qT resummation with DYTurbo

GDR QCD Workshop on *W*-mass

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F. Dattola

Introduction

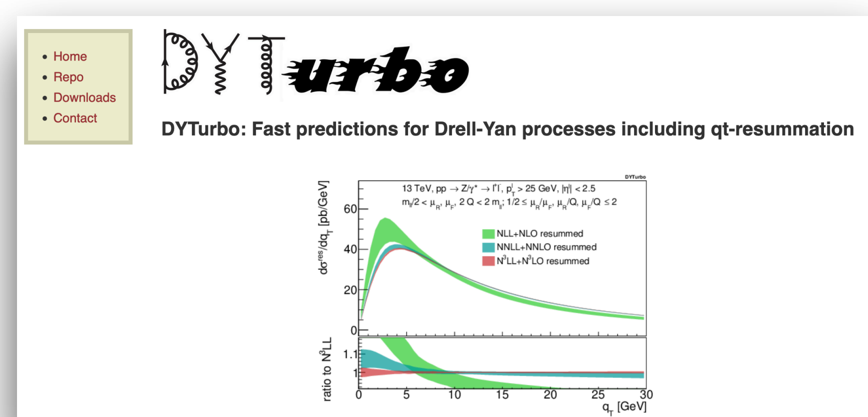
Foreword and references

- DYTurbo is a program for the calculation of fast and numerically precise predictions of QCD transverse-momentum resummed Drell-Yan (DY) cross sections

The **DYTurbo project** is developed and maintained by **S. Camarda et al**.

I am not one of the authors and my perspective is only that of a curious (experimentalist) user. As such, these slides provide insights based on user experience and practical application in the context of the *W*-mass measurement.

For technical details, implementation choices and future developments, I encourage you to consult Stefano and the other authors.



- Public code available on HEPForge
- A few references:
  - DYTurbo: Fast predictions for Drell-Yan processes
  - Drell-Yan lepton-pair production: qT resummation at N3LL accuracy and fiducial cross sections at N3LO
  - Drell-Yan lepton-pair production: qT resummation at approximate N4LL+N4LO accuracy

DYTurbo

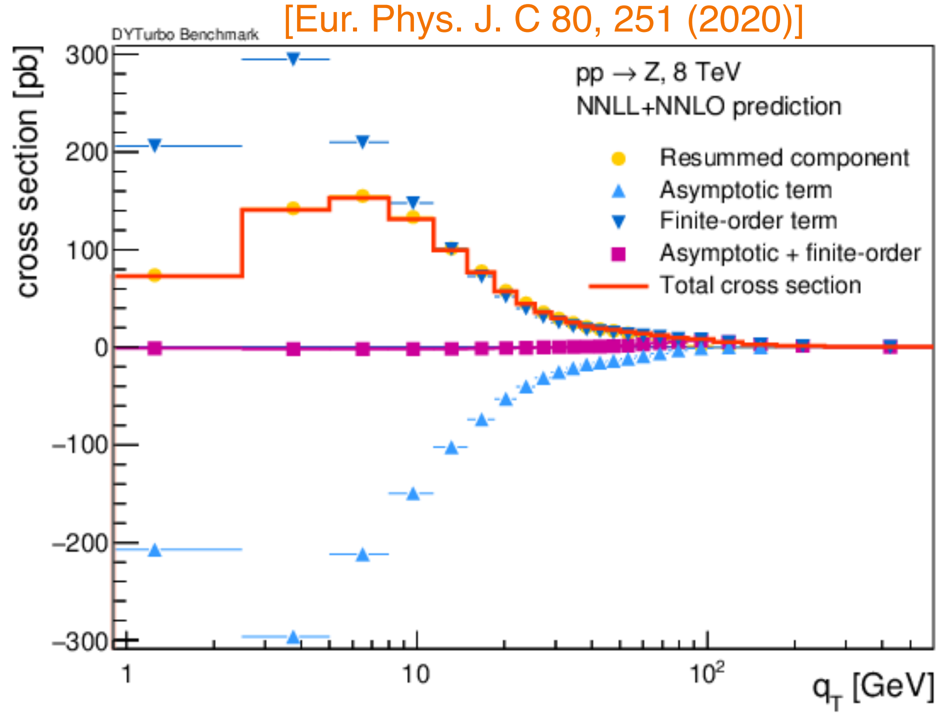
qT resummed cross section for *V*-boson production

- DYTurbo master formula for *V*-boson production cross section with q_T resummation

$$d\sigma^{V} = d\sigma^{res} - d\sigma^{asy} + d\sigma^{f.o.}$$

- $d\sigma^{res} \equiv d\hat{\sigma}^{V}_{LO} \times \mathscr{H}^{V} \times \exp\{\mathscr{G}(\alpha_S, L)\} \rightarrow$ resummed component of the cross section
- $d\sigma^{res} \equiv d\hat{\sigma}^{V}_{LO} \times \Sigma^{V}(q_T/Q) \rightarrow$ fixed order expansion of $d\sigma^{res}$
- $d\sigma^{f.o.} \equiv$ V + jet finite-order cross section integrated over final-state QCD radiation

[Eur. Phys. J. C 80, 251 (2020)]



DYTurbo

Analytic Resummation formalism in qT

- q_T resummation in impact parameter b space: $q_T \ll M \leftrightarrow Mb \gg 1 \rightarrow \ln(M/q_T) \gg 1 \leftrightarrow \ln(Mb) \gg 1$

$$\frac{d\hat{\sigma}}{dq_T^2} \overset{q_T \ll M}{=} \frac{M^2}{\hat{s}} \int \frac{d^2\mathbf{b}}{4\pi} e^{i\mathbf{b}\cdot\mathbf{q_T}} \mathcal{H}(\alpha_S) \otimes \exp\{\mathcal{G}(\alpha_S, L)\}$$

$$\mathcal{G}(\alpha_S, L) = L\, g^{(1)}(\alpha_S L) + g^{(2)}(\alpha_S L) + \frac{\alpha_S}{\pi} g^{(3)}(\alpha_S L) + \cdots \qquad \mathcal{H}(\alpha_S) = 1 + \frac{\alpha_S}{\pi}\mathcal{H}^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}^{(2)} + \cdots$$

- $\mathcal{H}(\alpha_S)$ $\equiv$ **hard-virtual** coefficient function in powers of α_S
- $\exp\{\mathcal{G}(\alpha_S, L)\}$ $\equiv$ **universal** (process independent) perturbative **Sudakov form factor**
  - $\mathcal{G}(\alpha_S, L)$ $\equiv$ **resummed logarithmic** $L \equiv \ln(M^2 b^2)$ **expansion** where the functions $g^{(n)}$ resum the $\alpha_S^k L^k$
- **Perturbative structure** $\rightarrow$ LL $(\sim \alpha_S^n L^{n+1})$: $g^{(1)}$, $(\hat{\sigma}^{(0)})$; NLL $(\sim \alpha_S^n L^n)$: $g^{(2)}$, $\mathcal{H}^{(1)}$; NNLL $(\sim \alpha_S^n L^{n-1})$: $g^{(3)}$, $\mathcal{H}^{(2)}$;
- $\tilde{\mathcal{W}}(b, m, y) = \mathcal{H}(\alpha_S) \otimes \exp\{\mathcal{G}(\alpha_S, L)\}$ $\rightarrow$ Back in q_T space: $\mathcal{W}(q_T, m, y) = \frac{m^2}{s} \int_0^\infty \mathrm{d}b\, \frac{b}{2} J_0(b q_T)\, \tilde{\mathcal{W}}(b, m, y)$

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- q_T resummation based on **Catani-de Florian-Grazzini formalism** [Nucl.Phys. B596 (2001) 299-312]

  - Process-dependence factorised in the hard-virtual function
  - Resummation scale $Q \sim M$: varied to estimate uncertainty from missing logs $\ln(M^2b^2) \rightarrow \ln(Q^2b^2) + \ln(M^2/Q^2)$
  - No soft scale ($\sim q_T$) in CdFG, only scale variations associated to renormalised divergences (μ_R, μ_F, Q)
  - Perturbative unitarity constraint enforcing correct total cross section

$$\ln(Q^2b^2) \;\rightarrow\; \widetilde{L} \equiv \ln(Q^2b^2+1) \;\Rightarrow\; \exp\left\{\alpha_s^n \widetilde{L}^k\right\}\Big|_{b=0} = 1 \;\Rightarrow\; \int_0^\infty dq_T^2 \left(\frac{d\hat{\sigma}}{dq_T^2}\right) = \hat{\sigma}^{(\mathrm{tot})}$$

  - Primed resummation orders (NLL', NNLL', ...) → log terms + non-logarithmic terms required to match F.O.

DYTurbo

Perturbative accuracy

[Credit S. Camarda]

	Virtual		Sudakov			Real
	$H[\delta(1-z)]$	H[z]	Cusp AD	Collinear, RAD	PDF	CT,V+jet
LL+LO	1	1	1-loop	0	const.	1
NLL+NLO	α_s	C1	2-loop	1-loop	LO	α_s
NNLL+NNLO	α_s^2	C2	3-loop	2-loop	NLO	α_s^2
N3LL+N3LO	α_s^3	C3	4-loop	3-loop	NNLO	α_s^3
N4LL+N3LO	α_s^4	C4	5-loop	4-loop	N3LO	α_s^4

Up to approximate N4LL′

Known analytically
Approximated numerically
Unknown, estimated with series acceleration
Not included

Predictions with DYTurbo

Modes of operation and applications

- DY cross section predictions require: integration over lepton kinematic variables and QCD radiation, plus convolutions and integral transforms

- **Two main modes of operation**: **Vegas** integration and **quadrature rules** based on interpolating functions

- **DY full-lepton phase space cross sections** in $(\mathrm{q_T}, y, M)$: **integration over kinematic variables with quadrature rules** available for all the terms up to (NNLO+NNLL) → **fast predictions**

- **General idea**: given an integral $\mathrm{I}f \equiv \int_0^1 \mathrm{d}^d x\, f(\vec{x})$ use a quadrature formula $\mathrm{I}f \approx \mathrm{Q_n}f \equiv \sum_{i=1}^{n} w_i f(x_i)$ with specially chose nodes x_i and weights w_i → *Deterministic* integration

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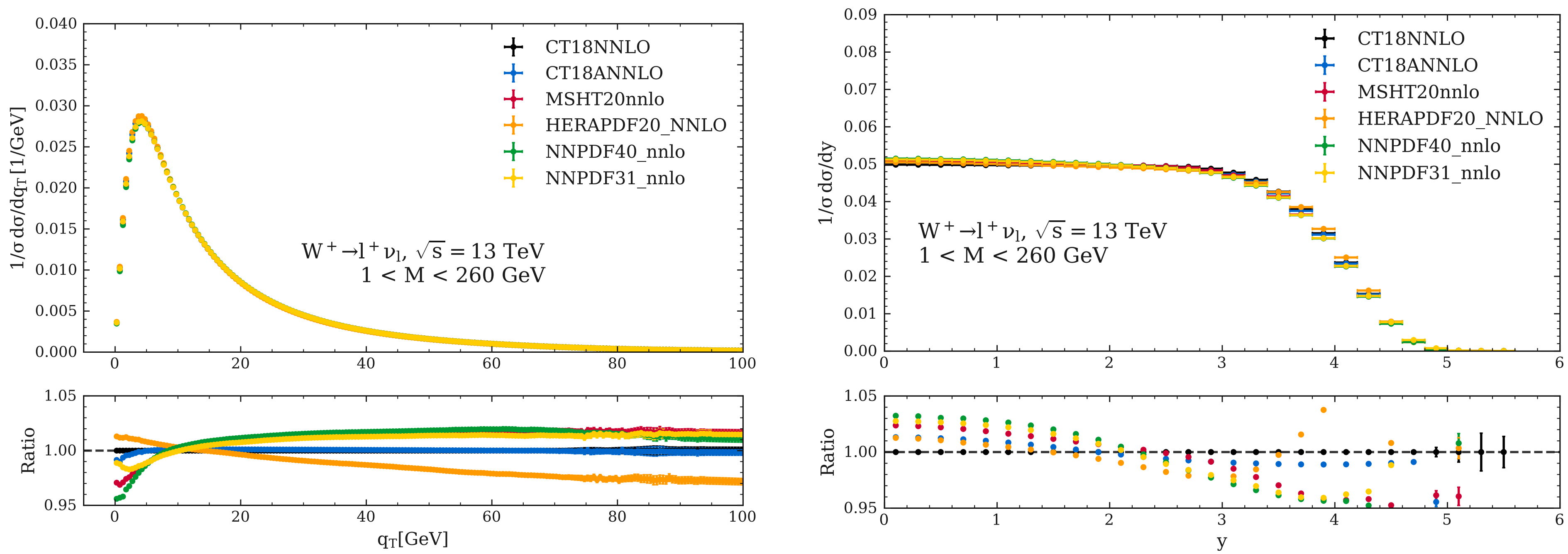
- **Application**: fast NNLO+NNLL computations of 2D (p_T, y) full phase-space *W,Z* distributions, for multiple PDF sets, integrating over fine intervals of p_T, y



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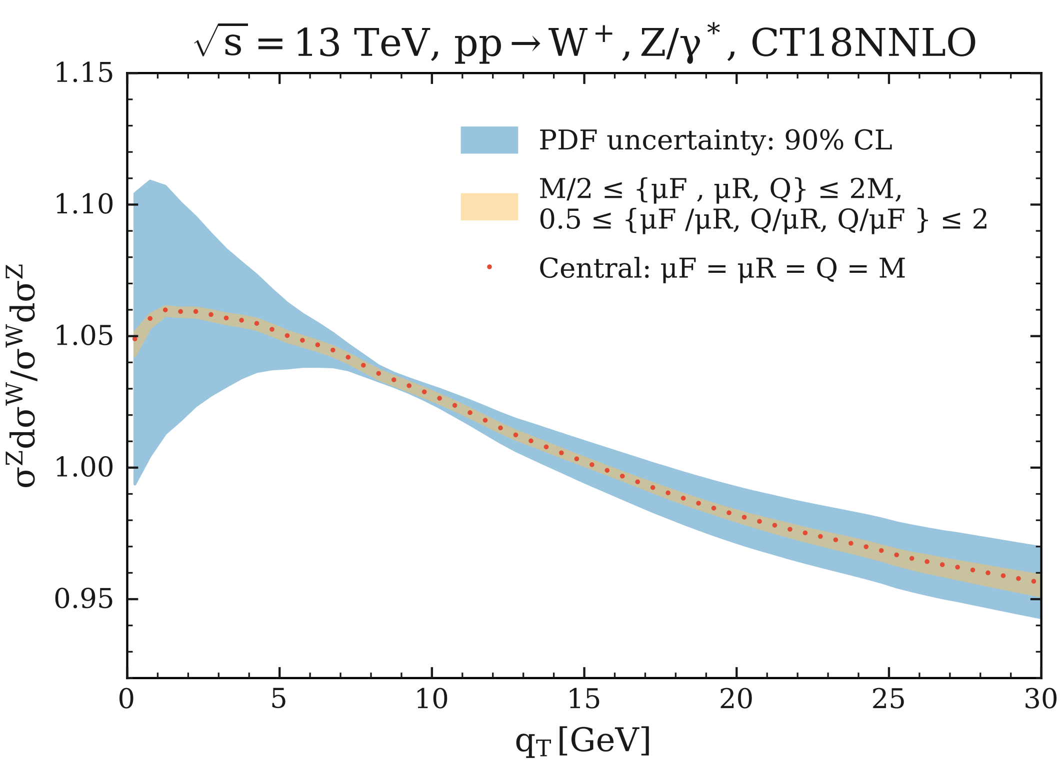
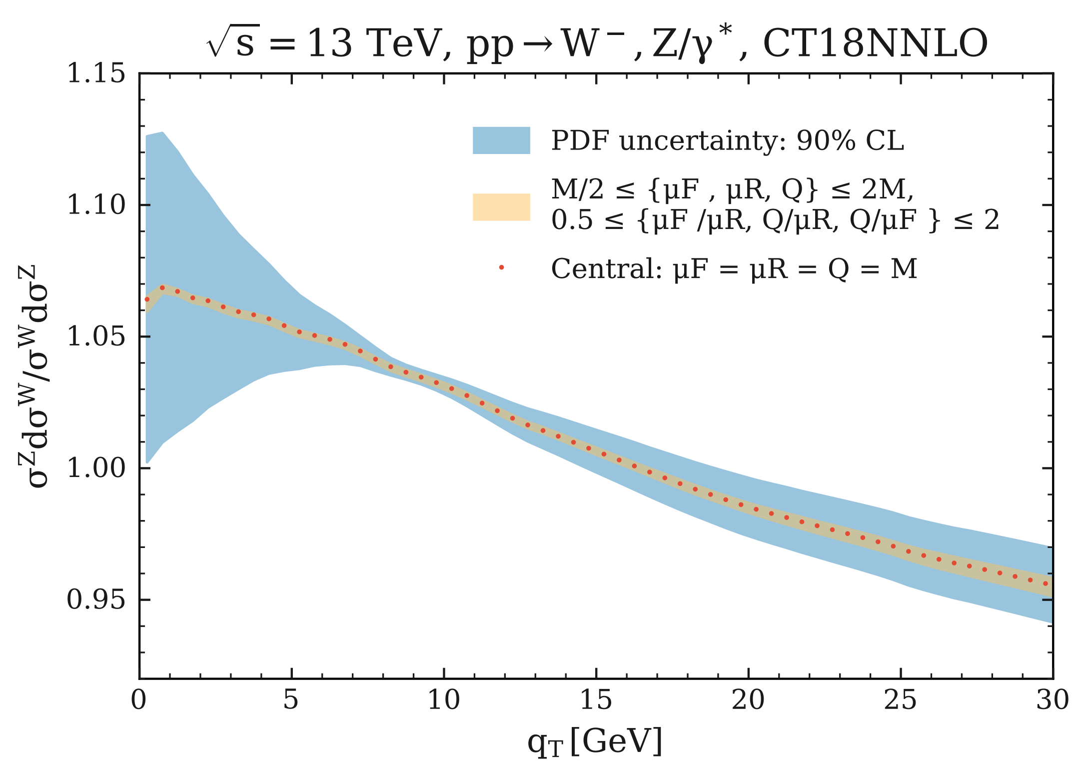
- **Application**: fast studies of W/Z q_T ratio at NNLO+NNLL including PDF and scale uncertainties





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- **Two main modes of operation**: **Vegas** integration and **quadrature rules** based on interpolating functions

- Finite order term in **DY fiducial cross sections and angular coefficients/helicity cross sections** computed with **VEGAS algorithm** → more versatile but slower (iterative adaptive importance sampling)

- **General idea**: Monte Carlo integration $\mathrm{I}f \equiv \int_0^1 \mathrm{d}^d x \, w(\vec{x}) \frac{f(\vec{x})}{w(\vec{x})}$ → must be able to sample from $w(\vec{x})$
    - Iteratively build up a piecewise constant weight function, represented on a rectangular grid
    - Each iteration consists of a sampling step followed by a refinement of the grid: many points are sampled where the grid's value is small

Predictions with DYTurbo

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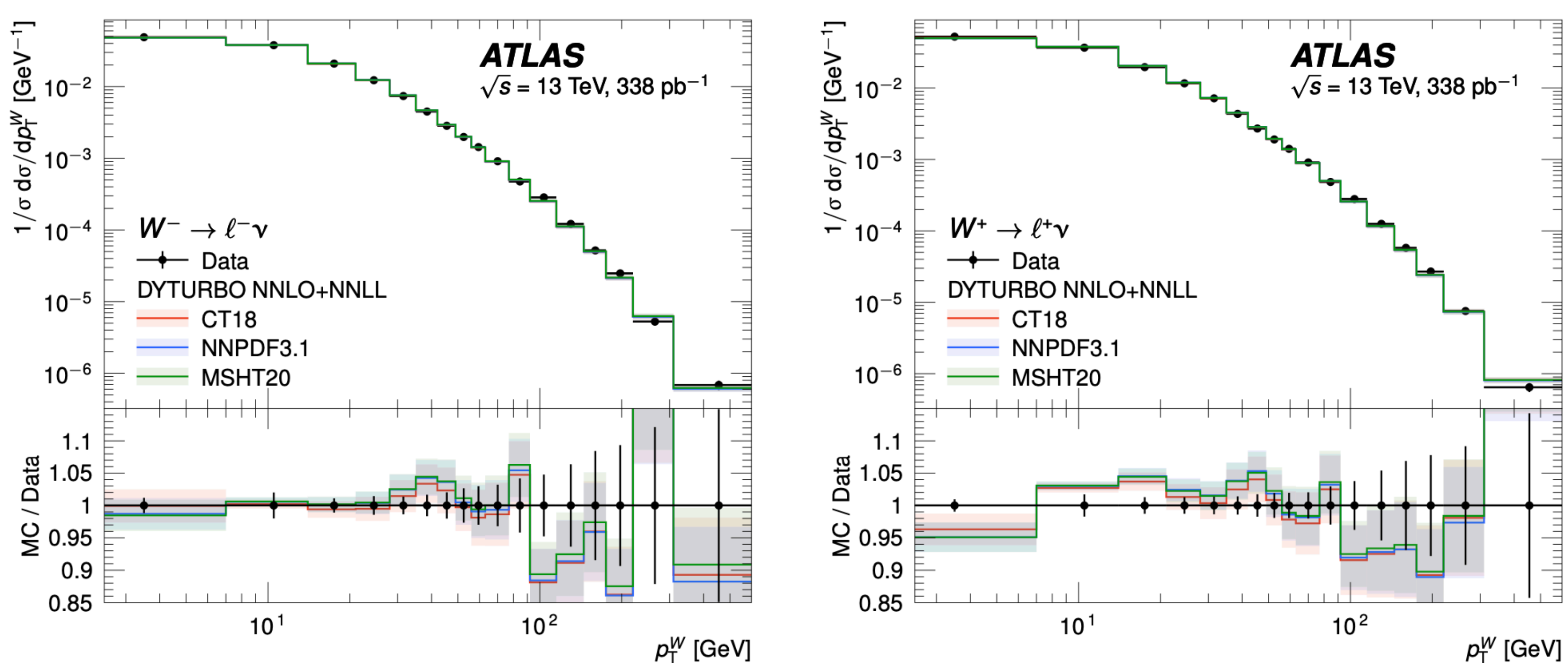
- **Application**: fiducial cross-section measurements, e.g. *"Precise measurements of W- and Z-boson transverse momentum spectra with the ATLAS detector using pp collisions at 5.02 and 13 TeV"* [Eur. Phys. J. C 84 (2024) 1126]



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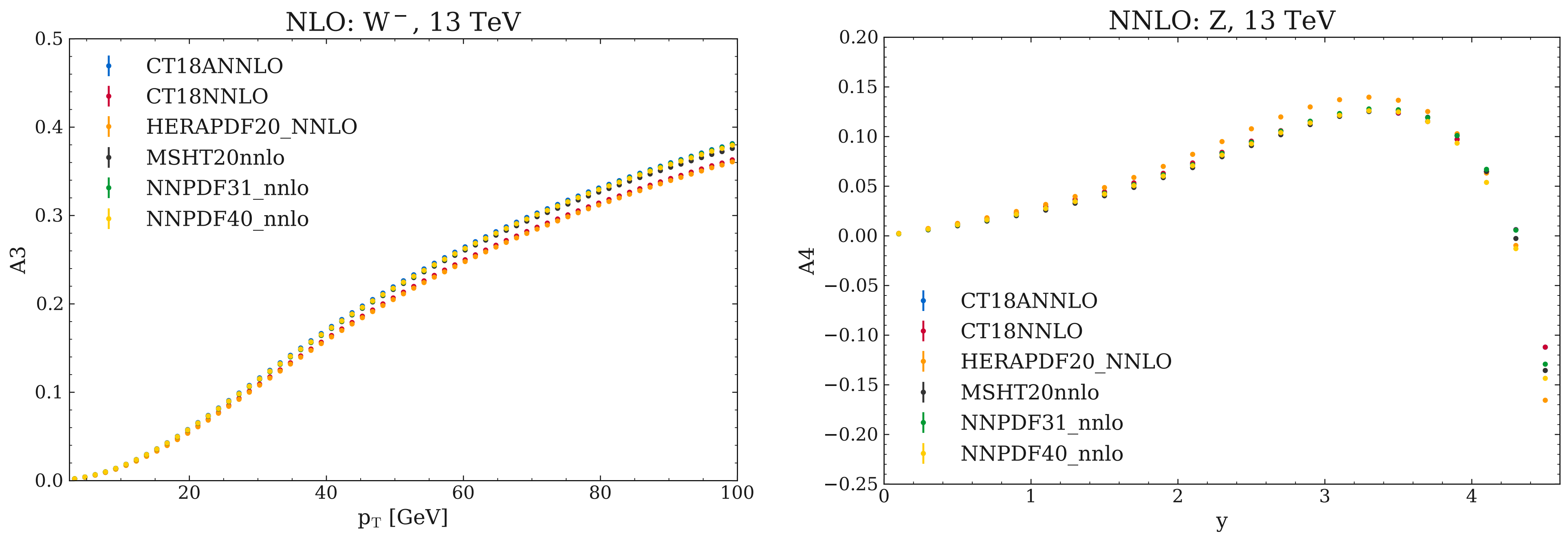
- **Application**: q_T, y predictions up to $\mathcal{O}(\alpha_S^2)$ (NNLO) for *W*,*Z* angular coefficients



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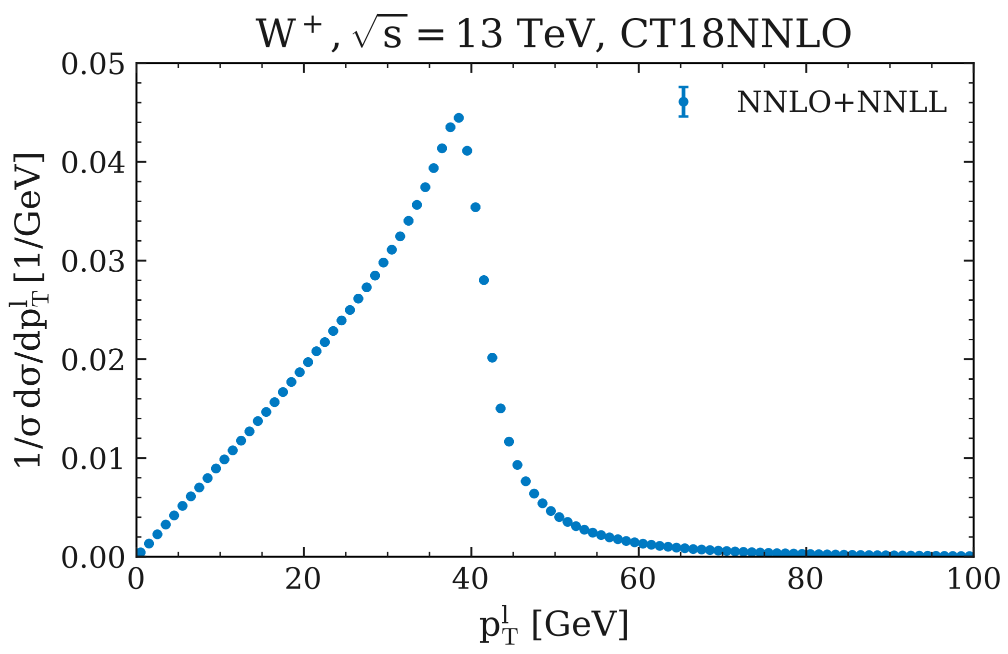
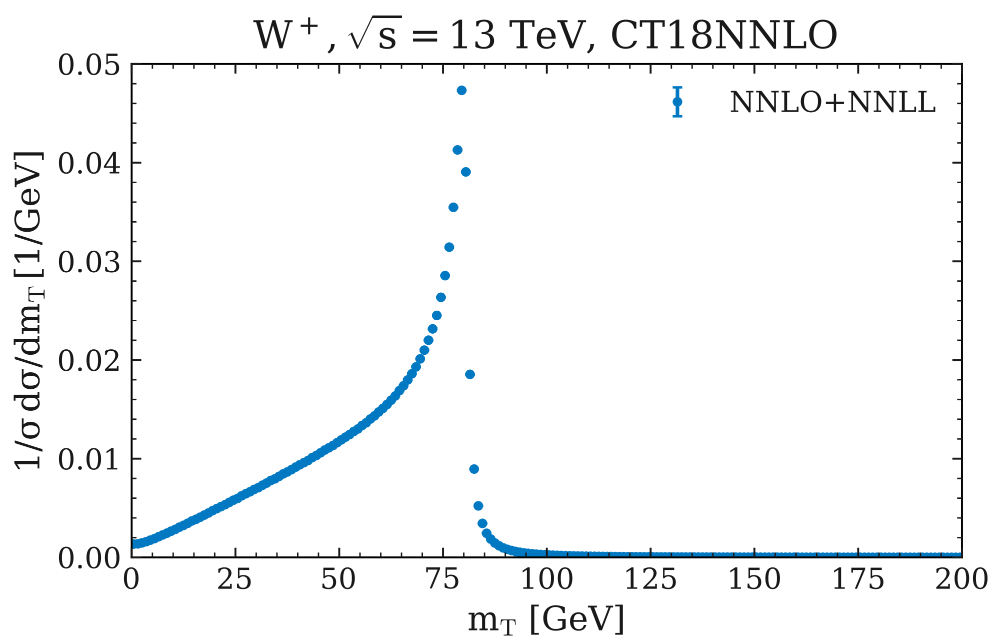
- **Application**: NNLO+NNLL predictions of **final-state kinematic observables**, i.e. p_T^l, p_T^ν, $\mathrm{m_T}$





Predictions with DYTurbo

Built-in tools

[Eur. Phys. J. C 80, 251 (2020)]

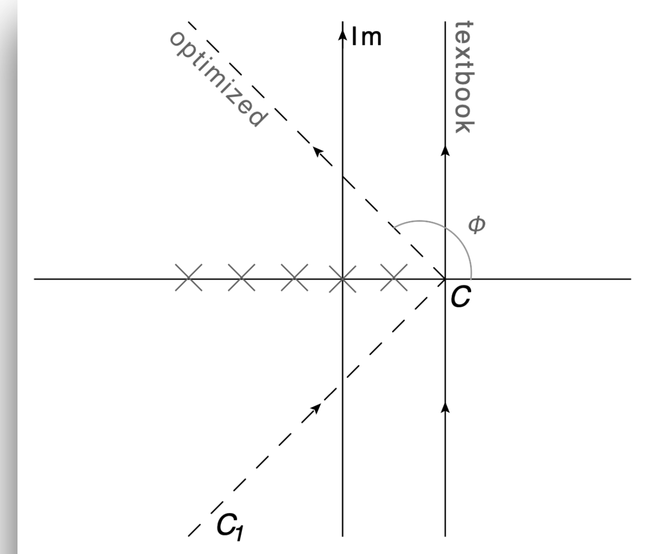
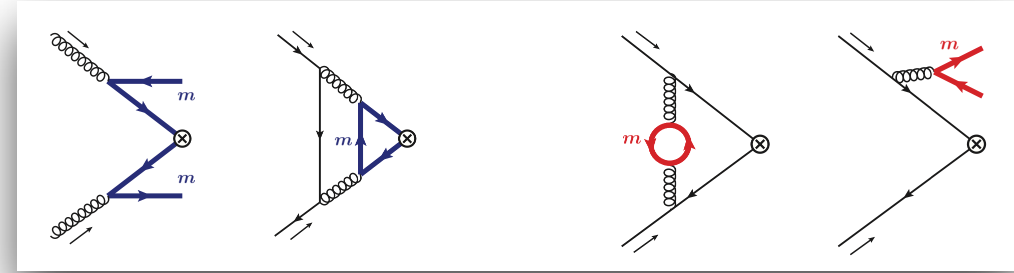


Figure 2 Standard and optimised integration contours in the complex plane for the inverse Mellin transform. The two contours intersect the real axis at the point c, and the optimised contour is bent by an angle $\phi > \pi/2$ with respect to the real axis. The crosses represent the poles of PDF parameterisation in Mellin space.

- Mellin transform
  - **Fast x-space to Mellin-space integral transform** based on Gauss-Legendre quadrature and optimised integration contours for a faster convergence of the Mellin inversion (e.g. bending on the negative real half plane)

- PDF evolution and flavour schemes
  - **FFN backward PDF evolution** based on Pegasus QCD algorithm: consistent with CdFG formalism where PDFs evolve backward from a hard scale
  - **VFN forward PDF evolution** from LHAPDF: evolve PDFs upward from a lower scale and generate heavy quark PDFs dynamically at thresholds



- QED and QED-QCD mixed corrections
  - With LL and NLL QED resummation effects

Predictions with DYTurbo

Built-in tools

- xFitter interface
  - Allows to compute DYTurbo q_T, y predictions on the fly within the QCD fit framework

xFitter

- Operational modes for Z/γ*
  - Full-lepton phase space cross sections in p_T,y,m: DYTurbo can provide the full fixed order or resummed prediction

```
&Data
  Name = 'CDF ZPT 1.96 TEV'
  IndexDataset =   101010
  Reaction  = 'NC ppbar'

  TheoryType     = 'expression'
  TermName = 'A','K','C'
  TermSource = 'DYTurbo','KFactor','KFactor'
  TermInfo = 'FileName=datafiles/tevatron/cdf/wzProduction/1207.7138/cdf196-n4ll.in',
             'FileName=datafiles/tevatron/cdf/wzProduction/1207.7138/qedisr.txt:FileColumn=3',
             'FileName=datafiles/tevatron/cdf/wzProduction/1207.7138/n3lo-afactor-r1-f1-q1.txt:FileColumn=3'
  TheorExpr= '(A+C)*K/1000'
```

  - Fiducial phase space cross sections in p_T,y,m: include V+jet APPLgrid, DYTurbo computes the rest

```
&Data
  Name = 'ATLAS Z pT 8 TeV'
  IndexDataset =   20
  Reaction =     'NC pp'

  TheoryType     = 'expression'
  TermName = 'A1','A2'
  TermSource = 'DYTurbo','APPLgrid'
  TermInfo = 'FileName=data/zpt8tev/dyturbo/z-8tev-nnll.in',
             'GridName=data/zpt8tev/applgrid/grid-40-6-15-3-Zjet_41_zpt8tev_peak.root'
  TheorExpr= 'A1+A2'
```

[Credit S. Camarda]

Treatment of non-perturbative QCD

DYTurbo phenomenological model

- $\mathcal{G}(\alpha_S, L)$ in the perturbative form factor becomes singular for $q_T \sim \Lambda_{QCD} \rightarrow$ non-perturbative regime

  - b_* **regularisation**: exp $\{\mathcal{G}(\alpha_S, L)\}$ dependence on b frozen close to singularity

$$b^2 \rightarrow b_*^2 = b^2 b_{\rm lim}^2/(b^2 + b_{\rm lim}^2) \text{ with } b_{\rm lim} \approx 2\,\text{GeV}^{-1}$$

  - Sudakov form factor modifed by including a **non-perturbative term**: $S(b) \equiv$ exp $\{\mathcal{G}(\alpha_S, L)\} \rightarrow S(b) \cdot S_{\rm NP}(b)$

- Baseline pheno model based on Collins-Rogers parametrisation $\rightarrow S_{\rm NP}(b\,|\,g_1, q, \lambda, g_0, b_{\rm lim}, Q_0)$

$$S_{\rm NP}(b) = \exp\left[-g_j(b) - g_K(b)\log\frac{m_{\ell\ell}^2}{Q_0^2}\right] \quad \begin{cases} g_j(b) = \dfrac{g\,b^2}{\sqrt{1+\lambda\,b^2}} + \text{sign}(q)\left(1 - \exp\left[-|q|\,b^4\right]\right) & \rightarrow \text{NP model for TMD PDF} \\ g_K(b) = g_0\left(1 - \exp\left[-\dfrac{C_F\alpha_s(b_0/b_*)b^2}{\pi g_0\,b_{\rm lim}^2}\right]\right) & \rightarrow \text{NP model for Collins-Soper kernel} \end{cases}$$

- $S_{\rm NP}$ includes 6 parameters which can be fitted to data or varied to assess an uncertainty
  - g_1 and q representing the leading quadratic and quartic terms, dominant at $p_T \sim$ 4–10 GeV
  - λ controlling the transition from Gaussian (quadratic) to exponential (linear)
  - g_0 controlling the very high b (very small p_T) behaviour
  - $b_{\rm lim}$ freezing the scale of α_S and Q_0 defining the starting scale of the TMD evolution

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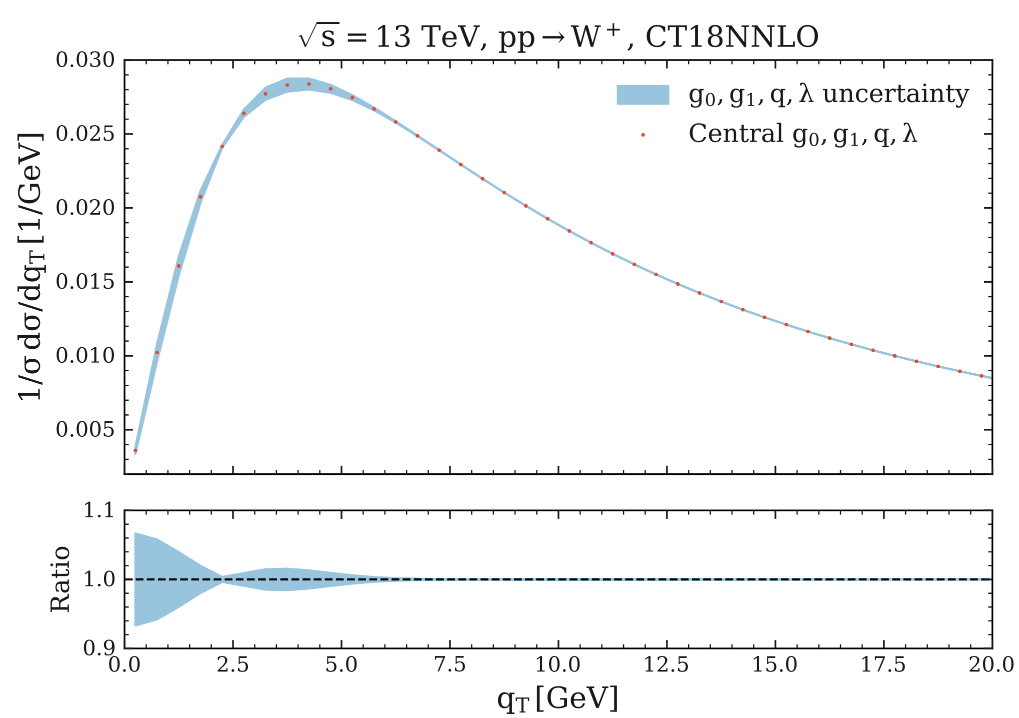
- Sudakov form factor modifed by including a **non-perturbative term**: $S(b) \equiv$ exp $\{\mathcal{G}(\alpha_S, L)\} \rightarrow S(b) \cdot S_{\rm NP}(b)$

- Interface between DYTurbo and xFitter allows **fits of TMD parameters** in CS parametrisation and others
- Can fit DYTurbo predictions to **low mass DY fixed target data** used in TMD analyses, e.g. E288, E605, E866/NuSea

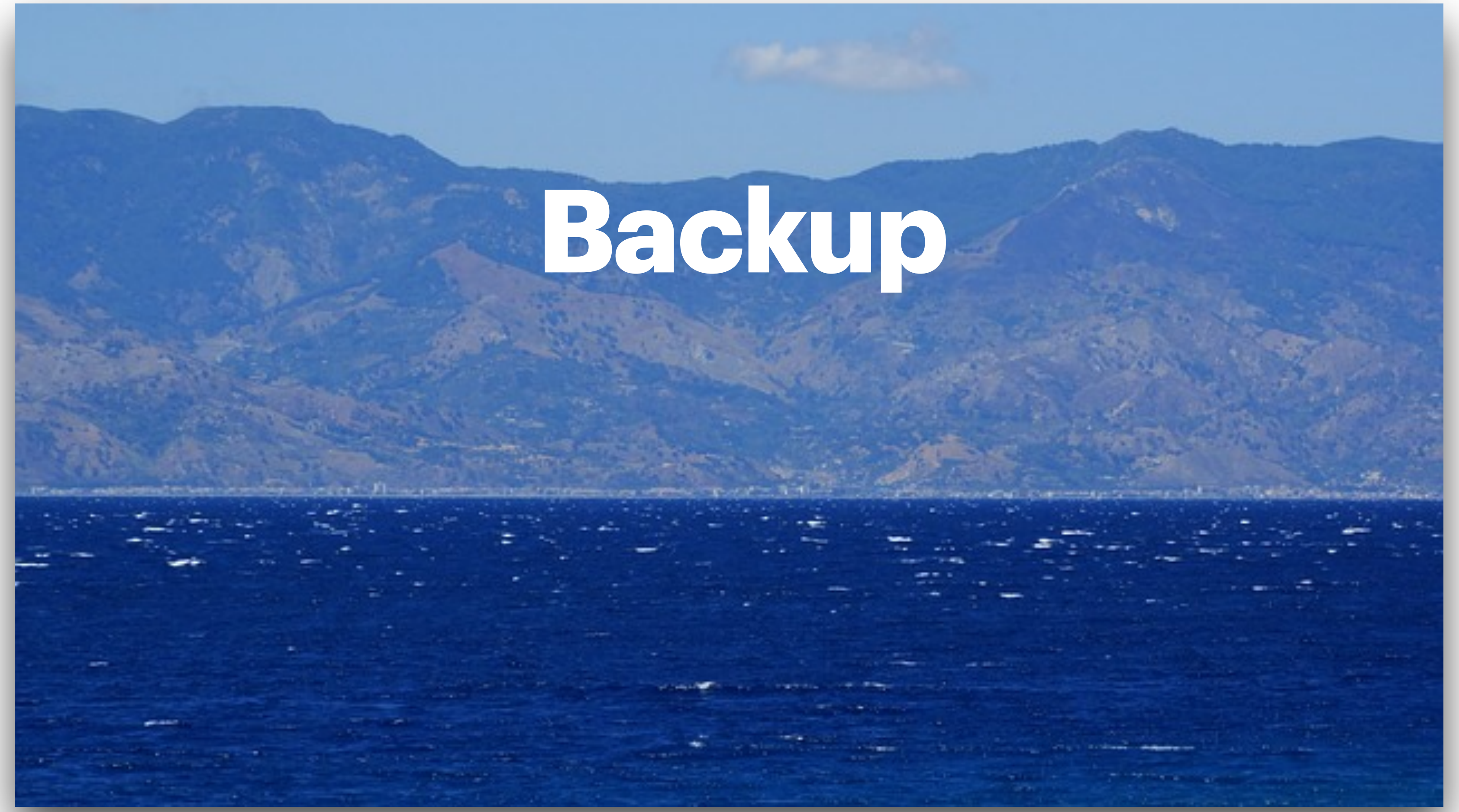
- **Application**: modelling of NP effects in *W*, *Z* q_T distributions for *W*-mass analysis



Conclusions

DYTurbo takeaways — from a user (experimentalist) perspective

- Very versatile in the context of DY precision measurements sensitive to q_T resummation corrections: the *W*-mass measurement is an ideal scenario for its application
- Reaches QCD perturbative accuracy needed to match current level of experimental precision in DY data
- Existing interface with xFitter makes it easy to use DYTurbo predictions in PDF and TMD fits
- Deterministic integration makes it very fast in many applications


Backup