Towards accuracy in parton showers

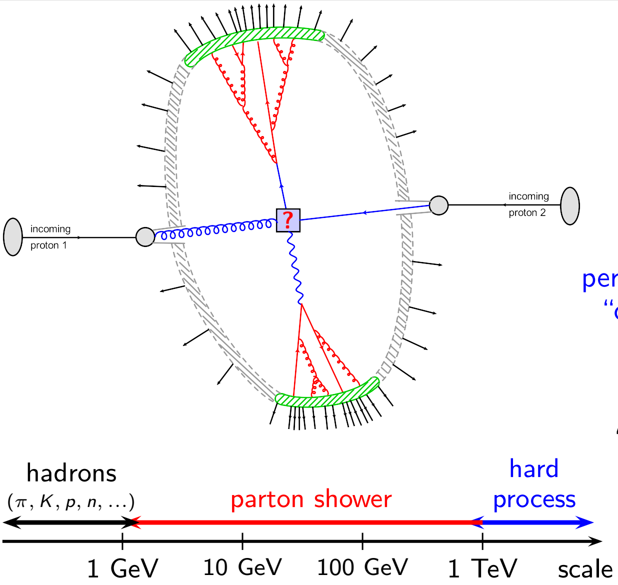
Gregory Soyez mostly based on work within PanScales:

arXiv:1805.09327, arXiv:1807.04758, arXiv:2002.11114, arXiv:2007.10355, arXiv:2011.10054, arXiv:2103.16526, arXiv:2109.07496, arXiv:2111.01161, arXiv:2205.02237, arXiv:2205.02861, arXiv:2207.09467, arXiv:2212.05076, arXiv:2301.09645, arXiv:2305.08645, arXiv:2307.11142, arXiv:2312.13275, arXiv:2402.05170, arXiv:2406.02664, arXiv:2409.08316, arXiv:2504.05377

IPhT, CNRS, CEA Saclay

GDR QCD W mass, IJCLab, Orsay, June 30-July 1 2025

Context: anatomy of a high-energy collision



Simulating a high-energy collision requires several ingredients

perturbatively "calculable"

- A hard process
- Parton shower (initial and final-state)

non-pert. "modelled"

- Hadronisation
- Multi-parton interactions

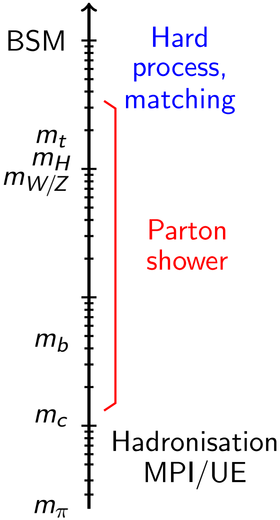
Context: physics at all scales

physics probed across many scales

$Q \equiv$ 100 GeV $\to$ 1 TeV

$Q \gg \mu_{\rm NP}$

$\mu_{\rm NP} \sim$ 1 GeV



"Standard" perturbative expansion

$$\alpha_s(Q) f_1(v) + \alpha_s^2(Q) f_2(v) + \alpha_s^3(Q) f_3(v) + \dots$$

LO NLO NNLO

expect logs between disparate scales

$\alpha_s \log^2 Q/\mu_{\rm NP},\ \alpha_s \log Q/\mu_{\rm NP}$

(double, single,...) logs to resum

Parton shower v. resummations

Resummation is a vast field $\Rightarrow$ let us take a concrete example: event shapes

For a generic shape v, the **analytic QCD prediction** is

$$\ln \Sigma(v_{\text{cut}}) \equiv \ln P(v < v_{\text{cut}}) = \frac{1}{\alpha_s} g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots$$

with $L = \log(v_{\text{cut}})$. [Working limit: $\alpha_s \ll 1$, $\alpha_s L \sim \text{cst}$]

All order resummation of logarithmically-enhanced terms:

- $\frac{1}{\alpha_s} g_1 = \alpha_s L^2 + \alpha_s^2 L^3 + \alpha_s^3 L^4 + \cdots \equiv$ leading-logs (LL)
- $g_2 = \alpha_s L + \alpha_s^2 L^2 + \alpha_s^3 L^3 + \cdots \equiv$ next-to-leading-logs (NLL)
- $\alpha_s g_3 = \alpha_s + \alpha_s^2 L + \alpha_s^3 L^2 + \cdots \equiv$ next-to-next-to-leading-logs (NNLL)

Side note: one can also work with $\alpha_s \ll 1$, $\alpha_s L^2 \sim \text{cst}$ (e.g. for $\Sigma(v_{\text{cut}})$ or multiplicity observables).
We then get an expansion $h_1(\alpha_s L^2) + \sqrt{\alpha_s} h_2(\alpha_s L^2) + \alpha_s h_3(\alpha_s L^2) + \ldots$.
h_1 resums double-logs (DL), h_2 next-to-double-logs (NDL), h_3 next-to-next-to-double-logs, etc...

Parton shower v. resummations

Resummation is a vast field ⇒ let us take a concrete example: event shapes

FIRST TAKE-HOME MESSAGE

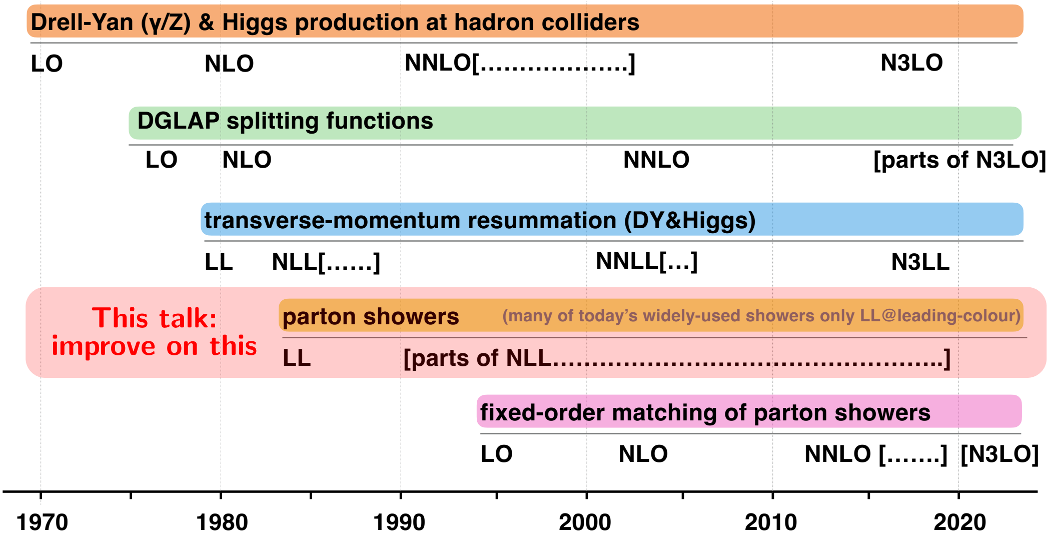
shower accuracy means logarithmic accuracy

(LL, NLL, NNLL, ...)

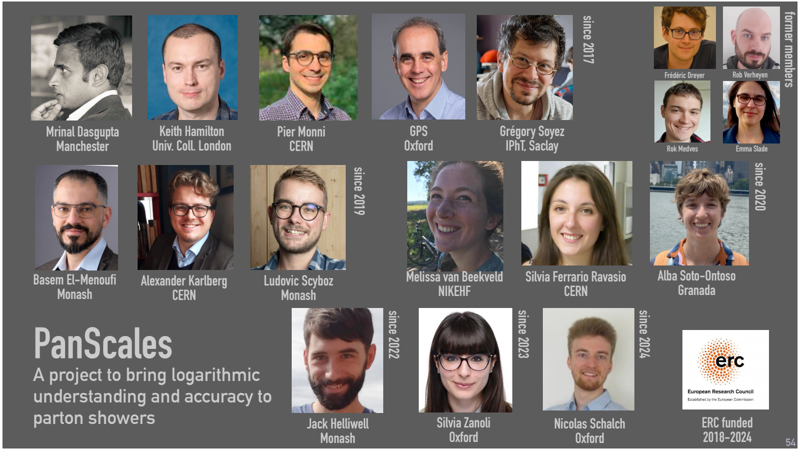
well-defined & systematically improvable

selected collider-QCD accuracy milestones

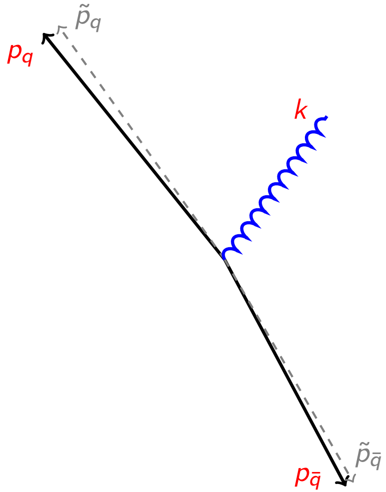
[slide from Gavin Salam (Moriond QCD 2023)]



1. An intuitive graphical representation
2. Ingredients of a (dipole) parton shower
3. Systematic approach to log accuracy
4. How to test the shower accuracy?
5. Achieving NLL
6. Towards NNLL



Take a gluon emission from a $(q\bar{q})$ dipole



Emission $(\tilde{p}_q \tilde{p}_{\bar{q}}) \to (p_q k)(k p_{\bar{q}})$:

$$k^\mu \equiv z_q \tilde{p}_q^\mu + z_{\bar{q}} \tilde{p}_{\bar{q}}^\mu + k_\perp^\mu$$

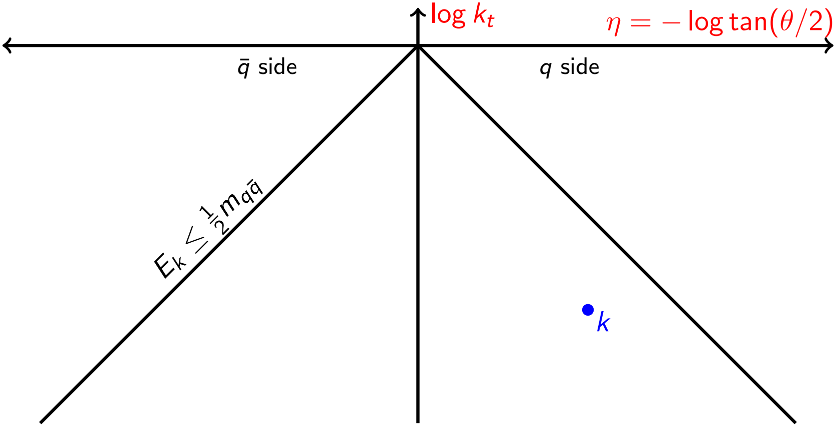
3 degrees of freedom:

- Rapidity: $\eta = \frac{1}{2} \log \frac{z_q}{z_{\bar{q}}}$
- Transverse momentum: $k_\perp$
- Azimuth: ϕ

In the soft-collinear approximation

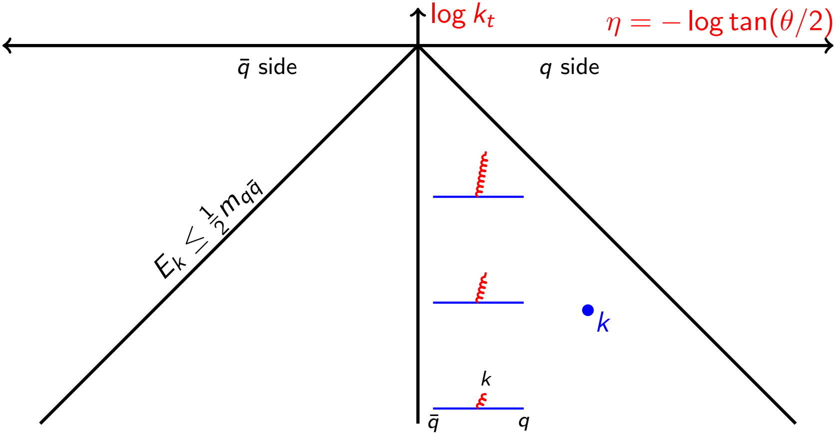
$$d\mathcal{P} = \frac{\alpha_s(k_\perp) C_F}{\pi^2} \, d\eta \, \frac{dk_\perp}{k_\perp} \, d\phi$$

Lund plane: natural representation uses the 2 “log” variables η and $\log k_\perp$

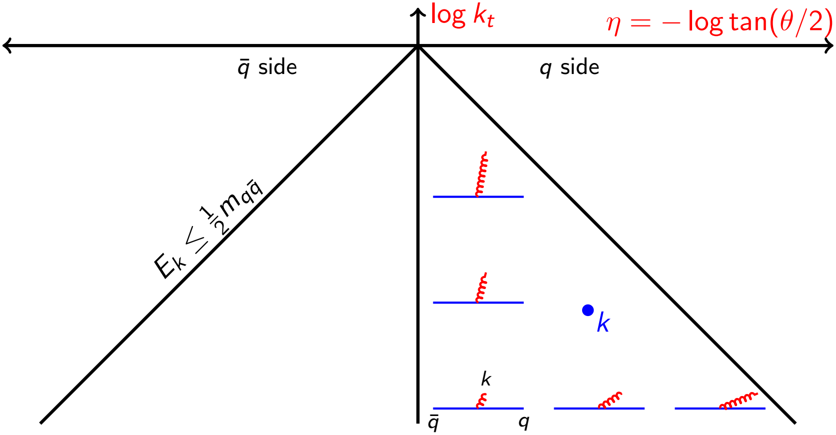


Basic features of QCD radiation: the Lund plane

Lund plane: natural representation uses the 2 "log" variables η and $\log k_\perp$

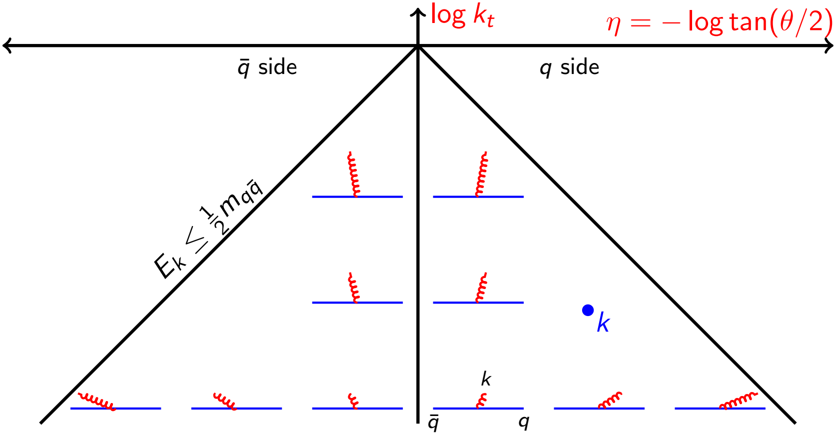


Lund plane: natural representation uses the 2 "log" variables η and $\log k_\perp$



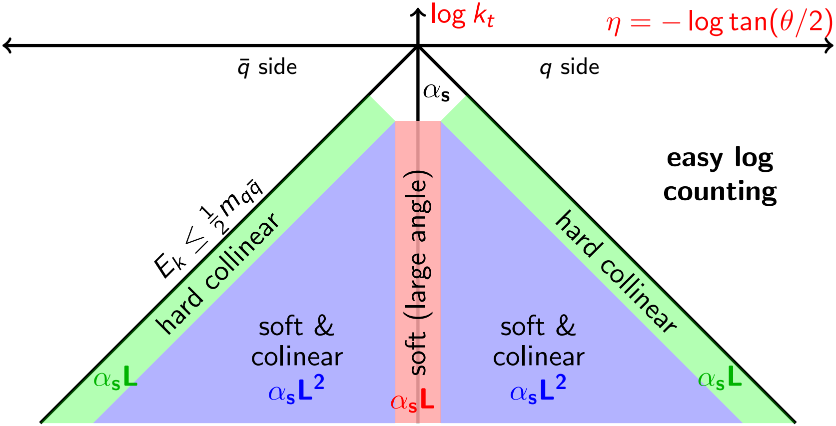
Basic features of QCD radiation: the Lund plane

Lund plane: natural representation uses the 2 "log" variables η and $\log k_\perp$

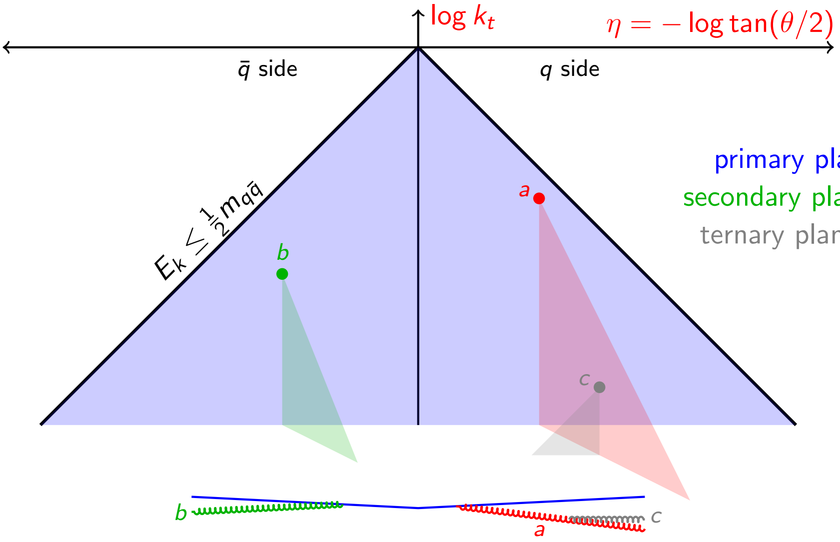


Basic features of QCD radiation: the Lund plane

Lund plane: natural representation uses the 2 “log” variables η and $\log k_\perp$



Multiple emissions in the Lund plane



primary plane

secondary plane(s)

ternary plane(s)

A (Dipole) Parton-Shower primer

Basic of parton showering in one slide

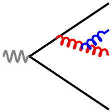
Dipoles at large-N_c

In the large-N_c limit, a gluon emission corresponds to a dipole splitting

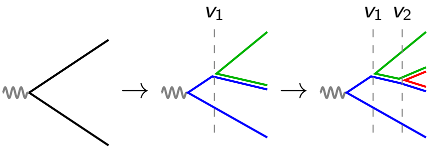
Mechanism: generate emissions one-by-one

ordering variable v (e.g. transverse momentum k_t)

Virtuals as Sudakov/unitarity/no-emission probability



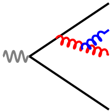
generated as



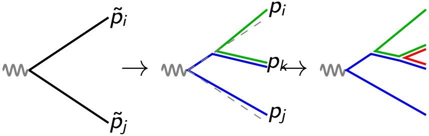
Basic of parton showering in one slide

Dipoles at large-N_c

In the large-N_c limit, a gluon emission corresponds to a dipole splitting



generated as



Mechanism: generate emissions one-by-one

ordering variable v (e.g. transverse momentum k_t)

Virtuals as Sudakov/unitarity/no-emission probability

Ingredient 1: Momentum map

How to go from
pre-branching momenta $(\tilde{p}_i, \tilde{p}_j)$
to post-branching (p_i, p_j, p_k)

Ingredient 2: Emission probability

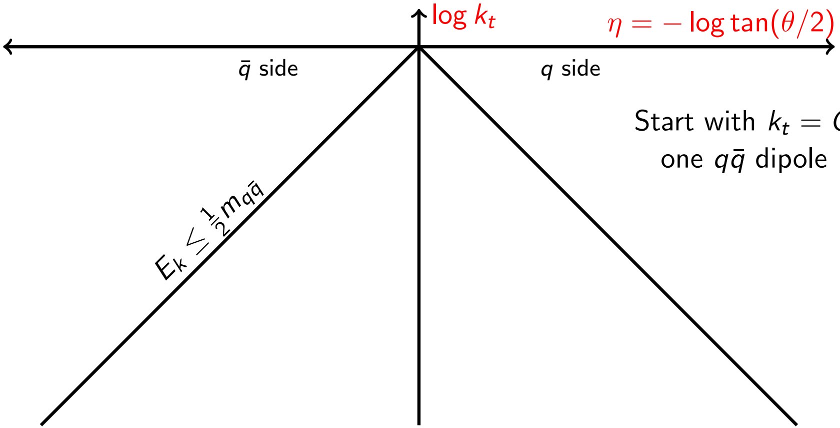
QCD-driven rate of emissions:

$$\frac{dP}{d\ln v d\eta} = \frac{\alpha_s(k_t) C_A}{\pi} g(\eta) P(z)$$

(for NLL, need 2-loop CMW $\alpha_s(k_t)$)

(Dipole) parton shower in the Lund plane

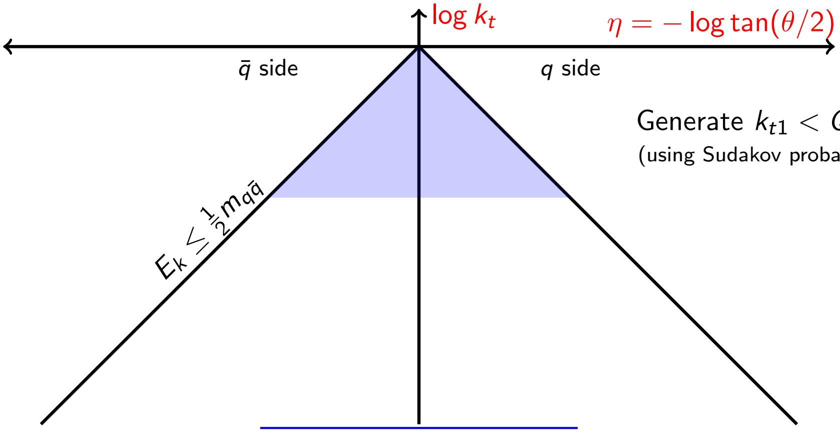
Ordering variable: transverse momentum k_t



Start with $k_t = Q$
one $q\bar{q}$ dipole

(Dipole) parton shower in the Lund plane

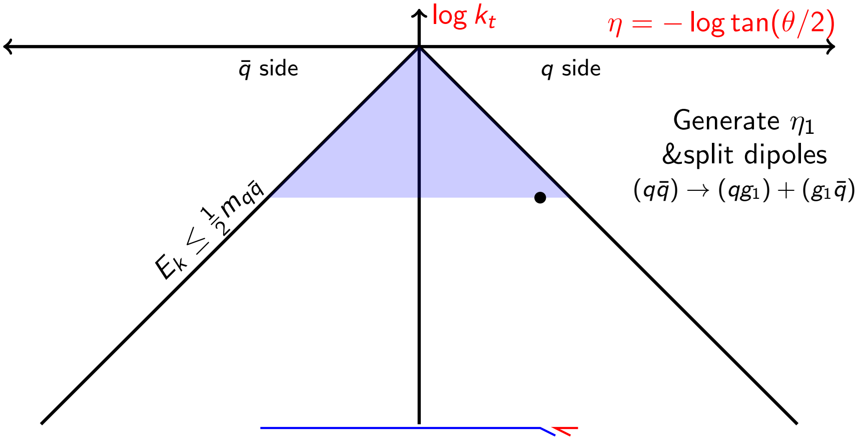
Ordering variable: transverse momentum k_t



Generate $k_{t1} < Q$
(using Sudakov proba)

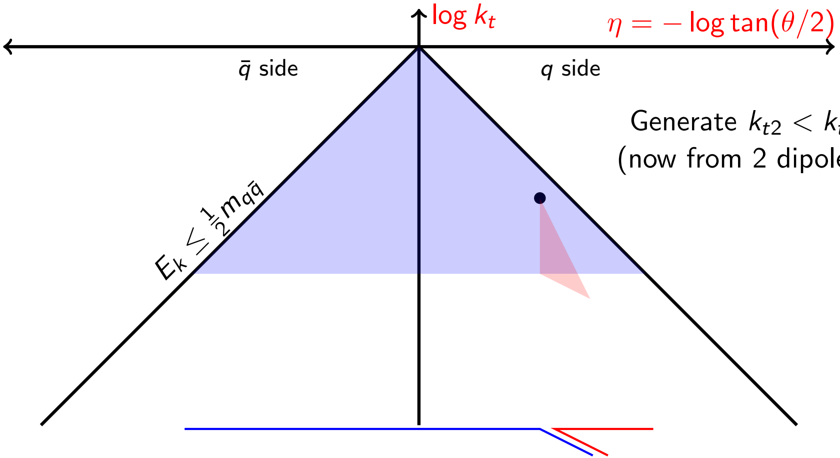
(Dipole) parton shower in the Lund plane

Ordering variable: transverse momentum k_t



(Dipole) parton shower in the Lund plane

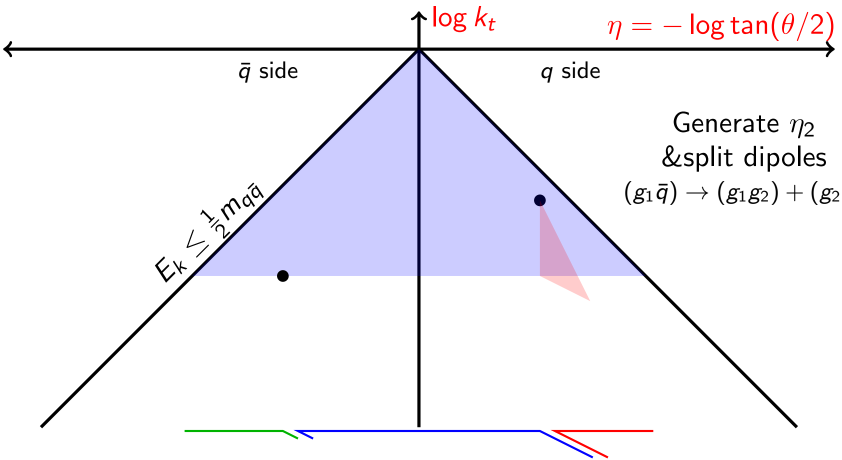
Ordering variable: transverse momentum k_t



Generate $k_{t2} < k_{t1}$
(now from 2 dipoles)

(Dipole) parton shower in the Lund plane

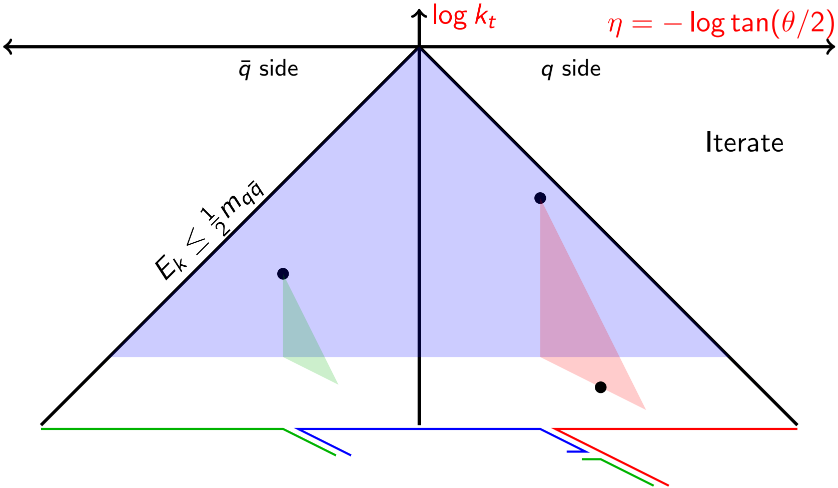
Ordering variable: transverse momentum k_t



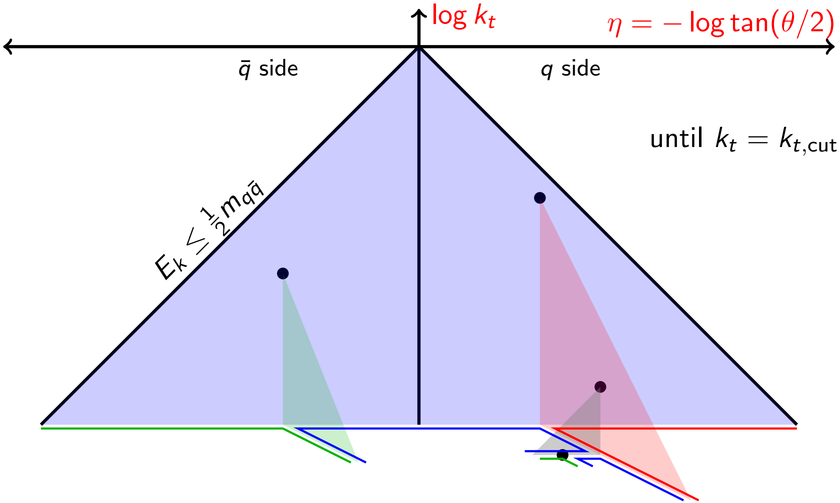
Generate η_2
&split dipoles
$(g_1\bar{q}) \to (g_1 g_2) + (g_2\bar{q})$

(Dipole) parton shower in the Lund plane

Ordering variable: transverse momentum k_t



Ordering variable: transverse momentum k_t



Physics result #1: an organising principle:

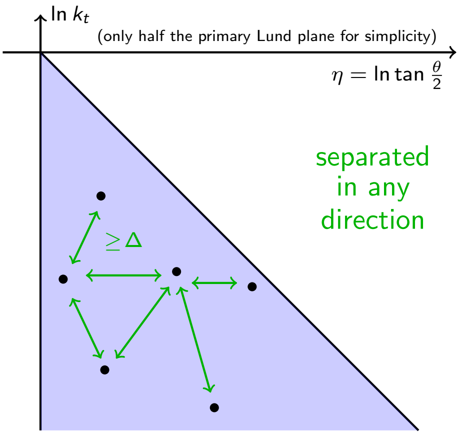
at a given (all-order) accuracy, what physics do we need to get right?

Systematic approach: accuracy $\leftrightarrow$ reproducing sets of MEs

target: describe disparate scales
at all-order perturbative QCD
$\Downarrow$
minimum: get the ME for an arbitrary number
of well-separated emissions

- If "log distance" Δ emissions factorise up to $\mathcal{O}(e^{-\Delta})$ corrections
- this achieves NLL accuracy

  in a way NLL can be viewed as the first meaningful order
- In particular, in a parton showers, an emission should not be affected by subsequent distant emissions

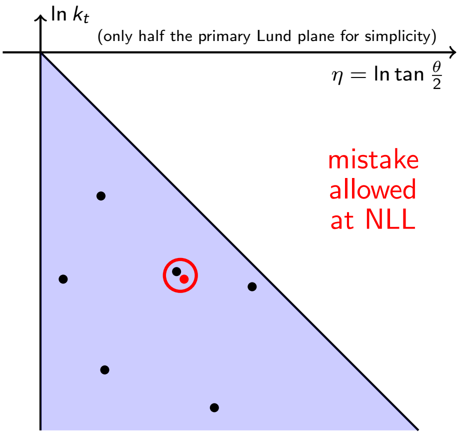


Systematic approach: accuracy $\leftrightarrow$ reproducing sets of MEs

target: describe disparate scales
at all-order perturbative QCD
$\Downarrow$
minimum: get the ME for an arbitrary number
of well-separated emissions

- If "log distance" Δ emissions factorise up to $\mathcal{O}(e^{-\Delta})$ corrections
- this achieves NLL accuracy

  in a way NLL can be viewed as the first meaningful order
- In particular, in a parton showers, an emission should not be affected by subsequent distant emissions



Systematic approach: accuracy $\leftrightarrow$ reproducing sets of MEs

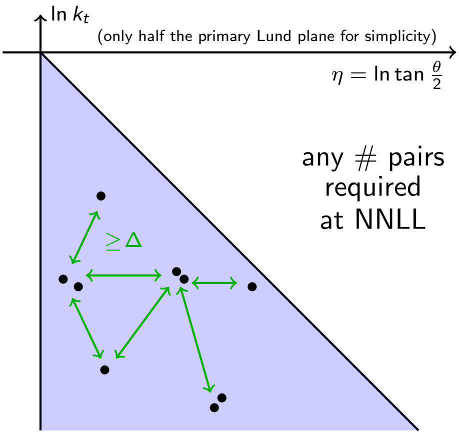
target: describe disparate scales at all-order perturbative QCD

$\Downarrow$

minimum: get the ME for an arbitrary number of well-separated emissions (NLL!)

Beyond NLL

- At NNLL we also want an arbitrary number of pairs of emissions
- N^{3}LL also requires triplets, etc...



Systematic approach: accuracy $\leftrightarrow$ reproducing sets of MEs

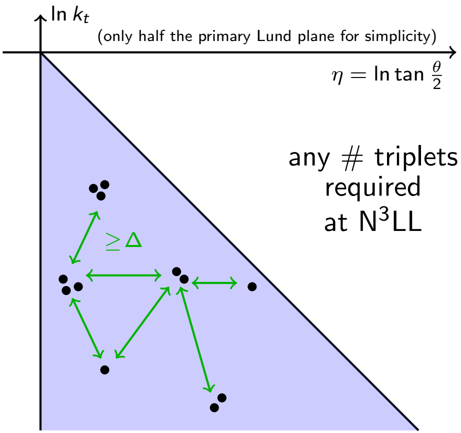
target: describe disparate scales
at all-order perturbative QCD

$\Downarrow$

minimum: get the ME for an arbitrary number
of well-separated emissions (NLL!)

Beyond NLL

- At NNLL we also want an arbitrary number of pairs of emissions
- N^{3}LL also requires triplets, etc...



Systematic approach: accuracy $\leftrightarrow$ reproducing sets of MEs

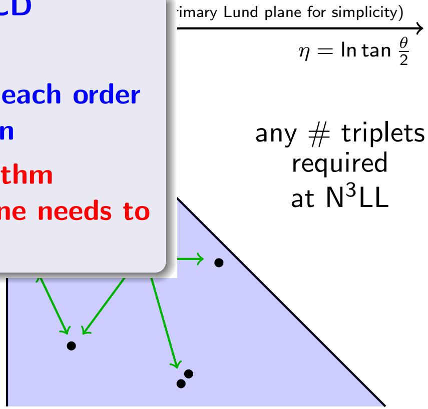
target: descri...
at all-order ...

minimum: get the M...
of well-separat...

- **Robust construction in pQCD**
- **Systematically improvable**
- **“only” a handful of ME at each order thanks to QCD factorisation**
- **difficulty: the shower algorithm generates spurious terms one needs to avoid/correct for**

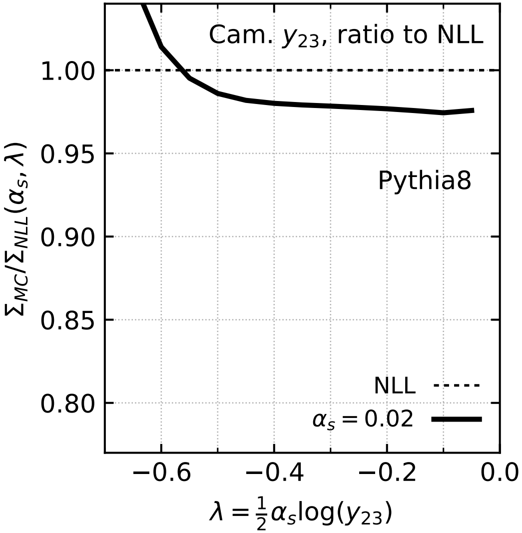
Beyond NLL

- At NNLL we also ...
  pairs of emissions
- N^{3}LL also requires triplets, etc...



any # triplets required at N^{3}LL

Testing log accuracy: a novel approach



Recall our regime: $\alpha_s \log(v) \sim 1$, $\alpha_s \ll 1$

Idea for NLL testing:

$$\frac{\Sigma_{\mathrm{MC}}(\lambda=\alpha_s L, \alpha_s)}{\Sigma_{\mathrm{NLL}}(\lambda=\alpha_s L, \alpha_s)} \quad \text{v.} \quad 1$$

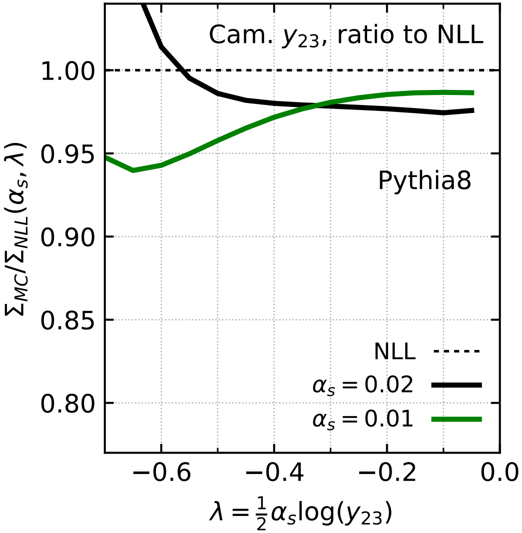
with $\lambda = \alpha_s L$

NLL deviations

or

subleading effects?

Testing log accuracy: a novel approach



Recall our regime: $\alpha_s \log(v) \sim 1$, $\alpha_s \ll 1$

Idea for NLL testing:

$$\frac{\Sigma_{\rm MC}(\lambda=\alpha_s L, \alpha_s)}{\Sigma_{\rm NLL}(\lambda=\alpha_s L, \alpha_s)} \quad \text{v.} \quad 1$$

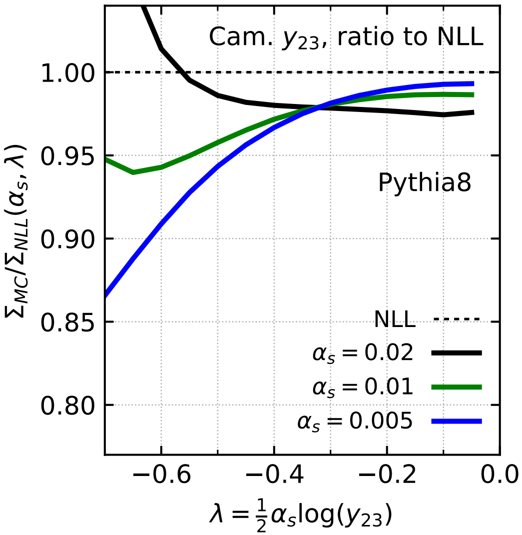
with $\lambda = \alpha_s L$

NLL deviations

or

subleading effects?

Testing log accuracy: a novel approach



Recall our regime: $\alpha_s \log(v) \sim 1$, $\alpha_s \ll 1$

Idea for NLL testing:

$$\frac{\Sigma_{\mathrm{MC}(\lambda=\alpha_s L, \alpha_s)}}{\Sigma_{\mathrm{NLL}(\lambda=\alpha_s L, \alpha_s)}} \quad \text{v.} \quad 1$$

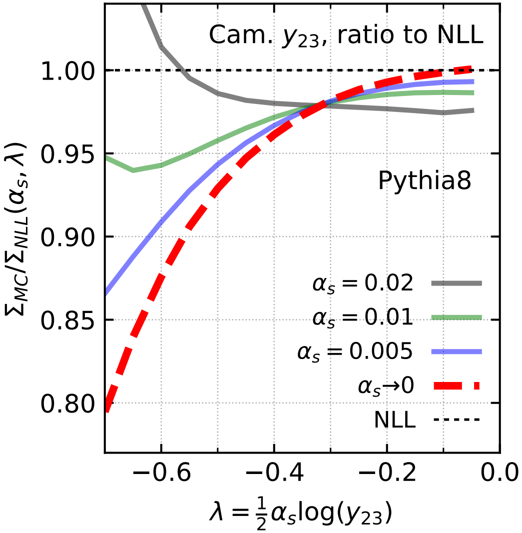
with $\lambda = \alpha_s L$

NLL deviations

or

subleading effects?

Testing log accuracy: a novel approach



Recall our regime: $\alpha_s \log(v) \sim 1$, $\alpha_s \ll 1$

Idea for NLL testing:

$$\frac{\Sigma_{\rm MC}(\lambda=\alpha_s L, \alpha_s)}{\Sigma_{\rm NLL}(\lambda=\alpha_s L, \alpha_s)} \xrightarrow{\alpha_s \to 0} 1$$

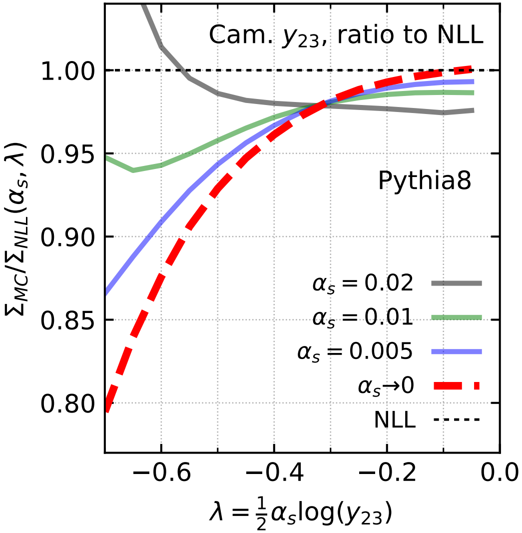
at fixed $\lambda = \alpha_s L$

NLL deviations

or

~~subleading effects?~~

Testing log accuracy: a novel approach



Recall our regime: $\alpha_s \log(v) \sim 1$, $\alpha_s \ll 1$

Idea for NLL testing:

$$\frac{\Sigma_{\rm MC}(\lambda=\alpha_s L, \alpha_s)}{\Sigma_{\rm NLL}(\lambda=\alpha_s L, \alpha_s)} \xrightarrow{\alpha_s \to 0} 1$$

at fixed $\lambda = \alpha_s L$

At N^{k}LL: test if

$$\frac{1}{\alpha_s^k} \frac{\log \Sigma_{\rm MC} - \log \Sigma_{\rm RESUM}}{\log \Sigma_{\rm RESUM}} \xrightarrow{\alpha_s \to 0} 0 \; ?$$

Physics result #2: NLL-accurate showers

Assessing accuracy: y_{23}

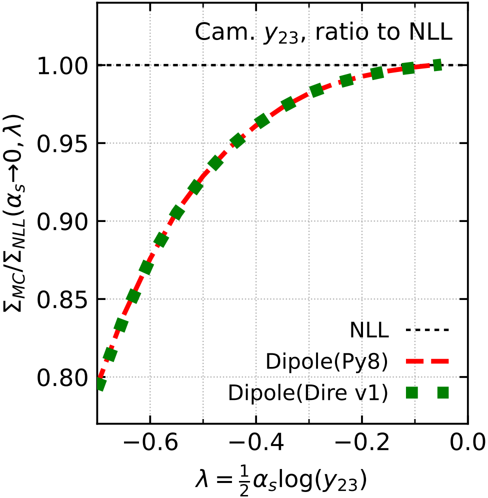
[M.Dasgupta,F.Dreyer,K.Hamilton,P.Monni,G.Salam,GS,20]

NLL if $\frac{\Sigma_{MC}(\lambda=\alpha_s L,\alpha_s)}{\Sigma_{NLL}(\lambda=\alpha_s L,\alpha_s)} \xrightarrow{\alpha_s\to 0} 1$

Failure of standard dipole showers

Pythia8, Dire(v1) deviate from NLL

Reason:
spurious recoil for commensurate-k_t emissions at disparate angles violates our NLL ME requirement



Assessing accuracy: y_{23}

[M.Dasgupta,F.Dreyer,K.Hamilton,P.Monni,G.Salam,GS,20]

NLL if $\frac{\Sigma_{MC}(\lambda=\alpha_s L,\alpha_s)}{\Sigma_{NLL}(\lambda=\alpha_s L,\alpha_s)} \xrightarrow{\alpha_s\to 0} 1$

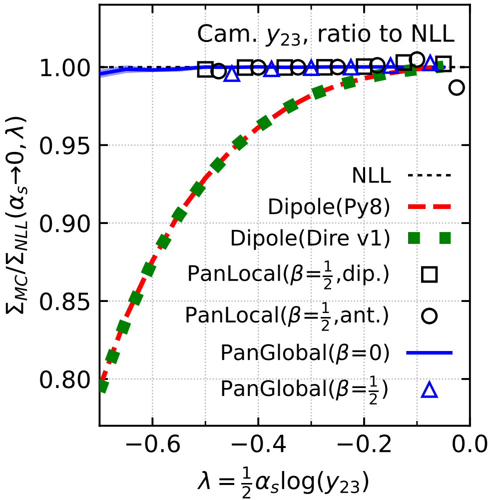
Failure of standard dipole showers

Pythia8, Dire(v1) deviate from NLL

New series of NLL-accurate showers

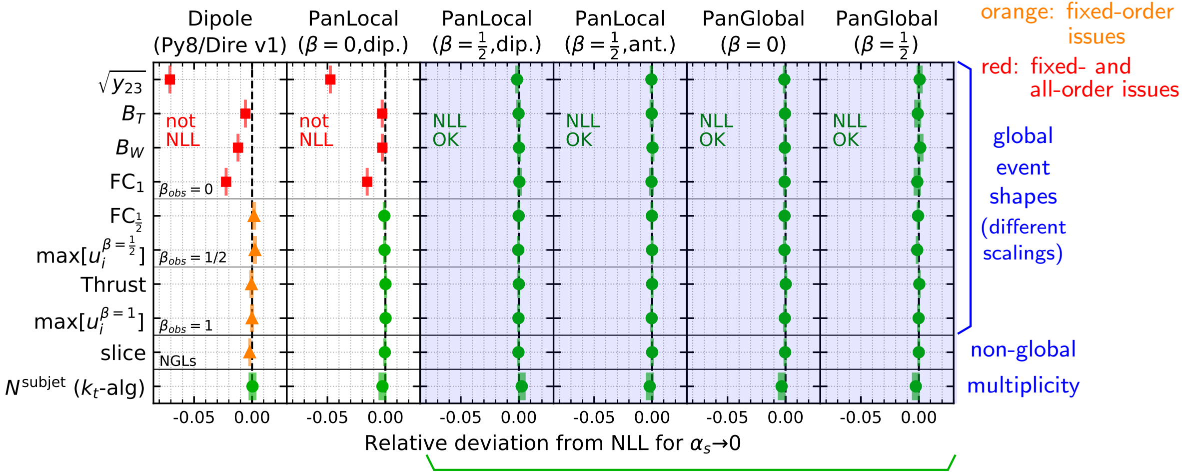
PanLocal($0 < \beta < 1$): local recoil (map) (dipole or antenna)

PanGlobal($0 \le \beta < 1$): global recoil (map)



Assessing accuracy: extensive observable list

[M.Dasgupta,F.Dreyer,K.Hamilton,P.Monni,G.Salam,GS,20]



PanLocal($0 < \beta < 1$) and PanGlobal($0 \leq \beta < 1$) get expected NLL (i.e. 0)

More progress with NLL-accurate showers

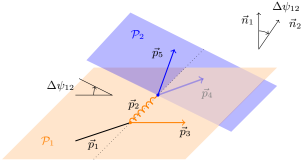
Beyond large N_c (backup)

(collinear & soft) spin correlations

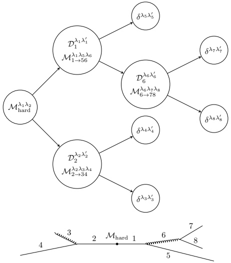
hadronic collisions DIS/VBF (backup)

Physics:

$\Delta\psi$ distribution due to spin correlations



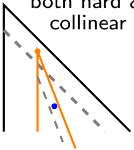
Solution: adapt the Collins-Knowles alg.



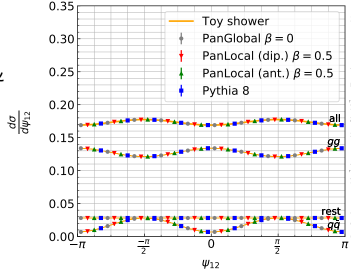
build and update a spin correlation tree as shower progresses

Tests:

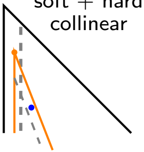
both hard & collinear



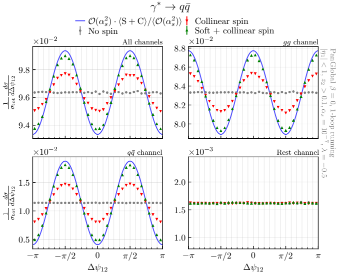
also EEEC v. analytics



soft + hard collinear



first all-order result



NLL now becoming the standard

PanScales

- ▷ Parton showers Beyond leading logarithmic accuracy [2002.11114]
- ▷ PanScales parton showers for hadron collisions: formulation and fixed-order studies [2205.02237]
- ▷ PanScales showers for hadron collisions: all-order validation [2207.09467]
- ▷ Next-to-leading logarithmic PanScales showers for Deep Inelastic scattering and Vector Boson Fusion [2305.08645]
- ▷ Colour and logarithmic accuracy in final-state parton showers [2011.10054]
- ▷ Spin correlations in final-state parton showers and jet observables [2103.16526]
- ▷ Soft spin correlations in final-state parton showers [2111.01161]
- ▷ Introduction to the PanScales framework, version 0.1 [2312.13275]

Various combinations of M.vanBeekveld, M.Dasgupta, B.El Menoufi, S.Ferrario Ravasio, K.Hamilton, J.Helliwell, A.Karlberg, R.Medves, P.Monni, G.P.Salam, L.Scyboz, A.Soto-Ontoso, G.Soyez, R.Verheyen

Apollo

- ▷ A partitioned dipole-antenna shower with improved transverse recoil [2403.19452]

C. Preuss

Deductor

- ▷ Summations of large logarithms by parton showers [2011.04773]
- ▷ Summations by parton showers of large logarithms in electron-positron annihilation [2011.04777]

Z.Nagy, D.Soper

Alaric

- ▷ A new approach to color-coherent parton evolution [2208.06057]
- ▷ New approach to QCD final-state evolution in processes with massive partons [2307.00728]
- ▷ alaric parton shower for hadron colliders [2404.14360]

Combinations of B.Assi, F.Herren, S.Höche, F.Krauss, D.Reichelt, M.Schönherr

CVolver

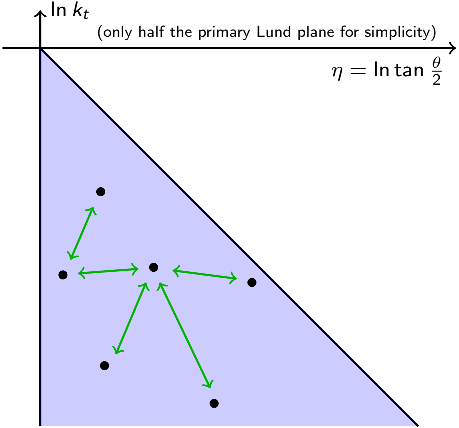
- ▷ Parton branching at amplitude level [1905.08686]
- ▷ Building a consistent parton shower [2003.06400]
- ▷ Improvements on dipole shower colour [2011.15087]

J.Forshaw, J.Holguin, S.Platzer

Physics result #3: towards NNLL-accurate showers

Rule of thumb:
LL $\equiv$ qualitative starting point
NLL $\equiv$ first quantitative order
NNLL $\equiv$ towards precision physics

NNLL: include pairs of emissions



(NNLL) accuracy $\leftrightarrow$ reproducing (extra) sets of MEs

NNLL: include pairs of emissions

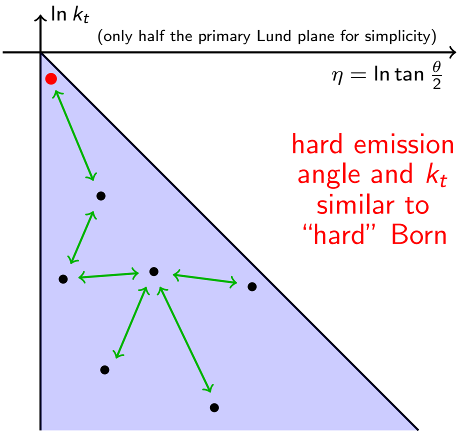
Matching

Get exact 3-jet LO (2-jet NLO) ME

$\equiv$ one hard emission (pair with the hard event)

Standard approaches work but require care to preserve NLL accuracy

[K.Hamilton,A.Karlberg,G.P.Salam,L.Scyboz,arXiv:2301.09645], [M.vanBeekveld,S.Ferrario Ravasio,J.Helliwell,A.Karlberg,G.P.Salam,L.Scyboz, A.Soto-Ontoso,G.Soyez,S.Zanoli,arXiv:2504.05377]



NNLL: include pairs of emissions

Matching

[K.Hamilton,A.Karlberg,G.P.Salam,L.Scyboz,arXiv:2301.09645], [M.vanBeekveld,S.Ferrario Ravasio,J.Helliwell,A.Karlberg,G.P.Salam,L.Scyboz, A.Soto-Ontoso,G.Soyez,S.Zanoli,arXiv:2504.05377]

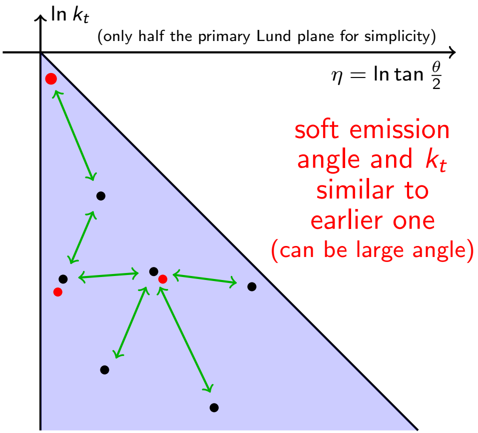
Double-soft corrections

Two soft emissions at commensurate angles and k_t (not necessarily collinear)

- Correction spurious shower ME $\rightarrow$ correct ME watch out for flavour channels and colour flows
- Need to get the correct virtual contributions (done through a modified K_{CMW})

Gain: state-of-the-art (next-to-single-log) non-global logs

[S.Ferrario Ravasio,K.Hamilton,A.Karlberg,G.P.Salam,L.Scyboz,GS,arXiv:2307.11142]



(NNLL) accuracy $\leftrightarrow$ reproducing (extra) sets of MEs

NNLL: include pairs of emissions

Matching

[K.Hamilton,A.Karlberg,G.P.Salam,L.Scyboz,arXiv:2301.09645],
[M.vanBeekveld,S.Ferrario Ravasio,J.Helliwell,A.Karlberg,G.P.Salam,L.Scyboz,
A.Soto-Ontoso,G.Soyez,S.Zanoli,arXiv:2504.05377]

Double-soft corrections

Two soft emissions at commensurate angles and k_t
(not necessarily collinear)

- Correction spurious shower ME $\rightarrow$ correct ME
  watch out for flavour channels and colour flows
- Need to get the correct virtual contributions
  (done through a modified K_{CMW})

Gain: state-of-the-art (next-to-single-log) non-global logs
[S.Ferrario Ravasio,K.Hamilton,A.Karlberg,G.P.Salam,L.Scyboz,GS,arXiv:2307.11142]

Achieved via a revised emission rate:

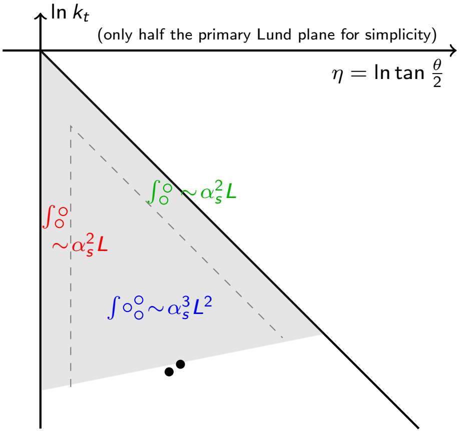
$$\frac{dP}{d\ln v\, d\eta} = \frac{\alpha_s(k_t)C_A}{\pi} \times M \times g(\eta)P(z)$$

Matrix elements

First emission: $M(k)$ corrects to the exact $\mathcal{O}(\alpha_s)$ ME (matching)

Next emissions: $M(k_1, k_2)$ corrects for double-soft ME

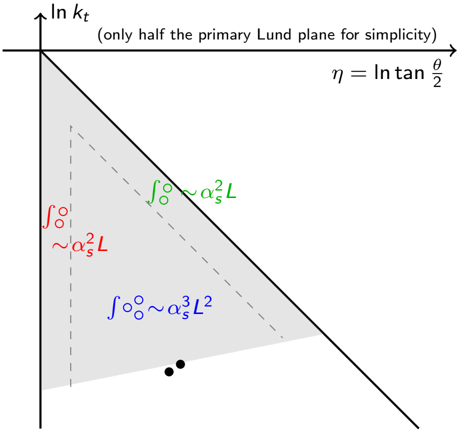
NNLL Sudakov: revised emission strenght



$$\alpha_s = \alpha_s^{(3\ell)}(k_t)\left[1 + \alpha_s \Delta K_1 + \alpha_s(B_2 + \Delta B_2) + \alpha_s^2 \Delta K_2\right]$$

- 3-loop running in the CMW scheme (K_1, K_2)
- B_2: analytic ($\sim$ NLO DGLAP + right $C_{\rm hc}^{(1)} \alpha_s(Q_{\rm hc})$)
- rest: analytic and shower assume different conserved different quantities ($\approx$ "scheme"). These absorb spurious $\alpha_s^2 L$ double-soft effects
- ΔK_2: 2 numbers (δ_y, $\delta_{\ln k_t}$) from num integration
- $\Delta K_1(y)$: (soft wide angles)
  shapes only need $\int dy \Delta K_1(y) \propto \delta_y$.
  Non-globals need full differential (sumrule!)
- $\Delta B_2(z)$: (hard-collinear)
  shapes only need $\int dz\, \Delta B_2(z) \propto \delta_y + \delta_{\ln k_t + \frac{\pi^2}{12}\beta_0}$
  Full differential for e.g. jet substructure.

NNLL Sudakov: revised emission strenght

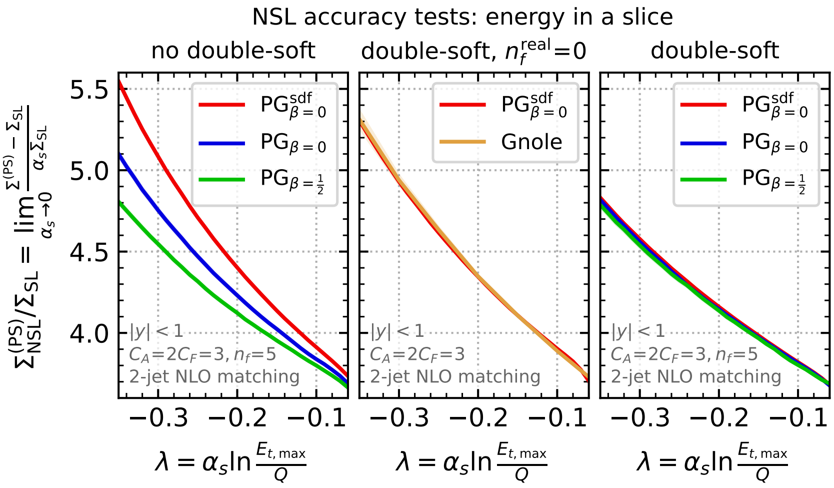


Full analytic proof of NNLL accuracy

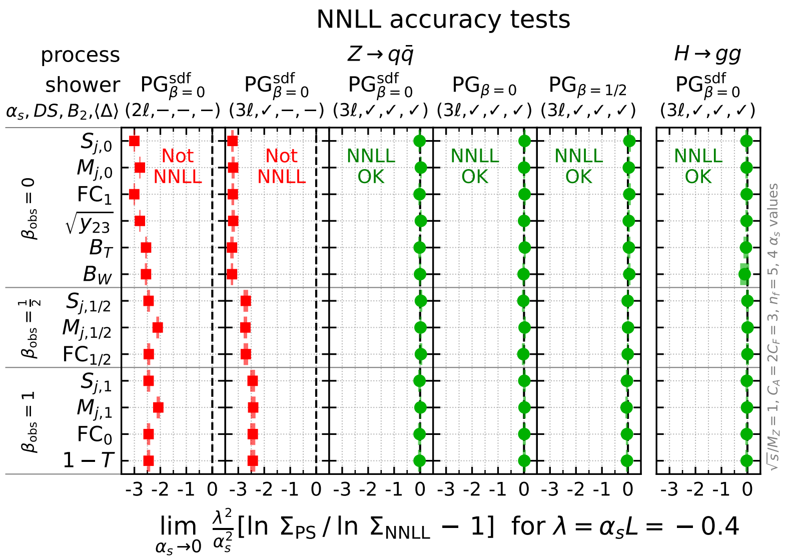
$$\alpha_s = \alpha_s^{(3\ell)}(k_t)\left[1 + \alpha_s \Delta K_1 + \alpha_s(B_2 + \Delta B_2) + \alpha_s^2 \Delta K_2\right]$$

- 3-loop running in the CMW scheme (K_1, K_2)
- B_2: analytic ($\sim$ NLO DGLAP + right $C_{hc}^{(1)}\alpha_s(Q_{hc})$)
- rest: analytic and shower assume different conserved different quantities ($\approx$ "scheme"). These absorb spurious $\alpha_s^2 L$ double-soft effects
- ΔK_2: 2 numbers (δ_y, $\delta_{\ln k_t}$) from num integration
- $\Delta K_1(y)$: (soft wide angles)
  shapes only need $\int dy \Delta K_1(y) \propto \delta_y$.
  Non-globals need full differential (sumrule!)
- $\Delta B_2(z)$: (hard-collinear)
  shapes only need $\int dz\, \Delta B_2(z) \propto \delta_y + \delta_{\ln k_t + \frac{\pi^2}{12}\beta_0}$
  Full differential for e.g. jet substructure.

Beyond NLL: double-soft corrections

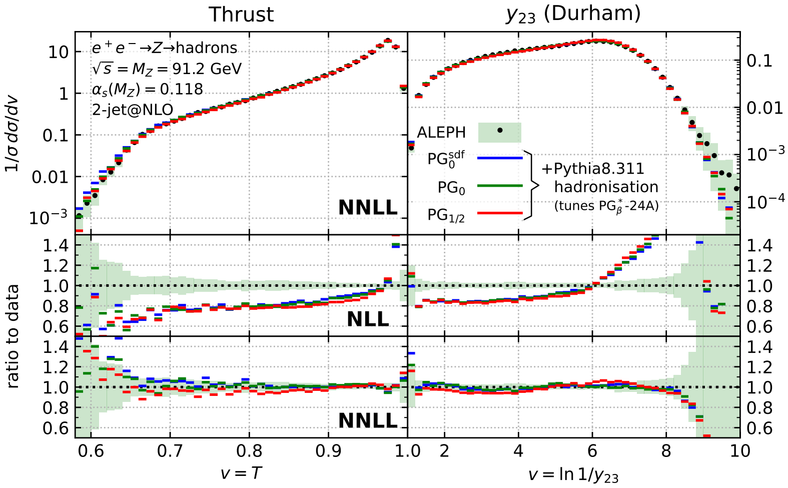


Successfully reproduce next-to-single (non-global) logs for emissions in a slice



explicit numerical test that we get g_3 (NNLL coefficient) right.

NNLL preliminary pheno

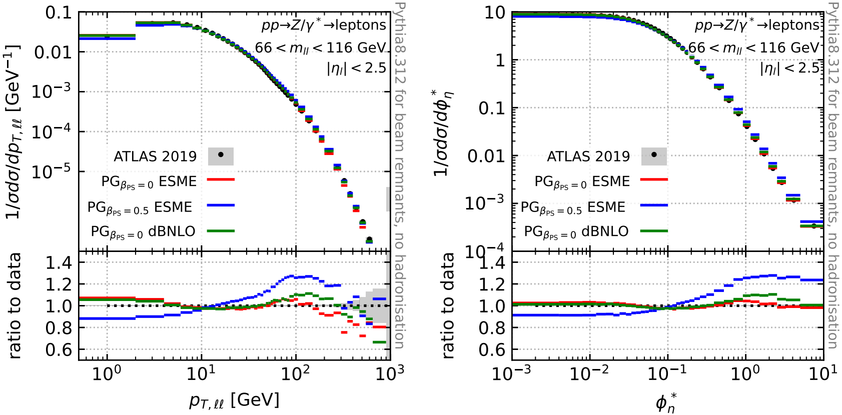


Quite good agreement with LEP data

- "physical" α_s
- NLL deviation from one could be seen as uncertainty
- NNLL expected to give better accuracy
- NP tuning (mostly) not sizeable

pp NLL+NLO preliminary pheno

M. van Beekveld, S. Ferrario Ravasio, J. Helliwell, A. Karlberg, G. P. Salam, L. Scyboz, A. Soto-Ontoso, G. Soyez, S. Zanoli, arXiv:2504.05377



ESME ≡ New matching method (fast, no negative weights)

Still some work needed to reach ≥NNLL+NNLO but going in this direction

Conclusions and perspectives

Recap of take-home messages

- Parton showers are a cornerstone of collider physics
- Parton showers accuracy $\equiv$ log accuracy
- Systematically improvable, can be tested analytically and numerically
- PanScales 2019-2023: NLL parton showers... several others now
- PanScales 2023-now: good NNLL progress (*ee* shapes, large angle non-globals)

Future

- NNLL in *pp* (LHC)
- NNLL hard-colliner (jet substructure)
- NNLL PanLocal
- more complex processes/(N)NLO
- Tuning
- Investigate phenomenology

. . . key steps towards NNLL were just O(5) years away

slide from Pier Monni

Birth of Herwig (with elements of NLL for global observables)

SIMULATION OF QCD JETS INCLUDING SOFT GLUON INTERFERENCE

G. MARCHESINI

Istituto di Fisica dell'Università di Parma
and
INFN, Sezione di Milano, Italy

B.R. WEBBER*

CERN, Geneva, Switzerland

Received 21 March 1983
(Revised 14 December 1983)

We present a new Monte Carlo simulation scheme for jet evolution in perturbative QCD which takes into account the results of recent analyses of soft-gluon interference. Therefore, this scheme accounts correctly not only for the leading collinear singularities, as in previous schemes, but also for leading infrared singularities. In this first paper we study the basic features of gluon jet evolution such as: (i) the interference effects and the corresponding depletion of the parton distributions in the soft region; (ii) the approach to asymptopia; (iii) the efficiency of colour screening (preconfinement), which has been questioned recently by Bjorken.

MONTE CARLO SIMULATION OF GENERAL HARD PROCESSES WITH COHERENT QCD RADIATION*

G MARCHESINI

Dipartimento di Fisica, Università di Parma, INFN, Gruppo Collegato di Parma, Italy

B R WEBBER

Cavendish Laboratory, University of Cambridge, Madingley Road, Cambridge CB3 0HE, UK

Received 8 February 1988

In this paper we extend our previous work on the simulation of coherent soft-gluon radiation to hard collisions that involve incoming as well as outgoing coloured partons Existing simulations correctly sum the leading collinear singularities for initial- and final-state radiation, and in some cases the leading infrared contributions from outgoing partons, but not those for incoming (or the interference between incoming and outgoing) Asymptotically, however, the leading infrared and collinear contributions are comparable, the bulk of gluon emission occurring in the soft region Furthermore, a correct treatment of leading infrared terms is necessary for the inclusive cancellation of singularities in the Sudakov form factor We show how such a treatment may be formulated in terms of an angular ordering procedure applicable to all hard processes We then describe a new Monte Carlo program which incorporates this procedure, together with other new features such as azimuthal correlations due to gluon polarization and interference The program is designed as a general-purpose event generator, simulating hard lepton–lepton, lepton–hadron and hadron–hadron scattering in a single package Simulation of soft hadronic collisons and underlying events is also included We present the predictions of the program for a wide variety of processes, and compare them with analytical results and experimental data

General principles for NNLL parton showers

A new standard for the logarithmic accuracy of parton showers

Melissa van Beekveld,[1] Mrinal Dasgupta,[2] Basem Kamal El-Menoufi,[3] Silvia Ferrario Ravasio,[4] Keith Hamilton,[5] Jack Helliwell,[6] Alexander Karlberg,[4] Pier Francesco Monni,[4] Gavin P. Salam,[6,7] Ludovic Scyboz,[3] Alba Soto-Ontoso,[4] and Gregory Soyez[8]

[1] *Nikhef, Theory Group, Science Park 105, 1098 XG, Amsterdam, The Netherlands*
[2] *Department of Physics & Astronomy, University of Manchester, Manchester M13 9PL, United Kingdom*
[3] *School of Physics and Astronomy, Monash University, Wellington Rd, Clayton VIC-3800, Australia*
[4] *CERN, Theoretical Physics Department, CH-1211 Geneva 23, Switzerland*
[5] *Department of Physics and Astronomy, University College London, London, WC1E 6BT, UK*
[6] *Rudolf Peierls Centre for Theoretical Physics, Clarendon Laboratory, Parks Road, Oxford OX1 3PU, UK*
[7] *All Souls College, Oxford OX1 4AL, UK*
[8] *IPhT, Université Paris-Saclay, CNRS UMR 3681, CEA Saclay, F-91191 Gif-sur-Yvette, France*

We report on a major milestone in the construction of logarithmically accurate final-state parton showers, achieving next-to-next-to-leading-logarithmic (NNLL) accuracy for the wide class of observables known as event shapes. The key to this advance lies in the identification of the relation between critical NNLL analytic resummation ingredients and their parton-shower counterparts. Our analytic discussion is supplemented with numerical tests of the logarithmic accuracy of three shower variants for more than a dozen distinct event-shape observables in $Z \to q\bar{q}$ and Higgs $\to gg$ decays. The NNLL terms are phenomenologically sizeable, as illustrated in comparisons to data.

A MODEL FOR INITIAL STATE PARTON SHOWERS

Torbjörn SJÖSTRAND

Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, IL 60510, USA

Received 25 February 1985

We present a detailed model for exclusive properties of initial state parton showers. A numerically efficient algorithm is obtained by tracing the parton showers backwards, i.e. start with the hard scattering partons and then successively reconstruct preceding branchings in falling sequence of spacelike virtualities Q^2 and rising sequence of parton energies. We show how the Altarelli–Parisi equations can be recast in a form suitable for this, and also discuss the kinematics of the branchings. The complete model is implemented in a Monte Carlo program, and some first results are presented.

Birth of Pythia

Parton showers beyond leading logarithmic accuracy

Mrinal Dasgupta,[1] Frédéric A. Dreyer,[2] Keith Hamilton,[3] Pier Francesco Monni,[4] Gavin P. Salam,[2,*] and Grégory Soyez[5]

[1] *Consortium for Fundamental Physics, School of Physics and Astronomy, University of Manchester, Manchester M13 9PL, United Kingdom*
[2] *Rudolf Peierls Centre for Theoretical Physics, Parks Road, Oxford OX1 3PU, UK*
[3] *Department of Physics and Astronomy, University College London, London, WC1E 6BT, UK*
[4] *CERN, Theoretical Physics Department, CH-1211 Geneva 23, Switzerland*
[5] *Institut de Physique Théorique, Université Paris-Saclay, CNRS, CEA, F-91191, Gif-sur-Yvette, France*

Parton showers are among the most widely used tools in collider physics. Despite their key importance, none so far has been able to demonstrate accuracy beyond a basic level known as leading logarithmic (LL) order, with ensuing limitations across a broad spectrum of physics applications. In this letter, we propose criteria for showers to be considered next-to-leading logarithmic (NLL) accurate. We then introduce new classes of shower, for final-state radiation, that satisfy the main elements of these criteria in the widely used large-N_C limit. As a proof of concept, we demonstrate these showers' agreement with all-order analytical NLL calculations for a range of observables, something never so far achieved for any parton shower.

Parton showering with higher-logarithmic accuracy for soft emissions

Silvia Ferrario Ravasio,[1] Keith Hamilton,[2] Alexander Karlberg,[1] Gavin P. Salam,[3,4] Ludovic Scyboz,[3] and Gregory Soyez[1,5]

[1] *CERN, Theoretical Physics Department, CH-1211 Geneva 23, Switzerland*
[2] *Department of Physics and Astronomy, University College London, London, WC1E 6BT, UK*
[3] *Rudolf Peierls Centre for Theoretical Physics, Clarendon Laboratory, Parks Road, Oxford OX1 3PU, UK*
[4] *All Souls College, Oxford OX1 4AL, UK*
[5] *IPhT, Université Paris-Saclay, CNRS UMR 3681, CEA Saclay, F-91191 Gif-sur-Yvette, France*

The accuracy of parton-shower simulations is often a limiting factor in the interpretation of data from high-energy colliders. We present the first formulation of parton showers with accuracy one order beyond state-of-the-art next-to-leading logarithms, for classes of observable that are dominantly sensitive to low-energy (soft) emissions, specifically non-global observables and subjet multiplicities. This represents a major step towards general next-to-next-to-leading logarithmic accuracy for parton showers.

General principles for a NLL parton shower
(formulated for e^+e^-, many extensions will follow)

[ca. 800 papers on the subject of event generators ....]

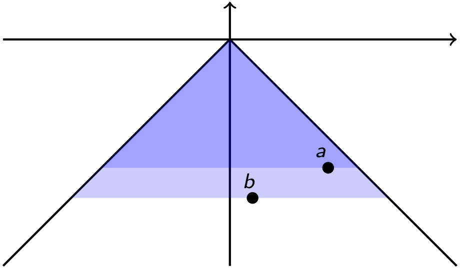
89

Backup

Different ordering variables...

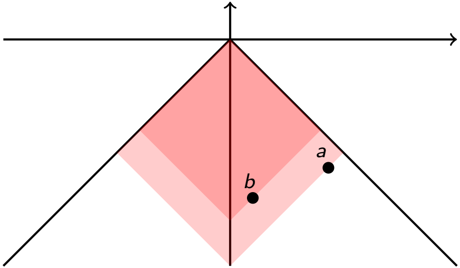
... can lead to different emission orderings

k_t (transv. mom.) ordering



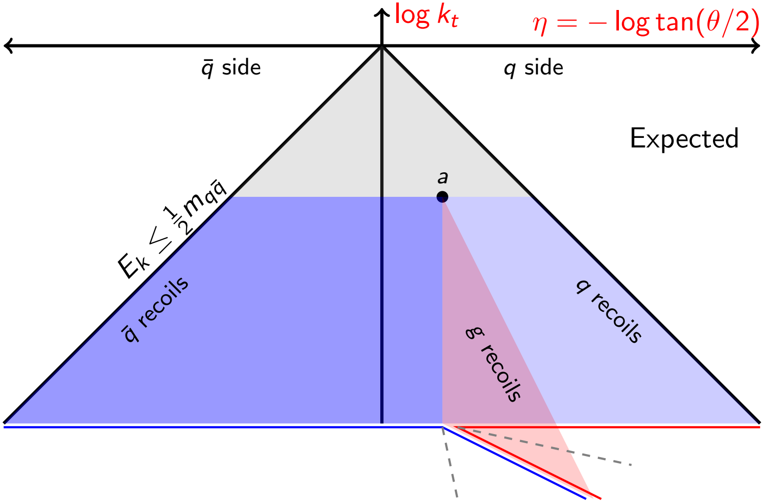
$k_{ta} > k_{tb}$
$\Rightarrow$ a emitted before b

q (virtuality) ordering



$q_b > q_s$
$\Rightarrow$ b emitted before a

Lund-plane representation: transverse recoil boundaries



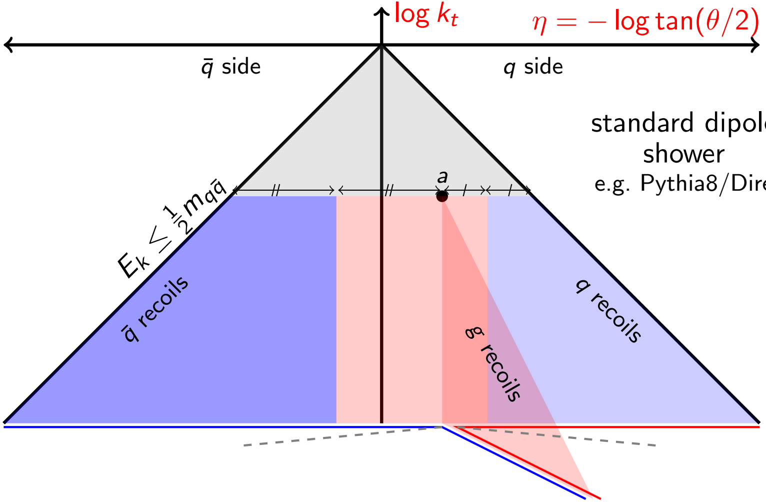
gluon a radiated at scale k_{ta} and angle θ_a

gluon b radiated at scale $k_{tb} \le k_{ta}$

Expected

a takes recoil iff $\theta_{ab} < \theta_a$

Lund-plane representation: transverse recoil boundaries



gluon a radiated at scale k_{ta} and angle θ_a

gluon b radiated at scale $k_{tb} \leq k_{ta}$

Expected

a takes recoil iff $\theta_{ab} < \theta_a$

standard dipole shower

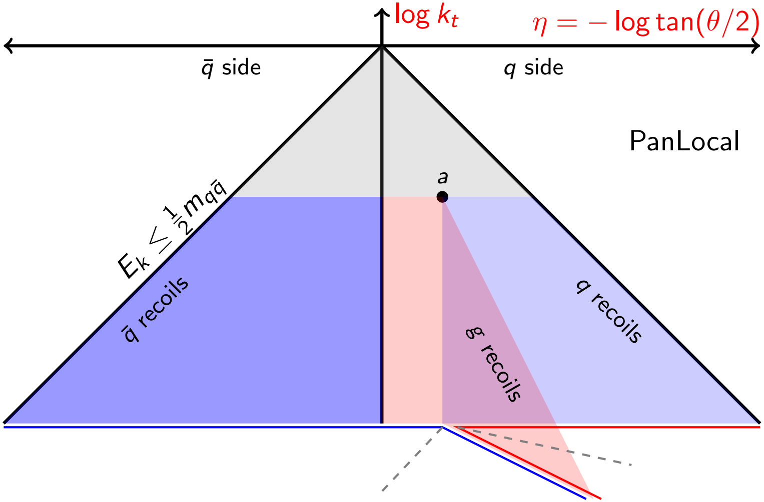
decided in dipole frame:

a takes recoil if

$$\theta_{bg}^{(\text{dip})} < \theta_{bq}^{(\text{dip})}$$

WRONG!

Lund-plane representation: transverse recoil boundaries



gluon a radiated at scale k_{ta} and angle θ_a

gluon b radiated at scale $k_{tb} \leq k_{ta}$

Expected

a takes recoil iff $\theta_{ab} < \theta_a$

PanLocal (step 1)

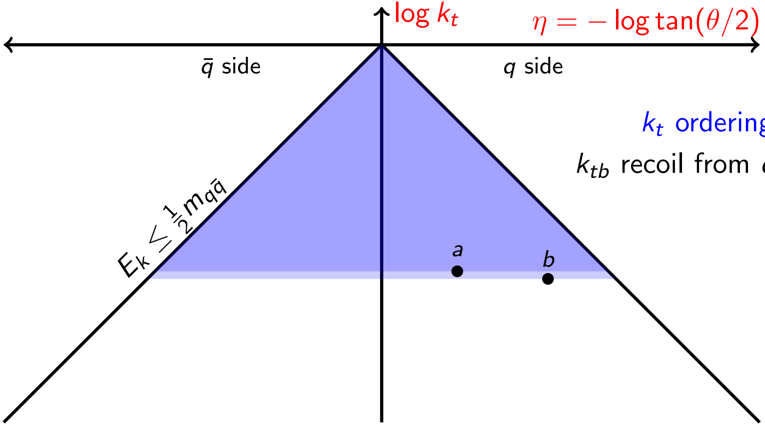
decided in event frame:

a takes recoil if

$$\theta_{bg} < \theta_{bq}$$

better but still WRONG!

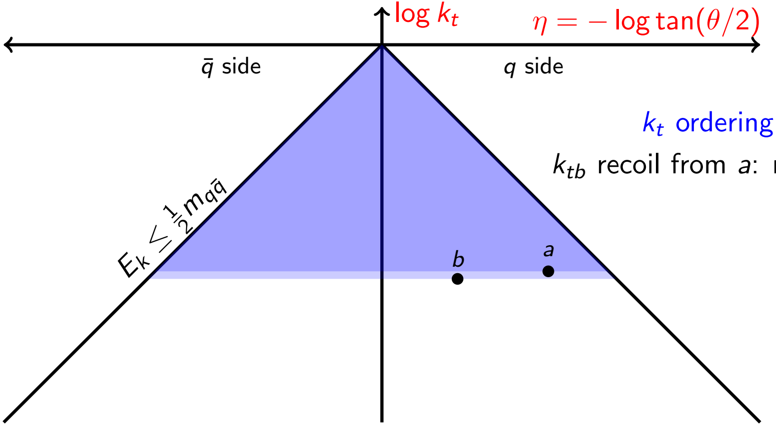
Lund-plane representation: PanLocal evolution variable



k_t ordering

k_{tb} recoil from q: OK

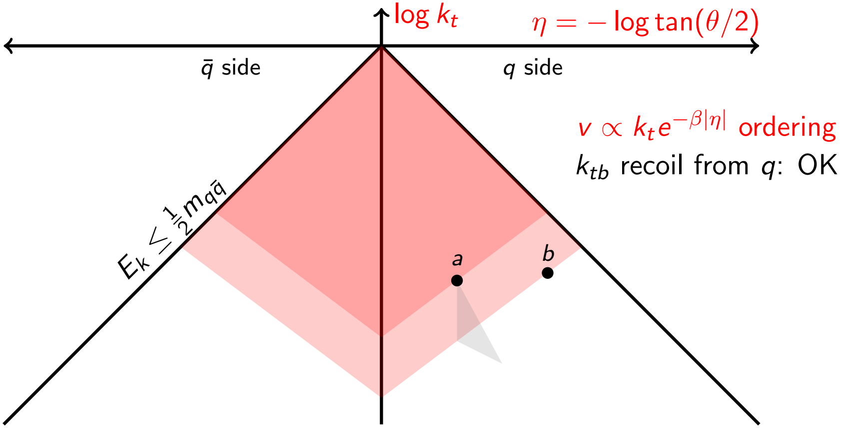
Lund-plane representation: PanLocal evolution variable



k_t ordering

k_{tb} recoil from a: not OK

Lund-plane representation: PanLocal evolution variable



commensurate k_t emissions generated from central to forward rapidities

$\Rightarrow$ no recoil issue

Kinematic maps

PanLocal (local $\perp$ recoil)

$$p_k = a_k \tilde{p}_i + b_k \tilde{p}_j + k_\perp$$
$$p_i = a_i \tilde{p}_i + b_i \tilde{p}_j - k_\perp$$
$$p_j = a_j \tilde{p}_i + b_j \tilde{p}_j$$

PanGlobal (global $\perp$ recoil)

$$p_k = r(a_k \tilde{p}_i + b_k \tilde{p}_j)$$
$$p_i = r(1 - a_k)\tilde{p}_i$$
$$p_j = r(1 - b_k)\tilde{p}_j$$

with r so as to conserve event Q^2
+ transverse boost to conserve event Q^μ.

Evolution variable v ($v \approx k_\perp \theta^\beta$)
Auxiliary variable(s): $\bar{\eta}$, ϕ
($\bar{\eta} \equiv$ rapidity in event frame) Define:

$$|k_\perp| = \rho\, v\, e^{\beta|\bar{\eta}|} \qquad \rho = \left(\frac{2\tilde{p}_i.Q\, \tilde{p}_j.Q}{Q^2\, \tilde{p}_i.\tilde{p}_j}\right)^{\beta/2}$$

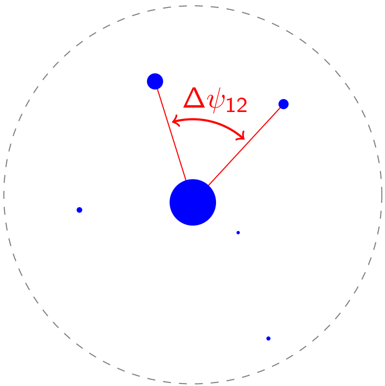
$$a_k = \sqrt{\frac{\tilde{p}_j.Q}{2\tilde{p}_i.Q\, \tilde{p}_i.\tilde{p}_j}}\, |k_\perp|\, e^{+\bar{\eta}},$$

$$b_k = \sqrt{\frac{\tilde{p}_i.Q}{2\tilde{p}_j.Q\, \tilde{p}_i.\tilde{p}_j}}\, |k_\perp|\, e^{-\bar{\eta}},$$

A striking example

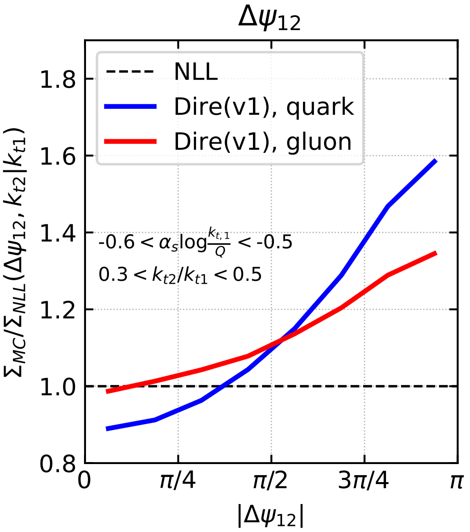
- Look at angle $\Delta\psi_{12}$ between two hardest "emissions" in jet

  (defined through Lund declusterings)



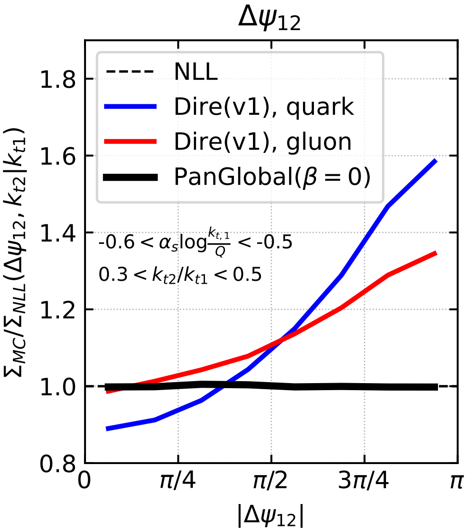
A striking example

- Look at angle $\Delta\psi_{12}$ between two hardest "emissions" in jet
  (defined through Lund declusterings)
- quite large NLL deviations in current dipole showers
- differences between quark and gluon jets



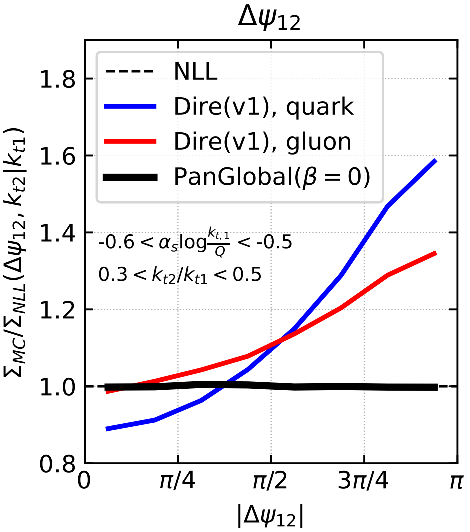
A striking example

- Look at angle $\Delta\psi_{12}$ between two hardest "emissions" in jet
  (defined through Lund declusterings)
- quite large NLL deviations in current dipole showers
- differences between quark and gluon jets
- PanScales showers (here PanGlobal) get the correct NLL



A striking example

- Look at angle $\Delta\psi_{12}$ between two hardest "emissions" in jet
  (defined through Lund declusterings)
- quite large NLL deviations in current dipole showers
- differences between quark and gluon jets
- PanScales showers (here PanGlobal) get the correct NLL
- ML could "wrongly/correctly" learn this



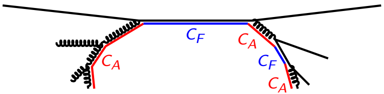
Beyond large N_c

(collinear & soft) spin correlations

hadronic collisions

Physics:

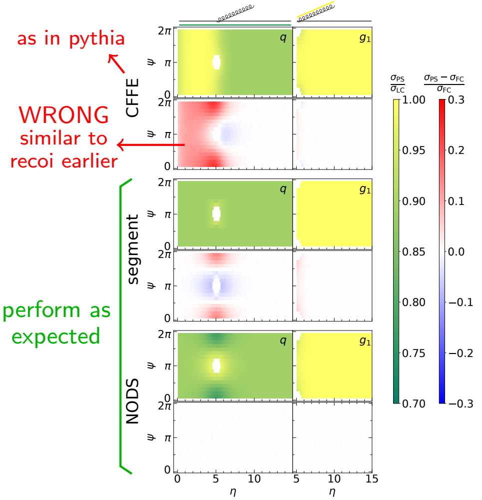
Keep track of the C_F–$C_A/2$ transitions



First generate assuming $C_A(/2)$, then correct in one of 2 ways:

1. segment
   factor $2C_F/C_A$ if in quark segment
   OK in the angular-ordered limit
2. NODS
   (soft) $q\bar{q}g$ matrix-element correction
   also OK for 2 emissions at $\sim$ angles

Fixed-order tests:



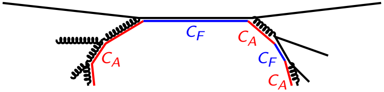
Beyond large N_c

(collinear & soft) spin correlations

hadronic collisions

Physics:

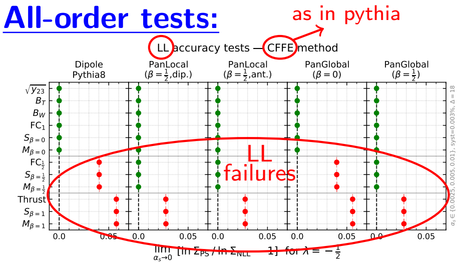
Keep track of the C_F–$C_A/2$ transitions

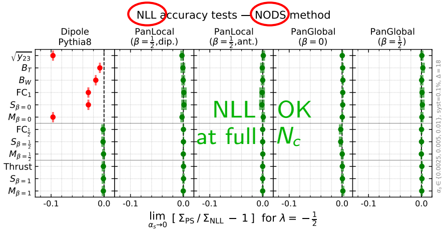


First generate assuming $C_A(/2)$, then correct in one of 2 ways:

1. segment
   factor $2C_F/C_A$ if in quark segment
   OK in the angular-ordered limit
2. NODS
   (soft) $q\bar{q}g$ matrix-element correction
   also OK for 2 emissions at $\sim$ angles

All-order tests:





Non-global logs: large-N_c + (full-N_c at $\mathcal{O}(\alpha_s^2)$)

Beyond large N_c

(collinear & soft) spin correlations

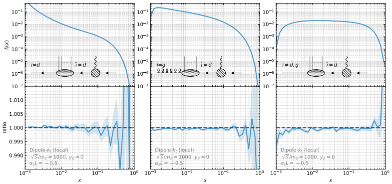
hadronic collisions

Physics:

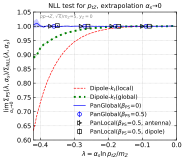
- hadron collision
  $\Rightarrow$ initial-state radiation
- Consider Drell-Yan
- existing showers have the same recoil issue as for final state
  earlier emission takes recoil instead of the Z
- fix is essentially the same
  (modulo kinematic differences)
- includes colour and spin
- so far limited to colour singlet production

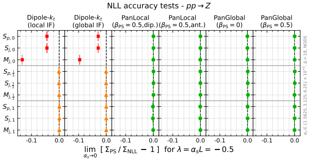
Tests:

explicit test of DGLAP



+ usual tests: Z-boson p_t, event shapes





+ multiplicity, non-globals, beyond large-N_c, spin

Matching within PanScales

Matching = exact fixed-order generator + parton shower resumming logs

Physics

Focus on e^+e^- collisions. We want

- ✓ exact $q\bar{q}g$ ($\mathcal{O}(\alpha_s)$) distributions
- ✓ maintain NLL accuracy

Benefit: "NNDL" accuracy for event shapes$^{(*)}$

$$\Sigma(L) = \underbrace{h_1(\alpha_s L^2)}_{\text{DL}} + \underbrace{\sqrt{\alpha_s} h_2(\alpha_s L^2)}_{\text{NDL}} + \underbrace{\alpha_s h_3(\alpha_s L^2)}_{\text{NNDL}} + \dots$$

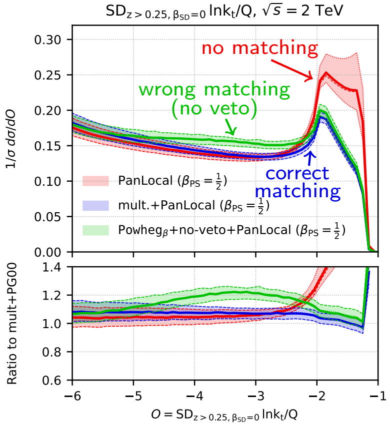
Implementation

Several possibilities:

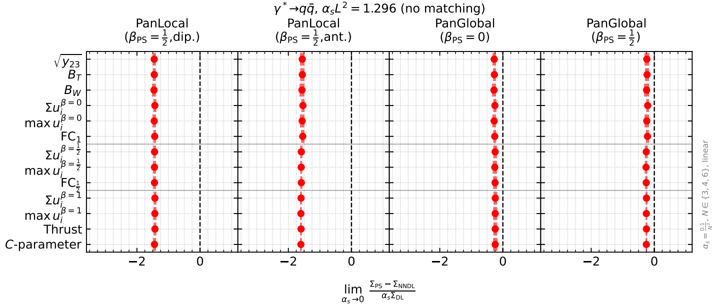
- simple multiplicative matching (accept first emission with probability $P_{\text{exact}}/P_{\text{shower}}$)
- MC@NLO-like matching
- POWHEG-like matching (with β scaling and careful veto to avoid double-counting when switching from POWHEG to the shower)

$^{(*)}$ Note: N^{k}LL expands $\ln \Sigma(\alpha_s L, \alpha_s)$ for "exponentiating" observables; N^{k}DL directly expands $\Sigma(\alpha_s L^2, \alpha_s)$
alternative viewpoint: N^{k}LL takes the limit $\alpha_s L \sim$ cst with $\alpha_s \ll 1$; N^{k}DL takes the limit $\alpha_s L^2 \sim$ cst with $\alpha_s \ll 1$
practical implication: NLL requires an arbitrary number of single-logs ($(\alpha_s L)^n$); NDL requires only one ($(\alpha_s L)(\alpha_s L^2)^n$)

Accuracy tests

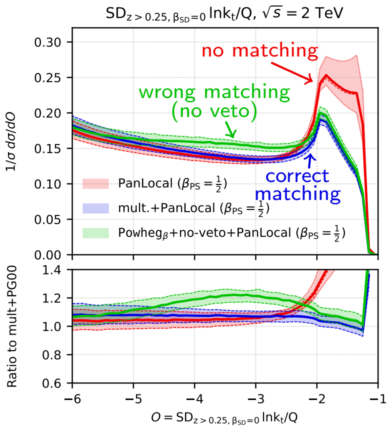


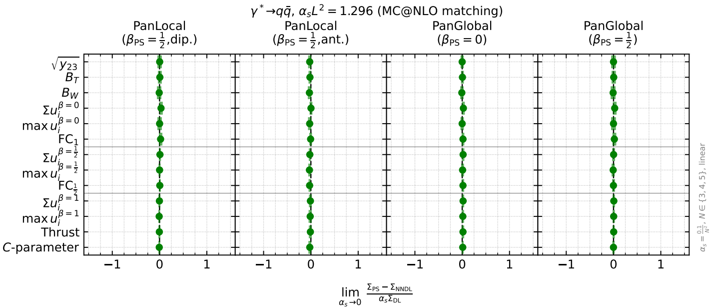
- visible effect at large k_t (right)
- spurious effect if not careful
- "correct" matching OK everywhere



- no matching $\Rightarrow$ wrong NNDL

Accuracy tests

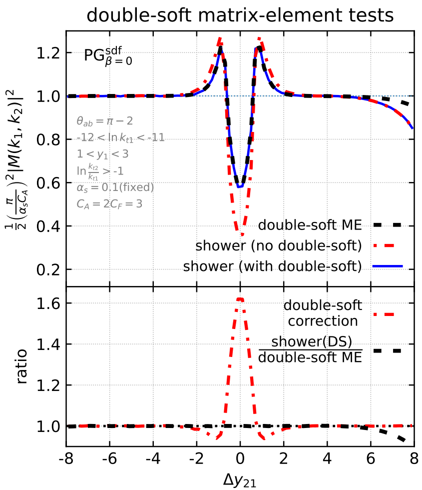


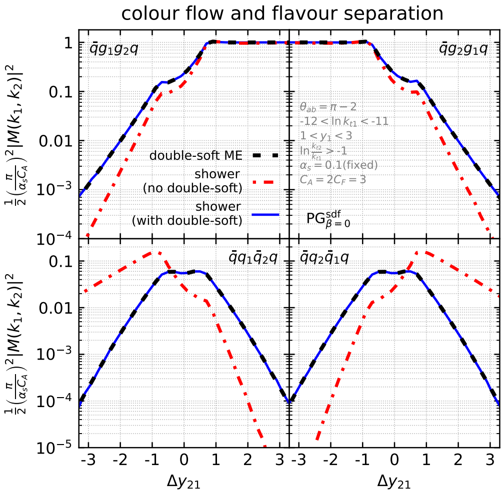


- visible effect at large k_t (right)
- spurious effect if not careful
- “correct” matching OK everywhere

- no matching $\Rightarrow$ wrong NNDL
- with matching $\Rightarrow$ OK at NNDL

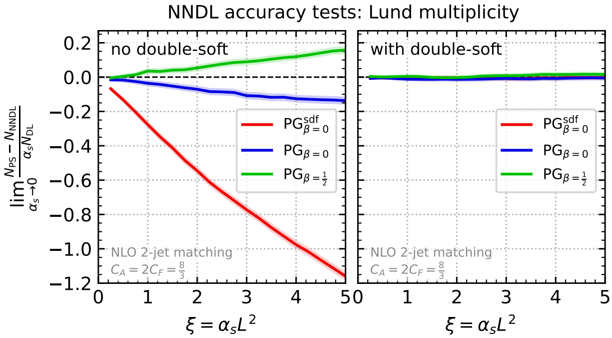
Extra double-soft results: matrix-element tests



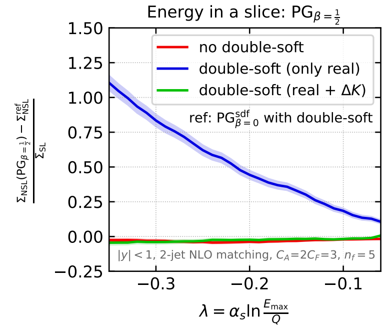


Correct reproduction of the double-soft matrix elements

Extra double-soft results: multiplicity, δK

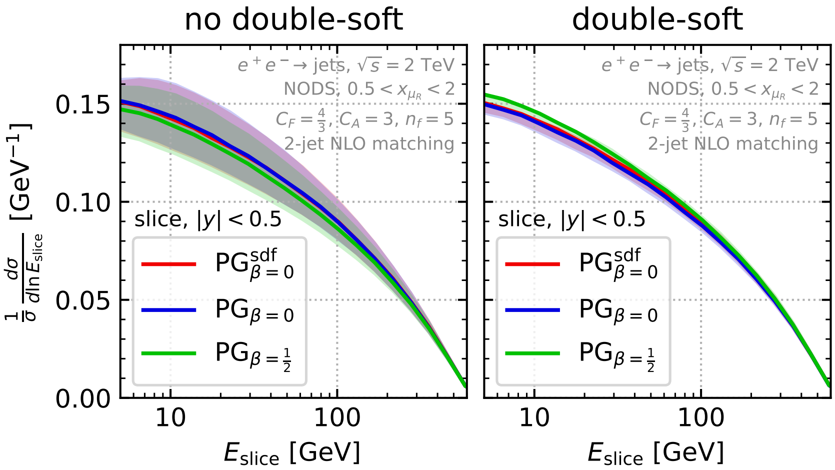


Reproduces NNDL multiplicity



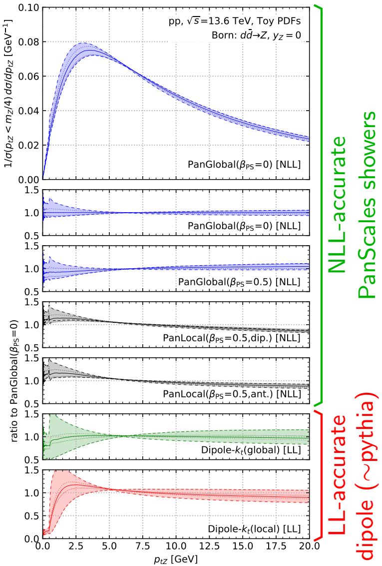
Requires the correct K_{CMW} prescription

Extra double-soft results: multiplicity, δK



No large shift of central value but large reduction of the uncertainty estimates

Example #1: Z-boson transverse momentum



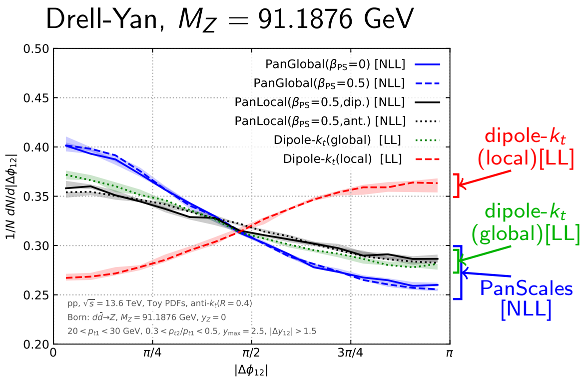
Uncertainties:

- renormalisation scale variation:
  for NLL-accurate showers include compensation term to maintain 2-loop running for soft emissions
- factorisation scale variations (note: use of toy PDFs)
- term associated with lack of matching for $k_t \sim M_Z$
- for LL showers: a term associated with spurious recoil for commensurate k_t's

Observations: Differences are relatively small except

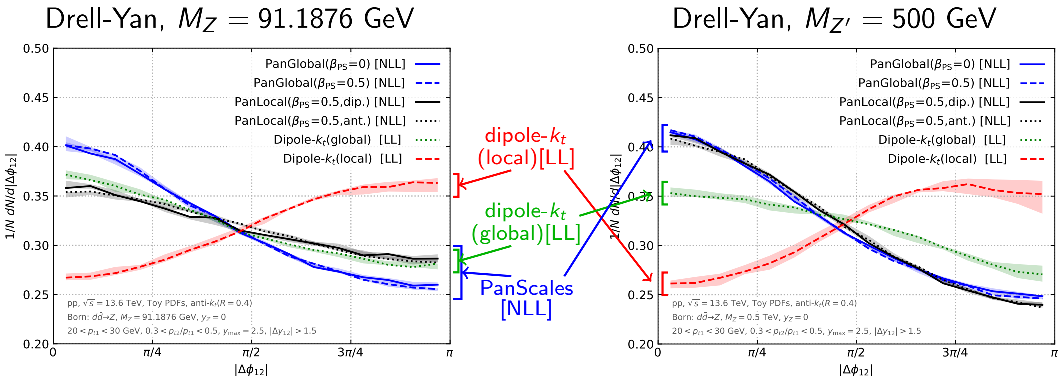
- at very small k_t for dipole-k_t (esp. w global recoil)
- NLL brings significant uncertainty reduction

Example #2: $\Delta\psi_{12}$



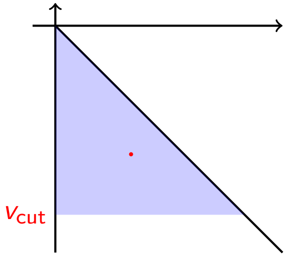
- Dipole-k_t with global recoil (LL) quite off
- All others [local dipole-k_t(LL) and PanScales(NLL)] similar

Example #2: $\Delta\psi_{12}$



- Dipole-k_t with global recoil (LL) quite off
- All others [local dipole-k_t(LL) and PanScales(NLL)] similar

- At higher scale: dipole-k_t(LL) $\neq$ PanScales(NLL)
- DANGER: false sense of control from lower-energy info!



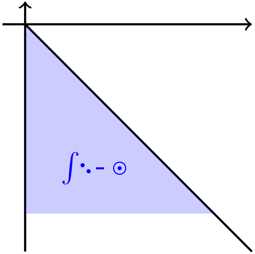
Soft-collinear:
$\mathcal{O}(\alpha_s L^2)$ + 1-ℓ α_s

In the soft-collinear approx

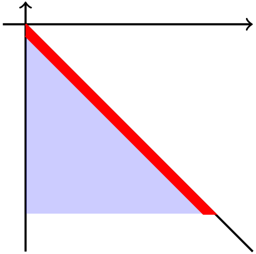
$$v_{\rm cut} \approx k_t e^{-\beta|\eta|}$$

(here $\beta = 0$)

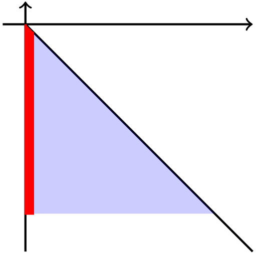
Log counting for NLL Event shapes



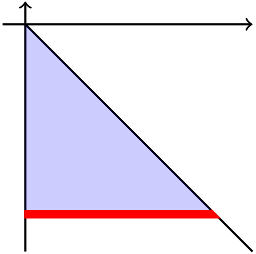
Soft-coll: 2-ℓ α_s + $\mathcal{O}(\alpha_s^2 L^2)$ R-V (CMW)



Hard collinear (virtual) (from $\alpha_s L$)

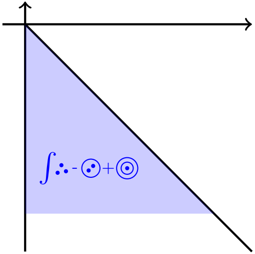


Soft large-angle (virtual)=0

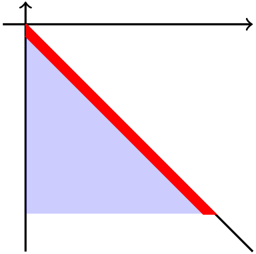


Multiple real emissions (from $\alpha_s^2 L^2$)

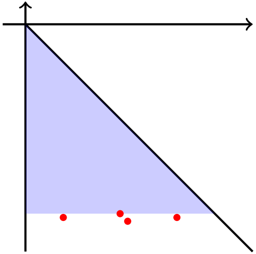
Log counting for NNLL Event shapes



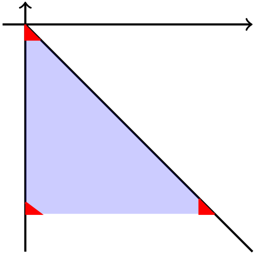
Soft-coll: 3-ℓ α_s + $\mathcal{O}(\alpha_s^3 L^2)$ R-V (CMW)



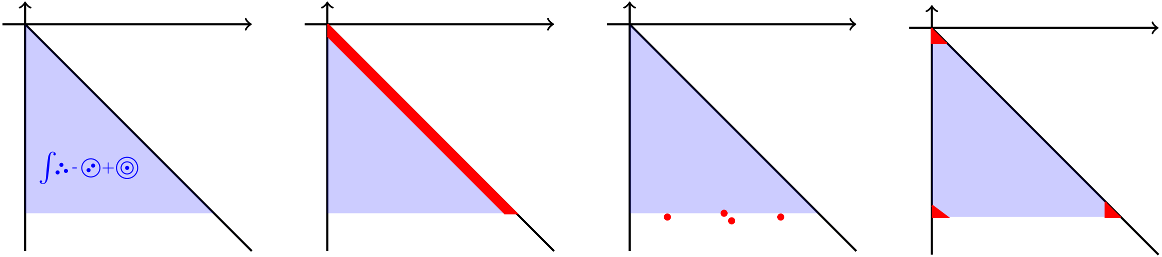
Hard collinear (virtual) $\mathcal{O}(\alpha_s^2 L)$ corrections



Multiple reals: $O(\alpha_s^2 L)$ double-soft



New $\mathcal{O}(\alpha_s)$ contributions



Soft-coll: 3-ℓ α_s + $\mathcal{O}(\alpha_s^3 L^2)$ R-V (CMW)

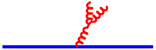
Hard collinear (virtual) $\mathcal{O}(\alpha_s^2 L)$ corrections

Multiple reals: $O(\alpha_s^2 L)$ double-soft

New $\mathcal{O}(\alpha_s)$ contributions

Freedon to reshuffle terms between different contributions

Example: double-soft k_1, k_2 emission

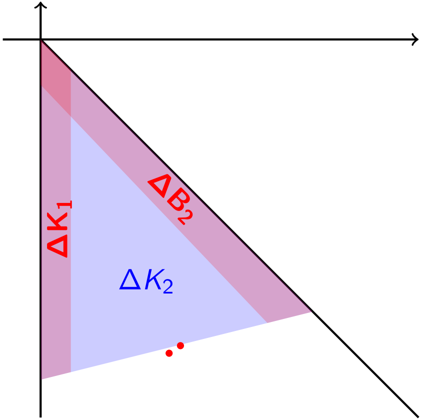


Typical approach:

- define a massless k_{1+2} with same $k_\perp$, η, ϕ as $k_1 + k_2$
- express the Sudakov using k_{1+2}
- treat $k_{1+2} \to k_1 + k_2$ as real double-soft correction

Shower Sudakov drifts

The shower does not take the same prescription:
- generate a first emission $\tilde{k}_1$
- generate a second branching $\tilde{k}_1 \to k_1, k_2$ (with correct k_1, k_2 matrix element)



NNLL shapes magic trick

NNLL: enoug to get (soft-coll) average drift between $\tilde{k}_1$ and k_{1+2} (in $k_\perp$ and y)!
$\longrightarrow$ defines ΔK_2, ΔK_1 and ΔB_2

Sumrules

For shapes, only $\int dy \Delta K_1$ ($\propto \langle y \rangle_{\text{drift}}$) matters

For exclusive observables (E in slice) full differential ΔK_1 needed $\Rightarrow$ powerful check

Same for triple-coll. region (not yet in PanScales)