Automated calculations for new physics searches

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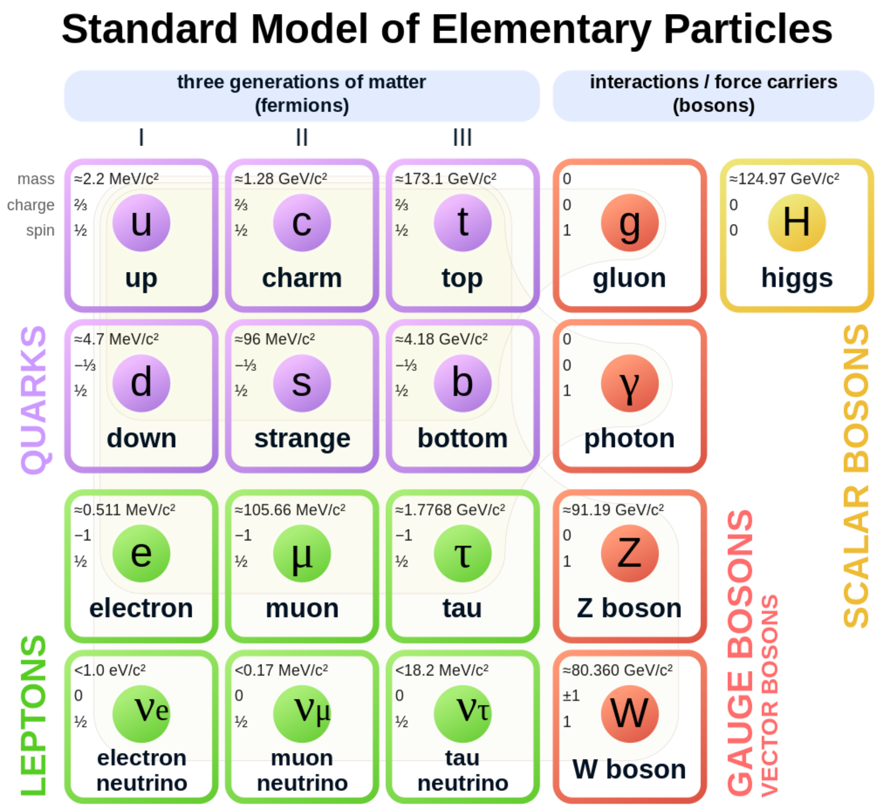
April 17 2025

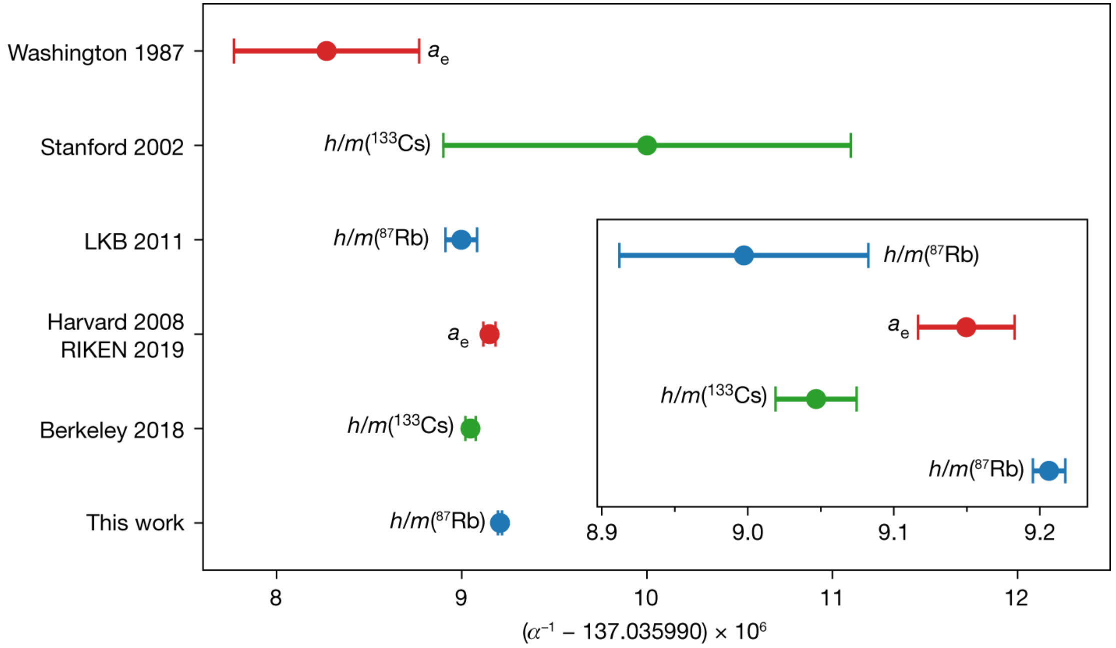
PhD Days

The Standard Model
...and its limits

Or why do we care

The Standard Model…



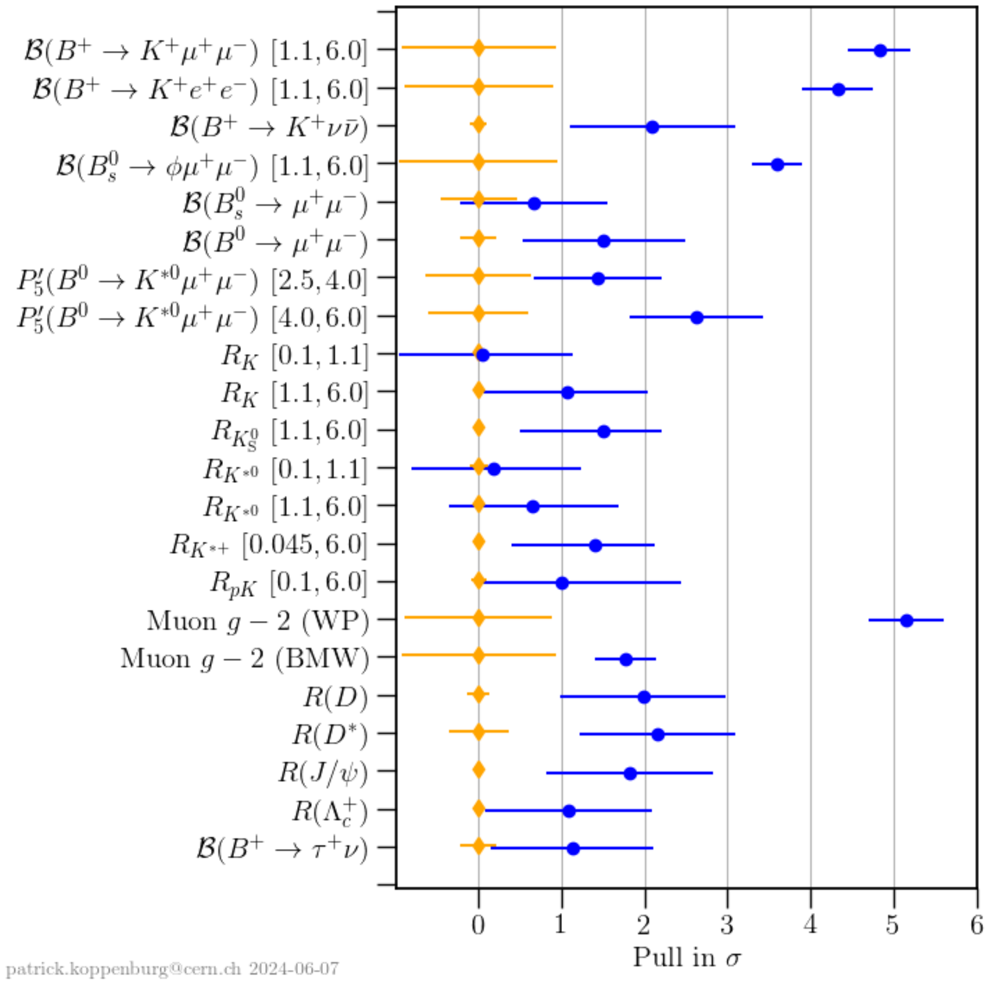


Morel et al. (2020) – Determination of the fine structure constant with an accuracy of 81 parts per trillion

… and its limits

The SM does not answer some important questions !

Need for BSM Physics



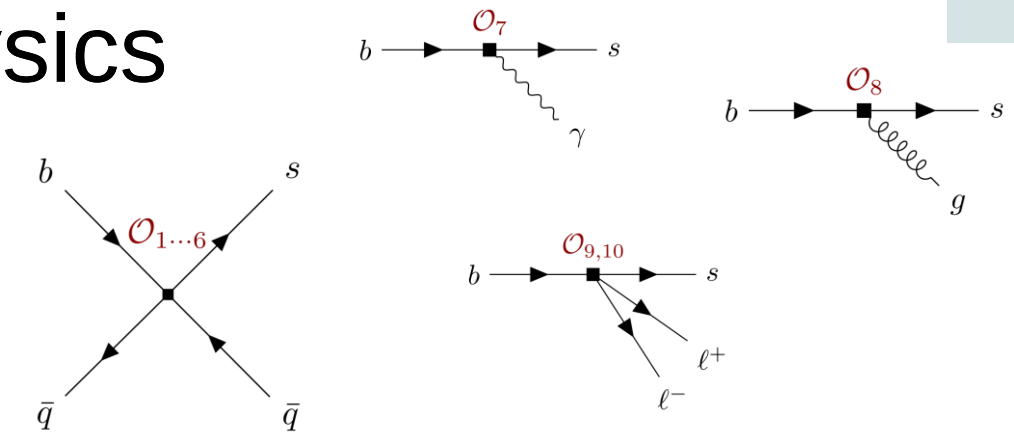
Patrick Koppenburg (2024) – Flavor anomalies

The Need For Automated Calculation

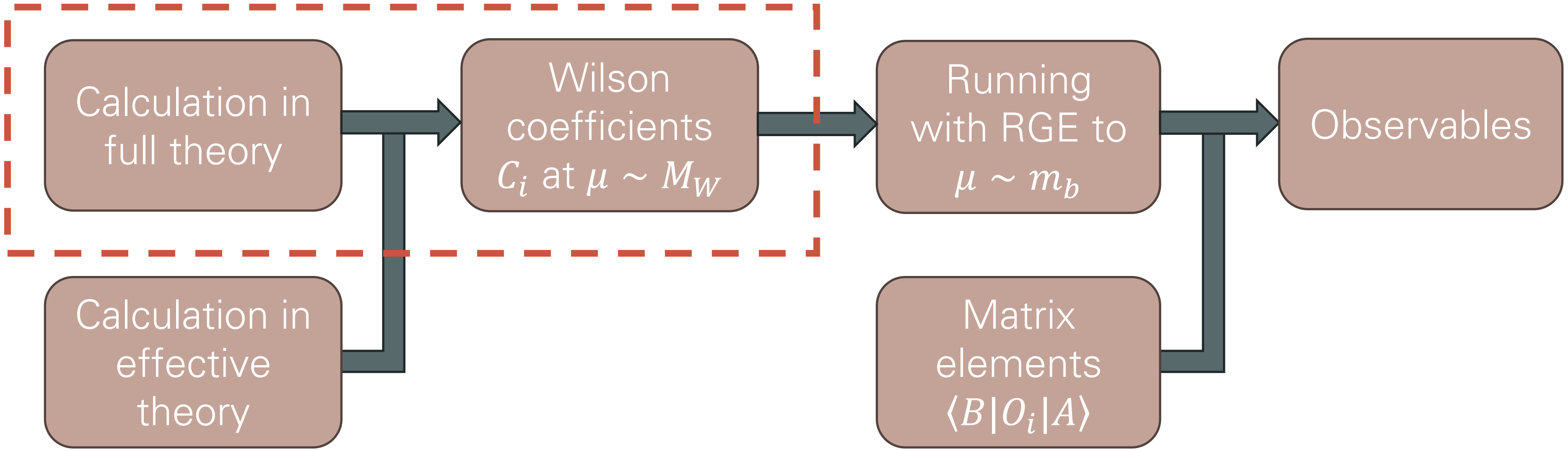
Or why are we paid

Calculations in flavour physics

$$L_{(B)SM} \to H_{\text{eff}}(b \to sX) = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_{i=1}^{10} C_i(\mu)O_i(\mu)$$



For each BSM model



Need for automated calculations !

Calculations in flavour physics

2.2.2 Box diagrams

In this part we will evaluate the two rightmost diagrams in fig. 1b. We use the same IR regularization as before, and we define our notations in fig. 3.

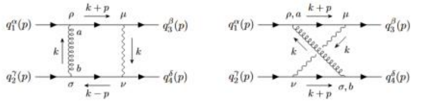


Figure 3: Notations for the box diagrams. Indices $\alpha, \beta, \gamma, \delta$ are color indices, μ, ν, ρ, σ are Lorentz indices and a, b are $SU(3)_C$ indices. Momentum conservation and color conservation by the W vertex have already been taken into account.

Uncrossed box Applying the Feynman rules to the uncrossed box diagram gives

$$\mathcal{M}^u = i\int\frac{d^4k}{(2\pi)^4}\left[\bar{u}_3^\beta\frac{ig}{2\sqrt{2}}V_{13}^*\gamma_L^\mu\frac{i(\not{k}+\not{p})}{(k+p)^2+i\lambda}ig_s\gamma^\rho T^a_{\beta\alpha}u_1^\alpha\right]\frac{-ig_{\rho\sigma}\delta^{ab}}{k^2+i\lambda} \times\left[\bar{u}_4^\delta\frac{ig}{2\sqrt{2}}V_{42}\gamma_L^\nu\frac{i(\not{k}-\not{p})}{(k-p)^2+i\lambda}ig_s\gamma^\sigma T^b_{\delta\gamma}u_2^\gamma\right]\frac{-ig_{\mu\nu}}{k^2-M_W^2+i\lambda}. \quad (2.40)$$

After some elementary simplifications, we arrive at

$$\mathcal{M}^u = i\mathcal{G}M_W^2g_s^2T^a_{\beta\alpha}T^a_{\delta\gamma}\int\frac{d^4k}{(2\pi)^4}\frac{\left[\bar{u}_3^\beta\gamma_L^\mu(\not{k}+\not{p})\gamma^\nu u_1^\alpha\right]\left[\bar{u}_4^\delta\gamma_{L\mu}(\not{k}-\not{p})\gamma_\nu u_2^\gamma\right]}{[(k+p)^2+i\lambda][(k-p)^2+i\lambda][k^2-M_W^2+i\lambda][k^2+i\lambda]}, \quad (2.41)$$

which we can split into

$$\mathcal{M}^u = i\mathcal{G}T^{\rho\sigma}I_{\rho\sigma}, \quad (2.42)$$

where

$$T^{\rho\sigma} = T^a_{\beta\alpha}T^a_{\delta\gamma}\left[\bar{u}_3^\beta\gamma_L^\mu\gamma^\rho\gamma^\nu u_1^\alpha\right]\left[\bar{u}_4^\delta\gamma_{L\mu}\gamma^\sigma\gamma_\nu u_2^\gamma\right], \quad (2.43)$$

$$I_{\rho\sigma} = M_W^2g_s^2\int\frac{d^4k}{(2\pi)^4}\frac{(k+p)_\rho(k-p)_\sigma}{[(k+p)^2+i\lambda][(k-p)^2+i\lambda][k^2-M_W^2+i\lambda][k^2+i\lambda]}. \quad (2.44)$$

At first sight, it is not obvious how to simplify the contractions between Dirac matrices in different fermion lines that appear in the expression of $T^{\rho\sigma}$. The Fierz identities can help us here, as they allow us to exchange fermions between lines, and in particular to put all the Dirac matrices within the same one. Writing the chirality projectors explicitly and using their anti-commutation relation, we can rearrange

$$\begin{aligned}T^{\rho\sigma} &= T^a_{\beta\alpha}T^a_{\delta\gamma}\left[\bar{u}_3^\beta\gamma^\mu(1-\gamma^5)\gamma^\rho\gamma^\nu u_1^\alpha\right]\left[\bar{u}_4^\delta\gamma_\mu(1-\gamma^5)\gamma^\sigma\gamma_\nu u_2^\gamma\right]\\ &= T^a_{\beta\alpha}T^a_{\delta\gamma}\left[\bar{u}_3^\beta\gamma^\mu\gamma^\rho\gamma^\nu(1-\gamma^5)u_1^\alpha\right]\left[\bar{u}_4^\delta(1+\gamma^5)\gamma_\mu\gamma^\sigma\gamma_\nu u_2^\gamma\right]. \quad (2.45)\end{aligned}$$

Using the Fierz identity (F1), we have then

$$(1-\gamma^5)u_1^\alpha\bar{u}_4^\delta(1+\gamma^5) = -\frac{1}{2}\left[\bar{u}_4^\delta\gamma^\tau(1-\gamma^5)u_1^\alpha\right]\gamma_\tau(1+\gamma^5), \quad (2.46)$$

and therefore

$$T^{\rho\sigma} = -\frac{1}{2}T^a_{\beta\alpha}T^a_{\delta\gamma}\left[\bar{u}_4^\delta\gamma^\tau(1-\gamma^5)u_1^\alpha\right]\left[\bar{u}_3^\beta\gamma^\mu\gamma^\rho\gamma^\nu\gamma_\tau(1+\gamma^5)\gamma_\mu\gamma^\sigma\gamma_\nu u_2^\gamma\right]. \quad (2.47)$$

Now we can "push" the projector to the right and use some Dirac algebra to reduce the rightmost bracket:

$$\begin{aligned}\left[\bar{u}_3^\beta\gamma^\mu\gamma^\rho\gamma^\nu\gamma_\tau(1+\gamma^5)\gamma_\mu\gamma^\sigma\gamma_\nu u_2^\gamma\right] &= \left[\bar{u}_3^\beta\gamma^\mu\gamma^\rho\gamma^\nu\gamma_\tau\gamma_\mu\gamma^\sigma\gamma_\nu(1-\gamma^5)u_2^\gamma\right]\\ &= -2\left[\bar{u}_3^\beta\gamma_\tau\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_\nu(1-\gamma^5)u_2^\gamma\right]\\ &= -8g^{\rho\sigma}\left[\bar{u}_3^\beta\gamma_\tau(1-\gamma^5)u_2^\gamma\right]. \quad (2.48)\end{aligned}$$

Putting everything together and exchanging back the two fermion lines with (F3), we have

$$T^{\rho\sigma} = 4g^{\rho\sigma}T^a_{\beta\alpha}T^a_{\delta\gamma}\left[\bar{u}_3^\beta\gamma_L^\mu u_1^\alpha\right]\left[\bar{u}_4^\delta\gamma_{L\mu}u_2^\gamma\right]. \quad (2.49)$$

To complete the calculation, we use the Fierz-like identity for the generators of $SU(3)$, giving

$$\begin{aligned}T^{\rho\sigma} &= 2g^{\rho\sigma}\left(\delta_{\alpha\delta}\delta_{\beta\gamma}-\frac{1}{N}\delta_{\alpha\beta}\delta_{\gamma\delta}\right)\left[\bar{u}_3^\beta\gamma_L^\mu u_1^\alpha\right]\left[\bar{u}_4^\delta\gamma_{L\mu}u_2^\gamma\right]\\ &= 2g^{\rho\sigma}\left(\mathcal{O}_1-\frac{1}{N}\mathcal{O}_2\right). \quad (2.50)\end{aligned}$$

Now, to evaluate $I_{\rho\sigma}$, we could use the usual technique with Feynman parameters right away, but the resulting Feynman integrals are a pain to evaluate. Instead, we can first simplify the integral using some general arguments. First, Lorentz invariance implies that $I_{\rho\sigma}$ can be written as

$$I_{\rho\sigma} = J_1(p^2, M_W)g_{\rho\sigma} - K_1(p^2, M_W)p_\rho p_\sigma, \quad (2.51)$$

in order to have the correct tensor structure. Contracting both sides with $g^{\rho\sigma}$, we have

$$4J_1 - p^2K_1 = M_W^2g_s^2\int\frac{d^4k}{(2\pi)^4}\frac{k^2-p^2}{[(k+p)^2+i\lambda][(k-p)^2+i\lambda][k^2-M_W^2+i\lambda][k^2+i\lambda]}, \quad (2.52)$$

from which we identify

$$J_1 = \frac{1}{4}M_W^2g_s^2\int\frac{d^4k}{(2\pi)^4}\frac{1}{[(k+p)^2+i\lambda][(k-p)^2+i\lambda][k^2-M_W^2+i\lambda]}, \quad (2.53)$$

$$K_1 = M_W^2g_s^2\int\frac{d^4k}{(2\pi)^4}\frac{1}{[(k+p)^2+i\lambda][(k-p)^2+i\lambda][k^2-M_W^2+i\lambda][k^2+i\lambda]}. \quad (2.54)$$

Note that in J_1 we canceled the $k^2+i\lambda$ factor in the denominator with the k^2 in the numerator, which doesn't pose any problem as we can freely add a $+i\lambda$ term in the numerator in the limit $\lambda\to 0$. In order to evaluate these integrals, we perform a Wick rotation $k^0\to k^0_E = ik^0$ (and similarly for p^0), so that $d^4k = id^4\mathbf{k}_E$, $k^2=-k_E^2$ and $(k\pm p)^2 = -(\mathbf{k}_E\pm\mathbf{p}_E)^2$. Omitting the E subscripts as we now only work with components of Euclidean vectors, and denoting by k the magnitude of the Euclidean vector $\mathbf{k}$, the integrals become

$$J_1 = i\frac{M_W^2g_s^2}{4}\int\frac{d^4\mathbf{k}}{(2\pi)^4}\frac{1}{(\mathbf{k}+\mathbf{p})^2(\mathbf{k}-\mathbf{p})^2(k^2+M_W^2)}, \quad (2.55)$$

$$K_1 = -iM_W^2g_s^2\int\frac{d^4\mathbf{k}}{(2\pi)^4}\frac{1}{k^2(\mathbf{k}+\mathbf{p})^2(\mathbf{k}-\mathbf{p})^2(k^2+M_W^2)}. \quad (2.56)$$

Note that we dropped the imaginary regulator as it is no longer needed in Euclidean spacetime. Splitting the euclidean volume form into $d^4\mathbf{k} = k^3dk\,d\Omega_4$, where $d\Omega_4$ is the 4-dimensional euclidean solid angle, we write

$$J_1 = i\frac{M_W^2g_s^2}{4(2\pi)^4}\int d\Omega_4\int_0^\infty dk\,\frac{k^3}{(\mathbf{k}+\mathbf{p})^2(\mathbf{k}-\mathbf{p})^2(k^2+M_W^2)}, \quad (2.57)$$

$$K_1 = -i\frac{M_W^2g_s^2}{(2\pi)^4}\int d\Omega_4\int_0^\infty dk\,\frac{k}{(\mathbf{k}+\mathbf{p})^2(\mathbf{k}-\mathbf{p})^2(k^2+M_W^2)}. \quad (2.58)$$

Let's compute J_1 first. In order to evaluate the k integral, we use partial fraction decomposition, writing

$$\frac{k^3}{(\mathbf{k}+\mathbf{p})^2(\mathbf{k}-\mathbf{p})^2(k^2+M_W^2)} = \frac{Ak+B}{k^2+\alpha k+p^2}+\frac{Ck+D}{k^2-\alpha k+p^2}+\frac{Ek+F}{k^2+M_W^2}, \quad (2.59)$$

where $\alpha = 2p\cos\theta$ and we chose to align $\mathbf{p}$ with the x^3 axis of the Euclidean space, so that θ corresponds to the second hyperspherical angular coordinate, ranging from 0 to π. Multiplying both sides by the denominator of the left-hand side and identifying by powers of k, we obtain the following system of equations:

$$A+C+E=0, \quad (2.60a)$$
$$B+D+F+\alpha(C-A)=0, \quad (2.60b)$$
$$(A+C)(M_W^2+p^2)+\alpha(D-B)+(2p^2-\alpha^2)E=1, \quad (2.60c)$$
$$\alpha M_W^2(C-A)+(B+D)(M_W^2+p^2)+(2p^2-\alpha^2)F=0, \quad (2.60d)$$
$$p^2M_W^2(A+C)+\alpha M_W^2(D-B)+Ep^4=0, \quad (2.60e)$$
$$p^2M_W^2(B+D)+Fp^4=0. \quad (2.60f)$$

Changing variables to $X_\pm = C\pm A$, $Y_\pm = D\pm B$, the system can be put in matrix form

$$\begin{pmatrix}1&0&1&0&0&0\\ p^2+M_W^2&\alpha&2p^2-\alpha^2&0&0&0\\ p^2M_W^2&\alpha M_W^2&p^4&0&0&0\\ 0&0&0&\alpha&1&1\\ 0&0&0&\alpha M_W^2&p^2+M_W^2&2p^2-\alpha^2\\ 0&0&0&0&p^2M_W^2&p^4\end{pmatrix}\begin{pmatrix}X_+\\ Y_-\\ E\\ X_-\\ Y_+\\ F\end{pmatrix}=\begin{pmatrix}0\\1\\0\\0\\0\\0\end{pmatrix}, \quad (2.61)$$

from which we read immediately that $X_- = Y_+ = F = 0$, hence $A = C$ and $B = -D$. Inverting the upper system yields after replacing α by its value

$$X_+ = 2A = 2C = \frac{1}{M_W^2}\frac{1}{\Delta}, \quad Y_- = 2D = -2B = \frac{1}{M_W^2}\frac{p(\beta-1)}{2\cos\theta}\frac{1}{\Delta}, \quad E = -\frac{1}{M_W^2}\frac{1}{\Delta}, \quad (2.62)$$

where $\beta = p^2/M_W^2$ and $\Delta = (1-\beta)^2+4\beta\cos^2\theta$. The integral J_1 then reads

$$J_1 = i\frac{g_s^2}{4(2\pi)^4}\int\frac{d\Omega_4}{\Delta}\int_0^\infty dk\left\{\frac{ak+b}{k^2+\alpha k+p^2}+\frac{ak-b}{k^2-\alpha k+p^2}-\frac{k}{k^2+M_W^2}\right\}, \quad (2.63)$$

where $a = 1/2$ and $b = p(1-\beta)/4\cos\theta$. Now, we evaluate the basic integrals that appear after the partial fraction decomposition. Only two of them are convergent, namely

$$I^{(0,+)} = \int_0^\infty\frac{dk}{k^2+\alpha k+p^2}, \quad I^{(0,-)} = \int_0^\infty\frac{dk}{k^2-\alpha k+p^2}. \quad (2.64)$$

To evaluate the first one, we replace α by its value, we complete the square and shift the momentum in the denominator to obtain

$$I^{(0,+)} = \int_0^\infty\frac{dk}{(k+p\cos\theta)^2+p^2(1-\cos^2\theta)} = \int_{p\cos\theta}^\infty\frac{dq}{q^2+p^2\sin^2\theta}, \quad (2.65)$$

for which an anti-derivative is known in terms of usual functions,

$$I^{(0,+)} = \frac{1}{p\sin\theta}\left[\arctan\left(\frac{q}{p\sin\theta}\right)\right]_{p\cos\theta}^\infty = \frac{1}{p\sin\theta}\left[\frac{\pi}{2}-\arctan\left(\frac{1}{\tan\theta}\right)\right]. \quad (2.66)$$

Finally, using the property of the arc-tangent,

$$\arctan x+\arctan\frac{1}{x} = \text{sgn}(x)\frac{\pi}{2}, \quad (2.67)$$

this reduces to

$$I^{(0,+)} = \frac{1}{p\sin\theta}\arctan(\tan\theta) = \frac{\theta}{p\sin\theta}. \quad (2.68)$$

The second integral, $I^{(0,-)}$, can be obtained from $I^{(0,+)}$ by changing $\theta\to\pi-\theta$, hence

$$I^{(0,-)} = \frac{\pi-\theta}{p\sin(\pi-\theta)} = \frac{\pi-\theta}{p\sin(\theta)}. \quad (2.69)$$

The other three integrals are divergent, only their sum is finite. To give expressions, we introduce a hard cutoff Λ and we let $\Lambda\to\infty$. We then have to evaluate

$$I^{(1,\pm)}(\Lambda) = \int_0^\Lambda dk\,\frac{k}{k^2\pm\alpha k+p^2}, \quad I^{(1,0)}(\Lambda) = \int_0^\Lambda dk\,\frac{k}{k^2+M_W^2}. \quad (2.70)$$

For the first integral, we rewrite the quotient by splitting it into

$$I^{(1,+)}(\Lambda) = \frac{1}{2}\int_0^\Lambda dk\,\frac{2k+2p\cos\theta}{k^2+2pk\cos\theta+p^2}-\int_0^\Lambda dk\,\frac{p\cos\theta}{k^2+2pk\cos\theta+p^2}. \quad (2.71)$$

The first integral admits a known anti-derivative, and the second one is the same as (2.64), hence

$$\begin{aligned}I^{(1,+)}(\Lambda) &= \frac{1}{2}\left[\ln(k^2+2pk\cos\theta+p^2)\right]_0^\Lambda - p\cos\theta\left[\frac{\theta}{p\sin\theta}+\mathcal{O}\left(\frac{1}{\Lambda}\right)\right]\\ &= \frac{1}{2}\ln\left(1+2\frac{\Lambda}{p}\cos\theta+\frac{\Lambda^2}{p^2}\right)-\frac{\theta}{\tan\theta}+\mathcal{O}\left(\frac{p}{\Lambda}\right), \quad (2.72)\end{aligned}$$

which, keeping only relevant contributions in the logarithm, reads

$$I^{(1,+)}(\Lambda) = \frac{1}{2}\ln\left(\frac{\Lambda^2}{p^2}\right)-\frac{\theta}{\tan\theta}+\mathcal{O}\left(\frac{p}{\Lambda}\right). \quad (2.73)$$

As before, the $I^{(1,-)}$ integral follows from replacing $\theta\to\pi-\theta$,

$$I^{(1,-)}(\Lambda) = \frac{1}{2}\ln\left(\frac{\Lambda^2}{p^2}\right)+\frac{\pi-\theta}{\tan\theta}+\mathcal{O}\left(\frac{p}{\Lambda}\right). \quad (2.74)$$

Finally, the last integral is easy to evaluate,

$$I^{(1,0)}(\Lambda) = \frac{1}{2}\int_0^\Lambda dk\,\frac{2k}{k^2+M_W^2} = \frac{1}{2}\ln\left(1+\frac{\Lambda^2}{M_W^2}\right) = \frac{1}{2}\ln\left(\frac{\Lambda^2}{M_W^2}\right)+\mathcal{O}\left(\frac{M_W^2}{\Lambda^2}\right). \quad (2.75)$$

Injecting these expressions in J_1 leads to

$$J_1 = i\frac{g_s^2}{4(2\pi)^4}\lim_{\Lambda\to\infty}\int\frac{d\Omega_4}{\Delta}\left\{\frac{1}{2}\left[\ln\left(\frac{\Lambda^2}{p^2}\right)+\frac{\pi-2\theta}{\tan\theta}\right]+(1-\beta)\frac{2\theta-\pi}{2\sin(2\theta)}-\frac{1}{2}\ln\left(\frac{\Lambda^2}{M_W^2}\right)\right\}, \quad (2.76)$$

and we see that the Λ dependence indeed cancel out between all the integrals, leaving a finite result,

$$J_1 = i\frac{g_s^2}{4(2\pi)^4}\int\frac{d\Omega_4}{\Delta}\left\{\frac{1}{2}\ln\left(\frac{M_W^2}{p^2}\right)+\frac{\pi-2\theta}{2\tan\theta}+(1-\beta)\frac{2\theta-\pi}{2\sin(2\theta)}\right\}. \quad (2.77)$$

In the leading log (LL) approximation, we are not interested in constant terms nor terms proportional to β (which is small), thus we only keep the first term in the integral,

$$J_1 = i\frac{g_s^2}{8(2\pi)^4}\ln\left(\frac{M_W^2}{p^2}\right)\int\frac{d\Omega_4}{\Delta}. \quad (2.78)$$

The 4-dimensional solid angle element reads $d\Omega_4 = \sin^2\chi\sin\theta\,d\chi\,d\theta\,d\phi$, where χ and θ range from 0 to π while ϕ ranges from 0 to 2π. As Δ only depends on θ, we may write

$$\int\frac{d\Omega_4}{\Delta} = \int_0^\pi d\chi\,\sin^2\chi\int_0^{2\pi}d\phi\int_{-1}^1\frac{d\cos\theta}{\Delta} = \pi^2\int_{-1}^1\frac{d\cos\theta}{\Delta}, \quad (2.79)$$

with

$$\int_{-1}^1\frac{d\cos\theta}{\Delta} = \int_{-1}^1\frac{du}{(1-\beta)^2+4\beta u} = \frac{1}{\sqrt{\beta}(\beta-1)}\arctan\left(\frac{2\sqrt{\beta}}{\beta-1}\right) = 2+\mathcal{O}(\beta). \quad (2.80)$$

Finally, the J_1 integral in the LL approximation reads

$$J_1 = \frac{i}{4}\frac{\alpha_s}{4\pi}\ln\left(\frac{M_W^2}{p^2}\right). \quad (2.81)$$

Getting back to Minkowskian vectors, we replace p^2 by $-p^2$. In the LL approximation the contribution of the $p_\rho p_\sigma$ term in $I_{\rho\sigma}$ is not relevant, thus the complete amplitude for the uncrossed box reads, using (2.50) and $g^{\rho\sigma}g_{\rho\sigma} = 4$,

$$\mathcal{M}^u = -2\mathcal{G}\frac{\alpha_s}{4\pi}\ln\left(\frac{M_W^2}{-p^2}\right)\left(\mathcal{O}_1-\frac{1}{N}\mathcal{O}_2\right). \quad (2.82)$$

Crossed box For the crossed box diagram (rightmost diagram in fig. 3), the Feynman rules give

$$\mathcal{M}^c = i\int\frac{d^4k}{(2\pi)^4}\left[\bar{u}_3^\beta\frac{ig}{2\sqrt{2}}V_{13}^*\gamma_L^\mu\frac{i(\not{k}+\not{p})}{(k+p)^2+i\lambda}ig_s\gamma^\rho T^a_{\beta\alpha}u_1^\alpha\right]\frac{-ig_{\rho\sigma}\delta^{ab}}{k^2+i\lambda} \times\left[\bar{u}_4^\delta ig_s\gamma^\sigma T^b_{\delta\gamma}\frac{i(\not{k}+\not{p})}{(k+p)^2+i\lambda}\frac{ig}{2\sqrt{2}}V_{42}\gamma_L^\nu u_2^\gamma\right]\frac{-ig_{\mu\nu}}{k^2-M_W^2+i\lambda}. \quad (2.83)$$

And after simplifications,

$$i\mathcal{M}^c = i\mathcal{G}M_W^2g_s^2T^a_{\beta\alpha}T^a_{\delta\gamma}\int\frac{d^4k}{(2\pi)^4}\frac{\left[\bar{u}_3^\beta\gamma_L^\mu(\not{k}+\not{p})\gamma^\nu u_1^\alpha\right]\left[\bar{u}_4^\delta\gamma_\nu(\not{k}+\not{p})\gamma_{L\mu}u_2^\gamma\right]}{[k^2+i\lambda][(k+p)^2+i\lambda]^2[k^2-M_W^2+i\lambda]}. \quad (2.84)$$

As before, we split this amplitude into

$$i\mathcal{M}^c = i\mathcal{G}T^{\rho\sigma}I_{\rho\sigma}, \quad (2.85)$$

where

$$T^{\rho\sigma} = T^a_{\beta\alpha}T^a_{\delta\gamma}\left[\bar{u}_3^\beta\gamma_L^\mu\gamma^\rho\gamma^\nu u_1^\alpha\right]\left[\bar{u}_4^\delta\gamma_\nu\gamma^\sigma\gamma_{L\mu}u_2^\gamma\right], \quad (2.86)$$

$$I_{\rho\sigma} = M_W^2g_s^2\int\frac{d^4k}{(2\pi)^4}\frac{(k+p)_\rho(k+p)_\sigma}{[k^2+i\lambda][(k+p)^2+i\lambda]^2[k^2-M_W^2+i\lambda]}. \quad (2.87)$$

The Fierz and gamma matrices identities allow us to calculate $T^{\rho\sigma}$,

$$\begin{aligned}T^{\rho\sigma} &= T^a_{\beta\alpha}T^a_{\delta\gamma}\left[\bar{u}_3^\beta\gamma^\mu(1-\gamma^5)\gamma^\rho\gamma^\nu u_1^\alpha\right]\left[\bar{u}_4^\delta\gamma_\nu\gamma^\sigma\gamma_\mu(1-\gamma^5)u_2^\gamma\right]\\ &= T^a_{\beta\alpha}T^a_{\delta\gamma}\left[\bar{u}_4^\delta\gamma_\nu\gamma^\sigma\gamma_\mu(1-\gamma^5)u_2^\gamma\right]\left[\bar{u}_3^\beta(1+\gamma^5)\gamma^\mu\gamma^\rho\gamma^\nu u_1^\alpha\right]\\ &= -\frac{1}{2}T^a_{\beta\alpha}T^a_{\delta\gamma}\left[\bar{u}_3^\beta\gamma^\tau(1-\gamma^5)u_2^\gamma\right]\left[\bar{u}_4^\delta\gamma_\nu\gamma^\sigma\gamma_\mu\gamma_\tau(1+\gamma^5)\gamma^\mu\gamma^\rho\gamma^\nu u_1^\alpha\right]\\ &= -\frac{1}{2}T^a_{\beta\alpha}T^a_{\delta\gamma}\left[\bar{u}_3^\beta\gamma^\tau(1-\gamma^5)u_2^\gamma\right]\left[\bar{u}_4^\delta\gamma_\nu\gamma^\sigma\gamma_\mu\gamma_\tau\gamma^\mu\gamma^\rho\gamma^\nu(1-\gamma^5)u_1^\alpha\right]\\ &= T^a_{\beta\alpha}T^a_{\delta\gamma}\left[\bar{u}_3^\beta\gamma^\tau(1-\gamma^5)u_2^\gamma\right]\left[\bar{u}_4^\delta\gamma_\nu\gamma^\sigma\gamma_\tau\gamma^\rho\gamma^\nu(1-\gamma^5)u_1^\alpha\right]\\ &= -2T^a_{\beta\alpha}T^a_{\delta\gamma}\left[\bar{u}_3^\beta\gamma^\tau(1-\gamma^5)u_2^\gamma\right]\left[\bar{u}_4^\delta\gamma^\sigma\gamma_\tau\gamma^\rho(1-\gamma^5)u_1^\alpha\right]. \quad (2.88)\end{aligned}$$

We then use Feynman parametrization to rewrite the integral $I_{\rho\sigma}$ as

$$I_{\rho\sigma} = 6M_W^2g_s^2\int_0^1 dx\,dy\,dz\,\delta(x+y+z-1)\,y \times\int\frac{d^4k}{(2\pi)^4}\frac{(k+p)_\rho(k+p)_\sigma}{\left[xk^2+y(k+p)^2+z(k^2-M_W^2)+i\lambda\right]^4}. \quad (2.89)$$

Using $x+y+z=1$ in the integral and shifting $k = q-yp$, this becomes

$$I_{\rho\sigma} = 6M_W^2g_s^2\int_0^1 dx\,dy\,dz\,\delta(x+y+z-1)\,y\int\frac{d^4q}{(2\pi)^4}\frac{(q+(1-y)p)_\rho(q+(1-y)p)_\sigma}{[q^2-V+i\lambda]^4}, \quad (2.90)$$

where $V = zM_W^2+y(y-1)p^2$. Using the tensor reduction procedure, we rewrite this integral as a sum of scalar integrals,

$$I_{\rho\sigma} = 6M_W^2g_s^2\int_0^1 dx\,dy\,dz\,\delta(x+y+z-1)\left\{\frac{y}{4}g_{\rho\sigma}I^{(4,1)}+y(1-y)^2p_\rho p_\sigma I^{(4,0)}\right\}, \quad (2.91)$$

which we evaluate using (1.81), leading to

$$I_{\rho\sigma} = i\frac{\alpha_s}{4\pi}\left\{-\frac{1}{2}g_{\rho\sigma}J_2(\beta)+\frac{p_\rho p_\sigma}{M_W^2}K_2(\beta)\right\}, \quad (2.92)$$

where as before $\beta = -p^2/M_W^2$ and

$$J_2(\beta) = \int_0^1 dx\int_0^{1-x}dy\,\frac{y}{(1-y)(1+\beta y)-x+i\lambda}, \quad (2.93)$$

$$K_2(\beta) = \int_0^1 dx\int_0^{1-x}dy\,\frac{y(1-y)^2}{[(1-y)(1+\beta y)-x+i\lambda]^2}. \quad (2.94)$$

These integrals are easier to evaluate upon exchanging the two variables, which leaves after a first evaluation (dropping the $i\lambda$ regulator as it is no longer needed)

$$\begin{aligned}J_2(\beta) &= \int_0^1 dy\int_0^{1-y}dx\,\frac{y}{(1-y)(1+\beta y)-x}\\ &= \int_0^1 dy\left\{y\ln[(y-1)(1+\beta y)]-y\ln[\beta y(y-1)]\right\}, \quad (2.95)\end{aligned}$$

both arguments in the logarithms are negative real, so we can combine the logs and write

$$J_2(\beta) = \int_0^1 dy\,y\ln\left(1+\frac{1}{\beta y}\right). \quad (2.96)$$

Integrating by parts, we are left with

$$J_2(\beta) = \frac{1}{2}\ln\left(1+\frac{1}{\beta}\right)+\frac{1}{2}\int_0^1 dy\,\frac{y}{1+\beta y}, \quad (2.97)$$

and using partial fraction decomposition in the leftover integral finally yields the final result,

$$J_2(\beta) = -\frac{1}{2}\ln\beta+\frac{1}{2\beta}+\frac{\beta^2-1}{2\beta^2}\ln(1+\beta). \quad (2.98)$$

In order to recover the Leading Log approximation, we expand this result around $\beta = 0$ up to $\mathcal{O}(1)$, which yields

$$J_2(\beta) = \frac{1}{2}\ln\left(\frac{1}{\beta}\right)+\frac{1}{2\beta}+\frac{1}{2}\left(1-\frac{1}{\beta^2}\right)\left(\beta-\frac{\beta^2}{2}+\mathcal{O}(\beta^3)\right) = \frac{1}{2}\ln\left(\frac{1}{\beta}\right)+\mathcal{O}(1). \quad (2.99)$$

For $K_2(\beta)$ we also swap the integrals before computing the inner x-integration, leading to

$$K_2(\beta) = \int_0^1 dy\,y(1-y)^2\left[\frac{1}{\beta y(y-1)}-\frac{1}{(y-1)(1+\beta y)}\right] = \frac{1}{\beta}\int_0^1 dy\,\frac{y-1}{1+\beta y}, \quad (2.100)$$

and we then use partial fraction decomposition to compute the y-integral, which yields

$$K_2(\beta) = -\frac{1}{\beta^2}+\frac{1+\beta}{\beta^3}\ln(1+\beta). \quad (2.101)$$

As we did with J_2, we finally expand this result around $\beta = 0$,

$$K_2(\beta) = -\frac{1}{\beta^2}+\left(\frac{1}{\beta^2}+\frac{1}{\beta^3}\right)\left(\beta-\frac{\beta^2}{2}+\frac{\beta^3}{3}\right) = \frac{1}{2\beta}+\mathcal{O}(1). \quad (2.102)$$

With the Feynman integrals evaluated, the momentum integrals now reads

$$I_{\rho\sigma} = i\frac{\alpha_s}{4\pi}\left\{-\frac{1}{4}g_{\rho\sigma}\ln\left(\frac{M_W^2}{-p^2}\right)+\frac{1}{2}\frac{p_\rho p_\sigma}{-p^2}\right\}. \quad (2.103)$$

The $p_\rho p_\sigma/p^2$ term is of $\mathcal{O}(1)$ and therefore not needed in the LL approximation. Injecting (2.88) and (2.103) in (2.85), we have

$$\mathcal{M}^c = -\frac{1}{2}\mathcal{G}\frac{\alpha_s}{4\pi}\ln\left(\frac{M_W^2}{-p^2}\right)T^a_{\beta\alpha}T^a_{\delta\gamma}\left[\bar{u}_3^\beta\gamma^\tau(1-\gamma^5)u_2^\gamma\right]\left[\bar{u}_4^\delta\gamma_\rho\gamma_\tau\gamma^\rho(1-\gamma^5)u_1^\alpha\right]. \quad (2.104)$$

Using some Dirac algebra, the Fierz identity (F3) and the generator product identity (1.39), this can be rewritten as

$$\mathcal{M}^c = \frac{1}{2}\mathcal{G}\frac{\alpha_s}{4\pi}\ln\left(\frac{M_W^2}{-p^2}\right)\left(\mathcal{O}_1-\frac{1}{N}\mathcal{O}_2\right). \quad (2.105)$$

Full box contribution Adding together the amplitudes for the crossed and uncrossed box, and their symmetric counterparts which contribute the same to the total amplitude, we arrive at

$$\mathcal{M}^{\text{box}} = 2(\mathcal{M}^u+\mathcal{M}^c) = -3\mathcal{G}\frac{\alpha_s}{4\pi}\ln\left(\frac{M_W^2}{-p^2}\right)\left(\mathcal{O}_1-\frac{1}{N}\mathcal{O}_2\right). \quad (2.106)$$

Calculations in flavour physics

E.2 Isospin asymmetry of $B \to K^*\gamma$

The isospin asymmetry Δ_0 in $B \to K^*\gamma$ decays arises when the photon is emitted from the spectator quark. The contribution to the decay width depends therefore on the charge of the spectator quark and is different for charged and neutral B meson decays:

$$\Delta_{0\pm} = \frac{\Gamma(\bar{B}^0 \to \bar{K}^{*0}\gamma) - \Gamma(B^\pm \to K^{*\pm}\gamma)}{\Gamma(\bar{B}^0 \to \bar{K}^{*0}\gamma) + \Gamma(B^\pm \to K^{*\pm}\gamma)} , \tag{450}$$

which can be written as [59]:

$$\Delta_0 = \mathrm{Re}(b_d - b_u) , \tag{451}$$

where the spectator dependent coefficients b_q take the form:

$$b_q = \frac{12\pi^2 f_B Q_q}{\overline{m}_b T_1^{B\to K^*} a_7^c} \left(\frac{f_{K^*}^\perp}{\overline{m}_b} K_1 + \frac{f_{K^*} m_{K^*}}{6\lambda_B m_B} K_{2q} \right) . \tag{452}$$

In the same way as for $b \to s\gamma$ branching ratio, the SUSY contributions induced by charged Higgs and chargino loops must be taken into account for the calculation of isospin symmetry breaking.

The functions K_1 and K_{2q} can be written in function of the Wilson coefficients C_i in the traditional basis (see Appendix D.2) at scale μ_b [59]:

$$\begin{aligned} K_1 &= -\left(C_6(\mu_b) + \frac{C_5(\mu_b)}{N} \right) F_\perp + \frac{C_F}{N} \frac{\alpha_s(\mu_b)}{4\pi} \left\{ \left(\frac{m_b}{m_B} \right)^2 C_8(\mu_b) X_\perp \right. \\ &\left. - C_2(\mu_b) \left[\left(\frac{4}{3} \ln \frac{m_b}{\mu_b} + \frac{2}{3} \right) F_\perp - G_\perp(x_{cb}) \right] + r_1 \right\} + (C_i \leftrightarrow C_i') , \end{aligned} \tag{453}$$

$$\begin{aligned} K_{2q} &= \frac{V_{us}^* V_{ub}}{V_{cs}^* V_{cb}} \left(C_2(\mu_b) + \frac{C_1(\mu_b)}{N} \right) \delta_{qu} + \left(C_4(\mu_b) + \frac{C_3(\mu_b)}{N} \right) \\ &+ \frac{C_F}{N} \frac{\alpha_s(\mu_b)}{4\pi} \left[C_2(\mu_b) \left(\frac{4}{3} \ln \frac{m_b}{\mu_b} + \frac{2}{3} - H_\perp(x_{cb}) \right) + r_2 \right] + (C_i \leftrightarrow C_i') , \end{aligned} \tag{454}$$

where $x_{cb} = \frac{m_c^2}{m_b^2}$ and $N = 3$ and $C_F = 4/3$ are colour factors, and:

$$\begin{aligned} r_1 &= \left[\frac{8}{3} C_3(\mu_b) + \frac{4}{3} n_f \Big(C_4(\mu_b) + C_6(\mu_b) \Big) - 8 \Big(N C_6(\mu_b) + C_5(\mu_b) \Big) \right] F_\perp \ln \frac{\mu_b}{\mu_0} + \ldots , \\ r_2 &= \left[-\frac{44}{3} C_3(\mu_b) - \frac{4}{3} n_f \Big(C_4(\mu_b) + C_6(\mu_b) \Big) \right] \ln \frac{\mu_b}{\mu_0} + \ldots . \end{aligned} \tag{455}$$

Here the number of flavours $n_f = 5$, and $\mu_0 = O(m_b)$ is an arbitrary normalization scale.

The coefficient a_7^c reads [60]:

$$\begin{aligned} a_7^c(K^*\gamma) &= C_7(\mu_b) + \frac{\alpha_s(\mu_b) C_F}{4\pi} \Big[C_2(\mu_b) G_2(x_{cb}) + C_8(\mu_b) G_8 \Big] \\ &+ \frac{\alpha_s(\mu_h) C_F}{4\pi} \Big[C_2(\mu_h) H_2(x_{cb}) + C_8(\mu_h) H_8 \Big] + (C_i \leftrightarrow C_i') , \end{aligned} \tag{456}$$

where $\mu_h = \sqrt{\Lambda_h \mu_b}$ is the spectator scale, and

$$G_2(x_{cb}) = -\frac{104}{27} \ln \frac{\mu_b}{m_b} + g_2(x_{cb}) , \tag{457}$$

$$G_8 = \frac{8}{3} \ln \frac{\mu_b}{m_b} + g_8 , \tag{458}$$

with

$$g_8 = \frac{11}{3} - \frac{2\pi^2}{9} + \frac{2i\pi}{3} , \tag{459}$$

$$\begin{aligned} g_2(x) &= \frac{2}{9} x \Big[48 + 30i\pi - 5\pi^2 - 2i\pi^3 - 36\zeta(3) + (36 + 6i\pi - 9\pi^2) \ln x \\ &+ (3 + 6i\pi) \ln^2 x + \ln^3 x \Big] \\ &+ \frac{2}{9} x^2 \Big[18 + 2\pi^2 - 2i\pi^3 + (12 - 6\pi^2) \ln x + 6i\pi \ln^2 x + \ln^3 x \Big] \\ &+ \frac{1}{27} x^3 \Big[-9 + 112i\pi - 14\pi^2 + (182 - 48i\pi) \ln x - 126 \ln^2 x \Big] \\ &- \frac{833}{162} - \frac{20i\pi}{27} + \frac{8\pi^2}{9} x^{3/2} , \end{aligned} \tag{460}$$

where $\zeta(3)$ is given in Eq. (21). The function $H_2(x)$ in Eq. (456) is defined as:

$$H_2(x) = -\frac{2\pi^2}{3N} \frac{f_B f_{K^*}^\perp}{T_1^{B\to K^*} m_B^2} \int_0^1 d\xi \frac{\Phi_{B1}(\xi)}{\xi} \int_0^1 dv \, h(\bar{v}, x) \Phi_\perp(v) , \tag{461}$$

where $h(u, x)$ is the hard-scattering function:

$$h(u, x) = \frac{4x}{u^2} \left[\mathrm{Li}_2 \left(\frac{2}{1 - \sqrt{\frac{u - 4x + i\varepsilon}{u}}} \right) + \mathrm{Li}_2 \left(\frac{2}{1 + \sqrt{\frac{u - 4x + i\varepsilon}{u}}} \right) \right] - \frac{2}{u} , \tag{462}$$

and Li_2 is the usual dilogarithm function given in Eq. (38).
$\Phi_\perp$ is the light-cone wave function with transverse polarization, which can be written in the form [61]:

$$\Phi_\perp(u) = 6u\bar{u} \left[1 + 3a_1^\perp \xi + a_2^\perp \frac{3}{2}(5\xi^2 - 1) \right] , \tag{463}$$

where $\bar{u} = 1 - u$ and $\xi = 2u - 1$, and Φ_{B1} is the distribution amplitude of the B meson involved in the leading-twist projection. Finally:

$$H_8 = \frac{4\pi^2}{3N} \frac{f_B f_{K^*}^\perp}{T_1^{B\to K^*} m_B^2} \int_0^1 d\xi \frac{\Phi_{B1}(\xi)}{\xi} \int_0^1 dv \frac{\Phi_\perp(v)}{v} . \tag{464}$$

The first negative moment of Φ_{B1} can be parametrized by the quantity λ_B such as

$$\int_0^1 d\xi \frac{\Phi_{B1}(\xi)}{\xi} = \frac{m_B}{\lambda_B} . \tag{465}$$

The convolution integrals of the hard-scattering kernels with the meson distribution amplitudes are as follows:

$$\begin{aligned} F_\perp &= \int_0^1 dx \frac{\phi_\perp(x)}{3\bar{x}} , \\ G_\perp(s_c) &= \int_0^1 dx \frac{\phi_\perp(x)}{3\bar{x}} G(s_c, \bar{x}) , \\ H_\perp(s_c) &= \int_0^1 dx \left(g_\perp^{(v)}(x) - \frac{g_\perp'^{(a)}(x)}{4} \right) G(s_c, \bar{x}) , \\ X_\perp &= \int_0^1 dx \, \phi_\perp(x) \frac{1 + \bar{x}}{3\bar{x}^2} , \end{aligned} \tag{466}$$

with $s_c = (m_c/m_b)^2$, and

$$G(s, \bar{x}) = -4 \int_0^1 du \, u\bar{u} \ln(s - u\bar{u}\bar{x} - i\epsilon) , \tag{467}$$

and the Gegenbauer momenta read [61]:

$$\begin{aligned} g_\perp^{(a)}(u) &= 6u\bar{u} \left\{ 1 + a_1^\parallel \xi + \left[\frac{1}{4} a_2^\parallel + \frac{5}{3} \zeta_3^A \left(1 - \frac{3}{16} \omega_{1,0}^A \right) + \frac{35}{4} \zeta_3^V \right] (5\xi^2 - 1) \right\} \\ &+ 6\,\tilde{\delta}_+ (3u\bar{u} + \bar{u} \ln \bar{u} + u \ln u) + 6\,\tilde{\delta}_- (\bar{u} \ln \bar{u} - u \ln u) , \end{aligned} \tag{468}$$

$$\begin{aligned} g_\perp^{(v)}(u) &= \frac{3}{4}(1 + \xi^2) + a_1^\parallel \frac{3}{2} \xi^3 + \left(\frac{3}{7} a_2^\parallel + 5\zeta_3^A \right) (3\xi^2 - 1) \\ &+ \left(\frac{9}{112} a_2^\parallel + \frac{105}{16} \zeta_3^V - \frac{15}{64} \zeta_3^A \omega_{1,0}^A \right) (3 - 30\xi^2 + 35\xi^4) \\ &+ \frac{3}{2} \tilde{\delta}_+ (2 + \ln u + \ln \bar{u}) + \frac{3}{2} \tilde{\delta}_- (2\xi + \ln \bar{u} - \ln u) . \end{aligned} \tag{469}$$

To compute $X_\perp$, the parameter $X = \ln(m_B/\Lambda_h)(1 + \varrho e^{i\varphi})$ is introduced to parametrize the logarithmically divergent integral $\int_0^1 dx/(1-x)$. $\varrho \leq 1$ and the phase φ are arbitrary, and $\Lambda_h \approx 0.5$ GeV is a typical hadronic scale. The remaining parameters are given in Appendix G.

Calculations in flavour physics

$\times\, \mathcal{O}(10)$ Wilson Coefficients

$\times\, \mathcal{O}(100)$ Relevant Observables

$\times\, \mathcal{O}(?)$ Interesting Models

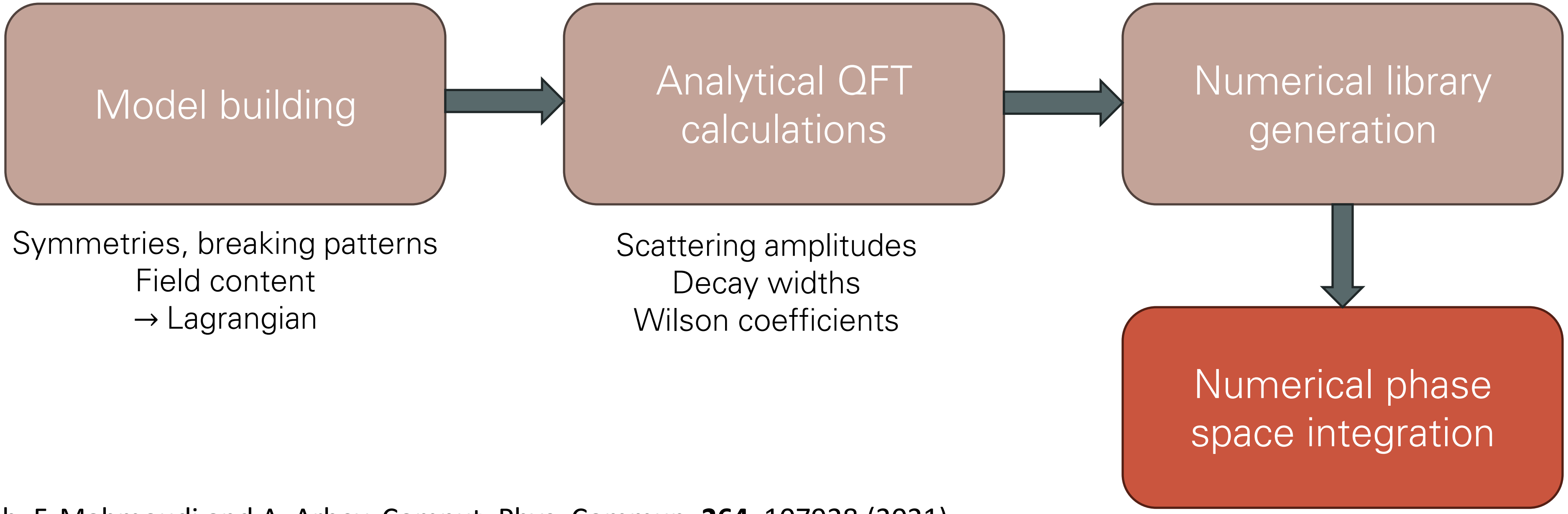

Technical
Difficulties
Please Stand By

The Tools

Or the reason why I’m going blind

MARTY (1.5)

MARTY



G. Uhlrich, F. Mahmoudi and A. Arbey, Comput. Phys. Commun. **264**, 107928 (2021)

MARTY (1.5)

Time for a live demo !

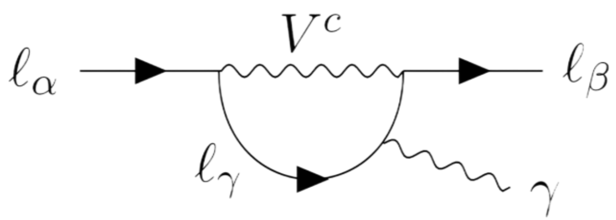
Calculation of $\sigma(e^+e^- \to h^0W^+W^-)$ in the SM.

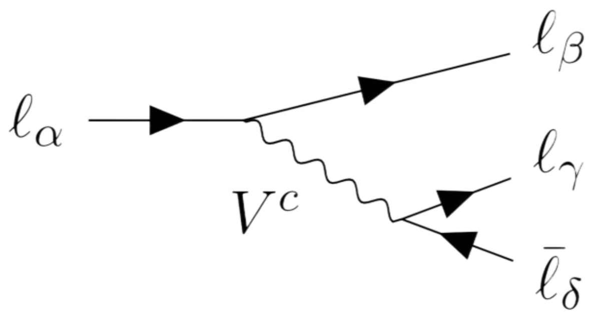
MARTY and the $SU(2)_\ell$ model

Darmé, Deandrea, Mahmoudi (2023) – Gauge $SU(2)_f$ flavor transfers [hep-ph:2307.09595]

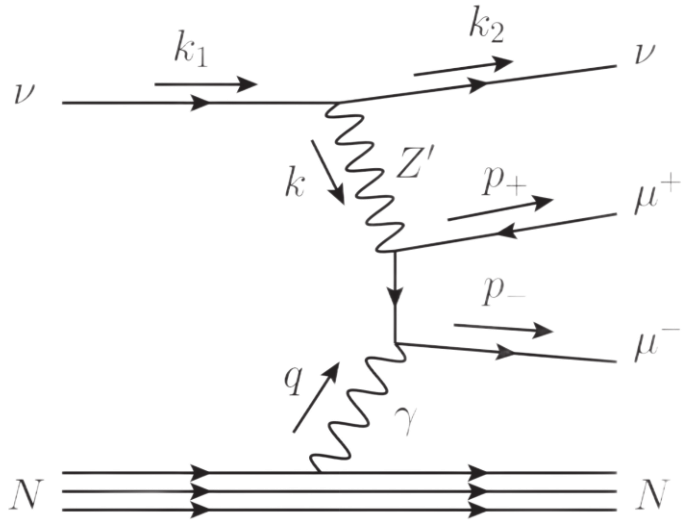
Add a SU(2) gauged flavor symmetry between second and third generations of leptons.

Constraints ?



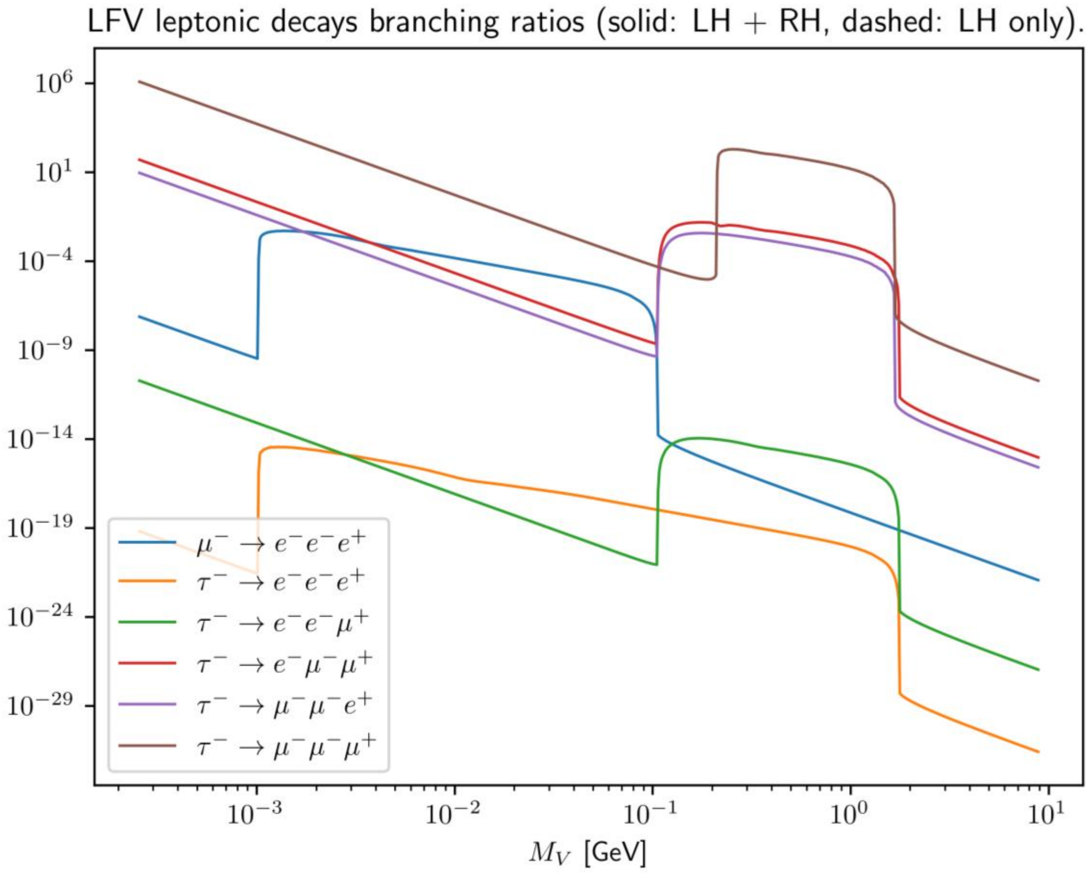


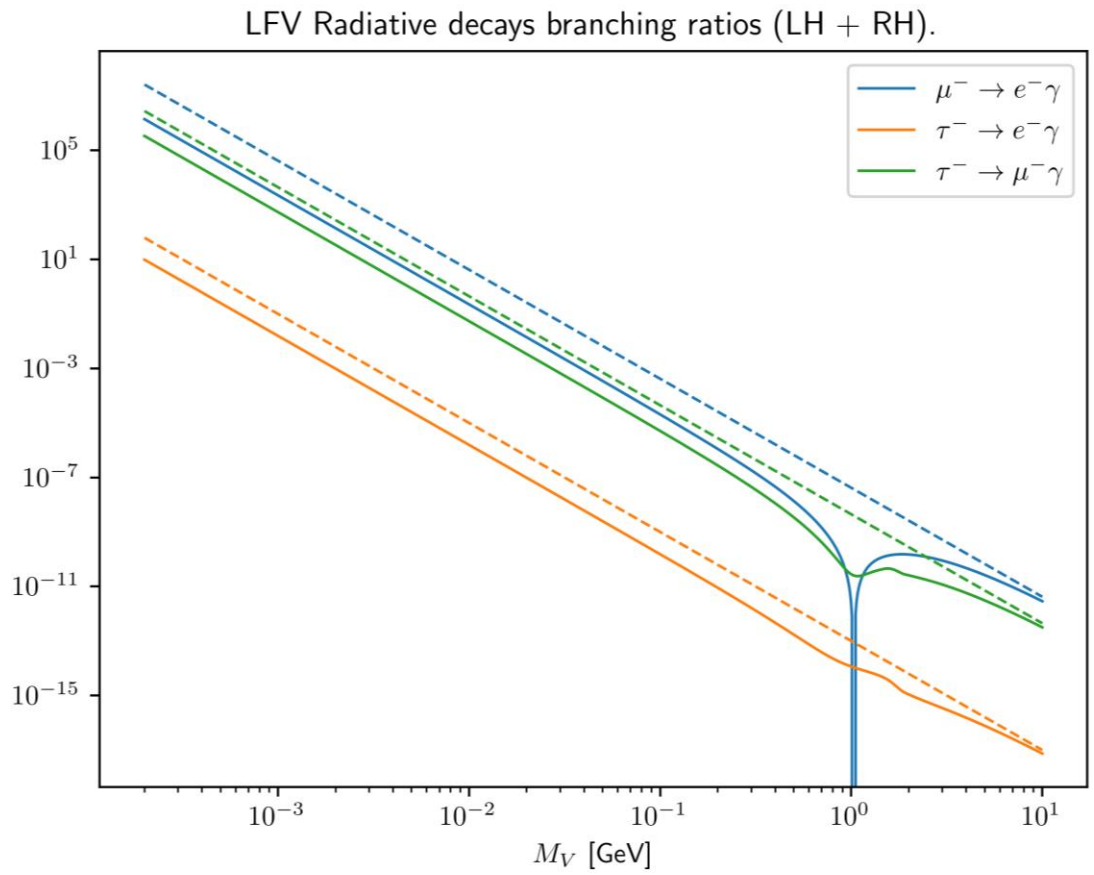
Lepton flavor violating decays



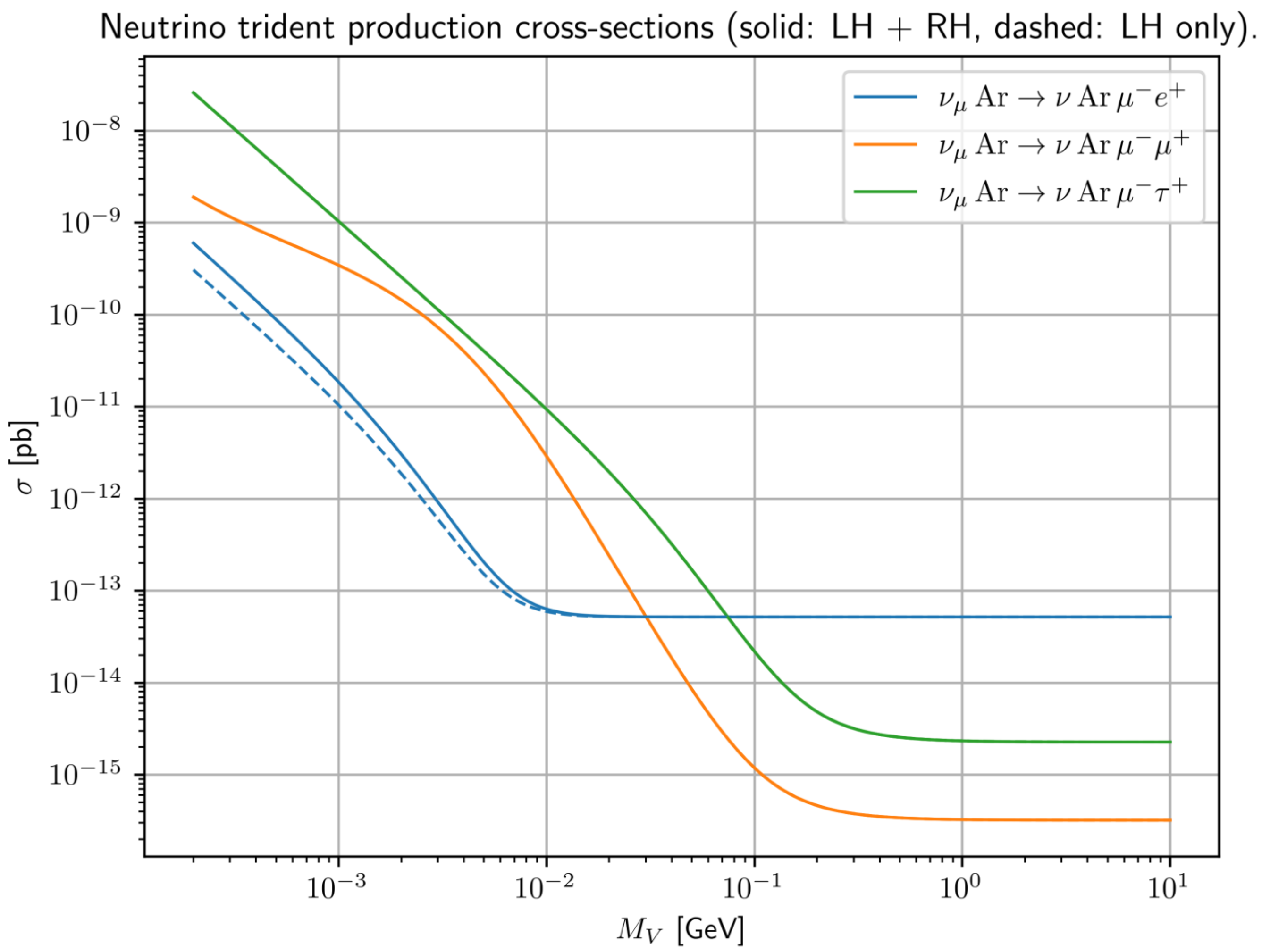
Neutrino trident production [hep-ph:1406.2332]

MARTY and the $SU(2)_\ell$ model



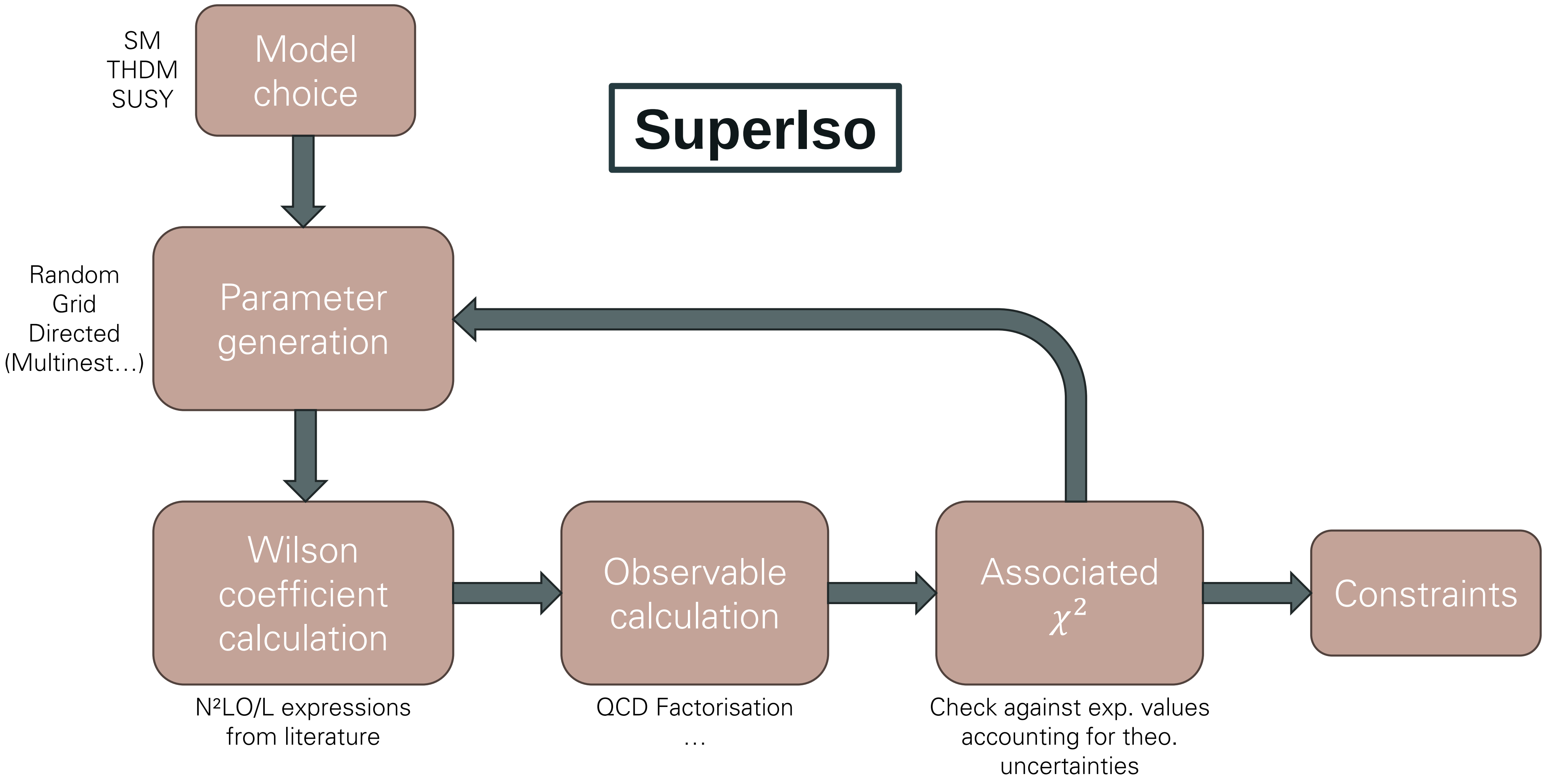


MARTY and the $SU(2)_\ell$ model

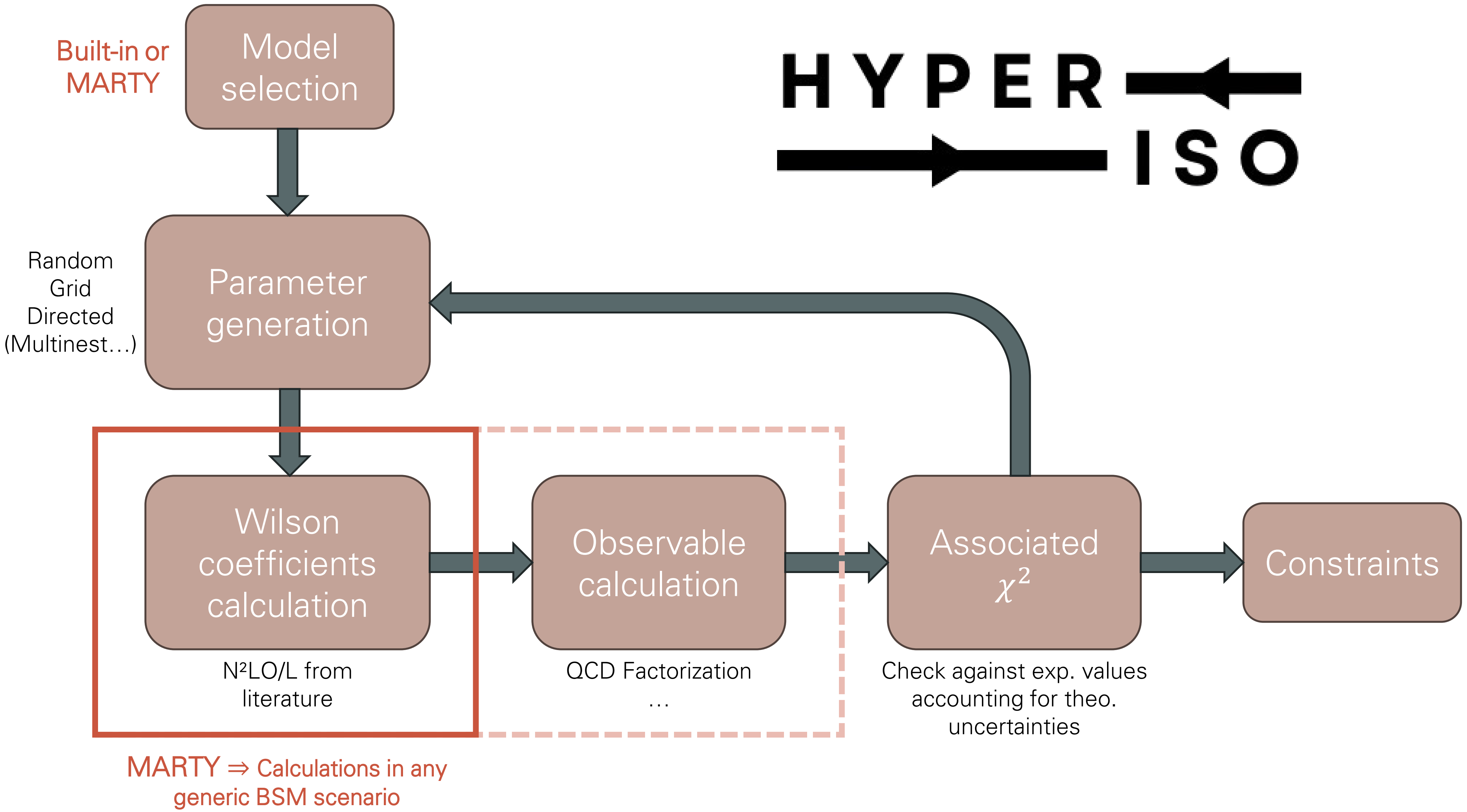


SuperIso

F. Mahmoudi, Comput. Phys. Commun. **178**, 745 (2008) [0710.2067]



HyperIso – MARTY



Thanks