Tomography of nucleon through dimeson photoproduction

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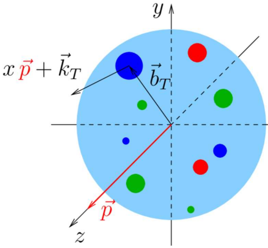
Physical content of Generalized Parton Distributions

- Off-diagonal matrix elements of bilocal operators at light-like separation.

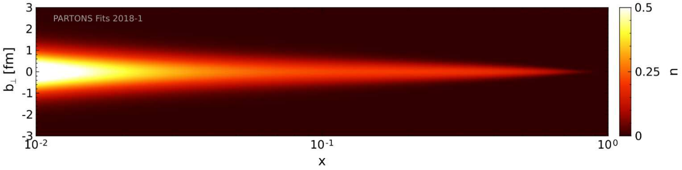
$$\frac{1}{P^+}\bar{u}(p_2)(H^q(x,\xi,t)\gamma^+ + \dots)u(p_1)$$
$$= \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p_2|\bar{q}(-\frac{z}{2})\gamma^+ q(\frac{z}{2})|p_1\rangle_{z_\perp=0,z^+=0}$$



- Connected to impact parameter distribution:

$$q(x,b_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-ib_\perp\cdot\Delta_\perp} H^q(x,0,t)$$

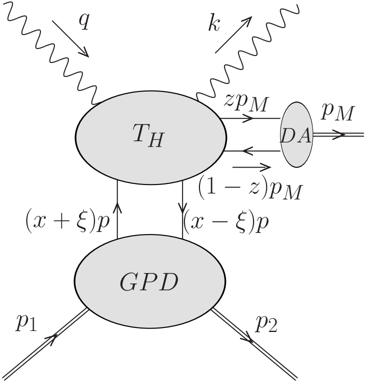
Moutarde, Sznajder, Wagner, 2018



- They give the total angular momentum carried by partons:

$$\int_{-1}^{-1} dx\, x(H^q(x,\xi,0) + E^q(x,\xi,0)) = 2J^q$$

An example of exclusive process: generalized Compton diffusion with meson production



Factorization recently proved Qiu, Yu, 2023

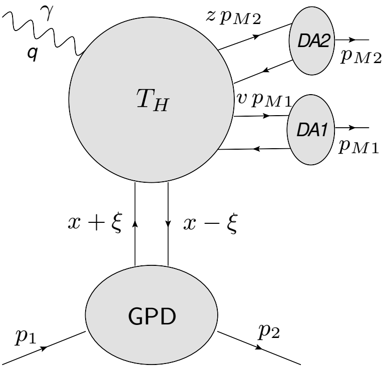
Duplančić, Nabeebaccus, Passek-Kumerički, Pire, Szymanowski, Wallon

Meson	Number of events
π^+	0.29-1.76 $\times 10^5$
π^-	0.53-1.33 $\times 10^5$

GPD	Meson	Number of events
Chiral-even	ρ^0_p	1.3-2.4 $\times 10^5$
	ρ^0_n	1.7-4.0 $\times 10^4$
	ρ^+_p	0.9-1.4 $\times 10^5$
	ρ^-_n	0.3-1.8 $\times 10^5$
Chiral-odd	ρ^0_p	2.1-4.2 $\times 10^4$
	ρ^0_n	1.0-2.6 $\times 10^4$
	ρ^+_p	3.5-6.7 $\times 10^4$
	ρ^-_n	3.5-6.8 $\times 10^4$

Results for JLab facilities

Dimeson photoproduction



Factorisation recently proved in Qiu, Yu, 2023

- Give access both to chiral even and chiral odd GPDs
- Violation of factorization expected for:

$$p\,\gamma \to p\,\pi^+\pi^-$$
$$p\,\gamma \to p\,\rho^+\rho^-$$
$$p\,\gamma \to p\,\pi^0\rho^0$$

- Lots of processes to study with no gluon in t channel, for example:

$$p\,\gamma \to n\,\pi^+\rho_0$$
$$p\,\gamma \to p\,\pi_0\pi_0$$
$$p\,\gamma \to n\,\pi_0\rho^+$$
$$p\,\gamma \to n\,\rho_0\rho^+$$
$$p\,\gamma \to p\,\rho_0\rho_0$$

- Leading Order, Leading twist

The goal: the unpolarized differential cross-section for any dimeson photoproduction

Goal: unpolarised differential cross-section of any dimeson photoproduction

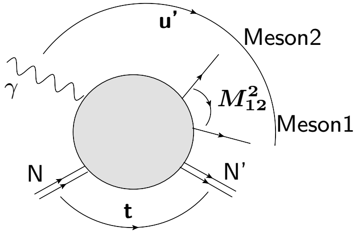
- $2 \to 3$ exclusive process, nucleon slightly deflected

  $\Rightarrow$ 3 independent degrees of freedom

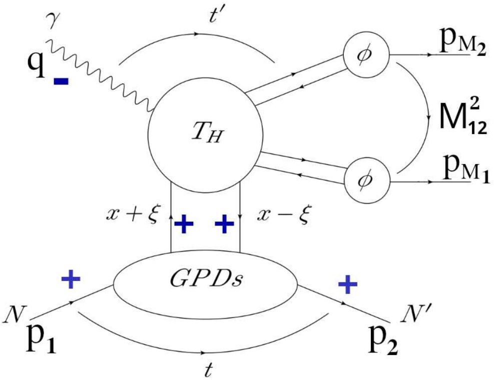
  We choose t, u' and M_{12}^2

- Longitudinal/ transverse vector mesons, and pseudo-scalar mesons

  $\Rightarrow$Vector, Axial or Transverse GPDs



Collinear Factorization of dimeson photoproduction



$p = \frac{\sqrt{s}}{2}(1,0,0,1) \quad \rightarrow +$

$q = \frac{\sqrt{s}}{2}(1,0,0,-1) \quad \rightarrow -$

$p_2 = (1-\xi)p$

$p_1 = (1+\xi)p \quad \Delta = p_2 - p_1$

$p_{M_1} = \alpha q - \frac{p_\perp^2}{\alpha s} p + \boldsymbol{p}_\perp$

$p_{M_2} = (1-\alpha) q - \frac{p_\perp^2}{(1-\alpha)s} p - \boldsymbol{p}_\perp$

- Hard scale present: $-p_\perp^2, s \gg \Lambda_{\text{QCD}}^2$
- Nucleon is slightly deflected: $-t \sim -\Delta_\perp^2 \ll -p_\perp^2$

Collinear Factorisation $\Rightarrow$
diagrams where the 3 pairs of quarks fly collinearly dominate

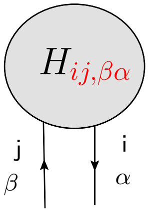
How do we get GPDs ?

- **Fierz Projection in spinor space**:

$$\langle p_2|\bar{q}_\alpha(-\frac{z}{2})q_\beta(\frac{z}{2})|p_1\rangle = \frac{1}{4}\gamma_{\mu\,\beta\alpha}\langle p_2|\bar{q}\gamma^\mu q|p_1\rangle + \frac{1}{4}(\gamma^5)\gamma_{\mu\,\beta\alpha}\langle p_2|\bar{q}\gamma^\mu\gamma^5 q|p_1\rangle$$
$$+\frac{1}{4}I_{\beta\alpha}\langle p_2|\bar{q}q|p_1\rangle + \frac{1}{4}\gamma^5_{\beta\alpha}\langle p_2|\bar{q}\gamma^5 q|p_1\rangle + \frac{1}{8}\sigma_{\mu\nu\,\beta\alpha}\langle p_2|\bar{q}\,\sigma^{\mu\nu}q|p_1\rangle$$

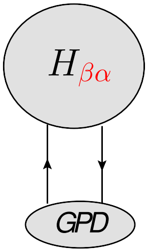
- **Fierz Projection in color space**:

$$\langle p_2|\bar{q}_i\gamma^+ q_j|p_1\rangle = \frac{1}{N_c}\langle p_2|\bar{q}\gamma^+ q|p_1\rangle\delta_{ij} + 2T^a_{ij}\overbrace{\langle p_2|\bar{q}\,{}^tT^a\gamma^+ q|p_1\rangle}^{=0}$$



$$\langle p_2|\bar{q}_{i,\alpha}(-\tfrac{z}{2})q_{j,\beta}(\tfrac{z}{2})|p_1\rangle$$

$\Rightarrow$



$$\frac{1}{N_c}\gamma_{\mu\,\beta\alpha}\langle p_2|\bar{q}(-\tfrac{z}{2})\gamma^\mu q(\tfrac{z}{2})|p_1\rangle$$

What kind of GPD do we get ?

- **Vector GPD:** $H^q, E^q \leftrightarrow \gamma^+$

$$\frac{1}{P^+}\bar{u}(p_2)(H^q(x,\xi,t)\gamma^+ + E^q(x,\xi,t)\frac{i\sigma^{+\mu}\Delta_\mu}{2M})u(p_1) = \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p_2|\bar{q}(-\frac{z}{2})\gamma^+ q(\frac{z}{2})|p_1\rangle_{\substack{z_\perp=0\\ z^+=0}}$$

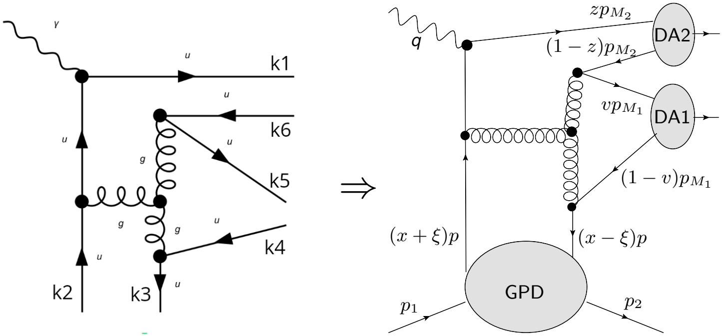
- **Axial GPD:** $\tilde{H}^q, \tilde{E}^q \leftrightarrow \gamma^+\gamma^5$

$$\frac{1}{P^+}\bar{u}(p_2)\left(\tilde{H}^q(x,\xi,t)\gamma^+\gamma^5 + \tilde{E}^q(x,\xi,t)\frac{\gamma^5\Delta^+}{2M}\right)u(p_1) = \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p_2|\bar{q}(-\frac{z}{2})\gamma^+\gamma^5 q(\frac{z}{2})|p_1\rangle_{\substack{z_\perp=0\\ z^+=0}}$$

- **Transverse GPD:** ${H_T}^q, \tilde{H}_T^q, {E^q}_T, \tilde{E}_T^q \leftrightarrow \sigma^{+j}$

$$\frac{1}{P^+}\bar{u}(p_2)\left(H_T^q(x,\xi,t)\sigma^{+j} + \dots\right)u(p_1) = \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p_2|\bar{q}(-\frac{z}{2})\sigma^{+j} q(\frac{z}{2})|p_1\rangle_{\substack{z_\perp=0\\ z^+=0}}$$

Creation of diagrams using Feynart



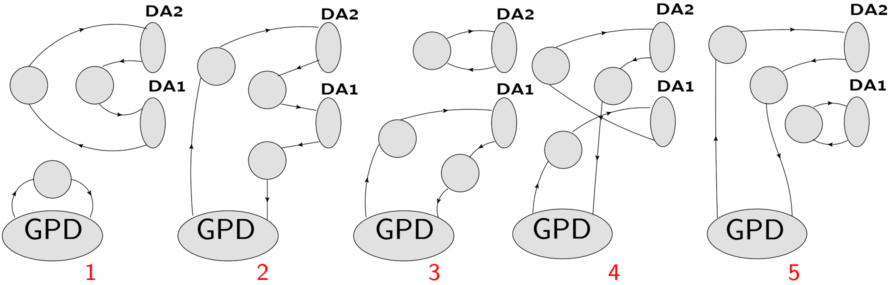
$$\frac{\boxed{\bar{u}(k1)}\,\not{q}(q1)(\not{k}2+\not{k}3+\not{k}4+\not{k}5+\not{k}6)\gamma_\mu\,\boxed{u(k2)\bar{u}(k3)}\,(\not{k}3+\not{k}4+2\not{k}5+2\not{k}6)\,\boxed{v(k4)\bar{u}(k5)}\,\gamma^\mu\,\boxed{v(k6)}}{(k3+k4)^2(k5+k6)^2(k3+k4+k5+k6)^2(k2+k3+k4+k5+k6)^2}$$

$$\Downarrow$$

$$\frac{\mathrm{Tr}\left(\not{q}(q1)(\not{k}2+\not{k}3+\not{k}4+\not{k}5+\not{k}6)\gamma^\mu\,\boxed{GPD}\,(\not{k}3+\not{k}4+2\not{k}5+2\not{k}6)\,\boxed{DA1}\,\gamma_\mu\,\boxed{DA2}\right)}{(k3+k4)^2(k5+k6)^2(k3+k4+k5+k6)^2(k2+k3+k4+k5+k6)^2}$$

Sorting and storage of diagrams

136 diagrams grouped into 5 configurations



14 combinations of Dirac structures $\Rightarrow$ 1480 diagrams stored

$$\underset{\{\text{diag},\,\text{Dirac struct},\,\text{conf},\,\text{photon}\}}{\underset{\uparrow}{\{41,1,1,\{1,4\}\}}} \rightarrow \frac{-eg^4\left(\mathrm{Nc}^2-1\right)\left(\bar{\varepsilon}_q\cdot\bar{p}_\perp\right)\left(\xi\left(-2\alpha^2 z+\alpha v(2z-1)+2\alpha z^2+\alpha-2vz+v-z\right)+(\alpha-1)x(2\alpha z-1)\right)\overset{\overset{\text{Prefactor of diagram}}{\uparrow}}{\mathrm{f}(\{41,1,1,\{1,4\}\},\mathrm{N1},\mathrm{N2},\{\mathrm{M1},\mathrm{t1}\},\{\mathrm{M2},\mathrm{t2}\})}}{128(\alpha-1)\mathrm{Nc}^3 s^2 v\xi^3(z-1)z(v-\alpha(v+z-1))(\alpha(v+z-1)-vz)\left(-\frac{\xi(\alpha(v+z-1)-2vz+v)}{\alpha(v+z-1)-v}+\imath\epsilon+x\right)}$$

Integration over x, z,v

$$\frac{eg^4 v(z-1)(2\alpha(z-1)+1)\left(\varepsilon_q \cdot p_\perp\right) f(\{110,1,2,\{5,6\}\},\mathrm{N1},\mathrm{N2},\{\mathrm{M1},\mathrm{t1}\},\{\mathrm{M2},\mathrm{t2}\})}{81\alpha s\xi(s-\alpha s)(\alpha(-z)+\alpha+z-1)(\imath\epsilon+x-\xi)(-\imath\epsilon+x+\xi)}$$

$\Downarrow$ Partial Fraction over x

$$\frac{eg^4 v(2\alpha(z-1)+1)\left(\varepsilon_q \cdot p_\perp\right) f(\{110,1,2,\{5,6\}\},\mathrm{N1},\mathrm{N2},\{\mathrm{M1},\mathrm{t1}\},\{\mathrm{M2},\mathrm{t2}\})}{162(\alpha-1)^2\alpha s^2\xi^2(\imath\epsilon+x-\xi)} - \frac{eg^4 v(2\alpha(z-1)+1)\left(\varepsilon_q \cdot p_\perp\right) f(\{110,1,2,\{5,6\}\},\mathrm{N1},\mathrm{N2},\{\mathrm{M1},\mathrm{t1}\},\{\mathrm{M2},\mathrm{t2}\})}{162(\alpha-1)^2\alpha s^2\xi^2(-\imath\epsilon+x+\xi)}$$

$\left(\frac{1}{x-\xi\pm\imath\epsilon} = \mathrm{p.v.}\frac{1}{x-\xi} \mp \imath\pi\delta(x-\xi)\right)$ $\Downarrow$ (N1=p, N2=n, {M1,t1}={ρ^+,$\|$},{M2,t2}={ρ^0,$\|$})

$$\frac{eg^4(v-1)v^2(z-1)z(2\alpha(z-1)+1)\varepsilon_q\cdot p_\perp(\mathrm{Qd}\,\mathrm{H}^d(-\xi)-\mathrm{Qd}\,\mathrm{H}^d(x)+\mathrm{Qu}\,(\mathrm{H}^u(-\xi)-\mathrm{H}^u(x)))}{9(\alpha-1)^2\alpha\xi^2 s^2(\xi+x)} + \frac{eg^4(v-1)v^2(z-1)z(2\alpha(z-1)+1)\varepsilon_q\cdot p_\perp(-\mathrm{Qd}\,\mathrm{H}^d(\xi)+\mathrm{Qd}\,\mathrm{H}^d(x)+\mathrm{Qu}\,(\mathrm{H}^u(x)-\mathrm{H}^u(\xi)))}{9(\alpha-1)^2\alpha\xi^2 s^2(x-\xi)}$$
$$+\frac{eg^4(v-1)v^2(z-1)z\left(\log\left(\frac{1-\xi}{\xi+1}\right)-i\pi\right)(2\alpha(z-1)+1)\varepsilon_q\cdot p_\perp(\mathrm{Qd}\,\mathrm{H}^d(\xi)+\mathrm{Qu}\,\mathrm{H}^u(\xi))}{9(\alpha-1)^2\alpha\xi^2 s^2} - \frac{eg^4(v-1)v^2(z-1)z\left(\log\left(\frac{\xi+1}{1-\xi}\right)+i\pi\right)(2\alpha(z-1)+1)\varepsilon_q\cdot p_\perp(\mathrm{Qd}\,\mathrm{H}^d(-\xi)+\mathrm{Qu}\,\mathrm{H}^u(-\xi))}{9(\alpha-1)^2\alpha\xi^2 s^2}$$

We perform a numerical integration in x, v, z in the end

A more sophisticated case

$$-\frac{ieg^4(v-1)^2\left(\left(\overline{p}_\perp\cdot\overline{\epsilon}_q\right)\left(\alpha^2\xi s\overline{\epsilon}_{M2}{}^j-2\overline{p}_\perp^j\left(\overline{p}_\perp\cdot\overline{\epsilon}_{M2}\right)\right)+(\alpha-2)\alpha\xi s\left(\overline{\epsilon}_q{}^j\left(\overline{p}_\perp\cdot\overline{\epsilon}_{M2}\right)+\overline{p}_\perp^j\left(\overline{\epsilon}_{M2}\cdot\overline{\epsilon}_q\right)\right)\right)f(\{115,10,5,\{5,4\}\},\mathrm{N1},\mathrm{N2},\{\mathrm{M1},\mathrm{t1}\},\{\mathrm{M2},\mathrm{t2}\})}{108\alpha^4\xi s^3((\alpha-1)v-\alpha z+1)(-\xi+x+i\epsilon)(\xi+x-i\epsilon)\left(\frac{\xi-\xi v(\alpha-2z+1)+(\alpha-2)\xi z}{(\alpha-1)v-\alpha z+1}+x+i\epsilon\right)}$$

$$=$$

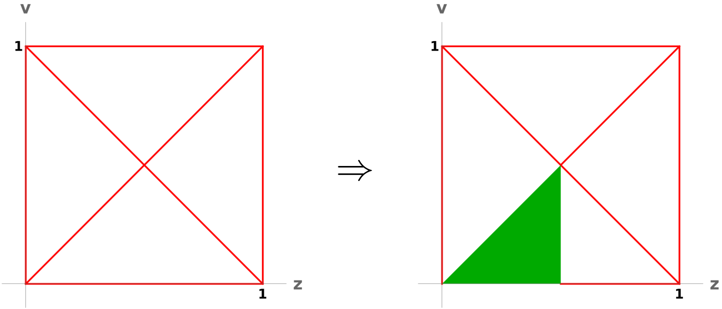
$$+\frac{-ieg^4(v-1)\left(\left(\overline{p}_\perp\cdot\overline{\epsilon}_q\right)\left(\alpha^2\xi s\overline{\epsilon}_{M2}{}^j-2\overline{p}_\perp^j\left(\overline{p}_\perp\cdot\overline{\epsilon}_{M2}\right)\right)+(\alpha-2)\alpha\xi s\left(\overline{\epsilon}_q{}^j\left(\overline{p}_\perp\cdot\overline{\epsilon}_{M2}\right)+\overline{p}_\perp^j\left(\overline{\epsilon}_{M2}\cdot\overline{\epsilon}_q\right)\right)\right)f(\{115,10,5,\{5,4\}\},\mathrm{N1},\mathrm{N2},\{\mathrm{M1},\mathrm{t1}\},\{\mathrm{M2},\mathrm{t2}\})}{432\alpha^4\xi^3 s^3\boxed{(z-1)}(-\xi+x+i\epsilon)}$$

$$+\frac{ieg^4(v-1)^2\left(\left(\overline{p}_\perp\cdot\overline{\epsilon}_q\right)\left(\alpha^2\xi s\overline{\epsilon}_{M2}{}^j-2\overline{p}_\perp^j\left(\overline{p}_\perp\cdot\overline{\epsilon}_{M2}\right)\right)+(\alpha-2)\alpha\xi s\left(\overline{\epsilon}_q{}^j\left(\overline{p}_\perp\cdot\overline{\epsilon}_{M2}\right)+\overline{p}_\perp^j\left(\overline{\epsilon}_{M2}\cdot\overline{\epsilon}_q\right)\right)\right)f(\{115,10,5,\{5,4\}\},\mathrm{N1},\mathrm{N2},\{\mathrm{M1},\mathrm{t1}\},\{\mathrm{M2},\mathrm{t2}\})}{432\alpha^4\xi^2 s^3(\xi+x-i\epsilon)(\xi z(\alpha+v-1)-\alpha\xi v)}$$

$$-\frac{ieg^4(v-1)((\alpha-1)v-\alpha z+1)\left(\left(\overline{p}_\perp\cdot\overline{\epsilon}_q\right)\left(\alpha^2\xi s\overline{\epsilon}_{M2}{}^j-2\overline{p}_\perp^j\left(\overline{p}_\perp\cdot\overline{\epsilon}_{M2}\right)\right)+(\alpha-2)\alpha\xi s\left(\overline{\epsilon}_q{}^j\left(\overline{p}_\perp\cdot\overline{\epsilon}_{M2}\right)+\overline{p}_\perp^j\left(\overline{\epsilon}_{M2}\cdot\overline{\epsilon}_q\right)\right)\right)f(\{115,10,5,\{5,4\}\},\mathrm{N1},\mathrm{N2},\{\mathrm{M1},\mathrm{t1}\},\{\mathrm{M2},\mathrm{t2}\})}{432\alpha^4\xi^2 s^3\boxed{(z-1)}(\xi z(\alpha+v-1)-\alpha\xi v)\left(\frac{\xi+(\alpha+1)\xi(-v)+\xi z(\alpha+2v-2)}{(\alpha-1)v-\alpha z+1}+x+i\epsilon\right)}$$

Partial Fraction $\rightarrow$ spurious divergences $z=1, z=0, v=1, v=0, v=z, v=1-z$

How to deal with spurious divergences



$$\int_0^1 dz \int_0^1 dv\, \mathcal{A}(z,v) = \int_0^{\frac{1}{2}} dz \int_0^z dv (\mathcal{A}(z,v) + \mathcal{A}(v,z) + \mathcal{A}(1-v,1-z)$$
$$+ \mathcal{A}(1-z,1-v) + \mathcal{A}(1-v,z) + \mathcal{A}(z,1-v) + \mathcal{A}(1-z,v) + \mathcal{A}(v,1-z))$$

Generalized Parton Distribution modelling

What do we choose for $H(x,\xi,t), \tilde{H}(x,\xi,t), H_T(x,\xi,t)$?

A GPD model has to satisfy:

- Forward limit: $H^q(x,\xi=0,t=0) = q(x)\Theta(x) - \bar{q}(-x)\Theta(-x)$
- Reduction to elastic form factors: $\int_{-1}^{1} dx H^q(x,\xi,t) = F_1^q(t)$
- Polynomiality:

$$\int_{-1}^{1} dx x^m H^q(x,\xi,t) = \sum_{i=0}^{\lfloor \frac{m}{2} \rfloor} (2\xi)^{2i} A^q_{i,m}(t) + mod(m,2)(2\xi)^{m+1} C_{m+1}(t)$$

see Mezrag: Introductory lecture on Generalised Parton Distributions

But still many possibilities.

Generalized Parton Distribution modelling

We stick to Radyushkin Double Distribution Ansatz

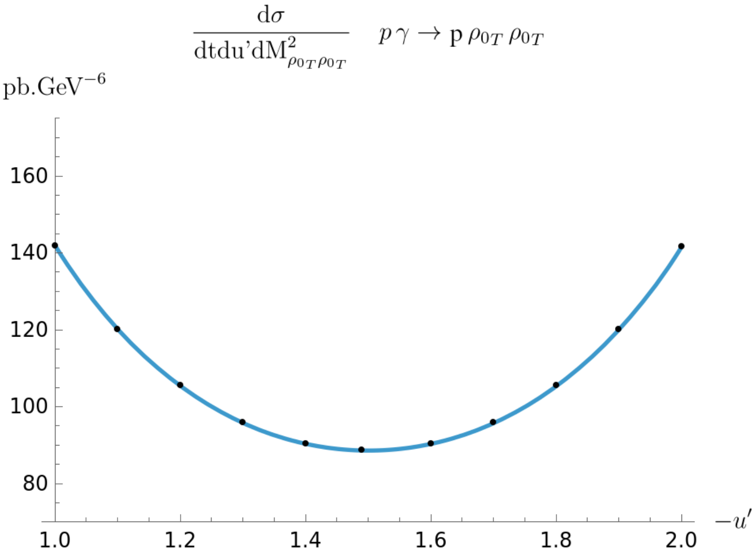
$$H^q(x,\xi,t) = \int_{\{|\beta|+|\alpha|\leq 1\}} d\beta\, d\alpha\, \delta(\beta+\xi\alpha-x) F^q(\beta,\alpha,t) = \mathrm{Radon}[F]$$

(we discard the D-term)

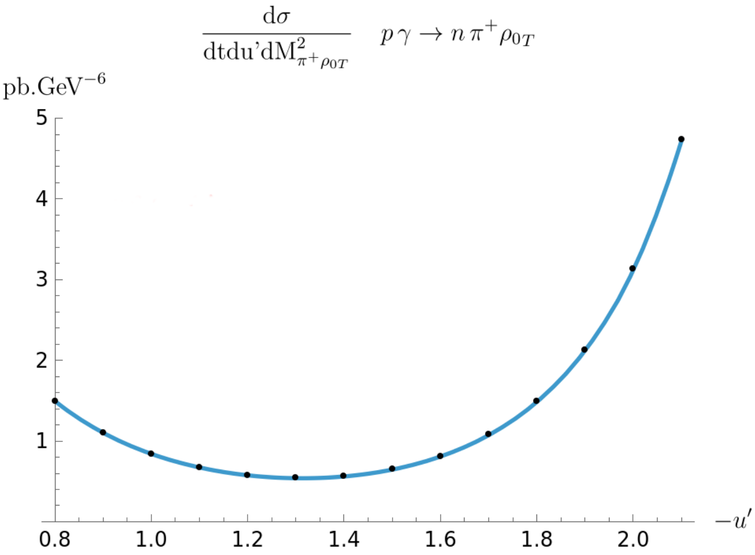
- $F^q(\beta,\alpha,t) = (\Pi(\beta,\alpha) q(\beta)\Theta(\beta) - \Pi(-\beta,\alpha)\bar{q}(-\beta)\Theta(-\beta))\,\frac{C^2}{(t-C)^2}$
- $\tilde{F}^q(\beta,\alpha,t) = (\Pi(\beta,\alpha) \Delta q(\beta)\Theta(\beta) - \Pi(-\beta,\alpha)\Delta\bar{q}(-\beta)\Theta(-\beta))\,\frac{C^2}{(t-C)^2}$
- $F_T^q(\beta,\alpha,t) = (\Pi(\beta,\alpha) \delta q(\beta)\Theta(\beta) - \Pi(-\beta,\alpha)\delta\bar{q}(-\beta)\Theta(-\beta))\,\frac{C^2}{(t-C)^2}$

Problem: we don't know precisely Δq and δq

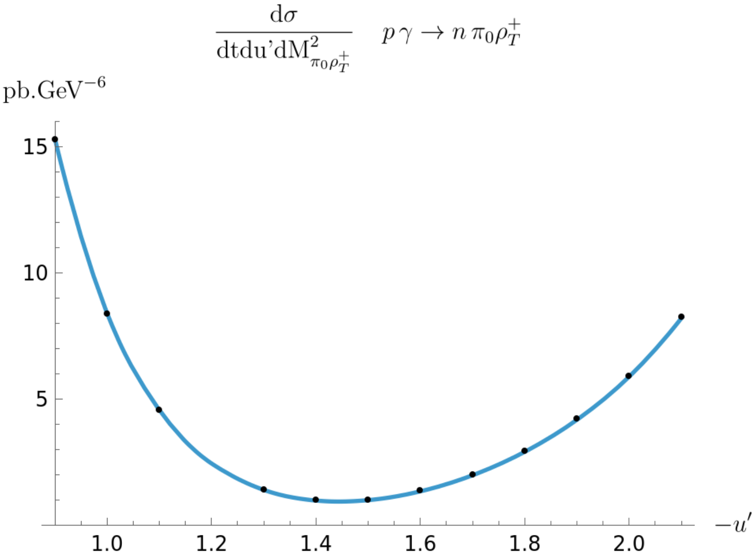
$p\,\gamma \to p\,\rho_{0T}\rho_{0T}$ at JLab $S_{\gamma N} = 20$ GeV^2 $M^2_{\rho_{0T}\rho_{0T}} = 3$ GeV^2



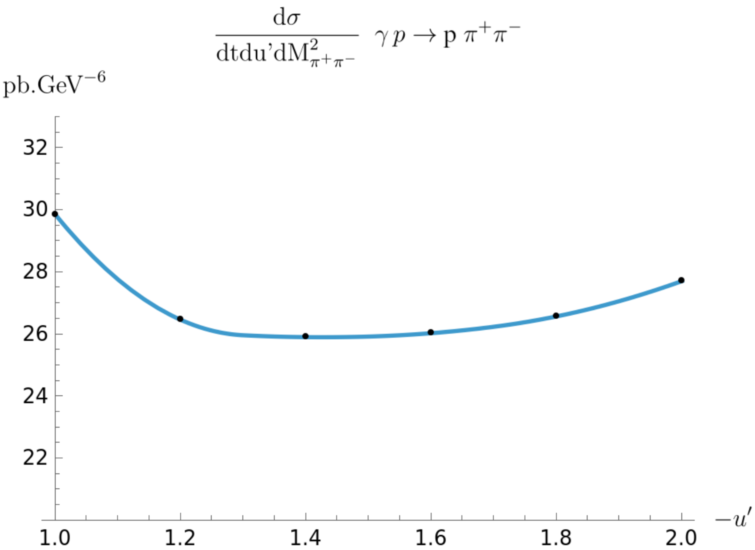
$p\,\gamma \to n\,\pi^+ \rho_{0T}$



$p\,\gamma \to n\,\pi^0 \rho_T^+$



$p\,\gamma \to n\,\pi^+\pi^-$



Conclusion

- Dimeson photoproduction is an interesting channel that gives access to both chiral even and chiral odd generalized parton distributions
- More than 20 processes can now be studied automatically.
- Preliminary cross-sections seem sufficient for JLab kinematics.
- We intend to complete the integration for all processes in the various kinematics of UPC at LHC and EIC. We will test different models of GPDs and DAs and study polarization asymmetries