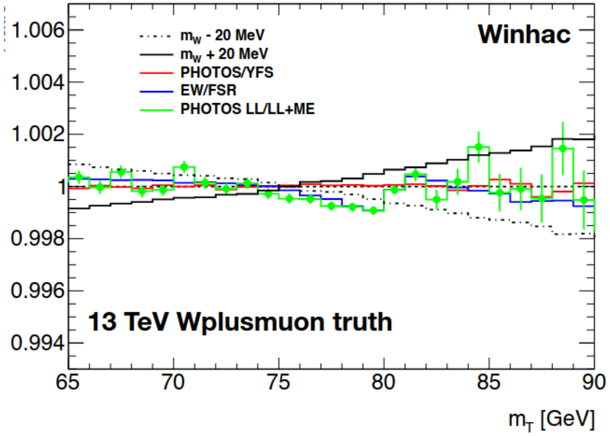
- Electroweak uncertainties play at the 2-3 MeV level; a proper correlation treatment will have a visible effect on combined results



Winhac:
LL / LL + ME:
LL-type exponentiation/LL+ ME correction - similar to MEC off?
PHOTOS/YFS FSR comparison: similar to FSR horace
Full EW/ YFS: similar to EW virtual

Differences TBU!





Xuewei Jia

Next combinations

- Measurements presently available, and considered for combination:
  - D0 ‘13 (offsets)
  - LHCb ‘21 (partial profiling)
  - ATLAS ‘24, CMS ‘24 (profiled measurements)

    (+LEP legacy)
- Foreseen on ~one year timescale
  - ATLAS low-mu
  - Others?
- Still a goal to publish these results as experimental collaborations, instead (or in addition) to the current working group

Next combinations

- First exercise, with NNPDF3.1. Not even preliminary; just to give an idea of what to expect:

BLUE: combination of all the measurements using only the uncertainty decomposition.

MatrixCMB: ATLAS and CMS provides NPs covariance matrix and offsets while LHCb, D0, CDF and LEP are offset-like measurements.

	BLUE					MatrixCMB					
Combination	m_W	σ_{total}	σ_{PDF}	χ^2	Prob(χ^2)	m_W	σ_{total}	σ_{PDF}	χ^2	n.d.f	Prob(χ^2)
CMS + ALTAS (CA)	80356.7	8.3	2.9	0.3	58.21%	80357.9	8.2	2.5	101.11	101	47.81%
CA + LHCb (CERN)	80356.9	7.9	2.5	0.3	85.71%	80357.6	7.9	2.2	101.13	102	50.57%
CERN + D0 (CERN-D)	80358.6	7.6	2.7	0.8	84.18%	80359.6	7.5	2.3	101.81	103	51.47%
CERN-D0 + LEP (CERN-DL)	80359.4	7.4	2.6	1.1	89.47%	80360.4	7.3	2.2	102.04	104	53.60%
CERN-DL + CDF	80381.0	6.5	3.3	38.1	0.00%	80386.7	6.1	2.8	142.89	105	0.82%

Andres Pinto

Perspectives

- Projects
  - A CONF note in the next months, including ATLAS and CMS ‘24?
  - A collaboration-signed publication in 1-2 years?

- Working group and documentation
  - https://twiki.cern.ch/twiki/bin/view/LHCPhysics/LHC-TEV-MWWG
  - Combination : Eur. Phys. J. C (2024) 84: 451
  - Statistical methods : Eur.Phys.J.C 84 (2024) 6, 593