On the concept of Interaction in Quantum Mechanics

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INTRODUCTION

Everybody knows that Quantum Mechanics (QM) starts with

$$H=H_0+\mathbf{V}$$

But nobody will never tell you what **V** is !
This is a question I've recurrently heard, during … many years of scientific work

The simplest – and probably the truest - answer could be

« V is what we have to add to H_0 to describe the phenomenology(*) »

Notice that this requires to be able to solve « exactly »

$$H\,|\psi\rangle = E\,|\psi\rangle \qquad (1)$$

Otherwise, the required **V** will depend on the way we (dont) solve (1) and this game would be meaningless !
Thus, there is a close link between the topic of this lecture and the – difficult! - art of properly solving the QM equations: the so called Few-Body Physics

(*) By « phenomenology » I mean bound states of several particles and 2-body scattering experiments

In 2015 I gave in Japan a 26h Lectures trying to clarify this point.
This was for a community of « experts » in Nuclear and Particle Physics: PhD, Postdocs and seniors researchers.

I wil try to do the same in 3h with a group of master students.

The faillure is guaranteed…but I will try anyway :

- because it is a fundamental question

- for the pleasure to discuss with you !

I'm counting on all of you to make it (at least a bit !) fun !!!

I will follow the historical approach…and will arrive where I can !

I apologize in advance for not reaching the end !!!

I. About the interactions in Non Relativistic QM (1)

In Quantum – as well as in Classical – Mechanics, an interaction is :

“something” that prevents a plane wave to remain … a plane wave”

This is **true since Galileo**, and crossed several scientific revolutions without any distortion

In Classical Mechanics (CM) “something” is realized by a “Force”, a function F such that

$$dv/dt=F(x,t)$$

Not only it changes the plane waves but tells us “in which way”, “how much” it changes

In non relativistic Quantum Mechanics (NRQM) “something” is realized by an **operator V that does not commute with H_0** (and so it has no $|k\rangle$ as eigenvectors) : **the potential**

While $H_0 |k\rangle = c |k\rangle$ ($\langle x|k\rangle = e^{ikx}$)

One has $V |k\rangle \neq c |k\rangle$

and so $H |k\rangle = (H_0+V) |k\rangle = \sum_{k'} c_{k'} |k'\rangle \neq c |k\rangle$

Rm: Under H, the time-evolution of $|k\rangle$ is never « stationnary », that is $|k(t)\rangle \neq c |k(0)\rangle$
This is only true for eigenstates of H

About the interactions in Non Relativistic QM (2)

From where does V come from ?

QM started with the Hydrogen atom with **V** taken from Classical Mechanics, i.e. from a force between point-like charged particles (static)

$V(r) = - 1/r$ (no QM here, know since 1785 !!! …)

The results were fantastically successful in H, H_2^+, H_2, He… (also H^-, already in 1929…) …even if they could not be true !

The "guide" provided by **Classical Mechanics** disappeared quite fast due to the emergence of new phenomena, beyond atomic physics
- spin of particles S
- non electromagnetic interactions ("strong" with nuclei, "weak" with neutrinos…)

What to take as V(r) ????

Since [x,p]#0, any function of x makes the job !!!

So we can take it out of the hat

This is **absolutely legal**, provided one respects some space-time (translation, rotation, P, T) or internal symmetries (C, isospin)

About the interactions in Non Relativistic QM (3)

From this point of view, things interact …because they go into an inhomogeneous region of the space (if V was constant, nothing happens)

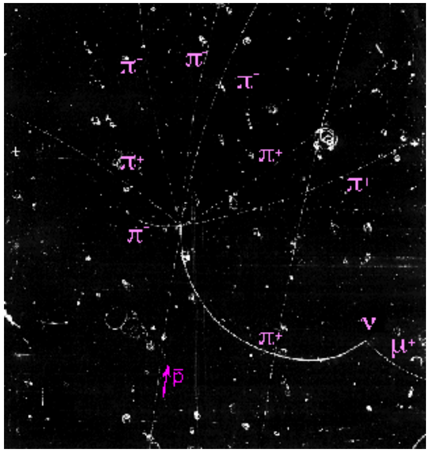
This “out of hat” approach, though leading remarkable success, **has not been very fertile** to describe the new physical phenomena, specially in the subatomic world

Transforming matter into light $e^+ + e^- \longrightarrow \gamma + \gamma$

Transforming “speed’ into matter $p + p \longrightarrow p + p + (p + \bar{p}) + (n + \bar{n}) \longrightarrow X + d + \bar{d}$

or the contrary…

$$p + \bar{p} \longrightarrow \pi^+ + \pi^- + \pi^+ + \pi^- + \pi^+ + \pi^-$$



A new formalism must be found !!!

About the interactions in Non Relativistic QM (4)

According to Dirac[(*)], **"the problem of finding the right H comes from our lack of imagination to quit the classical world".**
He did in 1928 with his equation, incorporating Lorentz invariance (relativity) in QM, describing the electron spin, its coupling to electromagnetic field …and predicting antimatter! .

He "simply" wrote H - "out of his hat" - as a 4x4 matrix $H = \vec{\alpha} \cdot \vec{p} + \beta m$

Setting $\gamma^\mu = \begin{pmatrix} \gamma^0 = \beta \\ \vec{\gamma} = \beta\vec{\alpha} \end{pmatrix} \quad \partial_\mu = \begin{pmatrix} \partial_t \\ \vec{\nabla} \end{pmatrix}$

one has $i\hbar\partial_t\Psi = H\Psi \iff (i\gamma^\mu\partial_\mu - m)\Psi = 0$

NB. The **above sentence** was written in 1982, as a kind of testament (+1984), and concerned even the progress of Quantum Field Theory !
Unfortunately there are no so many Dirac's in the history….

Even worst, this way of doing does not provide any "way of thinking", i.e. **a philosophy of natural elementary processes**…which is our secret hope !

(*) Requirements of a fundamental Theory, P.A.M. Dirac, Eur. Journ. of Phys. 5 (1984) 65

II. Interactions in Quantum Field Theory

(1)

The first « physical description » of an interaction appeared with the advent of Quantum Field Theory (QFT) : Heisenberg, Born, Jordan (1926), Dirac (1927)…

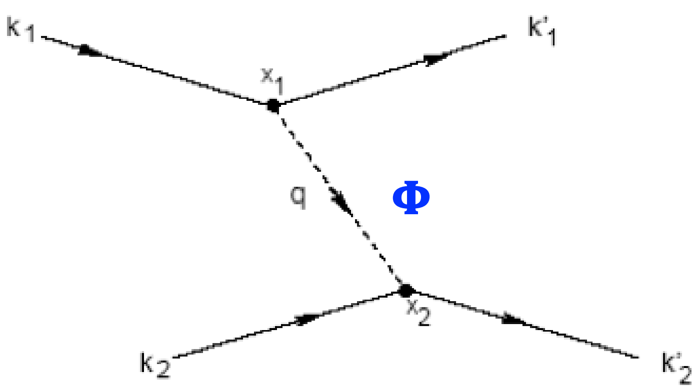
Fields are space-time operators written as superposition of creation and annihilation free-particle states operators

$$\Psi(x) = \sum_{\vec{k}} N_k \left\{ a_k e^{-ikx} + a_k^\dagger e^{+ikx} \right\} = \sum_{\vec{k}} N_k \left\{ a_k e^{i(\vec{k}\vec{x}-\omega t)} + a_k^\dagger e^{-i(\vec{k}\vec{x}-\omega t)} \right\}$$

acting on a « vacuum »

$$a_k^\dagger \; |0> \; = \; |k>$$
$$a_k \; |k> \; = \; |0>$$

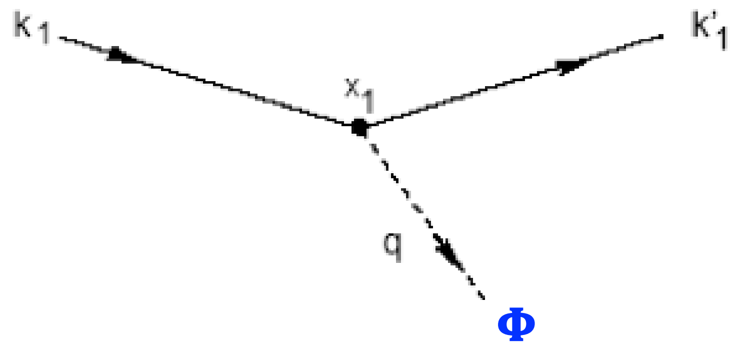
Interaction between particles of field **Ψ** **results from « exchange » of** field **Φ** particle



A « mediator » is required !!!

II. Interactions in Quantum Field Theory

The basic piece of this lego **(vertex x_1)** is **"Destruction-Création-Création"**

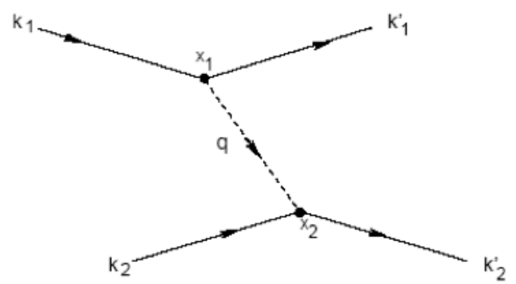


This is realized by the « elementary process »…. contained in the field product

$$a^\dagger_{k'_1}\ b^\dagger_q\ a_{k_1} \quad \rightarrow \quad a^\dagger_{k'_1} e^{-ik'_1 x_1}\ b^\dagger_q e^{iqx_1}\ a_{k_1} e^{ik_1 x_1} \quad \rightarrow \quad \Psi(x_1)\Phi(x_1)\Psi(x_1)$$

And the same for vertex x_2. Setting all together :
Particle $|k_1>$ emits a quanta $|q>$ (vertex x_1) which is absorbed by $|k_2>$ (vertex x_2)
Their initial states have been modified in this process : **they have interacted !**

$k'_1=k_1-q$
$k'_2=k_2+q$



QFT allows to compute the probability amplitude of this process : $A(k_1,k_2 \rightarrow k'_1,k'_2)$

Interactions in Quantum Field Theory

The « basic lego » (supplied with a « strength » or «coupling constant »)

$$g\, \Psi(x)\, \Phi(x)\, \Psi(x)$$

represents the interaction (or coupling) among fields Ψ and Φ

This defines the interaction « Lagrangian » or « Hamiltonian » densities :

$$\mathcal{L}_I(x) = g\, \Psi(x)\, \Phi(x)\, \Psi(x)$$
$$\mathcal{H}_I(x) = -g\, \Psi(x)\, \Phi(x)\, \Psi(x)$$

Since $\mathcal{H}_I(x)$ is a scalar quantity, the way in wich fields Ψ and Φ are coupled depends on their space-time structure, i.e. their behaviour with respect to Poincaré Group (*) **: mainly rotation (J) and parity (P)**

Scalars	$J^P=0^+$	$\Phi(x)$	σ, H
Pseudoscalars	$J^P=0^-$	$\varphi(x)$	π
Vectors (4d space-time)	$J^P=1^-$	$A^\mu(x)$ $\mu=0,1,2,3$	γ, g
Pseudovectors (4d)	$J^P=1^+$	$d_\mu\varphi(x)$	
Dirac spinors (4d # space-time)	$J^P=1/2$	$\Psi_\alpha(x)$ $\alpha=1,2,3,4$	e, q

(*) Lorentz + Rotation + Translation + R,P,T

Interactions in Quantum Field Theory

Fermion fields (s=1/2, as e,p,..) are represented by Dirac spinors

$$\Psi_k(x) = u(k)e^{-ikx} \quad u(k) = \begin{bmatrix} u_1(k) \\ u_2(k) \\ u_3(k) \\ u_4(k) \end{bmatrix} \qquad \bar{u}(k) = [u_1^*(k), u_2^*(k), -u_3^*(k), u_4^*(k)]$$

The most popular couplings among fermions are named according to **exchanged particle**

Scalar $$\mathcal{L}_s(x) = -g_s\, \bar{\Psi}(x)\, \Phi(x)\, \Psi(x)$$

Pseudoscalar $$\mathcal{L}_{ps}(x) = -g_{ps}\, \bar{\Psi}(x)\, i\gamma_5\, \varphi(x)\, \Psi(x)$$

alternatively $$\mathcal{L}_{pv}(x) = \frac{f_{pv}}{m}\, \bar{\Psi}(x)\, \gamma^\mu \gamma_5\, \partial_\mu \varphi(x)\, \Psi(x)$$

Vector $$\mathcal{L}_v(x) = -g_v \bar{\Psi}(x)\, \gamma_\mu A_\mu(x)\, \Psi(x) + \frac{g_t}{2M} \bar{\Psi}(x)\, \sigma^{\mu\nu} F_{\mu\nu}(x)\, \Psi(x)$$

where γ_μ= 4(4x4 matrices) and

$$\begin{aligned} F_{\mu\nu}(x) &= \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) \\ \sigma^{\mu\nu} &= \frac{i}{2}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu) \end{aligned}$$

At this level, the Field interactions are – as V in NRQM – « guessed »
« GAME » : built scalars out of the theoretical objects we have at our disposal.

Latter on, one found other « dogmas » (Gauge Invariance) to discard many of them ….

Gauge Invariance as a « universal recipe » to obtain the physical theories

(Illustrative exemple from NRQM)

The free Schrodinger equation

$$i\hbar\partial_t\Psi(x,t) = \frac{p^2}{2m}\Psi(x,t) \qquad p = \frac{\hbar}{i}\nabla$$

is invariant with respect to the transformation $\Psi \to \Psi' = e^{i\theta}\Psi(x,t)$ ***θ***=constant

However if the « gauge angle » ***θ*** depend on (x,t) (« local gauge »)

$$\Psi'(x,t) = e^{i\theta(x,t)}\Psi(x,t)$$

these invariance is lost :

$$i\hbar\partial_t\Psi = \frac{p^2}{2m}\Psi \qquad \Longleftrightarrow \qquad i\hbar\partial_t\Psi' = \left[\frac{(\vec{p}-\hbar\vec{\nabla}\theta)^2}{2m} - \hbar\partial_t\theta\right]\Psi'$$

Inspired by this result one can show that by introducing two interacting fields $(\phi, \vec{A})$

$$i\hbar\partial_t\Psi = \left[\frac{(\vec{p}-q\vec{A})^2}{2m} + q\phi\right]\Psi$$

the local invariance of the equation is recovered …provided that in this transformation

$\Psi' = e^{i\frac{q}{\hbar}\Lambda}\Psi$ these fields transform as
$$\begin{aligned} \phi' &= \phi - \partial_t\Lambda \\ \vec{A}' &= \vec{A} + \nabla\Lambda \end{aligned}$$

The interaction between particle arises out of a local gauge symmetry group U(1) !!!
Better than out of a hat !

These astonishing result, here obtained in the framework of NRQM, is valid in QFT.
Different theories differ mainly by their gauge group !

- QED obtained by imposing to the free Dirac equation the U(1) group (10)

$$\mathcal{L}_D = \bar{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi \qquad \Longrightarrow \qquad \mathcal{L}_D = \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi \qquad D_\mu = \partial_\mu + iqA_\mu \tag{13}$$

- QCD obtained by imposing to the free Dirac equation for quarks (6 flavors $s = \{u, d, c, s, t, b\}$ and 3 colors c each)

$$\mathcal{L}_Q = \sum_{s=1}^{6} \bar{\Psi}_s(i\gamma^\mu \partial_\mu - m)\Psi_s = \sum_{s=1} (\bar{\Psi}_s^r, \bar{\Psi}_s^b, \Psi_s^g) \begin{pmatrix} i\gamma^\mu \partial_\mu - m_s & & \\ & i\gamma^\mu \partial_\mu - m_s & \\ & & i\gamma^\mu \partial_\mu - m_s \end{pmatrix} \begin{pmatrix} \Psi_s^r \\ \Psi_s^b \\ \Psi_g^v \end{pmatrix} \tag{14}$$

a SU(3) color group

$$\Psi_s' = U(\theta)\Psi_s \qquad U(\theta) = e^{-i\frac{g}{2}\,\theta\cdot\lambda} \in SU(3) \qquad \theta\cdot\lambda = \sum_{c=1,8} \theta_c \lambda_c$$

where $\theta = \{\theta_1, \ldots, \theta_8\}$ 8 real paramèters and λ_α Gell-Man matrices

In order to make (14) invariant under the SU(3) transformation one should simply replace

$$\partial_\mu \to D_\mu + igA_\mu \qquad A_\mu(x) = \sum_{c=1}^{8} \frac{1}{2}\lambda_c A_\mu^c$$

which reads

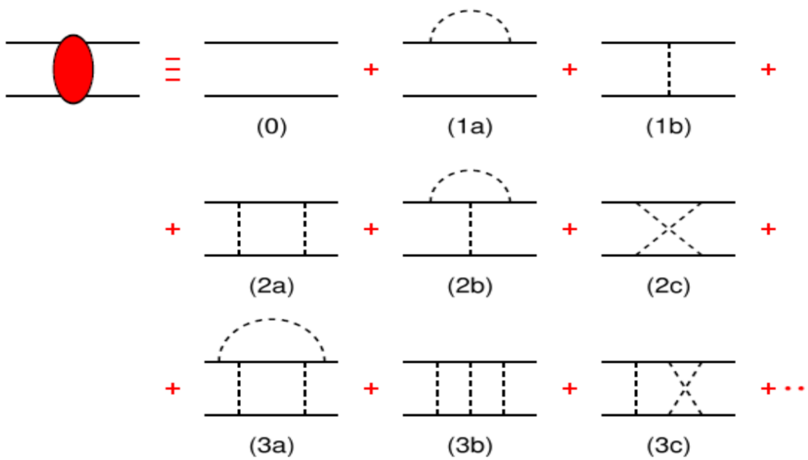
$$\mathcal{L}_Q = \sum_{s=1}^{6} \bar{\Psi}_s(i\gamma^\mu D_\mu - m)\Psi_s$$

Interactions in Quantum Field Theory

- QFT provides a « natural » framework to understand what « interaction » means

- This requires a « particle exchange » (vector, mediator)
  It « propagates on space-time » according to the QM rules

- The interaction process we have considered is the simplest that one can imagine
  There is an infinity of processes leading to « changing k's »



They are ordered by increasing powers of g (perturbative expansion)
If g is small it can be fine, if g is large … what one neglects is not negligeable !
The interaction is the sum of all of them (red bulk)….but **nobody knows how to compute it !**

And where is the « potential » in all that ????

Coulomb potential from QED

QFT was developped in the framework of Electrodynamics (QED), i.e. electromagnetic interaction between charged particles.
In this framework the interaction is produced by exchange of photons **$A^\mu(x)$**

Dirac fermion fields **$\Psi(x)$** (e,p,..) were coupled to **$A^\mu(x)$** by the (classically inspired) term

$$\mathcal{L}_I(x) = -\, j_\mu(x) A^\mu(x) = -q\ \bar{\Psi}(x)\gamma_\mu\Psi(x)\ A^\mu = [.,.,.,.] \begin{bmatrix} . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \end{bmatrix} \begin{bmatrix} . \\ . \\ . \\ . \end{bmatrix}$$

γ_μ= 4(4x4 matrices)

After some (sever) approximations the « scattering amplitude » in CM system $k_1=-k_2=k$ reads

$$A(k_1,k_2 \to k_1',k_2') = A(k,k') \approx -\frac{1}{2\pi^2}\frac{\alpha(\hbar c)}{\vec{q}^{\,2}} \equiv V(\vec{k},\vec{k}')$$

By Fourier transform one gets the usual Coulomb potential $V_C(r) = -\dfrac{\alpha(\hbar c)}{r}$

Potential in k-space ≡ Leading term of the Scattering Amplitude (Born)

First QFT application, and first achievement – and consistency test - of QED !!!

QFT provides not only a « philosophy of the interaction » but a way to obtain « potentials »

Coulomb potential from QED

(2)

Main approximations are $\bar{u}(k')u(k) \approx 1$ $\qquad q^2 = q_0^2 - \vec{q}^{\,2} \approx -\vec{q}^{\,2}$

If one keep the full spinor structure u(k) and the q_0-dependence, the results badly differ from Coulomb. The Resulting V is :

- Non local
- L,S,J, and E-dependence
- Singular ($1/r^3$) terms appear which are not integrable in Schrodinger (or other) equations.
- Spin-Spin and Spin-Orbit (LS) terms…

However by treating that « perturbatively » over the non-relativistic 1/r solution, one obtains the right corrections up to very high accuracy (α^2, α^4 …)

GREAT SUCCES (*) !

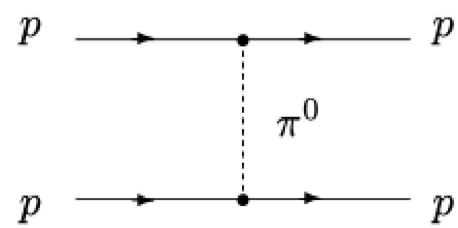
Recipe was immediately applied to other interactions …

1934 Fermi **Weak Interactions (without mediator, « contact theory »)**
1935 Yukawa **Strong Interactions**

(*) Notice that all this formalism is limited to elementary (pointlike) particles !!!

Yukawa Theory of Nuclear Interactions (1935)

In analogy with QED, Yukawa postulated for Nuclear forces



$$\mathcal{L}_Y(x) = g\, \Psi(x)\, i\, \gamma_5\, \Phi(x)\, \Psi(x)$$

$\Psi(x)$ = Nucleons (p,n)

$\Phi(x)$ = A new particle as mediator, the pion π, PS « meson » with $m_\pi \backsim 100$ MeV

The resulting potential (« one-pion exchange » OPEP) was quite peculiar :

$$V_\pi(x) = \frac{m_\pi}{3}\frac{g^2}{4\pi}\left(\frac{m_\pi}{2M}\right)^2 \{[Y(x) - 4\pi\delta(\vec{x})]\,(\vec{\sigma}_1\cdot\vec{\sigma}_2) + \chi_T(x)Y(x)S_{12}(\hat{r})\}\,(\vec{\tau}_1\cdot\vec{\tau}_2) \qquad x = m_\pi r$$

$$Y(x) = \frac{e^{-x}}{x} \qquad \chi_T(x) = 1 + \frac{3}{x} + \frac{3}{x^2}$$

- V depend on L,S,J (Spin-spin term,…), while Coulomb V is the same for all LSJ !
- Range of Nuclear interaction given by **$r_0=1/m_\pi=1.4$ fm**
- Non conserving L: S_{12} a tensor operator acting on S=1 states, mixing L waves (SD,PF,…) responsable for the deuteron (**d=p-n**) quadupole moment (deformation)
- Non integrable singularities at r=0 !
- Huge value of the coupling constant **$\alpha_s=g^2/4\pi=14$ (compared to 1/137 in QED !)**

π was found (1947) at expected mass, **d** deformed, and everybody was impressed by the

GREAT SUCCES OF QFT

Nuclear forces become more and more complicate : from OPEP to OBEP

Contrary to QED, nuclear phenomenology was not described by $\mathbf{V_\pi}$ even by $\mathbf{V_\pi}$+$\mathbf{V_{\pi\pi}}$
In order to describe the NN scattering data and the simplest nucleus (d=pn) other contributions were needed.

They were supposed to be due to the « exchange » of other (heavier) mesons :

The ω **meson** (vector) was predicted[(*)] …and found at the expected mass m_ω =780 MeV
The ρ **meson** (vector) was predicted[(**)]…and found at the expected mass m_ρ=769 MeV
The σ **meson** (scalar) was predicted …and (more or less) found with m_σ=500 MeV
….

Again a VERY GREAT SUCCES…

The « paradigma » was $V_{NN} = \mathbf{V_\pi} + \mathbf{V_{\pi\pi}} + \mathbf{V_\omega} + \mathbf{V_\rho} + \mathbf{V_\sigma} + \ldots$

However the initial simplicity - its beauty - was lost, the number of parameters (g's and cut-off singularities) was dramatically increased (1, 10, 20, 40…) and the potentials became shamefully complicated (as Ptolomean celestial Mechanics)

Worst than that…

(*) Y. Nambu, Phys Rev 106 (1957) 1366 , (**) J.J. Sakurai, Phys Rev 119 (1960) 1784

DIFFICULTIES IN DESCRIBING THE NUCLEAR CHART WITH V_{NN} (1)

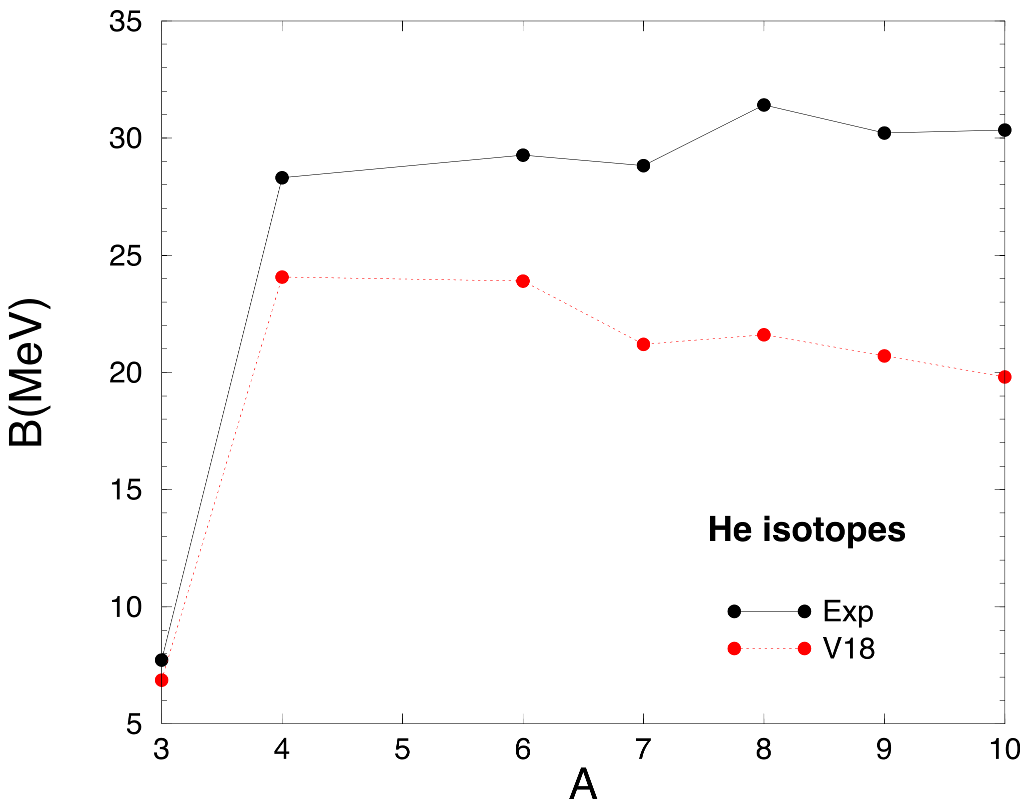
NN scattering observables and **d (p-n)** properties were described almost perfectly
With 40 parameters it is almost « tautological », but still it's hard to do better !!!

As soon as one was able**(*)** to accurately compute A=3 systems (^3H,^3He) one founds that none of V's reproduced even the simplest data (binding energies) : the « underbinding problem »

Exemple

Binding energies of He isotopes with Argonne v18 NN potential **(χ^2/data=1.01)**

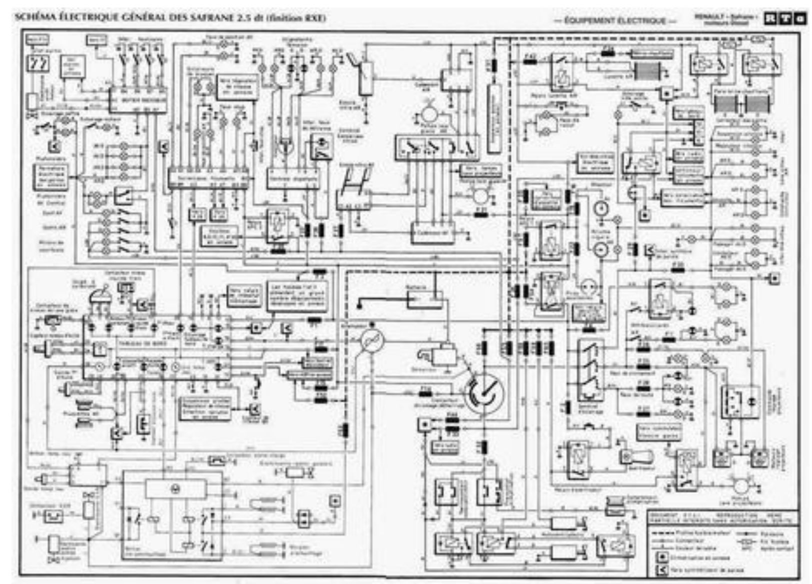
Missing E increases with A
For ^8He is 10 MeV, 1 MeV per nucleon !



(*) Started with Faddeev equations for 3-body (1960)… took 30 years to fix A=3 and 10 more for A=4

WHY (THRORETICAL) NUCLEAR PHYSICS IS SO DIFFICULT ?

From the simplicity of Yukawa theory to… « l’usine à gaz »



SEVERAL (MAJOR) REASONS

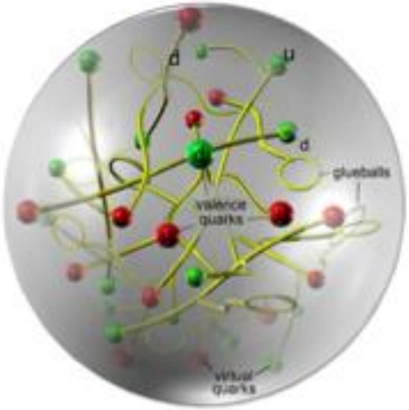
1. **Many-body forces** : When solving A>2 we assume $V=\sum_{ij} V_{ij}(r_{ij})$ (1)

   Things are more complicate : there are terms in V not reducibles $V(r_1,r_2,r_3,\ldots)$

2. For strong interaction (α_s >1), **V is very different from $\mathcal{L}_Y(x)$!!!**
   Even if V's are inspired by QFT, **QFT is much more than V**
   Nobody knows what is the prediction of the Full Yukawa theory, even for **d** !

3. **Pointlike particle are always assumed,** even if solving exactly QFT, (unavoidable !)

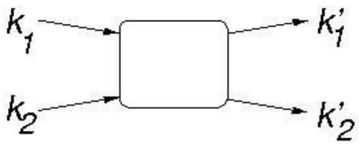
We know now that N is a very complicated object (not known in Yukawa time !)
Any atempt to describe Nuclei in terms of pointlike constituents is hopeless.
R(N)=0.8 fm and R(^{4}He)=1.6 !!!

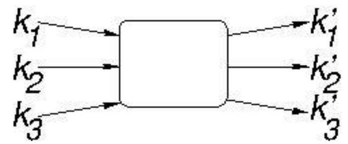


We will review points 1 and 3 and present the (desperate) solution proposed by EFT

1. THREE BODY FORCES

If we rely on QFT it is clear that in the 3-body Hamiltonian there are terms giving rise to to interactions $V(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)$ not reducible to sums of $V_{ij}(\mathbf{r}_i-\mathbf{r}_j)$: these are the 3N forces V_{3N}





This was already pointed out as soon as in ….1939 !!!

H. Primakoff and T. Holstein, Phys. Rev. 55 (1939) 1218

« the description of electrons in atomic systems by the customary two-body potentials is an excellent approximation »

However for nuclei $V_3=(v/c)V_2$ and $V_m=(v/c)^{m-2}V_2$ m=4,5,… and conclude with :

« The usual description of nuclei in terms of two-body potentials cannot therefore be considered satisfactory, except in the case of the deuteron »

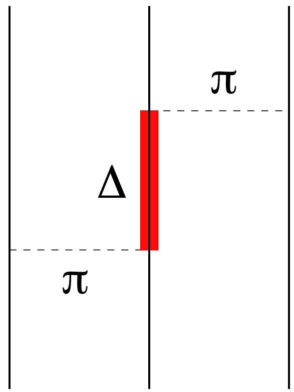
as well as in

L. Janossy, Proc. Cambridge Phil. Soc. 35 (1939) 616

V_3 gives small contribution at $r > \lambda_c=0.2$ fm, but comparable to V_2 at $r<0.1$ fm

The first attempts to get explicit expression were :

Fujita-Miyazawa (1957)



First attempt to quantify 3-body forces in nuclei was

J. Fujita and H. Miyazawa, Progr. Theor. Phys. 17(1957) 360

They consider P-wave process π+N → Δ → π+N (2π-exchange)

Input from π+N → Δ experimental data

They found that this contribution was attractive :

- ΔE=-0.22 MeV in 3H (perturbatively and simplifed wf)
- Also attractive and larger in heavier nuclei

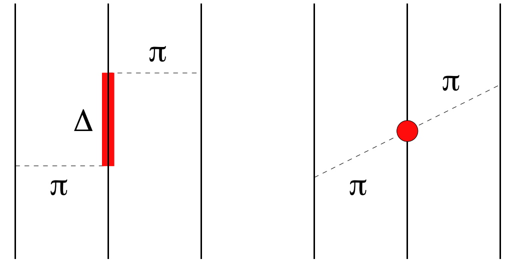
…. but was also attractive in nuclear matter

This contribution cannot account for the underbinding in ^{3}H (B_{exp}=8.48 MeV)

It was later found that (V_{NN}+V_{NNN}) missed more than 1 MeV :

Reid 68	7.23 MeV	ΔE=1.25 MeV
Paris 84	7.38 ”	ΔE=1.10 ‘’

Tucson-Melbourne (1979)



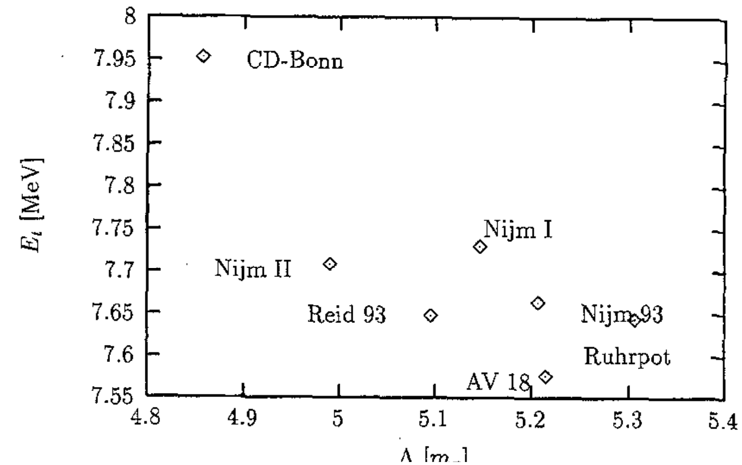
S.A. Coon, et al, Nucl. Phys. A317 (1979) 242-278
Same physics but with S+P waves (same contribution)

More involved calculations **but** uncontrolled parameters… opening a **« Pandora box »**
Mainly a « vertex form factor **Λ** » acting as cut-off, required to make calculations finite

This TM 3NF contribution was larger than in FM and also attractive.
In infinite nuclear matter **ΔE_3=-2,7(3) MeV** or **ΔE_3= -1.9(2) MeV …** depending on **Λ**

B(^{3}H) depends also on **Λ** ⟹ adjust **Λ** to get rigth B !!! (not a great achievement ….)
In its turn **Λ** depends on V_{NN} !

V_{NN}	B(3H)	Λ/m_π	$V_{NN}+V_{NNN}$
CD-Bonn	7.953 (8.014)	4.856 (4.79)	8.483
Nijm II	7.709	4.990	8.477
Reid 93	7.648	5.096	8.480
Nijm I	7.731	5.147	8.480
Nijm 93	7.664	5.207	8.480
AV18	7.576 (7.685)	5.215 (5.107)	8.479
Ruhrpot	7.644	5.306	8.459



From A. Nogga, D. Huber, H. Kamada, W. Glockle, PLB 409 (1997) 19

Brasil 3NF (1983)

H.T. Coelho, T.K. Das, M.R. Robilotta, Phys. Rev. C28 (1983) 1812

- Chiral dynamics
- New terms
- Strong dependence on cut off (x_0)

If x_0 fixed by ^3H and ^4He elastic em form factor, 3NF contributions remain small

All these QFT inspired approaches led to attractive 3BF

GOOD for B(^3H) !!!

….while the infinite matter nuclear equation od states (EoS) seems to badly require (short range) repulsion

This motivated the following step :

Urbana 3NF « saga » (I)

Started an ambitious program building a family of V_{NNN} in conjunction with the local V_{NN} from Urbana (81), Argonne Av14 (84) and Av18 (95) aiming to simultaneously describe

- **Light nuclei** (Variational, GFMC)
- **Infinite nuclear mater**

Their main contribution was to **incorporate an ad-hoc short range repulsion** - absent in QFT inspired models (FM, TM, Brasil) - but required to produce an acceptable EoS

URBANA IV-V-VI (1983)

J. Carlson, V. R. Pandharipande and R. B. Wiringa, Nucl. Phys. A401 (1983) 59

Depend on 3 parameters but only one was adjusted to B(3H) with Urbana VNN

Version	$A_{2\pi}$	U_0	U_c	3H	3He	4He
TM				8.1(1)	7.4(1)	29.3(5)
U − IV	−0.0289	0	0	8.5(1)	7.7(1)	31.4(3)
U − V	−0.0333	0.003	0	8.1(1)	7.4(1)	29.1(4)
U − VI	−0.0467	0	38000	8.4(2)	7.6(2)	30.9(8)
Exp.				8.48	7.72	28.3

If ^{3}He is fited… ^{4}He is **OVERbound** by 3 MeV

Urbana 3NF « saga » (II)

URBANA VII (1986)

R. Schiavilla, V.R. Pandharipande and R.B. Wiringa, Nucl. Phys. A449, (1986) 219

Little stronger repulsion than U-V (U_0=-0.0038 instead of U_0=-0.003)
gives **reasonable A=3,4 energies simultaneously**

Version	$A_{2\pi}$	U_0	U_c	3H	3He	4He
U − IV	− 0.0289	0	0	8.5(1)	7.7(1)	31.4(3)
U − V	− 0.0333	0.003	0	8.1(1)	7.4(1)	29.1(4)
U − VI	− 0.0467	0	38000	8.4(2)	7.6(2)	30.9(8)
U − VII	− 0.0333	0.0038		8.48(10)	7.82(10)	28.2(4)
Exp.				8.48	7.72	28.3

URBANA VIII (1991)

R. Schiavilla, V.R. Pandharipande and R.B. Wiringa, Nucl. Phys. A449, (1986) 219

Still variational calculations but with AV14
It seems its only glory was to reproduce B(^{3}H) when used with Av14

Urbana 3NF « saga » (III)

URBANA IX (1995)

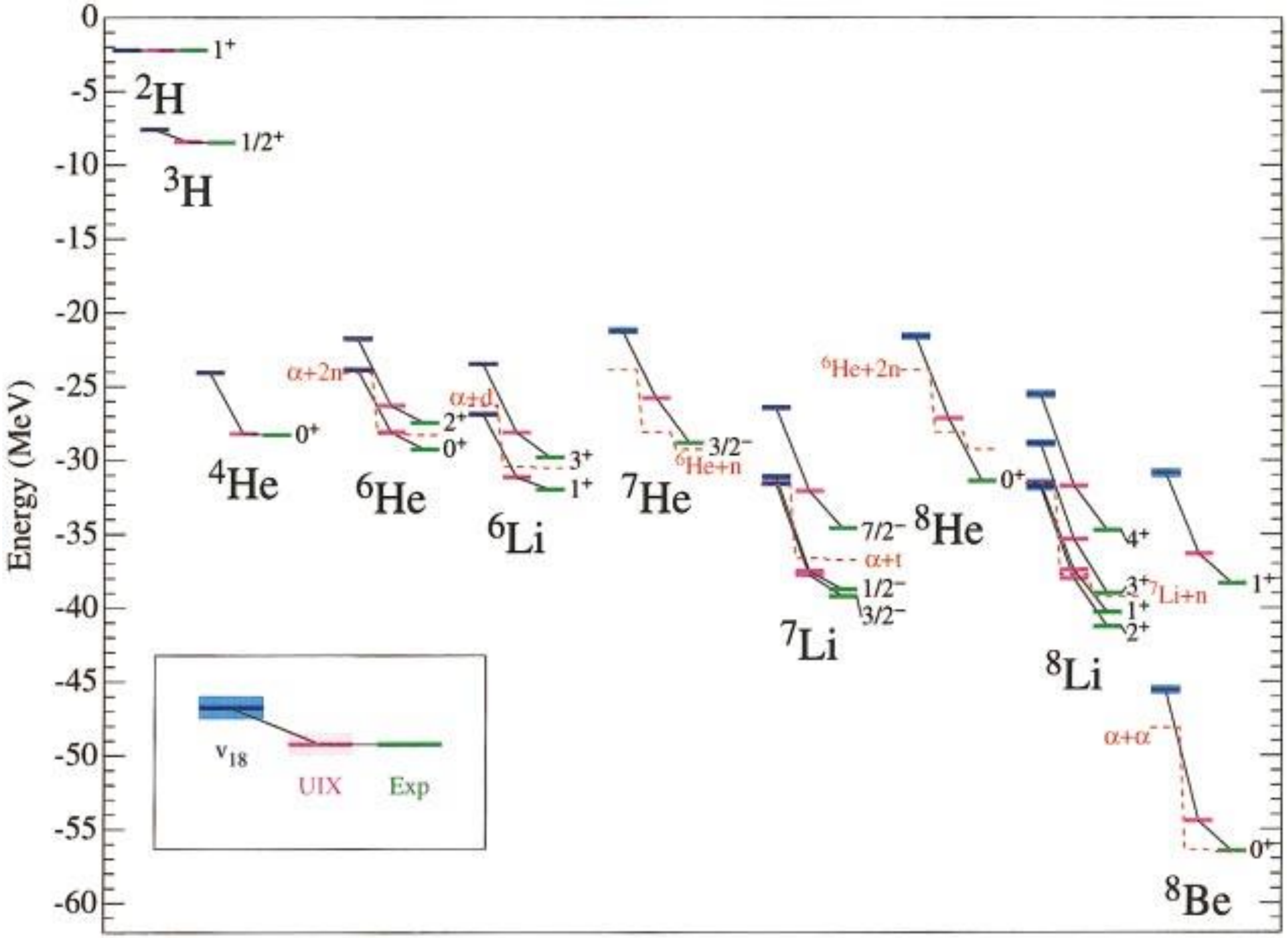
B. S. Pudliner, V. R. Pandharipande, J. Carlson, R. B. Wiringa, Phys. Rev. Lett. 74, 4396 (1995)

Serious things start with a new 3NF parametrisation **UIX + Av18 + GFMC**

Version	NN	$A_{2\pi}$	U_0	U_c	3H	3He	4He
U − IV	U81	− 0.0289	0	0	8.5(1)	7.7(1)	31.4(3)
U − V		− 0.0333	0.0030	0	8.1(1)	7.4(1)	29.1(4)
U − VI		− 0.0467	0	38000	8.4(2)	7.6(2)	30.9(8)
U − VII	U81	− 0.0333	0.0038		8.48(10)	7.82(10)	28.2(4)
U − VIII	v14	− 0.0280	0.0050		8.46		28.3(2)
U − IX	v18	− 0.0293	0.0048		8.47(2)		28.3(1)
Exp.					8.48	7.72	28.3

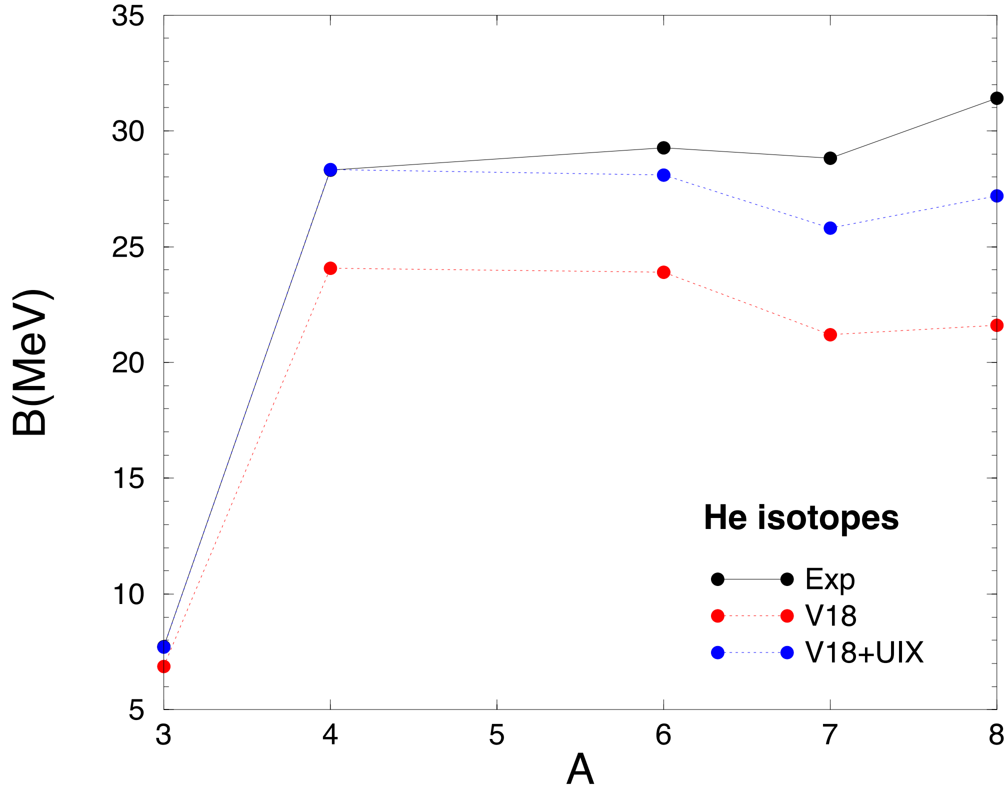
The interesting thing is that, progressively GFMC was able to reach A=6,8,10,12….

providing us with a **fascinating series of light nuclei spectra**



S. C. Pieper, V. R. Pandharipande, R. B. Wiringa, J. Carlson,Phys. Rev C 64, 014001 (2001)

Analyzing them with some detail, what happened was quite interesting !



The same that happened between U-V and U-VII happens again with U-IX :
a new readjustement is needed….but there are no more parameteres in the game !!!

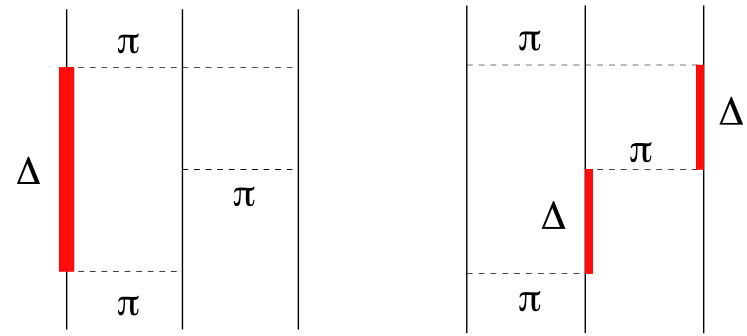
So what ????

Urbana-Illinois 3NF « saga » (2001)

In fact it already started in

S. C. Pieper, V. R. Pandharipande, R. B. Wiringa, J. Carlson,Phys. Rev C 64, 014001 (2001)

when the same GMC authors realized the problem



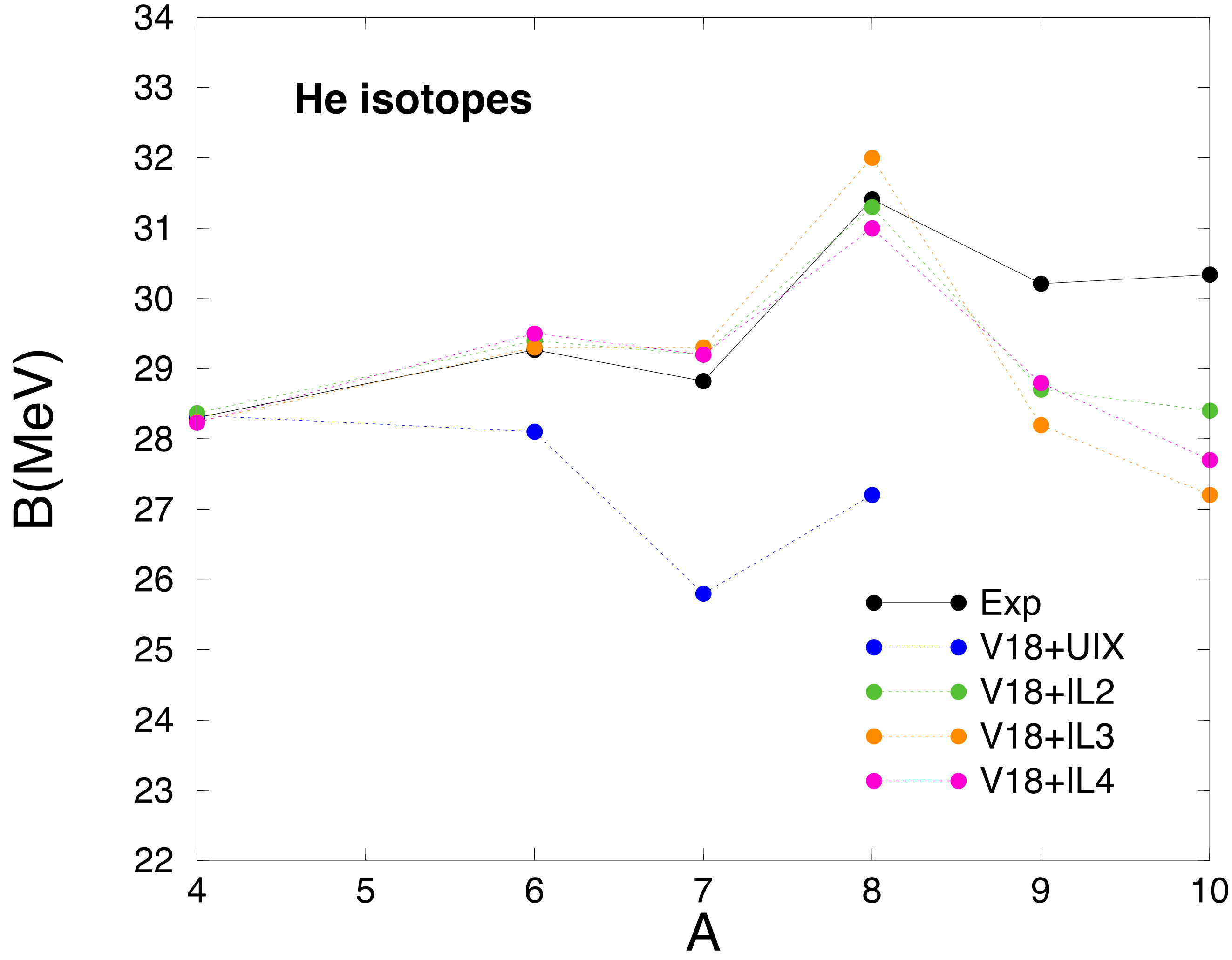
Illinois saga incoporates 3π–echange terms
and since work in paralel with CD Av18
has also a (small) T=3/2 component

They have 5 parameters (4+1) and several (IL1,IL2,….IL7) versions

The description with respect U-IX is certainly improved in this way

It will be dangerous to detail their analytic structure….

But similar things happens when one move away from the region
where the 3NF parameters are adjusted…



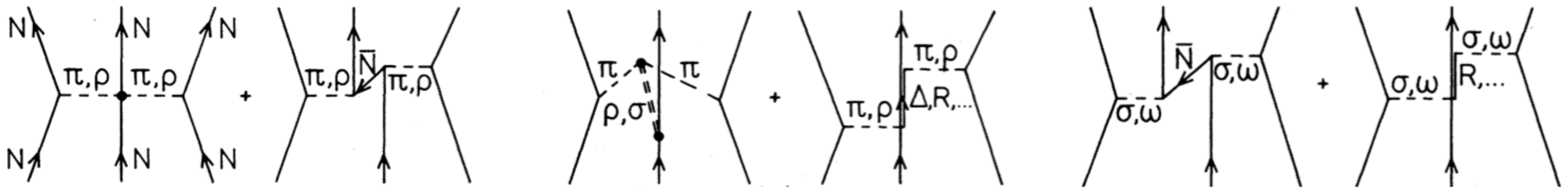
He isotopes
B(MeV)
A
Exp
V18+UIX
V18+IL2
V18+IL3
V18+IL4

Its seems we are damned to start the job
…. as many times as we make any progres on it !!!

In ancien Greece this was known as the **« Sisyphus effect »**

All that start being quite uncontrolled….

I. Why not include self-energy graph and so many others: NN pairs, ρ, σ… ?



Consistent three-nucleon forces in the nuclear many-body problem
P. Grange, A. Lejeune, M. Martzolff, J.F. Mathiot, Phys. Rev. C40 (1989) 1040

All of them exist, in the same foot !

II. If everything that is possible should be included we are not yet done !

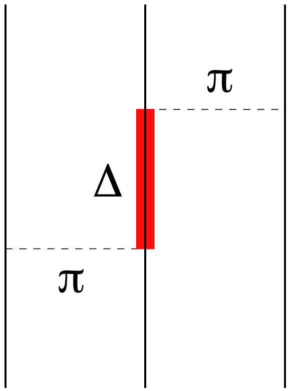
What is the hierarchy ?

Things start being quite uncontrolled….

III. N and intermediate states should be separated in space-time to give them some meaning

Wahat 3N+Δ+π means, squeezed in a box of 1.5 f ?

They are as close to eahother….as they are big !



IV. Most of involved Feynmann graphs contain divergent quantities : regularize them means new parameters of no any physical meaning

……………….

………..

3. INTERACTION BETWEEN COMPLEX SYSTEMS … TREATED AS POINTLIKE PARTICLE

An exemple taken from Atomic Physics….

Positronium **(e^+e^-)** spectrum depends only on two parameters (m,α)
This is true for the infinity of bound states, and whatever the required accuracy
At the lowest order (V=-1/r) it gives :

$$E_n = -\frac{1}{4n^2}\, m\,\alpha^2$$

and nothing new appears when including higher order QED corrections (10-12 digits !!!)

It remains true for describing few eV **e^+e^-** collision as well as for energies 10^6 times greater !
In fact the first « new parameter » appears at 10^8 eV… and it is « trivial » : m_μ

$$e^+ + e^- \to \mu^+ + \mu^-$$

This beatiful simplicity remains also when we compute a **Ps**+**Ps** collision
It is a 4-body problem **(e^+e^-)**+**(e^+e^-)** that includes many « asymptotic channels »
But its solution still depends on the very same parameters (m,α)
…. **provided we solve it exactly**, as a 4-body problem !!!

The same would be true for Ps^+ + Ps^+ collision…except that in this case **we do not know** (yet) **how to solve it !** (N=6)…. and **here starts the drama**

This is also the case if we want to describe **the system of two ^{4}He « noble gases » atoms**

These atoms have « lost their nobility » by combining into dimers, trimers, tetramers, either with themselves, or with their ^{3}He « cousins » (…so quite a relative lost)

These are « periferal states », like quasi-satellites, without sharing their electrons (no co-valence !) and imply B≈10^{-7} eV and sizes ≈100 - 1000 A !

To describe these – purely electromagnetic – systems :
- we consider ^{4}He atom as a pointlike particle,
- we introduce an atom-atom «potential »
- and we inserted in a two-body Schrodinger equation

i.e. **we consider as simple** (point-like) **what is in fact complicate**

How to obtain this interatomic potential ?

We have to invent it ! …vaguely guided by Van der Waals forces

- $1/r^6$ attraction (charge fluctuations in neutral atoms)

- $1/r^{12}$ repulsion (Pauli principle between electrons)

$$V(r) = V_0 \left(\frac{R_0^6}{r^{12}} - \frac{1}{r^6} \right)$$

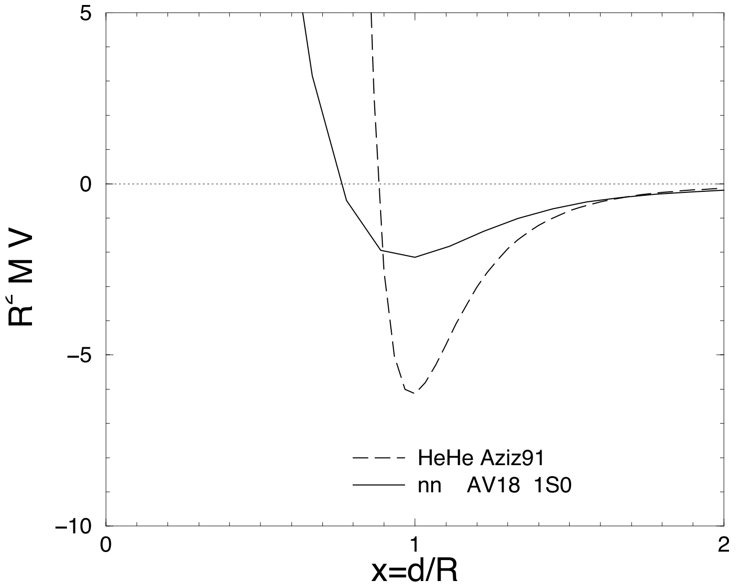
Plus two parameters (V_0,R_0)

Soon appeared that these 2-parameter form was unable to reproduce experimental results: dimers, trimers,.. And atomic collisions

A more complicated radial dependence was invented …with new parameters

Exemple: He-He Aziz potentiel **V_A**
R.A. Aziz et al., J. Chem. Phys 94 (1991) 8047
works well but at the price of .. **12 parameters !**

A more recent version has already > **20 parameters** !
W. Cencek et al, CPL 459, 183 (2008)



V(He-He) is the - complicate - prize to pay for our incompetence (or laziness ?)

In reality V « does-not exist », it comes from nothing, it is « invented « on purpose !!!

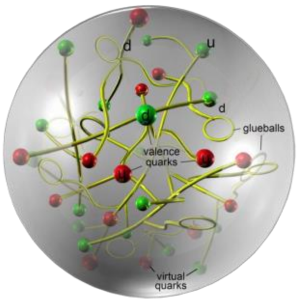
Everything depends on (m,α) an « exact » 6-body solution will provide an accurate result

This « parameter explosion » happens in an very simple case, when compared to V_{NN} (no SS, no LS, no T…)
It illustrates well what happens in the - much more complicated - Nuclear case

Conclusion: « Complexity » follows in fact from our « simplifications » !

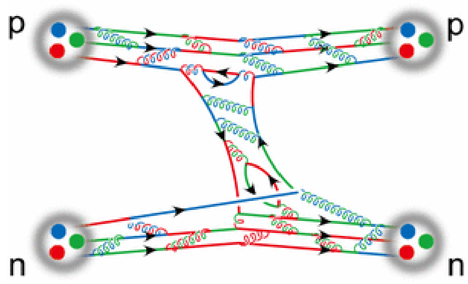
Physics is simple … our incompetence make it complicate
Quite a **paradoxical situation** which manifests in atom-atom collisions
… and become almost ridiculous in Nuclear Physics :

Nucleons are VERY COMPLICATED objects

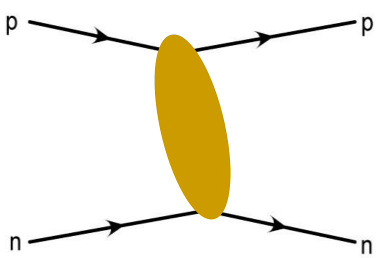


(here is the problem …. !)

this results into very complicated interactions



If we force ourselves to describe them by "potentials" V between simple point-like objets......



One ends up – obliged by phenomenology – **with "monsters" having 40 parameters.....**

Is there any reasonable solution ?

1. No !
   Describing complicate things in terms of simple ideas interacting with ad-hoc V will be always a limited game: we can play it but…a tower will never kill a King !
   V is a parametrization of our ignorance

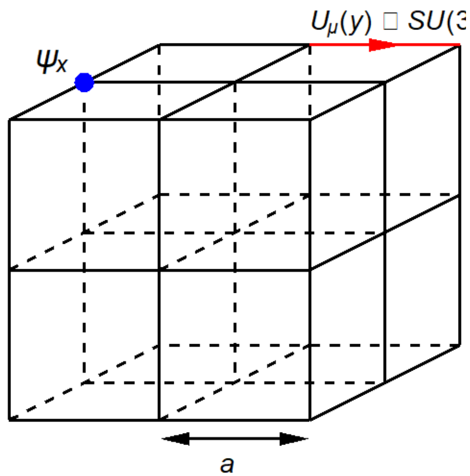
2. Return to elementary fields…by solving « exactly » QFT **(no any V !)**
   This is the LQCD philosophy
   It is possibe…but useless for practical purposes

3. The EFT « solution » for Nuclear Physics

In what follows I will comment on points 2 and 3

LQCD : an attempt to solve QFT in “its full glory” (non peturbative)

« Lattice QCD calculations » , although the method is in principle applicable to any QFT
Uses Feynman path-integral formulation of QFT in a discretized Euclidean space-time lattice $V=L^3xT$ and Monte Carlo integration techniques.



On each « link »
4 SU(3) matrices

(gluons)

On each site
3 x 4 x N_f complex « fields »

(quarks)

$$U_\mu(x) = \exp\left\{\frac{iag}{2}\sum_{c=1}^{8}\int_0^1 d\tau A_\mu^c(x + a\tau\mu)\lambda^c\right\} \in SU(3)$$

$$q_f(x) = \begin{pmatrix} q_f^b(x) \\ q_f^r(x) \\ q_f^v(x) \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} \cdots \\ \cdots \\ \cdots \\ \cdots \end{pmatrix} \\ \begin{pmatrix} \cdots \\ \cdots \\ \cdots \\ \cdots \end{pmatrix} \\ \begin{pmatrix} \cdots \\ \cdots \\ \cdots \\ \cdots \end{pmatrix} \end{pmatrix}$$

Free q lagrangian $\quad L_D = \Psi(x)(\gamma_\mu\partial_\mu + m)\Psi(x)$

Discret derivatives $\quad 2a\,\partial_\mu\Psi(x) = \Psi(x+\mu) - \Psi(x-\mu)$

Gauge inv restored

$$\bar\Psi_x\Psi_{x+\hat\mu} \rightarrow \bar\Psi_x\ U_\mu(x)\ \Psi_{x+\hat\mu}$$
$$\bar\Psi_x\Psi_{x-\hat\mu} \rightarrow \bar\Psi_x\ U_{-\mu}(x)\ \Psi_{x-\hat\mu}$$

Some sizes

L	T	V
24	48	660 000
32	64	2 100 000
48	96	10 600 000
64	128	32 200 000
96	192	169 600 000

Parameters:

- "bare" quark masses $\mathbf{m_l} = m_u = m_d$, m_s , ...

  To control the "physical value" of m_l , one computes m_π ($m_\pi^2 = B\, m_q$)

  If m_π=140 MeV… m_l is the right one ! But it is almost never he case !!!

- one parameter **β** that controls the "lattice spacing"

  one goes down to a=0.05 fm (**Errors** due to discretisation: o(a), o(a2),…)

- Lattice **L** (**Errors** due to "finite volume" L x a fm)

- $\mathbf{n_f}$ = number of quarks in the loops in unquenched calculations ($\mathbf{n_f}$=0,2,2+1,2+1+1,…)

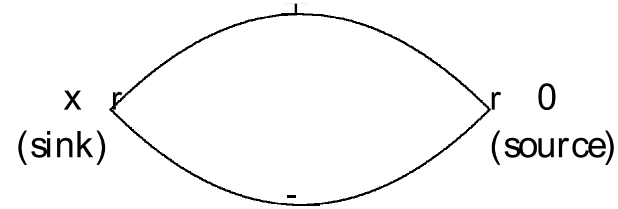
Since $a \to 0$ the, only free paramters are the bare quark masses.

In Nuclear physics $m_u = m_d = m$: one single parameter !!!!
Nuclear physics recovers the simplicity of Atomic Physics

In Hadronic physics $\mathbf{m, m_s, m_c}$

COMPUTING OBSERVABLES

Meson masses



Consider space-time propagation of $\bar{q}q$

Compute correlator between currents J

$$C(t) = \sum_{\vec{x}} < 0 \mid J(x) J^\dagger(0) \mid 0 >$$

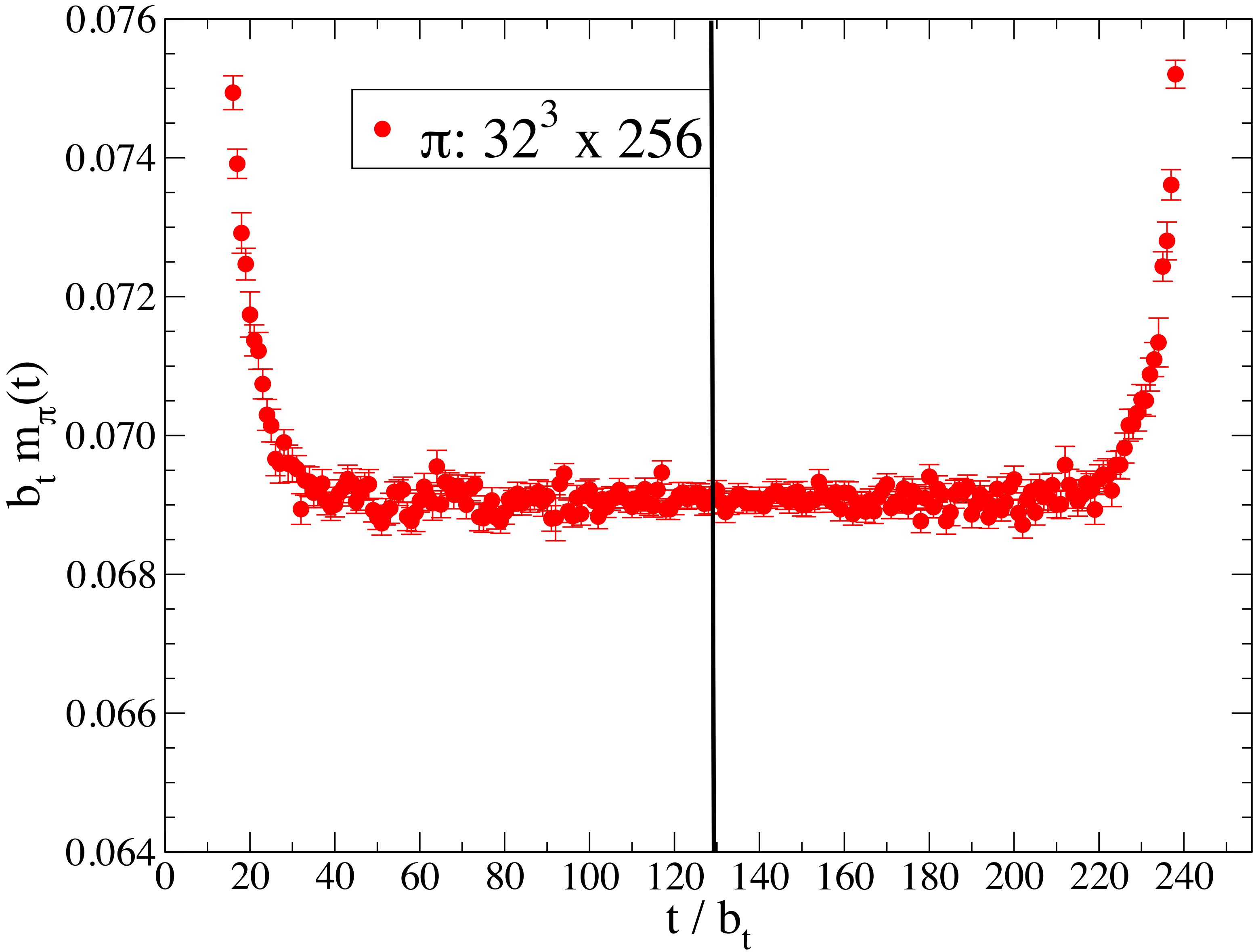
Simplest case $\quad J(x) = \bar{u}(x) d(x)$

$$\begin{aligned}
C(t) &= \sum_{\vec{x}} < 0 \mid \bar{u}(x) d(x) \bar{d}(0) u(0) \mid 0 > \\
&= -\sum_{\vec{x}} < 0 \mid d(x) \bar{d}(0) \mid 0 >< 0 \mid u(0) \bar{u}(x) \mid 0 > \\
&= -\sum_{\vec{x}} \mathrm{Tr}\left[S_d(x,0) S_u(0,x)\right] \\
&= -\sum_{\vec{x}} \mathrm{Tr}\left[S_d(x,0)\gamma_5 S_u^\dagger(x,0)\gamma_5\right]
\end{aligned}$$

Exo: $J_P(x) = i\bar{u}(x)\gamma_5 d(x) \qquad C_P(t) = \sum_{\vec{x}} \mathrm{Tr}\left\{\left[\gamma_5 S_d(x,0)\right]\left[\gamma_5 S_u(x,0)\right]^\dagger\right\}$

On another hand $\quad < 0 \mid O_1(t) O_2(0) \mid 0 >= \sum_n < 0 \mid O_1 \mid n >< n \mid O_2 \mid 0 > e^{-E_n t} \sim e^{-E_0 t}$

This provides an effcient way to compute meson masses $\qquad aM_{eff}(t) = \log \dfrac{C(t)}{C(t+1)}$



π: 32^3 x 256
b_t m_π(t)
t / b_t
0.076
0.074
0.072
0.070
0.068
0.066
0.064
0
20
40
60
80
100
120
140
160
180
200
220
240

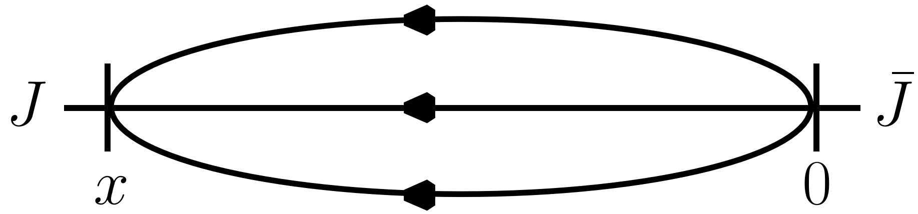
Baryon masses (N)

First step: "built " a N (in fact a $J^\pi=1/2^+$ state) by combining 3 q fields

Create N at y=0

Propagate N from y→x

Annihilate N at x



$N_a \equiv \bar{\epsilon}^{ijk}(u^i C\gamma_5 d^j)u^k_a$ $\qquad$ $\overline{N}_a \equiv \epsilon^{ijk}(\bar{u}^i C\gamma_5 \bar{d}^j)\bar{u}^k_a$

$$C_{\alpha\beta}(x,y) = \langle 0 \,|\, N_\alpha(x)\overline{N}_\beta(y) \,|\, 0\rangle = \sum C^{abcd}_{\alpha\beta}\langle 0 \,|\, q_\alpha(x) q_a(x) q_b(x) \bar{q}_\beta(y) \bar{q}_c(y) \bar{q}_d(y) \,|\, 0\rangle$$

It is a v.e.v. of a product of 6 quark fields q(x)

Wick Th: sum of products of **q propagators** ("contractions")

$$S^{cc'}_{ss'}(x) =< 0|q^c_s(x)\bar{q}^{c'}_{s'}(0)|0 > \qquad D^{c'c}_{s's}(x,y)\; S^{cc"}_{ss"}(y) = \delta^{c'c"}\delta^{s's"}\delta(x)$$

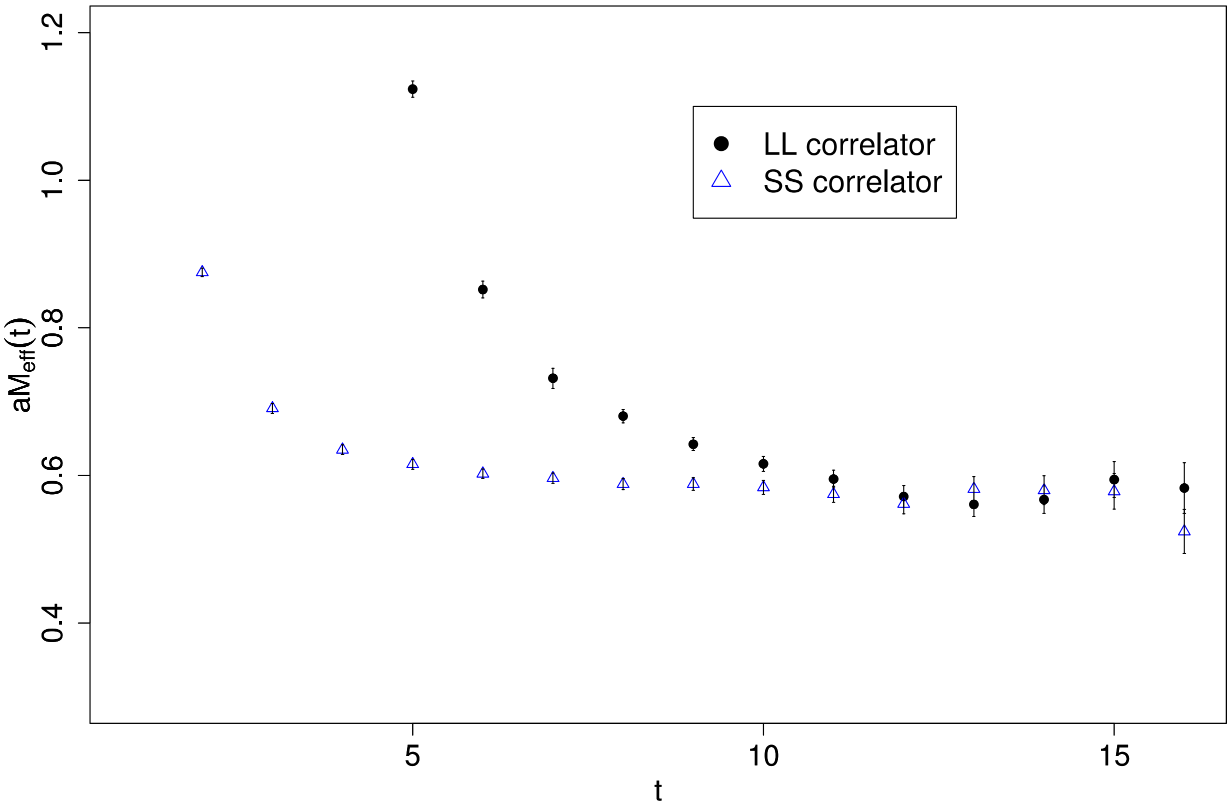
N mass is extracted from matrix elements of this correlator 4x4 (y=0)

$$\mathrm{Tr}\,[C_{\alpha\beta}(t)] = \mathrm{Tr}\left[\sum_{\vec{x}} C_{\alpha\beta}(\vec{x},t)\right] \sim e^{-aM_N t}$$

The method can be extended to (6A) q fields and access to A-baryon system

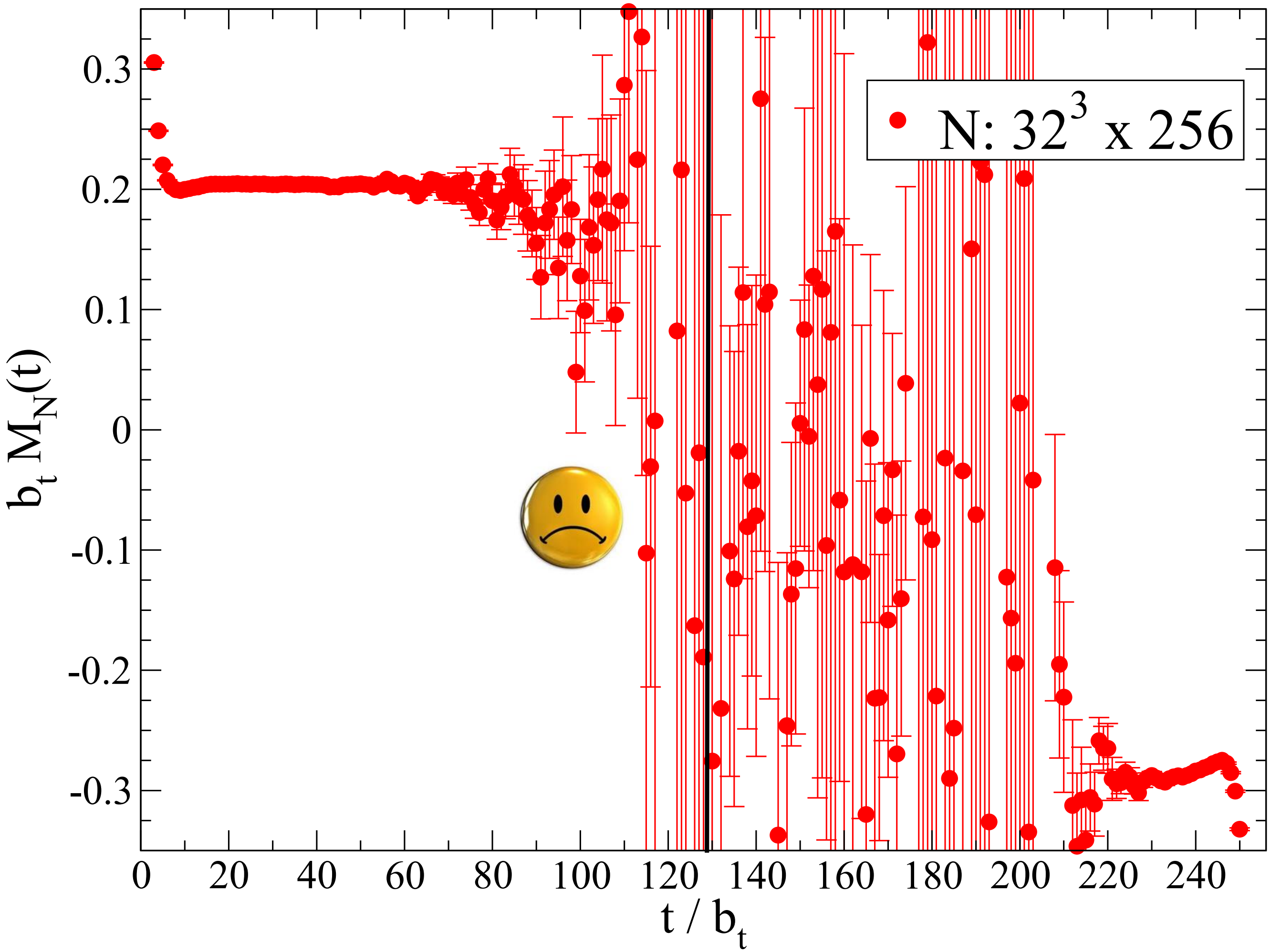
Example: N

$$aM_{eff}(t) = \log \frac{C(t)}{C(t+1)}$$



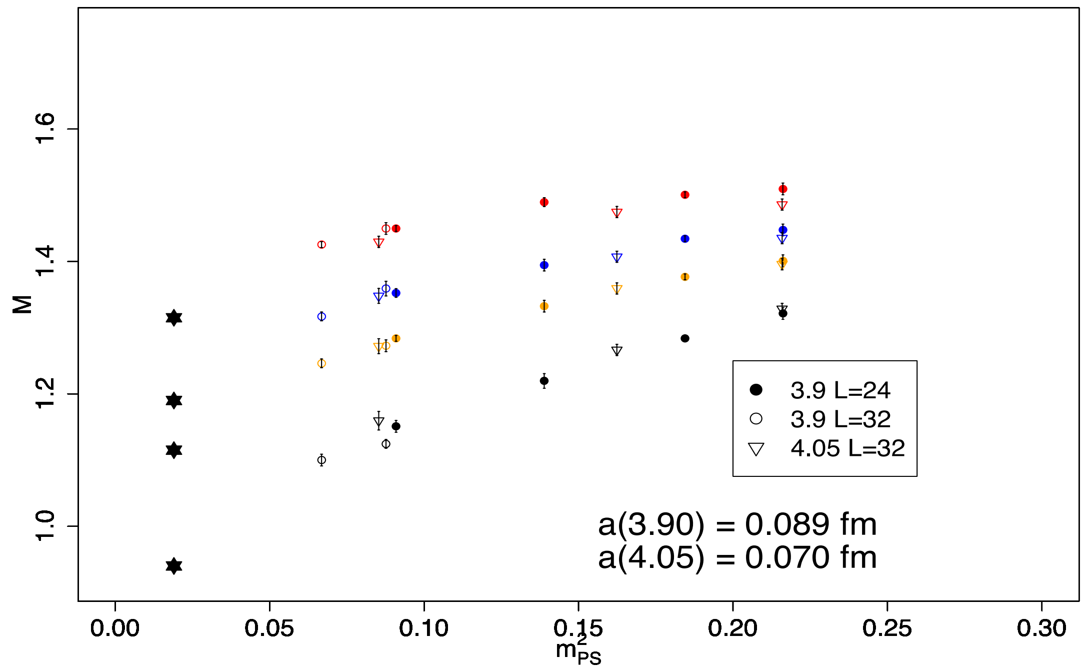
Same quality for other baryons (ground state !)

Signal to noise $\sim e^{-(M_N - \frac{3}{2} M_\pi)t}$



I.2 Baryon Ground states

Despite less beautiful signal, they reach an « accuracy era » (<1%) since several groups (BMW, ETMC) compute with physical $m_q(m_\pi)$ and lower (!) avoiding hazardous chiral extrapolations



$$M_N^{p^3}(m_\pi) = M_N^{(0)} - 4c_N^{(1)}\, m_\pi^2 - \frac{3g_A^2}{16\pi f_\pi^2}\, m_\pi^3$$

$$M_\Lambda^{p^3}(m_\pi) = M_\Lambda^{(0)} - 4c_\Lambda^{(1)}\, m_\pi^2 - \frac{g_{\Lambda\Sigma}^2}{16\pi f_\pi^2}\, m_\pi^3$$

$$M_\Sigma^{p^3}(m_\pi) = M_\Sigma^{(0)} - 4c_\Sigma^{(1)}\, m_\pi^2 - \frac{2g_{\Sigma\Sigma}^2 + \frac{g_{\Lambda\Sigma}^2}{3}}{16\pi f_\pi^2}\, m_\pi^3$$

$$M_\Xi^{p^3}(m_\pi) = M_\Xi^{(0)} - 4c_\Xi^{(1)}\, m_\pi^2 - \frac{3g_{\Xi\Xi}^2}{16\pi f_\pi^2}\, m_\pi^3$$

$m_\pi^2 = B \times m_q$

Highly non trivial result: 1 GeV N mass comes « out of nothing »

If $m_q=0$ M_N is almost the same : Is Higgs boson the real origin of mass ?

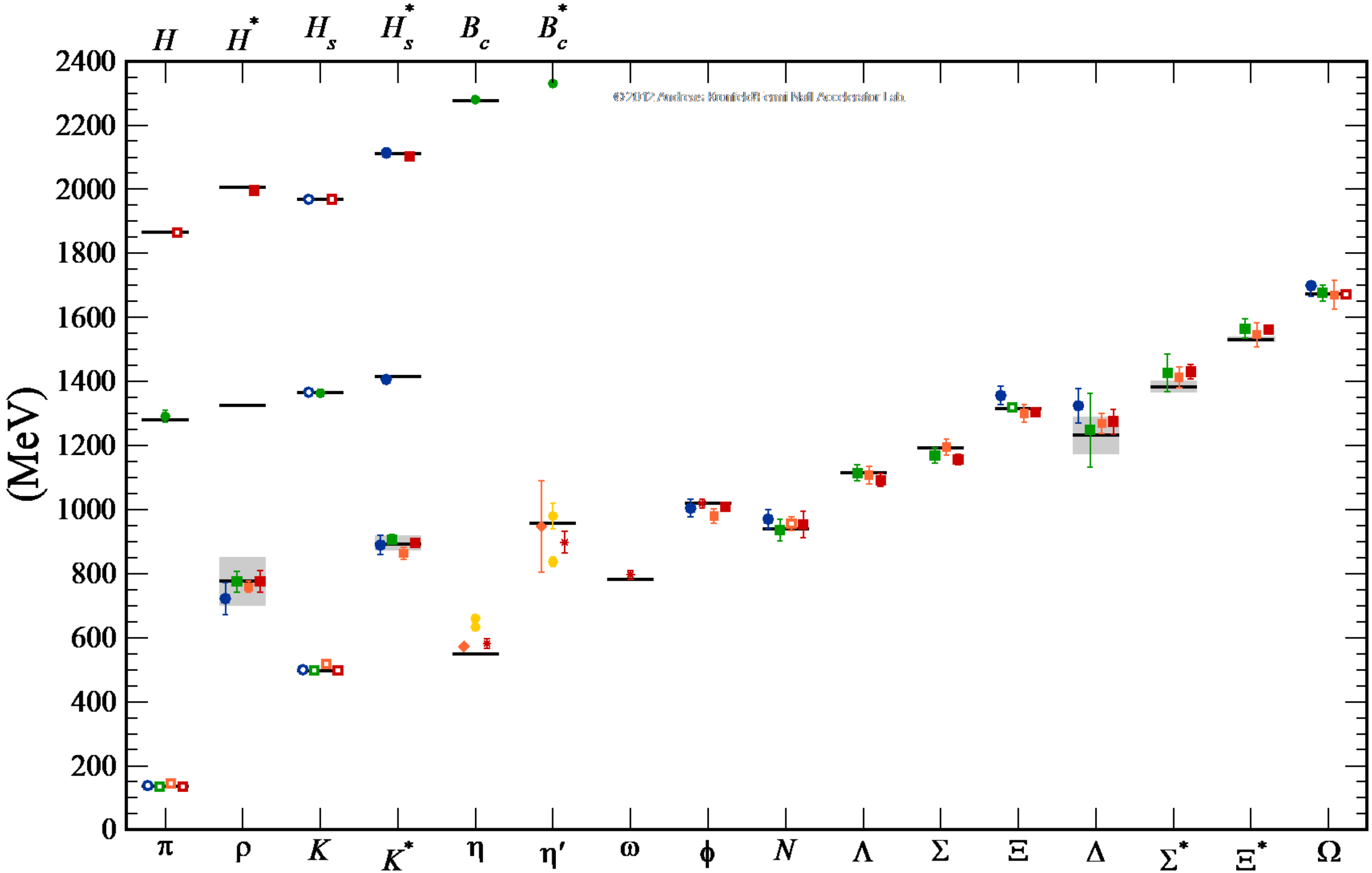
M(atom) = M_A+ M_e - 10^{-5} — **alll life on earth come from this thiny difference**

M(nucleus)= M_p+M_n - 10^{-2} — **all stars light by this 1%**

M(N) = $3m_q$ + 10^{+2} !!!! — **QCD is a real epistemological breakdown !!!**

Summary of different collaborations

Results firmly stablished and independent of the discretisation schemes



A. S. Kronfeld, Annu. Rev. Nucl. Part. Sci. 62, 265–284, (2012)

Ground state masses are entering now a precision era…

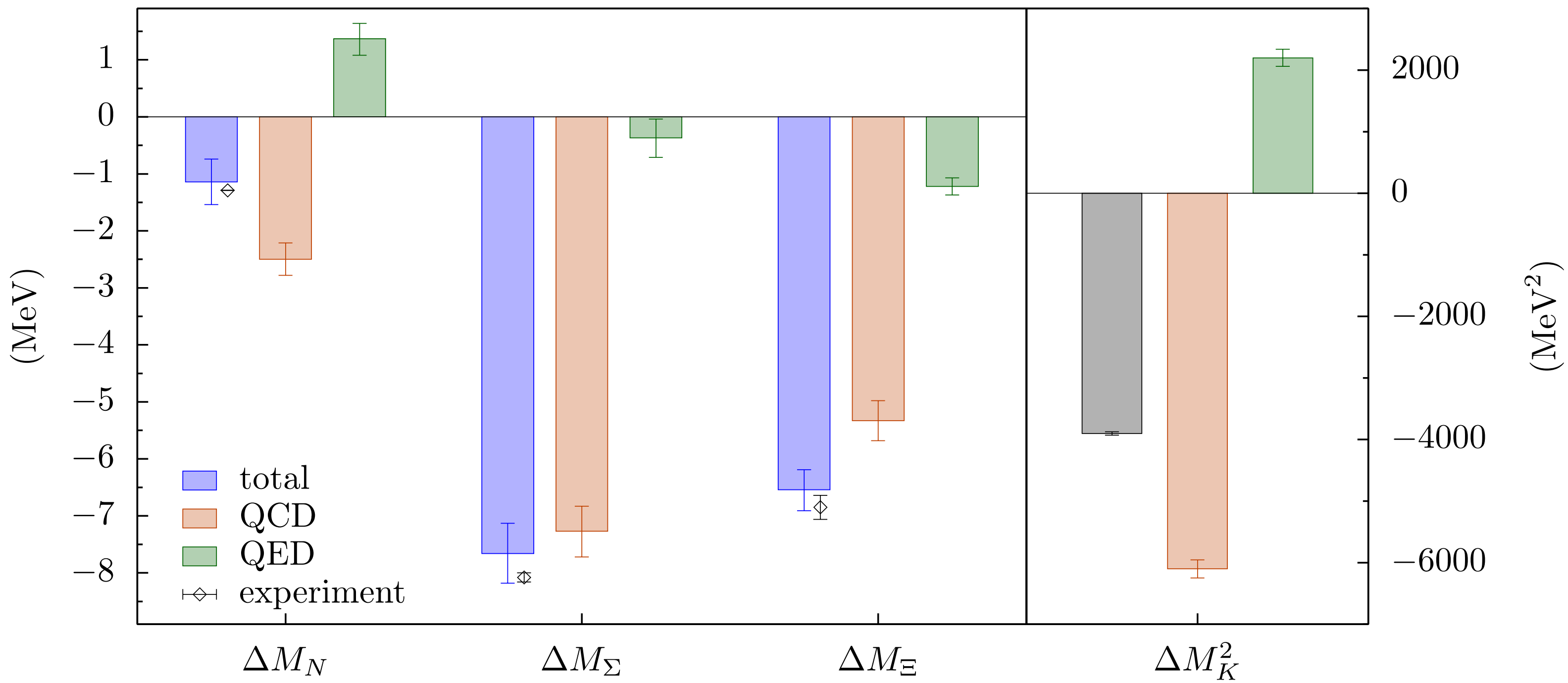
Isospin symmetry is broken in the most recent calculations

- m_u # m_d
- Incorporate electromagnetic effects between quarks (QCD+QED “quenched”)

The mass difference in isospin multiplets (N,Σ,Ξ,K) has been calculated (10^{-3})

Of particular interest is M_n-M_p which governs the weak decay and stability of nuclear chart

It results from a cancellation of opposite tendencies (if m_u=m_d , M_p>M_n … still atomes ?)



ΔM_N=M_p-M_n

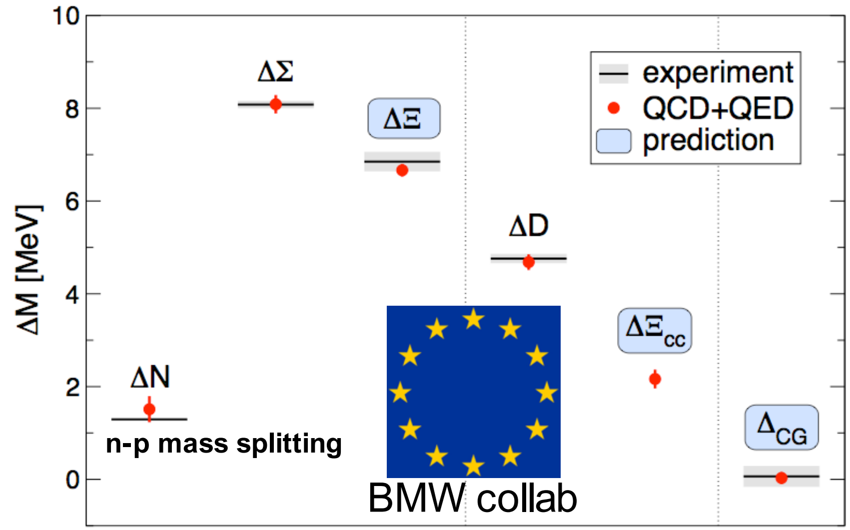
A. Portelli, PhD CPT Marseille 2012 (BMW)

Borsanyi et al, Phys. Rev.Lett. 111 (2013) 252001

This is a very impressive result

- For its accuracy

Borsany et al.
Science 347 (2015) 1452



- For the « physico-philosophico content » : quarks (m_u<m_d) are there to compensate e.m. and let matter be stable !

- For the huge underlying work it represents

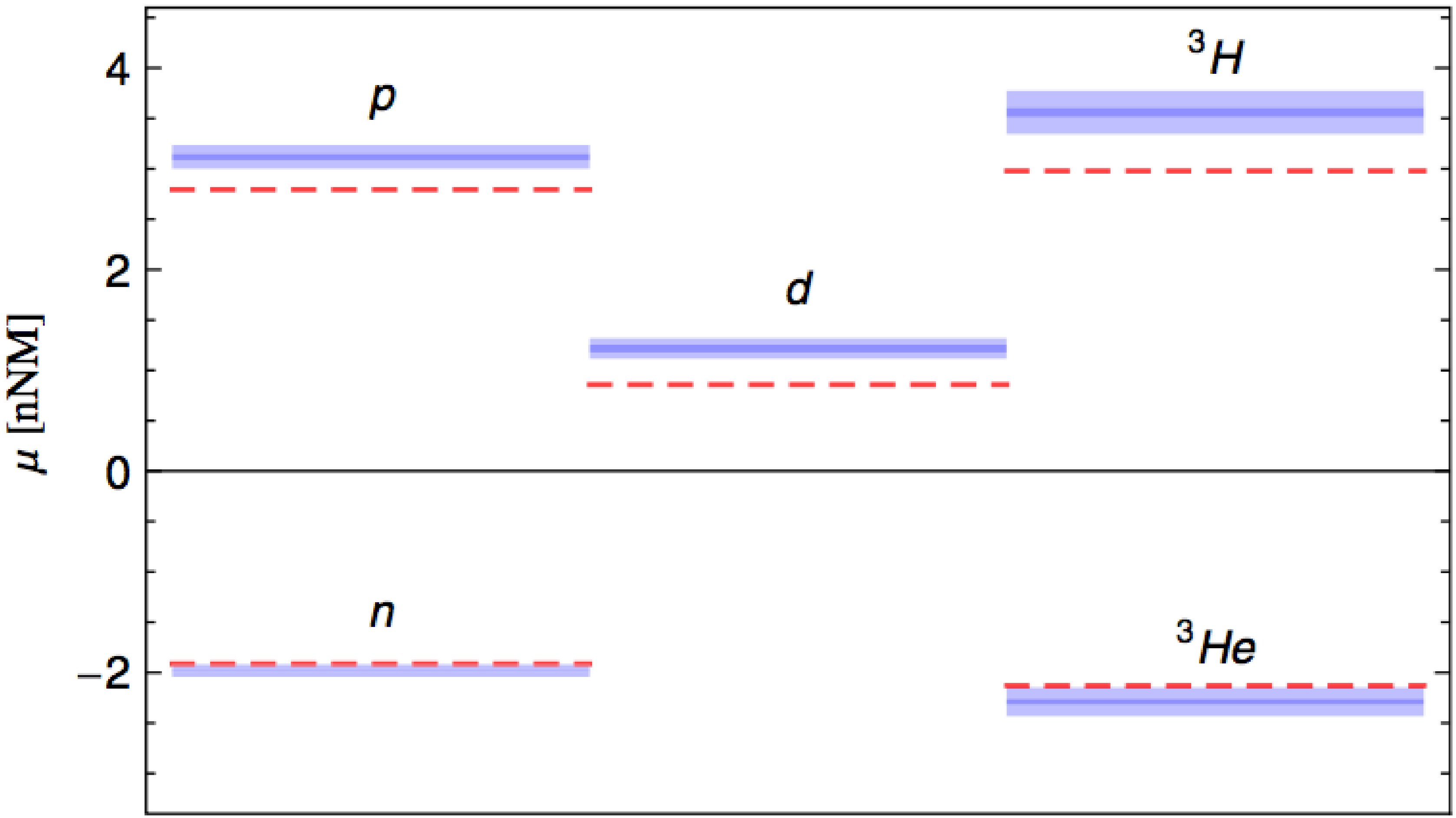
But… it remains a bit disapointing since LQCD gives ΔM(QCD) = 2.52(17)(24) MeV

From (FLAG 13) : m_u=2.16(11) MeV
m_d=4.68(16) MeV

and so M(udd)-M(udd) = 11.52-9.00= **2.52 MeV**

Hell !

Not only M, but also magnetic nuclear moments… out of nothing

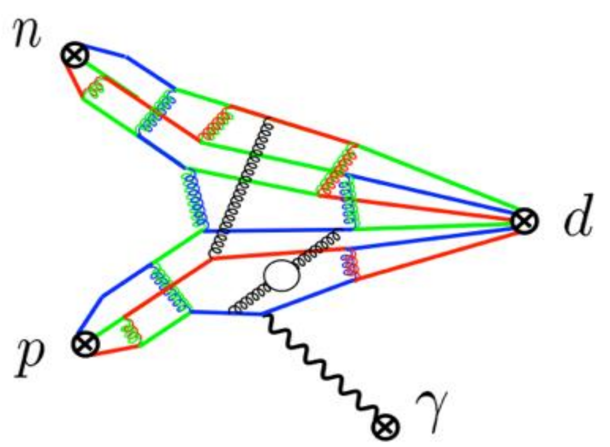


Beane, *et al.* (NPLQCD), Phys.Rev. Lett.113, 2014.

As well as (simple) nuclear reaction…

Radiative capture n+p →d + γ

(with some EFT ingredients)



σ^{LQCD} = 334.9(5.3) mb $v = 2,200$ m/s

σ^{expt} = 334.2(0.5) mb

The same calculation with point-like N gives 306 mb

NPLQCD, PRL115 (2015) 132001

A very very great achievement !

Hadronic Physics recovers - conceptually and in practice – the simplicity of atomic physics (one paramter)… apart from some « technical » aspects of LQCD !

Next step should be to exend it to Nuclear Phsyics…and get rid of these devilish nuclear potentials !
Already announced by NPLQCD ….but turned to be fake ! Run too fast It is very dangerous!!!

For the moment even deuteron is not yet firmly stablished !

Unfortunately this expected achievement – if it comes at all - will not very useful to the Nuclear Physics community….
« Models » and « Potentials » will remain mandatories for usual nuclear physics beyond A=4.
For « true nuclei » (Ca, Pb, U…) the « traditional potential approach » will remain forever: number of Wick contractions is 2 for N … but 10^{1494} for ^{235}U

THE EFT "SOLUTION" FOR BUILDING NUCLEAR POTENTALS

The EFT theories applied to Nuclear Physics started with Weinberg's seminal works early 90's

Nuclear forces from chiral lagrangians
S. Weinberg, Phys. Lett. B 251, 288 (1990)
Effective chiral lagrangians for nucleon-pion interactions and nuclear forces
S. Weinberg, Nucl. Phys. B363 (1991)

They are based on the « unproved but suposed to be true » « folk theorem » !

The idea was to keep only **non-relativistic N** and **π** and let them **interact with the most general possible lagrangian consistent with isospin and (broken) chiral symmetry**
These are the only remaining from QCD which « inspired » them.

There is **an infinity of such lagrangians** but since it was aimed to be a **low-energy theory** these terms will be classified in increasing order of some **small parameter Q/M** where

- **Q** is the scale for the momenta involved in this EFT (all p>Q are « integrated out »)
- **M** « a QCD scale ».

The (not very helpful!) lagrangian density is

$$
\begin{aligned}
L_{\pi N} = & -\frac{1}{2}\frac{1}{D}\left\{\frac{1}{D}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi} + m_\pi^2\vec{\pi}^2\right\} + \bar{N}\left\{i\partial_0 - m_N\right\}N \\
& - \bar{N}\left\{\frac{2}{D F_\pi} g_A \vec{\tau}(\vec{\sigma}\cdot\vec{\nabla})\vec{\pi}) + \frac{2}{D F_\pi^2}\vec{\tau}\cdot(\vec{\pi}\times\partial_0\vec{\pi})\right\}N \\
& - \frac{1}{2}C_S(\bar{N}N)(\bar{N}N) - \frac{1}{2}C_T(\bar{N}\vec{\sigma}N)(\bar{N}\vec{\sigma}N) + \ldots
\end{aligned}
$$

$$D = 1 + \frac{\vec{\pi}^2}{F_\pi^2} \qquad F_\pi = 195\ \mathrm{MeV}$$

This approach was justified by S. Weinberg as an attempt to put some order in the meson theory :

Due to « the incresing number of paramters in the meson theory as more ans more meson types are included »

He was aware from the very beging that

« It is not clear which expansion will be more useful in dealing with the two-nucleon problem, but the expansion in powers of momenta gives far more specific informa- tion about multi-nucleon potentials »

Let us see what happens some years latter…

Weiberg ideas were immeditately developped by his PhD students
C. Ordonez and U. van Kolck, Phys. Lett. B 291, 459 (1992)

They added other terms to the initial Weinberg Lagrangian

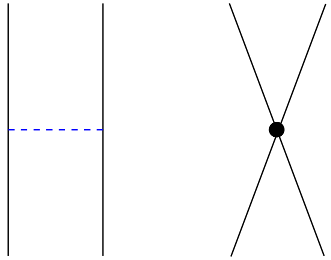
$$L_{\pi N} = L_0 + L_1 + L_2 + \ldots$$

and obtained the higher order corrections in **Q/M**

In terms of $\mathbf{V_{NN}}$ one gets

<u>At leading order</u> (LO) on **Q/M** **(already in Weinberg 90)**

$$V_{NN}^{(LO)}(x) = V_\pi + [C_S + C_T(\vec{\sigma}_1 \cdot \vec{\sigma}_2)]\delta(\vec{x})$$



OPEP + (attractive) **contact terms to be regularized** (**Λ**) **[2 parameters + Λ]**

Quite a primary description: no any repulsive core, no LS, no tensor..

<u>At next to leading order</u> (NLO) [(Q/M) corrections] nothing new appears…

At next-to leading order N2LO [$(Q/M)^2$ corrections]

- **NN** : **Full operator structure due to *ω*, *ρ*, *σ*, ..** mesons apears

$$
\begin{aligned}
V_2^a \;=\;& -\frac{2g_A}{F_\pi^2}(\tau_1\cdot\tau_2)\frac{(\vec\sigma_1\cdot\vec q)(\vec\sigma_2\cdot\vec q)}{q^2+m_\pi^2}\left[A_1q^2+A_2k^2+\frac{2g_A}{m_N}\frac{\vec q\cdot\vec k}{\sqrt{q^2+m_\pi^2}}\right]\\
&+\;[C_3q^2+C_4k^2](\vec\sigma_1\cdot\vec\sigma_2)+\frac{iC_5}{2}(\vec\sigma_1+\vec\sigma_2)(\vec q\times\vec k)\\
&+\;C_6(\vec\sigma_1\cdot\vec q)(\vec\sigma_2\cdot\vec q)+C_7(\vec\sigma_1\cdot\vec k)(\vec\sigma_2\cdot\vec k)
\end{aligned}
$$

introducing 7 new parameters (**A_1,A_2,C_3,C_4,C_5,C_6,C_7**) + **Λ's**

- **NN** : contribution due to **2π-exchange** terms
 (complicated analytical structure)

$$
-\frac{1}{128}\left(\frac{g_A}{F_\pi}\right)^4\int\frac{d^3l}{(2\pi)^3}\frac{1}{\omega_+^3\omega_-}\left[\left(\frac{3}{\omega_-}+\frac{8\tilde{t}_1\cdot\tilde{t}_2}{\omega_++\omega_-}\right.\right.
$$

- **NNN forces** generated by

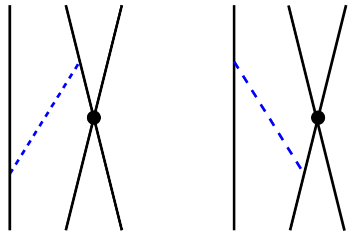
$$
{}^2+4\left(\frac{3}{\omega_++\omega_-}+\frac{8\tilde{t}_1\cdot\tilde{t}_2}{\omega_-}\right)\boldsymbol{\sigma}_1\cdot(
$$

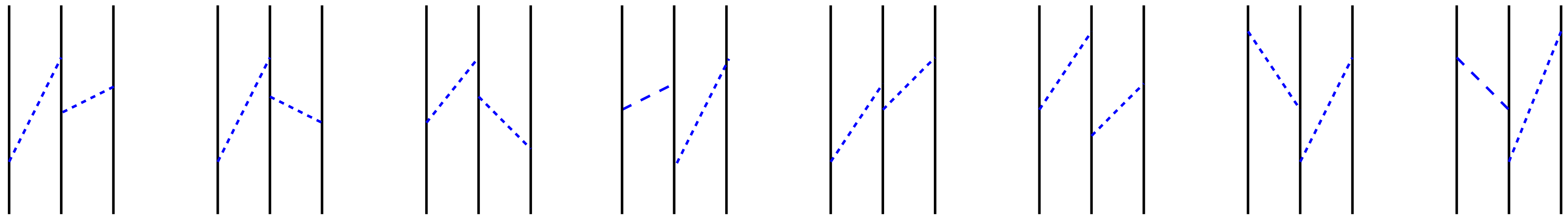
t $|l|\sim Q$. Other one-loop graph

$$
\begin{aligned}
V_{3N} \;=\;& -\frac{1}{2}\left(\frac{2g_A}{F_\pi}\right)^2(\vec\tau_1\cdot\vec\tau_2)(\vec\tau_2\cdot\vec\tau_3)(\sigma_1\cdot\nabla_1)(\sigma_2\cdot\nabla_2)(\sigma_2\cdot\nabla_2)(\sigma_3\cdot\nabla_3)\left[\hat Y(r_{12})Y(r_{23})+\hat Y(r_{23})Y(r_{12})\right]\\
&-\;\delta(\vec r_{12})\left(\frac{2g_A}{F_\pi}\right)^2[C_S+C_T(\vec\sigma_1\cdot\vec\sigma_2)]\,(\sigma_2\cdot\nabla_2)(\sigma_3\cdot\nabla_3)Y(r_{23})\;+\;\text{Perms}
\end{aligned}
$$

They have the standard 2π-exchange form, **without introducing new parameters**

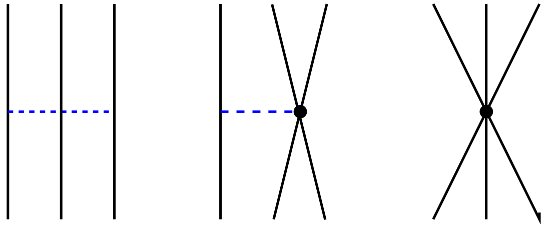
NB: In the **t-ordered** perturbation theory used in non-relativistic QM this corresponds to 10 different contributions to V_{NNN}





At N3LO [(Q/M)³ corrections]

New **3NF** appears from the t-ordered of



$$V_3^c = 2\left(\frac{2g_A}{F_\pi^2}\right)^2 \frac{(\vec\sigma_1\cdot\vec q\,')(\vec\sigma_3\cdot\vec q)}{(q^2+m_\pi^2)(q'^2+m_\pi^2)}\left[(\vec\tau_1\cdot\vec\tau_3)(B_1(\vec q\cdot\vec q\,')+m_\pi^2 B_3)+B_2\vec\tau_2\cdot(\vec\tau_1\times\vec\tau_3)\vec\sigma_2\cdot(\vec q\times\vec q\,')\right]$$
$$-\;2\left(\frac{g_A}{F_\pi^2}\right)^2\frac{1}{q^2+m_\pi^2}(\vec\sigma_3\cdot\vec q)\,[(\vec\tau_1\cdot\vec\tau_3)(D_1\vec\sigma_1+D_2\vec\sigma_2)\cdot\vec q+(\vec\tau_2\cdot\vec\tau_3)(D_2\vec\sigma_1+D_1\vec\sigma_2)\cdot\vec q]$$
$$+\;E_1+E_2(\vec\sigma_2\cdot\vec\sigma_3)+\frac{E_3}{2}(\vec\tau_1\times\vec\tau_3)[(\vec\sigma_1\cdot\vec\sigma_2)+(\vec\sigma_2\cdot\vec\sigma_3)]+\dots$$

This time with 7 new parameters : **B_1, B_2, D_1, D_2, E_1, E_2, E_3** + **Λ's**

………..

………..

The physical content of this EFT approach is quite poor (non relativistic N and **π**)

but

one obtains this way a systematic « algorithm » to generate new terms in the NN+NNN+NNNN+… Hamiltonian, according to some hierarchical « counting scheme »

This is certainly the origin of EFT planetary success, as pointed out by Weinberg himself

« It is in dealing with multi-nucleon potentials that our approach becomes most useful »

At this point, **two main « EFT Schools »** appeared :

I

Those trying to keep a rigourous QFT approach to the problem, by using renomalization techniques and sending $\Lambda \to \infty$

Limited application (hard with tensor force)

Not very accurate predictions, limited N2LO and to light nuclei

II

Those pushing the Q/M expansion to produce increasingly accurate V_{NN}+V_{NNN}+ V_{NNNN} …
with a Λ-dependence of the results (smoothed by renormalisation techniques)

Forgetting the low-energy origin of all that and describing NN processes above 300 MeV !

« EFT » $V_{NN+NNN+..}$ « constructors »

IDAHO	: **Machleidt, Samarruca, Entem,…**
BOCHUM-BONN-JUELICH	: **Epelbaum, Meisner, ….**

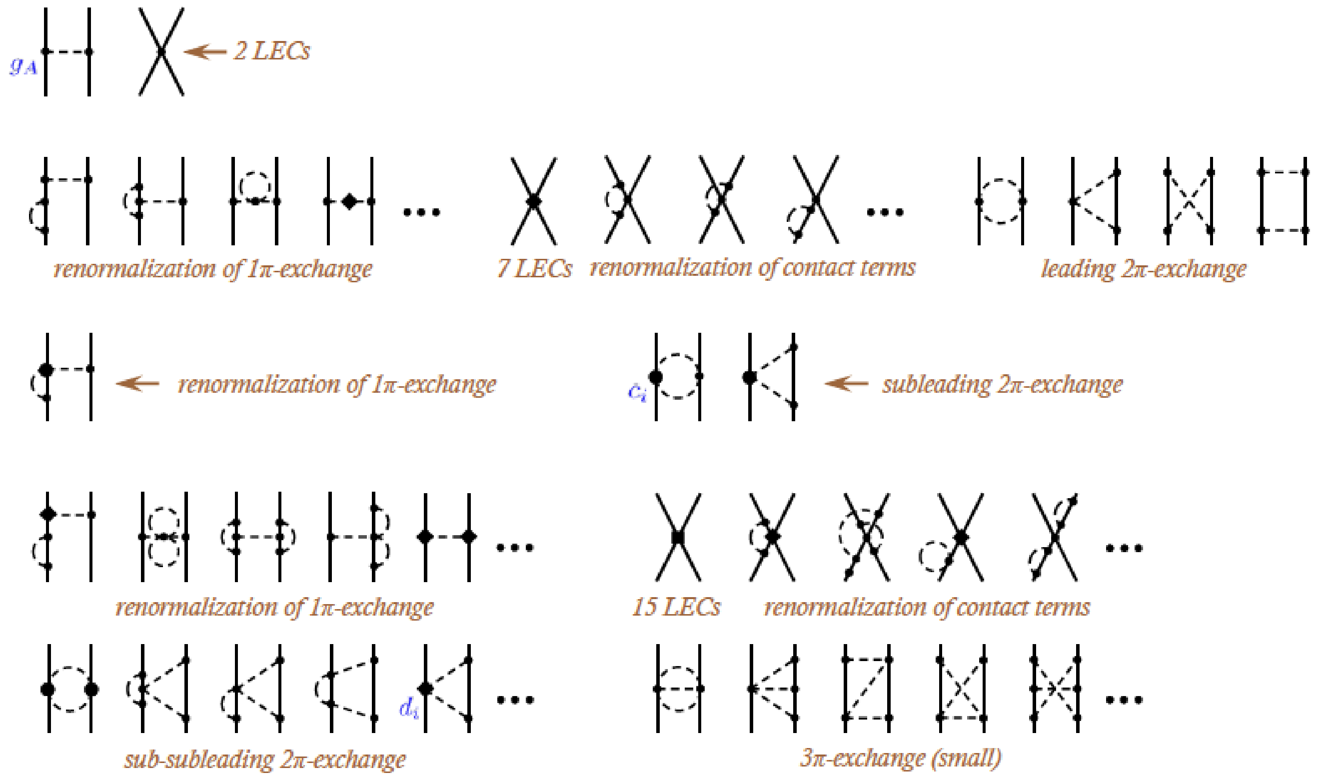
…..(sorry for so many others)

Soon - forced by NN phenomenology - the situation became again quite complicated…. generating "monsters" with 30-40 parameters.....

V_{NN} ensure an almost perfect description of NN data (T_{Lab}<300MeV) X^2/datum=1.01

Example : N3LO NN “potential” : pion+ “contact terms” (regularized)

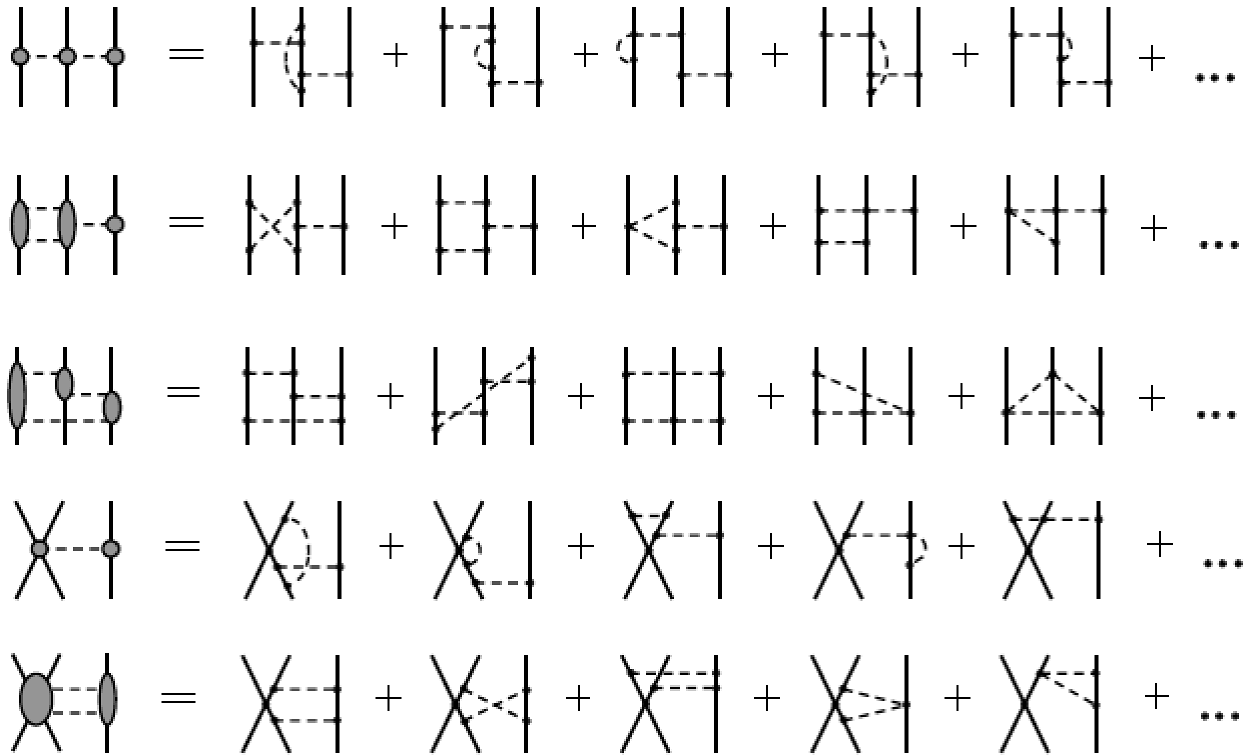
E. Epelbaum Ecole Joliot Curie 2010 (since it has been improved with N4LO and N5LO)



+ isospin-breaking corrections…

…. but were also unable to reproduce ^{3}He and ^{4}He binding energies

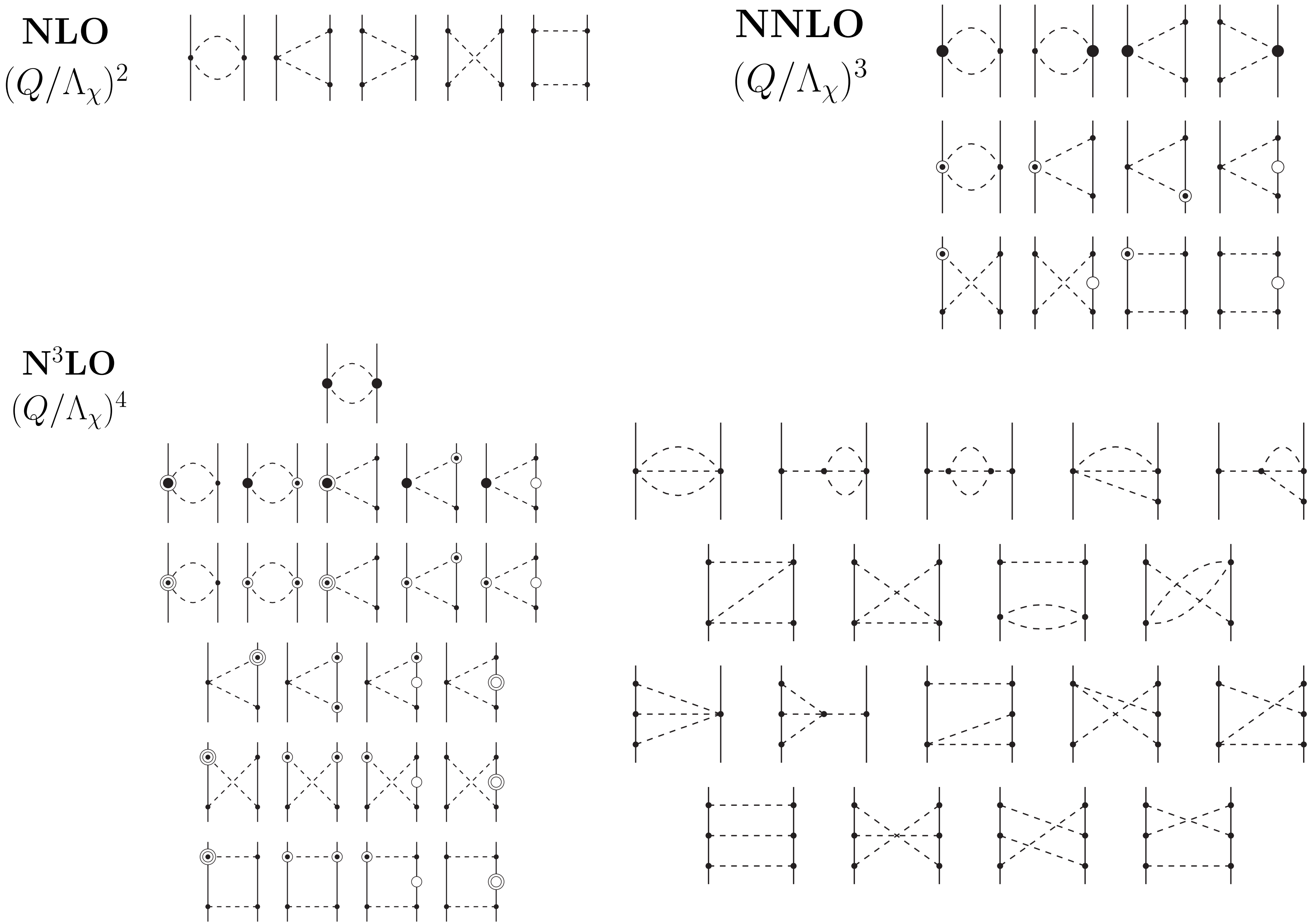
For which one still has to add "some" corrections (3NF)…with corresponding parameters



Some improvement were achieved (e.g. n-^{3}H elastic cross section in the peak region)

but continue to fail in some n-^{2}H polarisation observables (Ay), low energy A=4 resonances, 3- and 4-body breakup…

Some further improvements (N3LO IDAHO Potential Machleid, Entem 2011)



Some selected 3NF

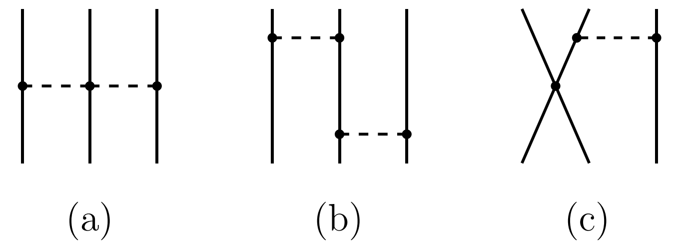


Figure 17: Three-nucleon force diagrams at NLO. Notation as in Fig. 1.

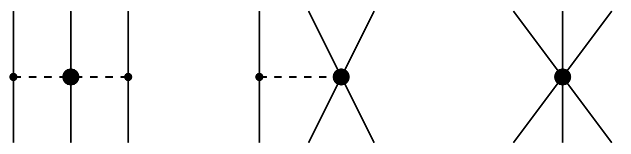


Figure 18: The three-nucleon force at NNLO. From left to right: 2PE, 1PE, and contact diagrams. Notation as in Fig. 1.

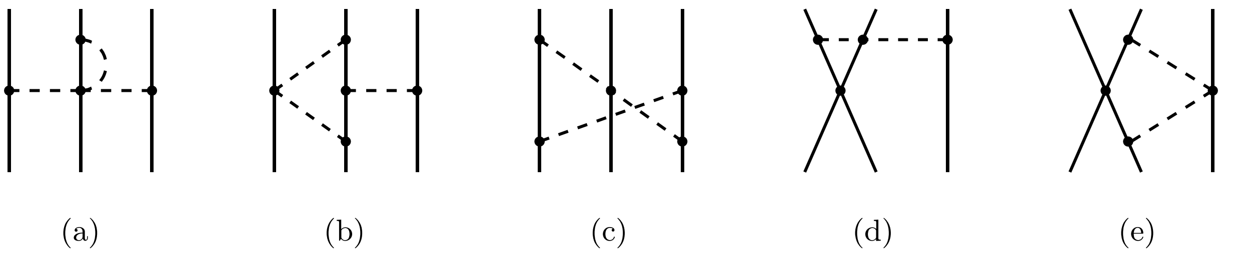


Figure 19: Leading one-loop 3NF diagrams at N^{3}LO. We show one representative example for each of five topologies, which are: (a) 2PE, (b) 1PE-2PE, (c) ring, (d) contact-1PE, (e) contact-2PE. Notation as in Fig. 1.

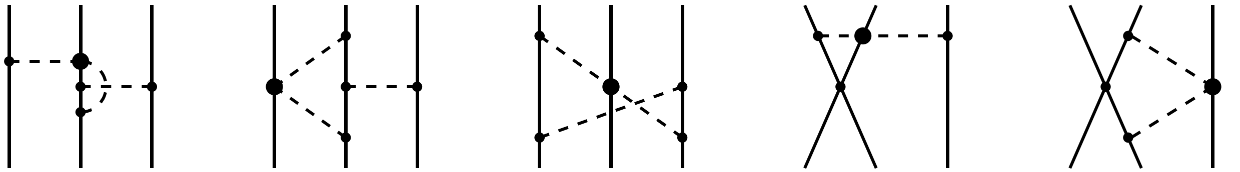


Figure 20: Sub-leading one-loop 3NF diagrams which appear at N^{4}LO. Notation as in Fig. 1.

Leading order 4NF

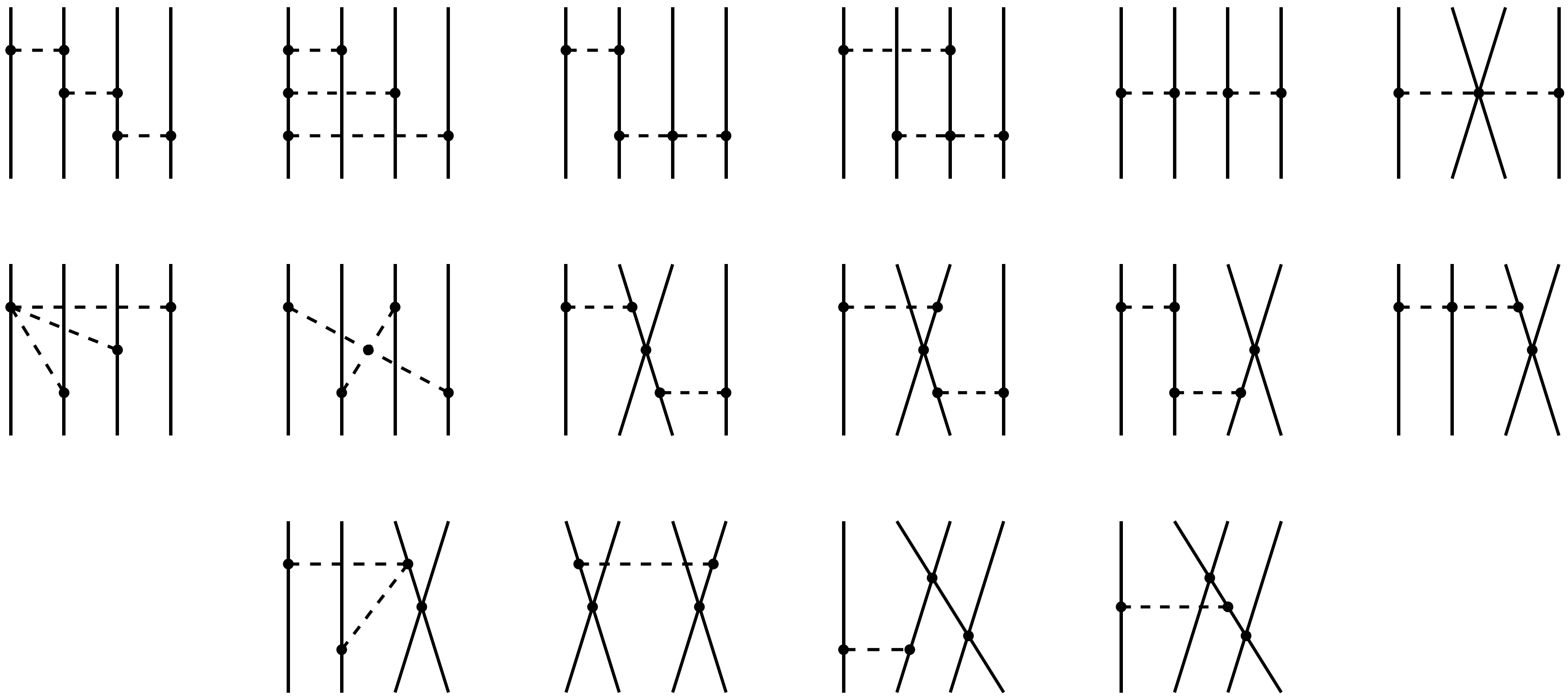


Figure 22: Leading four-nucleon force at N^3LO.

In France we could ask : "A-t-on perdu la tête ?"

" Wouldn't be a little bit crazy ? "

Apart from the luxuriant variety of diagrams above described, som remarks are needed

I. The EFT expansion is not finsihed: the state of the art is now N5LO

see e.g. R. Machleidt Int.J.Mod.Phys. E26 (2017) no.01n02, 1740018)

No clear evidence that this expansion will converge and – if at all - when !

II. « big dots » represent contact interactions… between quite extended objets $r_N \cong 0.9$ fm ! assumed to be point-like

III. Each **line** represents a « propagation » of an object (**N**, π) for which there is neither space nor time to propagate on a nuclei : like pictures illustrating a tale

^{4}He as a size of 1.5 fm : as we see in LQCD, a N on a (1.5 fm)^3 box is not really a N

.... Is all that really reasonable ?

Since it is "a bit fictitious", would it not be possible to make it simpler ???

Some simplifications …

The very same authors that developed EFT in 90's were well conscious about the unpleasant developments and try to give consistency to some simplified versions
A step forward in the simplicity was done by "pionless EFT"

" pionless EFT "

No mesons ! only NN+NNN contact interaction (regularized by cut-off **Λ** or renormalisation)
Radical conceptual bareness: "nuclei exist …because N's touch eachother"
At LO : 2 (NN) + 1 (Reg) +1 (3N parameters (without CSB)

$$V_{NN}^{LO}(k,q) = C_1 + C_2(\sigma_1 \cdot \sigma_2) \qquad V_{NNN}^{LO} = \sum_{i<j<k} C_{NNN}^{LO}\left[\tau_i \cdot \tau_j + \tau_i \cdot \tau_k + \tau_j \cdot \tau_k\right]$$

At NLO : <u>add</u> 4 NN + 1 3N

$$V_{NN}^{NLO}(k,q) = D_1 q^2 + D_2 k^2 + (\sigma_1 \cdot \sigma_2)[D_3 q^2 + D_4 k^2]$$

No tensor, no spin-orbit,… but simple (8 parameters)

Meaningful only at LO and NLO : going furher make no sense …otherwise "go to 1"
Nice review in P.F. Bedaque, U. van Kolck, Ann. Rev. Nucl. Part. Sci. 52 (2002) 339

Can we do physics without any physical content ?

In conclusion …

The complexity of Theoretical Nuclear Physics describing the simplest nuclei_seems **unavoidable** - whatever the approach one tries - **if you ask your theory to be accurate** (<1%) **in all the observables and over a wide region of energy** (T_{lab}<300 MeV)

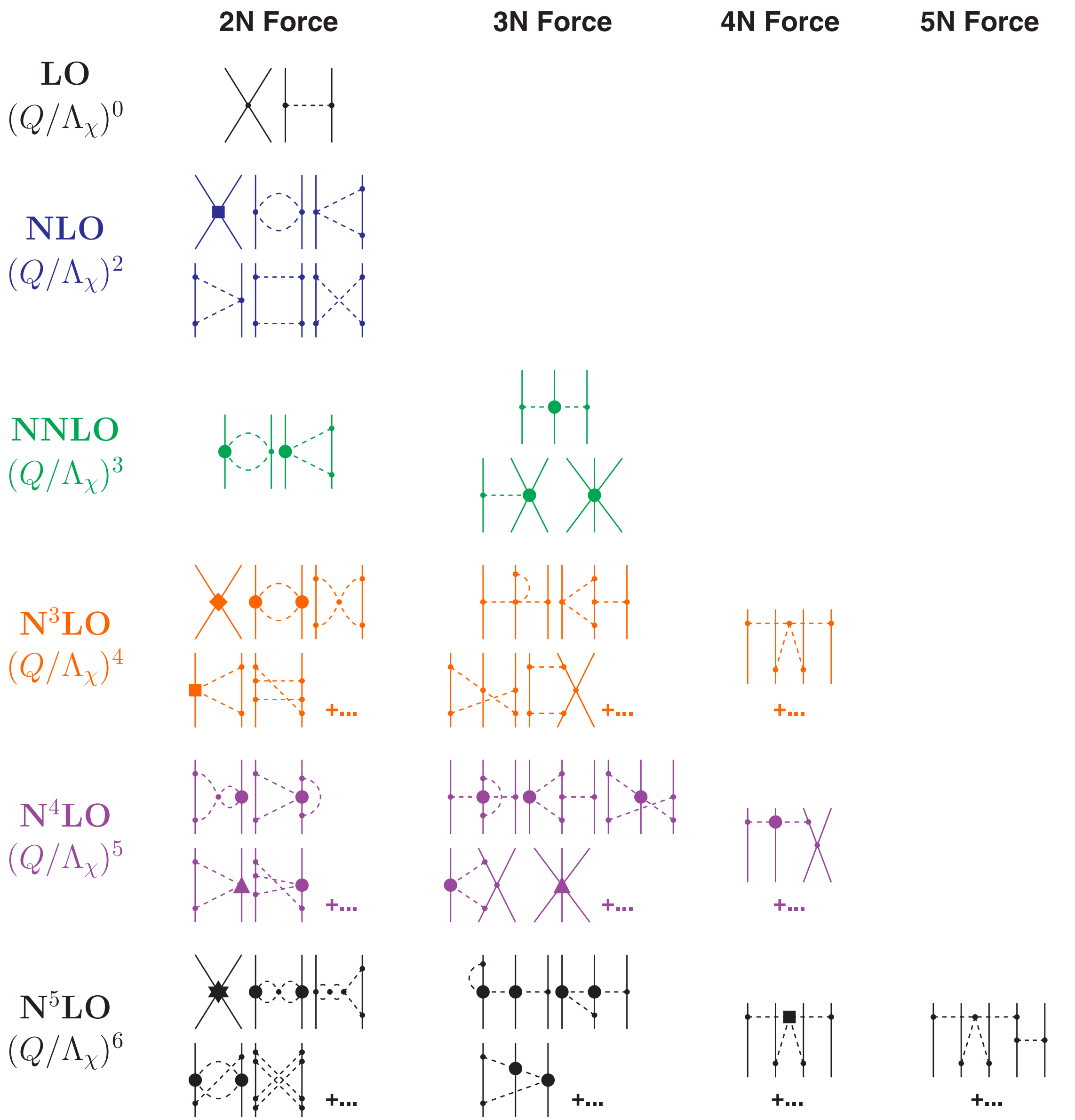
There is no other example in field of physics (in science?) where one deals with 40 parameters
The epistemological contents of such a description is very doubtfull !

This unique complexity is present even before the complexity inherent to the solution of the N-body problem could manifest as well

EFT approach, even by removing all meson contents, does not solve this problem
The only real advantage with respect to the traditional "meson exchange" is to provide a rationale for building this complexity.

It looks more and more like a parameter factory with very little physical content …to parametrize the same complexity.

Entem Machleidt



SUMARY

Fundamental physics (QED,QCD,WI) as described by QFT is **astonishingly simple**
It deals with point-like particles interacting by exchanging point-like « mediators » and depends on a **very small number of parameters**.

The « interaction » is given by terms of coupled fields. This coupling is derived from « gauge invariance », the « new dogma » in theoretical physics.

The « potential » is not a well-defined object in the QFT framework.
The usual definition in terms of scattering amplitude at lowest order contains only a very small part of the interaction.
It is meaningful only when the coupling constant is very small (QED, atomic physics)

The solution of the full QFT problem is limited to very simple cases, mainly QCD for qq and qqq bound states.
They describe the full ensemble of mesons and baryons…with only one or two parameters (essentially m_u=m_d and m_s)

The same simplicity should apply in reproducing the nuclei, in terms of their « elementary constituents », i.e q's.
This was announced in 2015 (2H,3H,3He,3He)…but turned to be « fake »

SUMARY

This simplicity is lost when – either for ignorance or laziness – we make some « approximations ».

For instance :

- When we replace the QFT by some simplified interaction, potentials V :« quark model », EFT

- When we consider complex systems as point–like particles (as in NN, 4He-4He atoms) interacting by « potentials », even QFT-inspired

These potentials become soon terribly complicated, if we asked them to describe the physical phenomena, since in fact they hidde an underlying complexity

They seem however unavoidables, the prize to pay, if we want to go much faster than our technical capabilities allows us to go