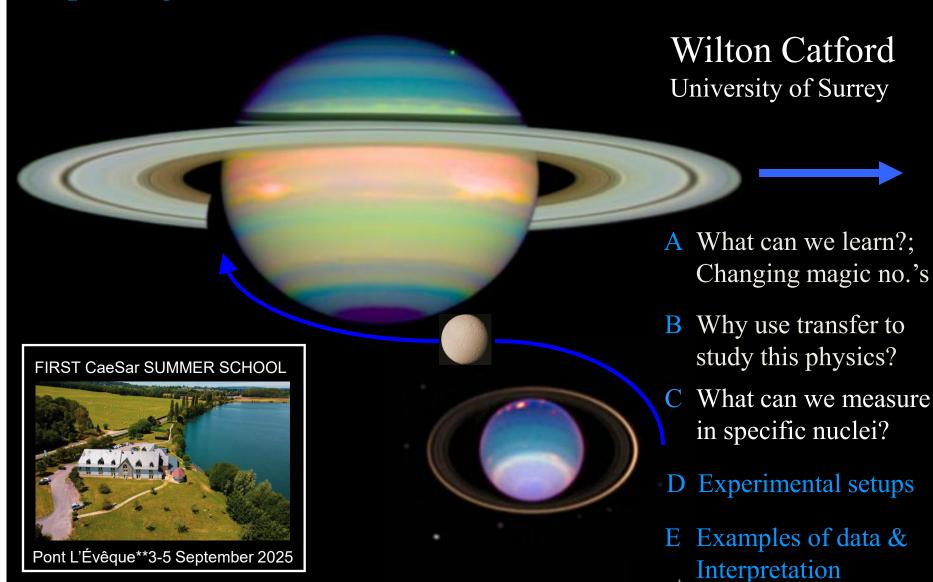
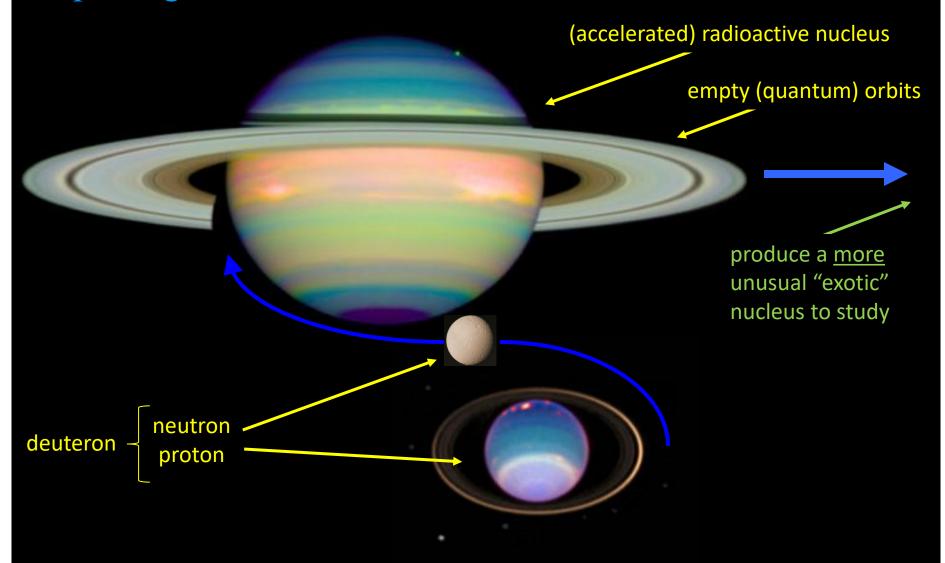
Exploring Unusual Nuclei with Nucleon Transfer Reactions

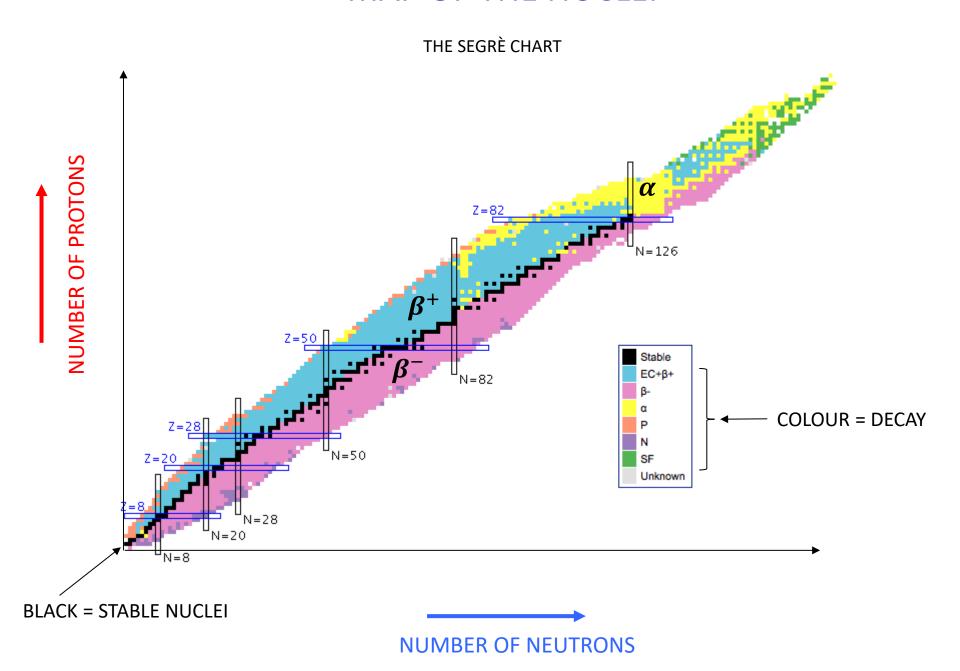


SATURN HST/ IR 1998, TETHYS VOYAGER2 1981, URANUS HST/ IR 1986

Exploring Unusual Nuclei with Nucleon Transfer Reactions

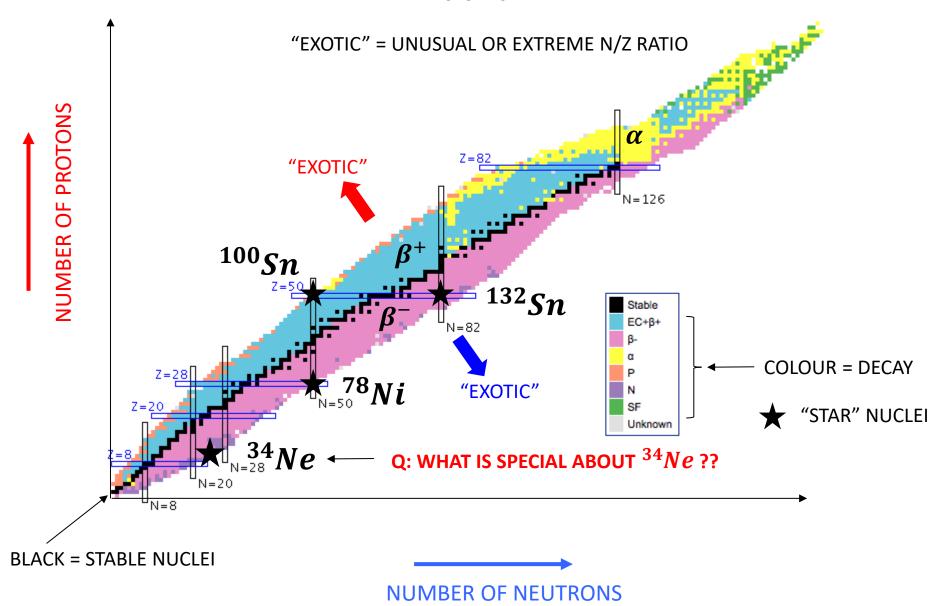


MAP OF THE NUCLEI

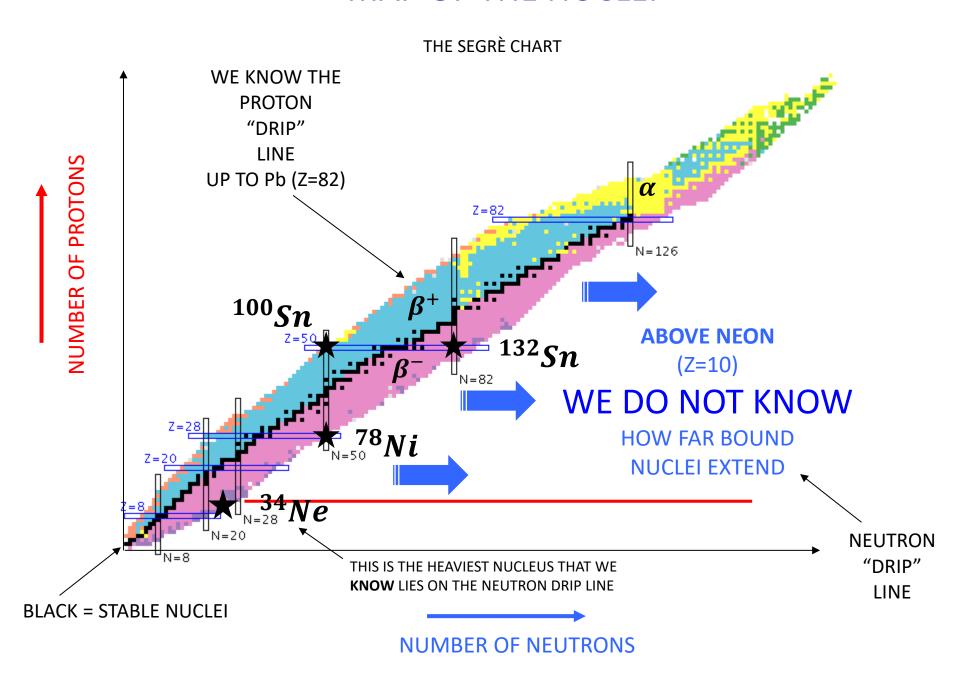


MAP OF THE NUCLEI



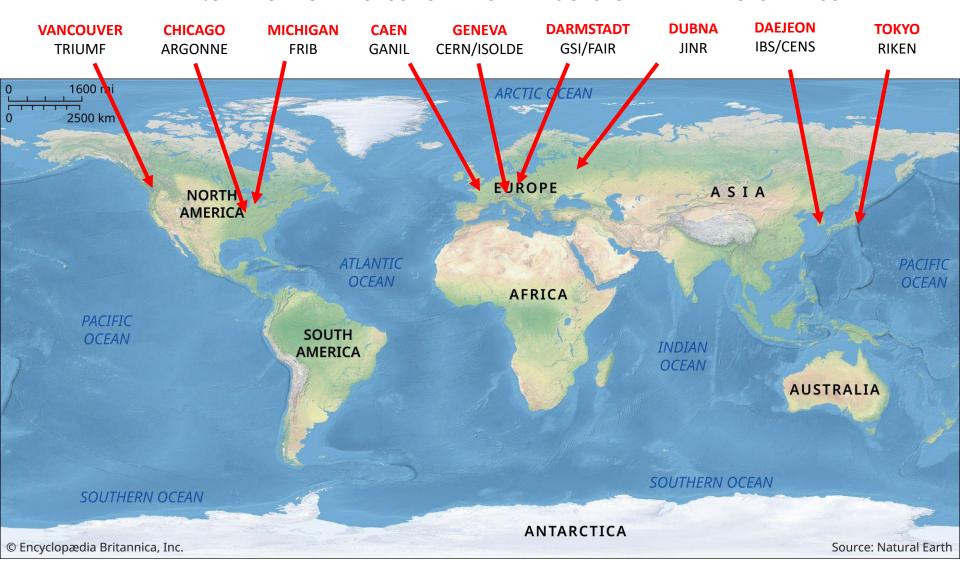


MAP OF THE NUCLEI



MAP OF THE WORLD

EVERY MAJOR LABORATORY PRODUCING INTENSE BEAMS OF SHORT-LIVED RADIOACTIVE NUCLEI



MAP OF THE WORLD

AN IRRELEVANT ASIDE



This is where I am from

SO HOW DO WE MAKE A BEAM OF RADIOACTIVE NUCLEI?

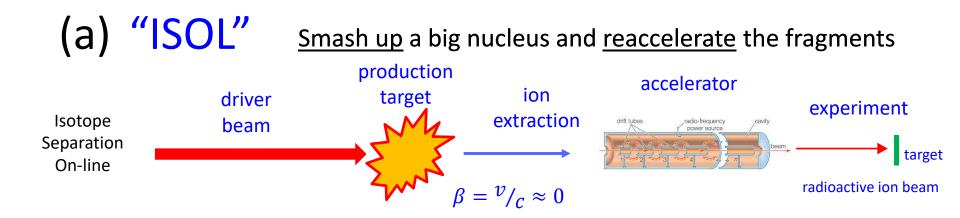


Q: WHY DO I TALK JUST ABOUT RADIOACTIVE **BEAMS** ??

(AND NOT ABOUT TARGETS MADE OF THESE RADIOACTIVE NUCLEI ??)

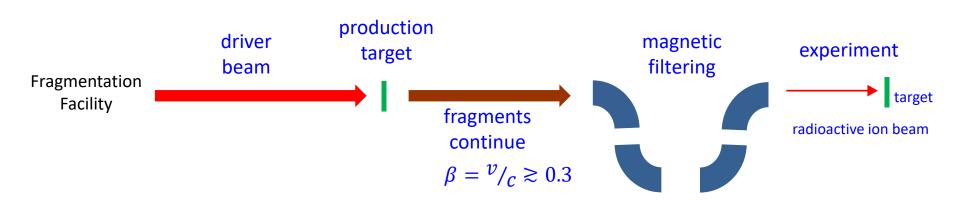


SO HOW DO WE MAKE A BEAM OF RADIOACTIVE NUCLEI?



(b) "IN FLIGHT"

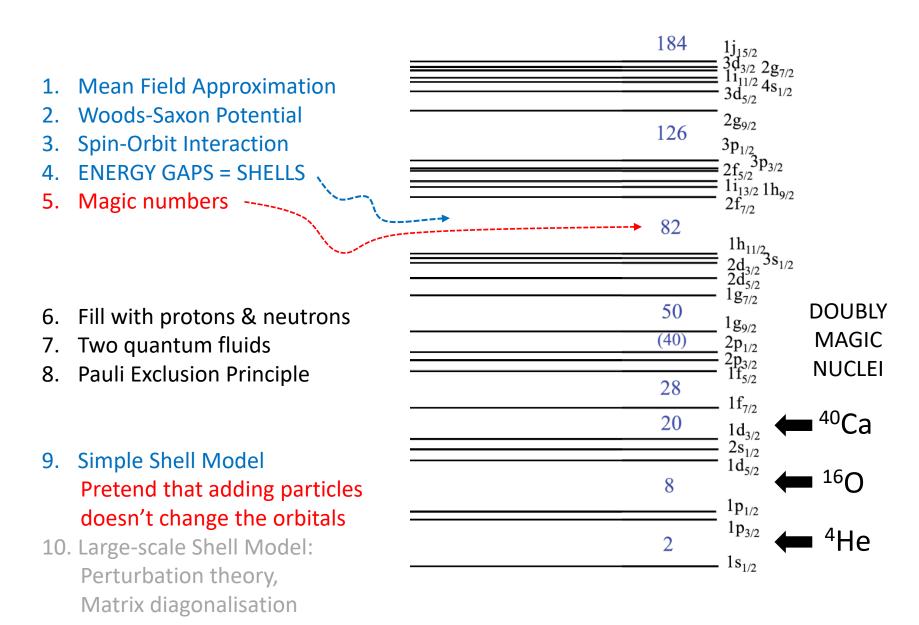
Tear pieces out of a big nucleus and filter the "good guys" magnetically



- Motivation: nuclear structure reasons for transfer
- What quantities we actually measure
- What reactions/energies can we choose to use?
- Inverse Kinematics
- Implications for Experimental approaches
- Why do people make the choices that they do?
- Example experiments and results



SINGLE PARTICLE STATES in the nuclear shell model...



Changes – tensor force, p-n

Residual interactions <u>move</u> the mean field levels

Magic numbers "migrate", changing stability, reactions, collectivity...

Watch as we reduce the proton number...

proton filling affects neutron orbitals

Changes – tensor force, p-n

Residual interactions <u>move</u> the mean field levels

Magic numbers "migrate", changing stability, reactions, collectivity...

Watch as we reduce the proton number...

proton filling affects neutron orbitals

Changes – tensor force, p-n

Residual interactions <u>move</u> the mean field levels

Magic numbers "<u>migrate</u>", changing stability, reactions, collectivity...

Similarly... neutron filling affects proton orbitals

Changes – tensor force, p-n

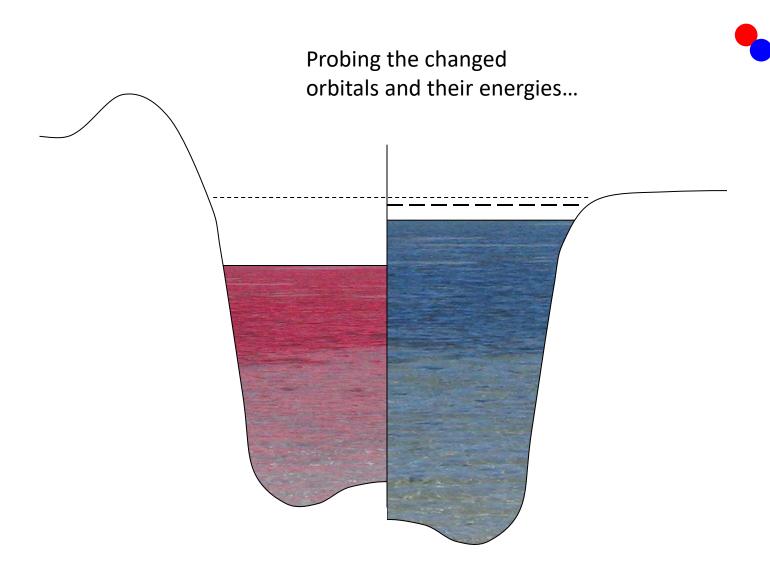
Residual interactions <u>move</u> the mean field levels

Magic numbers "migrate", changing stability, reactions, collectivity...

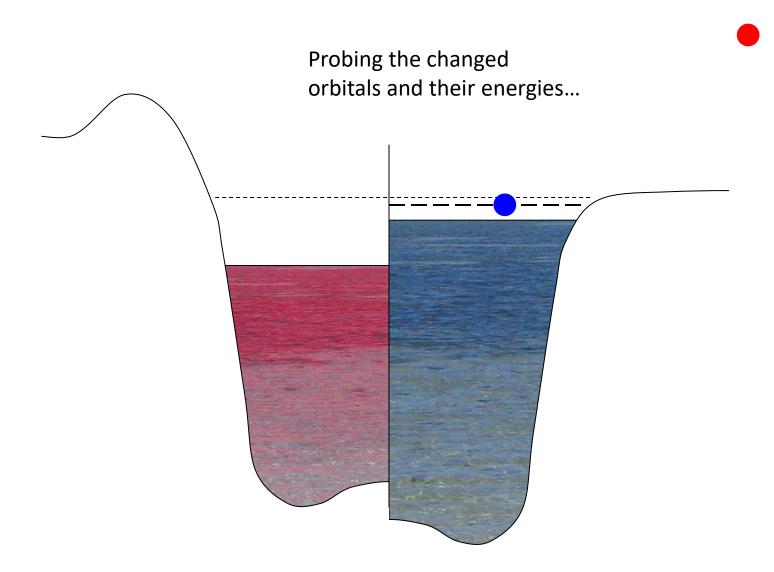
Similarly...

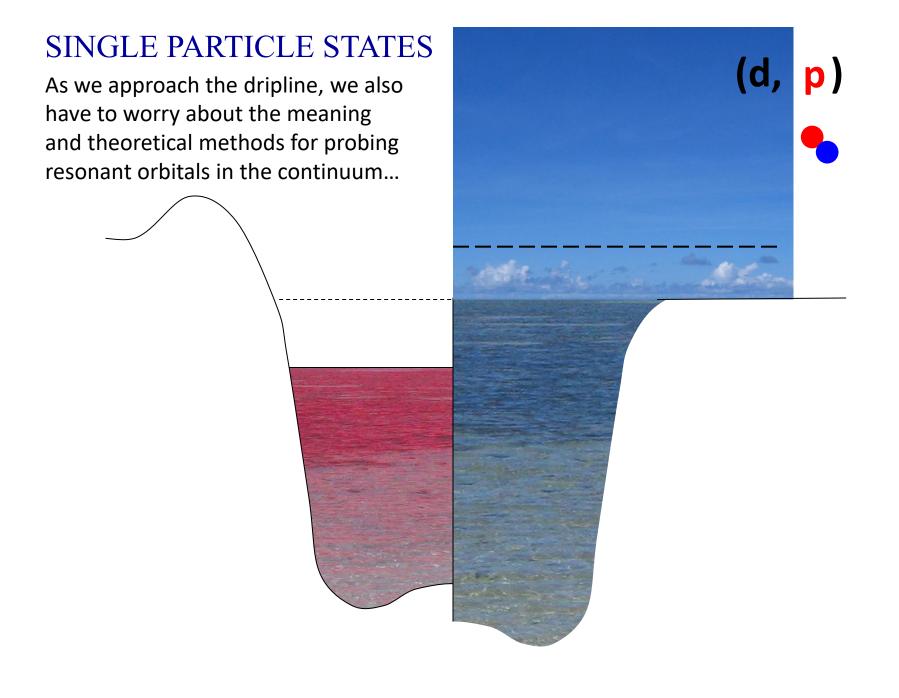
neutron filling affects
proton orbitals

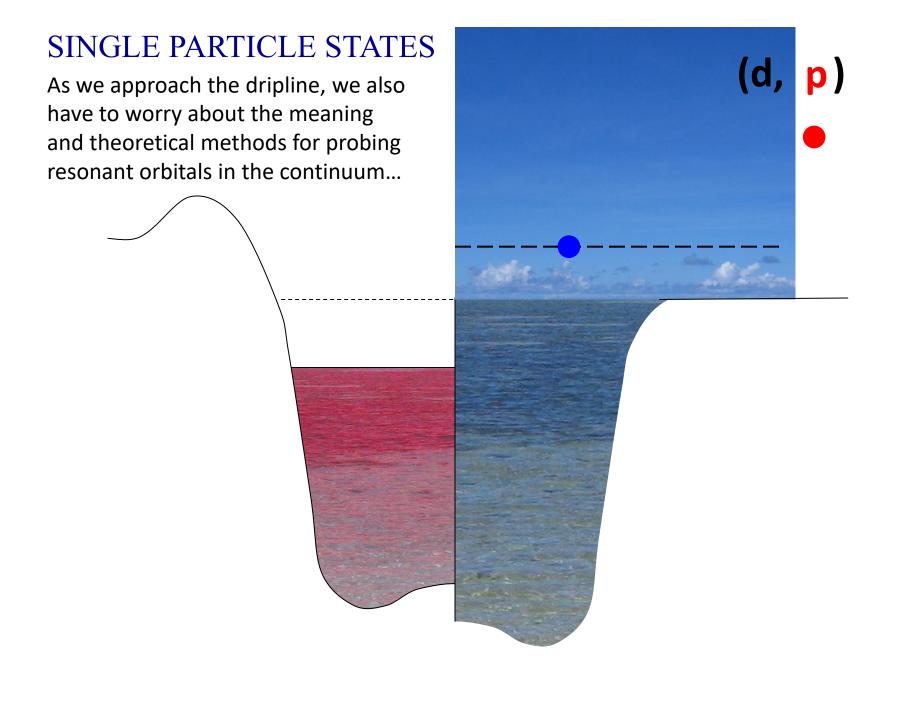
(d, p)



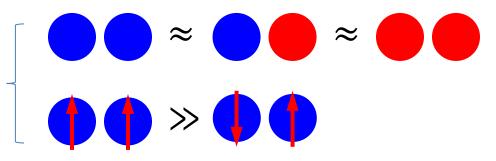
(d, p)



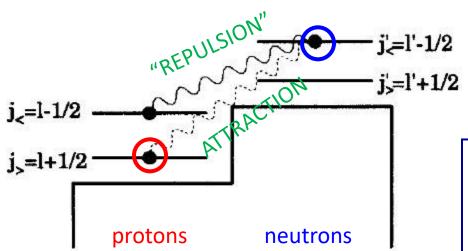




Nucleon-Nucleon Interaction:



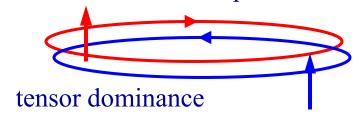
Changing Magic Numbers:



T. Otsuka et al., Phys. Rev. Lett. 97, 162501 (2006).

T. Otsuka *et al.*, Phys. Rev. Lett. **87**, 082502 (2001).

attractive p-n interaction



Nuclei are quantum fluids comprising two distinguishable particle types...

They separately fill their quantum wells...

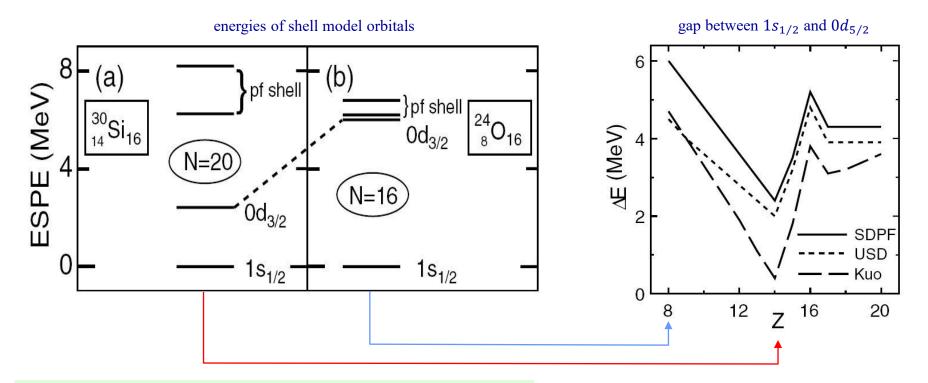
Shell structure emerges...

Valence nucleons interact...

This can perturb the orbital energies...

The shell magic numbers for p(n) depend on the level of filling for the n(p)

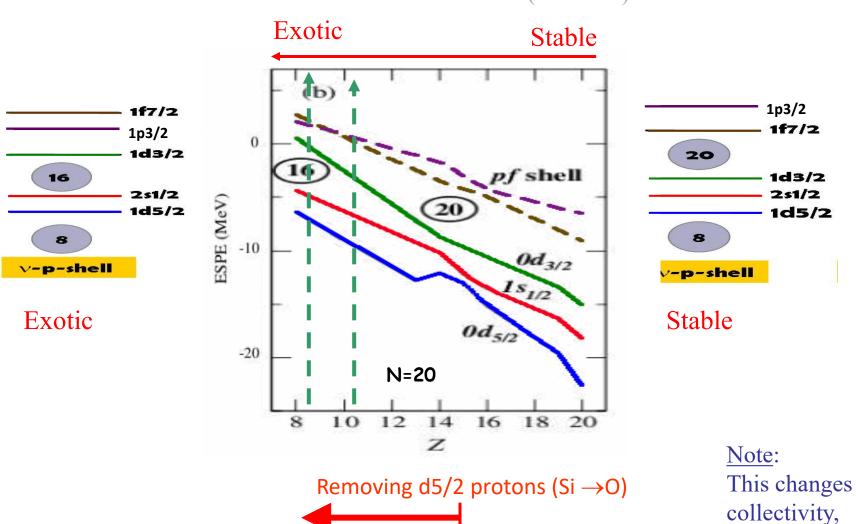
Changing Magic Numbers



As the occupancy of the $j_{>}$ orbit $d_{5/2}$ is reduced in going from (a) 30 Si to (b) 24 O, then the attractive force on $j_{<}$ $d_{3/2}$ neutrons is reduced, and the orbital rises relatively in energy. This is shown in the final panel by the $s_{1/2}$ to $d_{3/2}$ gap, calculated using various interactions within the Monte-Carlo shell model.

The trend varies for different orbitals in nuclei, as we go more exotic...

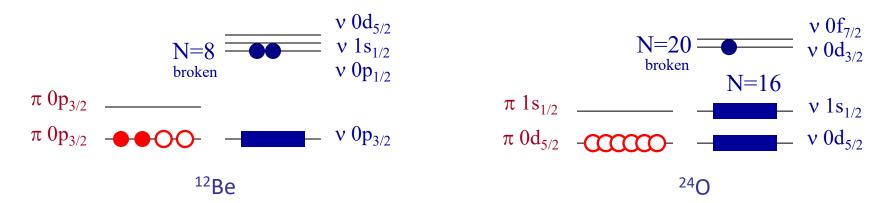
Utsuno et al., PRC,60,054315(1999) Monte-Carlo Shell Model (SDPF-M)



gives relative rise in v(d3/2)

also...

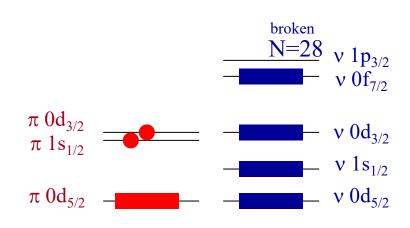
Changing Magic Numbers: proton-deficient examples



In the lighter nuclei (A<50) a good place to look is near closed proton shells, since a closed shell is followed in energy by a j $_{>}$ orbital. For example, compared to 14 C the nuclei 12 Be and 11 Li (just above Z=2) have a reduced π (0p_{3/2}) occupancy, so the N=8 magic number is lost.

Similarly, compared to 30 Si, the empty π (0d_{5/2}) in 24 O (Z=8) leads to the breaking of the N=20 magic number.

Another possible extreme is when a particular neutron orbital is much more complete than normal.





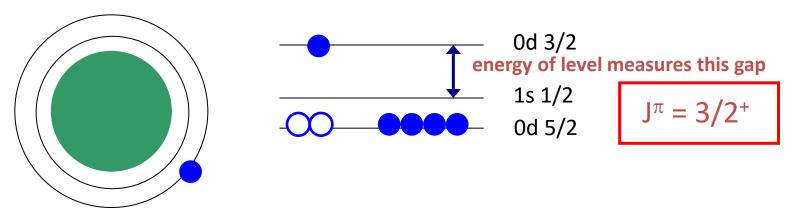
Nuclear states are not in general pure SP states, of course

For nuclear states, we measure the **spin** and **energy** and the magnitude of the single-particle component for that state (the **spectroscopic factor**)

Next slide example: low-lying 3/2⁺ states in ²¹O

Single-particle States MIXING with states with other structures

Example of population of single particle state: ²¹O



The mean field has orbitals, many of which are filled.

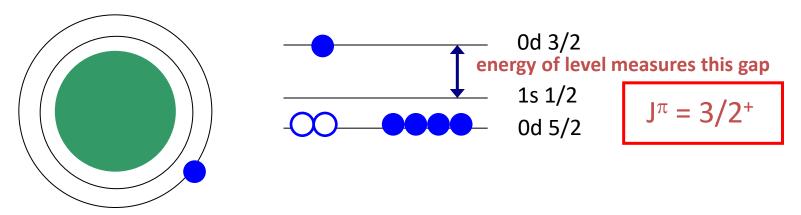
We probe the energies of the orbitals by transferring a nucleon

This nucleon enters a vacant orbital

In principle, we know the orbital wavefunction and the reaction theory

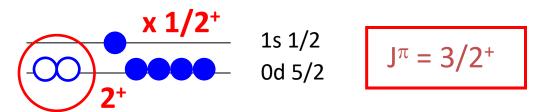
Single-particle States MIXING with states with other structures

Example of population of single particle state: ²¹O



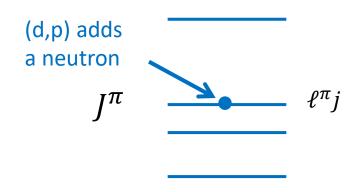
The mean field has orbitals, many of which are filled.
We probe the energies of the orbitals by transferring a nucleon
This nucleon enters a vacant orbital
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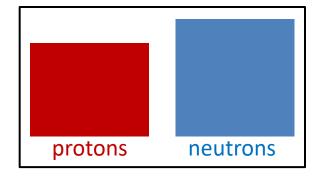
But not all nuclear excited states are single particle states...



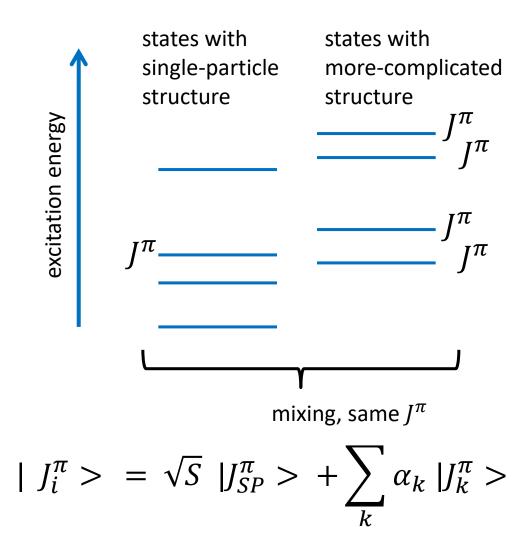
We measure how the two 3/2⁺ states share the SP strength when they mix

Single-particle States MIXING





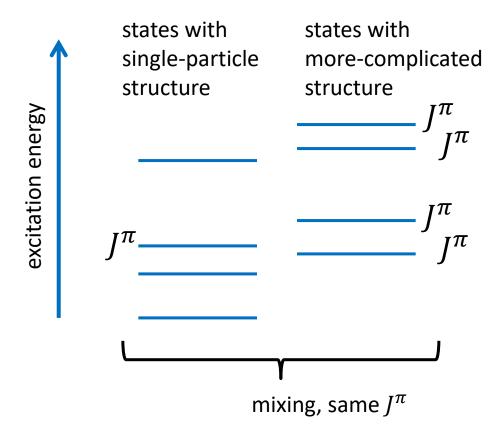
single-particle state, unperturbed core (idealized situation)



$$S = |\langle J_{SP}^{\pi} | J_i^{\pi} \rangle|^2$$

spectroscopic factor = overlap with pure SP state

Single-particle States MIXING



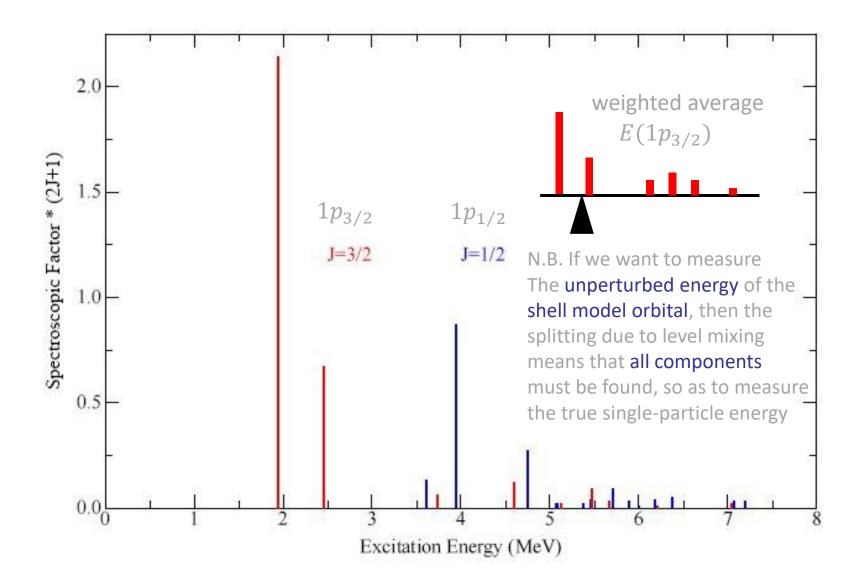
$$|J_i^{\pi}\rangle = \sqrt{S} |J_{SP}^{\pi}\rangle + \sum_{k} \alpha_k |J_k^{\pi}\rangle$$

- we measure transferred ℓ_n from ${}^{d\sigma}\!/{}_{d\Omega}$
- we measure gamma-decays
- we aim to identify J and π
- we model the transfer yield for S=1
- we deduce S from the observed yield

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spectroscopic factor
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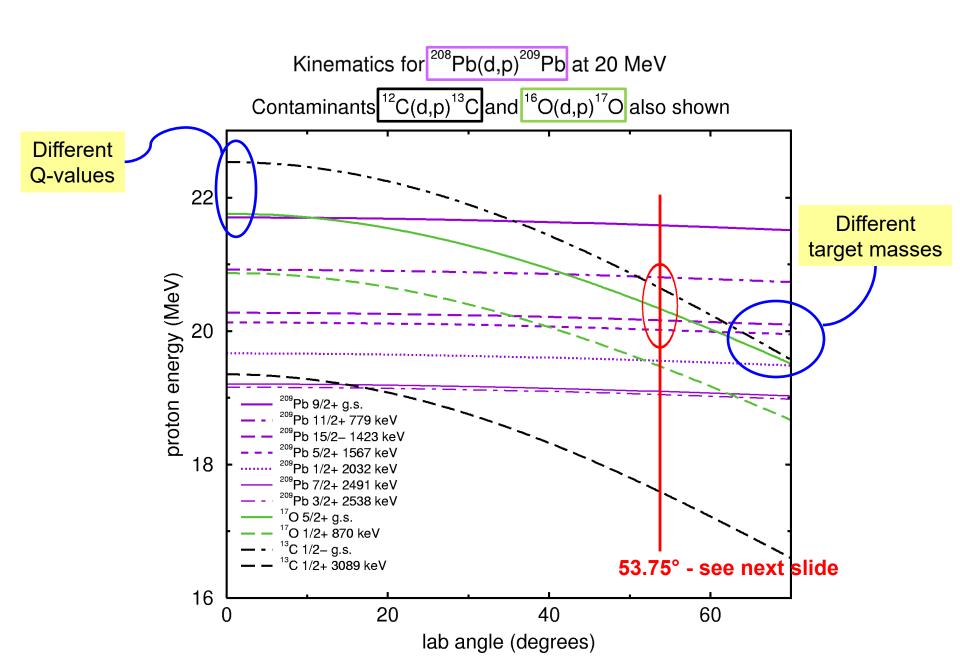
Single-particle States – SPLITTING of strength due to MIXING



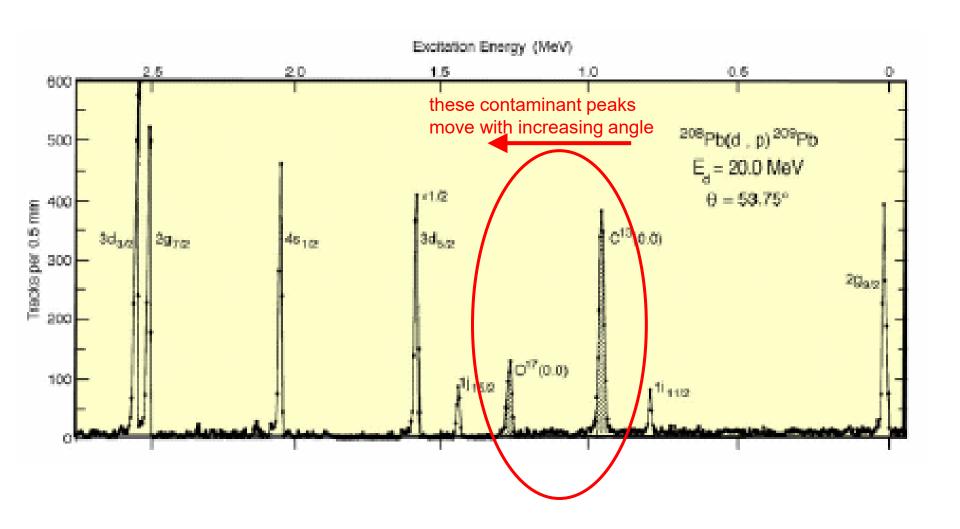
All p-wave ($\ell = 1$) spectroscopic strengths in ${}^{40}Ca(d, p){}^{41}Ca$

Plot: John Schiffer

An Experiment to Study Neutron Orbitals Above Doubly Magic ²⁰⁸Pb...



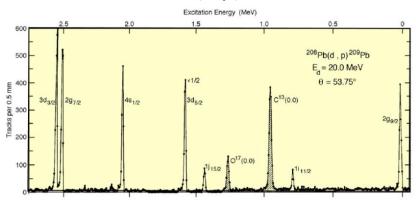
An Experiment to Study Neutron Orbitals Above Doubly Magic ²⁰⁸Pb...

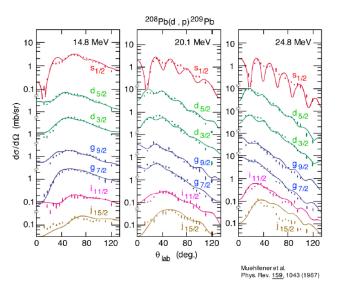


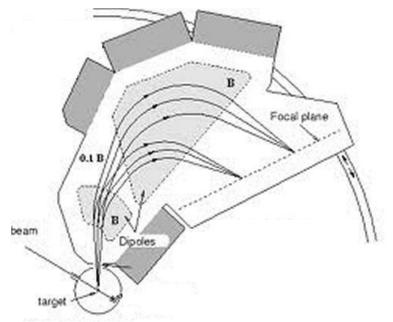


1950's 1960's

²⁰⁸Pb(d,p)²⁰⁹Pb 1967







Deuteron beam + target Tandem + spectrometer >10¹⁰ pps (stable) beam

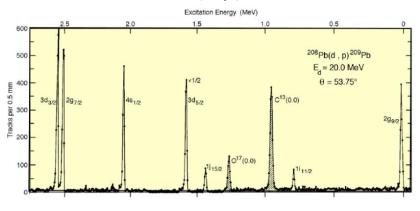


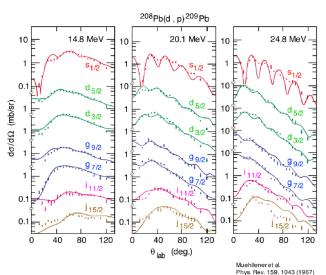


1950's 1960's

STABLE NUCLE

²⁰⁸Pb(d,p)²⁰⁹Pb 1967



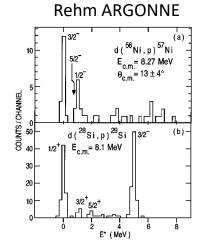


1990's

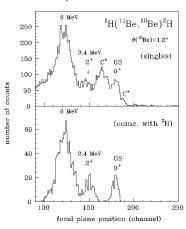
2000's, 2010's, 2020's

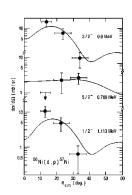
radioactive ion beam

1998 d(⁵⁶Ni,p)⁵⁷Ni



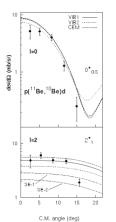
1999 p(11Be,d)10Be Fortier/Catford GANIL



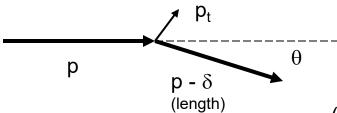


Beam 10⁶ weaker, experiments 10⁶ more difficult (?!)

i.e. fewer statistics



Measuring Spin, or at least... Angular Momentum Transfer



Cosine rule, 2nd order:

$$\theta^2 = \frac{(p_t/p)^2 - (\delta/p)^2}{1 - (\delta/p)}$$

But
$$p_t \times R \ge \sqrt{\ell(\ell+1)}$$
 \hbar (R = max radius)

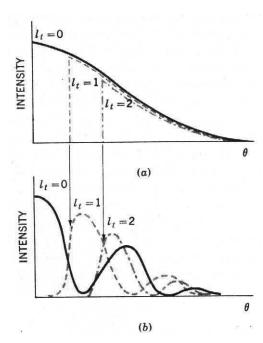
So
$$\theta^2 \ge \frac{\ell(\ell+1) \hbar^2 / p^2 R^2 - (\delta/p)^2}{1 - (\delta/p)}$$

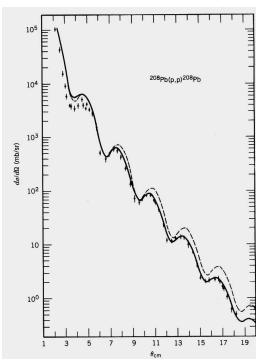
or
$$\theta \geq \text{const} \times \sqrt{\ell(\ell+1)} \text{ neglecting } (\delta/p)$$

$$\theta_{\text{min}}$$
 \approx const \times ℓ

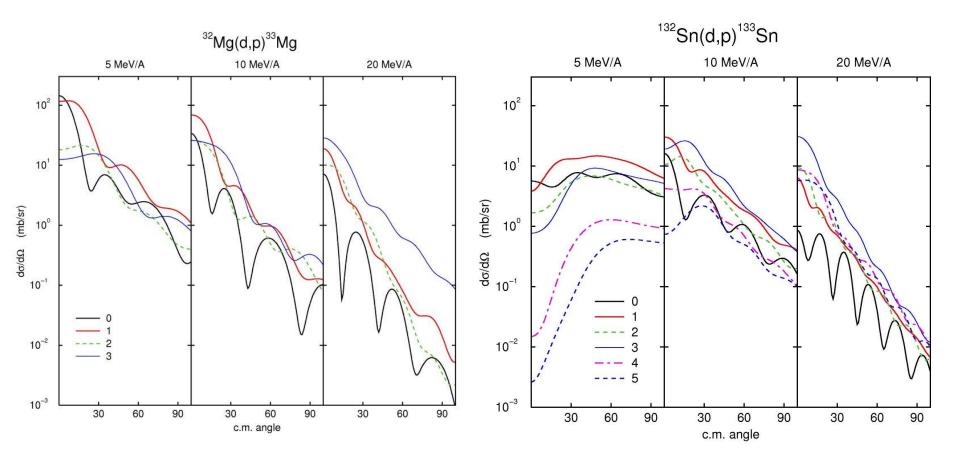
<u>Diffraction structure</u> also expected (cf. Elastics)

<u>PWBA</u> ⇒ spherical Bessel function, $\theta_{\text{peak}} \approx 1.4 \sqrt{\ell(\ell+1)}$

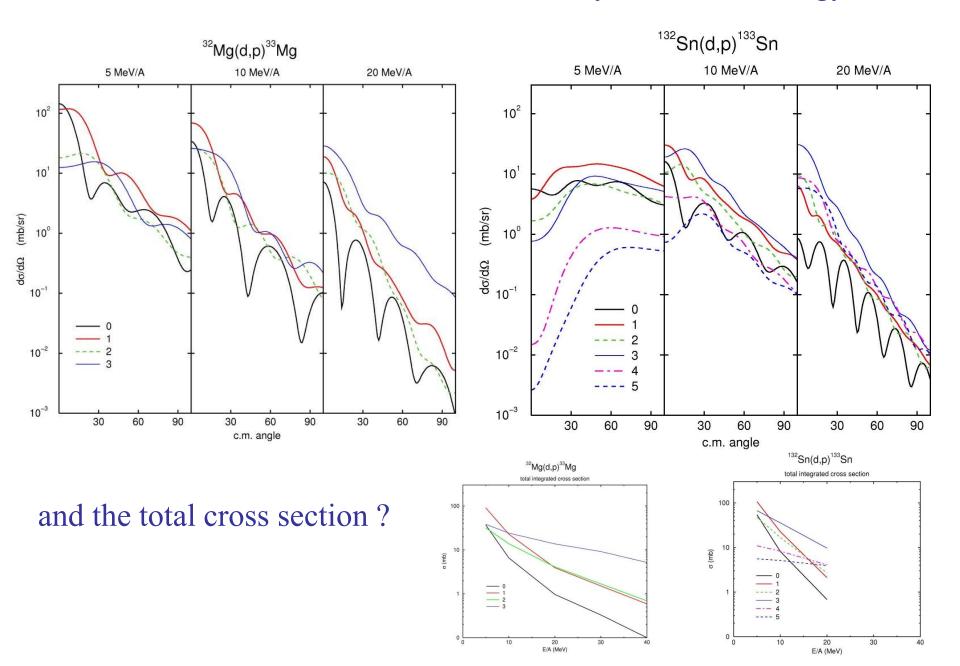




How does the differential cross section vary with beam energy?

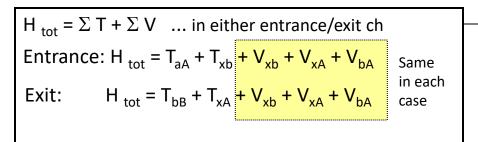


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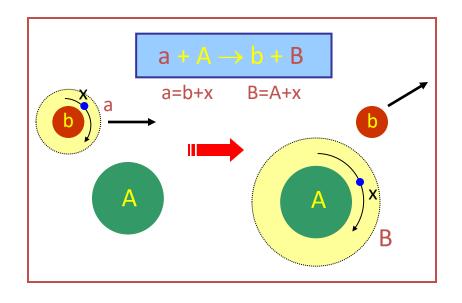


Distorted Wave Born Approximation – Outline (1 of 3)

e.g. (d,p) with a deuteron beam (following H.A.Enge Chap.13 with ref. also to N. Austern book)

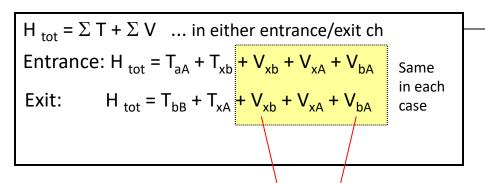


→ But the final scattering state can be written approximately as an outgoing DW using the optical potential for the exit channel:



Distorted Wave Born Approximation – Outline (2 of 3)

e.g. (d,p) with a deuteron beam (following H.A.Enge Chap.13 with ref. also to N. Austern book)



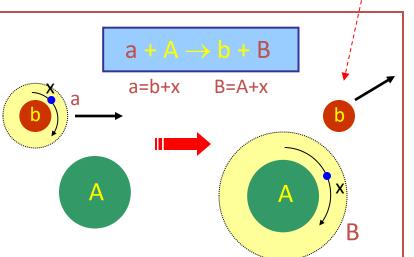
But the final scattering state can be written approximately as an outgoing DW using the optical potential for the exit channel:

$$|\psi_{f}\rangle \approx |\phi_{b}\rangle |\phi_{B}\rangle \chi^{-}_{bB}$$
 Internal outgoing wave functions distorted wave

In the optical model picture, $V_{xb} + V_{bA} \approx U_{bB}$ (= $V^{opt}_{bB} + i W^{opt}_{bB}$), the optical potential)

And the final state, we have said, can be \int approximated by an eigenstate of $U_{\rm bB}$

The transition -inducing interaction is



$$V_{\text{int}} = H_{\text{entrance}} - H_{\text{exit}} = V_{xb} + V_{yA} + V_{bA} - (V_{xb} + V_{bA}) - V_{A}$$
$$= V_{xb} + V_{bA} - U_{bB}$$

Remnant term ≈ 0 if x < < A i.e. $V_{int} \approx V_{xh}$ which we can estimate reasonably well

$$T_{f,i}^{DWBA} = \langle \phi_b \phi_B \chi_{bB}^{-} | V_{xb} | \chi_{aA}^{+} \phi_a \phi_A \rangle$$

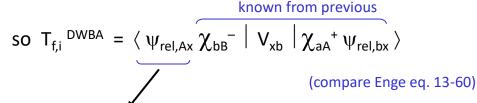
$$= \phi_x \phi_A \psi_{rel,Ax} = \phi_x \phi_b \psi_{rel,bx}$$

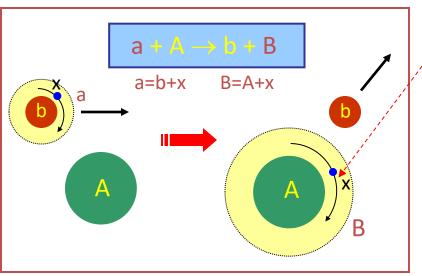
so
$$T_{\text{i,f}}^{\text{ DWBA}}$$
 = $\langle\;\psi_{\text{rel,Ax}}\;\chi_{\text{bB}}^{-}\;\big|\;V_{\text{xb}}^{}\;\big|\;\chi_{\text{aA}}^{}^{+}\,\psi_{\text{rel,bx}}^{}\;\rangle$

known as radial form factor often simple, for the transferred nucleon

e.g. if a = d

Distorted Wave Born Approximation – Outline (3 of 3)





The wave function of the transferred nucleon x, orbiting A, inside of B:

radial wave function u(r) given by $\psi(r) = u(r)/r$

V(r) given by Woods-Saxon; depth determined by known binding energy

 u_{nlj}^*

Woods-Saxon:

$$V(r) = \frac{-V_0}{1 + e^{-(r-r_0A^{1/3})/a}}$$

The radial wave function In the Woods-Saxon potential represents the shell orbital

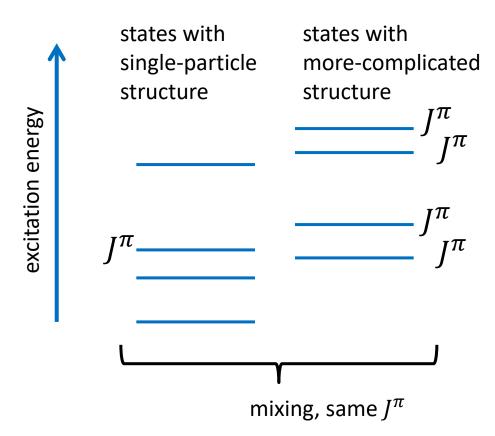


S measures the <u>occupancy</u> of the shell model orbital... the *spectroscopic factor*

 $S = (S^{1/2})^2$ is a factor that scales the predicted DWBA cross section for a pure single-particle state and is determined by comparison between DWBA and experiment

So, in summary:

$$\left(\frac{d\sigma}{d\Omega}\right)_{experiment} = S \times \left(\frac{d\sigma}{d\Omega}\right)_{DWBA}$$



$$\mid J_i^\pi> \ =\ \sqrt{S}\ \mid J_{SP}^\pi> \ + \sum_k \alpha_k \mid J_k^\pi>$$
 we measure transferred ℓ_n from ${}^{d\sigma}/{}_{d\Omega}$

- we measure gamma-decays
- we aim to identify J and π
- we model the transfer yield for S=1
- we deduce S from the observed yield

$$S = |\langle J_{SP}^{\pi} | J_i^{\pi} \rangle|^2$$

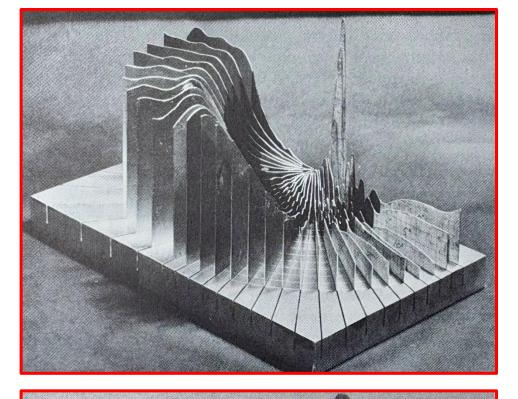
spectroscopic factor
= overlap with pure SP state

Photographs of Distorted Waves

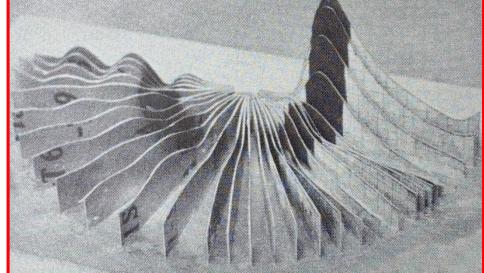
From: N. Austern

Direct Nuclear Reaction Theories

Beam of α's on ⁴⁰Ca 18 MeV from left



Beam of p's on ⁴⁰Ca 40 MeV from left

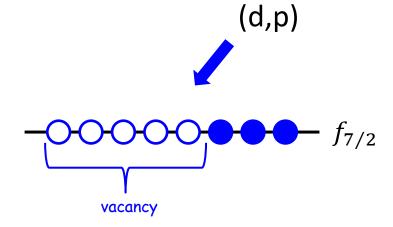


What's plotted: modulus $|\chi^{(+)}|$ of the incoming optical model wavefunction Dark zone: 10%-90% region of the potential

SUM RULES for Single-Nucleon Transfer

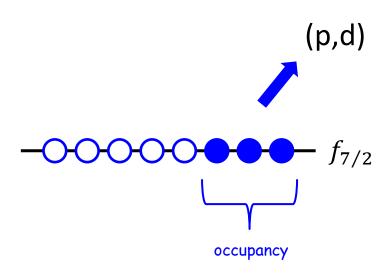
For adding a nucleon to a given j-shell the sum rule gives the vacancy in the shell

Number of Holes =
$$\sum_{i} \left(\frac{2T_{f}^{i} + 1}{2T_{0} + 1} \right) \left(\frac{2J_{f}^{i} + 1}{2J_{0} + 1} \right) S_{i}$$



for removing a nucleon from a given j-shell it gives the occupancy of the shell, with the sum running over all final states i.

Number of Particles =
$$\sum_{i} \left(\frac{2T_f^i + 1}{2T_0 + 1} \right) S_i$$



Note that only one value of isospin $T_f (= T_0 + 1/2)$ is allowed for neutron adding or proton removing reactions, and two values $T_f (= T_0 \pm 1/2)$ for neutron removal or proton adding.

S = spectroscopic factor J = spin T = isospin i = sum over all states

Adapted from: John Schiffer, Argonne

Some Physics that Complicates Transfer Interpretation



We do not compare like-with-like when we compare theory and experiment Left: chalk; Right: cheese – they are not the same

Spectroscopic Factor

Shell Model: overlap of $|\psi (N+1)\rangle$ with $|\psi (N)\rangle_{core}\otimes n (\ell j)$

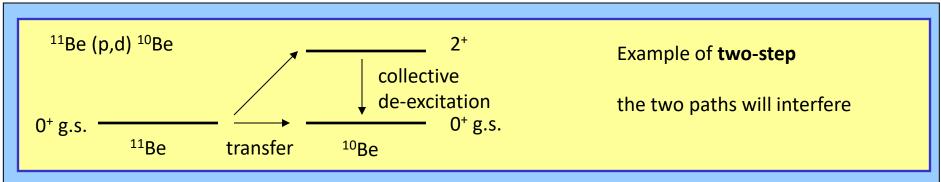
Reaction: the observed yield is not just proportional to this S, because in T the overlap integral has a radial-dependent weighting or sampling

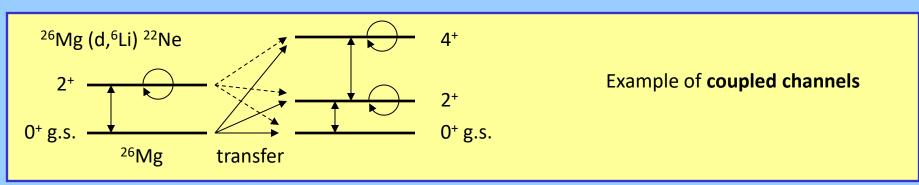
Many-body theory of $d + A(N, Z) \rightarrow B(N + 1, Z) + p$

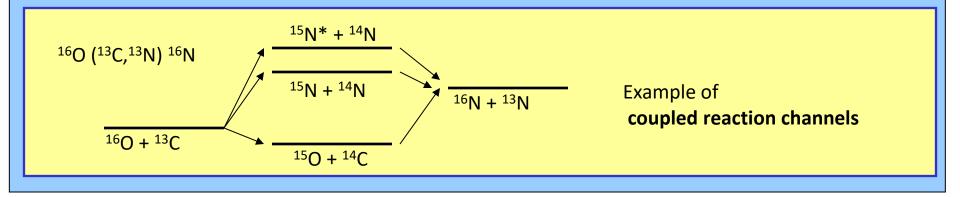
overlap integral
$$\phi_n^{BA}(\vec{r}_n) = \sqrt{N+1} \int d\xi_A \phi_B^*(\xi_A, \vec{r}_n) \phi_A(\xi_A)$$
 spectroscopic factor $S^{AB} = \int d\vec{r}_n \mid \phi_n^{AB}(\vec{r}_n) \mid^2$
$$T_{d,p} = \langle \chi_p^{(-)} \phi_n^{BA} \mid V_{np} \mid \Psi_{\vec{K}_d} \rangle$$

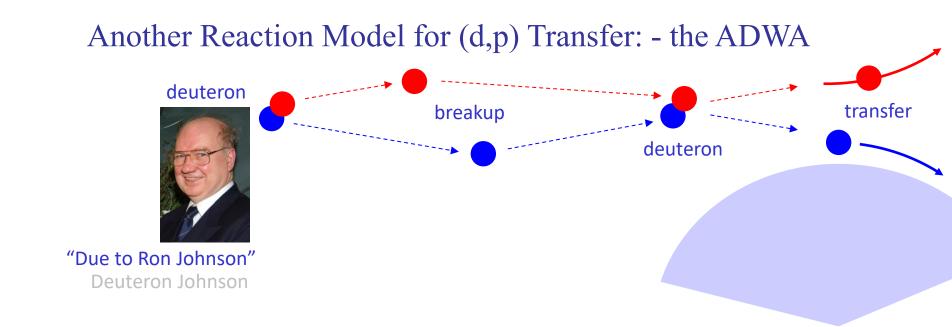
Hence the yield, and hence deduced spectroscopic factor, depends on the radial wave function and thus the geometry of the assumed potential well for the transferred nucleon, or details of some other structure model

Some Other Physics that can Complicate Transfer Calculations









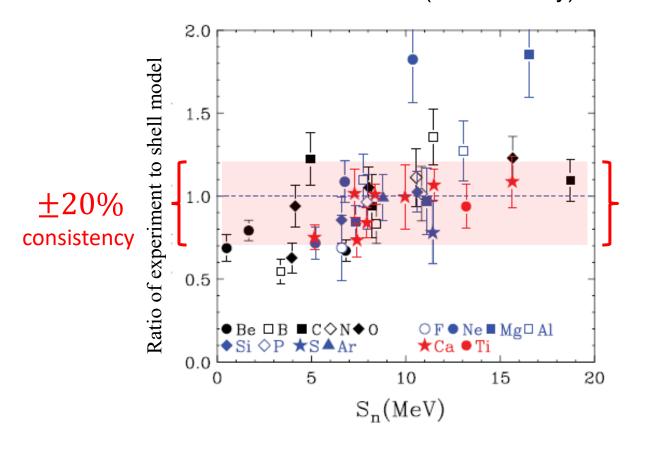
Another Reaction Model for (d,p) Transfer: - the ADWA deuteron breakup transfer deuteron ADIABATIC DISTORTED WAVE APPROXIMATION (BORN)

<u>Johnson-Soper Model:</u> an **alternative to DWBA** that gives a simple prescription for taking into account coherent *entangled* **effects of deuteron break-up** on (d,p) reactions [1,2]

- does not use deuteron optical potential uses *nucleon-nucleus optical potentials* only
- formulated in terms of adiabatic approximation, which is sufficient but not necessary [3]
- uses parameters (overlap functions, spectroscopic factors, ANC's) just as in DWBA
- [1] Johnson and Soper, PRC 1 (1970) 976
- [2] Harvey and Johnson, PRC 3 (1971) 636; Wales and Johnson, NPA 274 (1976) 168
- [3] Johnson and Tandy NPA 235 (1974) 56; Laid, Tostevin and Johnson, PRC 48 (1993) 1307

Another Reaction Model for (d,p) Transfer: - the ADWA

A CONSISTENT application of ADWA gives 20% agreement with large basis SM for well-understood (near-stability) nuclei



80 spectroscopic factors Z = 3 to 24 Jenny Lee et al.

Tsang et al PRL 95 (2005) 222501

Lee et al PRC 75 (2007) 064320

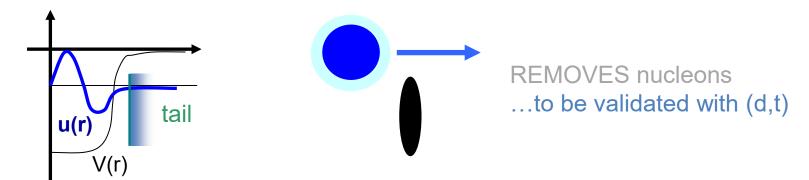
Delaunay at al PRC 72 (2005) 014610

... so we can compare experiment and theory in a reliable fashion



A Different Type of Reaction to Study Similar Things

- Given what we have seen, is transfer the BEST way to isolate and study single particle structure and its evolution in exotic nuclei?
 - TRANSFER decades of (positive) experience, makes nuclei more exotic
 - KNOCKOUT* high cross section, requires orbitals to be occupied



- (e,e'p) ambitious for general RIB application, requires occupied orbitals
- (p,p'p) more practical than (e,e'p) for RIB, requires occupied orbitals

A: YES!

and don't forget:
heavy ion transfer (9Be),
3,4He-induced reactions

Summary of single-particle studies via <u>transfer</u> and <u>knockout</u>

Each of these processes can probe single-particle structure:

- measure the occupancy of single-particle (shell model) orbitals (spectroscopic factors)
- identify the angular momentum of the relevant nucleon.





We can therefore identify the distribution of single-particle strength across nuclear states and this <u>allows detailed comparisons</u> with the predictions of our best <u>nuclear structure model</u>: the <u>nuclear shell model</u>.

Summary of single-particle studies via <u>transfer</u> and <u>knockout</u>

Each of these processes can probe single-particle structure:

- measure the occupancy of single-particle (shell model) orbitals (spectroscopic factors)
- identify the angular momentum of the relevant nucleon.

With knockout we can probe:

- occupancy of single-particle (shell model) orbitals in the projectile ground state
- identify the angular momentum of the **removed** nucleon
- hence, identify the s.p. level energies in odd-A nuclei produced from even-even projectiles

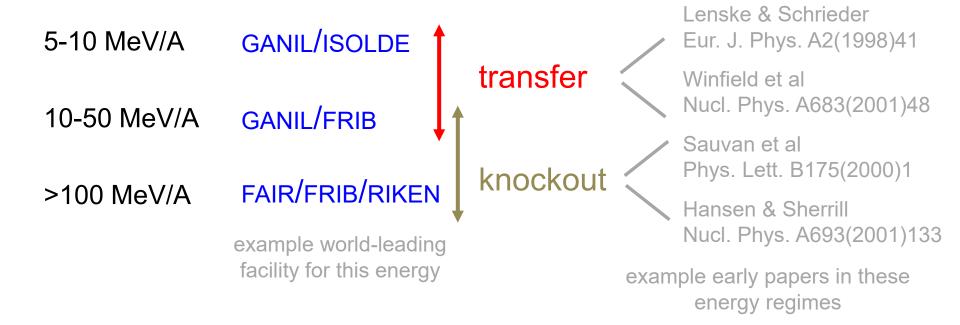
and the projectile-like particle is detected essentially at zero degrees

With <u>transfer</u> we can probe:

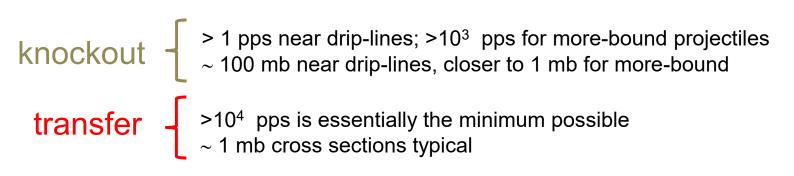
- occupancy of single-particle (shell model) orbitals in the original nucleus A ground state
 or distribution of s.p. strength in all final states of A–1 or A+1 nucleus
 that is, can add a nucleon to the original nucleus, e.g. by (d,p)
- identify the angular momentum of the transferred nucleon
- hence, identify the s.p. level energies in A-1 or A+1 nuclei produced from even-even nuclei
- identify the s.p. purity of coupled states in A-1 or A+1 nuclei produced from odd nuclei

and the **scattered particle** is detected, with most yield being at small centre-of-mass angles

Energy Regimes Best Matched to Transfer and Knockout



Intensity Regimes Best Matched to Transfer and Knockout



Some additional comments about <u>transfer reactions in general</u>...

The nucleon having to "stick" places kinematic restrictions on the population of states:

- the reaction Q-value is important (for Q large and negative, higher ℓ values are favoured)
- the degree (ℓ-dependent) to which the kinematics favour a transfer is known as *matching*

Various **types of transfer** are employed typically, and using different mass probe-particles:

- light-ion transfer reactions: (probe $\leq \alpha$ say) ... (d,p) (p,d) (d,t) (d, 3 He) also (3 He, α) etc.
- heavy-ion transfer reactions: e.g. (13C,12C) (13C,14C) (17O,16O) (9Be,8Be)
- two-nucleon transfer: e.g. (p,t) (t,p) (9 Be, 7 Be) (12 C, 14 C) (d, α)
- alpha-particle transfer (or α-transfer): e.g. (⁶Li,d), (⁷Li,t), (d,⁶Li), (¹²C,⁸Be)

Some additional comments specifically about <u>light-ion transfer</u>... $p, d, t, {}^{3}He, \alpha$

... induced by <u>radioactive ion beams</u>...

... where the light ion is the <u>target</u>

Light-ion induced reactions give the clearest measure of the <u>transferred</u> ℓ , and have a <u>long history</u> of application in experiment and a highly refined theory.

Thus, they are attractive to employ as an essentially <u>reliable tool</u>, now that radioactive beams of sufficient intensity have become available.

To the **theorist**, there are <u>some new aspects</u> to address, near the drip lines.

To the **experimentalist**, the transformation of reference frames is a <u>much bigger problem!</u>

The new experiments need hydrogen (or He) nuclei as targets & the beam is <u>much heavier</u>. This is <u>inverse kinematics</u>, and the energy-angle systematics are completely different.



A PLAN for how to study nuclear STRUCTURE:

- Use **transfer reactions** to identify strong single-particle states, measuring their spins and strengths
- Use the energies of these states to compare with theory
- Refine the structure (e.g. shell model, ab initio) theory
- Improve the extrapolation to very exotic nuclei
- Hence learn the structure of very exotic nuclei
- N.B. The **shell model** is arguably the best theoretical approach for us to confront with our results, but it's **not the only one**.

 The experiments are needed, no matter which theory we use.
- N.B. Transfer (as opposed to knockout) allows us to study orbitals that are empty, so we don't need quite such exotic beams.

