UNITARITY VIOLATION IN THE LEPTON SECTOR

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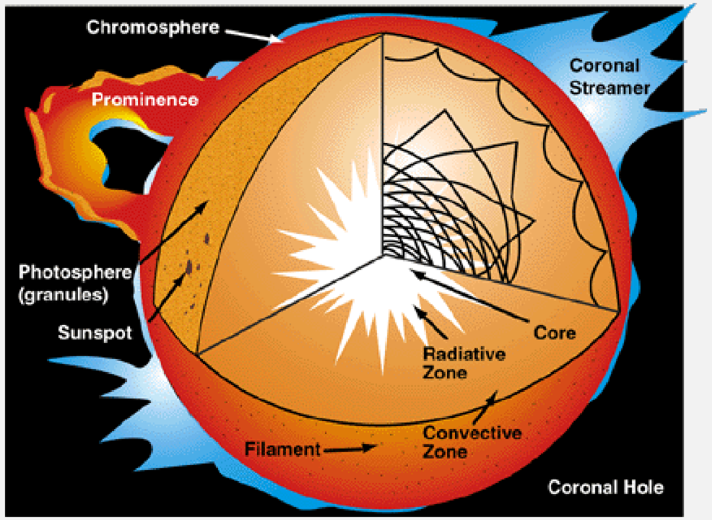
NEUTRINO OSCILLATIONS

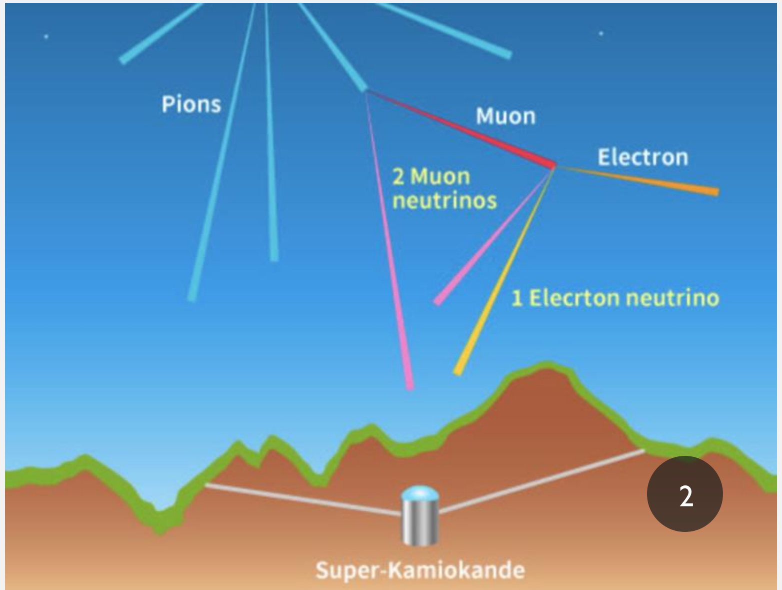
- Solar neutrino problem (70's)

Predicted νe flux from the solar standard model

Incompatibility

Measured νe flux from the Homestake experiment





- Similarly, atmospheric neutrino problem…

NEUTRINO OSCILLATIONS

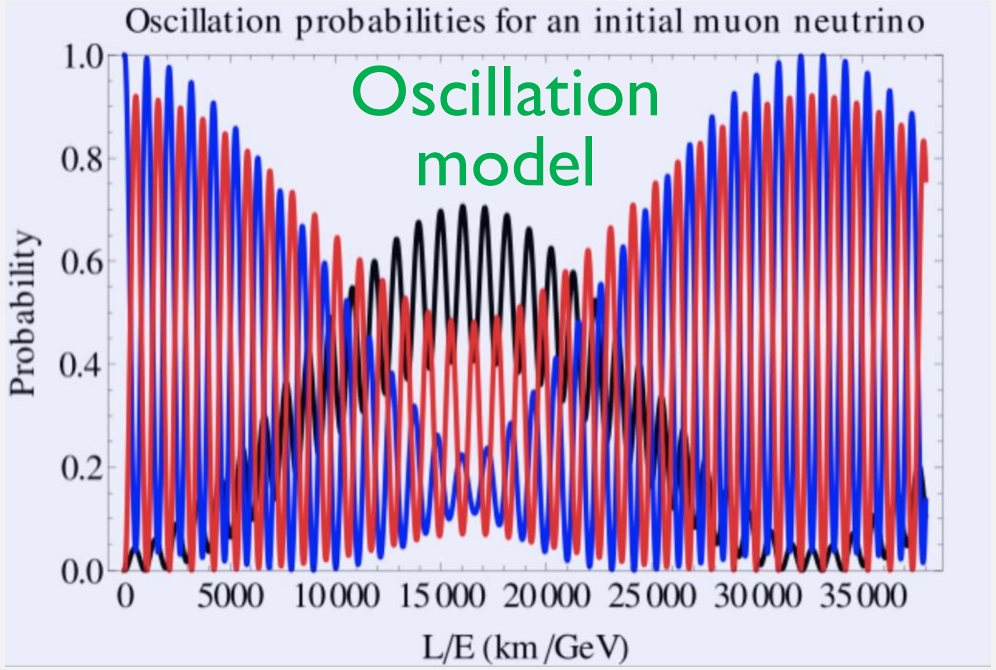
Solution: neutrinos can oscillate during propagation

- Mixing (flavour eig. ≠ mass eig.)

$$|\nu_\alpha\rangle = \sum_{i=1}^{3} U_{\alpha k}|\nu_k\rangle$$

- Schrödinger equation

$$|\nu_k(L)\rangle = e^{\frac{i m_k^2 L}{2E}}|\nu_k(L=0)\rangle$$



Free parameters:

θ_{12}, θ_{23}, θ_{13}, δ_{CP}, Δ(mij^2)

PMNS MATRIX
(STANDARD MIXING PARADIGM)

- PMNS matrix

$$U\alpha i = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix}$$

- 3 x 3 matrix because
  3 flavour eig. & 3 mass eig.

- The PMNS matrix is unitary by definition

$$U^\dagger U = UU^\dagger = I$$

UNITARITY CONDITIONS

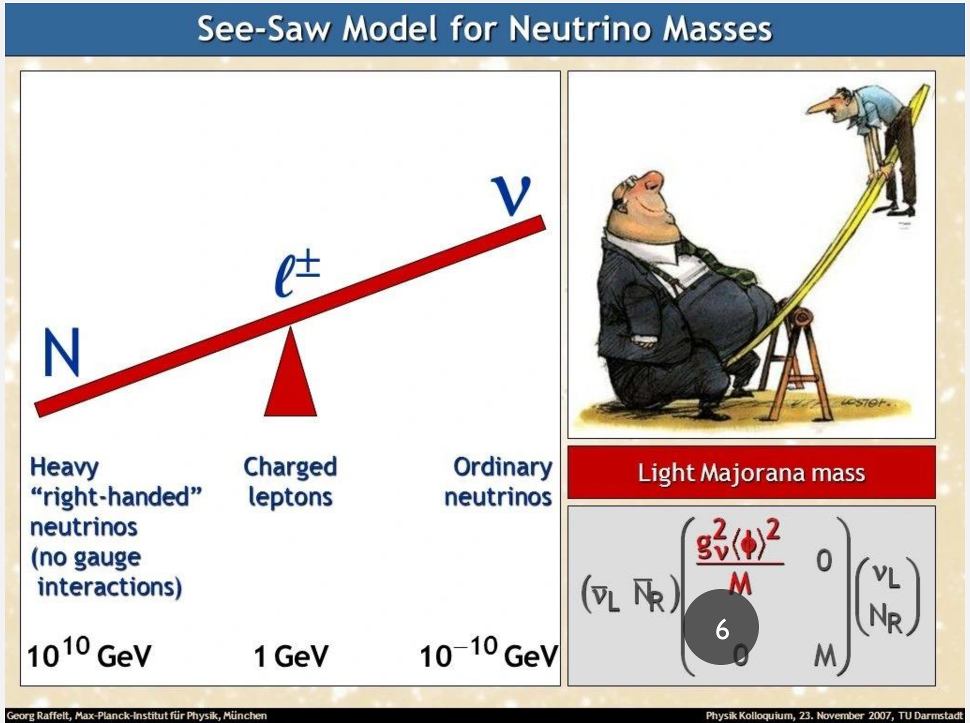
- Row normalisation: $\sum_{i=1}^{3} |U_{\alpha i}|^2 = 1$
- Row closure: $\sum_{i=1}^{3} U_{\alpha i}^* U_{\beta i} = 0 \quad (\alpha \neq \beta)$
- Column normalisation: $\sum_{\alpha=e,\mu,\tau} |U_{\alpha i}|^2 = 1$
- Column closure: $\sum_{\alpha=e,\mu,\tau} U_{\alpha i}^* U_{\alpha j} = 0 \quad (i \neq j)$

UNITARITY VIOLATION (MOTIVATION)

- Neutrino mass generation mechanism still unknown

- Only *for a few* number of mass generation models the lepton mixing matrix is unitary

- Unitarity violation non-measurable for some models (type1A seesaw), *but* measurable for others (different seesaw variations)…



DIFFERENT SCENARIOS

- Assume the Lepton Mixing Matrix U_{LMM} is a sub-matrix of a larger mixing matrix $\mathcal{U}$
- There are three scenarios:
  - ➢ $\dim(U_{LMM}) = \dim(\mathcal{U})$:
    - ❖ Standard (PMNS case): $U_{LMM} = \mathcal{U}$
  - ➢ $\dim(U_{LMM}) < \dim(\mathcal{U})$:
    - ❖ "Sub-matrix" case: $\mathcal{U}$ is unitary, U_{LMM} is not
    - ❖ "Agnostic" case: neither $\mathcal{U}$ nor U_{LMM} are unitary

UNITARITY TRIANGLES

- There are several ways to test the unitarity of the **Lepton Mixing Matrix**
- The most popular are the so called "**unitarity triangles**" (though not the most powerful)

Row closure condition: $\sum_{i=1}^{3} U_{\alpha i}^{*} U_{\beta i} = 0 \quad (\alpha \neq \beta)$

$$\Longrightarrow 1 + \frac{U_{\alpha i}^{*} U_{\beta i}}{U_{\alpha k}^{*} U_{\beta k}} = - \frac{U_{\alpha j}^{*} U_{\beta j}}{U_{\alpha k}^{*} U_{\beta k}}$$

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Complex number 1

Complex number 2

UNITARITY TRIANGLES

- By defining :

$$\begin{cases} (\rho_{\alpha\beta} + i\eta_{\alpha\beta})^{(i)} \equiv -\dfrac{U^*_{\alpha i} U_{\beta i}}{U^*_{\alpha k} U_{\beta k}} \\ (\rho^{'}_{\alpha\beta} + i\eta^{'}_{\alpha\beta})^{(i)} \equiv 1 + \dfrac{U^*_{\alpha j} U_{\beta j}}{U^*_{\alpha k} U_{\beta k}} \end{cases}$$

(Can be represented as the apex of a triangle of vertices (0, 0), (1, 0), (rho, eta))

- A possible unitarity test is to look if these two quantities coincide on the complex plane

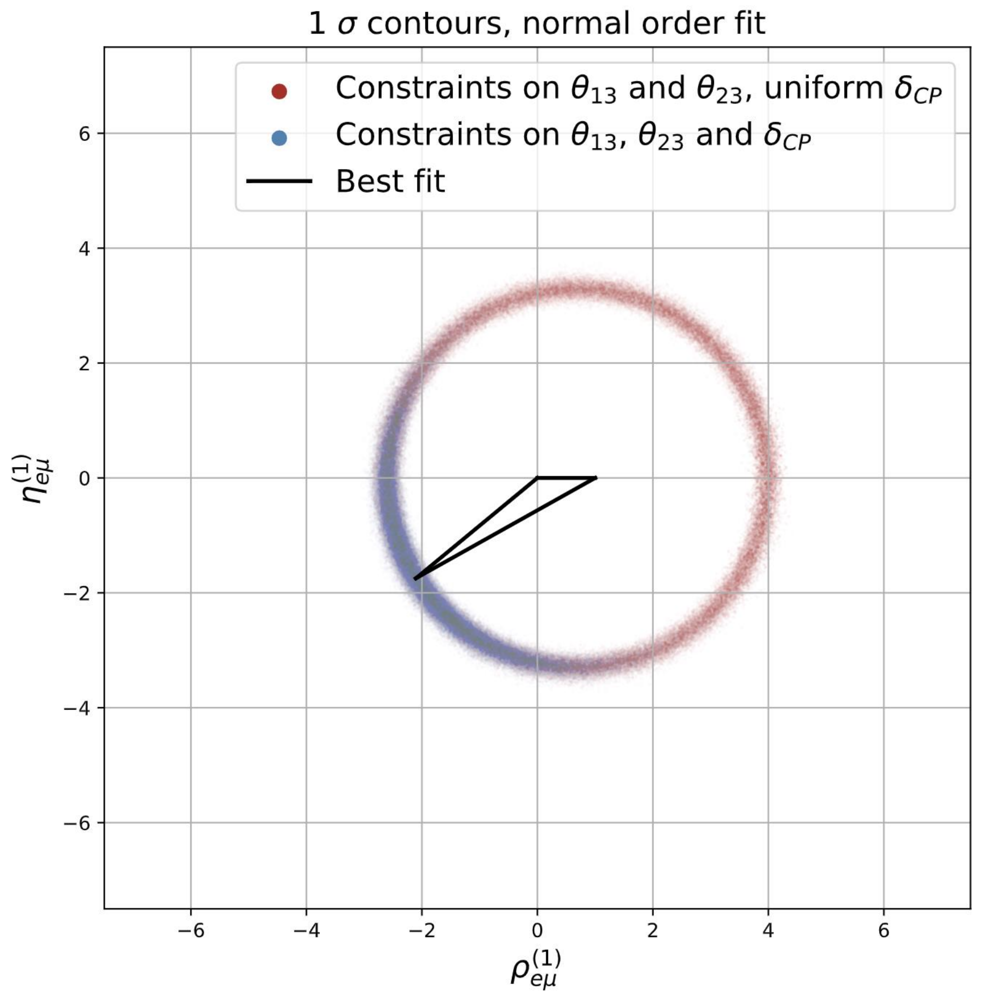
- Similarly, one can do this for the **column closure conditions**

- In total, 6 independent tests can be performed

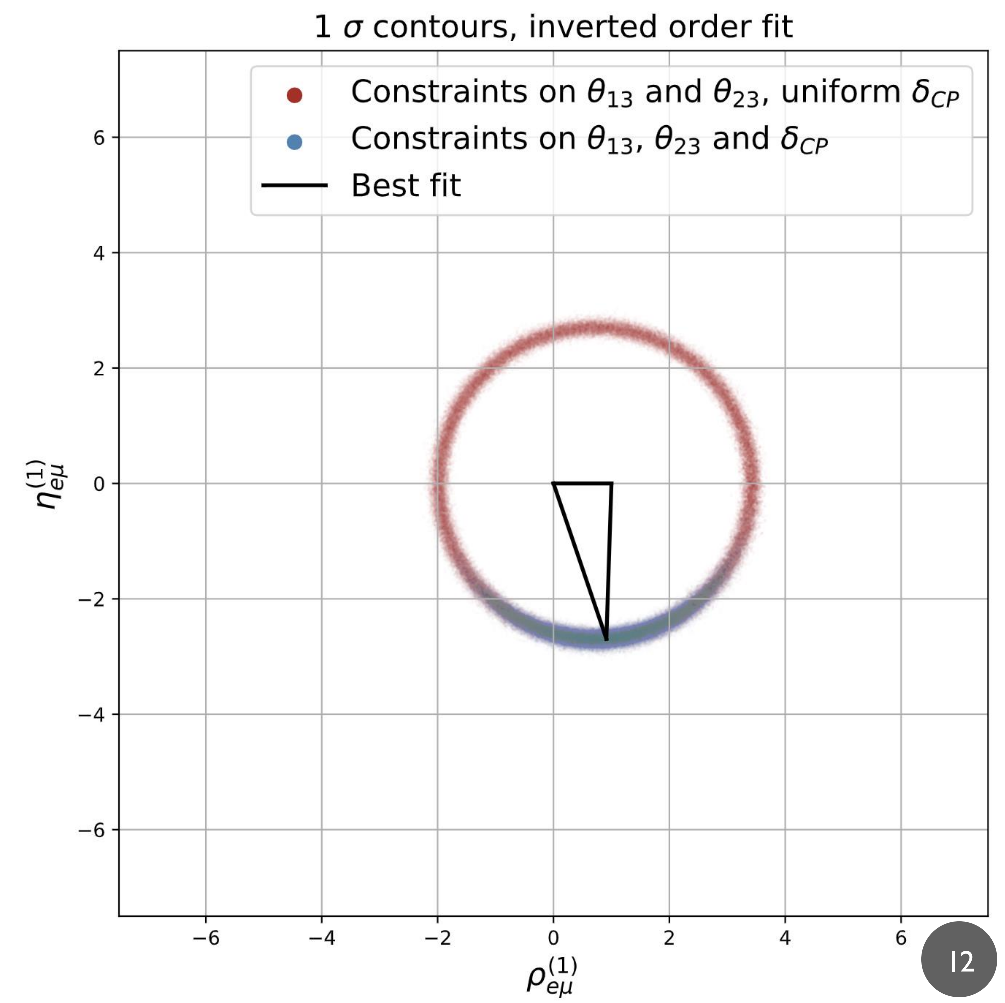
TOY MODEL

- We want to compute $$(\rho_{e\mu} + i\eta_{e\mu})^{(1)} \equiv -\frac{U_{e1}^* U_{\mu 1}}{U_{e3}^* U_{\mu 3}}$$
- We assume $U_{LMM} = \mathcal{U}$ (PMNS case)
- To approach T2K sensitivity, the parameters were chosen as:
  - $\boldsymbol{\theta}_{12}$ = bfv (from *nufit.org*)
  - $\boldsymbol{\theta}_{13}$ = bfv ± 1sigma (gaussian model)
  - $\boldsymbol{\theta}_{23}$ = bfv ± 1sigma (gaussian model)
  - $\boldsymbol{\delta}_{CP}$ = constant pdf (between $-\boldsymbol{\pi}$ and $\boldsymbol{\pi}$) **or** bfv ± 1sigma

RESULTS



1 σ contours, normal order fit
Constraints on θ13 and θ23, uniform δCP
Constraints on θ13, θ23 and δCP
Best fit
η(1)eμ
ρ(1)eμ



1 σ contours, inverted order fit
Constraints on θ13 and θ23, uniform δCP
Constraints on θ13, θ23 and δCP
Best fit
η(1)eμ
ρ(1)eμ

RESULTS - DISCUSSION

- This is the most common "test" which is presented in the litterature for the LMM
- BUT: This is actually not a true unitarity test because:

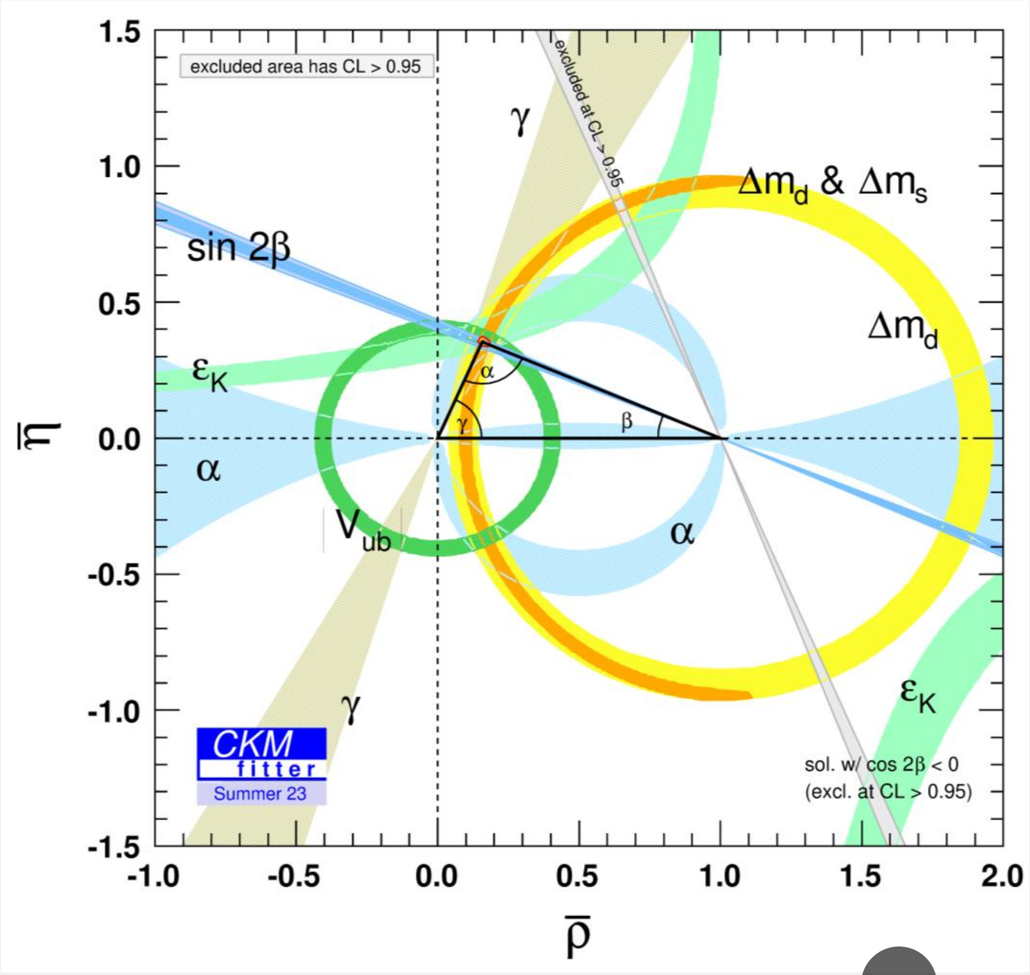
  - The constrained parameters are the mixing angles in the pmns picture, which assumes unitarity
  - The two quantities to compare are *by construction* the same in the pmns picture

$$(\rho_{e\mu} + i\eta_{e\mu})^{(1)} \equiv -\frac{U_{e1}^{*}U_{\mu 1}}{U_{e3}^{*}U_{\mu 3}} = 1 + \frac{U_{e2}^{*}U_{\mu 2}}{U_{e3}^{*}U_{\mu 3}} \equiv (\rho_{e\mu}^{'} + i\eta_{e\mu}^{'})^{(1)}$$

This is why only one of them is represented

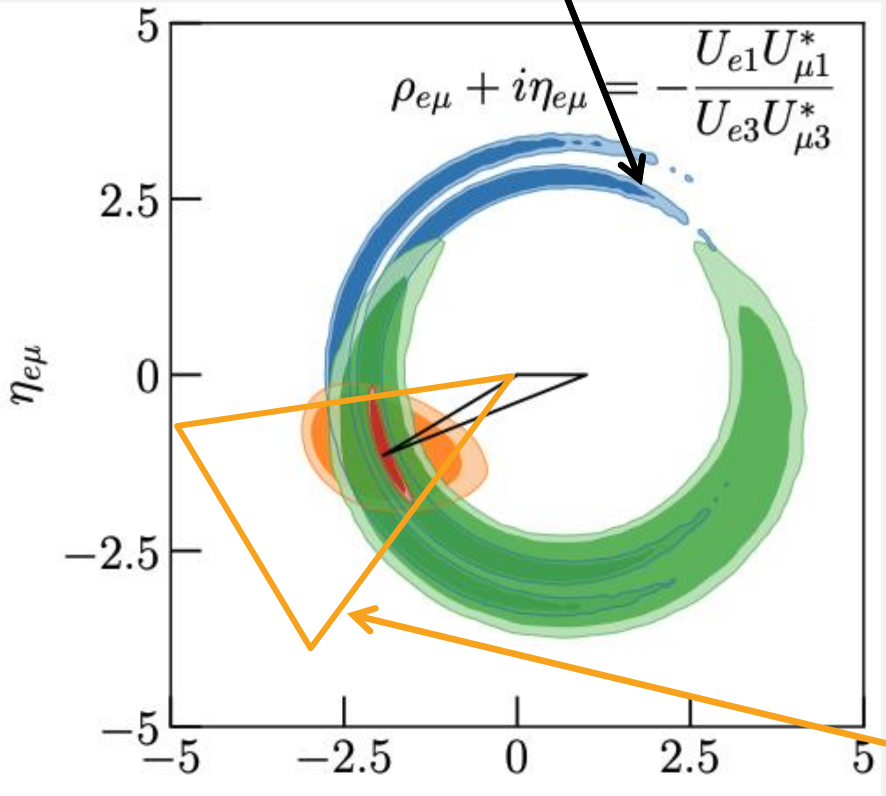
UNITARITY TRIANGLES IN FLAVOR PHYSICS

- Combination of multiple probes
  - e.g. V_{ub} measured from B to K transition makes a circular constraint
- Definition of triangle angles α β
  - Gamma: measurement via D → K^0_S makes a line constraint
- What combination of measurement is necessary in neutrino physics?

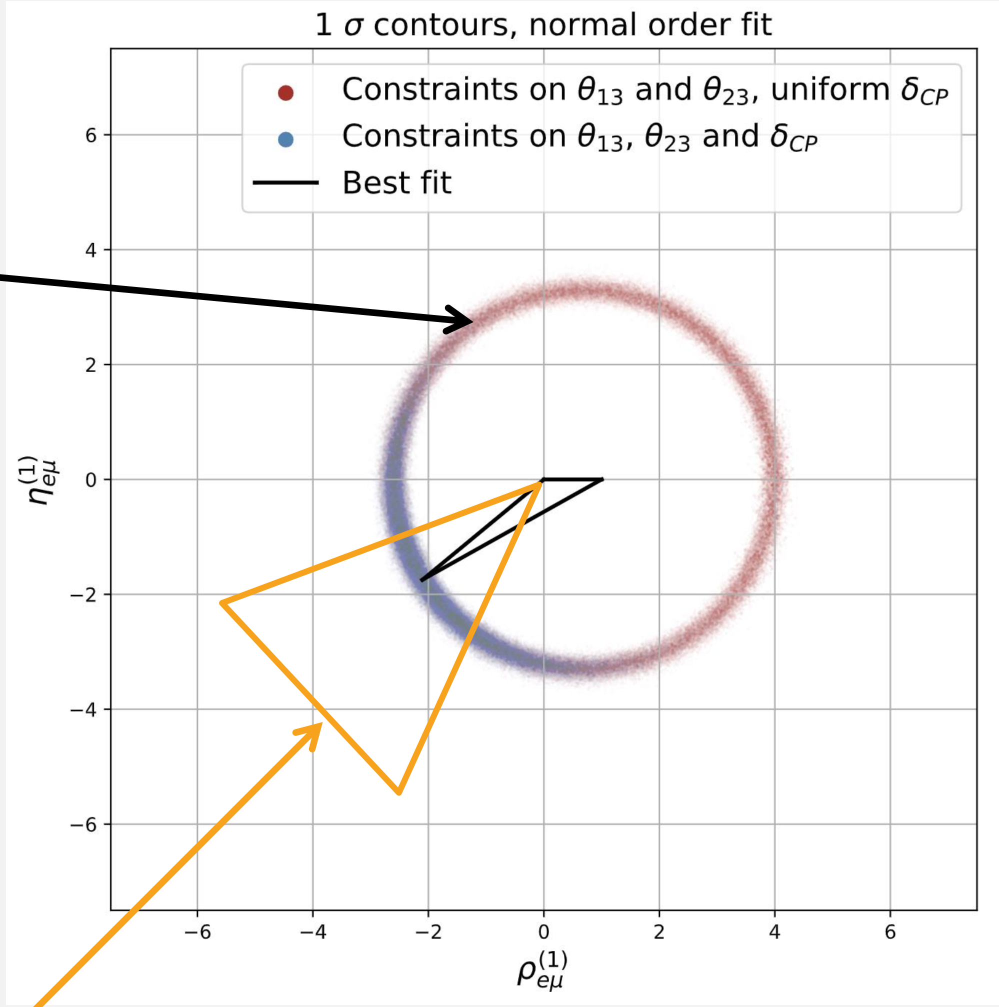


GOING BEYOND MIXING ANGLES

Disappearance (CP conserving constraints) seem to give a circle



https://arxiv.org/pdf/2004.13719



Appearance (CP violation constraints) seem to give a cone/line

DISAPPEARANCE

- <u>Reminder:</u>
  A circle is defined by a radius R and a center (x,y)
- Let's compute

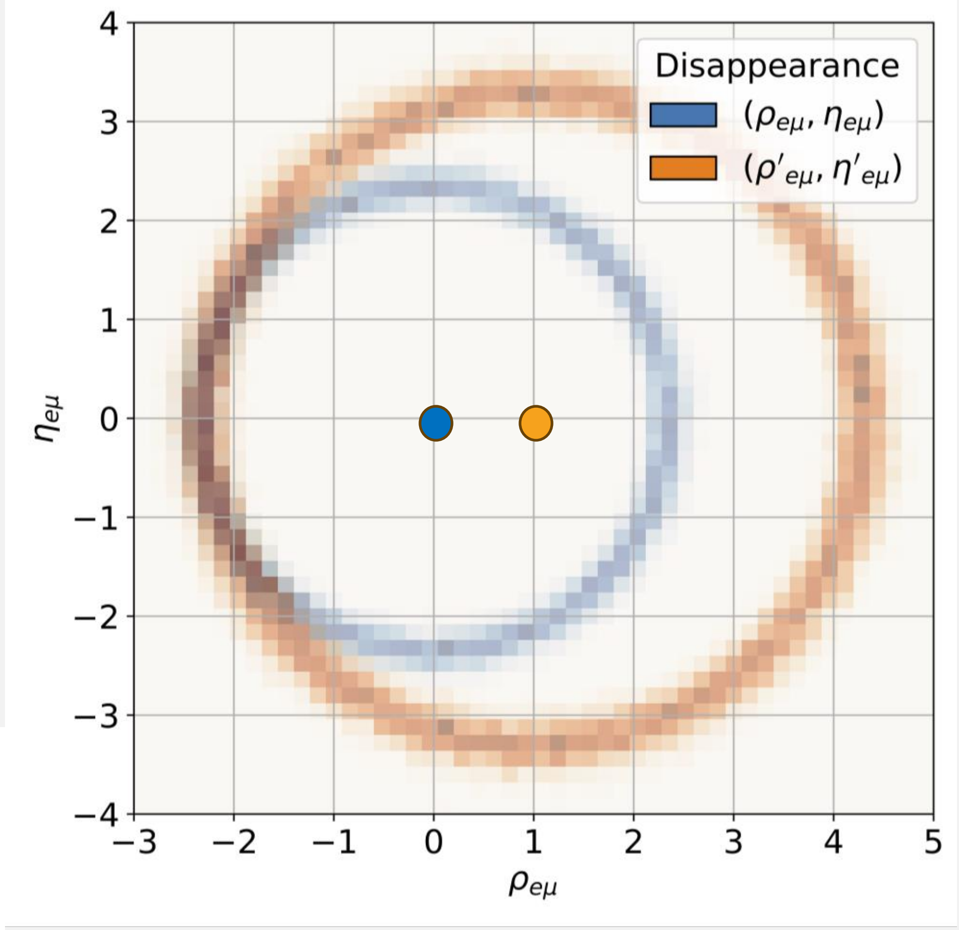
$$R^2 = \rho_{e\mu}^2 + \eta_{e\mu}^2 = \frac{|U_{\mu 1}|^2 |U_{\mu 2}|^2 |U_{e1}|^2 |U_{e2}|^2}{|U_{\mu 2}|^2 |U_{\mu 3}|^2 |U_{e2}|^2 |U_{e3}|^2} = \frac{A_{\mu,12} \boxed{A_{e,12}}}{A_{\mu,23} \boxed{A_{e,23}}}$$

$$R'^2 = (\rho'_{e\mu} - 1)^2 + \eta'^2_{e\mu} = \frac{|U_{\mu 1}|^2 |U_{\mu 2}|^2 |U_{e1}|^2 |U_{e2}|^2}{|U_{\mu 1}|^2 |U_{\mu 3}|^2 |U_{e1}|^2 |U_{e3}|^2} = \frac{A_{\mu,12} \boxed{A_{e,12}}}{A_{\mu,13} \boxed{A_{e,13}}}$$

- Requires multiple disappearance amplitude terms

$$P_{ee} = \left(\sum_k |U_{ek}|^2\right)^2 - \boxed{4|U_{e1}|^2|U_{e2}|^2} \sin^2(X_{12}) - \boxed{4|U_{e1}|^2|U_{e3}|^2} \sin^2(X_{13}) - \boxed{4|U_{e2}|^2|U_{e3}|^2} \sin^2(X_{23}),$$

$$P_{\mu\mu} = \left(\sum_k |U_{\mu k}|^2\right)^2 - 4|U_{\mu 1}|^2|U_{\mu 2}|^2 \sin^2(X_{12}) - 4|U_{\mu 1}|^2|U_{\mu 3}|^2 \sin^2(X_{13}) - 4|U_{\mu 2}|^2|U_{\mu 3}|^2 \sin^2(X_{23}),$$



APPEARANCE

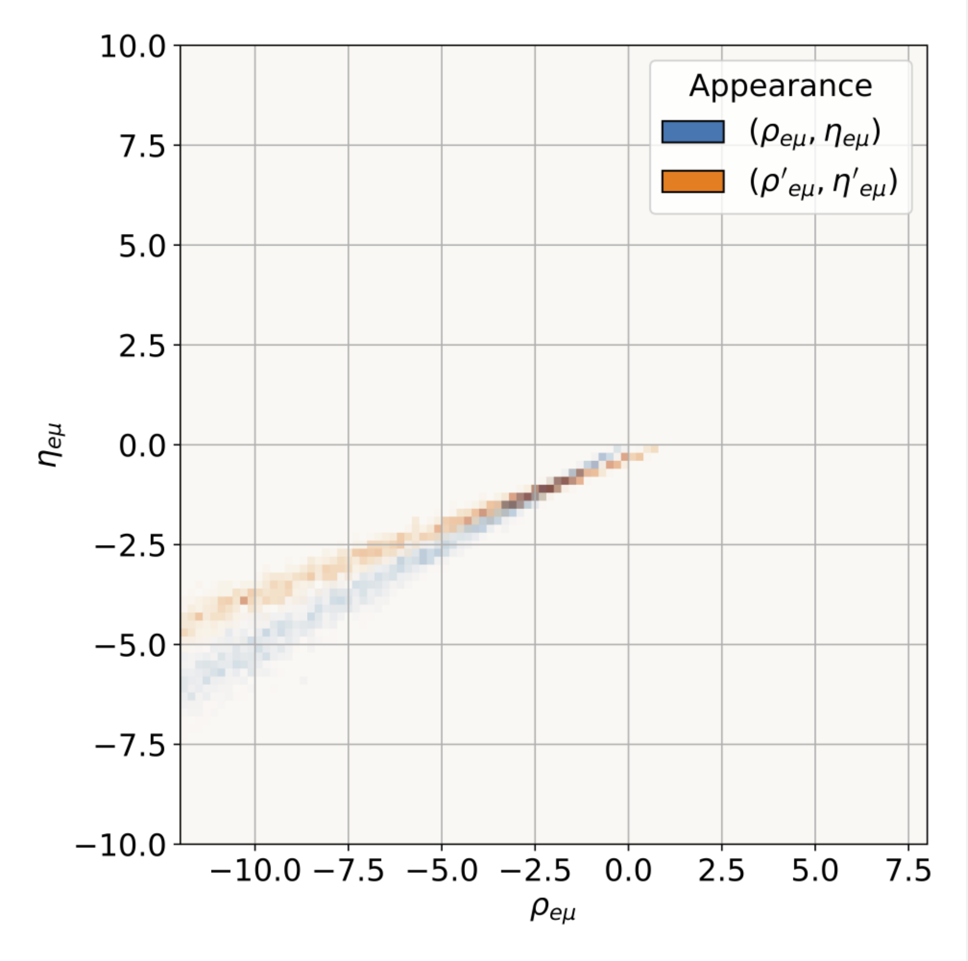
- <u>Reminder:</u>
  A linear function is defined with a slope a: y = a x
- What would a be?

$$\frac{\eta_{e\mu}}{\rho_{e\mu}} = \frac{\mathcal{I}m\left(-\frac{U_{e1}U^*_{\mu1}}{U_{e3}U^*_{\mu3}}\right)}{\mathcal{R}e\left(-\frac{U_{e1}U^*_{\mu1}}{U_{e3}U^*_{\mu3}}\right)} = \frac{\mathcal{I}m\left(\frac{U_{e1}U^*_{\mu1}U^*_{e3}U_{\mu3}}{U_{e3}U^*_{\mu3}U^*_{e3}U_{\mu3}}\right)}{\mathcal{R}e\left(\frac{U_{e1}U^*_{\mu1}U^*_{e3}U_{\mu3}}{U_{e3}U^*_{\mu3}U^*_{e3}U_{\mu3}}\right)} = \frac{\boxed{\mathcal{I}m\left(U_{e1}U^*_{\mu1}U^*_{e3}U_{\mu3}\right)}}{\boxed{\mathcal{R}e\left(U_{e1}U^*_{\mu1}U^*_{e3}U_{\mu3}\right)}},$$

$$= \frac{\sin(-\phi_{e3})}{\cos(-\phi_{e3})} = -\tan\phi_{e3},$$

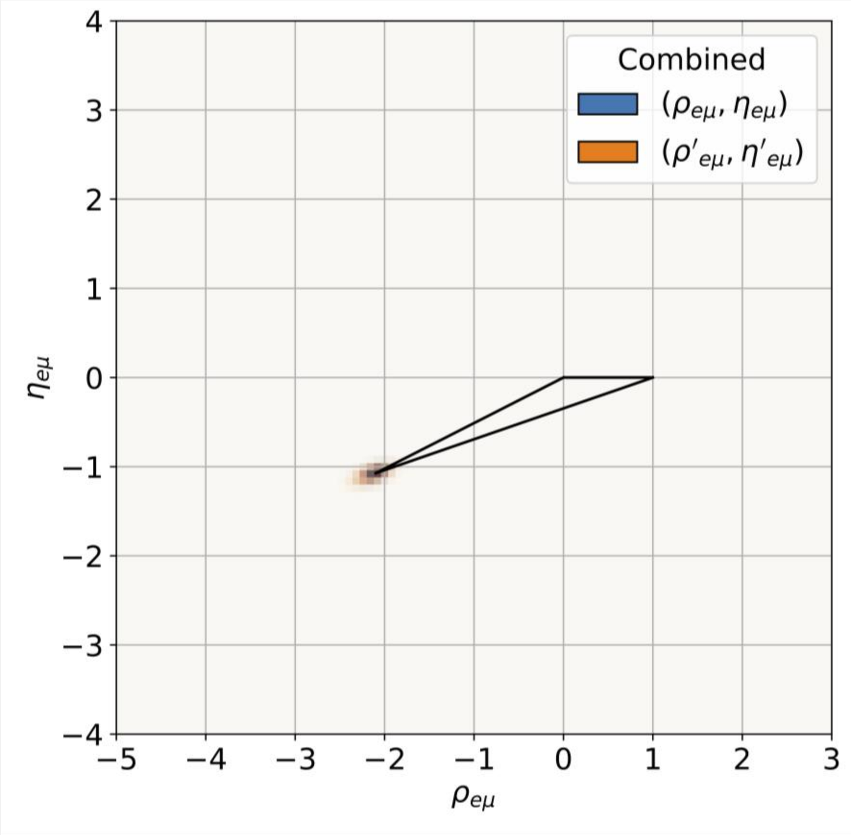
$$U \equiv \begin{pmatrix} |U_{e1}| & |U_{e2}|\, e^{i\phi_{e2}} & |U_{e3}|\, e^{i\phi_{e3}} \\ |U_{\mu1}| & |U_{\mu2}| & |U_{\mu3}| \\ |U_{\tau1}| & |U_{\tau2}|\, e^{i\phi_{\tau2}} & |U_{\tau3}|\, e^{i\phi_{\tau3}} \end{pmatrix},$$

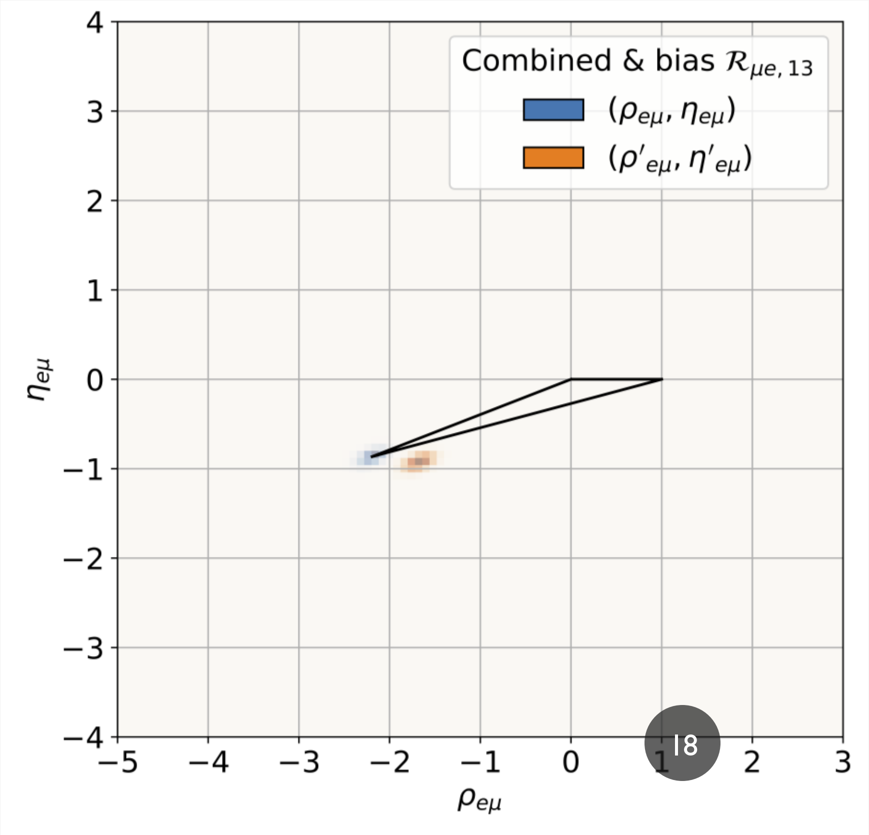
$$\begin{aligned} P_{\mu e} = \; & - 4\mathcal{R}e\left(U^*_{\mu1}U_{e1}U_{\mu2}U^*_{e2}\right)\sin^2(X_{12}) + 2\mathcal{I}m\left(U^*_{\mu1}U_{e1}U_{\mu2}U^*_{e2}\right)\sin(2X_{12}) \\ & - \boxed{4\mathcal{R}e\left(U^*_{\mu1}U_{e1}U_{\mu3}U^*_{e3}\right)}\sin^2(X_{13}) + \boxed{2\mathcal{I}m\left(U^*_{\mu1}U_{e1}U_{\mu3}U^*_{e3}\right)}\sin(2X_{13}) \\ & - 4\mathcal{R}e\left(U^*_{\mu2}U_{e2}U_{\mu3}U^*_{e3}\right)\sin^2(X_{23}) + 2\mathcal{I}m\left(U^*_{\mu2}U_{e2}U_{\mu3}U^*_{e3}\right)\sin(2X_{23})\,. \end{aligned}$$



PUTTING THINGS TOGETHER

- <u>Reminder:</u>
  The intersection of a circle with a line is 1 (or 2) points
- Testing the unitarity is making sure that the blue and orange dots are compatible !





BEYOND THIS PRELIMINARY WORK

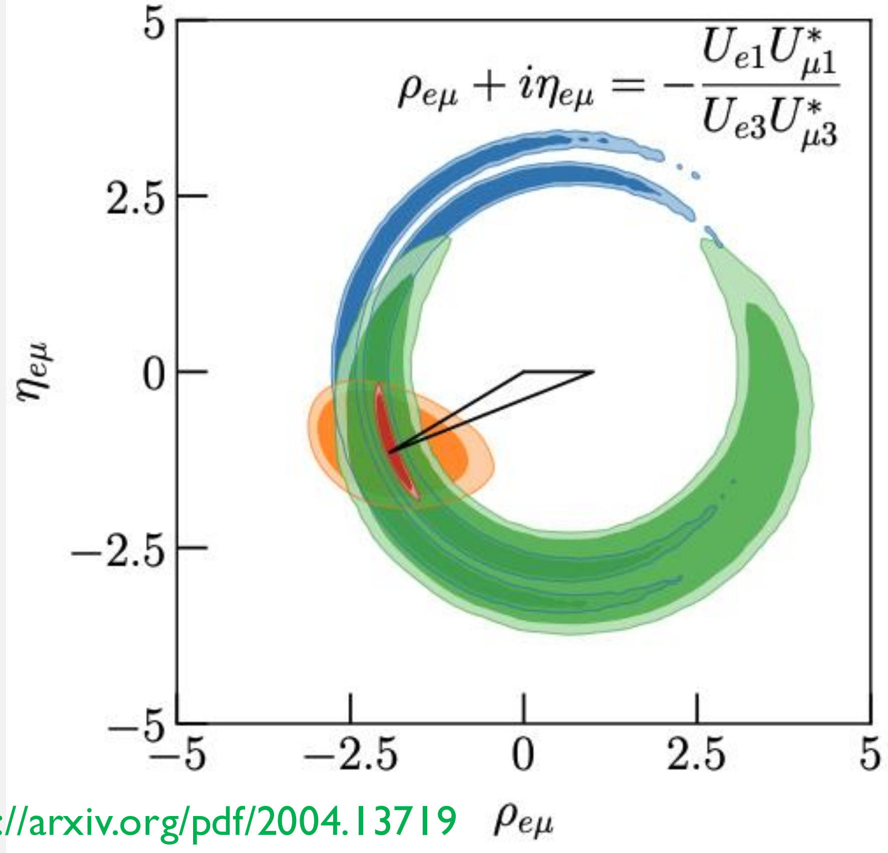
A tentative To-Do list

- Triangle defined by (e,mu) seems to be the most feasible in terms of constraints with reactor/solar/accelerator measurements
  - Is this statement true? Write the other unitarity constraints (row and column) in terms of the oscillation measurements
- What do the current/future experiments constrain?
  - Derive the current constraints from reactor/accelerator/solar from recent paper and update this/other triangle(s)
  - Compare with Ellis et al. paper and Lorenzo's plots where the unitarity is assumed
  - What would non-unitarity look like?
  - How to disentangle between the 13 and 23 amplitude measurements (12 mass splitting << 13 mass splitting)?
- Transition toward direct experimental constraint on the oscillation amplitudes
  - Implementation in P-Theta of the generic LMM oscillation probability model (never done but straightforward)
  - Direct constraint on the oscillation amplitudes (independent from reactor constraints)

BACKUP SLIDES

COMPATIBILITY WITH THE LITTERATURE

- The mass ordering has considerably less impact using better models (fits with correlations)
- Overall, the results of our (very) simple model do not seem to be far from the litterature



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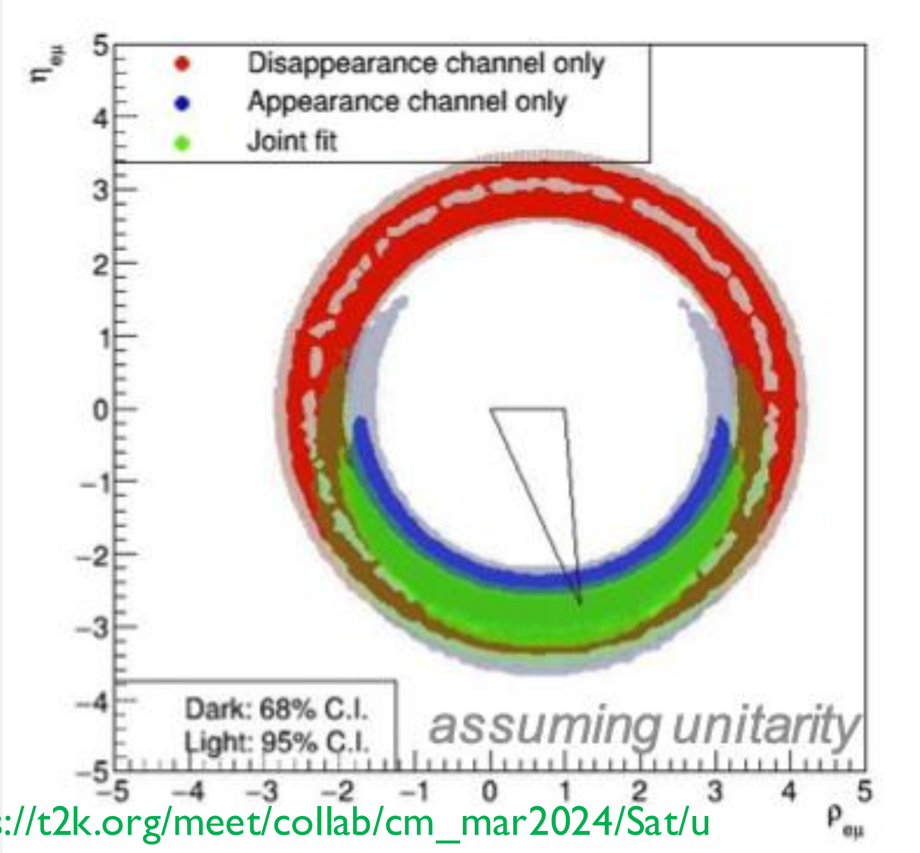

https://t2k.org/meet/collab/cm_mar2024/Sat/unitarity-triangles/view?searchterm=marvin+pfa

MAGNITUDES AND PHASES PARAMETRISATION

- This parametrisation of **U**Lmm applies to the agnostic case where no unitarity is assumed

$$U_{\mathrm{LMM}} \equiv \begin{pmatrix} |U_{e1}| & |U_{e2}|\, e^{i\phi_{e2}} & |U_{e3}|\, e^{i\phi_{e3}} \\ |U_{\mu 1}| & |U_{\mu 2}| & |U_{\mu 3}| \\ |U_{\tau 1}| & |U_{\tau 2}|\, e^{i\phi_{\tau 2}} & |U_{\tau 3}|\, e^{i\phi_{\tau 3}} \end{pmatrix}$$

- Results on slide 18:
  - The unitarity triangles only need the magnitudes of the matrix elements (and not the phases)
  - For the results shown here a 5% error on the magnitudes was arbitrarily chosen
  - This gives us an idea of what a true unitarity triangle test would look like **if we could directly measure these quantities**

MEASURED QUANTITIES

- The unitarity test that interests us requires the measurement of Uei and Umui for i = 1, 2, 3

Experiment	PMNS Quantity	LMM Quantity
Solar Neutral Current	1	$(\lvert U_{e1}\rvert^2 + \lvert U_{e2}\rvert^2)N_2^2 + \lvert U_{e3}\rvert^2 N_3^2$
Solar Charged Current	$\sin^2\theta_{12}\cos^4\theta_{13} + \sin^4\theta_{13}$	$\lvert U_{e2}\rvert^2 (\lvert U_{e1}\rvert^2 + \lvert U_{e2}\rvert^2) + \lvert U_{e3}\rvert^4$
KamLAND	$\cos^4\theta_{13}\sin^2(2\theta_{12})$	$4\lvert U_{e1}\rvert^2 \lvert U_{e2}\rvert^2$
Daya Bay	$\sin^2(2\theta_{13})$	$4\lvert U_{e3}\rvert^2 (\lvert U_{e1}\rvert^2 + \lvert U_{e2}\rvert^2)/N_e^2$
Sterile Neutrino $P_{\alpha\beta}$ $(\alpha \neq \beta)$	0	$\lvert t_{\alpha\beta}\rvert^2$
OPERA	$\cos^4\theta_{13}\sin^2(2\theta_{23})$	$4\lvert U_{\mu 3}\rvert^2 \lvert U_{\tau 3}\rvert^2 / N_\mu^2$
Long-baseline $P_{\mu e}$ (T2K, NOvA, DUNE, T2HK)	$\sin^2\theta_{23}\sin^2(2\theta_{13})$	$4\lvert U_{e3}\rvert^2 \lvert U_{\mu 3}\rvert^2 / N_\mu^2$
Long-baseline $P_{\mu\mu}$ (T2K, NOvA, DUNE, T2HK)	$4\cos^2\theta_{13}\sin^2\theta_{23}(1-\cos^2\theta_{13}\sin^2\theta_{23})$	$4\lvert U_{\mu 3}\rvert^2 (\lvert U_{\mu 1}\rvert^2 + \lvert U_{\mu 2}\rvert^2)/N_\mu^2$

T2K

Table 1. Quantities to which each experiment is sensitive: using the PMNS parameterization when unitarity is assumed (center column), using the MP parametrization when unitarity is not assumed (right column).