MIXED QCD-EW CORRECTIONS TO THE DRELL-YAN PROCESS

GDR QCD workshop on W mass
Orsay, 01/07/2025

Simone Devoto

European Research Council
Established by the European Commission

GHENT UNIVERSITY
FACULTY OF SCIENCES

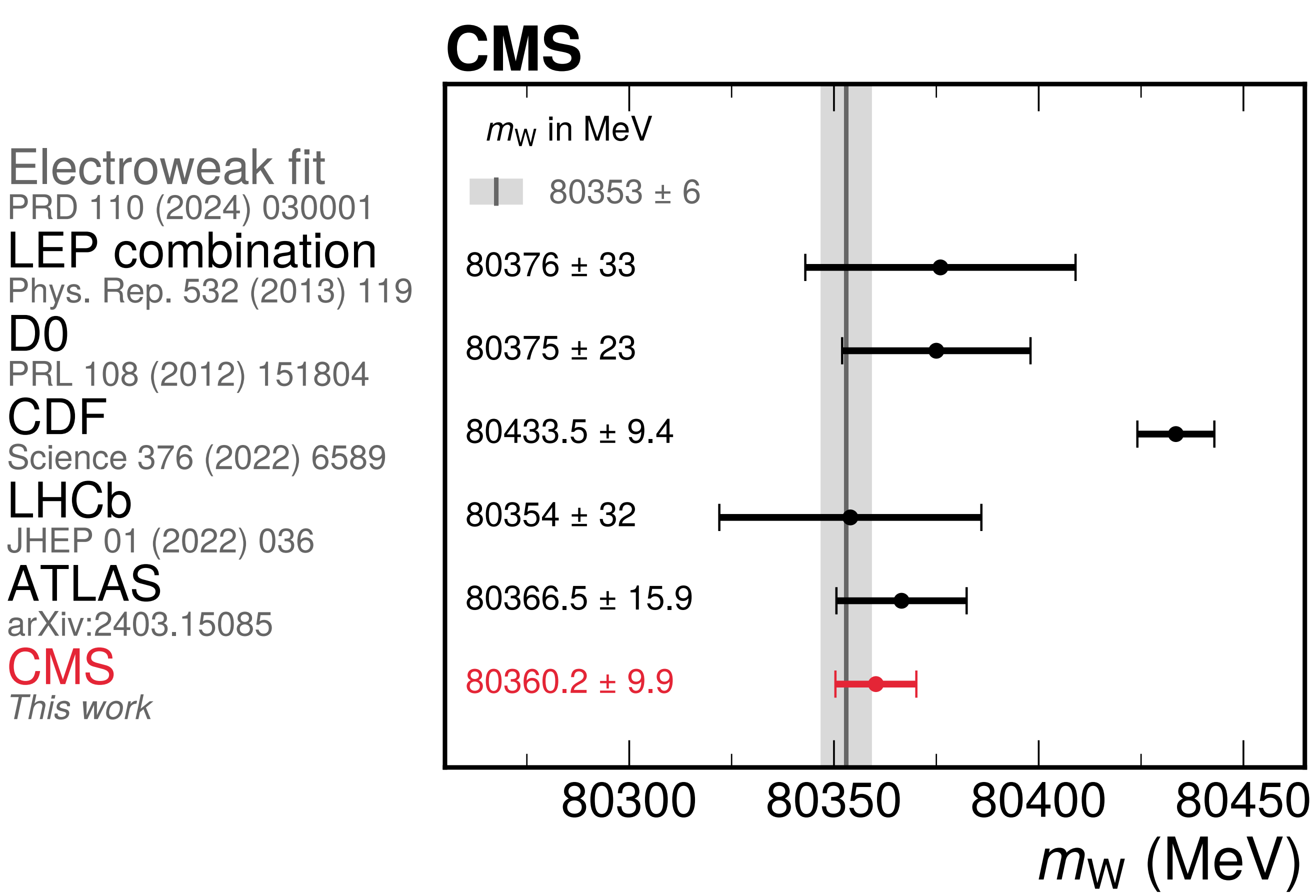
In collaboration with:
T. Armadillo, R. Bonciani, L. Buonocore,
M. Grazzini, S. Kallweit, N. Rana, A. Vicini

MOTIVATIONS

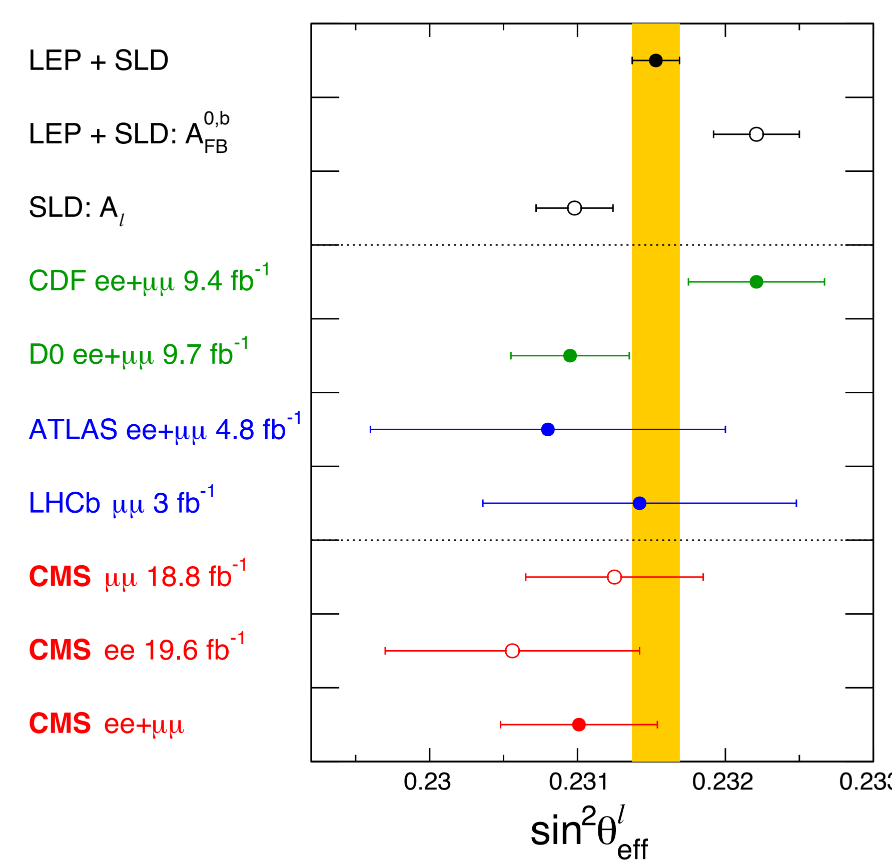
Standard Model Precision Studies

- Extremely precise measurement of W boson mass (10 MeV uncertainties!);
- Determination of $\sin^2\theta_{\text{eff}}^{\text{lep}}$ is becoming competitive with LEP result: 0.23152(16);
- Crucial process for PDF determination.

Can we provide SM theoretical predictions matching experimental accuracy?



[CMS collaboration, arXiv:2412.13872]



[CMS collaboration, arXiv:1806.00863]

MIXED QCDxEW CORRECTIONS

$$q(p_1) + \bar{q}(p_2) \to l^-(p_3) + l^+(p_4) \quad = \quad \sigma^{(0,0)}$$ *Drell-Yan (1970)*

MIXED QCDxEW CORRECTIONS

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$$+ \alpha_S\, \sigma^{(1,0)} + \alpha\, \sigma^{(0,1)}$$

$$+ \alpha_S^2\, \sigma^{(2,0)} + \alpha\, \alpha_S\, \sigma^{(1,1)} + \alpha^2\, \sigma^{(0,2)}$$

$$+ \alpha_S^3\, \sigma^{(3,0)} + \ldots$$

QCD MIXED EW

credits: xkcd (2605)

MIXED QCDxEW CORRECTIONS

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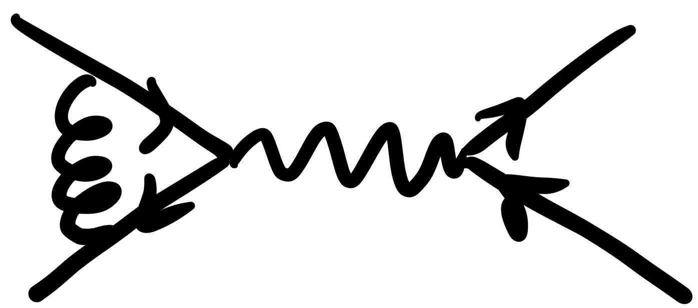
QCD CORRECTIONS



$$+ \alpha_S\, \sigma^{(1,0)} + \alpha\, \sigma^{(0,1)}$$

$$+ \alpha_S^2\, \sigma^{(2,0)} + \alpha\, \alpha_S\, \sigma^{(1,1)} + \alpha^2\, \sigma^{(0,2)}$$

$$+ \alpha_S^3\, \sigma^{(3,0)} + \ldots$$

- **Dominant effect**;
- Known up to N3LO.

NLO:
[G.Altarelli, R.Ellis, G.Martinelli Nucl.Phys.B 157 (1979)];

NNLO:
[R.Hamberg, T.Matsuura, W.van Nerveen, Nucl. Phys. B 359 (1991)];
[C.Anastasiou, L.J.Dixon, K.Melnikov, F.Petriello, hep-ph:0306192];
[S.Catani, L.Cieri, G.Ferrera, D.de Florian, M.Grazzini arXiv:0903.2120];

N3LO:
[C.Duhr, F.Dulat, B.Mistlberger arXiv:2007.13313];
[X.Chen, T.Gehrmann, N.Glover, A.Huss, T.Yang, and H.Zhu arXiv:2107.09085];
[S.Camarda, L.Cieri, G.Ferrera arXiv:2103.04974];
[X.Chen, T.Gehrmann, N.Glover, A.Huss, P.Monni, E.Re, L.Rottoli, P.Torrielli arXiv:2203.01565];.

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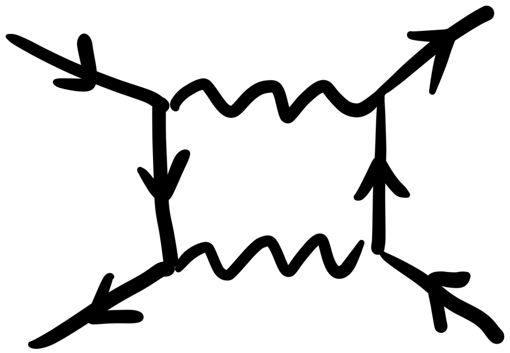
EW CORRECTIONS



- Subdominant with respect to QCD: **physical counting $\alpha_S \simeq \alpha^2$**;
- NLO corrections known;
  [U.Baur, O.Brein, W.Hollik, C.Schappacher, D.Wackeroth, hep-ph:0108274];
  [S.Dittmaier, M.Kramer, hep-ph:0109062];
  [U.Baur, D.Wackeroth, hep-ph:0405191];
- NNLO corrections still missing (available Sudakov high energy approximation).
  [B. Jantzen, J.H.Kühn. A.A.Penin, V.A.Smirnov, hep-ph:0509157];

MIXED QCDxEW CORRECTIONS

$$q(p_1) + \bar{q}(p_2) \to l^-(p_3) + l^+(p_4) \quad = \quad \sigma^{(0,0)}$$

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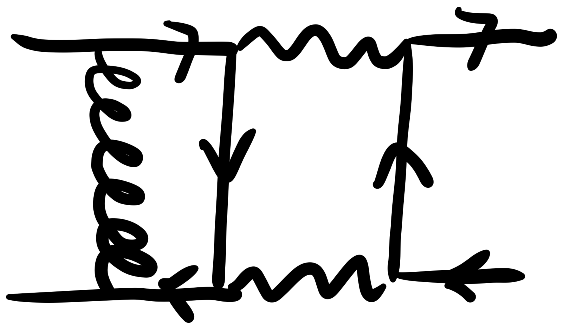
MIXED CORRECTIONS



NLO QCD and NLO EW separately large: what about mixed?

By physical counting, expected size comparable with N3LO QCD!

[R. Bonciani, L. Buonocore, M. Grazzini, S. Kallweit, N. Rana, F. Tramontano, A.Vicini, arXiv:2106.11953]

[T. Armadillo, R. Bonciani, L. Buonocore, SD, M. Grazzini, S. Kallweit, N. Rana, A.Vicini, arXiv:2412.16095]

[F. Buccioni, F. Caola, H. Chawdhry, F. Devoto, M.Heller, A. von Manteuffel, K. Melnikov, R. Röntsch, C. Signorile-Signorile, arXiv:2203.11237]

RECENT PROGRESSES IN MIXED CORRECTIONS

- **Theoretical Developments**
  - 2-loop virtual Master Integrals with internal masses *[U. Aglietti, R. Bonciani, arXiv:0304028, arXiv:0401193],[R. Bonciani, S. Di Vita, P. Mastrolia, U. Schubert, arXiv:1604.08581], [M.Heller, A.von Manteuffel, R.Schabinger arXiv:1907.00491], [M.Long,R,Zhang,W.Ma,Y,Jiang,L.Han,,Z.Li,S.Wang, arXiv:2111.14130],[X.Liu, Y.Ma, arXiv:2201.11669]*
  - Altarelli-Parisi splitting functions including QCD-QED effects *[D. de Florian, G. Sborlini, G. Rodrigo, arXiv:1512.00612]*
  - Renormalisation *[G.Degrassi, A.Vicini, hep-ph/0307122],[S.Dittmaier,T.Schmidt,J.Schwarz, arXiv:2009.02229], [S.Dittmaier, arXiv:2101.05154]*
- **On-shell Z and W production**
  - pole approximation of the NNLO QCD-EW corrections *[S.Dittmaier, A.Huss, C.Schwinn, arXiv:1403.3216, 1511.08016]*
  - analytical total Z production cross section including NNLO QCD-QED corrections *[D. de Florian, M.Der, I.Fabre, arXiv:1805.12214]*
  - fully differential on-shell Z production including exact NNLO QCD-QED corrections *[M.Delto, M.Jaquier, K.Melnikov, R.Roentsch, arXiv:1909.08428] [S.Hasan, U.Schubert, arXiv:2004.14908]*
  - analytical total Z production cross section including NNLO QCD-EW corrections *[R. Bonciani, F. Buccioni, R.Mondini, A.Vicini, arXiv:1611.00645], [R. Bonciani, F. Buccioni, N.Rana, I.Triscari, A.Vicini, arXiv:1911.06200], [R. Bonciani, F. Buccioni, N.Rana, A.Vicini, arXiv:2007.06518, arXiv:2111.12694]*
  - fully differential Z and W production including NNLO QCD-EW corrections *[F. Buccioni, F. Caola, M.Delto, M.Jaquier, K.Melnikov, R.Roentsch, arXiv:2005.10221], [A. Behring, F. Buccioni, F. Caola, M.Delto, M.Jaquier, K.Melnikov, R.Roentsch, arXiv:2009.10386, 2103.02671]*
- **Complete Drell-Yan**
  - neutrino-pair production including NNLO QCD-QED corrections *[L. Cieri, D. de Florian, M.Der, J.Mazzitelli, arXiv:2005.01315]*
  - 2-loop amplitudes *[M.Heller, A.von Manteuffel, R.Schabinger, arXiv:2012.05918],[T.Armadillo, R.Bonciani, SD, N.Rana, A.Vicini, arXiv:2201.01754; arXiv:2412.16095] ,*
  - NNLO QCD-EW corrections to neutral-current DY including leptonic decay *[R.Bonciani, L.Buonocore, M.Grazzini, S.Kallweit, N.Rana, F.Tramontano, A.Vicini, arXiv:2106.11953],[F.Buccioni, F.Caola, H.Chawdhry, F.Devoto, M.Heller,A.von Manteuffel, K.Melnikov, R.Röntsch, C.Signorile-Signorile, arXiv:2203.11237] [T. Armadillo, R. Bonciani, L. Buonocore, SD, M. Grazzini, S. Kallweit, N. Rana, A. Vicini, arXiv:2412.16095]*
  - NNLO QCD-EW corrections to charged-current DY including leptonic decay (2-loop contributions in pole approximation). *[L.Buonocore, M.Grazzini, S.Kallweit, C.Savoini, F.Tramontano, arXiv:2102.12539]*

CONTENTS

- **Phenomenological application**:
  - Neutral Current Drell-Yan;
  - Forward-Backward Asymmetry;
- **Two-loop Amplitudes**:
  - Theoretical challenges;
  - Our workflow;
  - Neutral Current Drell-Yan;
  - Charged Current Drell-Yan.

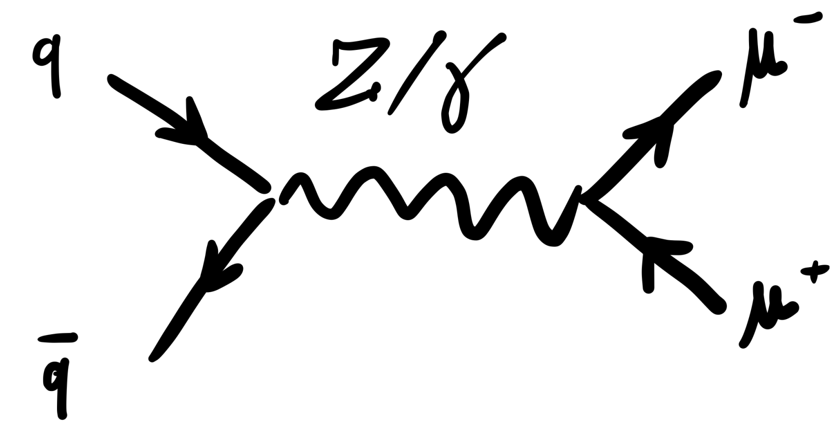
PHENOMENOLOGY

EXPLORING THE STANDARD MODEL

NEUTRAL CURRENT DRELL-YAN

[T. Armadillo, R. Bonciani, L. Buonocore, SD, M. Grazzini, S. Kallweit, N. Rana, A. Vicini, arXiv:2412.16095]

$$q(p_1) + \bar{q}(p_2) \to \mu^-(p_3) + \mu^+(p_4)$$



- IR singularities handled by **q_T-subtraction formalism**;

[S. Catani, M. Grazzini (2007)] [L.Buonocore, M. Grazzini, F.Tramontano (2019)]

$$d\sigma^F_{(N)NLO} = \mathcal{H}^F_{(N)NLO} \otimes d\sigma^F_{LO} + \left[d\sigma^{F+jets}_{(N)LO} - d\sigma^{CT}_{(N)LO} \right]$$

- Final-state soft singularities are regularised by the **muon mass** m_μ;
- Process implemented in the **MATRIX** framework.

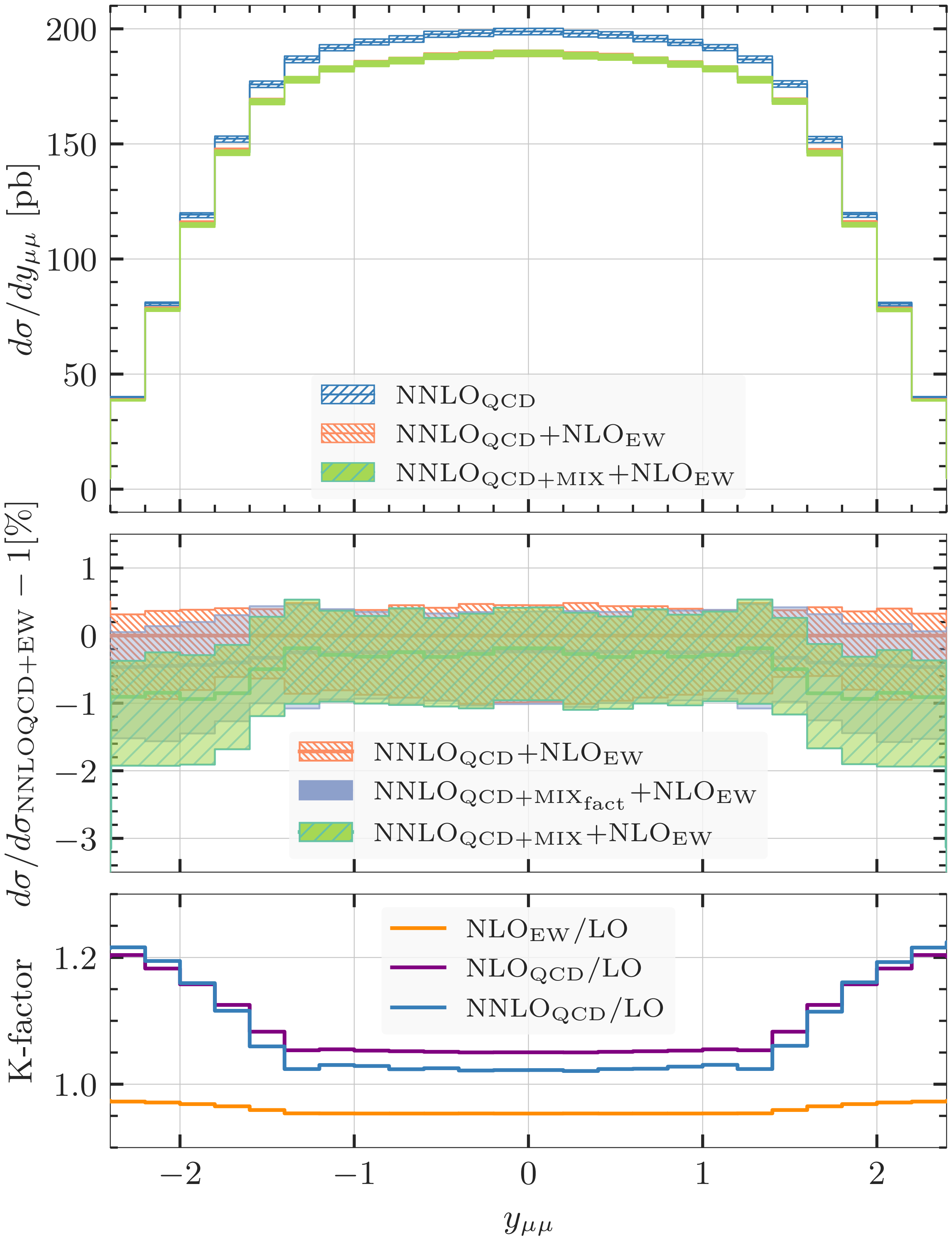
MATRIX

Munich -- the MUlti-chaNnel Integrator at swiss (CH) precision --
Automates qT-subtraction and Resummation to Integrate X-sections

https://matrix.hepforge.org

DIFFERENTIAL RESULTS

[T. Armadillo, R. Bonciani, L. Buonocore, SD, M. Grazzini, S. Kallweit, N. Rana, A. Vicini, arXiv:2412.16095]



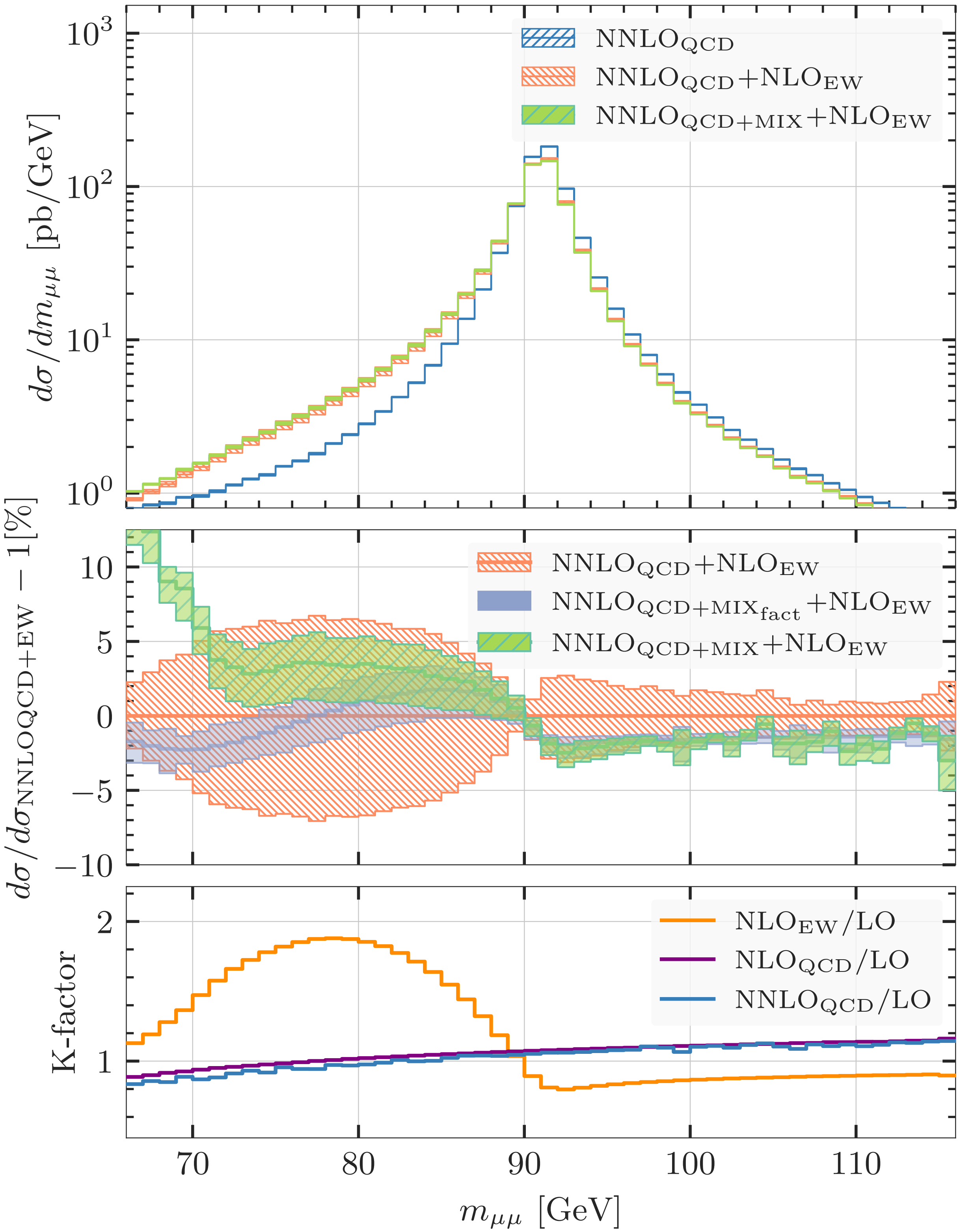
- We compare exact results MIX with the **factorised approximation** $\mathrm{MIX_{fact}}$

$$\frac{d\sigma^{(1,1)}_{\mathrm{fact}}}{dX} = \left(\frac{d\sigma^{(1,0)}}{dX}\right) \times \left(\frac{d\sigma^{(0,1)}_{q\bar{q}}}{dX}\right) \times \left(\frac{d\sigma_{\mathrm{LO}}}{dX}\right)^{-1};$$

- We observe a **shape distortion**: the strongest effect comes from NNLO QCD, but it is **emphasised** by the **mixed corrections**;
- Effect of the mixed corrections up to the **$\mathcal{O}(1\%)$ level**;
- Shape distortions are relevant for **PDF determinations**.

DIFFERENTIAL RESULTS

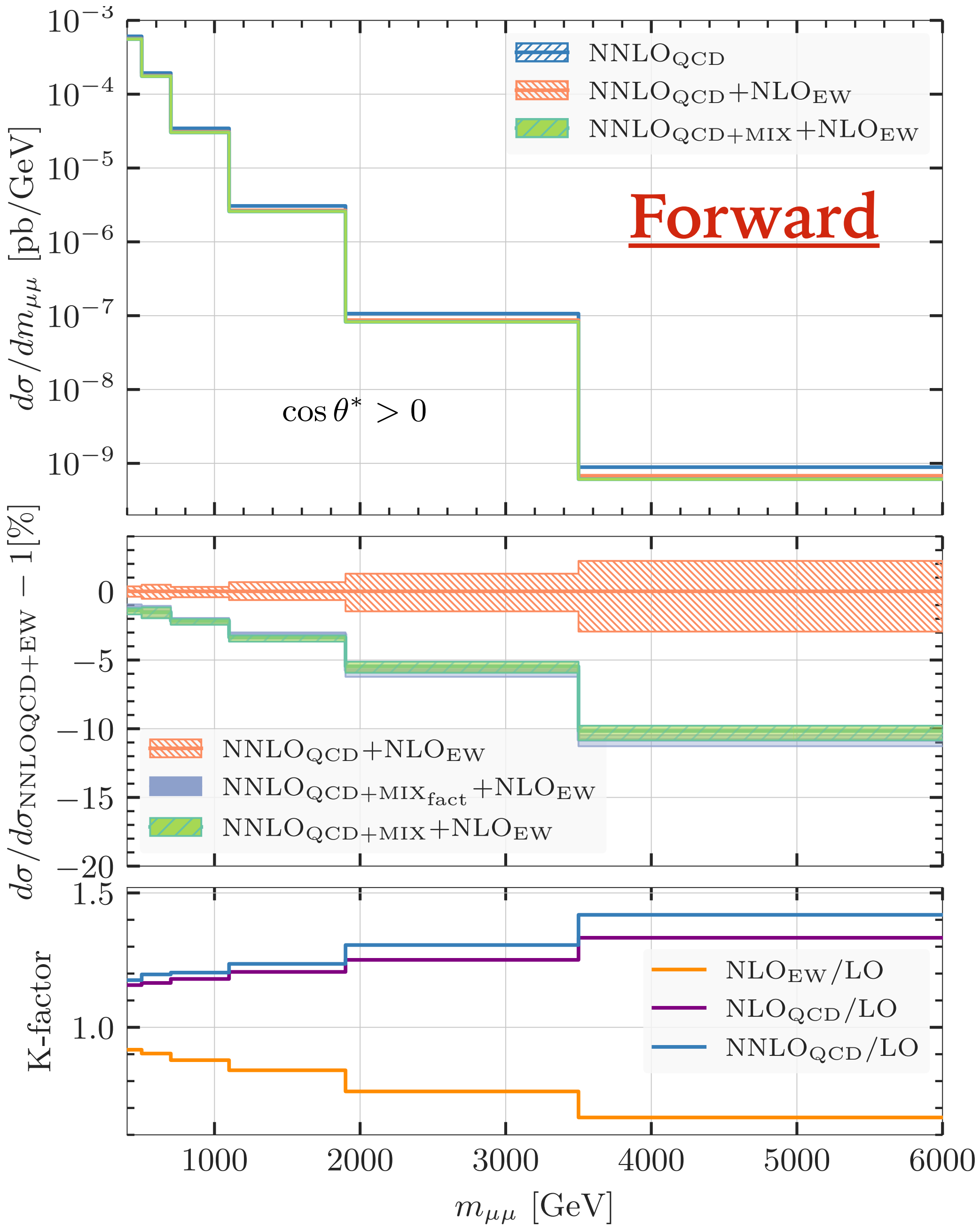
[T. Armadillo, R. Bonciani, L. Buonocore, SD, M. Grazzini, S. Kallweit, N. Rana, A. Vicini, arXiv:2412.16095]

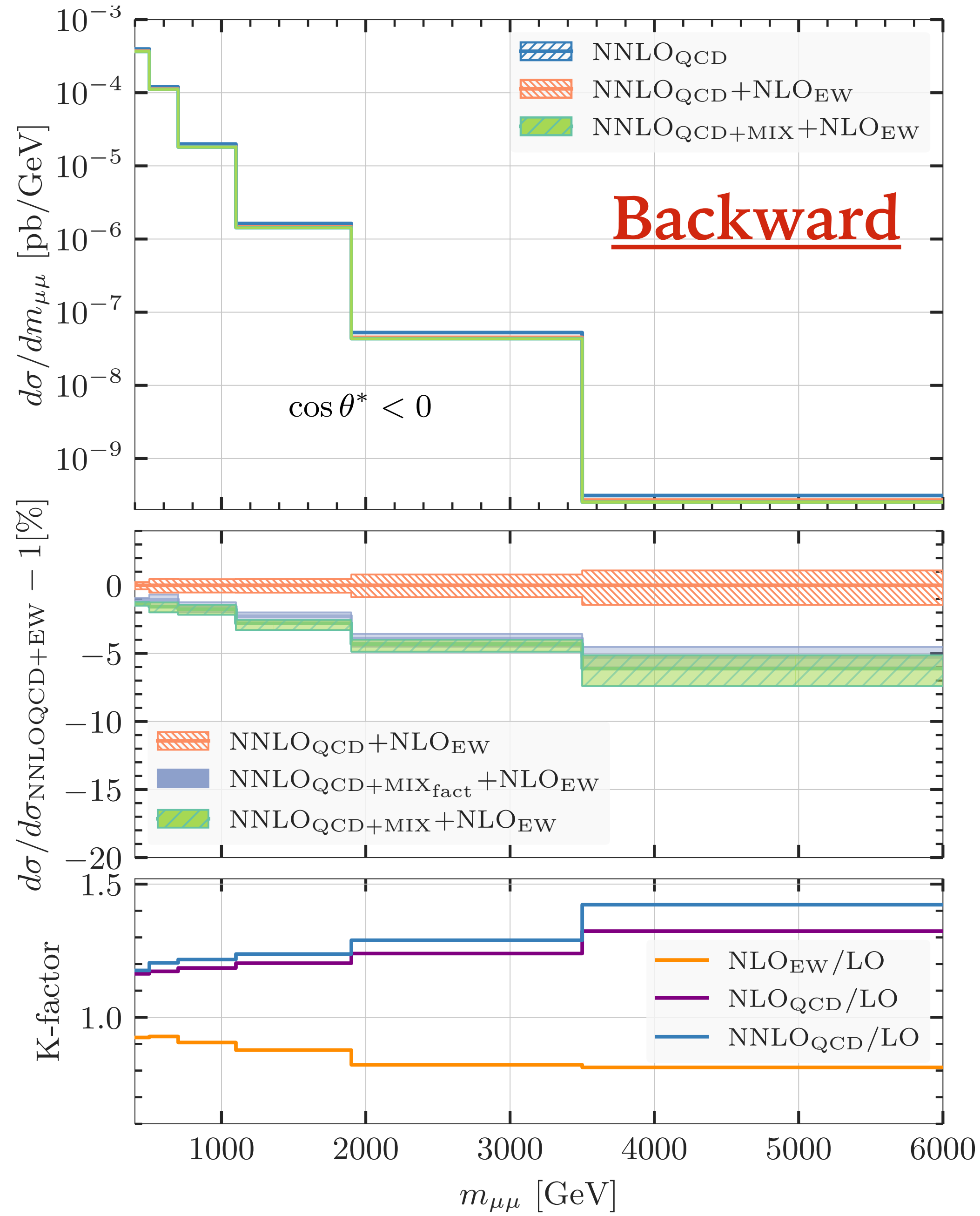


- **Mild** shape distortion from QCD corrections;
- **Strong** shape distortion from EW corrections, which generate a **radiative tail** before the peak;
- The **mixed corrections** provide a non-trivial **shape distortion**;
- The **factorised approximation** does **not** give a good description of the resonant region.

FORWARD-BACKWARD ASYMMETRY

[T. Armadillo, R. Bonciani, L. Buonocore, SD, M. Grazzini, S. Kallweit, N. Rana, A. Vicini, arXiv:2412.16095]

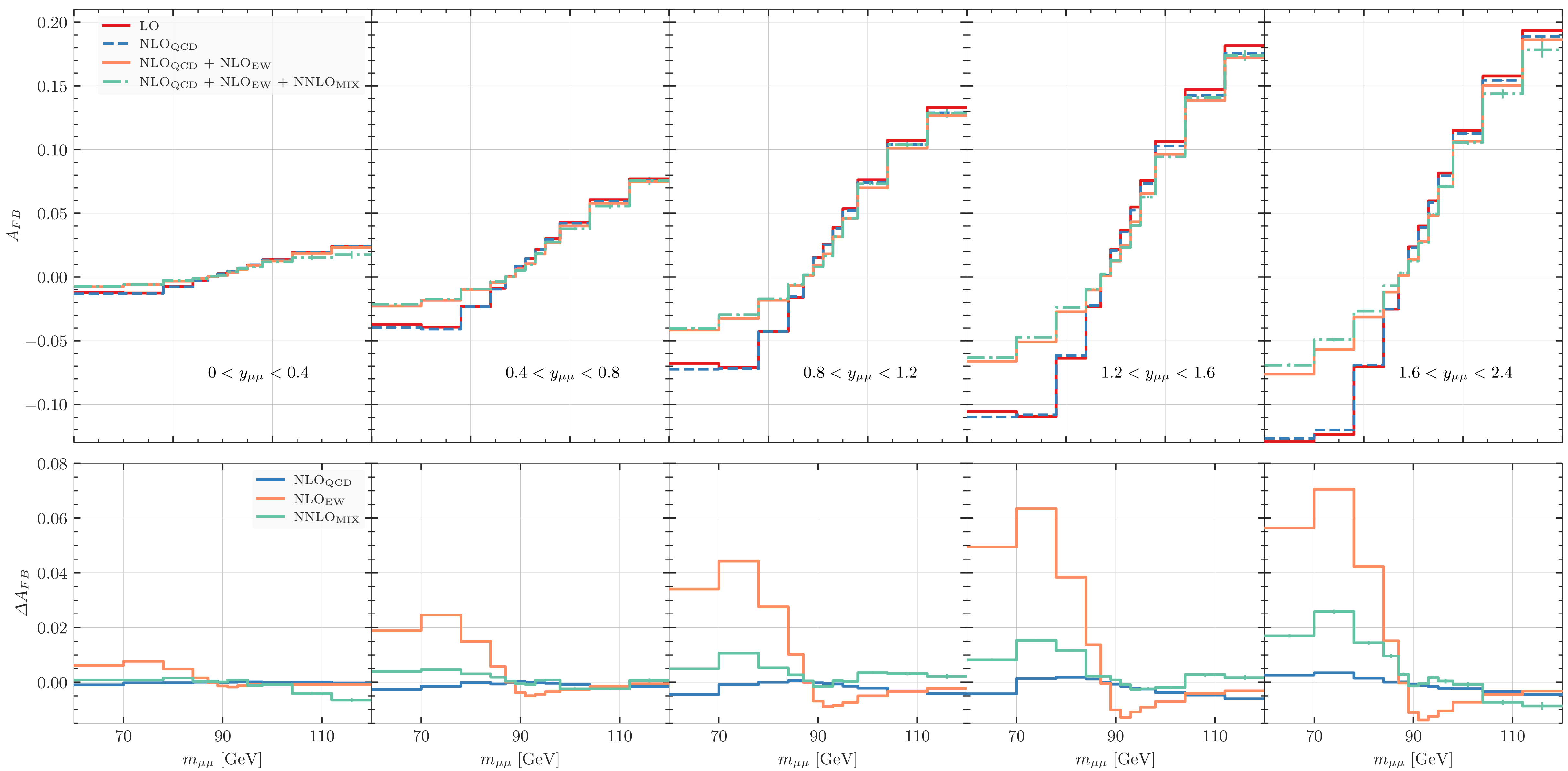




$$A_{FB}(m_{\mu\mu}) = \frac{F(m_{\mu\mu}) - B(m_{\mu\mu})}{F(m_{\mu\mu}) + B(m_{\mu\mu})}$$

FORWARD–BACKWARD ASYMMETRY

[T. Armadillo, R. Bonciani, L. Buonocore, SD, M. Grazzini, S. Kallweit, N. Rana, A. Vicini, arXiv:2412.16095]



- Significant effect from **mixed corrections** to the **forward-backward asymmetry**;
- This measurement has a direct impact on $\sin^2\theta^{\ell}_{eff}$.
- We apply the cuts used by the CMS collaboration for the extraction of $\sin^2\theta^{\ell}_{eff}$:

  $p_{T,1} > 25\,\text{GeV}$; $p_{T,2} > 15\,\text{GeV}$; $|y_\mu| < 2.4$; $60 < m_{\mu\mu} < 120\,\text{GeV}$. [CMS collaboration, arXiv:1806.00863]

TWO-LOOP AMPLITUDES

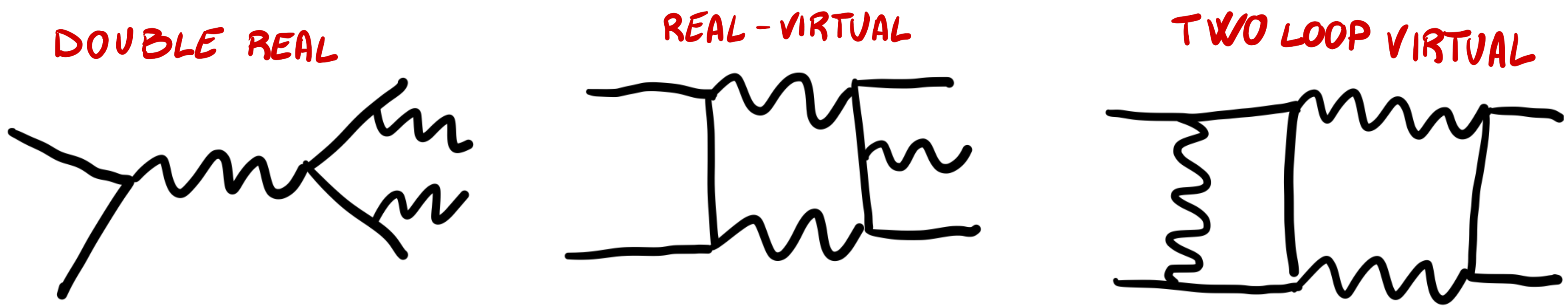
THEORETICAL CHALLENGES

THEORY BOTTLENECKS

What makes NNLO challenging?

<u>Subtraction procedure</u>

<u>Two loop amplitudes</u>



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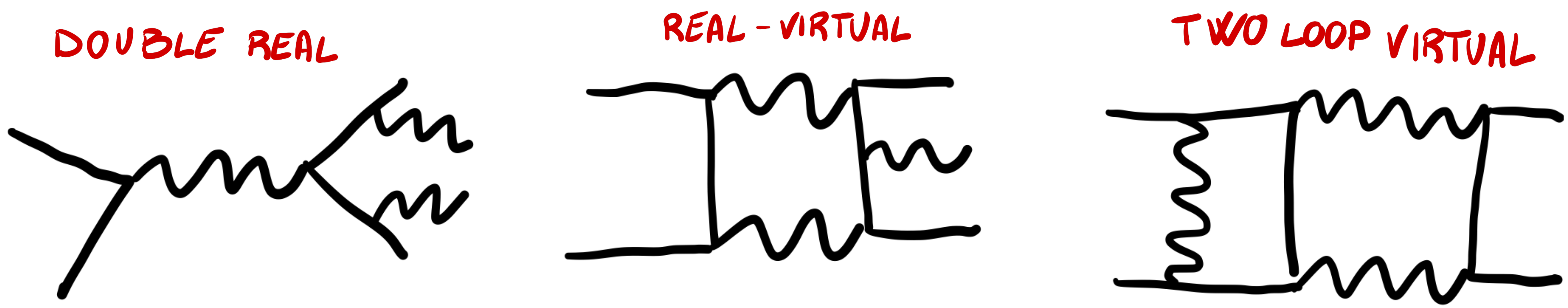


What makes EW corrections challenging?

- additional internal **massive lines**;
  additional scales in the problem (m_Z, m_W, m_H…) bring additional complications!
- treatment of γ_5;
  how can γ_5 be consistently used in dimensional regularisation?
- need for the **complex mass scheme**;
  requires to analytically continue the master integrals on the complex plane!

OUR WORKFLOW

THE BUILDING BLOCKS

ABISS

Private Mathematica package

FeynArts

[T.Hahn arXiv:0012260]

Kira

[P. Maierhöfer, J. Usovitsch and P. Uwer; arXiv:1705.05610]

SEASYDE

Public Mathematica package

OUR WORKFLOW

$$|\mathscr{M}|^2 = \sum_k c_k I_k$$

c_k=*coefficients*

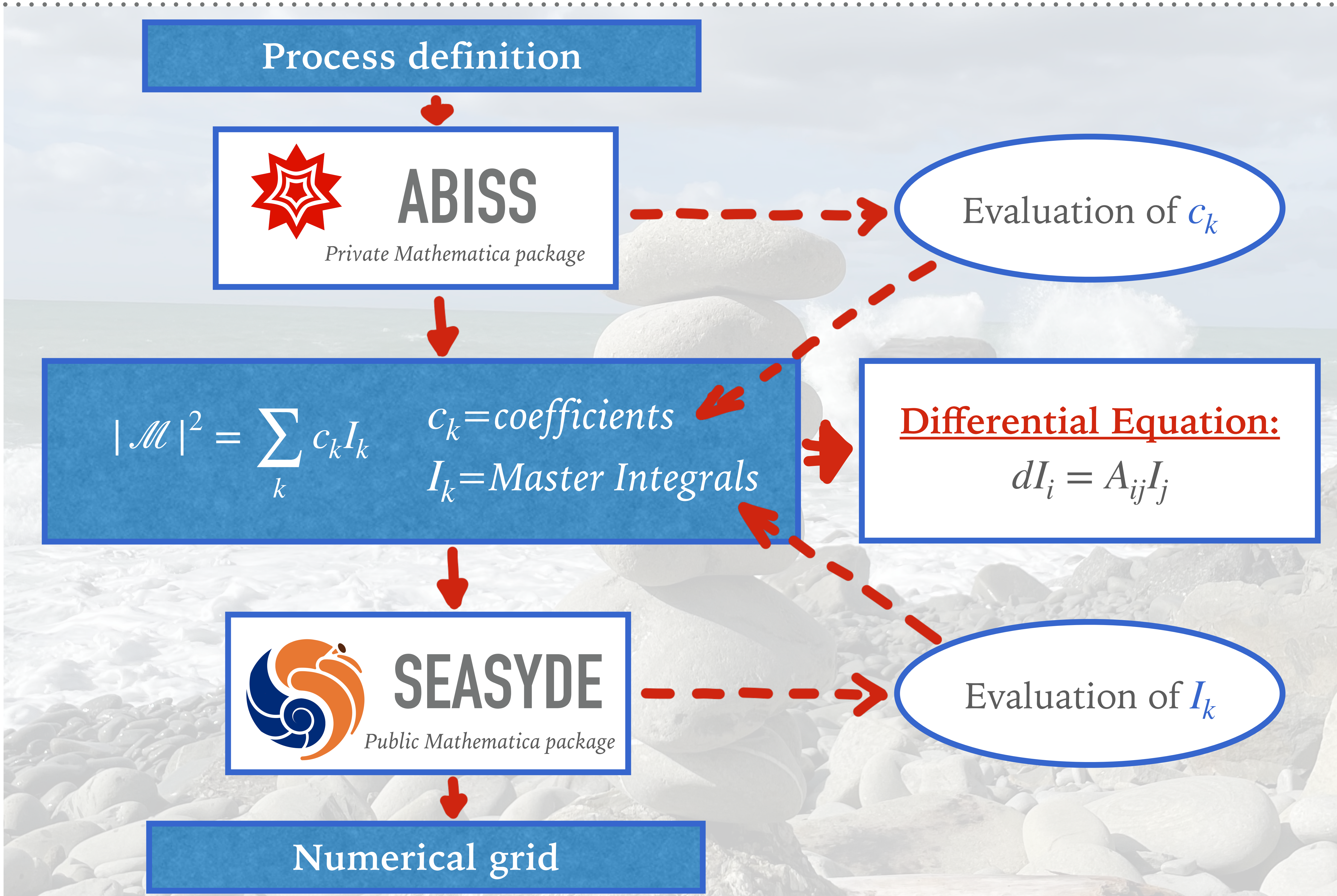
I_k=*Master Integrals*

→

Differential Equation:

$$dI_i = A_{ij} I_j$$

OUR WORKFLOW



SEASYDE - EVALUATING I_k

[T. Armadillo, R. Bonciani, SD, N.Rana, A.Vicini, arXiv:2205.03345]

Numerical Result

The result of the master integral is provided as a numerical **grid**.

Analytical Result

The result of the master integral can be expressed in closed form as a combination of elementary and special functions, whose **power expansion is known**.

Semi-Analytical Result

The result of the master integral can be expanded as a power series at every point of its domain, but **without** any additional functional relations.

We solve the master integral with **series expansion**!

SEASYDE – EVALUATING I_k

[T. Armadillo, R. Bonciani, SD, N.Rana, A.Vicini, arXiv:2205.03345]

SEASYDE

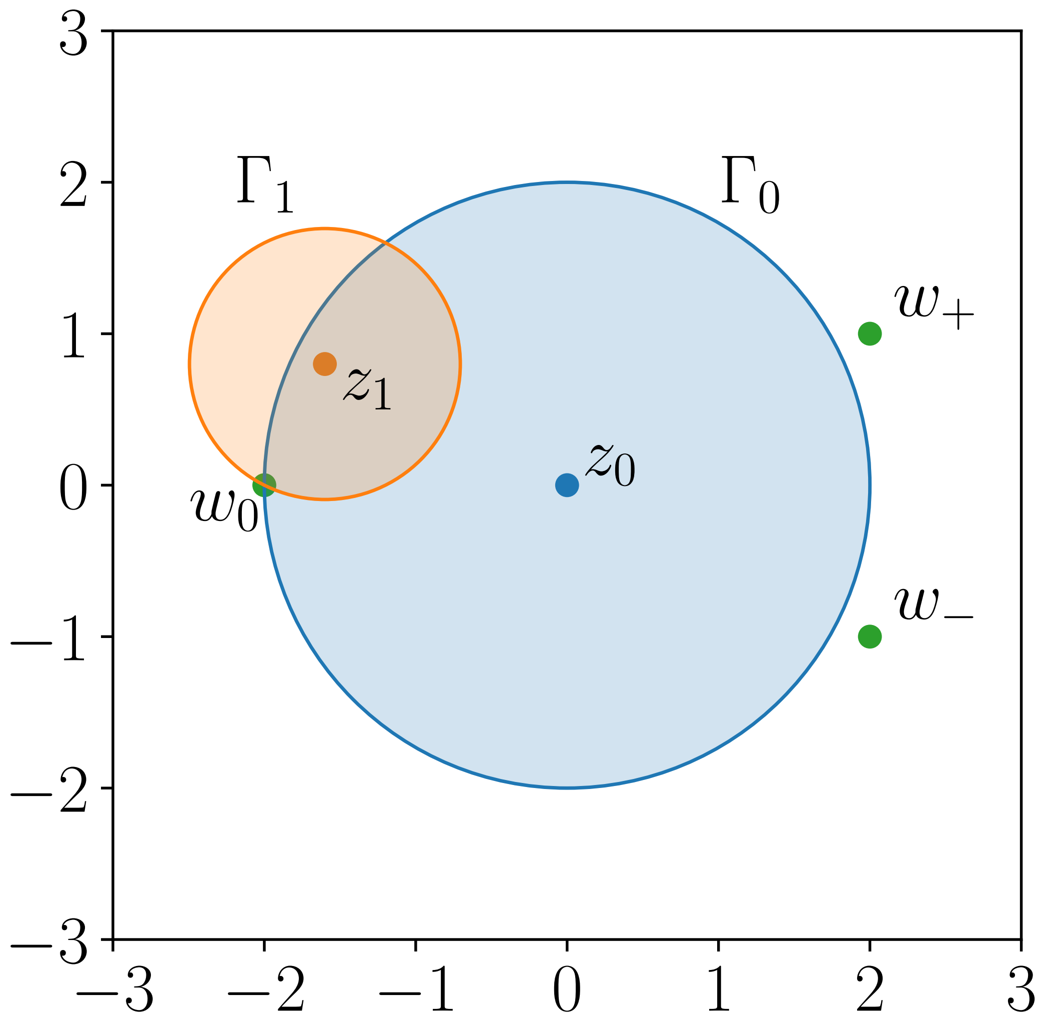
The latest version of SEASYDE can be downloaded from: github.com/TommasoArmadillo/SeaSyde

- SEASYDE(Series Expansion Approach for SYstem of Differential Equations) is a MATHEMATICA package for solving the system of differential equation, associated to the Master Integrals of a given topology.
- SEASYDE can handle any **system of coupled differential equations**.
- Method implemented in the Mathematica package DiffExp for real kinematic variables *[F. Moriello, arXiv:1907.13234]*, *[M. Hidding, arXiv:2006.05510]* (see also AMFLOW *[X. Liu and Y.-Q. Ma, arXiv: 2201.11669]*; LINE *[R.M. Prisco, J. Ronca, F. Tramontano, arXiv:2501.01943]*)
- By using the auxiliary mass flow, AMFLOW can provide **boundary conditions**.

COMPLEX PLANE

[T. Armadillo, R. Bonciani, SD, N.Rana, A.Vicini, arXiv:2205.03345]

- We generalised the series expansion method to **arbitrary complex-valued masses** → **complex plane** of the kinematical invariants!
- The radius of convergence of the series is limited by the presence of **poles**;
- "Transport" of the boundary conditions need to consider **branch-cuts**.



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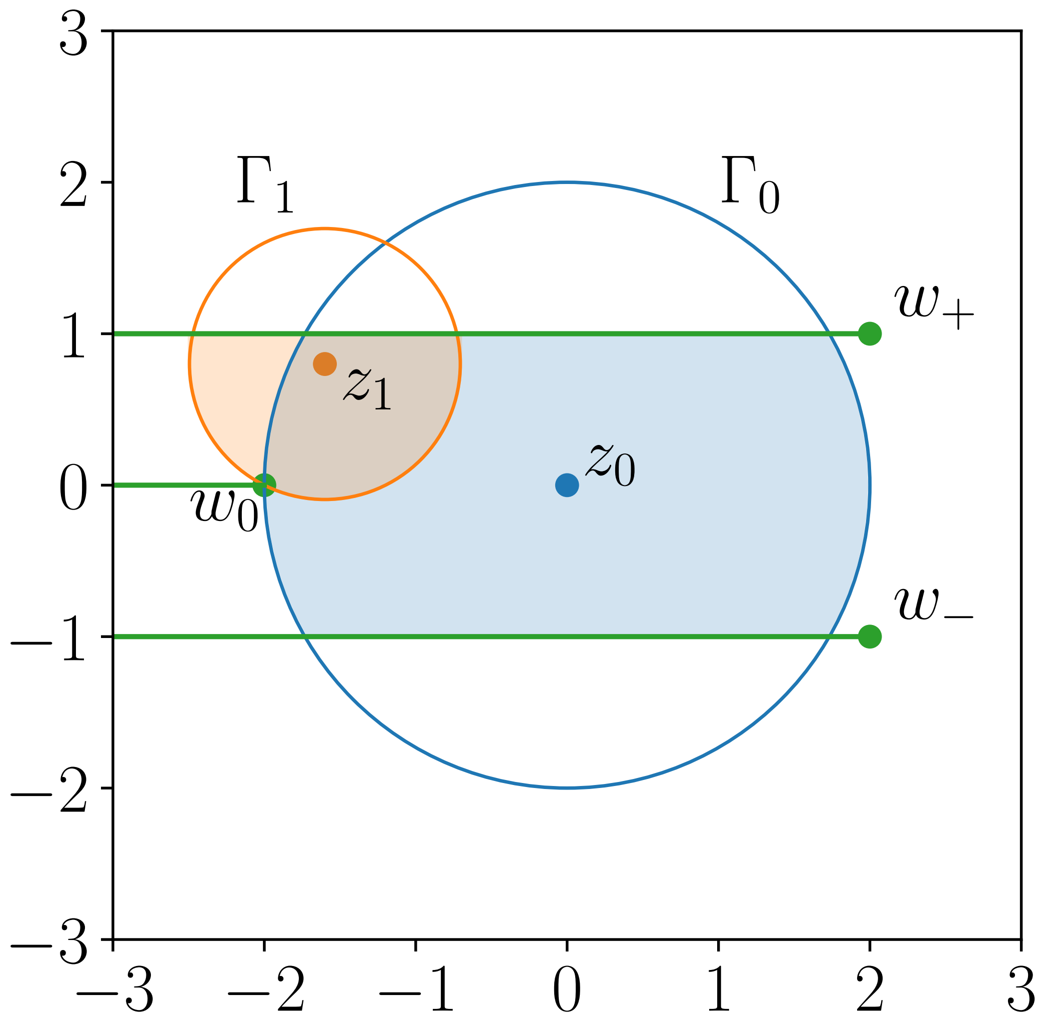

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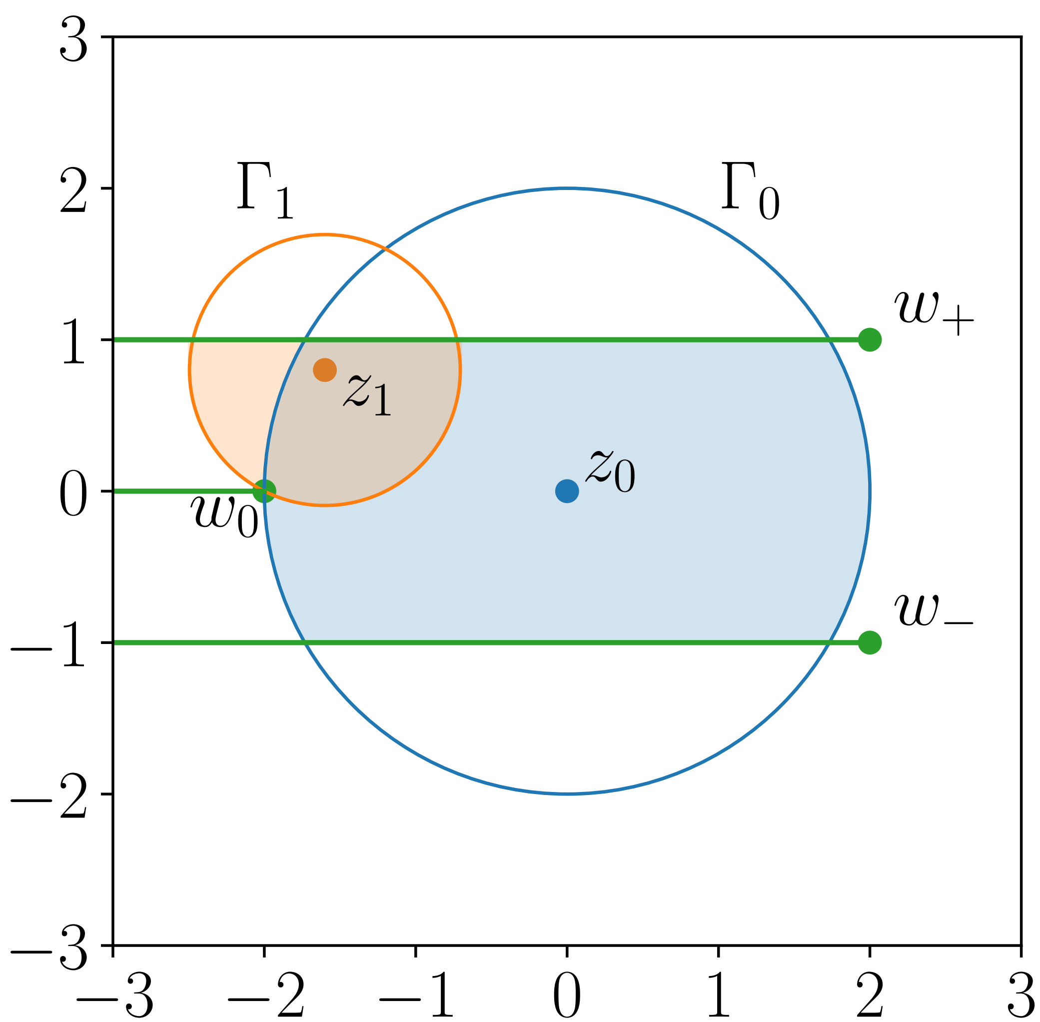

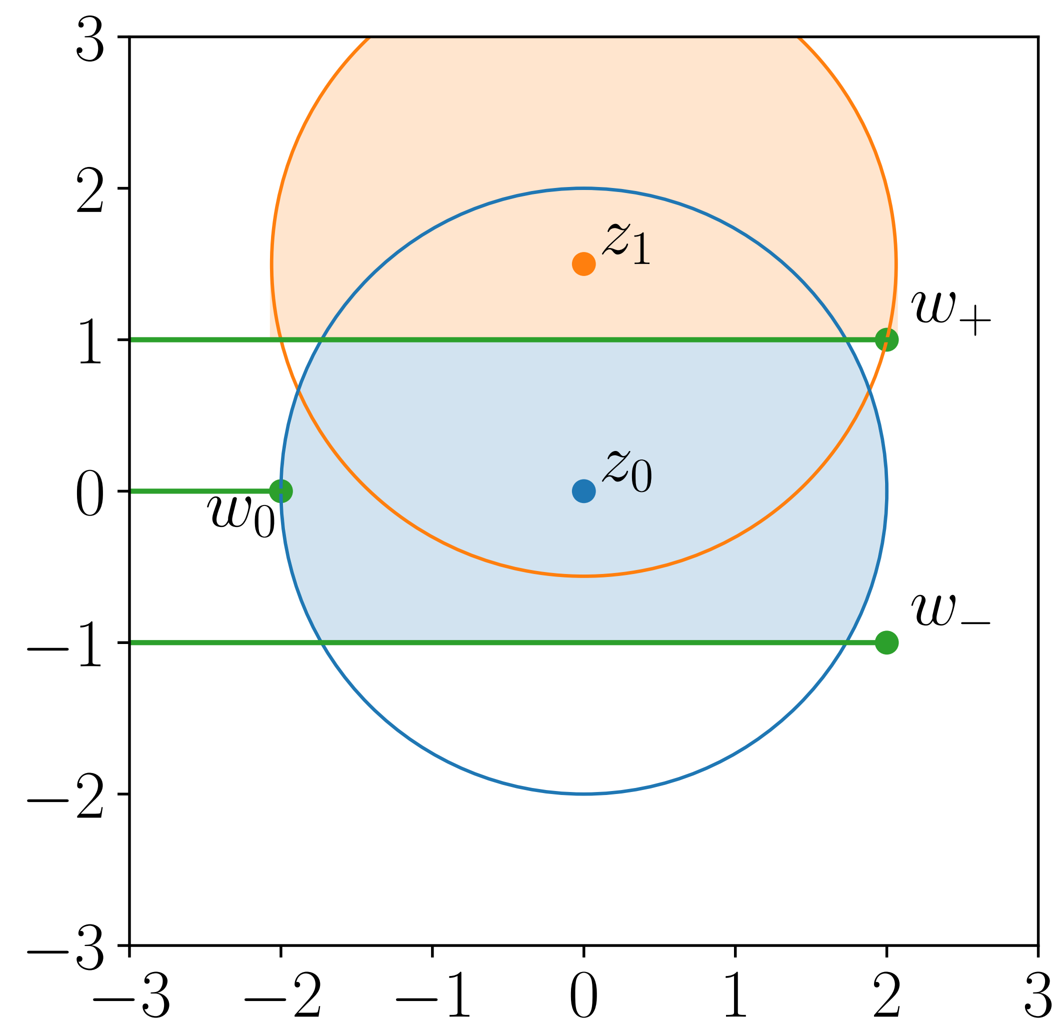

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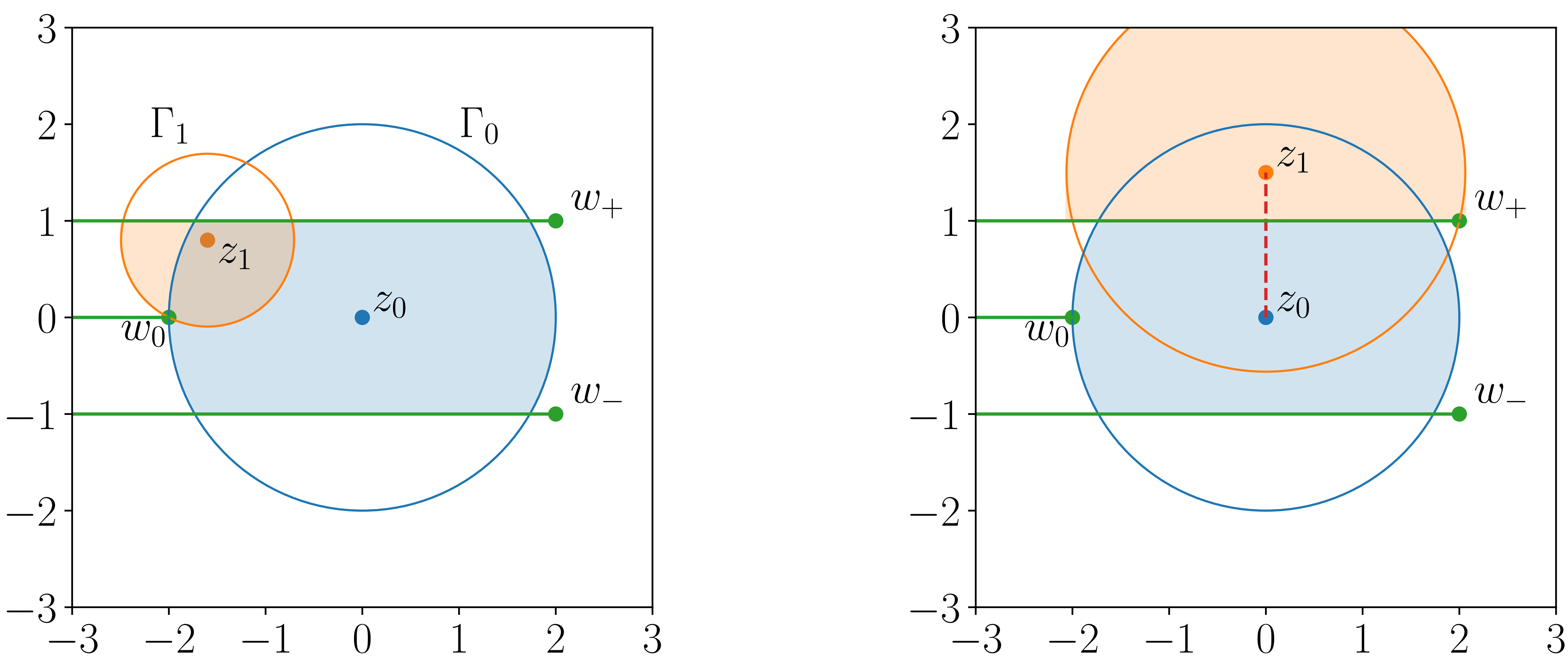

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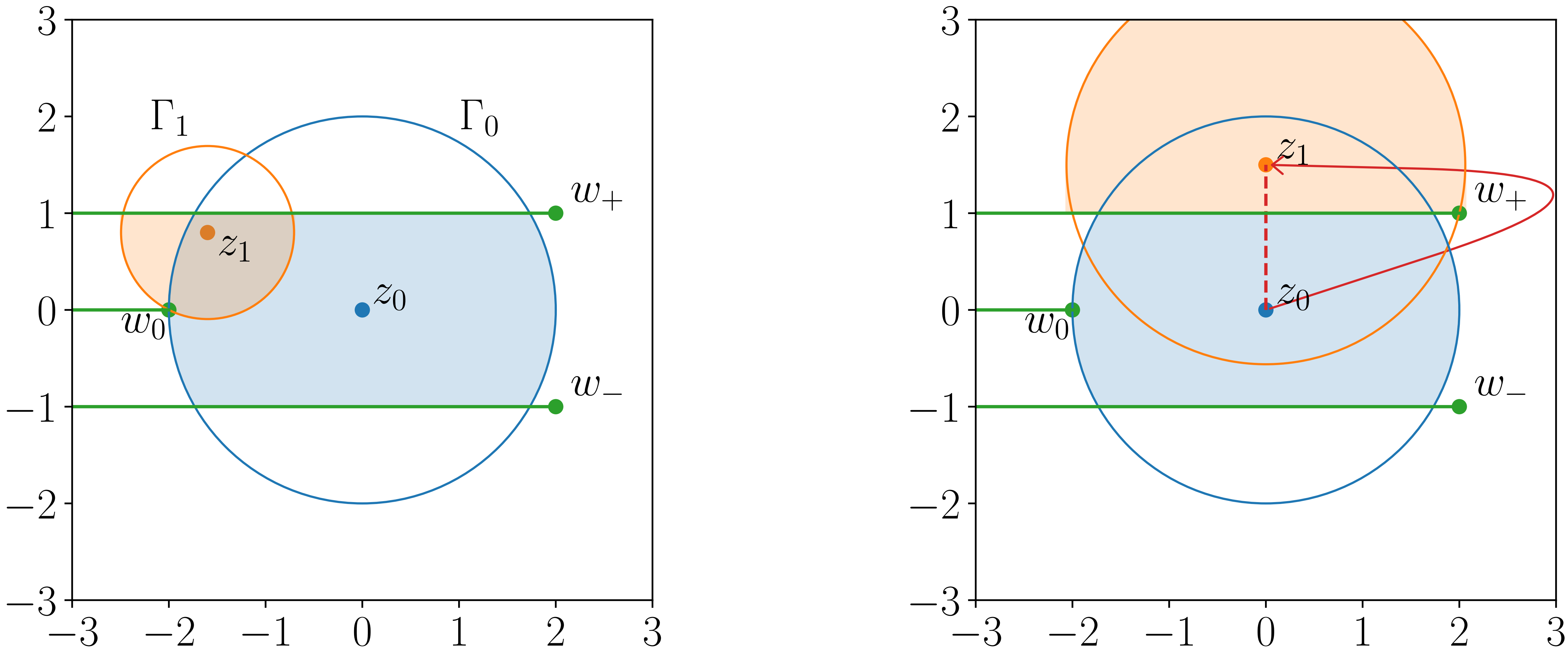
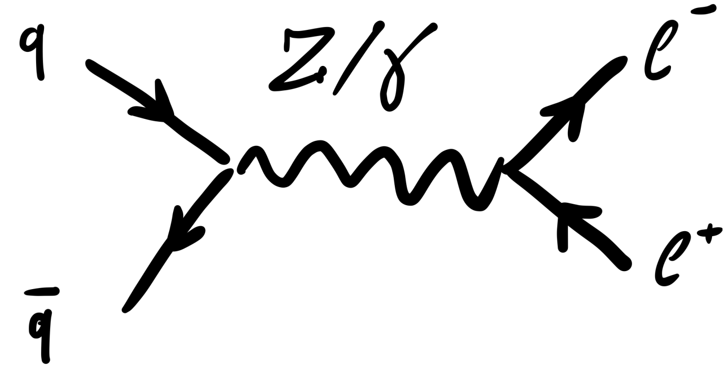


COMPUTATIONAL FRAMEWORK

[T. Armadillo, R. Bonciani, SD, N.Rana, A.Vicini, arXiv:2201.01754]

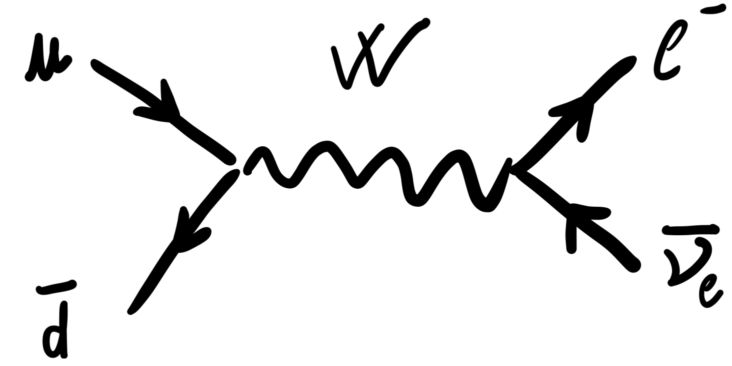
NCDY

$$q(p_1) + \bar{q}(p_2) \rightarrow l^-(p_3) + l^+(p_4)$$



CCDY

$$u(p_1) + \bar{d}(p_2) \rightarrow \nu_\ell(p_3) + l^+(p_4)$$



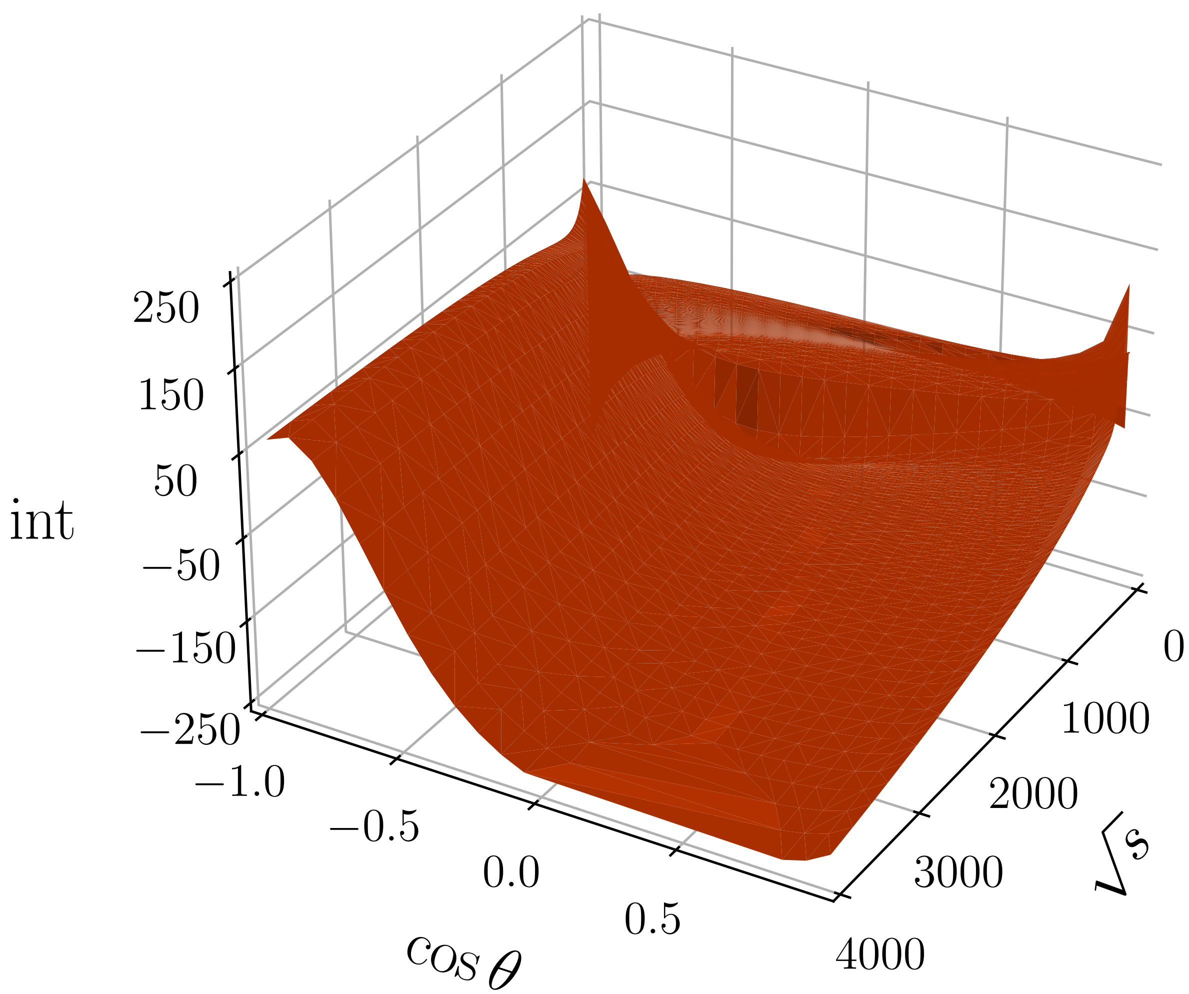
- IR singularities handled by **q_T-subtraction formalism** (straightforward implementation of **any other framework** by replacing the **subtraction operator**);
- Final-state collinear singularities regularised by the **lepton mass**;
- **small lepton mass limit**: consider the ratio $m_l/\sqrt{s}$ and keep only **logarithmic terms** $\sim \log\left(m_l/\sqrt{s}\right)$.

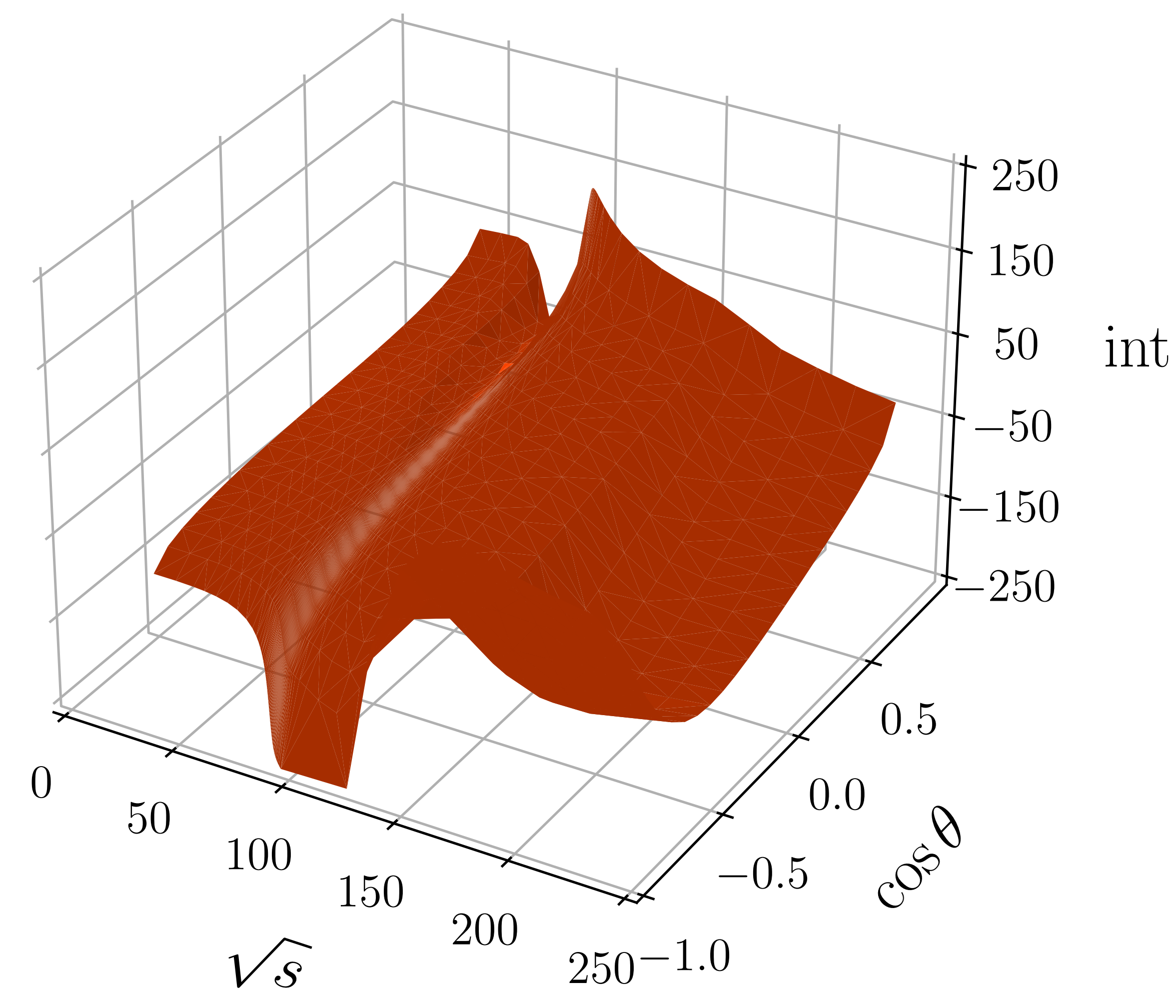
- When dealing with intermediate unstable particles, such as W and Z, it is useful to perform the calculations in the **complex-mass scheme**;
- We introduce the complex mass $\mu_V^2 = m_V^2 - i\Gamma_V m_V$ for both the Z and W bosons.

NCDY – NUMERICAL GRIDS

[T. Armadillo, R. Bonciani, SD, N.Rana, A.Vicini, arXiv:2201.01754]

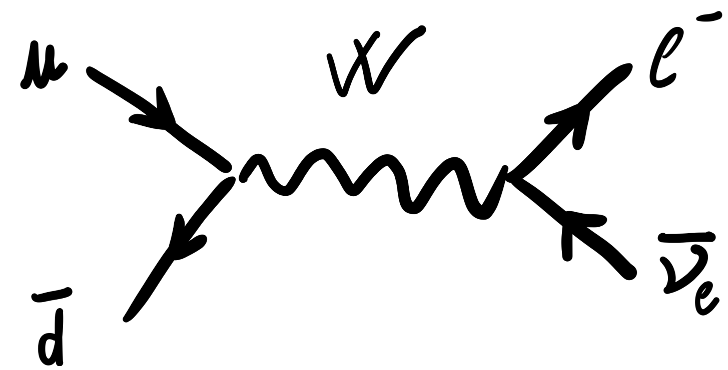
- After subtracting IR and UV divergences, we obtain the **hard function**;
- **Publicly available** as a MATHEMATICA notebook;
- Subtraction of the IR poles done in the **qT-subtraction** formalism;
- Production of the grid (3250 points) required $\mathcal{O}$(12h) on a 32-cores machine;
- Interpolation of the grid with excellent accuracy requires **negligible time**.



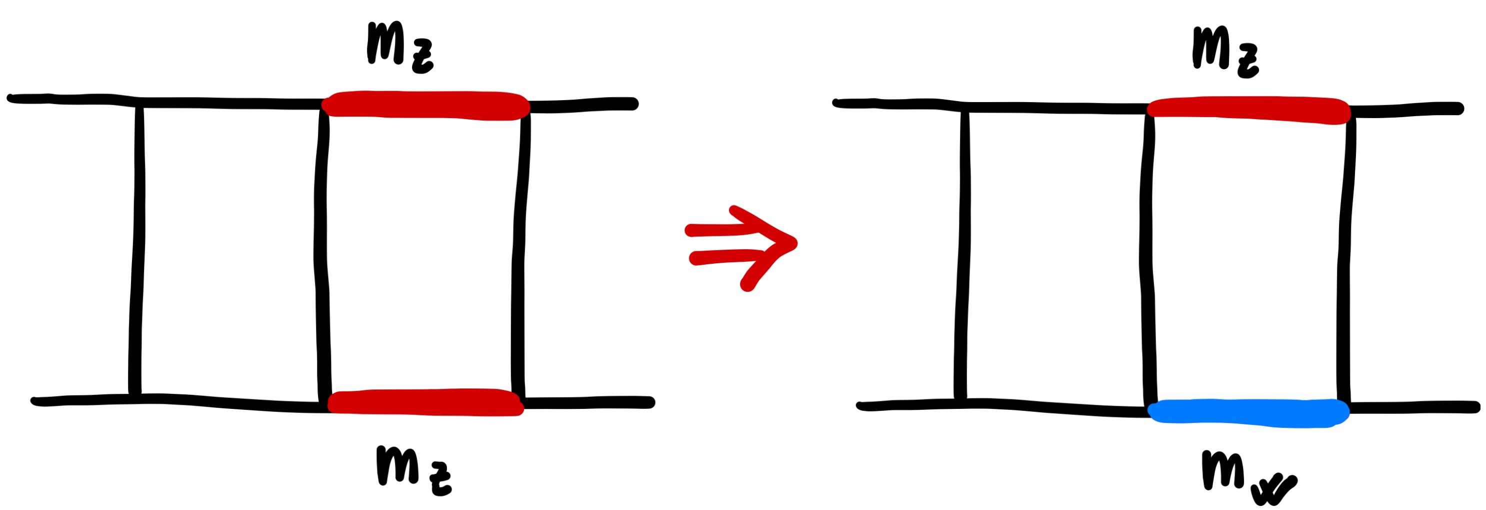


CCDY – BASIS OF MASTER INTEGRALS

[T. Armadillo, R. Bonciani, SD, N.Rana, A.Vicini, arXiv:2405.00612]



- Computationally **similar** to neutral current Drell-Yan;
- Extra complexity coming from new diagrams where **two different internal massive lines** appear:

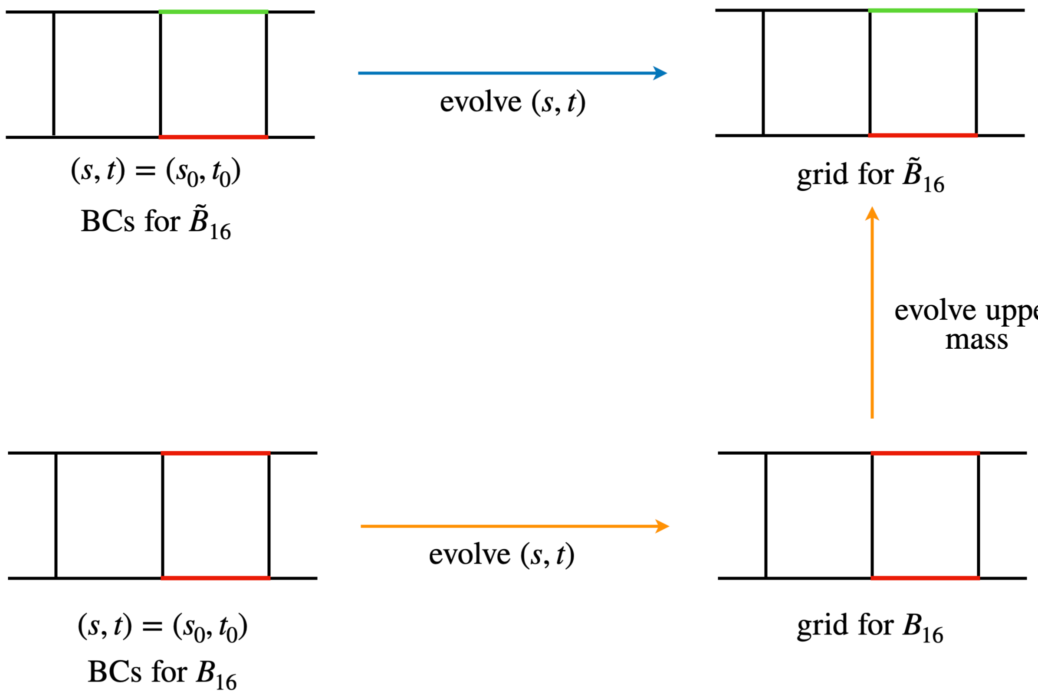


- **56** Master integrals with two internal masses:
  - 34 **identical** to the ones appearing in the neutral current case;
  - 22 **generalisation** of masters with two massive lines to the case of different masses;

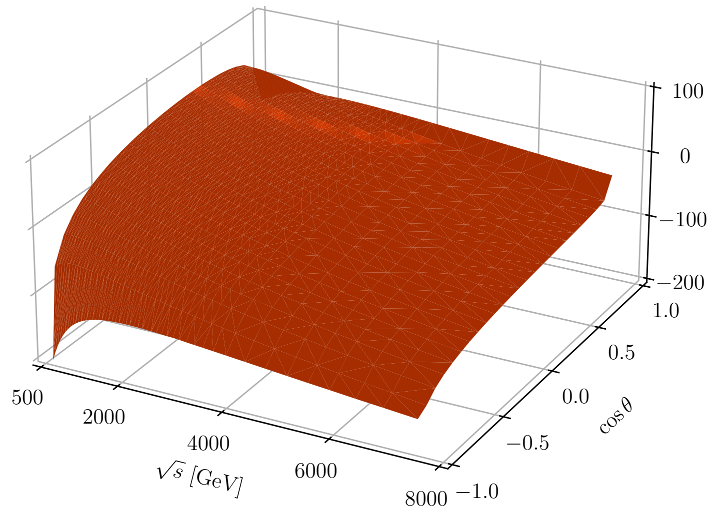
CCDY – NUMERICAL GRIDS

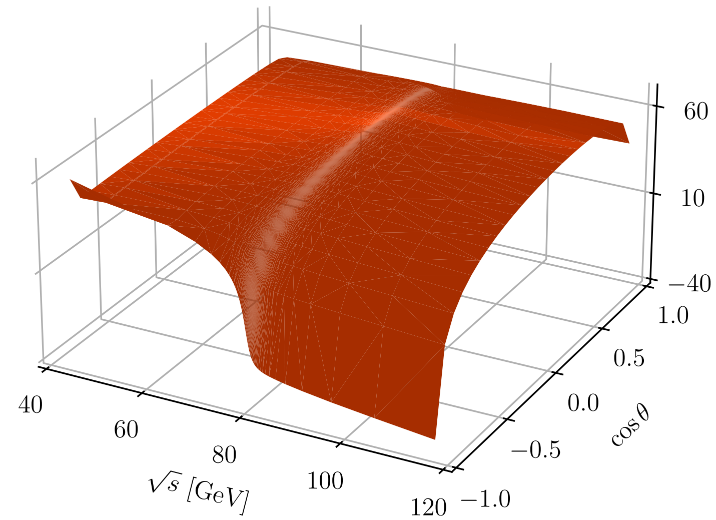
[*T. Armadillo, R. Bonciani, SD, N.Rana, A.Vicini, arXiv:2405.00612*]

- Two possible approaches:
  - Compute the boundary conditions with **AMFLOW** and use the **differential equations in *s* and *t*** to build the grid;
  - Use the grid of the neutral-current Drell-Yan as a boundary condition and use the **differential equation w.r.t. one of the masses**.



UV renormalisation & **IR subtraction** are analogous to the neutral current case.



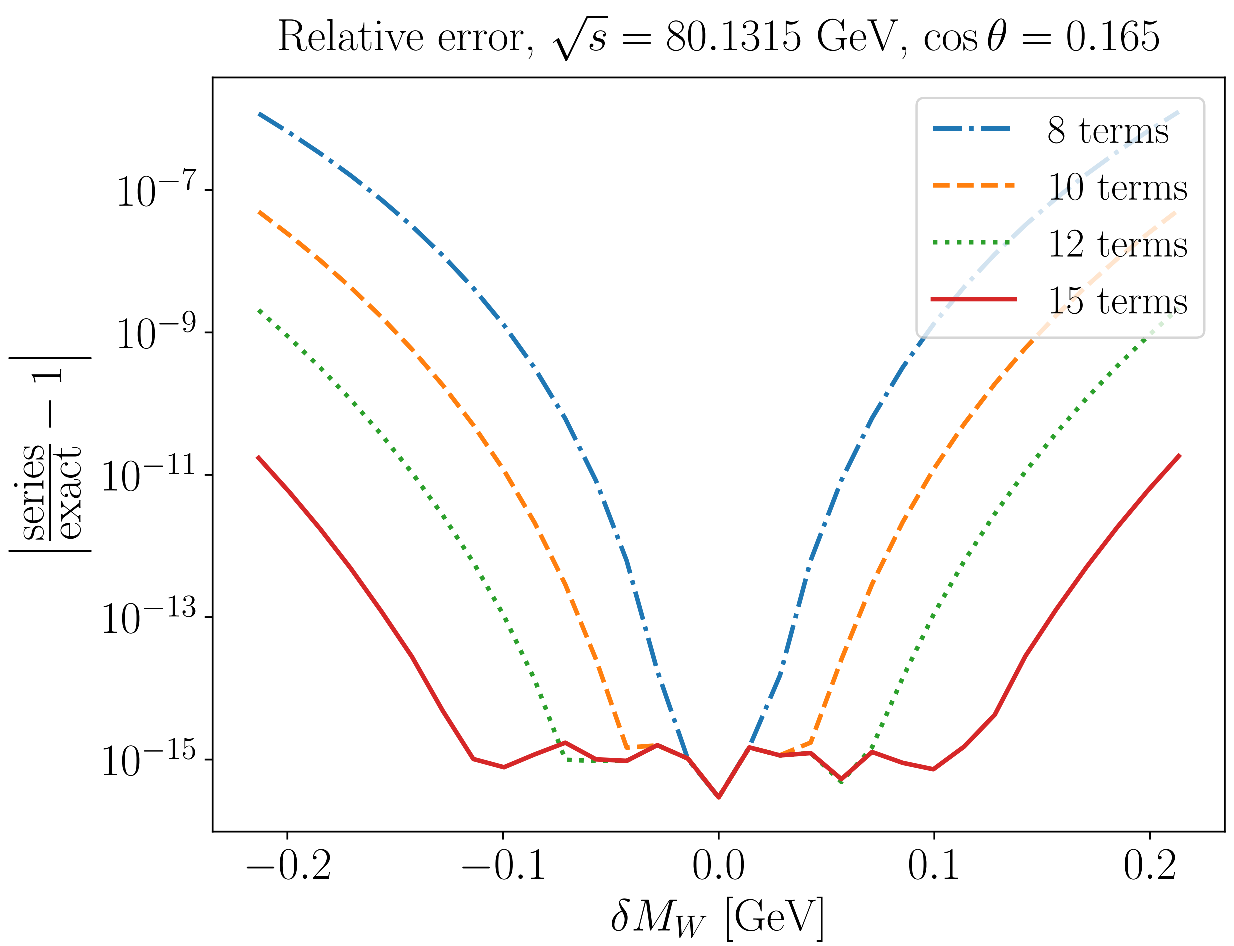


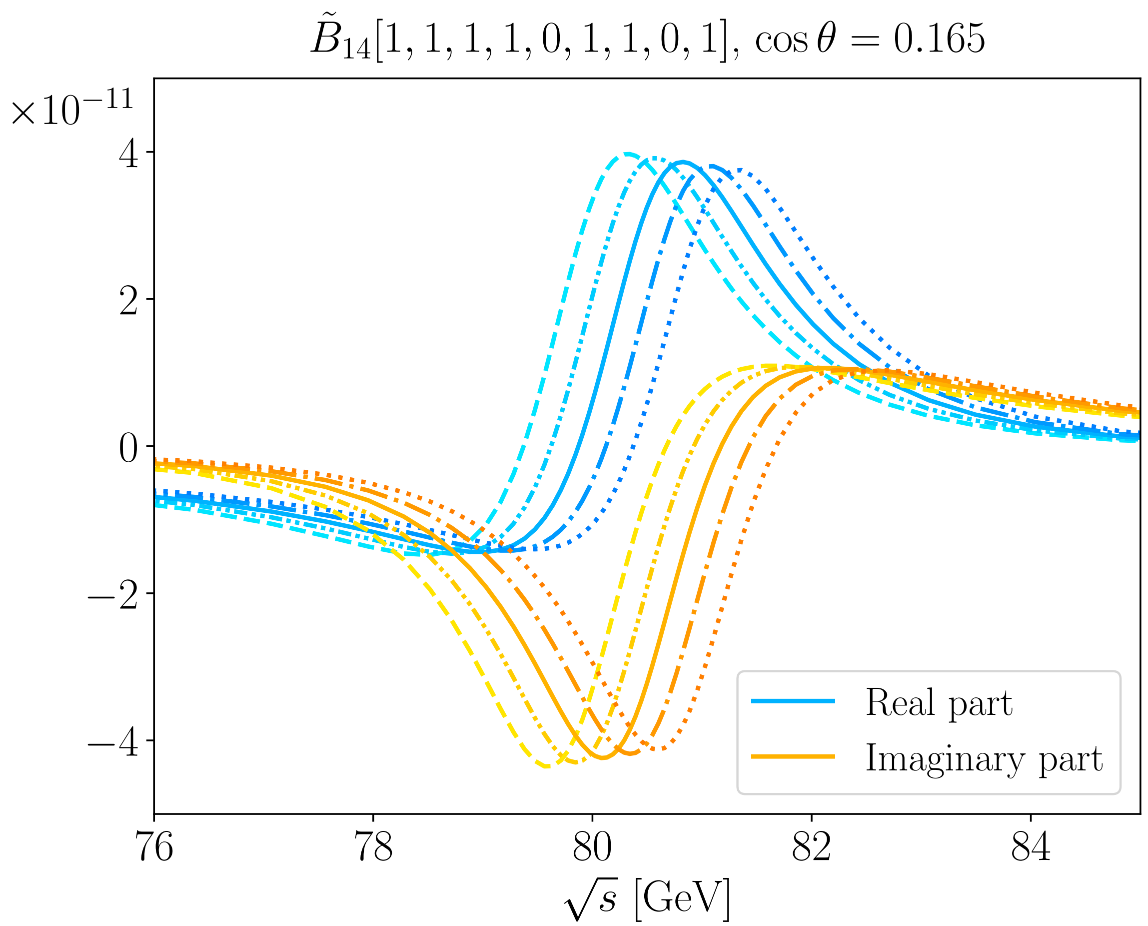
CCDY – NUMERICAL GRIDS

[T. Armadillo, R. Bonciani, SD, N.Rana, A.Vicini, arXiv:2405.00612]

- The mass evolution offers the possibility to also provide the **additional dependence on the W-boson mass**;
- Useful for frontier studies, such as M_W determination.

$\delta M_W = M_W - \bar{M}_W$ $\bar{M}_W \rightarrow$ **reference value**



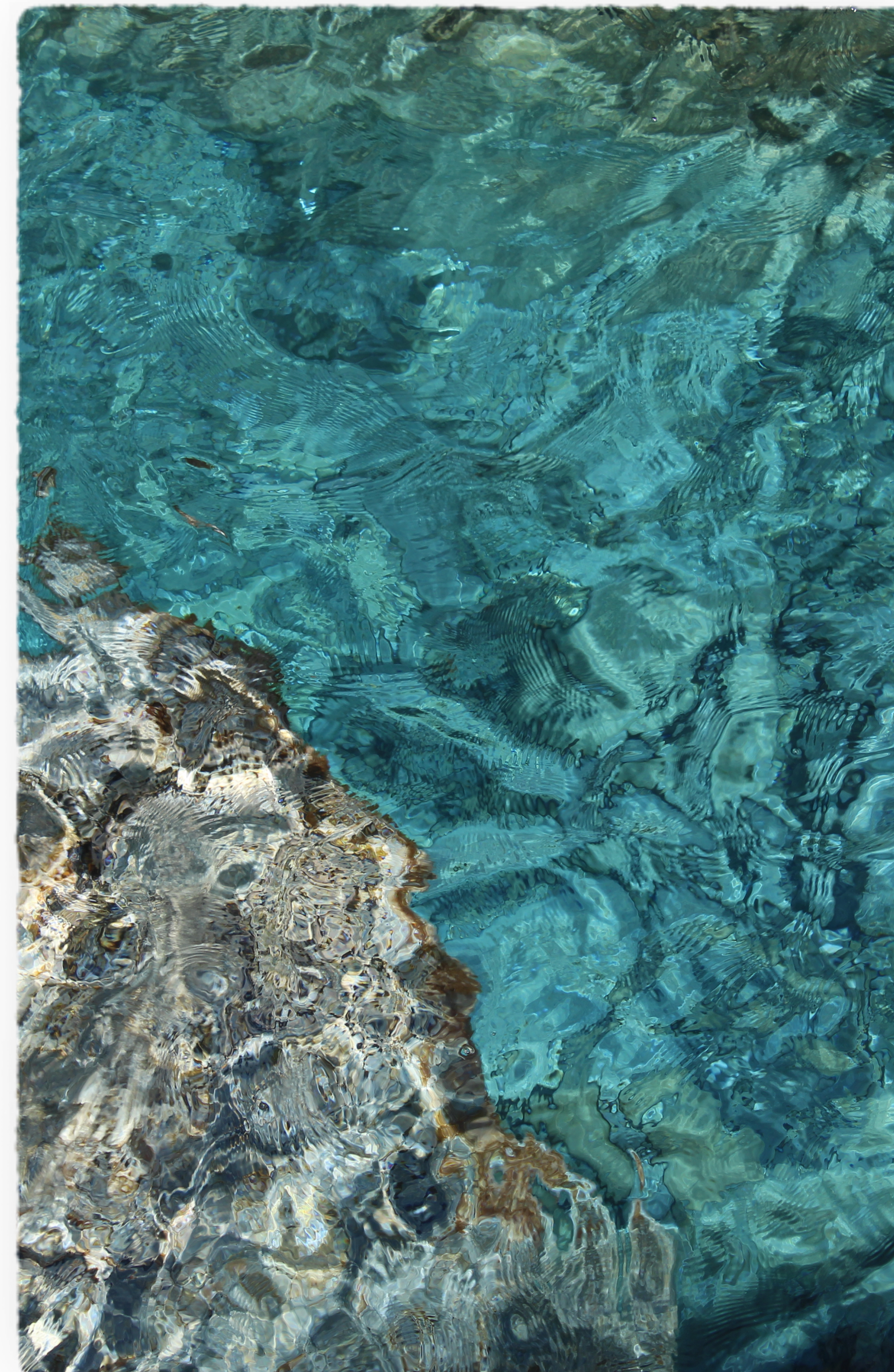


SUMMARY & OUTLOOK

- **EW corrections** will be of crucial importance for the future LHC scientific program;
- We developed a **framework** which relies on ABISS for the evaluation of rational coefficients and SEASYDE for the evaluation of the Master Integrals;
- We successfully applied our framework to the computation of **mixed QCD-EW corrections** to the Drell-Yan process.

SUMMARY & OUTLOOK

- Phenomenological studies for **charged-current Drell-Yan**;
- Towards **NNLO EW**?



SUMMARY & OUTLOOK

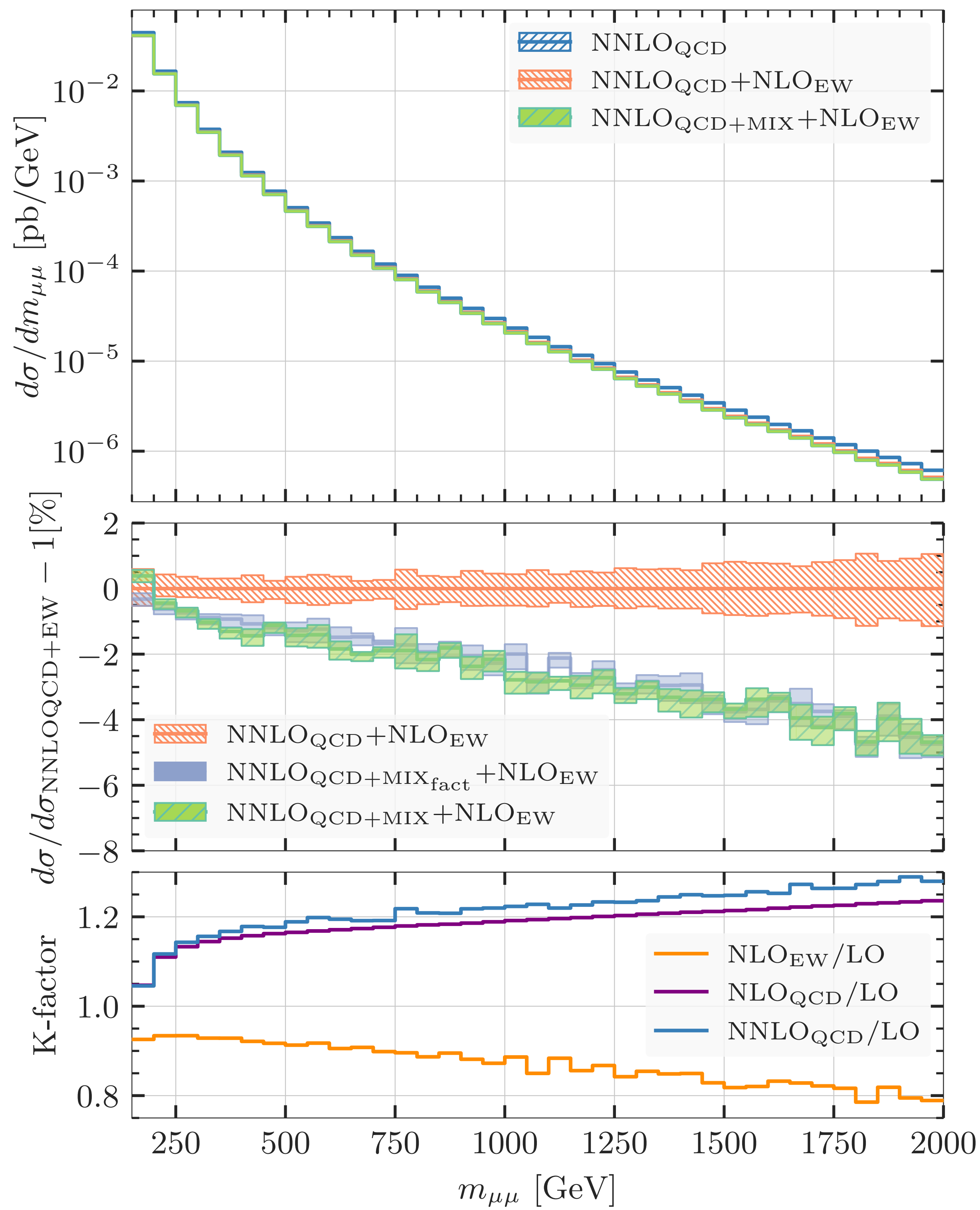
- Phenomenological studies for **charged-current Drell-Yan**;
- Towards **NNLO EW**?

THANKS!

BACKUP SLIDES

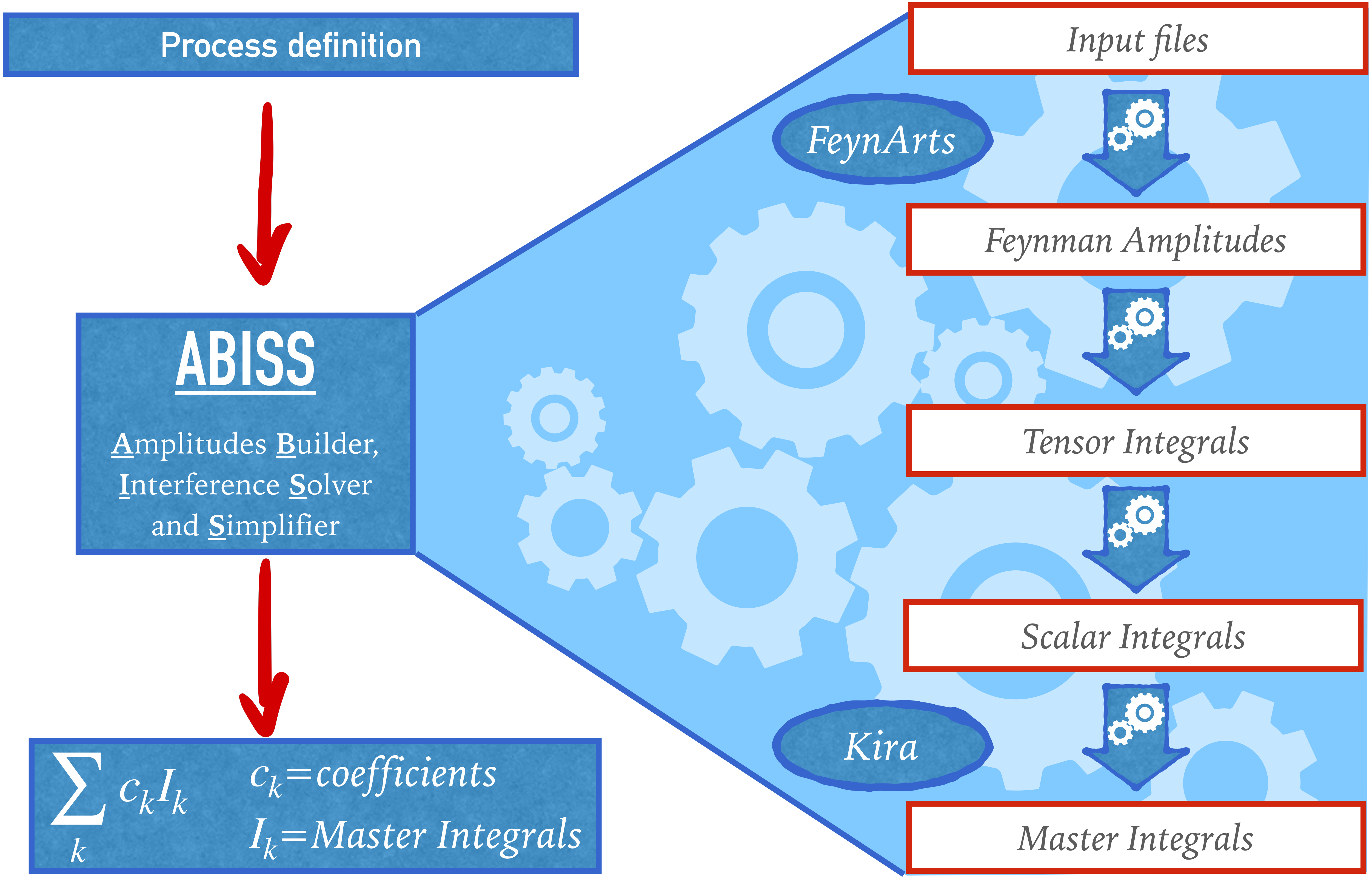
DIFFERENTIAL RESULTS

[T. Armadillo, R. Bonciani, L. Buonocore, SD, M. Grazzini, S. Kallweit, N. Rana, A. Vicini, arXiv:2412.16095]



- We examine the **high invariant-mass region**, following cuts from *[CMS collaboration, arXiv:2103.02708]*: $p_{T,\mu^\pm} > 53\,\text{GeV}$, $|y_\mu| < 2.4$, $m_{\mu\mu} > 150\,\text{GeV}$
- The **mixed corrections** provide a relevant contribution, **increasing at higher invariant mass** and larger than the statistical error expected by the end of **HL-LHC**;
- The **factorised approximation** gives a **good description** of the high-invariant mass region.

ABISS – EVALUATING c_k



SOLVING D.E. B SERIES EXPANSION

Method implemented in the Mathematica package DiffExp for real kinematic variables [*F.Moriello, arXiv:1907.13234*], [*M.Hidding, arXiv:2006.05510*] (see also AMFLOW [*X. Liu and Y.-Q. Ma, arXiv: 2201.11669*])

A Simple Example

$$\begin{cases} f'(x) + \frac{1}{x^2 - 4x + 5} f(x) = \frac{1}{x+2} \\ f(0) = 1 \end{cases}$$

$$f_{hom}(x) = x^r \sum_{k=0}^{\infty} c_k x^k$$

$$f'_{hom}(x) = \sum_{k=0}^{\infty} (k+r)\, c_k\, x^{(k+r-1)}$$

$$\begin{cases} r c_0 = 0 \\ \frac{1}{5} c_0 + c_1 (r+1) = 0 \\ \frac{4}{25} c_0 + \frac{1}{5} c_1 + c_2 (2+r) = 0 \\ \ldots \end{cases}$$

$$f_{hom}(x) = 5 - x - \frac{3}{10} x^2 + \frac{11}{150} x^3 + \ldots$$

$$f_{part}(x) = f_{hom}(x) \int_0^x dx' \underbrace{\frac{1}{(x'+2)} f_{hom}^{-1}(x')}_{\text{Expanded around } x' = 0}$$

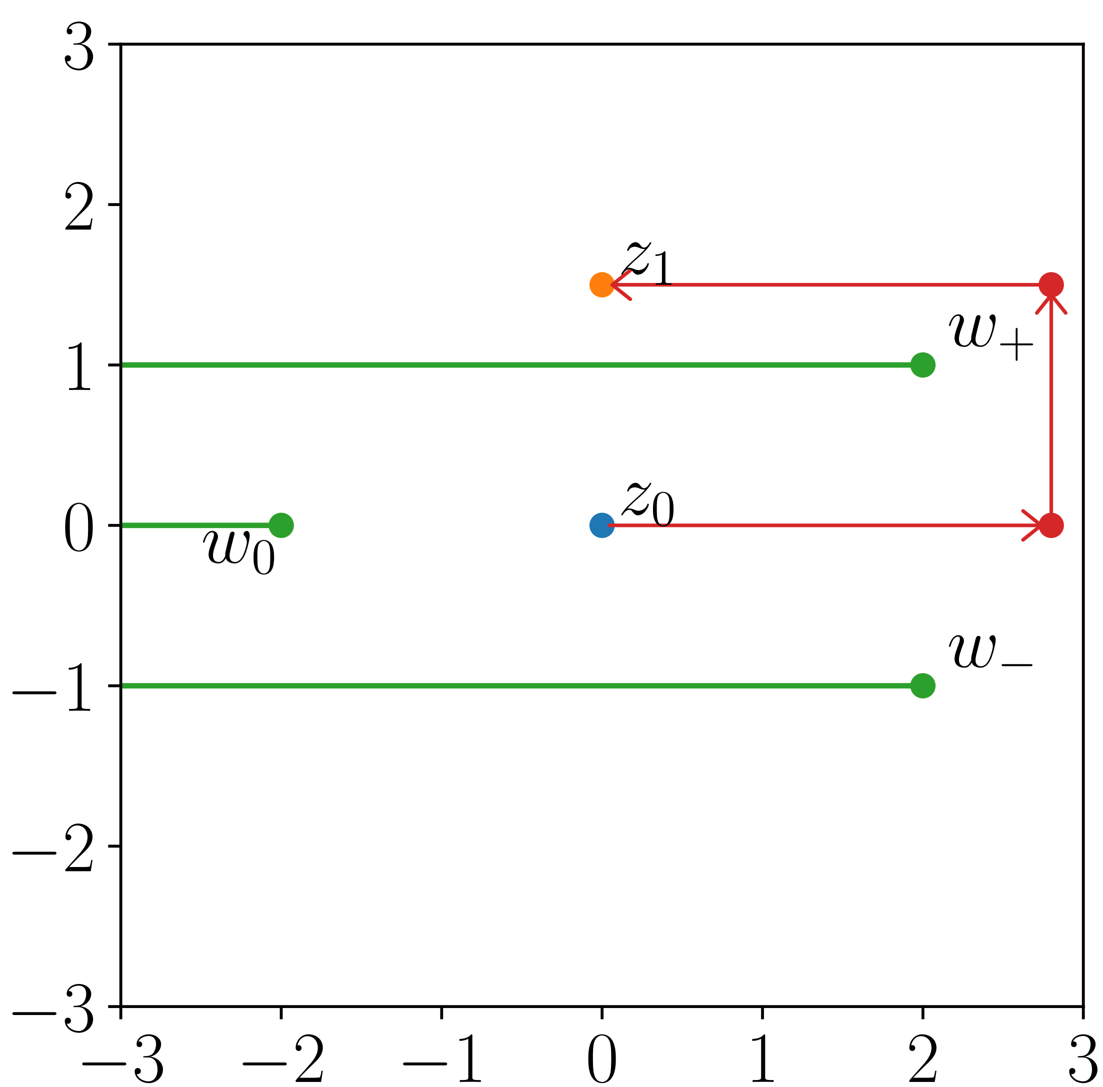
$$= \frac{1}{2} x - \frac{7}{40} x^2 + \frac{2}{75} x^3 + \ldots$$

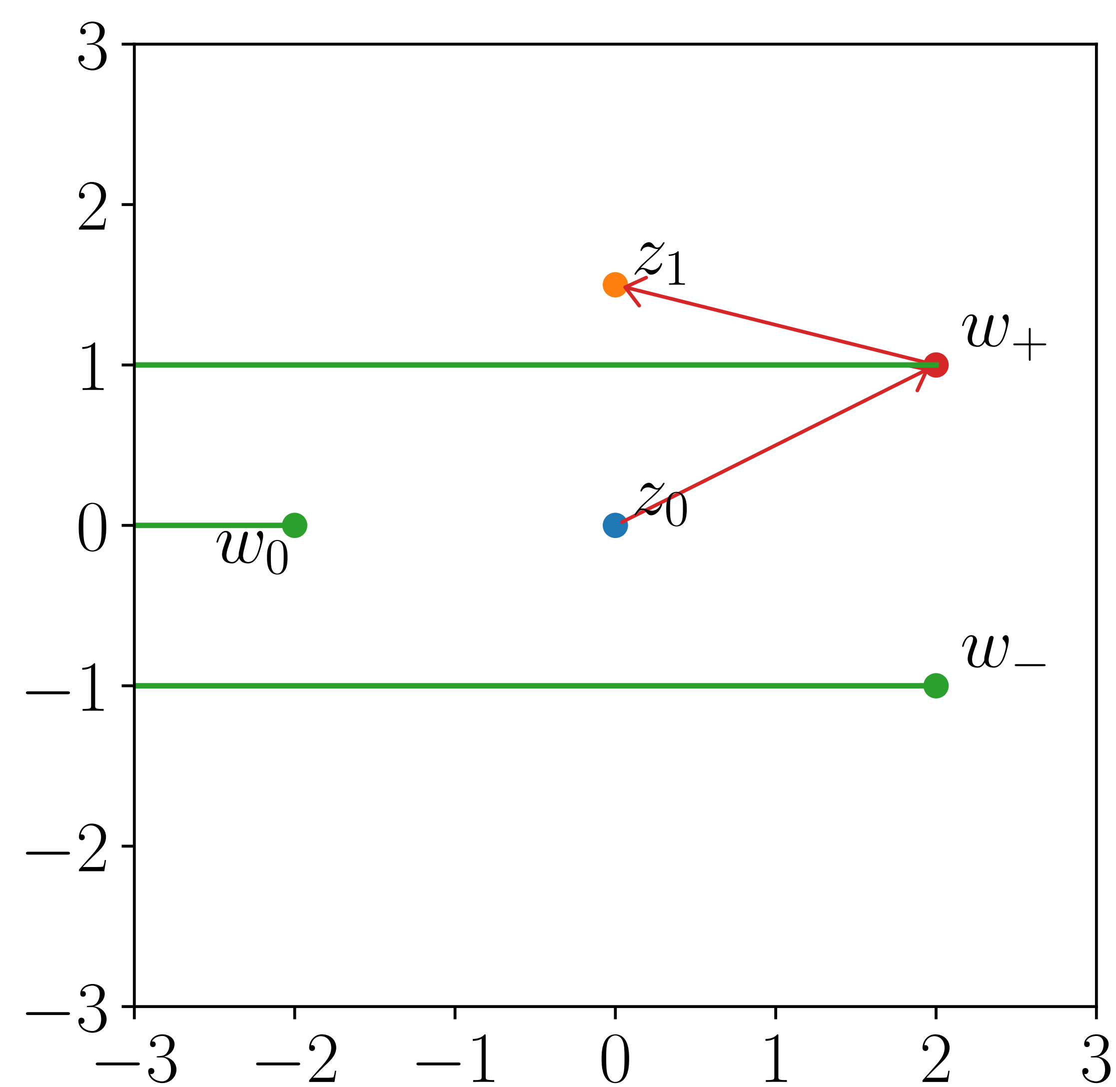
$$f(x) = f_{part}(x) + C\, f_{hom}(x)$$

$$f(0) = 1 \rightarrow C = \frac{1}{5}$$

TAYLOR VS LOGARITHMIC EXPANSION

- **Taylor expansion**: **avoids** the singularities;
- **Logarithmic expansion**: uses the singularities as **expansion points**.
- Logarithmic expansion has larger convergence radius but requires longer evaluation time. **We use Taylor expansion as default**.





TREATMENT OF γ_5

γ_5 is not well defined in a non integer number of dimensions!

	ANTICOMMUTATION $\{\gamma_\mu, \gamma_5\} = 0$	CYCLICITY OF THE TRACE
't Hooft and Veltmann *Nucl. Phys. B 44 (1972) 189–213*	✘	✔
Kreimer et al. *Phys. Lett. B 237 (1990) 59–62*	✔	✘

For neutral-current Drell Yan proven that at 2loops the two prescriptions yield:

➤ **different** scattering amplitudes; ➤ **same** finite corrections after subtraction.

[M. Heller, A. von Manteuffel, R. M. Schabinger and H. Spiesberger, arXiv:hep-ph/2012.05918]

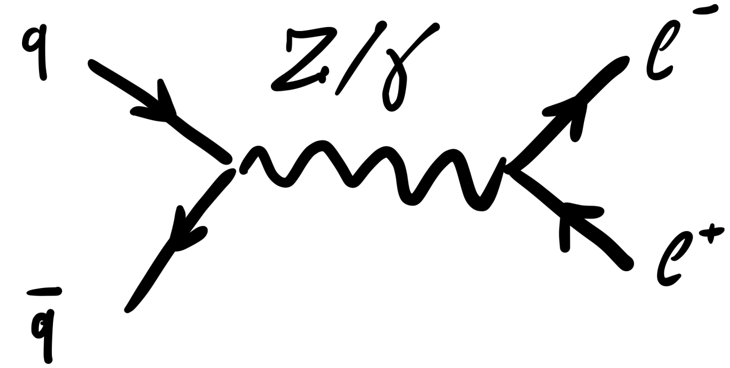
Our procedure:

1. Use anticommutation relation, bring all γ_5 at the end of the Dirac trace;
2. Use $\gamma_5^2 = 1$, end up with zero or one γ_5 in each Dirac trace;
3. Replace the (single) leftover γ_5 with the relation: $\gamma_5 = \frac{i}{4!}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma$.

NCDY - BASIS OF MASTER INTEGRALS

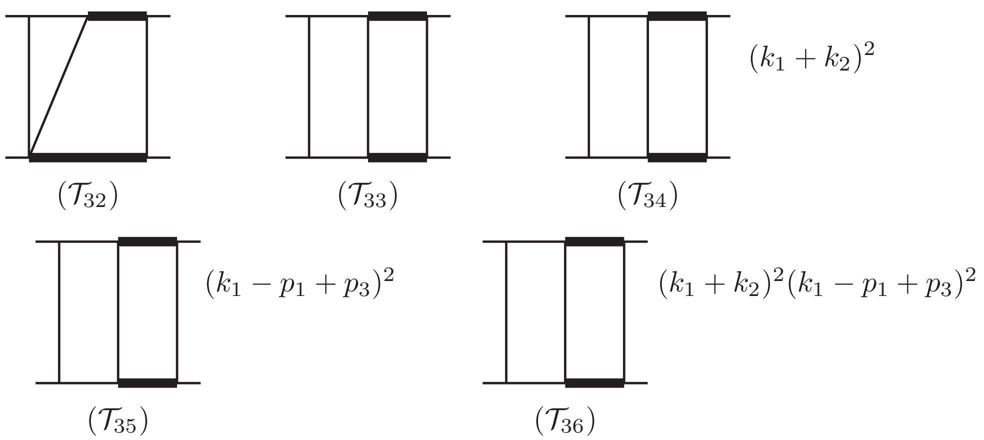
[T. Armadillo, R. Bonciani, SD, N.Rana, A.Vicini, arXiv:2201.01754]

Basis of Master integrals composed by:



- MIs relevant for the **QCD-QED corrections**, with massive final state; *[R.Bonciani, A.Ferroglia, T.Gehrmann, D.Maitre, C.Studerus, arXiv:0806.2301, 0906.3671]*
- MIs with 1or 2 internal mass relevant for the **EW form factor**; *[U.Aglietti, R.Bonciani, hep-ph/0304028, hep-ph/0401193]*
- 31 MIs with 1 mass and **36 MIs with 2 masses** including boxes, relevant for the **QCD-EW corrections to the full Drell-Yan**.

5 box integrals are in **Chen-Goncharov** representation; *[R. Bonciani, S. Di Vita, P. Mastrolia, U. Schubert, arXiv:1604.08581]*



difficult numerical evaluation requires alternative strategy.

closed form in terms of GPLs found but not public *[M.Heller, A.von Manteuffel, R.M. Schabinger, arXiv:1907.00491]*

NCDY - NUMERICAL GRIDS

[T. Armadillo, R. Bonciani, SD, N.Rana, A.Vicini, arXiv:2201.01754]

- 31 out of 36 masters known in terms of GPLs: **validation** of SEASYDE.
- 5 out of 36 masters are a genuine SEASYDE **prediction**;
- solution can be computed with **arbitrary number of significant digits**.